PRIVATE TOLL ROADS:

A DYNAMIC EQUILIBRIUM ANALYSIS

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April 1997

Acknowledgments: We would like to thank Erik Verhoef for extensive and very insightful comments on two earlier drafts, as well as participants in the workshop "Road traffic congestion and congestion policies" at the Free University of Amsterdam, April 8, 1997. Lindsey would also like to thank the Department of Spatial Economics at the Free University for research support and hospitality while the paper was being completed.

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ABSTRACT

In recent years there has been a surge of interest in private toll roads as an alternative to public free-access road infrastructure. Private toll roads have gained favour for a variety of reasons, including their potential to alleviate traffic congestion, shrinking public funds for road construction and maintenance, and growing acceptance of the user-pay principle.

This paper takes the profitability of private toll roads as given, and focuses on their allocative efficiency. The model features one origin and one destination linked by two parallel routes that can differ in capacity and free-flow travel time. Congestion takes the form of queueing. Individuals decide whether to drive, and if so on which route and at what time. Three private ownership regimes are considered: a private road on one route and free access on the other route, competing private roads, and a mixed duopoly with a private road competing with a public toll road. The efficiency gain (measured by social surplus) in each regime is measured relative to the efficiency gain derived from applying first-best optimal tolls on both routes.

Private toll roads are generally found to enhance efficiency. The efficiency gain is greater when tolls are varied over time to eliminate queueing, when competing routes are also tolled, when no private road has a dominant fraction of total capacity, and when a private road does not suffer a significant travel time disadvantage. Paradoxically, a mixed duopoly can be less efficient than a private duopoly. Price leadership by a public toll road operator avoids this possibility, although leadership typically yields little efficiency gain.

KEYWORDS: private toll roads, bottleneck congestion, departure time, second-best pricing.

1. INTRODUCTION

Interest is growing in private toll roads as an alternative to public untolled road infrastructure. Private toll roads have gained favour for a variety of reasons, including their potential to alleviate traffic congestion, shrinking public funds for road construction and maintenance, and growing acceptance of the user-pay principle.[1] A number of private roads are in operation, or under consideration, in Europe, the U.S. and elsewhere (Gomez-Ibanez and Meyer 1993).

A few analytical studies of private toll roads are found in the literature. Verhoef *et al*. (1996) consider a network with one origin and destination connected by two parallel routes. They examine two private ownership regimes: a regime in which one route is private and the other is free access, and a second regime in which one private firm controls both routes. Verhoef *et al*. compare the efficiency properties of the two regimes *vis à vis* three other regimes: free-access on both routes, public pricing on one route with free access on the other (second-best pricing), and public pricing on both routes (first-best pricing). The public authority is assumed to maximize social surplus. Their model allows for elastic demand as well as route choice, but it is static in that trip timing is not considered. Thus, pricing takes the form of flat tolls; i.e. tolls that do not vary over time.

Braid (1996) endogenizes trip timing by using the Vickrey (1969) bottleneck model of queueing congestion during the morning commute. Braid considers the usage of time-varying tolls that eliminate queueing, but limits attention to public pricing of one route. Similar analyses are found in Bernstein and El Sanhouri (1994)

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[1] Arguments for private roads are found in Geltner and Moavenzadeh (1987), Poole (1988), Roth (1991, 1996) and Nijkamp and Rienstra (1995) *inter alios*.

and Liu and McDonald (1996). A preliminary investigation with the bottleneck model of pricing on a private toll road is found in Chu (1990).

The economic viability of private roads is assessed in a detailed study by Viton (1995). In his model a private toll road competes with a free-access road. Route choice decisions are governed by strong idiosyncratic individual preferences between routes.[2] Viton concludes that the private road is highly profitable under a range of assumptions about the mix of vehicle types, costs of travel time and other parameters.

Competition between private roads is addressed in DeVany and Saving (1980). In their model a fixed number of firms operate identical roads under steady-state conditions with elastic demand. Firms take the full price of travel incurred by customers (toll plus waiting time cost) as given. The competitive equilibrium is shown to be efficient in the sense of maximizing social surplus (see also Vickrey 1968). de Palma (1992) also examines competition between two private roads when travellers make only route choice decisions. Depending on the relative travel times of the two routes, the Nash equilibrium may be more or less efficient than the free-access equilibrium.

This paper follows in the spirit of the above literature by studying the allocative efficiency of private toll roads on a network with two routes in parallel connecting a single origin-destination (OD) pair. As in Braid (1996), Vickrey's bottleneck model is adopted. Previous work on private roads is extended by treating endogenously all three travel decisions: whether to drive, and if so when and by what route. In

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[2] Idiosyncratic preferences may arise because of differences between travellers in the locations of their origins and destinations relative to the two routes.

addition, a broader range of ownership *cum* pricing regimes is considered, including Nash and Stackelberg equilibria for a mixed duopoly.

Except where indicated otherwise it is assumed that both public and private operators implement time-varying tolls to eliminate queueing. In defence it can be noted that AVI and smartcard technology now permit automatic time-of-day toll payment at high levels of reliability and at relatively low costs, both for system operation and for drivers. Time-of-day pricing was studied extensively as part of Hong Kong's electronic road pricing experiment, as well as for London, England and the Randstad area of the Netherlands. Time-of-day pricing has been implemented on California's Highway 91 demonstration toll road, and has been proposed for France's intercity toll highways. And it has been implemented in the form of step tolls on Trondheim's toll ring, on the Maine Turnpike and on Highway 407 near Toronto.

Motivation for using time-varying tolls to alleviate congestion is obvious in the case of public roads. As Mohring (1985) demonstrates, profit-maximizing firms also have an incentive to internalize the congestion costs borne by users of their facilities.[3] With one qualification to be noted, private firms in the model here suppress congestion efficiently, although they typically toll off too many trips.

The paper is organized as follows. Section 2 outlines the model. Section 3 explains how equilibrium departure time, route choice and trip-making decisions are derived, and sets up the framework for measuring efficiency. Section 4 describes the properties of equilibria in which one of the two routes is tolled; Section 5 does the

[3] On this point see also DeVany and Saving (1980), Small (1992, p.141) and Verhoef *et al*. (1996).

same for equilibria with tolling on both routes. Sensitivity analysis is conducted in Section 6, and Section 7 concludes.

2. THE MODEL

The model is similar to that described in Arnott *et al*. (1990a, 1990b). Travel occurs from a single origin to a single destination connected by two routes, A and B. (Each route may comprise a set of parallel lanes or roads provided they have the same free-flow travel cost.) Route R, R $\in$ {A,B}, has a deterministic flow capacity of s_R.[4] If the arrival rate of vehicles exceeds s_R, a queue develops. A driver departing from the origin at time t and choosing Route R experiences a travel time $T_R(t) = \overline{T_R} + Q_R(t)/s_R$, where $\overline{T_R}$ is free-flow travel time and $Q_R(t)$ is the number of vehicles in the queue. Travellers have the same preferred arrival time, t^*, at the destination. They incur a schedule delay cost $D(t_a - t^*)$ if they arrive at time t_a instead, where $D(\cdot)$ is quasiconvex, $D(0) = 0$ and $D(x) \geq 0,\ x \neq 0$.

