Winners and Losers in Spatial Duopoly Markets; the Relevance of a Value Transfer Approach

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Abstract
The effect of economic behaviour of different actors on the size of a market area is a classical subject of study in regional economics and over the years many studies have been published on this subject. Regional market differences are not only relevant in (location-) allocation theory, but they may also offer intriguing research opportunities for value transfer analysis. In our study a fresh look will be taken at the key forces determining the size of a market area and its implications and possibilities for value transfer. We will address in particular the combination of price competition (modelled by using a modified Bertrand duopoly model) and transport costs. In the paper, Hotelling's duopolistic allocation model will act as the basis for further theorizing. The reasons for the emergence of regional duopoly markets will be investigated by using the willingness-to-pay of households. A modified duopolistic model to analyze also the effects of choice behaviour under conditions of congestion will be placed in the context of earlier research on these types of models, while its implications for value transfer will be traced. Throughout the paper, a strictly formal approach will be adopted in order to identify the general conditions under which value transfer is feasible.

1. Introduction

Economics is - like the natural sciences - in search for structural relationships ("laws") among observed phenomena. Seen from this perspective, economics can be regarded as a nomothetic discipline. It is increasingly realized however, that real-world phenomena may exhibit complex (i.e., non-linear dynamic) behaviour. The study of such phenomena requires the use of sophisticated analytical tools. The variety of issues to be investigated is large, and hence the search for fine-tuned scientific methods may be time consuming and costly. In addition, sometimes studies are partly replicated, as it is often assumed that economic phenomena take place in a unique (non-repetitive) environment. In many cases, insufficient use is made of the stock of existing knowledge on common elements in previous research. Clearly, repetition of research is costly and inefficient. In the light of limited research budgets, several ways have been sought which may economize on the cost of scientific research, while still meeting the prior research objectives. One of the methods to increase research efficiency is to use cumulated knowledge from previous research experiments for a new, similar type of study. However, many pitfalls do exist, when such a research strategy is implemented. In this context, it is, for example, often forgotten that the straightforward application of physical laws (or their analogies in the social sciences) is limited to a well-defined situation. Also other factors restrict the transfer of earlier obtained results. Besides methodological concerns, also technical issues with respect to knowledge transfer may play an important role. For example, the application of a study undertaken in a different, but comparable environment may generate non-satisfactory results and then the question is whether these types of problem can be lifted or mitigated. Furthermore, not only the application of a model in comparable, but nevertheless distinct, environments may be problematic, but also the application of the same model to the same problem at another point in time or space can generate inadequate results. For instance, it is often forgotten that social science phenomena take place in an irreversibly evolving dynamic setting.

Within the neoclassical framework non-market valuation techniques are used to determine the value of a site. In many occasions the value of a site depends on the size of the market area. For example, to determine the value of an improvement of water quality in a particular area, Desvousges *et al.* (1992) multiply the per household benefit of this improvement with the number of households living in this area. However, the size of a market area is not a "static" entity. Many factors (such as (price-) competition, travel cost etc.) may have a significant influence of the size of the market area and thus, finally, on the (change of the) site-value obtained. One of the first applications of value transfer was the transfer of the value of a recreational site toward similar sites. According to Desvousges *et al.* (1992) and Bateman *et al.* (1995), the variable market area plays a important role. First of all, it influences the value obtained for a particular site. Secondly, when this value or measurement techniques is transferred via value transfer, in applications where the size of a market area is not a constant, it may reduce the performance of this form of this technique.

Economics has for a long time made systematic attempts at studying the behaviour of actors under systematically varying conditions, contextual variables are supposed to be constant during the analytical experiment. The question is then how valid certain analytical derivations are in a real-world situation and to which extent certain results can be transferred to other situations. Since the 1960s, several systematic attempts have been undertaken to study various forms of knowledge transfer. It is noteworthy that recently several studies have been published in the economic literature with the aim to test the performance of so-called *value transfer*. Thereto, results from independently undertaken (case-) studies are compared with results generated by a systematic transfer method: one of the independent studies is chosen as a study site and serves as a benchmark for value transfer, while the other site is treated as a policy site. Distinct studies show that usually the values obtained for the policy site via value transfer do not meet the results obtained from the independently undertaken study at that site. Clearly, numerous cases can be imagined for which value transfer may be a relevant exercise. This paper will offer a contribution to the discovery of new insights on the application of value transfer in a spatial duopoly market system. The paper considers value transfer in the context of modern theoretical economic developments. We will discuss in particular two important topics in a spatial duopolistic market that are of critical relevance for value transfer, viz:

☐ the global stability of (parameter-) values of the spatial duopoly model,
☐ the size of the market area in a duopoly situation.

Particular attention will be given to the size of a market area in relation to value transfer. Value transfers in economics have traditionally been approached via non-market valuation techniques (see Bergland *et al.*, 1995 and Bateman *et al.*, 1995). In this context, the general idea is that the theory underlying perfectly operating markets also create useful insights into markets which do not work perfectly. In a recent study by Bal and Nijkamp (1997), it was argued that in such a situation use is made of intra-disciplinary value transfer. In the next sections a theoretical approach will be adopted to investigate the relationship between existing market area theories and value transfer analysis in an intra-disciplinary setting. Hotelling's duopolistic allocation problem will serve as basis for our theorizing and discussion of the market sizes in a spatial duopoly.

Like any relatively new field of research, several gaps in knowledge have to be filled in deriving transferable inferences and existing knowledge has to be reconsidered, given the evolution of the analytical framework in which value transfer is placed. It is illustrative, for example, that over time the use of two distinct names - benefit transfer and value transfer - in the scientific literature has caused some methodological confusion. Since value transfer - as a name - covers more than benefit transfer, we prefer to use the latter nomenclature. On the theoretical side however, a firm theoretical foundation for value transfer is still lacking to our knowledge, although the well-known *ceteris paribus* argumentation in economics has clarified several issues. For the evaluation of applied results via an empirical application of value transfer a solid theory may be necessary. In the present paper, two subjects important for value transfer will be extensively studied: in the first place the global stability of (parameter-) values, and in the second place the size of a market area.

The paper has the following structure. After the previous introduction (Section 1) the basics of value transfer will be presented in Section 2. Section 3 will then consider the global stability of parameter values, where use will be made of the theory of convergent sequences. The economic analysis of the size of the market area will explicitly be discussed in Section 4, and further extended in a theoretical framework in Section 5. Especially the combination of price competition (via Hotelling's duopolistic allocation model) and transport costs will be intensively studied. In the final section we will summarize the main results of the paper and present ideas which may serve as a basis for further research.

2. Value Transfer: Basics of the Research Method

Cumulating knowledge with the aim to build up a theory underlying the explanation of a phenomenon or event requires an effort and hence is costly. On the basis of a formal analysis, Bal and Nijkamp (1997) showed the pay-off between the level of cumulated knowledge and the set of phenomena which were in compliance with this situation. It was also shown that more properties have to be known to describe a more complex phenomenon. Hence, the size of the body of knowledge will have to increase with the increased complexity of the constellation (the framework and structure of a phenomenon under study) observed. The rising complexity of our world requires that increasingly more knowledge is needed to derive a theory, the final objective of scientific research. Thus, the cumulation of knowledge from a complex system requires a significant effort and leads to high research costs. In this context, efficiency means to meet the research objectives at the least possible cost. This can *inter alia* be achieved by a proper research design for each separate study, but also by using prior knowledge from previously undertaken studies for a new and independent, but similar study. The latter strategy is called value transfer. Over the past three decades this form of research method has intensively been studied in economics. The name value transfer covers all techniques which aim to transfer cumulated knowledge to a new study on a similar phenomenon (see Bal and Nijkamp, 1997).

