

Evolutionary Game Theory and the Modelling of Economic Behavior ¹

Gerard van der Laan²
Xander Tieman³

November 6, 1996

¹This research is part of the Research-program "Competition and Cooperation" of the Faculty of Economics and Econometrics, Free University, Amsterdam.

²G. van der Laan, Department of Econometrics, Free University, De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands.

³A.F. Tieman, Department of Econometrics and Tinbergen Institute, Free University, De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands.

Abstract

Since the 1950's economists applied game theoretical concepts to a wide variety of economic problems. The Nash equilibrium concept has proven to be a powerful instrument in analyzing the outcome of economic processes. Since the late 1980's economists also show a growing interest in the application of evolutionary game theory. This paper discusses the main concepts of evolutionary game theory and their applicability to economic issues. Whereas traditional game theory focusses on the static Nash equilibria as the possible outcomes of the game, evolutionary game theory teaches us to model explicitly the behavior of individuals outside equilibrium. This may provide us with a better understanding of the dynamic forces within a society of interacting individuals.

Key words: Noncooperative symmetric bimatrix game, Evolutionary Stable Strategy, Replicator Dynamics, Metastrategy, Stable Population.

1 Introduction

Starting with the famous Nash equilibrium for noncooperative games, in the 1950's game theory became a popular field of research. The general feeling was that it would be possible to solve a lot of previously unsolvable problems using game theory. Although game theory indeed was applied to a wide variety of problems, attention for game theory diminished throughout the 1960's and the early 1970's. In the late 1970's and the early 1980's game theory boomed again, especially after Ariel Rubinstein's 1982 article in which he proved that a particular noncooperative bargaining model has a unique subgame perfect equilibrium. At the end of the 1980's however, there was a general feeling of discomfort about the use of the notion of Nash equilibrium to predict outcomes. The mode of research at the time was to use a different and very specific notion of Nash equilibrium for every problem and players were assumed to be hyperrational. The number of Nash equilibrium refinements had grown enormously and it was not clear in advance which refinement best suited which situation. What was clear was the fact that in the real world people did not act in the way behavior was postulated in the models. Before a new decline in interest in game theory could take place, game theorists picked up the idea of evolution from biology. In retrospect the 1973 article by the biologists Maynard Smith and Price in which they defined the concept of Evolutionary Stable Strategies, was the most important in transferring evolutionary thinking from biology to game theory. The book 'Evolution and the Theory of Games' by Maynard Smith [29] explicitly introduced evolutionary selection pressure in a game theoretic setting. The notion of evolution of strategies in a repeatedly played game closely resembles certain models in which players learn from past behavior, thus facilitating the shift of attention in the field towards evolutionary models.

In recent years evolutionary models have become increasingly popular in game theory and other fields of economics. Special issues of leading journals have been devoted to the subject, e.g. the issue of *Games and Economic Behavior* vol. 3 (1991) introduced by Selten [43], the issue in *Journal of Economic Literature* vol. 57 (1992) introduced by Mailath [26], and the issue in *Journal of Economic Behavior and Organization* vol. 29 (1996). Also at many conferences in the field of game theory or economics special sessions are devoted to the subject, see for instance the session introduced by Van Damme [12] as reported in the *European Economic Review* vol. 38, 1994.

In this survey article we start by reviewing the technical basics of the standard (Nash) equilibrium theory in section 2. This theory will be linked to the notion of Evolutionary Stable Strategies in section 3. In section 4 the set of differential equations specifying a biological evolutionary process, the replicator dynamics, will be discussed. Following that we discuss the economic significance of the ESS and the replicator dynamics in the section 5, 6 and 7. It turns out that ESS is no more than just another Nash equilibrium refinement and that although replicator dynamics

seem to describe certain biological (non-rational) evolutionary processes very well, in economics these dynamics do not do as well, since they do not take into account that economic subjects possess some form of boundedly rational behavior. In section 8 we draw attention to what we believe to become a fruitful alternative branch of modelling, the local interaction models. In these models players only interact with a small subgroup of all other players and the spatial distribution of the players is crucial. Finally we pose some concluding remarks in section 9.

2 Nash equilibrium

Noncooperative game theory has become a standard tool in modelling conflict situations between rational individuals. Such a model describes the set of strategies of each individual or player and the payoff to each player for any the strategy profile, the list of strategies chosen by the players. The concept of Nash equilibrium is the cornerstone in predicting the outcome of a game. In a Nash equilibrium each player's strategy maximizes his utility given the strategies played by the other players. In many situations it appears that the Nash equilibrium is not unique. Therefore many papers in game theory have been devoted to the issue of equilibrium selection. Refining the concept of Nash equilibrium allows to discard certain Nash equilibria as not satisfying certain type of rational behavior. In this section we review some of the standard Nash equilibrium theory and its refinements. This enables us in the next section to show how the basic concepts of evolutionary game theory fit into this framework.

The basic model in noncooperative game theory is known as the n -person game in normal form and is characterized by a $2n$ -tuple $\Gamma = (\Theta_1, \dots, \Theta_n; \hat{u}_1, \dots, \hat{u}_n)$, where for each $j \in \{1, \dots, n\}$, Θ_j denotes a nonempty finite set of m_j pure strategies of player j , indexed by $(j, 1), \dots, (j, m_j)$, and $\hat{u}_j: \Theta \rightarrow \mathbb{R}$ with $\Theta = \prod_{j=1}^n \Theta_j$ denotes the payoff function of player j , assigning a real number to each strategy profile $\theta = (\theta_1, \dots, \theta_n) \in \Theta$ of pure strategies. In the following we denote $I_n = \{1, \dots, n\}$. A mixed strategy for player j , $j \in I_n$ is a probability distribution over the set Θ_j of pure strategies and can be represented by a vector $x_j \in \mathbb{R}^{m_j}$, with its k th coordinate x_{jk} the probability assigned to pure strategy k by player j , $k \in I_{m_j}$. The set of all mixed strategies of player j is the $(m_j - 1)$ -dimensional unit simplex S^{m_j} defined as

$$S^{m_j} = \{x_j \in \mathbb{R}_+^{m_j} \mid \sum_{k=1}^{m_j} x_{jk} = 1\}.$$

The k th unit vector in $\mathbb{R}_+^{m_j}$ is denoted by e_j^k and is the vertex of S^{m_j} in which player j plays pure strategy k with probability one. The set $\hat{S} = \prod_{j=1}^n S^{m_j}$ is the mixed strategy space of the game. We now define for each player $j \in I_n$ the function $u^j: \hat{S} \rightarrow \mathbb{R}$ as the function assigning the expected value of the payoff for player j at

any mixed strategy profile $x = (x_1, \dots, x_n) \in \widehat{S}$, i.e.,

$$u^j(x) = \sum_{\theta \in \Theta} x(\theta) \widehat{u}(\theta),$$

where $x(\theta) = \prod_{j=1}^n x_{j\theta_j}$ denotes the probability with which the pure strategy profile $\theta = (\theta_1, \dots, \theta_n) \in \Theta$ is played under the mixed strategy profile $x \in \widehat{S}$. We also denote by $u^j(y_j, x_{-j})$ the expected payoff of player j when he plays his mixed strategy $y_j \in S^{m_j}$ and the other players $k \neq j$ play the mixed strategies $x_k \in S^{m_k}$. The value $u^j(e_j^k, x_{-j})$ is the expected marginal payoff for player j at $x \in \widehat{S}$ when j plays his k th pure strategy. Clearly, for any $x \in \widehat{S}$ and any $j \in I_n$ we have that

$$u^j(x) = u^j(x_j, x_{-j}) = \sum_{k=1}^{m_j} x_{jk} u^j(e_j^k, x_{-j}),$$

saying that $u^j(x)$ is the weighted sum over all expected marginal payoffs.