Let α denote the unit cost of travel time and $\tau_R(t)$ the toll paid if entering Route R at time t. Then the total travel cost for taking Route R at time t is $C_R(t) = \alpha T_R(t) + D(t + T_R(t) - t^*) + \tau_R(t)$. To assure an equilibrium in which the departure rate is finite it is assumed that $\alpha + D'(x) > 0$ for all realized values of x. Demand for travel is described by a conventional inverse demand curve, $p(N)$, where N is the number of travellers. In contrast to Viton (1995), individuals have no idiosyncratic preferences for routes.

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[4] The capacity of the route may be determined by a bottleneck – such as a bridge, tunnel, or lane drop – or may be constant along the whole route if the route is uniform in width and design standard.

3. EQUILIBRIUM CONDITIONS AND EFFICIENCY MEASUREMENT

Prospective travellers have three choices: whether to drive, and if so by what route and when.[5] To derive the equilibrium it is easiest to consider the departure time decision first, then route choice, and finally the number of drivers. To see that this procedure is valid, first note that for any given number, N_R, of individuals taking route R, no driver must be able to reduce travel cost by changing departure time. In equilibrium this yields a route cost function, $C_R(N_R)$. Next observe that for any given total number of drivers, N, no driver must be able to reduce travel cost by changing route. In equilibrium this defines a travel cost function, $C(N)$. Finally, individuals will drive if and only if their reservation price is at least $C(N)$. These conditions determine the equilibrium number of drivers.

3.1 Departure time equilibrium

With free access or flat tolls

The equilibrium departure time pattern in the bottleneck model under either free access or a flat toll was derived by Vickrey (1969). Throughout the departure period $[t_{0R}, t_{eR}]$ on Route R the bottleneck on R operates at capacity. Thus, if N_R individuals take Route R, the duration of the travel period is

$$t_{eR} - t_{0R} = \frac{N_R}{s_R}. \qquad (1)$$

In equilibrium the sum of queueing time cost and schedule delay cost, D_R, must be the same for every traveller. Because the first and last drivers both escape queueing, their schedule delay costs must both equal D_R:

[5] The decision whether to drive subsumes two choices: whether to travel, and if so by what mode.

$$D(t_{0R} + \overline{T}_R - t^*) = D(t_{eR} + \overline{T}_R - t^*) = D_R\,. \tag{2}$$

Together, eqns. (1) and (2) determine t_{0R}, t_{eR} and D_R. Equilibrium private travel cost is thus

$$C_R(N_R) = \alpha\overline{T}_R + D_R + \tau_R\,, \tag{3}$$

where τ_R is the flat toll (zero under free access).

With queue-eliminating tolls

To minimize schedule delay cost without allowing queueing, the departure rate on each route must be maintained at capacity throughout the travel period. This can be supported as an equilibrium by levying a toll that changes at the same rate as schedule delay cost but in the opposite direction, thereby eliminating the incentive to queue. At the end of the travel period the toll is back to its starting level.[6] Henceforth such a toll will be called a "no-queue toll". If τ_R^0 is the initial level of the toll, or "initial toll", it generates a revenue (henceforth "initial toll revenue") of $TR_R^0 = \tau_R^0 N_R$. Revenue generated by time variation of the no-queue toll (henceforth "variable toll revenue") is labelled TR_R^v. The private travel cost of an individual is again given by (3) with τ_R^0 in place of τ_R. Average social cost of travel is less than private cost by the average toll paid, $\tau_R^0 + TR_R^v / N_R$. The component TR_R^v / N_R constitutes an efficiency gain because queueing time (a real resource cost)

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[6] Outside the optimal travel period the toll must be high enough to deter individuals from travelling then. Given quasiconvexity of $D(\cdot)$, a simple solution is to maintain the toll outside the travel period at its level at the start of the travel period.

is replaced by monetary payment (a transfer with no resource cost) as the rationing device for travel times.

The following property of variable toll revenue will be used to establish subsequent results:

Proposition 1: $\frac{\partial TR_R^v}{\partial N_R} = N_R \frac{\partial C_R}{\partial N_R}$.

Prop. 1 is proved in Appendix (a).

3.2 Route choice equilibrium

According to Wardrop's (1952) Principle of equilibrium cost of travel between a given OD pair must be equal on all routes that are used, and equal or higher on unused routes. Attention is limited here to situations in which Routes A and B are both used, so that travel costs on them must be equal. For a given total number of travellers, N, the numbers who choose each route, N_A and N_B, are thus determined by the two conditions

$$N_A + N_B = N, \tag{4}$$

$$C_A(N_A) = C_B(N_B) = C(N), \tag{5}$$

where $C(N)$ is the simultaneous departure time and route choice equilibrium travel cost for given N.

3.3 Equilibrium travel volume

The equilibrium N is determined simply by the condition

$$p(N) = C(N). \tag{6}$$

3.4 Measuring efficiency

The efficiency of a given ownership *cum* tolling regime will be measured using social surplus. Costs of toll road construction, maintenance and operation, which may differ between public and private ownership, are ignored in the model. Social surplus thus includes consumers' surplus and toll revenue.[7] Let I_R be an indicator variable equal to zero if the toll on Route R is zero or flat, and equal to one if the toll is a no-queue toll. Social surplus, SS, can then be written

$$SS = \int_{n=0}^{N} p(n)dn - C(N)N + \tau_A^0 N_A + \tau_B^0 N_B + I_A TR_A^v + I_B TR_B^v. \qquad (7)$$

As benchmarks, social surplus in two polar cases will be calculated: free access equilibrium on both routes (referred to as FREE-FREE), and the first-best optimum in which both routes are public and tolled (PUB-PUB). The relative efficiency of a given toll regime, G, will be evaluated using the index

$$\omega^G \equiv \frac{SS^G - SS^{FREE-FREE}}{SS^{PUB-PUB} - SS^{FREE-FREE}} \leq 1.$$

This index has been used by Arnott *et al*. (1991), Braid (1996) and Verhoef *et al*. (1996) *inter alios*. Under free access $\tau_A^0 = \tau_B^0 = I_A = I_B = 0$. In the first-best

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[7] A more general formulation would allow for unequal weights on consumers' surplus and revenue. (See, for example, Nowlan, 1993.) For example, given the general dearth of public funds for infrastructure, and the deadweight loss in raising revenue with conventional tax instruments, the shadow value of toll revenue (at least on public toll roads) may exceed its face value. Indeed, the toll rings in Norway were introduced with the objective of generating revenue rather than congestion relief. Ramjerdi and Larsen (1991) conclude that the cordon toll in Oslo reduces social surplus, but is still welfare-enhancing if the marginal cost of public funds exceeds 16%. The actual figure, they claim, is about 40%, so that on second-best grounds the Oslo toll ring is preferable to no toll.

optimum, $I_A = I_B = 1$ and $\tau_A^0 = \tau_B^0 = 0$. The initial toll on each route is zero because time-variation of the toll fully internalizes the congestion externality that users impose on each other.[8] This is true even if demand is price elastic, even if the two routes differ in free-flow travel times, and regardless of the shape of the schedule delay cost function.[9]

4. TOLLING OF ONE ROUTE

In this section it is assumed that Route A is a toll road, with free access on Route B. This scenario is likely to be prevalent at least in the near future as isolated toll roads are introduced while the remainder of the road network remains untolled.