The origin of transfer of (parameter-) values to modelling other similar phenomena dates back to the 1960s when major research efforts were undertaken in water resources development in the USA; for an overview see Loomis (1992). Later on it was recognized that this approach offer a great intellectual challenge to economics. Instead of the use of single average willingness-to-pay (WTP) values on the demand for certain facilities or public goods, entire demand functions became the subject of transfer. Furthermore, these days value transfer analysis was often related to decision-making processes, as was illustrated recently by Button and Kerr (1996). In practice, most value transfer studies are undertaken with the aim to reduce research cost and to ease practically-oriented decision-making. At present, two major types of value transfer are frequently used in scientific research:

(i) a mean value of an endogenous variable is derived via a comparable analysis from previously undertaken studies and next transferred to other (comparable) situations or phenomena;
(ii) an entire benefit function is derived from accumulated knowledge and next transferred to a similar situation.

According to Bal and Nijkamp (1997) value transfer is essentially broader in scope and may be defined as follows: *it is a scientific analysis of a subject under study, which aims to use cumulated knowledge generated via previously undertaken similar types of research endeavours in order to draw inferences on hitherto unexplored cases. It serves to meet the formulated study objectives of a repeated study against the least possible research cost with the highest degree of confidence.*

Value transfer is thus a technique which attempts to transfer (parameter-) values from one case study toward another. The site that serves as origin of the estimated values is usually called the *study site*, while the site under consideration is known under the name *policy site*. According to Button and Kerr (1996), most value transfer analyses are undertaken with the aim to deal with some form of practically-oriented decision outcome. However, Bal and Nijkamp (1997) point out that also in fundamental scientific research the application of value transfer is not unusual. It has in the past become common practise in the social sciences that even for almost similar problems separate individual studies are initiated in order to derive the insights needed (see also Yin, 1994). All such new individual (case-) studies incorporate a high research cost. The renewed application of an already developed study to a similar case might lead to a significant cost saving due to the use of abundant information contained in earlier studies, while, at the same time, the desired research objectives can largely be met. In this context value transfer aims at the acquisition of the required scientific insights at relatively low costs due to the transfer of relevant information from previously undertaken studies to a new similar type of case study.

Transferring numerical knowledge in scientific investigations was, and still is, common practice. Data sets are used for different studies, parameters derived are used in other studies, and behaviourial elasticities are extensively exchanged (e.g., see Nijkamp and Pepping, 1997). This research strategy seems to be an efficient way of generating new insights and knowledge. It is interesting to point out that since the Cartesian era the methodological framework of scientific research is permanently adapted to the actual state of sciences, instead of the other way around where it ought to determine the progress of other fields of science. Nevertheless, a methodological framework adapted to scientific advances may serve as a stable structure for a relatively long period. For several years already, the Lakatosian methodological framework spans the context within which scientific research takes place (see Kastelijn, 1987, and Blaug, 1986). This Lakatosian scientific research programme consists of a hardcore surrounded by a protective ring of auxiliary hypotheses. It may be argued that this hardcore is irrefutable due to the protective ring induced by auxiliary hypotheses. The hardcore together with the auxiliary hypotheses form the testable set of theories. Where the hardcore is immunized due to the auxiliary hypotheses, a prevailing scientific research paradigm as a whole can, however, be replaced by a new one at some point in the future. Over the past decades, the methodological framework based on respectively the Popperian, Kuhnian and Lakatosian theories have been fairly robust, however.

Since the hardcore for economics has hardly changed over the past forty years, this stable methodological research environment has favoured the application of intra-disciplinary value transfers on a large scale basis. In essence, for decades already the hardcore of the conventional economic scientific research programme consists of methodological individualism, rational behaviour, constant tastes, perfect knowledge and foresight, independent decision-making, and free mobility of goods and actors. As a consequence, the principle of maximization under constraints within a general equilibrium environment is a proper tool of analysis. In this - for scientific research stable - methodological environment the rise in research efficiency due to the value transfer of a method of research within a field of science runs parallel to the methodological frameworks used over time. But it should be underlined that even if value transfers under strict conditions (e.g., *ceteris paribus*) are methodologically possible in scientific research, this does not automatically and necessarily result in promising or valid results. Consequently, relevant limitations have to be taken into consideration; see for details Bal and Nijkamp (1997).

Beside the role of methodology in an evolving research environment, another issue is important to consider when value transfer analysis is discussed: semantics. A general application of a formalized method on real-world issues is not as easy as thought most of the time. Like a language, it suffers from the problem of semantics. This problem of semantics has intensively been studied in relation to language. For example, Frege (1882) and Wittgenstein (1953) demonstrated the importance of a proper use of languages. Frege (1882) was able to show that there exists a one-to-one relationship between text languages and a formal language like logic. He could embed relationships into the logic system developed by him. Nevertheless, the non-unique meaning of words is still the problem that limits the use of languages in scientific analysis. The semantic problem undermines the one-to-one relationship between a formal language and a communication language (like English). This partly explains the problems inherent in modelling the functioning (of parts) of the real-world. Symbols in mathematics and logic are well-defined and have a unique meaning. Applying a deductive argumentation results normally in *sound* conclusions (except for certain well-known paradoxes). In economics, symbols used in (systems of) equations have, however, a meaning which is often not as abstract or formal as in strict formal sciences. If we define the symbol I to model investments we make a connection between a formal language and a situation found in the real-world. But we know that there are several types of investment, gross and net investments, investments in human capital and so on. In several ways this problem can be taken into account by an unambiguous definition and measuring system. But problematic is the incorporation of qualitative aspects such as tastes and opinion into a model. How to model investments in moral for example? Clearly, fuzzy set theory can be of some help in this case (see in this context Munda, 1997). Nevertheless, it is hard, and sometimes impossible, to describe the complete phenomenon observed in symbols in a one-to-one relationship. Thus, in situations like these we suffer a "semantic" problem, or a problem of semantic insufficiency. In the case of value transfer, we must consider differences in meanings carefully. What turned out to be the most proper definition of a phenomenon in the initial study, does not automatically capture the same phenomenon in another study, even if the environment in which this phenomenon takes place shows large similarities. In a spatial context, Leung (1997) also recognizes that networks with cumulated knowledge (such as GIS) are affected by this problem of semantics too!

From another perspective, language generates also insights which may be useful for value transfer. First of all, the use of language is highly important in large investment projects. The form of language used is mainly determined by several aspects. For example, due to the risk on financial and social grounds, the language used in the entire negotiation procedure and at the final stage in contracts is to some degree vague. This vagueness on purpose takes into account the strategic behaviour of participants and other interest groups during the decision-making process which results in the strategic use of information (i.e. knowledge) available, risk minimizing on the social side, etc.. Secondly, financial risks are reduced by setting-up well written contracts, risk sharing and so on. At the same time pressure groups are actively involved in the entire decision-making process. It can be concluded that all these activities influence the mid-term and final scientific reports, and the studies undertaken. Use of these studies for value transfer purposes could be more problematic due to the strategic importance and use of fuzzy concepts.