We denote the noncooperative n -person game in mixed strategies by $G = (I_n, \widehat{S}, u)$. For $n = 2$ this game reduces to a so-called two-player bimatrix game denoted by $G = (A, B)$, where A (respectively B) is the $m_1 \times m_2$ payoff matrix of player 1 (respectively 2), i.e., a_{ij} (b_{ij}) is the payoff to player 1 (2) when player 1 plays his pure strategy i and player 2 plays his pure strategy j . Clearly for $x \in \widehat{S} = S^{m_1} \times S^{m_2}$ we have that $u^1(x) = x_1^\top A x_2$ and $u^2(x) = x_1^\top B x_2$.

Within this framework it is assumed that each player behaves rational and searches to maximize his own payoff. This is expressed in the equilibrium concept due to John Nash [34], together with John Harsanyi and Reinhard Selten awarded with the 1994 Nobel Prize in economics. The Nash equilibrium concept is the most fundamental idea in noncooperative game theory. A strategy profile $x^* \in \widehat{S}$ is a Nash equilibrium if no player can gain by unilaterally deviating from it. This means that in the Nash equilibrium concept it is implicitly taken as given that the players make their choices simultaneously and independently.

Definition 2.1 (Nash equilibrium)

A strategy profile $x^* \in \widehat{S}$ is a **Nash equilibrium** for the game $G = (I_n, \widehat{S}, u)$ if

$$u^j(e_j^k, x_{-j}^*) \leq u^j(x^*) \text{ for any } j \in I_n \text{ and } k \in I_{m_j}.$$

According to the linearity of $u^j(y_j, x_{-j}^*)$ in the variables y_{jk} the definition implies that there is no (mixed) strategy $y_j \in S^{m_j}$ that gives player j a higher payoff than x_j^* , given that all the other players $h \neq j$ stick to their equilibrium strategy x_h^* . Any player plays a best reply to the strategies of the others. The best reply correspondence of player j assigns to any mixed strategy profile $x \in \widehat{S}$ the set of mixed strategies of player j yielding the highest payoff given the strategies of the other players.

Definition 2.2 (Best reply correspondence)

The mapping $\phi: \hat{S} \rightarrow S^{m_j}$ defined by

$$\phi(x) = \{y_j \in S^{m_j} \mid u^j(y_j, x_{-j}) \geq u^j(\hat{y}_j, x_{-j}) \text{ for all } \hat{y}_j \in S^{m_j}\}$$

is the **best reply correspondence** of player j . Any mixed strategy $y_j \in \phi(x)$ is a best reply for player j given the strategy profiles $x \in \hat{S}$.

Observe that $\phi(x)$ does not depend on x_j . Clearly $x \in \hat{S}$ is a Nash equilibrium if and only if $x_j \in \phi(x)$ for all players $j \in I_n$.

The elegance of the Nash equilibrium concept and the underlying rational behavior of the agents has inspired many economists in formulating economic problems as a noncooperative n -person game. Since the 1970's the Nash concept has been applied to a wide range of problems. However, in applying the concept game theorists became aware of a serious drawback of the Nash equilibrium, namely that a noncooperative n -person game may have many Nash equilibria. So, one particular arbitrarily chosen equilibrium does not make much sense as a prediction of the outcome of the problem. However, in many cases not all outcomes are consistent with the intuitive notion about what should be the outcome of the game. Therefore, from the seventies on several game theorists have addressed the problem of equilibrium selection by putting more requirements on the rational behavior of the players. Assuming highly rational players may eliminate the less intuitive outcomes. Several results have been obtained along this line of research. For an excellent survey on Nash equilibrium refinements we refer to Van Damme [11]. Here we only consider the concepts of perfect and proper Nash equilibrium for normal form games. The notion of ‘trembling hand’ perfect Nash equilibrium of a normal form game has been introduced by Reinhard Selten [41] and is one of the most fundamental results in the theory of Nash equilibrium refinements. For some real number $1 > \mu > 0$, a completely mixed strategy profile $x \in \hat{S}$, (i.e. $x_{jk} > 0$ for all j and k), is a μ -perfect Nash equilibrium if $x_{jk} \leq \mu$ when $e_j^k \notin \phi(x)$. So, at a μ -perfect Nash equilibrium each pure strategy is played with a positive probability, but only pure strategies in the best reply set can have a higher probability than μ . So the players are allowed to make errors, but the probability that a non optimal strategy will be played is bounded by μ . A perfect equilibrium is now defined as the limit of a sequence of μ perfect equilibria when μ goes to zero.

Definition 2.3 (Perfect Nash equilibrium)

A strategy profile $x^* \in \hat{S}$ is a **perfect Nash equilibrium** for the game $G = (I_n, \hat{S}, u)$ if for some sequence $\mu^r > 0$, $r \in \mathbb{N}$, converging to zero, there exists a sequence of μ^r -perfect Nash equilibria converging to x^* .

From the definition it follows immediately that any Nash equilibrium with completely mixed strategies is a perfect Nash equilibrium. Furthermore, Selten [41] proved that any game $G = (I_n, \hat{S}, u)$ has a perfect Nash equilibrium, even if the game has no Nash equilibrium in completely mixed strategies.

Theorem 2.4 (Selten Theorem)

Any game $G = (I_n, \hat{S}, u)$ has at least one perfect Nash equilibrium. Moreover, the set of perfect Nash equilibria is a subset of the set of Nash equilibria.

Although the notion of perfect equilibrium wipes out Nash equilibria which are not robust with respect to small probabilities of mistakes by error making players, Myerson [32] argued that the probability with which a rational player plays a strategy by mistake will depend on the detrimental effect of the non optimal strategy. More costly mistakes will be less probable than less costly mistakes. To further refine the set of perfect Nash equilibria, Myerson introduced the notion of proper equilibrium. For some real number $1 > \mu > 0$, a completely mixed strategy profile $x \in \hat{S}$ is a μ -proper Nash equilibrium if for any player $j \in I_n$, $x_{jk} \leq \mu x_{jh}$ if $u^j(e_j^k, x_{-j}) < u^j(e_j^h, x_{-j})$ for all $k, h \in I_{m_j}$. Again, at a μ -proper Nash equilibrium each pure strategy is played with a positive probability. Moreover, if some pure strategy k is a worse response against the strategies of the other players than a certain strategy h , then the probability that strategy k is played is at most μ times the probability with which strategy h is played.

Definition 2.5 (Proper Nash equilibrium)

A strategy profile $x^* \in \hat{S}$ is a **proper Nash equilibrium** for the game $G = (I_n, \hat{S}, u)$ if for some sequence $\mu^r > 0$, $r \in \mathbb{N}$, converging to zero there exists a sequence of μ^r -proper Nash equilibria converging to x^* .

Again it follows immediately that any interior (i.e. completely mixed) Nash equilibrium is a proper Nash equilibrium. Moreover, each proper Nash equilibrium satisfies the conditions of a perfect equilibrium, because the notion of properness further restricts the set of allowable mistakes. Myerson [32] proved that any game $G = (I_n, \hat{S}, u)$ has at least one proper Nash equilibrium.

