TI 2023-075/VI
Tinbergen Institute Discussion Paper

Green Technology Adoption and Skill Reallocation

Sacha den Nijs[1]
Stefanos Tyros[2]

1 Vrije Universiteit Amsterdam and Tinbergen Institute
2 Vrije Universiteit Amsterdam and Tinbergen Institute

Green Technology Adoption and Skill Reallocation

Sacha den Nijs*† Stefanos Tyros*†

November 16, 2023

Abstract

The green transition towards a carbon neutral economy constitutes a technological transformation, in which firms need to invest in new, clean technologies. In order to be operated, these new technologies often require a set of technology-specific skills that differ from those that were relevant for dirtier predecessors. We model the green technology investment decision of firms in a labour market with search frictions and two-sided heterogeneity. We find that the presence of skill mismatch negatively affects the expected productivity of the new, green technologies, delaying their adoption and resulting in higher emission intensity in production compared to a counterfactual with no labour market frictions. The induced effect is of first-order, making it policy-relevant. This slower diffusion of green technologies in the presence of skill heterogeneity results in workers with green skills being locked in brown jobs. We also find that an accelerated greening of the economy leads to larger labour market transitions. More specialised new technologies further reinforce these effects, and so does a slower pace of technological progress. Finally, our model results suggests that retraining policies are a useful policy tool both in the absence as well as the presence of a carbon tax or investment subsidies.

Keywords: Green technology adoption, skill sorting, carbon tax, retraining, decarbonisation, search and matching model

JEL Codes: J23, J24, O33, Q52

*Vrije Universiteit Amsterdam and Tinbergen Institute. Corresponding author: Stefanos Tyros, stefanos.tyros@vu.nl

†We are grateful to Pieter Gautier, Björn Brügemann, Gerard van der Meijden, and Henri de Groot for their valuable feedback and guidance. We also want to thank all the discussants and colleagues in multiple conferences that have helped with their questions and comments to greatly improve this paper.

1 Introduction

To mitigate climate change, economies need to rapidly reduce their carbon footprint. Such a transition requires investing in new, low-carbon technologies, turning *brown jobs* green (Rieff and Peschner, 2019; Montt et al., 2018). These green technologies often require different sets of technology-specific skills from employees to be operated, compared to the technologies currently in use. For example, construction workers will need to be able to install solar panels, maintenance workers need to work with new energy-saving devices, and car mechanics must handle battery fluids instead of engine oils. Hence, firms that have to invest in new, environmentally-friendly technologies might have to retrain their employees or search for new hires. ILO (2018) indicates that, globally, 25 million *green jobs* are expected to be created in the green transition, where around 20 million of these jobs potentially require retraining of workers. In this paper we study how the decision by firms to adopt new, green technologies and the existence of firm-worker skill mismatch interact in a labour market with search frictions and imperfect skill sorting.

We build on Hornstein et al. (2007) and Gautier and Teulings (2015) and model the interaction between firms' green technology adoption decision and two-sided heterogeneity. In doing so, we assume that available technologies become more energy efficient, or 'greener', with time and study their diffusion across the economy. In our baseline model, skills are not in aggregate shortage, but are sorted imperfectly. Hornstein et al. (2007) set up a search and matching model with a vintage capital structure and sunk investments that leads to heterogeneous firm productivity and vacancies. In our model, we interpret these capital vintages as technologies with higher energy efficiency, as in Mulder et al. (2003), and extend their model to include two-sided heterogeneity. Following Gautier et al. (2010) and Gautier and Teulings (2015), the skill mismatch between the production technology used and the worker, decreases productivity. Hence, green technology investments come with a risk: incumbent workers might not have the appropriate technology-specific skills for the new technology.

We show that imperfect skill sorting slows down the decarbonisation of the economy. Due to search frictions and skill mismatch there is a quantitatively significant increase in the distance of technologies-in-use from the frontier, on top of the one observed in Hornstein et al. (2007) due to the presence of sunk investments. More than half of this effect is attributable to imperfect skill sorting. It is larger when technological progress is slow, new energy efficiency technologies are more specialised, or labour markets are tighter. The underlying mechanism is that expected skill mismatch reduces the expected productivity of new green technologies, as the firms must take the cost of retraining or looking for a new worker into account. Hence, the incentive to update the old technology is lower and, therefore, firms delay investment in clean technologies. As a result, workers that have the skills to operate green technologies are *locked in* brown jobs.

Although the increased distance from the technological frontier induces a level, rather than a pace, effect on green technology diffusion, it is of great importance. The reason is that the green transition is not an indefinite process but one in which the total emissions over a finite period of time are of importance. Therefore, a larger distance from the technological frontier translates into higher emissions intensity in production during the green transition and, therefore, worse climate outcomes. The extension of our model that accounts for aggregate skill shortages indicates that our baseline estimates are a lower bound of the effect labour market frictions can have on the green transition.

Finally, we study various relevant policy measures. In many countries, subsidies for green technology investments are made available to firms: in our framework, a subsidy reduces the effect that sunk investment costs have on the adoption decision, but does not address the labour market frictions and skill mismatch. In addition, we study the effect of a carbon tax within our framework. We find that it stimulates investment in green technologies but, again, without addressing skill sorting directly. The carbon tax, however, increases unemployment duration and the unemployment rate. Hence, we also discuss how a policy mix addressing both climate and firing externalities may need to include retraining policy both in the absence and in the presence of a carbon tax. When a carbon tax has not been implemented, retraining can also accelerate decarbonisation. When it has, retraining is needed to reduce the *firing externality* of faster technology adoption, which results in lower political acceptability of climate change mitigation policies (Vona, 2019).

This paper builds on the theoretical literature of technological change, technology adoption and labour market outcomes, and applies it specifically to the green transition. Historically, concerns regarding job losses due to technological change have been persistent. Using search and matching models for the labour market, Mortensen and Pissarides (1998) study how technological change affects the amount of jobs, and Acemoglu (1998) shows that an increasing supply of skills can induce skill-biased technological change. Hornstein et al. (2007) use a capital vintages framework in a search and matching model and explain differences between the U.S. and EU labour market. More recently, the focus of labour economics studying technological change has shifted towards a task-based approach. Acemoglu and Restrepo (2018) use such an approach to model automation, while Duernecker (2014) studies investments and skill shortages that affect productivity. In our paper we build a framework that describes the interaction of new technology adoption and skill mismatch in a labour market with frictions.

In particular, we study the green transition: new technologies or capital vintages are more energy efficient than the older ones, allowing us to study the green technology adoption decision of the firm. This approach can help understand the so-called 'energy efficiency paradox' or 'gap' as described by Jaffe and Stavins (1994) and Hirst and Brown (1990): firms do not exploit all privately-profitable energy efficiency investments available to them based on a simple net present value analysis, considering only the direct energy costs saved due to the investment. Various studies, such as Solnørdal and Foss (2018), Backlund et al. (2012), and Sorrell et al. (2006) have investigated this gap by studying various barriers to investing in energy efficiency technologies, where competence and skilled staff play a role. In the Investment Survey of the EIB (2020), about 70% of surveyed EU firms indicate that the availability of skilled staff is an obstacle to investment in new technologies. Our work points out that imperfect skill sorting is an additional factor with a first-order effect on the speed of decarbonisation.

Finally, our study provides new insights on the labour market effects of the green transition. Various papers have studied the emergence of new green jobs (Garrett-Peltier, 2017), and the required green skills (Consoli et al., 2016; Tyros et al., 2023). At the same time, Vona et al. (2018) show that the general transferable *green skills*, required by green jobs, are not necessarily in aggregate shortage. In most broad occupation groups, workers in brown jobs have the necessary general transferable skills to undertake green jobs. This indicates that skill sorting, and not only aggregate skill shortage, is an important factor giving rise to mismatches between incumbent workers and newly adopted green technologies. Our model shows how the pace of green technology adoption affects labour market transitions, or the amount of people moving between employment to unemployment. This is crucial as such labour market outcomes can affect the political acceptability of the green transition (Vona, 2019).