Finally, another feature of strategic information has been left untouched here thus far. Strategic information is knowledge that is used by one of the actors to influence the behaviour of other actors. As a consequence, strategic information leaves room for Game Theory. Since the development of Game Theory by Von Neumann and Morgenstern (1947) many research efforts has been undertaken to implement this method in economic research; see for example Eichberger (1993), Friedman (1986), Tirole (1988) and Varian (1992). We emphasize here deliberately the importance of Game Theory, as this approach in general and a Bertrand duopoly in particular will serve as framework for our theoretical analysis concerning the size of a market area will be further discussed in Section 5.

3. The Role of Global Stability in Value Transfer

In the past two decades, the use of dynamic economic models has brought forward some magnificent insights into the behaviour of well-known economic models which had been studied via comparative statics before, see also Nijkamp and Reggiani (1997). In these type of studies the time-varying behaviour of a dynamic system, given its initial conditions, is investigated. Global stability is not necessarily present in such dynamic models (see Devaney, 1986). Our analysis of dynamic models in the context of a multi-regional economic system has been undertaken amongst others by De Grauwe *et al.* (1993). Their results show that also in multi-regional economic models stability is not necessarily present. This questions of course the valid use of estimated (parameter-) values for value transfer purposes. Bal and Nijkamp (1997) discuss the important role of the *ceteris paribus* condition in the context of value transfer. Under which conditions can results of such models be used in other situations (semi-controlled experimentation)? One of the implications of the *ceteris paribus* condition is that the average benefit from an activity - or the parameters of the benefit function - at the policy site are those values which are estimated, given the knowledge accumulated from the study site(s). A successful application of benefit function transfer requires therefore the equality of parameters of the study site and the policy site. It is remarkable however, that in a cross-state study concerning fishing, undertaken by Loomis (1992), the null hypothesis of the existence of equality of coefficients between two sites has been rejected, not only for the average benefit per activity but also for the benefit functions. This is a major drawback for the application of value transfer in its present form and may also hamper its use for a case of a spatial duopoly. Therefore, in the present section we will focus on this particular topic addressed by Loomis. Thereto, we will first consider the stability of parameter values in general. Use will be made of Cauchy's theory of convergent sequences.

The implicit assumption which legitimizes value transfers is that parameters derived for a particular decision-making process (or study) at a certain point in time or at one location can be employed in other equivalent or largely similar decision situations or studies. Basically, value transfer relies on the "global" prediction power of the initially developed model (where "global" is meant in the sense of stability). However, one should keep in mind that the stability condition means a convergence toward a real value in the limit, i.e. there is a *tendency* toward the optimum; with every iteration an adjustment closer to the optimum takes place. Thus, the dynamic behaviour tends to satisfy Cauchy's principle of convergence; for a introduction to this issue we refer to Malik and Arora (1992). Only fixed points will be mapped on their initial positions. Hence, the path of adjustment describes a dynamic process; see for example Stokey *et al.* (1989). From this point of view, the position along these trajectories for mutually independent identical systems already determines the performance of value transfer. Therefore, it is interesting to take a closer look at this issue. Let us consider a simple example in which a unique global equilibrium is present together with identical systems which are subject to value transfer. Then the position of equivalent unique global stable systems determines the performance of value transfer. A transfer of parameter values from one system to the other is no problem, given the specific constellation of our simple example. It is assumed that the systems are identical, and that the functional form and parameter values are the same among the systems. However, the initial point from which the trajectories have started may be significantly different. But, when at a later point in time, the state of all systems have values close to - but not exactly equal to - that of the unique global equilibrium, a value transfer becomes more fruitful. In the sense of prediction power of the future state of the systems, the results via value transfer are more reliable than in any other situation. However, our case is more problematic when we have to deal with (initial and future) states of the system consisting of completely different positions with respect to the unique global equilibrium or when the systems face a multi-equilibrium setting. In all these situations, the position of the system (its state in each point in time) solely determines the prediction power of value transfer for future states. When systems are also different in structure (thus when the systems have different initial conditions), the prediction performance will be even more difficult given the increased complexity of the analytical structure. Clearly, value transfer does not look very promising in this situation.

In practice, an exact description of the environment of the system at hand cannot be expected, which makes an assessment of the reliability of value transfers an even more tedious matter. Another route must therefore be chosen to derive insights into the differences and similarities of the initial conditions among systems. For example, indications about the variance in parameters can be obtained by the application of meta-analysis. In meta-analysis, several - especially formalized - methods and techniques do exist and can be applied to derive operational insights into differences between systems, e.g. site valuation methods. In this context we may refer to Van den Bergh *et al.* (1997) and Carson *et al.* (1996). There is certainly a need for a more formal analytical approach.

A formal approach will have be adopted to generate useful insight into the performance of value transfer on economic modelling. Thereto, we define an economic system, S, with the following general specification:

$$S \equiv S(t,x_0,x_t,g,X,\Omega) \qquad \textbf{(1)}$$

where

t is time, which is a proper subset of $\mathbb{I}$ (integers) or $\mathbb{R}$,

x_0 is the vector of initial values of the exogenous variables of the system,

Ω the space $\mathbb{R}^n$

x_t the vector containing the values of the exogenous variables at time t,

X the sequence, given x_0 and the system function g,

g the system function, which generates the sequence X.

Let us first compare two identical systems, S_i and S_j. For the sake of simplicity, we assume that these systems are time-invariant and can be described by difference equations. Now, when their initial values are equal, it can be shown that the endogenous values are equal over time too!

Proof

The endogenous values induced by both systems, X^i respectively X^j, can only be obtained as long as

$$S_i \equiv S(t,x_0,g_i^t(x_0),X^i,\Omega) = S(t,x_0,g_j^t(x_0),X^j,\Omega) \equiv S_j \quad \forall t \tag{2}$$

To obtain **(2)** a look must be taken on the initial conditions of the defined economic system. First of all, we have assumed that $x_0^i=x_0^j$. Thereto, the systems are assumed to be identical. As a consequence, their system function must be equal, i.e. $g_i^t(\cdot)=g_j^t(\cdot)$. If at t=0 their initial exogenous values are equal and if $g_k^t(\cdot)$, with k= {1,2}, denotes the t^{th} iterate of the system function, then we must compare the sequence generated by:

$$x_t^i = g_i^t(x_0), \quad x_t^j = g_j^t(x_0) \tag{3}$$

It follows that, for all points in time, the state of the system i equals the state of system j if and only if

$$x_0^i = x_0^j \ \wedge \ g_i^t(\cdot) = g_j^t(\cdot) \quad i \neq j, \quad \forall\, t \tag{4}$$

Q.E.D.

However, the previous proof does not automatically imply a convergence of both sequences toward a real-value, i.e. a unique equilibrium. Let us denote this unique equilibrium within Ω by γ, such that $\gamma \in \mathbb{R}$. When $x_0 \neq \gamma$ $\forall k$ and

$$|g_k^t(x) - \gamma| < \epsilon \qquad \forall\, n \geq m, \quad m,n \in t, \quad \gamma \in \mathbb{R} \ \wedge \ \epsilon > 0 \tag{5}$$

it follows that a sequence converges toward the unique equilibrium. **(5)** is a necessary and sufficient condition for convergence. From **(5)** it also follows that with each iteration the sequence further approaches γ. This can be described in a more proper formal way. We may write

$$\lim_{t \to +\infty} g_k^t(x) = \gamma \tag{6}$$

instead of **(5)**. In case it is unknown whether there exists an equilibrium point at all, Cauchy’s principle of convergence can be applied to determine wether the sequences of state values are convergent. A Cauchy sequence has the following form:

$$|g_k^{t+\tau}(x) - g_k^t(x)| < \epsilon \qquad \forall\, n \geq m, \quad \tau \geq 1, \quad \epsilon > 0 \tag{7}$$

Within a field of real numbers, every sequence is convergent when it is a Cauchy sequence. When, within a metric space, all possible sequences are Cauchy convergent, we deal with a compact space. Clearly, this feature is of significance in comparative analysis. To show now the relevance of systems with converging sequences of states in relationship with value transfer, we assume additionally that Ω is a closed compact space of dimension $\mathbb{R}^n$ in which a unique (global) equilibrium exists. When the initial values between system i and j are different, the distance between both sequences will decline with a rise in t.