4.1 Private tolling of Route A (PRIV-FREE)

A profit-maximizer on Route A, "Firm A", employs a no-queue toll and chooses τ_A^0 to maximize toll revenue subject to (4), (5) and (6). The solution[10] is

$$\tau_A^0 = -\frac{\partial TR_A^v}{\partial N_A} + N_A \frac{\partial C_A}{\partial N_A} - \frac{\dfrac{\partial C_B}{\partial N_B}}{\dfrac{\partial C_B}{\partial N_B} - p'(N)} \left[N_A p'(N) \right] . \tag{8}$$

The toll comprises three terms. The second term is the first-best flat congestion toll on Route A. The third term is a markup due to the exercise of market power by Firm A. As Route B becomes more congestible ($\partial C_B / \partial N_B$ rises) the markup approaches the standard monopoly markup for Route A: $-N_A p'(N)$. These two terms make up

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[8] See Arnott *et al.* (1990b, Theorem 2), Arnott *et al*. (1993) and Braid (1996, Sections 4 and 8).

[9] For a proof see Appendix (d).

[10] See Appendix (c).

the profit-maximizing toll in Verhoef *et al*. (1996, p.295, eqn. (8)). The first term reflects the loss in variable toll revenue caused by an increase in the initial toll. This term is absent in Verhoef *et al*. because they consider flat tolling. The tradeoff between initial and variable toll revenue in eqn. (8) is a crucial determinant of the efficiency of private road tolling, as to be shown in Section 5.4.

Given Prop. 1 the first and second terms in (8) cancel, leaving

$$\tau_A^0 = -\frac{\frac{\partial C_B}{\partial N_B}}{\frac{\partial C_B}{\partial N_B} - p'(N)} N_A p'(N) > 0. \tag{8′}$$

τ_A^0 is larger: (a) the larger is $\|p'(N)\|$, i.e. the less elastic is total demand; (b) the larger is N_A, i.e. the more traffic takes Route A and hence the less elastic is demand for usage of route A; and (c) the larger is $\frac{\partial C_B}{\partial N_B}$, i.e. the more congestible is Route B and hence the less scope it offers as an alternative to travelling on Route A. If demand is perfectly elastic, then $\tau_A^0 = 0$. But Firm A still collects variable toll revenue.

4.2 Public tolling of Route A (PUB-FREE)

Suppose now that the toll on Route A is set by a public operator (“Regulator A”) seeking to maximize social surplus. Because Route B is untolled, Regulator A’s problem is one of second best. The regulator can do no better than to employ a no-queue toll and to control the number of users on Route A by adjusting the initial toll.

Thus, the regulator sets τ_A^0 to maximize SS in (7) given $I_A = 1$ and $\tau_B^0 = I_B = 0$. The solution[11] is

$$\tau_A^0 = -\frac{\partial TR_A^v}{\partial N_A} + N_A \frac{\partial C_A}{\partial N_A} - \frac{-p'(N)}{\frac{\partial C_B}{\partial N_B} - p'(N)} \left[N_B \frac{\partial C_B}{\partial N_B} \right]. \qquad (9)$$

Eqn. (9) matches eqn. (2) in Verhoef *et al*. (1996, p.282) except for the first term, which is again absent with flat tolling. As in eqn. (8) describing Firm A's toll, the second term in (9) is the first-best flat congestion toll on Route A. The third term is a negative fraction of the first-best flat congestion toll on Route B. Once again the first and second terms in (9) cancel, leaving

$$\tau_A^0 = -\frac{-p'(N)}{\frac{\partial C_B}{\partial N_B} - p'(N)} \left[N_B \frac{\partial C_B}{\partial N_B} \right] < 0. \qquad (9')$$

In contrast to the private toll, the second-best initial toll is negative. This induces drivers off Route B, which is used less efficiently than Route A because of queueing. This benefit must be weighed against the fact that a negative toll encourages excessive total travel. As (9′) demonstrates, τ_A^0 is higher the less elastic is demand. The toll is zero if demand is perfectly elastic because usage of Route B then cannot be influenced through the toll set on Route A (Braid 1996, Verhoef *et al*. 1996).

4.3 Numerical comparison

To illustrate and compare PRIV-FREE, PUB-FREE and the two-route tolling regimes of Section 5, a numerical example will be used. Here, only base-case parameter values are considered; sensitivity analysis will be conducted in Section 6.

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[11] See Appendix (d).

In the base case the two routes are identical, with free-flow travel times $\overline{T}_A = \overline{T}_B = 40$ mins., and capacities $s_A = s_B = 4{,}000$/hr. These values are representative for urban arterials or freeways with two lanes in each direction. The unit cost of travel time is $\alpha = \$10/hr$. The schedule delay cost function is piecewise linear: $D(t_a - t^*) =$ $\beta\, Max[0, t^* - t_a] + \gamma\, Max[0, t_a - t^*]$, with $\beta = \$5/hr.$ and $\gamma = \$19.50/hr.$ (The ratio γ/β is consistent with empirical estimates by Small, 1982, Table 1.) The inverse demand curve is assumed to have the isoelastic form $p = p_0 N^{-1/\varepsilon}$, with $\varepsilon = 1.0$ and $p_0 = 1.2866 \times 10^5$; this results in an equilibrium trip cost of about \$12 with free access on both routes.

Columns 1-4 of Table 1 provide summary statistics for the four regimes FREE-FREE, PUB-PUB, PRIV-FREE and PUB-FREE. Changes in consumers' surplus, ΔCS, and changes in social surplus, ΔSS, are measured relative to FREE-FREE. (The lower part of Table 1, "Equilibria with flat tolls", will be discussed in Section 5.4.)

Consider first the two benchmark regimes. In the FREE-FREE regime 5,361 trips are made on each route. Consumers' surplus and social surplus are equal because no toll revenue is generated. In PUB-PUB route usage and consumers' surplus are both the same as in FREE-FREE because time-variation of the toll substitutes perfectly for queueing time cost, and the initial toll is zero, thus leaving travellers' private costs unchanged for given route usage. Revenue of \$28,590 (\$2.67 per vehicle) is raised in variable toll revenue. Social surplus exceeds consumers' surplus by this amount.