The rest of the paper is structured as follows. In Section 2 we present the baseline green technology adoption model. Next, in Section 3 we calibrate the balanced growth path and disentangle various effects on the delay of green technology adoption. Additionally, we extend the model and show that our baseline result qualitatively still holds. In Section 4 we study relevant policies: investment subsidies, a carbon tax, and retraining policy. Finally, in Section 5, we conclude.

2 The Baseline Model

We set up a search and matching model in the style of Mortensen and Pissarides (1994), describing a labour market with firm and worker-skill heterogeneity (Gautier et al., 2010), labour market frictions, and technologies-in-use of various ages using a capital vintages approach (Hornstein et al., 2007). We use this model to study the green transition as a technological transformation[1] in which the new greener technologies can require different technology-specific skills than the old technologies. We focus on skill sorting (not shortage) as workers in brown jobs mostly have the necessary general transferable skills to undertake green jobs (Vona et al., 2018). Greener technologies exogenously become available over time and we study their diffusion throughout the economy.

Production Production requires pairing a firm that owns capital of a certain technology with a worker, and yields a homogeneous output good. In every period, new, greener technologies become available, but are not necessarily instantaneously adopted by all firms. The productivity of a firm-worker match depends on the age of the technology-in-use and skill mismatch, x,[2] between the specific skills required to operate the technology and the worker's actual skills. We assume no substitution possibilities between energy and capital, that is, full complementarity. Thus, energy use in production is directly linked to the productivity of the technology embodied in the capital of the firm.

At time t, a firm using a technology of age a has $k(t, a)$ efficiency units of capital. The productivity of a firm-worker match with skills gap x is a share $f(x)$ of the productivity of an optimal match.[3] Following Hornstein et al. (2007), the level of disembodied technology productivity, $z(t)$, grows at rate ψ, and energy efficiency or technological change embodied

[1]The framework is generalised in Appendix A.4 to describe technological transformations more generally.

[2]That is, there are no workers with an absolute advantage, but every worker has a relative advantage over certain jobs.

[3]We assume that the skills gap and its effect on productivity is independent of the technology's age, and does not change over time.

in new technologies grows at rate η. Thus, the firm's production function is:

$$\begin{aligned} y(t,a,x) &= f(x)z(t)k(t,a)^{\omega} \\ &= f(x)z_0 e^{\psi t}\left[k_0 e^{\eta(t-a)}e^{-\delta a}\right]^{\omega}, \end{aligned} \tag{1}$$

where δ denotes the depreciation of capital and ω its output elasticity. At the balanced growth path (BGP) the economy grows at a rate $g = \psi + \omega\eta$ and productivity of the newest available technology increases at an effective rate of $\phi = \omega(\eta+\delta)$ (see Appendix A.1.1 for the full derivation). The price of the produced good is normalised to 1 and all units henceforth are normalised to the productivity of a perfectly matched new technology of age zero.

Following Gautier et al. (2010), we use a second-order Taylor expansion of $f(x)$ around $x = 0$. Therefore, the normalised productivity of a worker-firm match on the balanced growth path that uses a technology of age a and has a skills gap x is given by

$$y(a,x) = e^{-\phi a}\left[1 - \frac{1}{2}\gamma x^2\right] \quad x \in [0, 1/2], \tag{2}$$

where the linear term of the expansion disappears as $f(0) = 1$ is a maximum. The parameter γ is interpreted as the specialisation of the technology, i.e. how important the skills gap is in production. When $\gamma = 0$, skills become irrelevant, therefore our model reduces to the model of Hornstein et al. (2007). Figure 1 illustrates the productivity of a match for various (a, x) combinations. The reduction in relative productivity, as the technology ages, captures the fact that old technologies are relatively less productive than the new ones because they are more energy-consuming.[4]

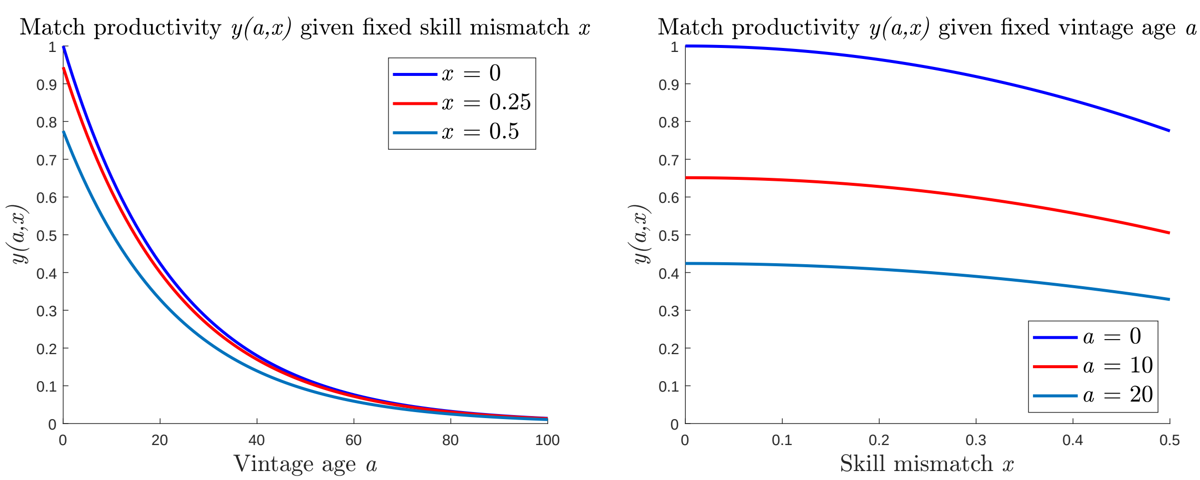

Figure 1: The production function of a match for various values of x and a, for $\gamma = 1.8$ and $\phi = 0.03$.

[4] Another possible interpretation is that brown goods produced with old technologies decline in relative demand compared to green goods.

Balanced Growth Path Our analysis focuses on the balanced growth path (BGP) of our model. Although the inclusion of transitional dynamics would offer a more precise picture of the green transition, focusing on the BGP allows for a more clear and quantifiable exposition. Moreover, the bulk of the green transition (outside its starting and end point) can be seen as a BGP where new green technologies are invented an adopted at a certain pace.

Skills Following Gautier et al. (2010) and Gautier and Teulings (2015) the skills of workers and those required by firms are uniformly distributed on a circle of unit circumference, i.e. $x \in [0, 1/2]$. This formulation implicitly assumes that there is no aggregate skill shortage in the economy, but that as a consequence of search frictions imperfect skill sorting arises. This is due to the fact that, given that meeting workers is a time consuming process during which no output is produced, firms are willing to hire workers that are not a perfect match for their technology, instead of searching for longer.

To make this clear it is important to differentiate between two types of skills. The first type consists of the general, transferable skills such as general engineering knowledge. General skills are not (to a very large extent) in aggregate shortage in the green transition, as Vona et al. (2018) show that most workers in brown jobs do have the necessary general skills to undertake green jobs. That is, the green transition is, largely, not skill-biased. They call these 'green skills' and group them in four categories: engineering and technical, operation management, monitoring, and basic science skills. The second type consists of the technology-specific skills, which give rise to the skills gap, x, between the worker and the firm. These skills are the ones required to undertake specific technology-related tasks. General skills facilitate those, but do not guarantee a good match with the technology-specific required skills. For example, a petroleum engineer does have the necessary general skills to undertake hydrogen engineering tasks due to their education and job experience but might be missing certain hydrogen-specific skills needed to transition to such a job.