Proof

Given the initial description of our example, it is clear that both systems face the same equilibrium (limit point). Therefore, we may write:

$$|g_i^t(x) - \gamma| - |g_j^t(x) - \gamma| = |g_i^t(x) - g_j^t(x)| \qquad \textbf{(8)}$$
$$\because\ |\eta| + |\beta| \geq |\eta + \beta| \ \wedge\ sign[g_k^t(x)] = sign[\gamma] \quad \forall\ t$$

In the light of **(6)**, we may now write

$$\lim_{t\to+\infty} [g_i^t(x) - g_j^t(x)] = 0 \qquad \textbf{(9)}$$

which generates the above proposition. **Q.E.D.**

Under certain conditions, starting from different initial states, the states of identical systems become less different when t is significantly large. They do not only converge toward one another, but also converge to the equilibrium at the same time. Systems that satisfy these conditions can be used in value transfer analysis. Systems with a state close to their equilibrium positions can be seen as systems having state values close to their limit point. So, we are able to make use of the algebra of limit points. Therefore, we may apply the following algebraic rules:

$$\lim(x_t^i + x_t^j) = \lim\ x_t^i + \lim\ x_t^j = \gamma^i + \gamma^j \qquad \textbf{(10)}$$

$$\lim(x_t^i x_t^j) = \lim\ x_t^i \cdot \lim\ x_t^j = \gamma^i \cdot \gamma^j \qquad \textbf{(11)}$$

$$\lim\left(\frac{x_t^i}{x_t^j}\right) = \frac{\lim\ x_t^i}{\lim\ x_t^j} = \frac{\gamma^i}{\gamma^j} \quad \text{if } \gamma^j \neq 0 \wedge x_t^j \neq 0\ \forall\ t \qquad \textbf{(12)}$$

These rules allow us to induce an algebraic handling of states of distinct systems. The only requirement is that all systems must generate states equal to the equilibrium position (i.e. fixed points) or must be close to these points. As a consequence, the assumption of a unique equilibrium does not necessarily hold, as is shown in Annex A to this paper. Reconsidering the proofs presented in Annex A, it is obvious that different system functions or multiple equilibria may cause different results. As long as they are close to a limit point, algebraic rules can be applied for our analysis.

4. Market Area and Value Transfer

It is noteworthy that Bal and Nijkamp (1998) show that when two "island economies" have a common labour market the speed of adjustment on sudden changes in one of the markets affects the utility obtained by each region. Important is that when two regions are economically competing with each other via a Bertrand duopolistic market form, both will prefer an instantaneous adjustment to the behaviour of the other region. In case two regions compete with each other via pricing, their market area will vary with the differences between the price set and the price levied by the competitor. Hence, when we interpret this in a game-theoretic setting, the size of the market area may depend on the speed of adjustment, i.e. the market form. The present sections will show the implications of the size of the market area on value transfer applications.

Many applications of the use of value transfer address the transfer of benefit functions in order to value the site under study, i.e. the policy site. Several attempts have been undertaken in this field; see for example Bergland *et al.* (1995), Bateman *et al.* (1995) and Parsons and Kealy (1994). As already indicated above, two classes of techniques exist that are able to accumulate knowledge from a collection of independent studies, i.e. the study site(s), viz. (i) a mean value of an endogenous variable, and (ii) an entire benefit function.

We will focus our attention now in particular on spatial-economic applications. In several studies the value of a total area depends on the estimated mean WTP value and the total population of a particular area. Therefore, in many situations where we want to apply value transfer we need to know the size of the market area. Many case studies focus on per-household or per capita benefits or values. Hence, to derive the value of a site, the per-household (per capita) benefits must be multiplied by the number of households (people) living in that area. Desvousges *et al.* (1992) discuss in this context the aggregate benefits due an improvement of water quality. As a consequence, the border which demarcates the relevant area must be determined. Following Desvousges *et al.* (1992) the demarcation line lies where the WTP for consumers is equal to zero. Loomis (1992) in this context suggests a zonal travel cost method to derive (regional) demand functions which incorporate variations in site characteristics. This brings us to Von Thünen's (1842) analysis of agricultural zoning; for a formal presentation in this context we may refer, for example, to Paelinck and Nijkamp (1975).

For empirical studies it is important to be aware of the fact that, in order to derive the size of the market area for the policy site, we need to transfer a function from the study site which includes the price due to induced travel cost as an independent (i.e. explanatory) variable of the WTP-function. Geographic Information Systems (GIS) can be of great help here. For example, Bateman *et al.* (1995) made use of a GIS system to take travel cost into account to derive the demand for woodland recreation in the United Kingdom. Such an information system has to be fed with empirical information. Part of the information stems from face-to-face questioning. With face-to-face interviews, for example per study site, the size of the relevant market area can be determined. When next a value transfer is undertaken, a problem may emerge however. Problematic is the determination of the size of the market area for the policy site, since basically no study of the new site has yet been made. The only solution is to initiate at least a small study which includes the estimation of the position of the demarcation line. For empirical studies Sturtevant *et al.* (1995) and

Pearce and Turner (1990) show how an extrapolation up to an WTP equal to 0 works. Techniques to figure out the position of the border line seem to be of great importance. For our theoretical analysis, we will assume that consumers are solely heterogeneous in their travel cost. Then we are able to determine the border of the market area, given a relevant congestion function in relation to a travel cost function. The next section will take a closer look at the theoretical core framework underlying the economic determination of the size of the market area.

5. Competition for Space and Value Transfer: Hotelling's Duopoly

Most studies on value transfer in the actual literature are mainly focusing on cases dealing with non-market pricing. Thereby, such studies center around different types of pricing: travel cost, hedonic pricing, contingent valuation and revealed preference valuation. Of course, value transfer can also be applied in market forms in which a price-setting takes place. It is noteworthy that a glance at spatial market price determination leads to some interesting insights. The market form and the value of the site (fully mirrored in our setting by the revenue function) seem to be interdependent. In particular, from basic microeconomic theory it is known that interactions among competitive sites may generate a price different from monopolistic price setting. The type of market form will, as a consequence, be reflected in the economic valuation based on market prices. Therefore, to place our analysis more in an economic context, a closer look at the fundamenals of allocation theory will be taken; we refer to Paelinck and Nijkamp (1975) and Tirole (1988) for more details.

The role of the market form with respect to pricing is well-known. A change in market form, for example induced by entry and exits or competition, will bring about a dynamic component into the analysis. No question that this is closely related to our earlier discussion of global stability in relationship with the performance of value transfer. Here, we will apply a simple analysis to generate a few ideas that are useful in a value transfer context. An incumbent firm in a site located at point 0 along the unit interval, is confronted with a new entry of a firm at an identical site at point 1. The rationality of this market entry as well as the chosen locations will not be considered in our analysis. As a consequence of this *ceteris paribus* condition, it depends on the conditions whether the site at point 0 can maintain monopolistic price-setting or not. It can be proven that when two sites face a situation in which price competition is possible the value of the site - in the sense of total revenue - declines compared to a monopolistic environment. To illustrate this point, use can be made of a modified version of a model known as Hotelling's duopolistic allocation problem. This proof is given in Annex B to this paper.