Now consider the two regimes in which Route A alone is tolled. In the PRIV-FREE regime Firm A sets an initial toll of \$2.19. Usage of Route A drops from 5,361 to 3,934, while usage of the free route rises to 6,138. Consumers' surplus drops by

	Benchmarks		Tolling of Route A		Tolling of both routes		
	1	2	3	4	5	6	7
	FREE-FREE	PUB-PUB	PRIV-FREE	PUB-FREE	PRIV-PRIV	PUB-PRIV	STACK-ELBERG
τ_A^0	0	0	\$2.19	-\$2.27	\$2.76	\$1.45	\$1.01
τ_B^0	0	0	0	0	\$2.76	\$2.49	\$2.40
N_A	5,361	5,361	3,934	6,866	4,594	5,317	5,563
N_B	5,361	5,361	6,138	4,589	4,594	4,278	4,173
N	10,721	10,721	10,072	11,455	9,188	9,595	9,736
TR_A^0	0	0	\$8,627	-\$15,558	\$12,704	\$7,722	\$5,634
TR_B^0	0	0	0	0	\$12,704	\$10,635	\$9,997
TR_A^v	0	\$14,295	\$7,698	\$23,452	\$10,499	\$14,063	\$15,395
TR_B^v	0	\$14,295	0	0	\$10,499	\$9,103	\$8,663
TR_A	0	\$14,295	\$16,325	\$7,893	\$23,203	\$21,785	\$21,029
TR_B	0	\$14,295	0	0	\$23,203	\$19,739	\$18,660
$TR_A + TR_B$	0	\$28,590	\$16,325	\$7,893	\$46,405	\$41,524	\$39,690
ΔCS	–	0	-\$8,037	\$8,512	-\$19,856	-\$14,281	-\$12,400
ΔSS	–	\$28,590	\$8,288	\$16,405	\$26,550	\$27,242	\$27,290
ω	–	1	0.290	0.574	0.929	0.953	0.954
$\%REDIST$	–	0	49.2	< 0	42.8	34.4	31.2
Equilibria with flat tolls							
τ_A	0	\$4.23	\$4.20	\$1.31	\$6.41	\$5.78	\$5.29
τ_B	0	\$4.23	0	0	\$6.41	\$6.18	\$6.00
$TR_A + TR_B$	0	\$35,980	\$11,140	\$5,897	\$48,886	\$46,471	\$44,417
ΔCS	–	-\$29,800	-\$15,186	-\$4,821	-\$43,828	-\$41,110	-\$38,983
ΔSS	–	\$6,179	-\$4,046	\$1,076	\$5,057	\$5,361	\$5,434
ω	–	1	-0.655	0.174	0.818	0.868	0.879
$\%REDIST$	–	82.8	136.3	81.8	89.7	88.5	87.8

Table 1: Summary statistics for ownership/tolling regimes (base-case parameters)[12]

[12] In each regime the first ownership reference is to Route A and the second to Route B.

\$8,037 but social surplus rises by \$8,288. A relative efficiency of $\omega^{PRIV-FREE}=0.290$ is achieved.

If Route A is public instead of private (PUB-FREE), the regulator sets an initial subsidy of \$2.27. But variable toll revenue exceeds the loss on initial toll revenue, so the public road still makes money. Social surplus rises by a respectable 0.574 of the first-best gain, nearly twice the gain from the private road. Moreover, travellers are better off with the public road than with free access, but worse off with the private road, and may oppose privatization for this reason. The percentage of toll revenue, $\%REDIST$, that has to be redistributed to travellers in PRIV-FREE to compensate them is about 50%.[13]

5. TOLLING OF BOTH ROUTES

In this section both routes are assumed to be toll roads. Section 5.1 analyzes competing private toll roads. Each firm treats the toll set by the other firm as given, so that the equilibrium concept is Nash equilibrium. Section 5.2 assumes Route A is public and Route B is private. In addition to a Nash equilibrium, a Stackelberg equilibrium is considered in which Regulator A sets the toll on Route A first, taking into account the response of Firm B.[14] In each of the three regimes, eqn. (8′)

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[13] It is assumed that compensation is effected in such a way that it does not induce additional travel.

[14] Various other two-route tolling regimes can be envisaged. One is a private roads monopoly. In Europe some private tunnel and bridge projects are either monopolies *de facto* or have been granted a monopoly position by contract (Nijkamp and Rienstra, 1995). In general, however, it seems unlikely that governments would allow control of all the roads connecting two points to be concentrated in the hands of a single private firm, at least if there is no alternative transport mode.

specifying the profit-maximizing initial toll remains applicable.[15] (For the toll on Route B, subscripts A and B are interchanged in (8′).) This is because none of the usage equilibrium conditions depends on who sets the tolls on either route.

5.1 Private roads duopoly (PRIV-PRIV)

The private roads duopoly equilibrium with the base-case parameter values is summarized in column 5 of Table 1. The equilibrium is symmetric, with firms setting a common toll that is higher than when only one route is privatized (PRIV-FREE), and appreciably higher than the zero toll of the first-best optimum (PUB-

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As Verhoef *et al.* (1996) demonstrate, the efficiency of monopoly is sensitive to the price elasticity of demand. (If the elasticity is less than unity the solution is undefined because revenue-maximizing tolls are unbounded.) Introducing time-varying tolls does not alter this feature of monopoly. For this reason, as well as to avoid proliferation in the number of regimes, monopoly will not be studied here.

A second possible regime is a collusive private roads duopoly, with collusion taking place either explicitly or tacitly. The outcome of such a regime would presumably be intermediate between the noncooperative duopoly equilibrium and the monopoly solution. A third regime is a private duopoly with one firm acting as Stackelberg leader and the other as follower. Yet another regime is a mixed duopoly with the private firm acting as leader, and the regulator as follower. This regime seems less plausible than having the regulator as leader. Where private roads have been authorized, the terms under which they can operate have typically been stipulated in advance (e.g. Built-Operate-Transfer legislation, speed limits, accessibility, rate of return regulation, etc.). Governments retain the prerogative of being first mover.

[15] While τ_B^0 does not appear in eqn. (8′) explicitly, it appears indirectly through its effect on N_A, N_B and N.

PUB).[16] Total usage is lower than in PRIV-FREE, but toll revenue is nearly three times as high. Consumers' surplus is lower, but social surplus is higher. Indeed, PRIV-PRIV achieves 0.929 of the first-best efficiency gain. The high efficiency is attributable to two properties of the duopoly equilibrium: queueing is eliminated, and the equal route split is optimal. Travel volume is below the first-best level, but only moderately so because competition between the routes, as well as the desire to generate variable toll revenue, deters firms from setting very high initial tolls.