In terms of modelling, the fact that workers in brown jobs have the general skills that green jobs require, implies that there is no aggregate skill shortage and that new green technologies have the same technology-specific skill distribution as brown ones do. Hence, the technology draw of firms, when they invest in a new technology, is taken from a stationary skill distribution. That is, the uniform unit circle is independent of the age of the technology. In Section 3.3 we extend our baseline model to also include aggregate skill shortage and show that our qualitative baseline results still hold and are a lower bound of the effect of labour markets on the green transition.

Timing The timing of the production process is as follows:

1. Entrant firms pay I to purchase capital of the newest available technology, i.e. $a = 0$. In what follows, these costs are sunk.

2. Firms with new and old technologies search for workers. Old firms use technologies of age $a > 0$.

3. Workers and firms randomly meet at a rate $\lambda = \lambda_0 u^a v^{1-a}$ per worker, where u denotes the unemployment rate and v the vacancy rate. We use $\theta = v/u$ to denote labour

market tightness. Before meeting, the firm does not know the technology-specific skills of the worker and the worker does not know the skills required by the firm nor the age of its technology.

4. After the meeting takes place, the skills gap x and the technology age a are observed, and the worker is employed by the firm if the match surplus is positive.

5. If matched, the worker-firm pair bargains over the match surplus. Given the monotonicity of productivity on a and x there is an age dependent reservation skills gap, $\overline{x}(a)$ (decreasing in a), with workers and firms accepting all matches with $x < \overline{x}(a)$.[5]

6. Workers start producing a stream of output $y(a, x)$ right after the match is formed. Over time, the technology ages (a increases) and thus becomes less productive relatively to the newest available technology.

7. Matches are exogenously destroyed at rate σ and endogenously destroyed when the technology reaches the reservation age $\overline{a}(x) = \overline{x}^{-1}(x)$ (i.e. the technology age at which the match surplus reaches zero).

8. After a match is destroyed, the firm retains its technology and searches for a new worker if and only if $a < a^*$. The scrapping age $a^* = \overline{a}(0)$ is the maximum age at which a match can be profitable. Beyond this age, the firm costlessly scraps its technology and exits the market.

9. If a matched technology reaches $a = a^*$ the match is endogenously destroyed and the firm exits the market.

10. The firm can re-enter the market by purchasing a new technology and hiring a new worker.

Importantly, in this baseline model firms always search for a new worker after updating their technology. In Appendix A.4 we extend the model to allow firms to retain their worker while updating their technology.

Value functions Following Hornstein et al. (2007), the flow value of employment at a firm that uses a technology of age a and has a skills gap of x, $V^E(a, x)$, is given by

$$\rho V^E(a, x) = w(a, x) - \sigma[V^E(a, x) - V^U] + V_a^E(a, x), \tag{3}$$

where $w(a, x)$ is the wage, the second term on the right-hand side is the expected loss from exogenous job destruction, and $V_a^E(a, x) = \frac{\partial V^E(a,x)}{\partial a}$ is the change in the value of employment due to the ageing of the technology.

The flow value of unemployment, V^U, equals the level of unemployment insurance benefits and/or home production, B, plus the expected gain from finding a job

$$\rho V^U = B + \frac{\lambda}{u} \int_{\Omega(a^*)} [V^E(a, x) - V^U] dF(a, x), \tag{4}$$

[5]Under the Nash bargaining solution, every decision is jointly taken and hence privately efficient. Therefore the reservation skills gap is identical for the firm and the worker.

where $F(a,x)$ is the distribution of meetings over a and x and $\Omega(a^*)$ its support, as shown in Figure 2.

The reservation skills gap $\overline{x}(a)$ is the point beyond which a job offer is not accepted by the worker as it does not pay more than his/her outside option

$$\rho V^U = \rho V^E(a, \overline{x}(a)). \tag{5}$$

The flow value for the firm of a filled job that uses a technology of age a at a skills gap x, $V^J(a,x)$, is given by

$$\rho V^J(a,x) = y(a,x) - w(a,x) - \sigma\left[V^J(a,x) - V^V(a)\right] + V_a^J(a,x), \tag{6}$$

where the flow value of an open vacancy, $V^V(a)$, is given by

$$\rho V^V(a) = \frac{2\lambda}{v}\int_0^{\overline{x}(a)} \left[V^J(a,y) - V^V(a)\right] dy + V_a^V(a), \tag{7}$$

where λ/v is the probability that a vacancy meets a worker. Finally, given free entry, firms enter in the market up to the point where the value of a vacancy with a new technology equals the entry fee

$$\begin{aligned} &V^V(0) = I \\ &\Rightarrow v\rho I = 2\lambda \int_0^{x^*} \left[V^J(0,y) - I\right] dy + V_a^V(0). \end{aligned} \tag{8}$$

This means that all expected firm profits in the economy are spent on new technologies.

Finally, firms and workers bargain over the division of the match surplus using Nash bargaining

$$w(a,x) = \text{argmax}_w \left[V^E(a,x) - V^U\right]^\beta \left[V^J(a,x) - V^V(a)\right]^{1-\beta}, \tag{9}$$

where β is the bargaining power of workers. Given that $\frac{\partial V^E}{\partial w} = -\frac{\partial V^J}{\partial w} = 1$ the surplus is divided as

$$\beta\left[V^J(a,x) - V^V(a)\right] = (1-\beta)\left[V^E(a,x) - V^U\right], \tag{10}$$

which determines the wage function, $w(a,x)$.

Distributions $g(a,x)$ denotes the joint distribution of a and x in existing matches, and $f(a,x)$ the distribution in worker-firm meetings in every period. Technology-specific required skills and worker skills are uniformly distributed on the unit circle independently of a, hence for a given a the skills gap distribution in a meeting is uniform as well:

$$f(a,x) = m(a)\cdot 2, \quad Sup_f = \{0 \le a \le a^*\} \times \{0 \le x \le 1/2\}, \tag{11}$$

where $m(a)$ is the age distribution of technologies in meetings. In a meeting, workers accept all offers with $x \le \overline{x}(a)$. Hence, at a given a, matches are also uniformly distributed up to $\overline{x}(a)$:

$$g(a,x) = \widetilde{g}(a)\cdot\frac{1}{\overline{x}(a)}, \quad Sup_g = \{0 \le a \le a^*, 0 \le x \le \overline{x}(a)\}, \tag{12}$$

where $\widetilde{g}(a)$ is the age distribution of matched technologies.

Figure 2 shows schematically the support of $f(a,x)$ and $g(a,x)$. The age of the oldest operating technology is $a^* = \overline{a}(0)$ and $x^* = \overline{x}(0)$ the largest skills gap in existing matches. The intersection of the two supports denotes accepted job offers, while the remaining support of $f(a,x)$ denotes the rejected ones.