The conclusion from Annex B is that *a change in a variable (in our case the travel cost per unit, t) may lead to a significant change in the valuation of the market area under study*.

We have assumed that the unit cost to supply the good to the customers is equal among the sites, in order to show more clearly the way competition takes place. If $c_i > c_j$, instead, site j would supply the entire market, while then site i's dominant strategy is to stay out of business; see Tirole (1988). Since the total revenue of each site depends on solely the travel cost, product differentiation takes place via travel cost. Thus, when t=0, there is a situation of perfect competition. A rise in travel cost will lead to a higher total revenue for each site. The consumer surplus (WTP), however, will decline when the travel cost rises, as will be shown in the next proofs. It is worth mentioning that, since the neoclassical methodological framework is based on homogeneous consumers, the previous analysis fits perfectly within this analytical research framework.

In the context of our simple Hotelling example, it can also be shown that there exists a border where the WTP equals the cost of the product. The consumer located at this position is indifferent to travel to the site or staying at home. The two sites are not necessarily located on the extremes of the unit interval. Returning to the initial Hotelling problem analyzed by Hotelling (1929), under well-known duopoly conditions, the two firms will choose a joint location at the centre of the market. It can be shown that a consumer at the left hand side of α has a travel cost equal to a consumer at the right hand side of α, under the condition that $0 \leq (x-\alpha) < \alpha < (\alpha+x) \leq 1$ where α represents the geographical position of the site. Thereby, we assume that t is such that $[\alpha-x, \alpha+x] \subset [0,1]$. In other words, the travel cost is such that, given the product price p, the total cost matches the consumer benefit somewhere on the unit interval which excludes α, in symbols: $(0,1)\backslash\alpha$.

Proof

The first consumer, positioned left from α at x, faces a cost equal to

$$p + t(\alpha - (\alpha - x)) = p + tx \tag{13}$$

while a second consumer, located right from site α at y, has to pay

$$p + t((\alpha + y) - \alpha) = p + ty \tag{14}$$

under the condition that

$$0 < (\alpha - x) < \alpha < (\alpha + y) < 1 \tag{15}$$

From this expression we can simply deduce that only when x=y, the cost for both consumers or customers travelling to the site is equal. Thus, there is a symmetry of cost around the business site α. As a consequence, there exist two points -positioned on an equal distance from the site- where the WTP is equal to the cost to be made for a visit. **Q.E.D.**

Thus far we have assumed linear travel cost functions. However, instead of a linear cost a (non-linear) strictly increasing convex cost function can also be used to approach the real-world situation in a more proper way, especially when congestion is taken into account. Such a travel cost function approaches the cost observed in real-life more properly; see also Mayeres *et al.* (1996). Hence, besides the cost per unit of distance, time plays a major role in the determination of the cost induced by travelling. Baaijens *et al.* (1997) show that the choice for a mode of transport mainly depends on the degree of independence in the first place and speed in the second place. The factor cost is of less importance. To incorporate cost induced by travel time, a functional relationship that links speed and the money cost per unit of time is required. Concerning speed, the determination of a congestion function based on speed-flow relationships is common practice; see e.g. Mayeres *et al.* (1996). The average speed is a function of the traffic conditions. Hence, an additional traveller will reduce the average speed when the physical link has a given limited capacity. Kirwan *et al.* (1995) conclude in their study that an exponential aggregate congestion function fits best in modelling the functional relationship between speed and the degree of use of a physical link. Given the congestion function, time can also be incorporated in the cost function which measures the travel cost per consumer. Thereto, the time required for travelling must be multiplied by a monetary value. Hague Consultancy Group (1990) provides empirical evidence concerning monetary valuations per unit of travel time for different transportation modes in the Netherlands. Altogether, the cost TC caused by travelling contains a cost per unit of distance and a cost due to travel time; in symbols:

$$TC(t,\omega,\delta,\psi,\nu,x) = tx + \nu\left[\omega + \delta e^{\psi x}\right] \text{ with } \omega,\delta,\psi,\nu > 0 \;\wedge\; x \in (0,1) \tag{16}$$

where t is again the unit transport cost, given a consumer located at x. Furthermore ω, δ and ψ are parameters of the congestion function, where obviously the speed in case of free flow is equal to α. The parameter ν represents the value of time. Since the initial formulated travel cost function is twice continuously differentiable (C^2) and the congestion function is also twice continuous differentiable, we can add these two functions together yielding a travel cost function of the generalized form:

$$TC = TC(t,\omega,\delta,\psi,\nu,x) \tag{17}$$

with the following properties:

$$\frac{\partial TC(\cdot)}{\partial x} > 0,\; \frac{\partial^2 TC(\cdot)}{\partial x^2} > 0 \tag{18}$$

Thus, a more proper functional form concerning the travel cost TC is a monotonic convex function which is strictly increasing. This assumption can be used to extend the previous analysis concerning the size of a market area.

Although factors like congestion prohibit the use of linear travel cost functions, it is easy to show that a symmetry of the market area around a site remains, given a strictly increasing convex travel cost function.

Proof

For simplicity (but without loss of generality) we assume that consumers face a specific form of a strictly increasing travel cost function; namely a quadratic travel cost; $TC(t,x^2)$. The travel cost functions are such that it satisfies $0 \le (\alpha - x) < \alpha < (\alpha + y) \le 1$. The first consumer faces a cost equal to

$$p + t(\alpha - (\alpha - x))^2 = p + tx^2 \tag{19}$$

The other consumer has to pay

$$p + t((\alpha + y) - \alpha)^2 = p + ty^2 \tag{20}$$

Again, similar to the linear cost case the cost among both consumers can only be equal if they have to travel an equal distance, i.e. x=y. **Q.E.D.**

It is good to notice that consumers at an equal distance from the site consume, in fact, an identical product. We will use this property to derive some further properties of the duopoly model. Without any problem, we may focus our further analysis on the consumer at the left hand side of α, knowing that due to symmetry the outcomes are also valid for the identical consumer on the right hand side of α. The position of the border, obviously, differs between the linear and the strictly increasing convex cost functions. It can now first be proven that the market area is smaller compared to the one generated by a linear travel cost function.

Proof

The strictly increasing monotonic travel cost function considered by us, $TC(t,x^2)$, is C^2 and has the following properties:

$$0 < \frac{\partial TC(t,x^2)}{\partial x} \; ; \; 0 < \frac{\partial^2 TC(t,x^2)}{\partial x^2} \tag{21}$$

Assume that the willingness-to-pay by a consumer is such that

$$0 < t < WTP(\cdot) \quad \wedge \quad 0 < \delta = \frac{\partial WTP(\cdot)}{\partial x} \tag{22}$$

Thus, the travel cost per unit of distance is smaller than the WTP, which induces a demand. The WTP is assumed to be equal among all consumers. Let TC(0)=0. The border of the market area, given the non-linear and linear travel cost function, must satisfy the following equation

$$TC(t,x^2) = WTP(\cdot) = TC(t,x) \tag{23}$$

Let us assume that there exist a consumer who is indifferent between travel and staying at home. Let us denote the indifferent consumer in case of a strictly increasing monotonic travel cost function (satisfying the properties of **(18)**) by x^n and, in the situation of a linear travel cost function, by x^l. Since

$$\frac{\partial TC(t,x)}{\partial x} < \frac{\partial TC(t,x^2)}{\partial x} \quad \forall \; x \tag{24}$$

it follows that

$$0 < x^n < x^l$$
$$\because \left[\frac{\partial TC(t,x^2)}{\partial x} - \frac{\partial TC(t,x)}{\partial x} \right] > 0 \quad \forall \; x \tag{25}$$

Thus, the relevant market area, given a travel cost function based on the properties **(18)**, is strictly smaller than in case of a linear travel cost function. **Q.E.D.**

Clearly, a strictly increasing convex travel cost function maintains the symmetry around the central site. However, the size of the population that will use this site will be lower.