Theorem 2.6 (Myerson Theorem)

Any game $G = (I_n, \hat{S}, u)$ has at least one proper Nash equilibrium. Moreover, the set of proper Nash equilibria is a subset of the set of perfect Nash equilibria.

The concepts of perfect and proper Nash equilibrium are nicely illustrated by a well-known example of Myerson [32]. Let the two-person bimatrix game be given by

$$A = B = \begin{bmatrix} 1 & 0 & -9 \\ 0 & 0 & -7 \\ -9 & -7 & -7 \end{bmatrix}.$$

This game has three Nash equilibria in pure strategies, namely each strategy profile in which the two players play the same pure strategy. However, the Nash equilibrium in which both players play the third strategy is very unlikely. This equilibrium is ruled out by the notion of perfectness, which only allows for the two equilibria in which

either both players play their first strategy or both players play their second strategy. The latter equilibrium is ruled out by the notion of properness. So, properness selects the equilibrium in which both players play the first strategy as the unique outcome of the game. This equilibrium is efficient and gives both players a payoff equal to one.

3 Evolutionary Stable Strategy

The theory of refinements has been proven very helpful to eliminate inadequate outcomes of the game. However, this theory also has its drawbacks. Not only many different concepts of refinements have been developed, the theory also assumes that players are acting according to a very high level of rationality. This leads Ken Binmore in his foreword to the monograph ‘Evolutionary Game Theory’ of Jörgen W. Weibull to the following conclusion:

However, different game theorists proposed so many different rationality definitions that the available set of refinements of Nash equilibria became embarrassingly large. Eventually, almost any Nash equilibrium could be justified in terms of someone or other’s refinement.

He then continues with:

As a consequence a new period of disillusionment with game theory seemed inevitable by the late 1980’s. Fortunately the 1980’s saw a new development. Maynard Smith’s book *Evolution and the Theory of Games* directed game theorists’ attention away from their increasingly elaborate definitions of rationality. After all, insects can hardly be said to think at all, and so rationality cannot be so crucial if game theory somehow manages to predict their behavior . . . the 1990’s have therefore seen a turning away from attempts to model people as hyper-rational players.

This brings us to the question what economists can learn from evolutionary game theory, introduced by biologists in studying the evolution of populations and the individual behavior of its members. Where has evolutionary game theory brought us and where might it be applicable?

Evolutionary or biological game theory originated from the seminal paper ‘The logic of animal conflict’ by Maynard Smith and Price [30], see also Maynard Smith [28] and [29]. Maynard Smith considers a population in which members are randomly matched in pairs to play a bimatrix game. The players are anonymous, that is any pair of players plays the same symmetric bimatrix game and the players are identical with respect to their set of strategies and their payoff function. So, for any member of the population, let m be the number of pure strategies and $S = S^m$ the set of mixed strategies. Furthermore, the payoff function $u: S \times S \rightarrow \mathbb{R}^2$ assigns to any pair of two players the payoff pair $(u_1(x), u_2(x))^\top$, $x \in S \times S$. The assumed

symmetry of the bimatrix game states that $u_1(x, y) = u_2(y, x)$ for any pair $(x, y) \in S \times S$, which states that the payoff of the first player when he plays $x \in S$ and his opponent plays $y \in S$ is equal to the payoff of the second player when the latter plays $x \in S$ and the first player plays $y \in S$. So, any pair plays the symmetric bimatrix game $G = (A, A^\top)$ with A an $m \times m$ matrix. Observe that it is not assumed that A is symmetric. In the following we denote with $v(x, y) = u_1(x, y) = u_2(y, x) = x^\top A y$ as the payoff of a player playing $x \in S$ against an opponent playing $y \in S$. In this way all members of the population are symmetric, except in their strategy choice. In the biological game theory it is not assumed that the members (or animals) in the population behave rationally. Instead it is assumed that any member is preprogrammed with an inherited, possibly mixed, strategy and that this strategy is fixed for life. Now, let $x \in S^m$ be the vector of average frequencies with which the strategies are played by the members of the population. So, x_j is the average probability or frequency over all members of the population that strategy j , $j \in I_m$, is played. Assuming that the population is very large, the differences between the expected strategy frequencies faced by different members are negligible and the average expected payoff of an arbitrarily chosen member of the population when paired at random with one of the other members is given by $v(x, x) = x^\top A x$. However, now suppose that a perturbation of the population occurs and that at random a (small) fraction ϵ of the population is replaced by individuals which are all going to play a so-called ‘mutant’ strategy q . Then the vector of average frequencies becomes

$$y = (1 - \epsilon)x + \epsilon q$$

and hence the average expected payoff of the incumbent individuals of the population when matched at random with a member of the perturbed population becomes

$$v(x, y) = x^\top A[(1 - \epsilon)x + \epsilon q] = (1 - \epsilon)v(x, x) + \epsilon v(x, q) \quad (1)$$

and the expected payoff of a mutant individual becomes

$$v(q, y) = q^\top A[(1 - \epsilon)x + \epsilon q] = (1 - \epsilon)v(q, x) + \epsilon v(q, q). \quad (2)$$

Now the population is said to be stable against mutants if for all $q \neq x$ there exists an $\epsilon(q) > 0$ such that for all $0 < \epsilon < \epsilon(q)$

$$v(x, y) > v(q, y), \text{ where } y = (1 - \epsilon)x + \epsilon q. \quad (3)$$

The reasoning above allows for several viewpoints. Originally the stability condition was only applied for monomorphic populations, i.e., populations in which all individuals are endowed with the same strategy x . In this framework the frequency x_j is the probability with which any member of the population plays pure strategy j . So, any incumbent individual plays x and hence the expected payoff of an incumbent is equal to the average expected payoff. A strategy x satisfying (3) for any $q \neq x$ with

some $\epsilon(q) > 0$ is called an **Evolutionary Stable Strategy (ESS)** and is the central equilibrium concept in the biological game theory about monomorphic populations as introduced by Maynard Smith and Price [30].

From another (opponent) viewpoint the stability condition can also be applied to a polymorphic population in which each member is preprogrammed with one of the pure strategies. Within this framework, originated from mathematicians like Taylor and Jonker [47], see also Zeeman [56], the frequency x_j is the fraction of members preprogrammed with the pure strategy j . Of course, also intermediate cases are possible in which multiple mixed strategies are present in the population.