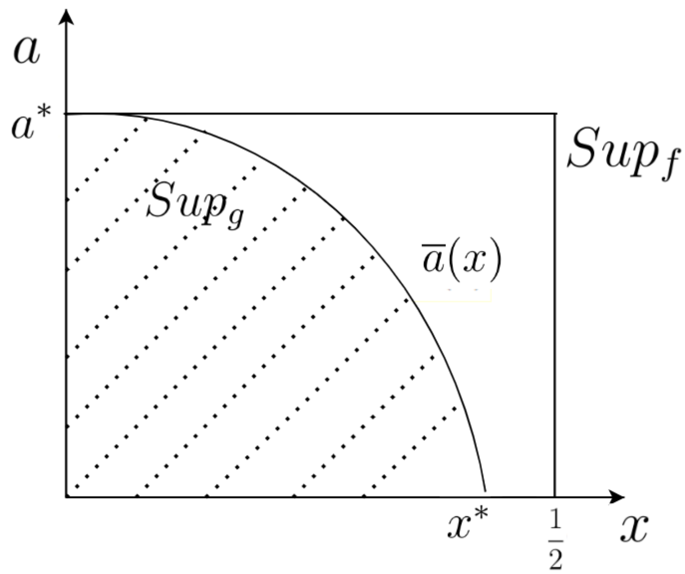

Figure 2: The support of the distributions $f(a,x)$ and $g(a,x)$

$\widetilde{F}(a)$ denotes the share of meetings in each period that lead to a match with a technology age below a, and $\widetilde{G}(a)$ the share of existing matches with technology age below a

$$\begin{aligned} \widetilde{F}(a) &= \int_0^a \left[\int_0^{\overline{x}(b)} f(b,x)dx \right] db, \\ \widetilde{G}(a) &= \int_0^a \left[\int_0^{\overline{x}(b)} g(b,x)dx \right] db. \end{aligned} \tag{13}$$

The corresponding density function of $\widetilde{G}$ is $\widetilde{g}$ and of $\widetilde{F}$ is given by

$$\widetilde{f}(a) = \frac{d\widetilde{F}(a)}{da} = 2m(a)\overline{x}(a) = f(a,x)\overline{x}(a). \tag{14}$$

Inflow-outflow equations There are two inflow-outflow equations that hold along the BGP. First, the age distribution of technologies-in-use must be constant over time. This implies that the amount of new technologies entering in a time period equals the amount being scrapped. Second, the age distribution of matches must be constant over time. Thus, the amount of workers leaving unemployment to start working with technologies of a certain age equals the amount of workers losing such a job.

The inflow-ouflow equation of technologies in the market with age below a reads

$$vm(0) = vm(a) + (1-u)\widetilde{g}(a), \quad 0 < a < a^*. \tag{15}$$

It states that at the BGP the inflow of technologies-in-use with age below a equals the outflow. The left-hand-side denotes that the inflow equals the number of vacancies with $a = 0$ (i.e. the number of new technologies purchased). The first term on the right-hand-side is the rate at which technologies in vacancies age beyond a, and the second term is the outflow rate of matched technologies that age beyond a, as explained in Appendix A.1.2.

Next, the inflow-outflow equation for matches with a technology age below a reads

$$\frac{\lambda}{1-u}\widetilde{f}(a) - \sigma\widetilde{g}(a) - e(a) = \dot{\widetilde{g}}(a) \tag{16}$$

$$\Rightarrow \lambda\widetilde{F}(a) = (1-u)\left[\sigma\widetilde{G}(a) + \widetilde{g}(a) + E(a)\right], \quad 0 < a \leq a^*. \tag{17}$$

The left-hand side is the inflow of matches with a technology age below a, given by the product of total number of meetings and the share of meetings that lead to a match with technology age below a. The right-hand side denotes the outflow due to exogenous destruction, ageing, and endogenous destruction. The endogenous destruction distribution, as derived in Appendix A.1.3, is given by

$$e(a) = -\frac{\widetilde{g}(a)}{\overline{x}(a)}\frac{d\overline{x}(a)}{da}. \tag{18}$$

Balanced Growth Path Equations (5), (8), and (17) characterise the BGP of the model $\{u, v, \overline{a}(x)\}$. In Appendix A.3 we present how we numerically solve these equations and calculate the BGP.

3 Balanced growth path analysis

We numerically solve[6] the system of equations which results in the balanced growth path (BGP) $\{u, v, \bar{a}(x)\}$. The model is calibrated to the U.S. labour market, and we use it to produce comparative statics that describe the behaviour of the BGP of the economy for different values of some important parameters

3.1 Calibration

The calibration of the model requires values for 11 parameters: ρ, δ, ω, η, σ, β, B, I, λ_0, a, and γ. For some of them we use estimates from the literature, while for others we target certain endogenous variables to calibrate them, based on Hornstein et al. (2007) and Gautier and Teulings (2015). Table 1 presents the former, and Table 2 the latter. We calibrate the model to the U.S. labour market and the values presented are on a per-year basis.

[6] The Matlab code is available upon request.

Table 1: Externally calibrated values

Parameter	Description	Value
γ	Specialisation	1.8
ρ	Discounting	0.02
η	Capital-embodied energy efficiency or technological change	0.013
ω	Capital income share in production	0.3
a	Cobb Douglas parameter matching function	0.5

Table 2: Calibrated parameter values

Parameter	Description	Value
λ_0	Matching efficiency	6
δ	Depreciation rate	0.13
β	Worker bargaining power	0.9
σ	Exogenous separation rate	0.03
I	Investment costs	1.9
B	Unemployment benefits replacement rate	0.05

With regards to the chosen parameters in Table 1, the specialisation parameter, γ, is taken from Gautier and Teulings (2015), and is based on the elasticity of complementarity between high- and low-skilled workers estimated by Katz and Murphy (1992). The parameter $\phi = \omega(\eta+\delta)$ is a combination of the capital income share in production, ω, the depreciation rate, δ, and the productivity, or energy efficiency, growth embedded in the capital technology, η. The capital income share, ω, is set to 0.3, in line with standard calibrations of macroeconomic growth models (e.g. Gomme and Lkhagvasuren, 2013). As we interpret η as the rate at which capital energy efficiency grows, it is expected to be smaller than the total capital embodied technology growth used in Hornstein et al. (2007). When considering energy efficiency improvements, the IEA (2021) report finds an annual rate of 1.3%. To achieve a Net Zero climate policy scenario, however, average yearly energy efficiency improvements need to be around 4%. For our main BGP calibration we consider the 1.3% value for the technological progress of available technologies and study the implications when this is increased to 4%.

Figure A.2 presents data from the U.S. Bureau of Economic Analysis on the average age of industry assets over time. It is evident that energy-intensive sector assets are used for a longer amount of time. Thus, we take the average technology age of the energy-intensive sector, $\tilde{a} = \int_0^{a^*} a\tilde{g}(a)da$, to equal 7.5 years, which fixes the depreciation rate, $\delta = 0.13$. Due to the sunk investment costs, to obtain this average technology age in the distribution of operating machines, the investment costs are set such that the realised average age equals

7.5. The other parameters are set to match certain empirical outcomes: a vacancy duration of 4-5 weeks (Andrews et al., 2008; Bassier et al., 2023), an unemployment duration of 4-5 months and an unemployment rate of 6% (see Figure A.1). For β and B we follow Hornstein et al. (2007).

The resulting balanced growth path is presented in Table 3. Here, a^*_{CE} is the competitive equilibrium scrapping age, that is, when there are no labour market frictions and firms can immediately find a worker which they pay the marginal product of labour (see Appendix A.1.4).

Table 3: Resulting BGP for main calibration

u	v	unemployment duration	vacancy duration	$\frac{\lambda}{v}$	a^*	a^*_{CE}	$\int_0^{a^*} a\tilde{g}(a)da$
0.06	0.04	4.6 months	6 weeks	8.1	15.2	11.4	7.5

3.2 Decomposition

In this subsection we study the balanced growth path of the model and uncover the interaction between labour markets and decarbonisation.

Green skills locked in brown jobs First, we investigate the effect of labour market frictions on the BGP. Frictions are quantified by the meeting technology efficiency, λ_0, where a larger value indicates fewer frictions. As λ_0 decreases, it becomes more difficult for firms to find a worker that matches the skill requirements of new technologies. As a result, the expected profits of updating their technology decline, making the continuation of production with the old technology more attractive. As a result, the scrapping age (the maximum age for which a technology is used by firms), a^*, increases. Figure 3 presents the age distribution of matched technologies for three values of λ_0. The $\lambda_0 = \infty$ case corresponds to a perfect competition frictionless market, where new technologies are instantaneously matched and all firms use their old technology until the scrapping age a^*_{CE}.