We make the plausible assumption that at the demarcation line of the market area the following condition is met:

$$WTP = p + TC(\cdot) \quad \text{or} \quad WTP - (p + TC(\cdot)) = 0 \tag{26}$$

The importance of this condition lies in the determination of the size of the market area. For many value transfer applications the market size must be derived, since the aggregate benefits consist of the household benefits (often the mean WTP when it comes to value transfer) times the number of households in that area. From this we may deduce the following conclusion. *If in a spatial duopoly of firms homogeneous consumers are assumed, then the assumption of a linear cost function as the basis of determining the market area leads to an over-estimate of the area (and hence the value of benefits) compared to a case of a cost function with a strictly increasing convex functional form.*

In our analysis thus far we have implicitly assumed that all consumers travel at the same moment in time. Given the character of the duopolistic game - a one shot game - supply and demand will take place at a (Nash-) equilibrium level. Our specified travel cost function was addressing such a situation. A slight modification in our assumption, however, makes it possible to consider the moment in time consumption takes place less rigidly. Thereto we will add a parameter in the travel cost function with the aim to introduce the degree of traffic that actually will take place at a certain moment in time. Let $\theta(\tau)$ be C^2 and denote the fraction of the consumers who are willing to visit a site at time τ. Then the total cost due to consumption by a consumer living at x is then equal to:

$$TC(t,\omega,\delta,\psi,\nu,\theta(\tau),x) = tx + \nu\left[\omega + \delta e^{\psi\theta(\tau)x}\right] \quad \text{with} \quad \theta(\tau),\tau \in \mathbb{R} \wedge 0<\theta(\tau)\leq 1 \tag{27}$$

Further on, for simplicity reasons this travel cost function may be written in the following generalized form:

$$TC(t,\theta(\tau),x) \equiv TC(t,\omega,\delta,\psi,\nu,\theta(\tau),x) \tag{28}$$

This function has the following properties:

$$\frac{\partial TC(t,\theta(\tau),x)}{\partial x} > 0, \quad \frac{\partial^2 TC(t,\theta(\tau),x)}{\partial x^2} > 0, \quad \frac{\partial TC(t,\theta(\tau),x)}{\partial\theta(\tau)} > 0, \quad \frac{\partial^2 TC(t,\theta(\tau),x)}{\partial\theta(\tau))^2} = 0 \tag{29}$$

Thus, the modified functional form concerning travel cost is a strictly increasing convex function to which a component is added for measuring the degree of physical link use per moment in time. Clearly, $\theta(\tau)$ can be considered as a stochastic variable. However, we will limit our analysis to two cases:

(i) the effect of peak and off-peak travelling on the size of the market area,

(ii) the effect of an increase of the population size on the size of the market area.

In order to show that the distinction of peak and off-peak travelling has an influence on the size of the market area, we assume that during peak hours (θ_p) more consumers are willing to go to a site than during off-peak hours, θ_o. The difference in the volume of traffic flows affects the amount of travel cost via the length of the timespan required to make the trip. Now we can infer that there exist a difference in travel cost between the two periods in time, resulting in a reduction of the size of the market area.

Proof:

A consumer located at x still has to pay the cost per distance, i.e. tx. This component is independent of the period in time travelled. So, we can limit ourselves to the time necessary to make a trip among these two time periods. A simple differentiation shows that

$$\left[\frac{\partial TC(t,x)}{\partial x} - \frac{\partial TC(t,\theta(\tau),x)}{\partial\theta(\tau)x}\right] = (1-\theta(\tau))\left[\frac{\partial TC(t,x)}{\partial x}\right] \leq \left[\frac{\partial TC(t,x)}{\partial x}\right] \quad \because \quad 0<\theta(\tau)\leq 1 \qquad \textbf{(30)}$$

When we exclude the case that all consumers want to travel at the same moment in time, i.e. $\theta\in(0,1)$, we may conclude that a lower volume of travellers reduces the total travel cost of the indifferent consumer who is most far located from the site. Assuming that during peak hours (θ_p) at the same moment in time more consumers are willing to go to a site than during other hours, θ_o, we find that peak-travelling induces a travel cost higher than would occur during off-peak hours. Given that

$$\tau_o \cap \tau_p = \varnothing \;\wedge\; \tau_o \cup \tau_p = \tau \qquad \textbf{(31)}$$

we find the following result:

$$\left[\frac{\partial TC(t,\theta(\tau_p),x)}{\partial\theta(\tau_p)x} - \frac{\partial TC(t,\theta(\tau_o),x)}{\partial\theta(\tau_o)x}\right] = (\theta(\tau_p)-\theta(\tau_o))\left[\frac{\partial TC(t,x)}{\partial x}\right] \leq \left[\frac{\partial TC(t,x)}{\partial x}\right] \qquad \textbf{(32)}$$
$$\because \quad 0<\theta(\tau_k)\leq 1,\; k=\{p,o\}$$

yielding

$$0 <(\theta(\tau_p)-\theta(\tau_o))\left[\frac{\partial TC(t,x)}{\partial x}\right] \quad \because \quad 0 < \theta(\tau_0) < \theta(\tau_p) \qquad \textbf{(33)}$$

Reminding that the demarcation line is the set of points where a consumer x is indifferent to travelling or not, it is clear that its position lies inward compared to the initial situation where we have considered solely the cost due to distance. From the previous derivations it follows that:

$$WTP - TC(t,\theta(\tau_k),x) < WTP - TC(t,x) \qquad \textbf{(34)}$$

Given the possible values of the parameters of the congestion function, see **(16)**, and the unequal τ-values for peak and off-peak hours, we may derive that the market area under peak travelling (M_p) is fully contained in the market area of the off-peak market area (M_o). As a consequence, we find

$$M_p \subset M_o \subset M_{initial} \qquad \textbf{(35)}$$

which completes the proof. **Q.E.D.**

Thus a strictly positive value of the cost of time leads to the final result **(35)**. As a consequence, when the cost of travel time is equal to 0, only distance determines the size of the market area. Basically, our approach concerning the effect of an increase of the population size on the size of the market area (i.e. (ii)) will be based on the assumption of an increased stock of the mode making travelling possible, given the initial capacity of the physical link. The physical capacity is completely mirrored by the functional form of the congestion function together with its parameter values. Of course, instead of starting from the increase of the population size, another strategic assumption can be used: reducing the available physical capacity while holding the population size unaltered. When more consumers make use of the existing physical link, more time will be required to complete the journey. Then it follows that a case of more consumers leads to higher travel costs.