Although both viewpoints are allowed, for the moment we restrict ourselves to the case of the monomorphic population. Now, suppose that $x \in S$ satisfies $v(q, x) \leq v(x, x)$ for all $q \in S$. Clearly, this is a sufficient and necessary condition for the strategy pair (x, x) to be a Nash equilibrium for the symmetric bimatrix game (A, A^\top) . In this equilibrium both players play the same strategy x . So, $(x, x) \in S \times S$ is a symmetric Nash equilibrium and we call x a symmetric equilibrium strategy. It is well-known that any symmetric bimatrix game has at least one symmetric equilibrium, see e.g. Weibull [55]. It should be noticed that a symmetric bimatrix game may also have non-symmetric Nash equilibria in which the two players use different strategies. Now, suppose that for a symmetric equilibrium strategy $x \in S$ the stronger condition $v(q, x) < v(x, x)$ holds for any mutant strategy $q \in S$. Then it follows from the equations (1) and (2) that there exists some $\epsilon(q) > 0$ such that equation (3) holds and the strategy $x \in S$ is also ESS. However, if the symmetric equilibrium strategy x is a completely mixed strategy, then $v(q, x) = v(x, x)$ for any $q \in S$. More precisely we have for any mixed strategy $x \in S$ that $v(q, x) = v(x, x)$ for any $q \in S$ such that for any j it holds that $q_j = 0$ if $x_j = 0$. So, when x is a mixed equilibrium strategy, $v(q, x) < v(x, x)$ does not hold for all $q \neq x$. However, in case that $v(q, x) = v(x, x)$, it follows from the equations (1) and (2) that equation (3) still holds if $v(q, q) < v(x, q)$, i.e., if the incumbent strategy x performs better against the mutant strategy q than the mutant against itself. This gives the following formal definition of ESS as originally formulated by Maynard Smith and Price [30]

Definition 3.1 (Evolutionary Stable Strategy)

*A strategy $\hat{x} \in S$ is an **Evolutionary Stable Strategy** of the symmetric bimatrix game $G = (A, A^\top)$ if it satisfies the two inequalities*

$$i) \quad q^\top A \hat{x} \leq \hat{x}^\top A \hat{x} \text{ for all } q \in S,$$

$$ii) \quad q^\top A q < \hat{x}^\top A q \text{ if } q^\top A \hat{x} = \hat{x}^\top A \hat{x} \text{ for all } q \neq \hat{x}.$$

Clearly, $x \in S$ satisfies the conditions of Definition 3.1 if and only if for any $q \in S$, x satisfies equation (3) for some $\epsilon(q) > 0$, see e.g. Weibull [55]. Furthermore, condition i) of Definition 3.1 shows that (x, x) is a Nash equilibrium for the bimatrix game $G = (A, A^\top)$ if x is ESS. However, the reverse is not true. If (x, x) is a Nash

equilibrium, then x is ESS only if x satisfies condition ii). So, not any symmetric equilibrium strategy is ESS. In other words, the ESS condition gives a refinement on the set of symmetric Nash equilibria. More precisely we have that if x is ESS, then (x, x) is a symmetric proper Nash equilibrium, see e.g. Van Damme [11], page 224. According to Weibull [55], page 42, we can say that an ESS satisfies the equilibrium condition and is ‘cautious’, i.e. is robust with respect to low-probability mistakes such that more costly mistakes are less probable than less costly mistakes. Again, the reverse implication is not true. When (x, x) is a symmetric proper Nash equilibrium, then x is not necessarily an ESS. Even stronger, not any symmetric bimatrix game has an ESS (for instance the unique symmetric equilibrium strategy of the Rock-Scissors-Paper game is not ESS, see Weibull [55], page 39), while it has been proven recently by Van der Laan and Yang [25] that any symmetric bimatrix game has a symmetric proper equilibrium. To summarize the above results let Δ^{NE} denote the set of symmetric Nash equilibrium strategies, Δ^{PE} the set of symmetric proper equilibrium strategies and Δ^{ESS} the set of Evolutionary Stable Strategies. Then we have that

$$\Delta^{ESS} \subset \Delta^{PE} \subset \Delta^{NE} \text{ and } \Delta^{PE} \neq \emptyset.$$

For further characterization results on the set of Evolutionary Stable Strategies we refer to e.g. Bomze [4], Van Damme [11] or Weibull [55]. In these references many results can be found with respect the structure of the set of ESS, the relation between this set and other refinements of the Nash equilibrium and with respect to conditions guaranteeing the existence of ESS. For instance, any nondegenerate symmetric bimatrix game with two pure strategies has at least one ESS. In this paper we also skip the discussion about other (weaker) evolutionary stability criteria. We only mention the weaker concepts of neutral stability introduced by Maynard Smith [29] and robustness against equilibrium entrants introduced by Swinkels [45]. Setwise evolutionary stability criteria have been given by e.g. Thomas [48] and Swinkels [45].

4 Replicator dynamics

Evolutionary game theory combines the static concept of Evolutionary Stable Strategy with the dynamic concept of replicator dynamics, a notion formalized by Taylor and Jonker [47], see also e.g. Zeeman [56], Bomze [4], Van Damme [11] and Weibull [55]. In the framework of replicator dynamics or *population dynamics*, we depart from the viewpoint that all individuals are preprogrammed to play a pure strategy. So, a strategy vector $x \in S$ has to be interpreted as the *state of the population* with x_j the proportion of individuals playing strategy j , $j \in I_m$, when paired with an opponent to play the symmetric bimatrix game $G = (A, A^T)$. Within this framework individuals are assumed to be paired at random and each member of the population is assumed to be engaged in exactly one contest at the time. Furthermore, the payoff to

an individual is assumed to represent fitness, measured by the number of offspring. So, more successful individuals get more offspring. Finally it is assumed that in this asexual world the individuals breed true, so that each child inherits its single parent's strategy. Then in the next generation the fraction of more successful members in the population will be higher and the fraction of less successful members will be lower. Modelling this process in continuous time results in differential equations known as the replicator dynamics.

Given a population state $x = x(t) \in S$ at time t , the expected payoff of a member playing strategy i , $i \in I_m$, is given by $e^{i\top}Ax$. The growth rate $\frac{\dot{x}_i}{x_i}$ of the share of i players is given by comparing the payoff or fitness of their strategy with the average fitness of the population $x^\top Ax$. This gives the system of differential equations

$$\dot{x}_i = (e^{i\top}Ax - x^\top Ax)x_i, \quad i \in I_m. \quad (4)$$

Taking for granted the theory of differential equations this system has a unique solution $x(t, x^0)$, $t \geq 0$, for any initial point $x^0 = x(0) \in S$. Furthermore, summing up the equations (4) over all $i \in I_m$ we get that $\sum_{i=1}^m \dot{x}_i = 0$ because $\sum_{i=1}^m x_i = 1$ and hence we have that S (and all its faces) is invariant, i.e., any trajectory starting in (a face of) S stays in (the same face of) S . So, $\sum_{i=1}^m x_i(t) = 1$ for all t and $x_i(t) = 0$ for any $t \geq 0$ if $x_i(\hat{t}) = 0$ for some $\hat{t} \geq 0$. The latter property says that if at a certain time the fraction of members playing strategy i is equal to zero, then it will always remain zero and it has always been zero. On the other hand $x_i(t) > 0$ for all $t \geq 0$ if $x_i^0 > 0$, so a strategy will survive for ever if it is available at $t = 0$. Of course, this does not exclude that $x_i(t)$ converges to zero if t goes to infinity, i.e., it may happen that a trajectory starting in the interior of S converges to the boundary. Now, a Nash equilibrium strategy $x \in S$ (of the monomorphic population in which all members play x) is said to be asymptotically stable for the polymorphic population (in which x_j denotes the fraction of members playing j), if there exists a neighbourhood X of x , i.e., an open set X in S containing x , such that any trajectory of the replicator dynamics starting at $x^0 \in X$ converges to x . The following result is due to Taylor and Jonker [47], see also Hines [19] or Zeeman [56], and can be seen as the basic result relating replicator dynamics and Evolutionary Stable Strategies.

Theorem 4.1

Every Evolutionary Stable Strategy $x \in S$ is asymptotically stable for the replicator dynamics given in equation (4).