5.2 Mixed duopoly: Nash equilibrium (PUB-PRIV)

If Route A is public and Route B is private the objective of Regulator A is to choose τ_A^0 to maximize SS in (7) given $I_A = 1$, $I_B = 1$ and τ_B^0. The solution[17] is

$$\tau_A^0 = \frac{-p'(N)}{\frac{\partial C_B}{\partial N_B} - p'(N)} \tau_B^0 > 0 \ . \tag{10}$$

τ_A^0 is positive to encourage usage of Route B. This is because Route B is overpriced by Firm B, in contrast to PUB-FREE where Route B is underpriced. Still, Regulator A stops short of matching τ_B^0 because raising τ_A^0 reduces total travel, which is below its optimal level.

The PUB-PRIV equilibrium is shown in column 6 of Table 1. Regulator A sets τ_A^0 below the equilibrium value for PRIV-PRIV, and this induces Firm B to reduce τ_B^0 slightly. Social surplus is higher than with a private duopoly, but only slightly.

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[16] In a recent paper Hackner and Nyberg (1996) study Bertrand price competition between congestible facilities. They too find that in a symmetric duopoly equilibrium price exceeds the first-best optimum.

[17] See Appendix (d).

Consumers are also better off than with private duopoly, but only moderately. Overall, the difference between PUB-PRIV and PRIV-PRIV is much smaller than the difference between PUB-FREE and PRIV-FREE. In part, this reflects the effects of competition in the PRIV-PRIV regime in curtailing exercise of market power, and in part the fact that with equal tolls on both routes the route split in PRIV-PRIV is optimal.

One intriguing feature of the PUB-PRIV equilibrium is that the public road earns a higher profit than the private road, even though the regulator's goal is to maximize social surplus rather than profit. This happens because, by setting a lower toll, the regulator draws traffic off the private route.

5.3 Mixed duopoly with leadership on the public road (STACKELBERG)

In Section 5.2 the regulator takes the toll set by the private firm as given, making no attempt to influence the firm's behaviour. Yet it may be possible, perhaps via binding legislation, for the regulator to commit itself to a particular toll. In choosing the toll, it can take into account its influence on the toll subsequently set by the private firm. The regulator then acts as a Stackelberg leader and the firm as a Stackelberg follower.

As Stackelberg leader Regulator A chooses τ_A^0 to maximize SS in (7) given $I_A = 1$, $I_B = 1$, and the reaction function of Firm B implicit in (8′) (with subscripts A and B interchanged).

Equilibria of this Stackelberg game for the base-case parameter values is shown in column 7 of Table 1. As expected Regulator A sets a lower toll than in the PUB-PRIV regime, and succeeds in inducing Firm B to reduce τ_B^0 slightly. But the difference in equilibria is small, and the gain in relative efficiency is tiny. Strategic pricing on the public road thus yields little benefit.

5.4 Comparison with flat toll equilibria

It is instructive to compare the equilibria of the various regimes with the corresponding equilibria when flat tolls are levied.[18] The latter equilibria are summarized at the bottom of Table 1. The index w now measures efficiency relative to the first-best optimum with flat tolling. Hence, in column 2, w is again unity.

Some striking differences are apparent: (a) Flat tolls are substantially higher than the initial values of the corresponding no-queue tolls. (Average values of the no-queue tolls are of course higher than the initial values.) Regulators prefer a higher flat toll because, with queueing, the marginal social cost of usage is greater. Firms also prefer a higher flat toll because they no longer face a tradeoff between initial toll revenue and variable toll revenue. (b) When both routes are tolled, total revenue is higher despite the fact that time-variation allows revenue to be extracted without raising drivers' costs. This happens because flat tolls are so much higher. (c) The absolute gains in social surplus from tolling are much smaller. (d) Relative efficiencies are also lower. Indeed, surplus in PRIV-FREE drops by nearly two thirds of the first-best welfare gain. And the efficiency of PUB-FREE is less than one third its value with no-queue tolls.

(e) In all tolling regimes except PUB-FREE travellers are substantially worse off than under free access. And in every regime travellers are worse off than with no-

[18] Flat toll reaction functions for firms and for regulators can be deduced from eqns. (A10) and (A12) respectively in the Appendix.

queue tolls. Again, this is a consequence of higher flat tolls. In fact, $\%REDIST$ exceeds 80% in all regimes. This is consistent with the frequent claim that the wealth transfers associated with tolling can dominate the efficiency gains. Indeed, once the amortization and operating costs of the toll system and the administrative costs of redistribution are accounted for, flat tolling may not yield a net welfare gain.

5.5 Strategic considerations

Before proceeding to the sensitivity analysis it is worthwhile to address the assumption that private firms employ no-queue tolling when the other route is tolled as well. It is conceivable that a private firm could benefit strategically by implementing a tolling scheme that allows some queueing. This is not the case in the model as it stands, in which both time variation and levels of tolls are chosen simultaneously on both routes. But the game could be extended to two stages, with operators choosing the structures of their tolls in the first stage, and the levels of the toll paths in the second stage.

To see where this two-stage game might lead, note from eqn. (8) that the reaction curve of Firm A shifts outward (i.e. away from the origin) if it chooses flat rather than no-queue tolling because the negative term involving TR_A^v drops out. But the reaction curve does not depend on the form of tolling employed on B. The same applies *mutatis mutandis* to Firm B. Thus, consider the PRIV-PRIV regime and suppose Firm A chooses flat rather than no-queue tolling in stage one. Because firm A's reaction curve shifts out, Nash equilibrium tolls on both routes will be higher, whether Firm B chooses flat or no-queue tolling. But Firm A forgoes variable toll revenue. The net impact on Firm A's profits depends on which effect dominates.

If Route A is public, Regulator A's reaction curve shifts in if flat rather than no-queue tolling is chosen in stage one for Route B (see eqn. (A12) in the Appendix). This is because diverting traffic from Route B is more desirable if B is inefficiently used. So if in the PUB-PRIV regime Firm B chooses flat rather than no-queue tolling three changes occur: Firm B's reaction curve shifts out, Regulator A's reaction curve shifts in, and Firm B forgoes variable toll revenue. The first change boosts Firm B's profit, but the other two changes reduce it. Again, the outcome depends on the relative strength of the competing effects.

As shown in Section 5.4, equilibrium profits in PRIV-PRIV can in fact be higher if both firms use flat rather than no-queue tolling.[19] Whether flat tolling is a Nash equilibrium of the two-stage game is another matter. Nevertheless, efficient tolling should not be assumed *a priori* of private firms if both routes are tolled. Note, finally, that the above reasoning is qualitatively unaffected if one or both of the candidate tolling schemes in stage one entail some, but less than fully efficient, time variation provided the terms $\partial TR_R^v / \partial N_R$ are smaller for the less efficient schemes (see eqns. (A10) and (A12) in the Appendix).

6. SENSITIVITY ANALYSIS

This section examines the effects of varying key model parameters. Except where indicated otherwise the Stackelberg equilibria are very similar to the PUB-PRIV equilibria, and hence not discussed.