Proof:

Making use of the improved congestion function we know that consumer x faces a travel cost equal to

$$TC(t,\theta(\tau),x) = tx + \nu\left[\omega + \delta e^{\psi\theta(\tau)x}\right] \quad \text{with} \quad \theta(\tau),\tau \in \mathbb{R} \wedge 0<\theta(\tau)\leq 1 \tag{36}$$

When all consumers travel at the same time, $\theta(\tau)$ has a value equal to 1. Let ζ now represent a factor meant to model the increase in link usage due to the rise in population. It is obvious that if and only if

$$TC(t,\theta(\tau),x,\zeta) = tx + \nu\left[\omega + \delta e^{\frac{1}{\zeta}\psi\theta(\tau)(\zeta x)}\right] \quad \text{with} \quad \theta(\tau),\zeta,\tau \in \mathbb{R} \wedge 0<\theta(\tau)\leq 1,\ \zeta > 1 \tag{37}$$

the speed along the link remains unchanged. Otherwise, the speed will drop, if ζ has a value higher than 1, which confirms our statement above. **Q.E.D.**

Given the specification of the congestion function it is clear that the indifferent consumer is the last unit that, when he would decide to travel to a site, yields a travel cost too high for any consumer not located between the demarcation line and the site. Obviously, for all consumers within the market area, it is beneficial to travel. Thus, the demarcation line is characterized by the following properties:

$$WTP - p - \left[tx + \nu\left(\omega + \delta e^{\psi x}\right)\right] = 0 \tag{38}$$

Therefore, we may conclude that by *including travel time into the spatial duopoly analysis in a case where congestion takes place, the market area is smaller compared to the market area solely based on distance costs*.

In our examples we have considered a non-cooperative one-shot price-setting game between two identical players, i.e. the sites, which can commercially be managed. However, (within same limits) various agreements exist which have an influence on the price generated by the market. For example, a binding contract may have an influence on the observed revenue of a site. The imposition of a binding contract will induce a change in the revenue obtained further on, which must be taken into account when it comes to value transfer. We refer for an interesting overview on the consequences of binding contracts to Binmore (1992).

In the light of the examples studied, the intriguing analytical question arises: what is the real value of a site? Or in different words, is there a market form which generates the "real-price" resulting in the real-value of a site? The logical answer is that the value of a site depends on the market form in which it acts. This is no problem for value transfer applications. However, a problem occurs when values of studies in the study site are based on different market structures. It is possibly hard (or impossible) to correct for these differences. A second problem is when values are transferred from a study site to a policy site with different market structures. Here, also a correction is required to generate proper results. It is noteworthy that markets have normally dynamic aspects. These aspects must be introduced into the value transfer method to make its results valid. Finally, it should be remarked that it is difficult to introduce socio-economic factors within a simple theoretical analysis. Nevertheless, in an abstract form socio-economic factors were taken into account via the quadratic travel cost function in our simple theoretical cases. By the assumption of equal purchasing power among consumers an ordering was generated in which the richest were positioned most far from the site.

The conclusion following from the previous analysis is that *the values obtained - via market pricing - from the study site can only be transferred properly to a policy site with an equal market structure. Otherwise, the values derived do not reflect the market characteristics of the policy site. Most theoretical studies are based on the assumption of homogeneous consumers and, as a result, over-estimate the size of the market area.*

6. Conclusion

From a theoretical stand point of view, we have investigated the stability conditions and market form dynamics to see how this may influence value transfer performance. It has been proven that interactive behaviour of actors must be taken into account when value transfer is to be applied. The relevant research techniques may rely on the size of a market area. For a proper application of value transfer analyses, the size of the market area is important. We have proven that the size of a market area in a Bertrand duopolistic market depends on the price set by the competitors. We have also shown that when instead of a linear travel cost function a (non-linear) strictly increasing convex cost function is used to model the consumers’ travel cost, the market area is smaller in size. Thus, the use of a linear travel cost function leads to an overestimation of the size of a market area. As a consequence, an overestimation of the total WTP value of a site is likely to take place in this context, while in many practically-oriented studies the value of an area depends on the estimated mean WTP value times the total amount of customers in a particular area. From our analysis it follows that when these values are transferred, a valid value for a new area (i.e. the policy site) can only obtained in case the size of this market area is determined correctly.

Annex A: Formal Proofs on Multiple Equilibria

We will consider in this annex two cases. First, we will analyze multiple equilibria and then we will focus on two different time-invariant systems.

Multiple equilibria may appear in a system for several reasons; see Devaney (1986) and Kehoe (1988). In such cases, a sensitivity on initial conditions is present, which is opposite to the situation with an unique stable global equilibrium. For simplicity reasons, we will assume that the system functions remain time-invariant and the space is compact and closed. As a consequence, the initial states determine the equilibrium value to which the state of a system will converge in the limit.

Proof

It is sufficient to show that a sequence can only converge to one limit point. Following Malik and Arora (1992), we assume that there are two limit points, γ and κ. We say that

$$0 < \frac{1}{3}|\gamma - \kappa| = \epsilon \quad \wedge \quad \gamma \neq \kappa, \quad \gamma,\kappa \in \mathbb{R} \tag{39}$$

Using a proof by contradiction we assume that a sequence we converge to both limit points. Hence:

$$|g_k^t - \gamma| < \epsilon, \quad \forall\, n \geq m_1 \quad \wedge \quad |g_k^t - \kappa| < \epsilon, \quad \forall\, n \geq m_2 \tag{40}$$

with m_1,m_2,n∈ℕ. Then it follows that

$$\begin{aligned} &|\gamma - \kappa| = \\ &|\gamma - g_k^t(x) + g_k^t(x) - \kappa| \leq |\gamma - g_k^t(x)| + |g_k^t(x) - \kappa| < 2\epsilon \end{aligned} \tag{41}$$

so

$$|\gamma - \kappa| < \frac{2}{3}|\gamma - \kappa|, \tag{42}$$

which is mathematically impossible. Thus, within a closed compact space of real numbers a sequence cannot converge toward two distinct real values. So, when the initial state of a system determines to which equilibrium (limit point) the sequence converges, two identical systems can only generate comparable state values when their trajectories converge toward the same equilibrium and t is sufficiently large. This completes the proof by contradiction. **Q.E.D.**

It should be noticed that by the assumption of a closed compact state we ensure that all sequences (one for each initial state x_0) converge. The interval of all x_0 values can therefore be split-up in subsets to which equilibrium their sequence converges.

The binding restriction for the application of the algebra of limit points is that systems must be close to these points. Equal system functions together with unequal initial states, as well as identical initial states together with unequal system function, yield their own sequence of state values. The first part has been proven above. Now, we will analyze a situation with two systems that differ in their system functions but have the same state as starting point. When system functions differ, the sequence generated will differ too! In fact, then we are dealing with two different systems. Reconsidering the transfer of benefit functions, this would mean that the parameter value and/or the functional form is different between the systems compared.

Proof
Assume that two unequal systems functions converge toward the same equilibrium (limit point). As a consequence the following must be valid:

$$|g_i^t(x) - \gamma| - |g_j^t(x) - \gamma| = |\gamma + g_i^t(x) - g_j^t(x) - \gamma| \quad i \neq j \tag{43}$$

which can only be equal to 0 if and only if

$$g_i^t(x) - g_j^t(x) = 0 \quad \forall\, t \tag{44}$$

As a consequence both functions must be identical. This contradicts with the assumption of the existence of two different system functions. **Q.E.D.**

Therefore, it can be concluded that systems can only generate different results if and only if they are sensitive to initial conditions.