As frictions increase, firms with new technologies need some time to find a suitable match, indicated by the gradual increase of $\widetilde{g}(a)$ close to zero. At the same time, old technologies whose match is destroyed (either endogenously or exogenously) may never manage to find a suitable match to re-enter the market, indicated by the gradual decrease of $\widetilde{g}(a)$ close to the scrapping age. Workers that have the necessary skills to match with a firm that owns a new, green technology can, due to frictions, be matched with an old, brown technology. Therefore, the extension of the lifetime of old technologies *locks-in* workers with green skills in brown jobs.

Skill mismatch and the speed of the green transition Figure 3 makes evident that the presence of frictions extends the lifetime of old, brown technologies. This is the first part

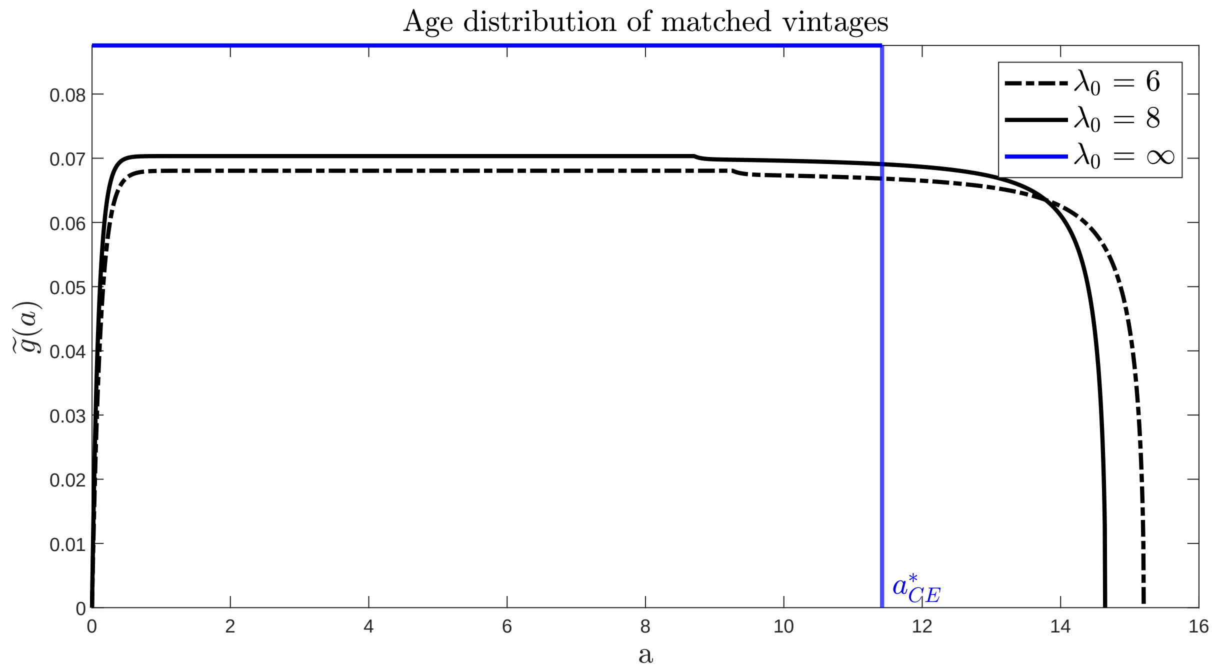

Figure 3: The age distribution of matches along the BGP for various levels of search frictions.

of the two-way relationship we study, between labour markets and decarbonisation: frictions slow down the transition. This effect of frictions can be decomposed into a *search effect*, related to the cost of waiting for a meeting with a worker, and a *matching effect*, related to the cost of not finding a worker that perfectly matches the technology.

Figure 4 shows that the scrapping age of old technologies, a^* declines, i.e. the pace of decarbonisation increases, when the technological progress or energy efficiency improvements η increases. The faster greener technologies become available, the earlier firms are incentivised to update their technologies.

Moreover, in the left panel, Figure 4 presents the decomposition of the scrapping age of technologies in the BGP in three different effects. The sunk investment effect gives rise to the scrapping age that would be observed in a frictionless competitive equilibrium. This is obtained by setting $\lambda_0 \to \infty$. The search effect is estimated by setting $\gamma \to 0$, i.e. its size is given by $a^*(\gamma = 0, \lambda) - a^*(\gamma = 0, \lambda \to \infty)$. Finally, the matching effect is the residual up to the observed scrapping age, i.e. its size is given by $a^*(\gamma, \lambda) - a^*(\gamma = 0, \lambda)$. The total effect of frictions is quantitatively significant: in our calibrated BGP it increases the scrapping age of technologies by roughly 35%. Moreover, roughly 2/3 of this attributed to the presence of imperfect skill sorting.

Finally, the right panel of Figure 4 presents the same decomposition, but for a higher level of specialisation. It shows that if green technologies are more specialised, the matching effect is stronger. This is a result of the fact that more specialisation makes it harder to find a good match for the green technology. This extends the use of the brown, older technologies in production further.

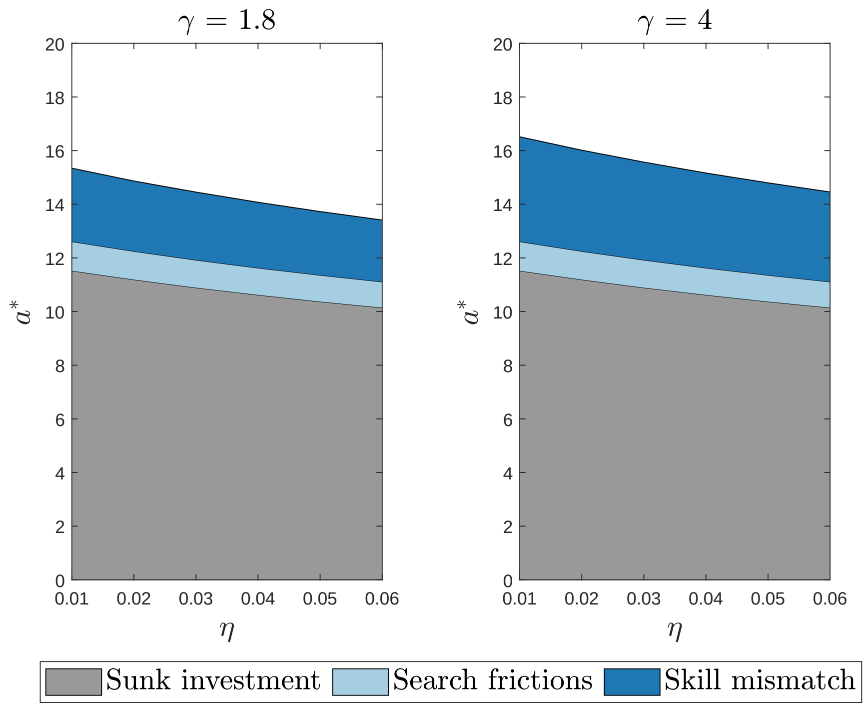

Figure 4: The scrapping age of technologies decomposed into the effects due to sunk investment costs, search, and skill mismatch, for different values of η and γ.

Decarbonisation and labour market transitions Figure 5 presents labour market transitions in the BGP of our model. $\lambda\widetilde{F}(a^*)$ denotes the inflow of workers into employment due to new matches, which in equilibrium equals the outflow, hence, the equilibrium labour market transitions. The x-axis denotes the rate of technological progress or energy efficiency and the y-axis the total labour market transitions. The two dotted lines present different levels of specialisation. The figure shows that a higher level of specialisation, γ,results in more transitions on the BGP. This is due to the fact that a more specialised firm experiences more difficulty to find a good match for its technology. Hence, matches are destroyed endogenously more often as technologies age, and firms look again to hire a different worker. As a result, the unemployment rate also increases.