The theorem says that for any ESS there is a neighborhood such that any trajectory of the replicator dynamics starting in this neighborhood converges to this ESS. However, the reverse is not true. The replicator dynamics may converge to a strategy not being ESS. Moreover, a trajectory does not always converge to some limit point x^* if t goes to infinity. It may happen that the trajectory path is a cycle on S or moves outwards

toward a hyperbola. For other properties of the trajectories, for instance that in the limit the replicator dynamics wipes out all strictly dominated strategies if initially all strategies are present, we refer the interested readers to e.g. Weibull [55] or Van Damme [11]. In these references many results are stated about the relation between (asymptotically) stable equilibria of the replicator dynamics and refinements of the Nash equilibrium, for instance that any asymptotically stable strategy is a perfect Nash equilibrium strategy. For a further discussion about the replicator dynamics see also Mailath [26] and Friedman [16].

Here we want to restrict ourselves to a more detailed discussion of the results for 2×2 symmetric matrix games. Let the payoff matrix A be given by

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

It is well known that the set of Nash equilibrium strategies is invariant with respect to adding the same constant to the payoffs of one player for a given pure strategy of the other player. From this it follows that we can distinguish three types of 2×2 games (see also e.g. Weibull [55] or Friedman [17]).

Type I: $a - c < 0 < d - b$. In this case the second strategy strictly dominates the first strategy and hence (e^2, e^2) is the unique Nash equilibrium of the symmetric game. Moreover, $x = e^2$ is ESS and it can be shown that the replicator dynamics converges to e^2 for any strict positive initial strategy x^0 . In case $d < a$ (and hence $c > b$) this type represents the well-known Prisoner's Dilemma Game (PDG) and the Nash equilibrium yields the non-efficient payoff d for both players. So, in this case the replicator dynamics lead to a non-efficient stable state and starting from a state x^0 close to e^1 the average payoff or fitness of the population is decreasing along the trajectory from almost equal to a at the starting point to d at the limit point. The case $d - b < 0 < a - c$ is similar with the first strategy as the dominant strategy.

Type II: $a - c > 0$ and $d - b > 0$. This class of games is known as Coordination Games (CG) and has three symmetric equilibrium strategies, namely the two pure strategies e^1 and e^2 and the mixed equilibrium strategy $x^* = \left(\frac{d-b}{a+d-b-c}, \frac{a-c}{a+d-b-c}\right)^\top$. The highest payoffs are obtained in the pure strategy equilibria. Although all three equilibrium strategies are proper, only the two pure equilibrium strategies are ESS and are asymptotically stable. The replicator dynamics converges to e^1 (resp. e^2) if $x_1^0 > (<) \frac{d-b}{a+d-b-c}$.

Type III: $a - c < 0$ and $d - b < 0$. A typical example of this class of games is the classical Hawk-Dove Game (HDG) of Maynard Smith and Price [30]. Taking $d = \frac{1}{2}b > 0$, $a < 0$ and $c = 0$, a member of the population gets in a contest with his rival a payoff of $a < 0$ if he fights (plays hawk) and 0 if not (plays dove) when his rival plays hawk, while the payoff is equal to $b > 0$, respectively $\frac{1}{2}b$ when his rival plays

dove. In this case there are two asymmetric equilibria, namely (e^1, e^2) and (e^2, e^1) . Furthermore there is one symmetric equilibrium (x^*, x^*) with x^* as given under Type II games. So, x^* is the unique symmetric equilibrium strategy. This strategy is also the unique ESS and the replicator dynamics converges to this strategy for any strictly positive initial strategy vector x^0 .

Observe that for type I and type II games the replicator dynamics converges to a pure evolutionary stable equilibrium strategy, whereas for type III games the dynamics converges to a mixed ESS. So, in this case the dynamics converges to a population in which a fraction x_1^* plays hawk and the others play dove. In such a stable population two types of individuals can be distinguished. Of course, in equilibrium both types have the same average fitness. We have also seen that for 2×2 games the replicator dynamics always leads to an ESS when starting at an interior point of the strategy space. So, for the 2×2 case the replicator dynamics always converges.

Until now we have discussed the notion of Evolutionary Stable Strategy and the concept of replicator or population dynamics. We now come to the question where this biological game theory has brought us. What is the meaning of this theory for economics? This question will be addressed in the next sections.

5 The economic meaning of Evolutionary Stable Strategy

We have seen that the concept of Evolutionary Stable Strategy is nothing less but also nothing more than another refinement of the Nash equilibrium concept. If an ESS exists, then it is a proper equilibrium strategy and therefore ESS is cautious and robust against trembles. Moreover, an ESS is stable against mutants and is asymptotically stable with respect to the replicator dynamics. This latter result shows that rational behavior is not necessary to obtain such a sophisticated equilibrium. A population of genetic players inherited with the behavior of their parents is able to reach an ESS when the offspring is determined by the fitness of the players. However, it is not easy to apply this result in economics.

First of all, the replicator dynamics always lead to Nash equilibria. Unfortunately, Nash equilibria often reveal bad outcomes for the society as a whole. They may suffer from the tragedy of the commons. This is nicely illustrated by the PDG. The Nash equilibrium yields the worst outcome, both players can obtain higher payoffs by playing the dominated strategy. In contrast, in empirical prisoner's dilemma-like situations cooperative behavior can be observed rather frequently. This has inspired many authors to develop game theoretic models in which the players cooperate by playing the dominated strategy. Many papers on repeated games are devoted to this topic. Unfortunately, evolutionary game theory is not very helpful in sustaining coop-

erative behavior. Even worse, in the Hawk-Dove game the Nash equilibrium selected by the ESS outcome is the worst possible Nash equilibrium. Taking $a = d = 0$ and hence $b > 0$ and $c > 0$, it can easily be seen that both players obtain more payoff in either of the two equilibria in pure strategies. So, ESS does not sustain cooperative behavior in type I or type III 2×2 games. This might explain that several theoretical papers on evolutionary games in economic journals have been focussed explicitly on coordination games. Indeed, for such games coordination is sustained by ESS. Even better, the most efficient outcome has the largest region of attraction, which means that to bring the population from one of the ESS outcomes in coordination games to the other, considerably less mutants are needed to go from the worst ESS to the best ESS than vice versa and therefore the most efficient outcome is very likely to occur. For further results on this topic we refer to e.g. Kandori, Mailath and Rob [21], Ellison [13] and [14], Young [54], Robson and Vega-Redondo [37] and Bergin and Lipman [2].

For type I and type III 2×2 games we may conclude that the replacement of the usual assumption of rationality in economics by the biological fitness criterium is not of any help sustaining cooperative behaviour, because basically both assumptions lead to best reply strategies and therefore the only possible outcomes are the Nash equilibria, all of which are Pareto inefficient. Other ideas have to be exploited to model the sustaining of cooperation. Recent literature in the field of sociology and psychology offers a way out by replacing the economic rationality approach by procedural rationality, see e.g. Simon [44], specifying a rule of behavior. The ‘Tit-for-Tat’ strategy, well-known from repeated game theory, can be seen as an example of such a rule of behavior. We return to this subject in section 8.

6 Learning and imitation as replicator dynamics

Even if we adapt the basic idea of replicator (population) dynamics and consider this as a useful tool to select one out of the set of Nash equilibria, the application to economics of the biological concept of fitness driving the dynamics is not straightforward. The genetic mechanism of natural selection has to be replaced by a social mechanism of learning and imitation. Here we encounter several problems.