A careful reader will have noticed that no optimization problem was defined for the analysis of this annex. Two reasons may serve as an answer. First of all, within the general equilibrium model stability is assumed throughout. From a theoretical point of view no difficulties arise. The second, more important reason is, that in empirical studies no certainty exists about the analytical foundation spanning the constellation. Currently, almost all value transfer applications deal with the empirical observed reality. Hence, the actual state of different systems and their dynamics must therefore come from analytical instruments available to apply an empirical study. Statistical and econometric techniques on the first level and rough set analysis and meta-analysis on the second level, may then generate the required insights necessary for more formal approaches, which ought to be possible given our presentation in this annex.

Annex B: Proof of a Modified Version of Hotelling's Duopolistic Allocation Problem

Proof
Assume a "linear city" consisting of consumers who are distributed with density 1 along the unit interval. We assume also that these consumers have a unit demand with respect to the homogenous good. Thus, the consumers can choose between two identical recreation sites, S_1 respectively S_2. These sites are placed on the extremes of the interval [0,1]. Each site can satisfy the entire demand. A visit to a site induces a travel cost of t per unit of distance. In concreto, this means for a consumer living at x, with $x \in (0,1)$, that a visit to S_1 will lead to a total travel cost of TC(t,x)=tx and a visit to S_2 generates a total travel cost equal to TC(t,(1-x))=t(1-x). For simplicity, we assume the travel cost per unit of distance to be equal among all consumers. If we assume that consumer x is indifferent between the two sites to spend a day-off, the following equation must be met:

$$p_1 + tx = p_2 + t(1-x) \tag{45}$$

where p_1 and p_2 are the prices to be paid for the good supplied by S_1 respectively S_2. This equation determines the demarcation line which indicates the size of the market area for each site. A market area satisfies two conditions: its intersection covers the entire market and the submarkets are mutually exclusive. In mathematical formulation:

$$M = \bigcup_{i=1}^{I} M_i \quad \wedge \quad M_i \cap M_j = \varnothing \quad \forall i \neq j \quad with\ i,j \in I \tag{46}$$

The borders determining the market areas are so-called demarcation lines, see Paelinck and Nijkamp. As a result, the demand for each site is then equal to

$$\begin{aligned} D_1(p_1,p_2,t) &= x = \frac{p_2 - p_1 + t}{2t} \\ D_2(p_1,p_2,t) &= 1-x = \frac{p_1 - p_2 + t}{2t} \end{aligned} \tag{47}$$

When c is the variable to express the unit cost -necessary to supply the good at each site- such that $c_1=c_2=c$, then the revenue functions are

$$\begin{aligned} \Pi_1(p_1,p_2,t,c) &= (p_1 - c)\left(\frac{p_2 - p_1 + t}{2t}\right) \\ \Pi_2(p_1,p_2,t,c) &= (p_2 - c)\left(\frac{p_1 - p_2 + t}{2t}\right) \end{aligned} \tag{48}$$

Thus, the revenue of each of the sites depends on the mutual market prices, the unit cost and the travel cost. The first order derivatives present the required insights about the relationship between the revenue function and these relevant variables. First of all, an increase in the unit cost of site k, k={1,2}, leads to a decline in the total revenue of site i since

$$\frac{\partial \Pi_i(\cdot)}{\partial c_i} = -\left(\frac{p_j - p_i + t}{2t}\right) \quad \forall\ i \neq j \tag{49}$$

When $p_i=p_j$, it follows that

$$\frac{\partial \Pi_i(\cdot)}{\partial c_i} = -\left(\frac{t}{2t}\right) = -\frac{1}{2} \tag{50}$$

The effect of a change in price induced by the other site has an influence on the revenue of both sites. Its effect on the revenue of site i, given the change in the price set by site j, is as follows:

$$\frac{\partial \Pi_i(\cdot)}{\partial p_j} = \frac{p_i - c_i}{2t} \quad \forall\ i \neq j \tag{51}$$

Under the conditions that $p_i \geq c_i$ and $t>0$ it is clear that

$$\frac{\partial \Pi_i(\cdot)}{\partial p_j} \geq 0 \qquad \textbf{(52)}$$

Thus, there exist a positive relationship between the revenue of site i and the price set by site j. Total revenue changes with a change in travel cost in the following way:

$$\frac{\partial \Pi_i(\cdot)}{\partial t} = \frac{1}{2}\left(\frac{(p_i - c_i)}{2t} + \frac{(p_i - c_i)(p_j - p_i + t)}{t^2}\right) \geq 0 \qquad \textbf{(53)}$$
$$\text{if } \; p_i \geq c_i > 0 \;\wedge\; t > 0$$

Since we are dealing with a Bertrand duopoly, we are interested in the reaction curves for each site. Thereto the first-order conditions must be derived. To derive the optimum the first order condition must be set equal to zero, i.e.

$$\frac{\partial \Pi_i(\cdot)}{\partial p_i} = \frac{p_j - p_i + t}{2t} = 0 \quad \text{for } \; i \neq j \qquad \textbf{(54)}$$

The reaction curve for site i on the pricing behaviour of site j is then as follows:

$$p_i = \frac{p_j + c + t}{2} \quad \text{with } \; i \neq j \qquad \textbf{(55)}$$

So when the price or unit cost of site j increases (decreases) or when the travel cost increases (decreases), the price of site i will rise (decline).

Given the specifications of our example, both sites will set a price which will result in a Bertrand optimum. This means that, in case the price set by a site is undercut by the other site, a consumer who would normally travel to the most near site will choose to travel to the other site, if the low price levels off the extra travel cost. If a linear travel cost function determines the cost of travelling and if the two sites are identical, for the site a price cut may be beneficial, if the extra amount of consumers levels off the loss of revenue due to a lower price compared to the initial price. The other site has an incentive to initiate the same strategy; otherwise, it will have no demand and no revenue. This (virtual) mutual price undercutting in our one-shot price-setting game leads to a price equal to the unit cost, i.e. $p_i=p_j=c$. No price will be set lower. This to avoid a negative revenue. If the market is at the Bertrand optimum, it follows that $p_i=p_j=c+t$, thereby inducing that the revenue functions of both sites are equal to:

$$\Pi_i(\cdot) = \Pi_j(\cdot) = \frac{t}{2} \qquad \textbf{(56)}$$

which implies that the total revenue of each site depends on the travel cost solely. Thus, the market value of each site depends on the market structure in which the price setting takes place.

There may exist a travel cost per unit which results in a non-duopoly market. As a consequence, not only the price setting will be different, but also the total revenue obtained by each site. If this revenue is used as an indicator of the value of the site, this value depends on the present market form. In other words, consumers on (0,x] travel to the site at 0 and consumers on [y,1) to the site at 1. It is clear that in this situation consumers on the interval (x,y) do not travel at all, since the cost exceed the benefits when a trip is undertaken. Hence, consumer x (respectively y) is indifferent between a visit at the site at 0 (respectively 1) and staying at home. In our example concerning the linear city it is obvious that the entire market is split-up into two equal-sized (sub-)market areas. Since the two sites will set equal prices and given the equation which determines the indifferent consumer, it follows that the demarcation line is located at x=½. As long as prices cannot be adjusted to catch-up the high travel cost per unit, no interaction among both sites will be possible. As a result, each site can behave as a price-setter, i.e. a monopolist. On many sites no price is charged. As a consequence no (price-) competition among the sites can take place. Travel cost will now determine solely the consumers behaviour. While the revenues in the duopoly situation equal those found in a perfect competitive market due to the possibility of price competition, they follow here from monopolistic price setting by each site. **Q.E.D.**

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