It also shows that as the productivity, or energy efficiency, growth of available new technologies, η, increases, so do labour market transitions. This is because a higher pace of technological advancement leads firms to update their technology more often, as shown in Figure 4. To do so, they have to search for new workers, as the skill requirements of the new technology are, often, different from those of the old one. As a result, the unemployment rate is higher and the vacancy rate lower. More specifically, reaching from today's growth rate of roughy 1%, to the roughly 4% that is required in order to reach a carbon neutral level according to the set goals, will cause labour market transitions to increase by roughly 5%.

This constitutes the second part of the two-way interaction: an increased pace of the green transition will lead to higher labour market transitions, in order to reallocate existing skilled workers to the jobs that need them.

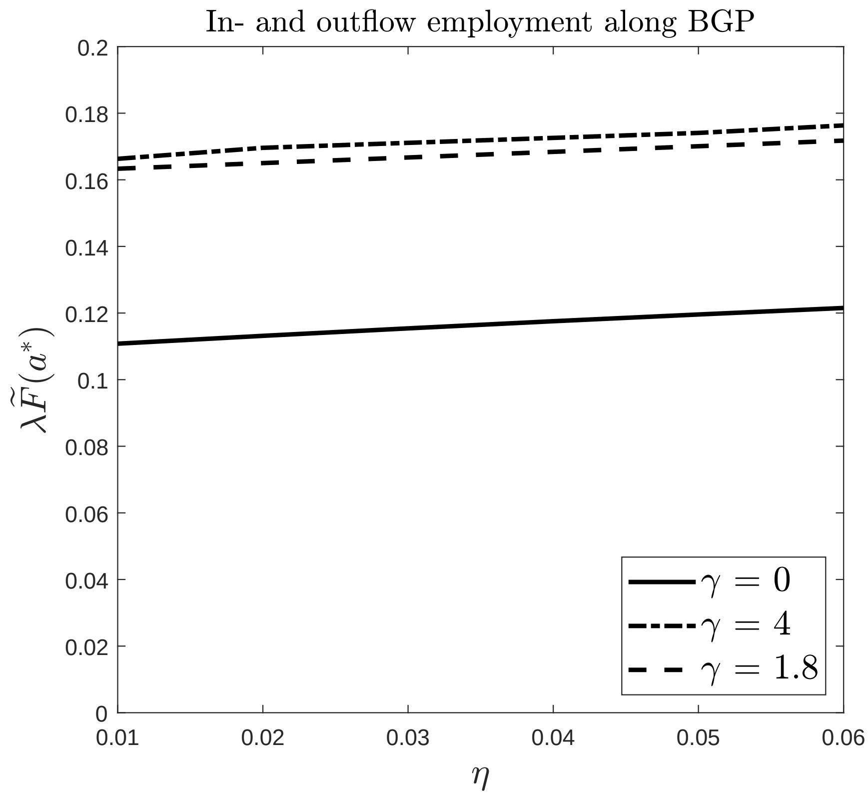

Figure 5: In- and outflow of employment along BGP for various values of η and γ.

3.3 Extensions

In Appendix A.4 we extend the model presented in Section 2 to account for more ways in which green technology adoption can interact with labour market frictions: the option of retaining the worker when updating, aggregate skill shortages and skill-biased technical change. As such, the model can also describe the interaction between technological transformations and skill mismatch in general, and not only for the green transition.

Here, we focus on two extensions that are relevant for the main results of our analysis. First, we consider worker retraining which can directly affect the skill mismatch in production. We represent retraining in the model as a percentage decrease of the skill mismatch, quantified by the retraining parameter ζ

$$y(a, x) = e^{-\phi a}\left[1 - \frac{1}{2}\zeta\gamma x^2\right].$$

This is equivalent to a reduction of the specialisation of jobs by a factor ζ. As seen in Figure 4 and Figure 5, this shrinks the matching effect on the scrapping age and reduces labour market transitions, as the matching process becomes more efficient.

Second, we analyse how our results change in the presence of aggregate skill shortage, on top of imperfect skill sorting. As discussed in Appendix A.4.3, the effect of skill shortage would be calibrated in the absence of labour market frictions. As such, the search and mismatch effects we have unveiled remain unaffected while the sunk cost effect declines in favour of the new shortage effect. As such, our estimations for the role of skill heterogeneity in the speed of the green transition constitute a lower bound, hence our main prediction of a first-order effect due to imperfect skill sorting is robust. Figure 6 is a simplified version of Figure 5 where we focus only on the decomposition of the scrapping age into its three main effects: the sunk investment, search, and mismatch effects. Figure 7 provides a schematic representation of how the decomposition changes in the presence of aggregate skill shortage, based on the discussion in Appendix A.4.3.

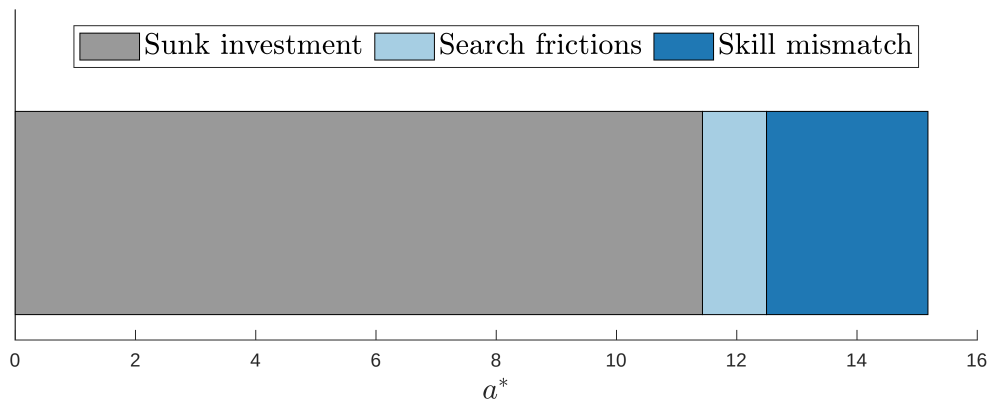

Figure 6: The composition of the effects in the main BGP calibration.

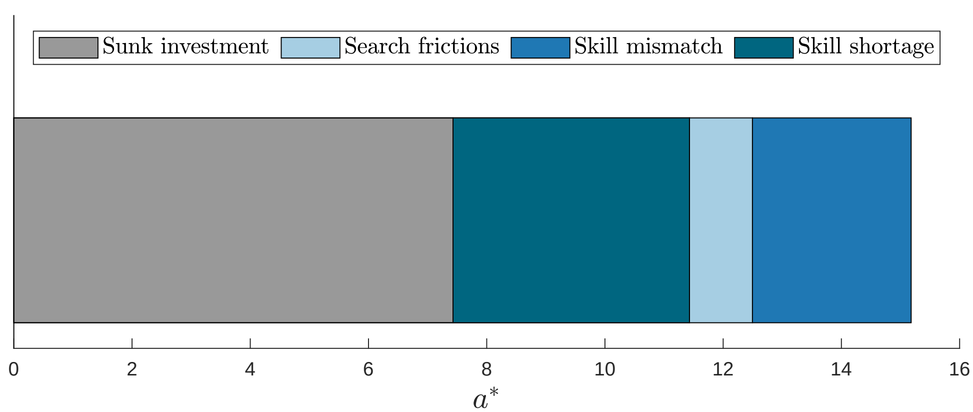

Figure 7: A schematic representation of the decomposition of the scrapping age of technologies in the presence of aggregate skill shortage.

4 Policy discussion

In this section we utilise our framework to investigate how various policies affect the speed of the green transition and the workers affected by it. The latter is crucial in order to minimise the distributional effects of climate policy, which decrease its political acceptability (Vona, 2019), and, thus, reduce their effectiveness and weaken the case for further interventions.

We first discuss the common policy that addresses the sunk investment effect, a capital investment subsidy. Second, we consider a carbon tax, increasing the cost of use of older, more energy-intensive technologies in production. Finally, we consider policy subsidizing the retraining of workers.