Firstly, natural selection by fitness results in a dynamic process in which frequencies of the strategies played by the members are adjusted through differences in the number of offspring. Members with more successful strategies get more offspring. So, the adjustment of the frequencies is an autonomous process. Individual members of the population do not make (rational) choices. But adjusting of behavior on an individual level and hence rational behaviour is an essential feature of learning. The implications of replacing the fitness mechanism by individual learning have been discussed in e.g. Crawford [7]. Crawford [9] considers the repeated play of coordination games and addresses the question whether experimental results reported by for

instance Van Huyck, Battalio and Beil [50] and [51] can be explained by a learning process for adjusting the strategies.

Secondly, in the replicator dynamics the strategy frequencies in the population are adjusted according to their fitnesses. As a consequence the frequencies of all strategies with a fitness above the average level are increasing, also when these strategies are not a best reply. Within a framework of learning and imitation this is not very realistic. We may expect that rational players in a process of learning or imitation will replace their strategies by best reply strategies. So, in such a process only the frequencies of the best reply strategies may be expected to increase. This raises the question whether a learning or imitation process can justify the replicator dynamics as defined in equation (4). The research on this topic has gone into several directions. A number of authors have addressed the form of the replicator dynamics. In the original system (4) the growth rate of the frequencies is given by the average expected payoff of the strategies. It has been shown that the weaker form of monotonicity of the growth rates in the expected payoffs is sufficient to preserve most of the results. Results along these lines have been obtained by e.g. Nachbar [33], Friedman [16], Matsui [27], Samuelson and Zhang [38] and Björnerstedt [6]. On the other hand, on the same issue research has been done addressing the question whether it is possible to formulate a learning or imitation process resulting in the replicator dynamics as given in equation (4), see e.g. Björnerstedt and Weibull [5], Gale, Binmore and Samuelson [18] and Schlag [40]. In the latter paper the players follow an imitative behavior, never imitate an individual that is performing worse than oneself, and imitate individuals doing better with a probability proportional to how much they perform better. It is shown that this behavioral rule results in an adjustment process that can be approximated by the replicator dynamics.

Finally, we would like to mention a third problem when replacing the genetic mechanism by a story of learning or imitation. To formulate this problem, we quote Mailath [26] when he discusses the problem of the often assumed bounded rationality of the players. He then states:

Players in these models are often quite stupid. For example, in evolutionary models, players are not able to detect cycles that may be generated by the dynamics. The criticism is that the models seem to rely on the players being implausibly stupid. Why are the players not able to figure out what the modeler can? Recall that in the traditional theory, players are often presumed to be computationally superior to the modeler.

Here we come to the question that when frequencies are adjusted by a learning process, why players are going to adapt their strategies according to a nonsophisticated process as the replicator dynamics? As we have seen before, convergence of the replicator dynamics can not be guaranteed. In fact, the replicator dynamics is equivalent with the Walrasian tatonnement process in general equilibrium theory and suffers from the same weaknesses. In general equilibrium theory several globally convergent

price adjustment processes have been designed, based on the well-known simplicial method introduced by Scarf [39]. In e.g. Van der Laan and Talman [23] and [24] and Kamiya [20] path-following globally convergent processes have been proposed. In these processes tatonnement-like rules are followed, but cycling is prevented by taking into account the starting price vector. In Van den Elzen and Talman [15], see also Van der Laan and Talman [22], the path-following technique is modified to a strategy adjustment process for noncooperative games in normal form. This adjustment process always converges to a perfect Nash equilibrium. In Yang [53], see also Yamamoto [52], such a process has been designed converging to a proper Nash equilibrium. It might be interesting to apply these results to the framework of a monomorphic population and to search for an interpretation of such sophisticated processes as learning or imitation processes.

7 Evolutionary stability in nonsymmetric games

Applying evolutionary game theory to economic situations we often encounter the problem that the conditions of a single symmetric population and pairwise random matching are not met. In many economic situations we have to deal with interaction between more than two individuals and/or with interaction between individuals from several distinct populations. Nevertheless, as argued by e.g. Friedman [17], evolutionary game theory can easily be adapted to model these features.

First, instead of pairwise matching, we consider the case that all players are interacting together, i.e., all players are ‘playing the field’. Now, the payoff to a player is determined by his own strategy and the strategies of all other players. However, we can still consider the payoff function as a fitness function and apply the replicator dynamics to this population. As an example we take a simple model of a pure exchange economy consisting of a large (infinite) number of agents and a fixed number of m commodities. Suppose that initially each individual is endowed with one unit of just one of the commodities. This endowment can be seen as the strategy of the individual inherited from his parents or obtained by education. Let x_j , $j = 1, \dots, m$, be the fraction of individuals endowed with commodity j . Furthermore, the utility of an agent obtained from a vector $y \in \mathbb{R}^m$ of commodities is given by $u(y)$. So, all agents are identical except for their initial endowments. Now, for this economy, suppose there is a unique Walrasian equilibrium price vector $p(x) \in \mathbb{R}^m$, i.e, at this price vector for each commodity j the total consumption equals the total initial endowment, so the average consumption of commodity j equals x_j , and each individual maximizes his utility given his income. Clearly, the Walrasian price vector $p(x)$ depends on the distribution x of the endowments. Within this framework the income $p_j(x)$ of an individual of type j , i.e., an agent endowed with commodity j , can be seen as the payoff to a type j individual. This payoff depends on his own endowment and the endowments of all others. Allowing for the adjustment of

endowments, for instance because agents die and are replaced by new born agents, we can apply the standard replicator dynamics by stating that the distribution of the endowments develops according to the system of differential equations

$$\dot{x}_j = [p_j(x) - \frac{1}{m} \sum_{k=1}^m p_k(x)x_k]x_j, \quad j \in I_m,$$

so that the fraction of individuals endowed with commodity j will increase (decrease) if $p_j(x)$ is above (below) the weighted average of the prices. By taking the weighted average $\frac{1}{m} \sum_{k=1}^m p_k(x)x_k$ we have that the system satisfies $\sum_{j=1}^m \dot{x}_j = 0$, so that along the solution trajectory the sum of the fractions x_j , $j \in I_m$, remains equal to one. Assuming that initially all commodities are available and that the process will converge, the economy develops to a situation in which all prices are equal. If initially a commodity has a very high price, there is a relatively shortage of this commodity and more agents are going to adopt this commodity. On the other hand, less individuals are going to adopt a low-priced commodity. It can be shown that the stable state in which all prices are equal maximizes social welfare. This result is in line with the observation of Nelson [35] in a recent survey article on evolutionary theory of economic change, that evolutionary economics should be seen as a dynamic generalization of conventional optimization and equilibrium theory. Analogously to the above evolutionary theory can be applied to the optimal choice of technologies. Crawford [8] considers fitness functions playing the field in stag-hunt games.

The second topic mentioned above is the assumption of a single symmetric population. In many situations interaction takes place between individuals from distinct populations. In biology, we may think of contests between the owner of a territory and an intruder; the analogy in economics is a market situation in which the incumbent has to compete with an entrant firm. An other example is a market with buyers and sellers. In a evolutionary setting this can be modelled by multiple (large) populations, where each population represents an (economically) distinct role. For detailed results on this topic we refer to e.g. Van Damme [11], Friedman [16] and [17], Cressman [10], Samuelson and Zhang [38], Swinkels [45] and Weibull [55]. Here we restrict ourselves to the basic ideas in a framework with only two distinct populations.