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[19] Consistent with this, Bulow *et al*. (1985) observe that transport firms may benefit from underinvestment in modern equipment because of reduced price competition.

6.1 Elasticity of demand

As the elasticity of demand rises, revenue on private toll roads decreases in all regimes because the scope for exercising monopoly power diminishes. The effect of demand elasticity on relative efficiency varies from regime to regime. For the two regimes in which both routes are tolled, PRIV-PRIV and PUB-PRIV, relative efficiency is insensitive to price elasticity. With tolling of only one route the dependence is more marked. For PRIV-FREE, relative efficiency rises monotonically and strongly with the elasticity. By contrast, for PUB-FREE relative efficiency falls monotonically.

6.2 Fraction of total capacity on Route A

The fraction of capacity on Route A, s_A / s, was varied while holding total capacity fixed at 8,000 vehicles/hr. As s_A / s rises, toll revenue in the PRIV-FREE and PUB-FREE regimes rises because Route A, the tolled route, gains an increasing fraction of the traffic. But as Figure 1 illustrates, these two regimes behave differently in terms of efficiency. Efficiency rises monotonically with s_A / s in PUB-FREE because a growing fraction of traffic takes the efficiently priced route. But in PRIV-FREE, efficiency rises to a maximum when the private road has just over half of total capacity, and then falls to a negative value as the capacity share approaches unity. This is because the efficiency gain derived from eliminating queueing is eventually outweighed by the tolling off of trips as the private firm takes advantage of its increasing monopoly power.

The behaviour of PRIV-FREE and PUB-FREE illustrated numerically here and in Section 6.1 holds generally in the sense made precise in the following proposition[20]:

Proposition 2: Suppose $\overline{T}_A = \overline{T}_B$. Then $\omega^{PRIV-FREE} \leq s_A/(s_A + s_B) \leq \omega^{PUB-FREE}$.

These inequalities are strict unless demand is perfectly price elastic.

[20] For a proof see Appendix (e).

According to Prop. 2, the relative efficiency of PRIV-FREE is bounded above by its share of total capacity, whereas the relative efficiency of PUB-FREE is bounded below by the same fraction. The common bound is approached asymptotically as demand becomes perfectly elastic. Prop. 2 holds independently of the functional form of the schedule delay cost function.[21]

The proof of Prop. 2 uses the fact that if the toll on Route A is set at its first-best level (an instance of third-best pricing) first-best efficiency gains are attained on A while usage of B is unaffected. This leaves $\omega = s_A / (s_A + s_B)$. If demand is perfectly elastic this is the best that can be done because usage of B cannot be influenced by alterring τ_A^0. Otherwise, further gains can be realized by reducing τ_A^0 to induce traffic off Route B and on to A, which is used more efficiently. This is indeed the policy followed in PUB-FREE. But in PRIV-FREE, Firm A sets a positive toll (unless demand is perfectly elastic, in which case the toll is zero; see eqn. (8′)), which boosts revenue but reduces ω below $s_A / (s_A + s_B)$.

Prop. 2 contrasts with Verhoef *et al*. (1996), who find that with flat tolls and identical routes $\omega^{PUB-FREE} \leq 1/2$, with equality only in the limit of perfectly elastic demand. Their results are derived for parameterized linear and travel costs curves, so a formal comparison with Prop. 2 cannot be made. Nevertheless, the lower relative efficiency of one-route tolling can be attributed, at least in part, to the fact that with a flat toll on Route A, usage of B rises above its free access level,

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[21] Braid (1996, Section 6) establishes the result $\omega^{PUB-FREE} > s_A / (s_A + s_B)$ under the assumptions that demand is perfectly inelastic, $s_A = s_B$, and where the schedule delay cost function takes the form $D(x) = a(-x)^g, t \leq 0;\ D(x) = a(bx)^g, t \geq 0,\ a > 0, b > 0, g \geq 1$.

exacerbating congestion on B. In this respect the tradeoff between congestion on tolled and untolled routes is less favourable with flat than with time-varying tolling. Efficiency of private one-route tolling is compromised in the same way. This demonstrates how the efficiency gains from pricing depend on the tolling scheme adopted (see also Tabuchi, 1993).

Consider now the effect of $s_A/(s_A+s_B)$ on the regimes with tolling of both routes. In the PRIV-PRIV regime a qualitatively similar pattern emerges as in PRIV-FREE (see Fig. 1): efficiency is greatest when each private road controls half of total capacity and drops as one route gains a dominant share. Still, private duopoly retains an appreciable efficiency advantage over private ownership of only one route.

Surprisingly, the mixed ownership regime PUB-PRIV actually performs less well than PRIV-PRIV when the public road has a small capacity share.[22] To see why, note that with private duopoly a higher toll is set on Route B than Route A when s_A / s is small because Route B gains the majority of traffic and faces less elastic demand. This results in a higher marginal social cost of travel on Route A than Route B. So if the government were to take over Route A, it would (treating τ_B^0 as given) raise τ_A^0. (Firm A preferred a lower toll to retain more traffic.) But because Firm B's reaction function is upward-sloping, it sets a higher toll in the new Nash equilibrium. This results in lower social surplus.[23,24]

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[22] By contrast, the Stackelberg regime always outperforms PRIV-PRIV.

[23] Because Regulator A raises τ_A^0, Firm B's profit is higher than in the private duopoly equilibrium. In this instance Firm B prefers to compete with a public road rather than with a private road.

The main conclusion from the sensitivity analysis here is that the efficiency of private toll roads can be undermined when one firm controls too large a fraction of total capacity. This is the case whether the other route is free-access, private, or public.

6.3 Relative free-flow travel times

As a final exercise the difference in free-flow travel times, $\overline{T}_B - \overline{T}_A$, was varied while keeping $\overline{T}_A + \overline{T}_B$ constant at 80 mins. One might think that shortening Route A relative to B has a similar effect to increasing the capacity of Route A relative to B. But there are two differences. First, the efficiency of PRIV-PRIV is insensitive to $\overline{T}_B - \overline{T}_A$, whereas a large drop in efficiency is evident in Figure 1 as capacities become unequal.

Second, whereas the efficiency of PRIV-FREE is an inverse U-shaped function of s_A / s, it is a U-shaped function of $\overline{T}_B - \overline{T}_A$.[25] Starting at zero (when Route A is so much longer that it is not used) efficiency drops initially below zero. This is because the toll on Route A exacerbates overusage of B relative to A. Once the time disadvantage of Route A is small enough, it acquires enough traffic for the efficiency gain from eliminating queueing on A to dominate. And once Route A is shorter than

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[24]A conceptually analogous result can beset cartels. (See, for example, Salant *et al*. 1983.) Consider a quantity-setting oligopoly. If a subset of the firms merge (or collude) and reduce output, their collective profits rise *cet. par*. But nonparticipants react by raising their outputs. Their response can be enough to leave the participants worse off than with no merger. Thus, as is the case here, the anticipated benefit to a player (or players) from taking an action may turn into a loss once other players respond. A general discussion of this point is found in Bulow *et al*. (1985).