Investment subsidy Climate policy over the last two decades has largely focused on directing resources towards subsidising green technologies. This is done to promote their adoption by lowering the relevant investment costs for firms. A subsidy that is an s share of the capital investment cost of the new technology, I, changes the Equation (8), the free entry condition, to

$$V^V(0) = 2\lambda \int_0^{x^*} \left[V^J(0, y) - I\right] dy + V_a^V(0) = (1 - s) * I \tag{19}$$

Table 4 presents the BGP for a capital investment subsidy of 20%. Labour market outcomes are similar to our baseline calibration in Table 3, except for the labour market tightness, which decreases, implying that there are more vacancies available per worker. The vacancy duration therefore increases. As expected, the scraping age of old technologies declines as the sunk investment effect is now smaller.

Table 4: Resulting BGP for main calibration with investment subsidy of $s = 0.2$.

u	v	unemployment duration	vacancy duration	$\frac{\lambda}{v}$	a^*	a^*_{CE}	$\int_0^{a^*} a\tilde{g}(a)da$
0.06	0.04	4.3 months	6.5 weeks	7.3	13.6	9.8	6.7

Thus, we find that subsidies reduce the sunk investment effect on the speed of adoption. The increased amount of vacancies and increased entry, however, can result in a higher energy use in the economy. This follows from the inability of subsidies to price the marginal price of emissions (see Goulder and Parry, 2008). Meanwhile, carbon pricing is the optimal instrument to address the externalities of carbon emissions.

Carbon tax A carbon tax prices the negative externality of CO_2 and other climate change inducing emissions. Within our setup, we model it as a tax on producing with an older technology, which is less energy efficient, and, thus, more polluting. Hence, the profit flow of a firm on which a carbon tax is levied reads

$$y(a, x) - ca. \tag{20}$$

We set $c = 0.02$. This is equivalent to a per technology-age year tax of 2% of the output of a perfectly matched, new technology. The resulting BGP is presented in Table 5, indicating an increased unemployment rate and decreased scrapping age compared to the no carbon tax case in Table 3. The unemployment rate and unemployment duration increase, as does the labour market tightness. Figure A.3 shows that the labour market transitions compared to Figure 5 increase. Fewer vacancies are created than in the case without a carbon tax: the investment costs stay the same, but the return on the investment decreases due to the shorter productivity of the vintage (the additional costs for older machines due to the carbon tax). There are thus large labour market implications of the carbon tax in this setup, taking into account the labour market frictions and imperfect skill sorting.

Table 5: Resulting BGP for main calibration with carbon tax of $c = 0.02$.

u	v	unemployment duration	vacancy duration	$\frac{\lambda}{v}$	a^*	a^*_{CE}	$\int_0^{a^*} a\tilde{g}(a)da$
0.08	0.01	7.5 months	3 weeks	14.6	10.5	8.6	2.8

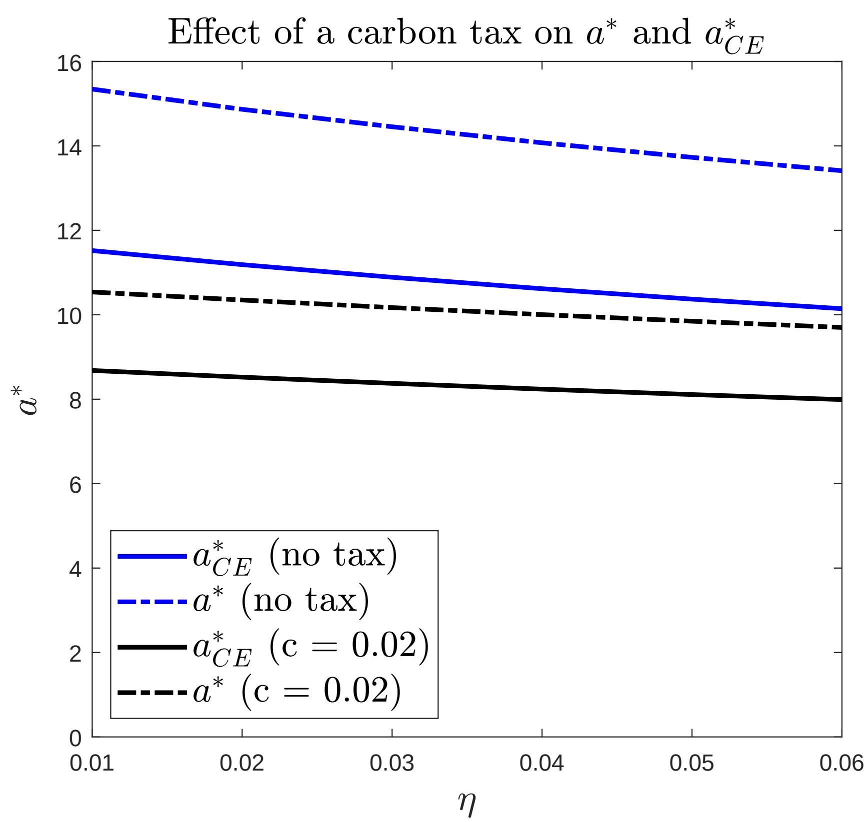

Figure 8: Effect of a carbon tax on the maximum technology age in use, both in competitive equilibrium and frictional labour market BGP.

Figure 8 presents the scrapping age, a^*, along with the frictionless scrapping age, a^*_{CE}, against the technological growth or energy efficiency parameter η. The blue lines, representing the case without a carbon tax, are the same as in the left panel of Figure 4. When a carbon tax is applied, shown by the black lines, both scrapping ages reduce. The carbon tax adds a cost to producing with an older technology, thus incentivising firms to update earlier. It is important to note, though, that the carbon tax acts on the total production output and, thus, does not directly act on the search and matching effects of the scrapping age.

The role of retraining As we discuss in Section 3.3, retraining acts in a way equivalent to a reduction in the specialisation of jobs and therefore reduces the skill mismatch effect. At the same time, as seen in Figure 5 it also reduces labour market transitions, as employers can opt for retraining instead of looking for a new worker when updating their technology.

The latter is important when accounting for another externality present in the green transition: labour market transitions have a personal cost (beyond income loss during unemployment) to fired workers that leads to decreased political acceptability of climate policy (Vona, 2019). This reduces the effectiveness of climate policies and weakens the case for further interventions. To address this concern, many policy initiatives have been developed to address socio-economic inequalities that may arise during the green transition. An example from the U.S. is New York's Clean Climate Careers Initiative, providing funds to train and prepare workers for jobs in the clean energy economy or the Just Transition Fund of the European Commission, allocated to regions in the EU that are expected to be negatively affected by the transition.

As such, the case for policies increasing incentives for retraining (such as retraining subsidies) is twofold: it accelerates green technology adoption by reducing the skill mismatch effect, and it reduces the firings externality. In the absence of an enforced carbon tax the marginal cost for the government of providing capital subsidies in accelerating green technology adoption and reducing emissions might become rather high. As a result, shifting part of the resources towards retraining subsidies instead can results in a more cost-effective policy, in case the reduction of the skill mismatch effect is large enough. At the same time, even in the presence of a carbon tax that forces polluters to internalise the cost of emissions, retraining subsidies have a role in reducing the firings externality. The increase of unemployment, its duration and labour market transitions can be counteracted by retraining and reducing the skills mismatch effect.

5 Conclusion

In this paper we explore the role of skill heterogeneity in the diffusion of green technologies in a labour market with search frictions. In our model, firms decide on investing in new, more energy efficient technologies, embodied in capital. The green transition is thought of as a large technological transformation where new technologies, which require less energy to produce than their predecessors, become available over time. Due to the sunk costs of capital investment, firms do not however immediately update but continue to produce with older technologies. In our model, which builds on Hornstein et al. (2007) and Gautier and Teulings (2015), a second effect arises: skill mismatch (between the worker's skills and the skills required to operate a certain technology) reduces the expected profitability of new, green technologies and as a result old, energy-intensive technologies remain in use for longer.