Let P^i denote the set of individuals in the monomorphic population i , $i = 1, 2$. In each contest a randomly chosen member of P^1 meets a randomly chosen member of P^2 to play the bimatrix game (A, B) . We first discuss the notion of Evolutionary Stable Strategy within the multiple population framework. As in the single population case, an ESS must be at least a Nash equilibrium. So, let $(x_1, x_2) \in \hat{S} = S^{m_1} \times S^{m_2}$ be a pair of Nash equilibrium strategies, i.e.,

$$\text{i) } x_1^\top A x_2 \geq q^\top A x_2 \text{ for all } q \in S^{m_1}, \quad (5)$$

$$\text{ii) } x_1^\top B x_2 \geq x_1^\top B q \text{ for all } q \in S^{m_2}. \quad (6)$$

To be evolutionary stable the Nash equilibrium strategies must be immune against

any mutant strategy in one or both populations. Suppose that some mutants playing strategy $q \in S^{m_1}$ appear in population P^1 . Analogously to equation 3 a Nash equilibrium (x_1, x_2) is said to be stable against q if the incumbent strategy x_1 performs better than the mutant strategy q . From the discussion in Section 2 we have seen that this condition holds if the incumbent strategy performs better against itself than the mutant strategy against the incumbent strategy or if the incumbent performs better against the mutant than the mutant against itself in case both strategies perform equally well against the incumbent strategy (see Definition 3.1, conditions i) and ii)). However, in the multiple population framework a mutant player in population P^1 only meets players from population P^2 and therefore cannot meet a colleague mutant in its own population. This implies that the second condition becomes redundant and hence for evolutionary stability the Nash equilibrium strategies must satisfy the stronger condition that the incumbent strategy of a population performs better against the other population than any mutant strategy. Hence we have the following definition.

Definition 7.1 (Evolutionary Stable Strategy Pair)

*A strategy pair $(x_1, x_2) \in \hat{S}$ is an **Evolutionary Stable Strategy Pair** of the asymmetric bimatrix game $G = (A, B)$ if it satisfies the Nash equilibrium conditions (5) and (6) with strict inequality.*

It has been noticed already by Selten [42] that an evolutionary stable strategy pair is not only stable when mutants appear in one of the populations but also if in both populations mutants appear simultaneously. More precisely, if (x_1, x_2) is evolutionary stable, then for any $q = (q_1, q_2) \in \hat{S}$ there exists an $\epsilon(q) \in (0, 1)$ such that for all $\epsilon < \epsilon(q)$ we have that either x_1 performs better against y_2 than q_1 does, or x_2 performs better against y_1 than q_2 does, or both, where $y_i = (1 - \epsilon)x_i + \epsilon q_i$, $i = 1, 2$. So, if mutants appear in both populations at the same time, the incumbent strategy performs better in at least one of the populations and hence within this population the mutants will die out in the limit. As soon as the fraction of mutants within this population is small enough, it follows from Definition 7.1 that in the other population the incumbent strategy performs better than the mutant strategy and that therefore also in the latter population the mutants will die out.

A Nash equilibrium satisfying the conditions of evolutionary stability is known in the traditional game theory as a strict Nash equilibrium and has the property that x_i , $i = 1, 2$ is the unique best reply against x_j , $j \neq i$. As discussed already in Section 2 this property implies that an evolutionary stable equilibrium is an equilibrium in pure strategies. This shows a serious weakness of evolutionary game theory in case of multiple populations. At any nonstrict equilibrium the populations are not stable against mutants playing alternative best replies. However, for a large class of games a Nash equilibrium in pure strategies does not exist and therefore evolutionary game theory fails to make any prediction about the outcome. For weaker stability concepts we refer to Swinkels [45].

There are several possibilities to generalize the replicator dynamics from the single population case to the multiple population case. The most common generalization has been proposed by Taylor [46] and is in case of a two-population given by

$$\begin{aligned}\dot{x}_{1j} &= [(e^{1j} - x_1)^\top A x_2] x_{1j}, & j \in I_{m_1}, \\ \dot{x}_{2j} &= [x_1^\top B (e^{2j} - x_2)] x_{2j}, & j \in I_{m_2},\end{aligned}$$

with e^{ij} the j th unit vector in \mathbb{R}^{m_i} , $i = 1, 2$. Clearly, this is a straightforward generalization of equation (4) and the solution path of these dynamics has about the same properties as in case of the replicator dynamics for single populations. In particular we have that every strict (evolutionary stable) Nash equilibrium is asymptotically stable and that strictly dominated strategies vanish along any interior solution path. However, since strict Nash equilibria often do not exist, the concept of asymptotic stability in asymmetric games is much less useful than in symmetric games. This has motivated Samuelson and Zhang [38] to search for more robust stability results by specifying alternative formulations for the dynamics. Unfortunately, even if the dynamics are requested to be payoff monotonic, i.e., a strategy with a higher payoff has a higher growth rate, a strictly dominated strategy may survive along the solution path if it is not strictly dominated by just one other pure strategy but only by mixed strategies. This brings Samuelson and Zhang to the conclusion that to get reasonable outcomes additional structure has to be placed on the evolutionary selection or learning process and that therefore theories of learning are an important area for further research.

For some illustrating examples of the application of the replicator dynamics to two player games we refer to Weibull [55] and Friedman [16]. The properties and weaknesses of the dynamics are very nicely demonstrated by the Entry Deterrence Game given by the payoff matrices

$$(A, B) = \begin{bmatrix} (2, 2) & (0, 0) \\ (1, 4) & (1, 4) \end{bmatrix}.$$

Player 2 is a monopolist on a market and wants player 1, the intruder, to stay out, which gives him a payoff of 4. In case player 1 enters, player 2 can yield in which case the market is shared (with payoff 2) or fight with payoff 0 to both players. This game has a unique strict Nash equilibrium, namely $x_1 = (1, 0)^\top$ (player 1 enters) and $y_1 = (1, 0)^\top$ (player 2 yields), giving both players a payoff of 2. Moreover, there is a continuum of Nash equilibria, namely any $(x_1, x_2) \in \tilde{S}$ with $x_{11} = 0$ (player 1 stays out) and $x_{22} \geq \frac{1}{2}$ (player 2 will fight with probability at least equal to $\frac{1}{2}$). Observe that for player 2 the strategy ‘fight’ is weakly dominated by ‘yield’. For most of the initial states the replicator dynamics converges to the unique strict equilibrium. However, for initial states with both x_{11} and x_{21} close to zero the replicator dynamics converge to a Nash equilibrium out of the set of equilibria in which player 1 stays

out. At such an equilibrium the evolutionary selection procedure does not wipe out the weakly dominated strategy ‘fight’.

To conclude this section we like to stress that random matching of the players is one of the basic assumptions of evolutionary game theory. Although this assumption might be appropriate within a biological framework, in a economic setting an agent often only interacts within a small subset of the other agents. Therefore opposite to the random matching, some authors have also discussed the implications of local matching rules in which each individual only interacts within a small group of friends or neighbors. Learning under local interaction has been studied for coordination games by e.g. Ellison [13] and [14] and Berninghaus and Schwalbe [3]. Ellison concludes that under local matching evolutionary forces are much stronger, that is that the system adjusts much faster. While under random matching convergence times are often incredibly long and therefore of little help for useful predictions on the outcome, under local interaction convergency may appear early in the process. Berninghaus and Schwalbe [3] show that the smaller the neighborhood group, the higher the probability to reach an efficient equilibrium. Nevertheless, in all these models we are still confined to the Nash equilibrium outcome and hence the worst possible outcome in Prisoner’s Dilemma Games. In the next section we will see that under sociological strategy adapting rules cooperation may emerge in Prisoner’s Dilemma Games with local interaction.