[25] Similar behaviour is displayed in Verhoef *et al*. (1996, Fig. 5).

B, the toll on A acts in the right direction on the route split (although it goes too far). Furthermore, in contrast to the effect of rising s_A / s, the congestibility of Route B, $\partial C_B / \partial N_B$, remains constant, which constrains exercise of market power on Route A. Indeed, even when Route A is 40 mins. shorter than B, $\omega^{PRIV-FREE}$ exceeds 0.75.

These results suggest that if a private road competes with an untolled alternative it is desirable for the private road to have a travel time advantage. If toll roads are built from scratch, this may be possible by designing them to a high standard and limiting access to maintain high speeds. However, where lack of space and/or zoning and environmental constraints prevent road construction along direct travel corridors, new private roads may suffer a travel time disadvantage.

7. CONCLUSIONS

This paper has analyzed the allocative efficiency of private toll roads *vis à vis* free access and public toll road pricing on a network with two parallel routes joining a common origin and destination. Prospective travellers decide whether to drive, and if so on which route and at what time. Congestion is assumed to take the form of queueing behind bottlenecks.

Private and public toll roads alike are found to generate substantially higher efficiency gains when tolls are varied over time to prevent queueing than when flat (time-invariant) tolling is adopted. Two competing private roads can yield most of the potential efficiency gains from first-best pricing if neither road has a dominant fraction of total capacity. A single private road competing with a free-access road tends to be most efficient if the two roads have approximately equal capacities and if the private road does not suffer a significant travel time disadvantage.

Consistent with most of the literature on road pricing it is found that tolling of private roads makes drivers worse off. However, with time-varying tolling toll revenues exceed the loss of consumers' surplus by a wide margin, so that ample scope exists for turning a potential Pareto improvement into an actual improvement.

Several assumptions in the paper could be relaxed in future work. First, congestion is assumed to take the form of queueing rather than flow congestion. Second, it is assumed that tolls can be varied continuously over time to eliminate queueing.[26] These two assumptions probably bias upwards the efficiency gains from tolling derived here.

Third, the welfare analysis was limited to two routes. This implicitly assumes that the rest of the road network, as well as any competing modes, operate under first-best conditions. Fourth, the costs of building, maintaining and operating toll roads were ignored. Fifth, travellers were assumed identical except for their reservation price for making an auto trip. Other dimensions of heterogeneity could be modeled, including value of time, costs of schedule delay, and trip-timing preferences. Finally, the ownership structure was taken as given, and private toll road firms were assumed to be profit-maximizers with freedom to set tolls as they please. In practice a variety of mixed private and public sector ownership and toll-setting arrangements have been implemented (Banister *et al.* 1995, Nijkamp and Rienstra 1995). Too, incumbent private toll road firms may practice limit pricing to deter entry by competitors. Indeed, by underpricing congestion public roads *de facto* deter entry by

[26] In lieu of varying tolls continuously, which may confuse drivers, it is possible to implement a series of step tolls. Laih (1994) has shown that with bottleneck queueing congestion a toll with n steps can achieve n/(n+1) of the efficiency gain of the smooth, queue-eliminating, toll.

private roads (Shmanske, 1996).

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APPENDIX

(a) Proof of Proposition 1

Let $D_R \equiv C_R - \alpha \overline{T}_R - \tau_R^0$ denote the portion of private user cost on Route R due to congestion. Under free access, or with a flat toll, an individual arriving at t_a suffers a queueing delay cost of $D_R - D(t_a - t^*)$. With a variable toll this cost is borne as a toll payment. Hence variable toll revenue is

$$TR_R^v = D_R N_R - \int_{t_a = t_{0R} + \overline{T}_R}^{t_{eR} + \overline{T}_R} s_R D(t_a - t^*) dt_a .$$

Differentiating:

$$\frac{\partial TR_R^v}{\partial N_R} = N_R \frac{\partial D_R}{\partial N_R} + D_R - s_R \left(D(t_{eR} + \overline{T}_R - t^*) \frac{\partial t_{eR}}{\partial N_R} - D(t_{0R} + \overline{T}_R - t^*) \frac{\partial t_{0R}}{\partial N_R} \right). \qquad \text{(A1)}$$

Given (1) and (2) in the text and $\frac{\partial D_R}{\partial N_R} = \frac{\partial C_R}{\partial N_R}$, (A1) reduces to $\frac{\partial TR_R^v}{\partial N_R} = N_R \frac{\partial C_R}{\partial N_R}$.

■

(b) Comparative statics of route usage with respect to initial toll on Route A

It is convenient to rewrite equilibrium conditions (4), (5) and (6) in the text as

$$p(N) = p(N_A + N_B) = C, \qquad \text{(A2)}$$

and

$$C_A^0(N_A) + \tau_A^0 = C_B^0(N_B) + \tau_B^0 = C, \qquad \text{(A3)}$$

where C_R^0 is the same whether the toll on Route R is flat or time-varying .

Differentiating (A2) with respect to τ_A^0 yields

$$\frac{\partial N_A}{\partial \tau_A^0} + \frac{\partial N_B}{\partial \tau_A^0} = \frac{1}{p'(N)} \frac{\partial C}{\partial \tau_A^0} = \frac{1}{p'(N)} \frac{\partial C_B}{\partial N_B} \frac{\partial N_B}{\partial \tau_A^0},$$

or

$$\frac{\partial N_A}{\partial \tau_A^0} = -\left(\frac{p'(N) - \partial C_B / \partial N_B}{p'(N)}\right)\frac{\partial N_B}{\partial \tau_A^0}\,. \tag{A4}$$

Differentiating (A3):

$$\frac{\partial \mathrm{C_A^0}}{\partial N_A}\frac{\partial N_A}{\partial \tau_A^0} + 1 = \frac{\partial C_B}{\partial N_B}\frac{\partial N_B}{\partial \tau_A^0} = \frac{\partial \mathrm{C}}{\partial \tau_A^0}\,. \tag{A5}$$

Define $Z \equiv \frac{\partial C_A^0}{\partial N_A}\left(\frac{p'(N) - \partial C_B / \partial N_B}{p'(N)}\right) + \frac{\partial C_B^0}{\partial N_B}$. Equations (A4) and (A5) yield

$$\frac{\partial N_A}{\partial \tau_A^0} = -\frac{1}{Z}\left(\frac{p'(N) - \partial C_B / \partial N_B}{p'(N)}\right). \tag{A6}$$

$$\frac{\partial N_B}{\partial \tau_A^0} = \frac{1}{Z}\,. \tag{A7}$$

And

$$\frac{\partial N}{\partial \tau_A^0} = \frac{\partial N_A}{\partial \tau_A^0} + \frac{\partial N_B}{\partial \tau_A^0} = \frac{1}{Z}\frac{\partial C_B / \partial N_B}{p'(N)}\,, \tag{A8}$$

$$\frac{\partial C}{\partial \tau_A^0} = \frac{\partial C_B}{\partial N_B}\frac{\partial N_B}{\partial \tau_A^0} = \frac{1}{Z}\frac{\partial C_B}{\partial N_B}\,. \tag{A9}$$

(c) Profit-maximizing toll on Route A

A private firm chooses τ_A^0 to maximize toll revenue:

$$\underset{\tau_A^0}{Max}\ TR_A = \tau_A^0 N_A + I_A TR_A^v$$

subject to (A6). The first-order condition is

$$N_A + \tau_A^0 \frac{\partial N_A}{\partial \tau_A^0} + I_A \frac{\partial TR_A^v}{\partial N_A}\frac{\partial N_A}{\partial \tau_A^0} = 0\,.$$

Using Prop. 1 and (A6) this yields

$$\tau_A^0 = -I_A \frac{\partial TR_A^v}{\partial \mathrm{N_A}} + N_A \frac{\partial C_A}{\partial N_A} - \frac{\partial C_B / \partial N_B}{\partial C_B / \partial N_B - p'(N)} N_A p'(N)\,. \tag{A10}$$

In regime PRIV-FREE, $I_A = 1$. Given Prop. 1, (A10) reduces to (8) in the text. With flat tolling, $I_A = 0$, and (A10) yields eqn. (8) in Verhoef *et al*. (1996).

(d) Socially optimal tolling of Route A

Regulator A chooses τ^0_A to maximize social surplus, taking τ^0_B and the structure of the toll on each route (I_A and I_B) as given:

$$\underset{\tau^0_A}{Max}\ SS = \int_{n=0}^{N} p(n)dn - C_A(N_A)N_A - C_B(N_B)N_B + \tau^0_A N_A + \tau^0_B N_B + I_A TR^v_A + I_B TR^v_B .$$

The first-order condition is

$$\frac{\partial SS}{\partial \tau^0_A} = p(N)\frac{\partial N}{\partial \tau^0_A} - (C_A - \tau^0_A + N_A \frac{\partial C_A}{\partial N_A})\frac{\partial N_A}{\partial \tau^0_A} - (C_B - \tau^0_B + N_B \frac{\partial C_B}{\partial N_B})\frac{\partial N_B}{\partial \tau^0_A}$$

$$+ I_A \frac{\partial TR^v_A}{\partial N_A}\frac{\partial N_A}{\partial \tau^0_A} + I_B \frac{\partial TR^v_B}{\partial N_B}\frac{\partial N_B}{\partial \tau^0_A} = 0 .$$

Using (A6), (A7) and (A8) this can be written

$$\frac{\partial SS}{\partial \tau^0_A} = \Big\{\ p(N)\frac{\partial C_B/\partial N_B}{p'(N)} - (C_B - \tau^0_B + N_B \frac{\partial C_B}{\partial N_B} - I_B \frac{\partial TR^v_B}{\partial N_B})$$

$$-(-C_A + \tau^0_A - N_A \frac{\partial C_A}{\partial N_A} + I_A \frac{\partial TR^v_A}{\partial N_A})\frac{p'(N) - \partial C_B/\partial N_B}{p'(N)}\ \Big\} = 0 . \qquad \text{(A11)}$$

Using (A2) and (A3), and after rearrangement:

$$\tau^0_A = -I_A \frac{\partial TR^v_A}{\partial N_A} + N_A \frac{\partial C_A}{\partial N_A} + \frac{-p'(N)}{\partial C_B/\partial N_B - p'(N)}\left[\tau^0_B - N_B \frac{\partial C_B}{\partial N_B} + I_B \frac{\partial TR^v_B}{\partial N_B}\right]. \qquad \text{(A12)}$$

In regime PUB-FREE, $I_A = 1$ and $\tau^0_B = I_B = 0$. Eqn. (A12) reduces to eqn. (9′) in the text. In regime PUB-PRIV and PUB-PUB, $I_A = I_B = 1$. Given Prop. 1, (A12) reduces to (10) in the text:

$$\tau_A^0 = \frac{-p'(N)}{\dfrac{\partial C_B}{\partial N_B} - p'(N)} \tau_B^0 . \tag{A13}$$

Finally, in regime PUB-PUB eqn. (A13) applies to Route B with route subscripts interchanged:

$$\tau_B^0 = \frac{-p'(N)}{\dfrac{\partial C_A}{\partial N_A} - p'(N)} \tau_A^0 . \tag{A14}$$

Equations (A13) and (A14) have the unique solution $\tau_A^0 = \tau_B^0 = 0$.

(e) <u>Proof of Proposition 2</u>

In the first-best optimum, $\tau_A^0 = \tau_B^0 = 0$. Private travel costs and route usage are the same as with free access. So the efficiency gain from tolling equals variable toll revenue on both routes: $\Delta SS^{PUB-PUB} = TR_A^v + TR_B^v$.

Consider tolling of Route A alone. Suppose $\tau_A^0 = 0$ and $I_A = 1$. Again private travel costs and route usage are the same as for free access, and $\Delta SS = TR_A^v$. Given equal free-flow travel times this is $s_A/(s_A + s_B)$ of the gain achieved in the first-best optimum.

Suppose now that τ_A^0 is adjusted. Given (A11) and Prop. 1, the marginal effect on social surplus is

$$\frac{\partial SS}{\partial \tau_A^0} = \frac{1}{Z}\left\{ p(N)\frac{\partial C_B/\partial N_B}{p'(N)} - (C_B + N_B\frac{\partial C_B}{\partial N_B}) + C_A^0\frac{p'(N) - \partial C_B/\partial N_B}{p'(N)} \right\}.$$

Given $p(N) = C_A^0 + \tau_A^0 = C_B$ and the definition of Z:

$$\frac{\partial SS}{\partial \tau_A^0} = -\frac{N_B\dfrac{\partial C_B}{\partial N_B} + \tau_A^0\dfrac{p'(N) - \partial C_B/\partial N_B}{p'(N)}}{\dfrac{\partial C_A}{\partial N_A}\dfrac{p'(N) - \partial C_B/\partial N_B}{p'(N)} + \dfrac{\partial C_B}{\partial N_B}} . \tag{A15}$$

The RHS of (A15) is negative for $\tau_A^0 \geq 0$. A private firm sets a positive initial toll unless $p'(N) = 0$ (see eqn. (8′) in the text). Hence $\omega^{PRIV-FREE} \leq s_A / (s_A + s_B)$, with the inequality strict unless $p'(N) = 0$. The RHS of (A15) is strictly negative at $\tau_A^0 = 0$ unless $p'(N) = 0$. Hence a regulator can increase surplus by setting a negative toll (eqn. (9′) in the text). So $\omega^{PUB-FREE} \geq s_A / (s_A + s_B)$, with the inequality strict unless $p'(N) = 0$. ■