8 Cooperation in Prisoner’s Dilemma Games with local interaction

Local interaction models are interesting because they often have some relationship to real world interaction situations and therefore have a nice economic interpretation. In every day (economic) life, typically not everyone will interact with all the other individuals present in a certain environment. Each individual has a (small) number of others with whom (s)he interacts consisting mostly of colleagues, friends, relatives and business associates, or in the case of firms other related firms. These colleagues and friends in their turn interact with their respective groups of relatives and friends and since these groups usually overlap, but are not identical, there is an indirect interaction through their respective groups of friends and colleagues between people who do know each other and who never actually meet.

Local interaction is typically modelled in a spatial way in which the people with whom one interacts are located nearby. The individuals with whom a member of the population interacts is called one’s neighbors. As stated above, in local interaction models there is an indirect influence on behavior of individuals by people that are not one’s neighbors via (a series of) other individuals. In real life the size of group of neighbors will vary across individuals. For reasons of simplicity however, the group

size is taken fixed in most of the models in the field. Modelling the spatial environment as a discrete 2-dimensional torus, it seems very plausible to think of every individual as having four (adjacent) or eight (adjacent and diagonal) neighbors, see e.g. Ellison [14]. In local interaction models a large number of games is played. At the beginning of every game an individual is selected and this individual is called the subject. This subject plays the game with (one or some of) his neighbors. A neighbor is either picked at random or there is some selection criterion in the model. On the basis of the payoff obtained from the game and further obtained information the players decide whether to change the action the next time they get to play the game, giving the individuals ample opportunity to learn. The specific way in which learning is modelled differs a lot between the models, but most modelers use some form of rational behavior with bounded recall (best reply dynamics). Most of the time these models are used to either select amongst Nash equilibria or to explain features of the emergence of cooperative behavior while this cooperative behavior is not one of the Nash equilibria of the game. However, to explain emergence of cooperative behavior the majority of local interaction models face a problem, since they need to expect too much rationality of the individuals. In other words, most models incorporate learning dynamics that are far more complicated than those used in every day life. In our opinion in general people do not optimize in most situation in which interaction takes place.

Based on literature in the field of sociology and psychology, in Tieman, Van der Laan and Houba [49] a fundamentally different approach has been chosen. In this paper the social environment has been modelled a 2-dimensional torus as described above. The dynamics is modelled as an adaption process on the level of the individual, as has Ellison [14], but instead of using best-reply dynamics, a sociological perspective has been chosen. This has been implemented by introducing a decision heuristic or metastrategy for all individuals on the torus, according to which they adapt the action they are playing. This metastrategy is an augmented version of the strategy Tit-for-Tat. Tit-for-Tat is a strategy specified for a game in which an individual can play one of two (pure) actions, cooperative or defective, and prescribes an individual to start the first game by playing the cooperative action. In the remainder of the game individuals simply play the action the opponent has played in the last interaction. So, an individual that uses the Tit-for-Tat strategy punishes an opponent who chooses the defective action, by playing defective in the next encounter. Tit-for-Tat is also a forgiving strategy: as soon as the opponent decides to play the cooperative action again, the Tit-for-Tat player will start playing cooperative again in the next game both players play. The Tit-for-Tat strategy became very popular after a tournament organized by Robert Axelrod [1], in which he invited a large number of scientists from different disciplines to send in computerprograms to take part in a competition of playing Prisoner's Dilemma Games. In the competition Tit-for-Tat turned out to be the winner by far and did better than far more complicated strategies.

In the model discussed in [49] the players play one of $k + 1$ possible actions,

which can be ordered from completely defective to completely cooperative. The metastrategy determines how players will play in the next game they participate in, based on their current action and the payoff they got from that action in comparison with the payoffs their neighbors got the last time they played the game as subjects. It is postulated that when a player obtains a payoff that is higher than the average payoff his neighbors got the last time they played the game as subjects, he feels as if he is in a ‘win’ situation. On the other hand, a player with a payoff lower than the average payoff of his neighbors feels as if he is in a ‘lose’ situation. Clearly, we have defined win and lose situations relative to the group of people one interacts with, mainly because we think this is what is happening in real life. People compare their own situation with the situation of their relatives.

Now, following Messick and Liebrand [31], it is assumed that a player in a win situation is going to play more cooperatively in the next game he participates in, whereas a player in a lose situation will play less cooperatively in the next game he is in. Since there are $k + 1$ possible actions in the game, playing more cooperative is incorporated as switching from action i , $i = 0, \dots, k - 1$ to action $i + 1$. If a player in a win situation already plays fully cooperative (action k), he does not change his action. Analogously, a player in a lose situation will change his action i , $i = 0, \dots, k$ into $i - 1$, whenever $i > 0$. When $i = 0$, his action will be left unchanged. The action of a player who is neither in a win, nor in a lose situation will also be left unchanged. Apart from this strategy resembling a multi-action version of Tit-for-Tat, there are also arguments of fairness, as described in Rabin [36] amongst others. Fairness is a concept in which people have ideas about what they should be getting out of a game (an aspiration level). If they reach this level, they are satisfied and are willing to cooperate. If they do not reach this level however, they feel they are being cheated on by the other players and they will act accordingly by not cooperating as much as they used to do.

The above described metastrategy is incorporated in a model in which the players are producers of heterogeneous, but substitutable goods. In this game the actions are the prices the producers set for their product. Every producer will compete against the neighbor who sets the lowest price in the neighborhood. This model results in a stable state of the population in which a high degree of cooperation (up to about 97% on average) is present, that is most producers set a price that is very near the price they would set would they form a cartel. Ofcourse these results depend on the exact specification of the metastrategy, but the general idea that cooperative behavior can be explained by interaction models incorporating sociological adaption rules is illustrated nicely here. Another interesting result of this model is the emergence of price wars. In the stable state of the population sometimes a producer starts lowering his price. Other producers in the neighborhood of this producer start losing customers and therefore are in a lose situation whenever they play against this producer with the lower price. Thus other producers in the neighborhood of this producer with a lower price also start to lower their price. In this manner a local

price war can emerge. After some time the general prices in such a neighborhood have declined considerably and more producers get to be in win situations again and convergence to the same stable situation the population was in before starts again. So, the model may explain the emergence of cooperation, even although cooperation is not a Nash equilibrium. We think that local interaction models with sociological learning rules are an important field for further research.

9 Concluding remarks

Evolutionary game theory puts the static Nash equilibrium concept in a dynamic setting. This dynamic framework may provide us with a better understanding about the stability of equilibria and selection mechanisms. However, the standard replicator dynamics has limited applicability in economic models and has to be replaced by a process of learning and imitation to be fruitful within the field of economic theory. In this way evolutionary game theory forces us to think more thoroughly on the behavior of players outside equilibrium and the game they are playing. As long as individuals are supposed to play best replies, at best sophisticated Nash equilibria will emerge. This may deepen our understanding on coordination problems, but does not provide us with a better insight in cooperation problems. To explain the emergence of cooperative behavior in the society as a whole, we have to allow for more elaborate strategies reflecting social rules in the behavior of individuals in an interactive environment.

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