We show that imperfect skill sorting slows down the adoption of green technologies. We decompose the extended use of older, dirtier machines into three effects. The first, and largest, is due to past technology investments being sunk (sunk investment effect). Firms will only update their technology when the expected gain from that is larger than the sunk cost of their new investment. This effect is well established in the literature. The other effects arise from the need of firms to look for new workers when updating their technologies due to their different skill requirements compared to the old ones. The second effect is due to the cost searching for a worker in labour markets with frictions (search effect). This increases the expected cost of technology updating, prolonging the usage of older technologies. The third effect is due to the cost of having to search for a worker again, in case the first did not have the correct skills to operate the technology of the firm (skill mismatch effect).

The extended use of dirtier technologies implies a higher energy-intensity in production, reducing the ability of economies to reach their emission targets. Therefore, policy makers should consider how the effectiveness of climate policy may be affected by imperfections in the labour market, and include these considerations in ex-ante analyses of such policies.

Using our model, we also investigate various policy instruments. As expected, we find that capital investment subsidies speed-up the updating of dirtier, older machines, by reducing the sunk investment effect. Moreover, we show that carbon taxes are effective in promoting green technology adoption. Both of these instruments, though, do not directly act upon the labour market effects of our decomposition and can even worsen labour market results, for example by inducing more firings.

As a result, our analysis indicates that a suitable policy mix should also include retraining policies. These reduce the firings externality that arises from a faster green technology adoption and, hence, the socio-economic implications of climate policies. Our results therefore support the design of complimentary policies to standard climate policy that promote the retraining of workers, such as the Just Transition Fund of the European Commission, New York's Clean Climate Careers Initative, aand Scotland's Climate Emergency Skills Action Plan 2020-2025.

References

Acemoglu, D. (1998): "Why do new technologies complement skills? Directed technical change and wage inequality," *The Quarterly Journal of Economics*, 113, 1055–1089.

Acemoglu, D. and P. Restrepo (2018): "The race between man and machine: Implications of technology for growth, factor shares, and employment," *American Economic Review*, 108, 1488–1542.

Andrews, M. J., S. Bradley, D. Stott, and R. Upward (2008): "Successful Employer Search? An Empirical Analysis of Vacancy Duration Using Micro Data," *Economica*, 75, 455–480.

Backlund, S., P. Thollander, J. Palm, and M. Ottosson (2012): "Extending the energy efficiency gap," *Energy Policy*, 51, 392–396.

Bassier, I., A. Manning, and B. Petrongolo (2023): "Vacancy Duration and Wages," *IZA Discussion Paper*.

Consoli, D., G. Marin, A. Marzucchi, and F. Vona (2016): "Do green jobs differ from non-green jobs in terms of skills and human capital?" *Research Policy*, 45, 1046–1060.

Duernecker, G. (2014): "Technology adoption, turbulence, and the dynamics of unemployment," *Journal of the European Economic Association*, 12, 724–754.

EIB (2020): *EIB Investment Survey*, European Investment Bank.

Garrett-Peltier, H. (2017): "Green versus brown: Comparing the employment impacts of energy efficiency, renewable energy, and fossil fuels using an input-output model," *Economic Modelling*, 61, 439–447.

Gautier, P. A. and C. N. Teulings (2015): "Sorting and the output loss due to search frictions," *Journal of the European Economic Association*, 13, 1136–1166.

Gautier, P. A., C. N. Teulings, and A. V. Vuuren (2010): "On-the-job search, mismatch and efficiency," *Review of Economic Studies*, 77, 245–272.

Gomme, P. and D. Lkhagvasuren (2013): *Calibration and simulation of DSGE models*, Edward Elgar Publishing.

Goulder, L. H. and I. W. Parry (2008): "Instrument choice in environmental policy," *Review of Environmental Economics and Policy*, 2, 152–174.

Hirst, E. and M. Brown (1990): "Closing the efficiency gap: barriers to the efficient use of energy," *Resources, Conservation and Recycling*, 3, 267–281.

Hornstein, A., P. Krusell, and G. L. Violante (2007): "Technology-policy interaction in frictional labour-markets," *Review of Economic Studies*, 74, 1089–1124.

IEA (2021): *Energy Efficiency 2021*, OECD Publishing.

ILO (2018): *World Employment and Social Outlook 2018: Greening with jobs*, International Labour Organisation (ILO).

Jaffe, A. B. and R. N. Stavins (1994): "The energy paradox and the diffusion of conservation technology," *Resource and Energy Economics*, 16, 91–122.

Katz, L. F. and K. M. Murphy (1992): "Changes in relative wages, 1963–1987: supply and demand factors," *The Quarterly Journal of Economics*, 107, 35–78.

Montt, G., K. S. Wiebe, M. Harsdorff, M. Simas, A. Bonnet, and R. Wood (2018): "Does climate action destroy jobs? An assessment of the employment implications of the 2-degree goal," *International Labour Review*, 157, 519–556.

Mortensen, D. T. and C. A. Pissarides (1994): "Job creation and job destruction in the theory of unemployment," *Review of Economic Studies*, 61, 397–415.

——— (1998): "Technological progress, job creation, and job destruction," *Review of Economic Dynamics*, 1, 733–753.

Mulder, P., H. L. de Groot, and M. W. Hofkes (2003): "Explaining slow diffusion of energy-saving technologies; a vintage model with returns to diversity and learning-by-using," *Resource and Energy Economics*, 25, 105–126.

Rieff, J. and J. Peschner (2019): *Employment and Social Developments in Europe 2019*, European Commission.

Solnørdal, M. T. and L. Foss (2018): "Closing the energy efficiency gap-A systematic review of empirical articles on drivers to energy efficiency in manufacturing firms," *Energies*, 11, 518.

Sorrell, S., J. Schleich, and S. Scott (2006): "The Economics of Energy Efficiency: Barriers to Cost-Effective Investment," *Energy Studies Review*, 14, 186–192.

Tyros, S., D. Andrews, and A. D. Serres (2023): "Doing green things: skills, reallocation, and the green transition," *OECD Economics Department Working Papers*, No. 1763.

Vona, F. (2019): "Job losses and political acceptability of climate policies: why the 'job-killing' argument is so persistent and how to overturn it," *Climate Policy*, 19, 524–532.

Vona, F., G. Marin, D. Consoli, and D. Popp (2018): "Environmental regulation and green skills: An empirical exploration," *Journal of the Association of Environmental and Resource Economists*, 5, 713–753.

A Appendix

A.1 Proofs

A.1.1 Productivity match on balanced growth path

As the skill mismatch is constant over time, the derivation of the growth rate on the balanced growth path and the resulting match productivity is similar:

$$\begin{aligned} y(t,a,x) &= f(x)z(t)k(t,a)^{\omega} \\ &= f(x)z_0e^{\psi t}\left[k_0e^{\eta(t-a)}e^{-\delta a}\right]^{\omega} \end{aligned} \tag{A.1}$$

Rewriting, assuming that $k_0 = 1$ and $z_0 = 1$:

$$y(t,a,x) = e^{(\psi+\eta\omega)t}e^{-(\eta\omega+\delta\omega)a}f(x) \tag{A.2}$$

Thus, setting $g = \psi + \eta\omega$ and $\phi = \omega(\eta + \delta)$:

$$y(t, a, x) = e^{gt} e^{-\phi a} f(x) \tag{A.3}$$

Since we study the balanced growth path, we can divide by e^{gt} and obtain the stationary output, that is equivalent to Equation (2):

$$y(a, x) = e^{-\phi a} f(x) \tag{A.4}$$

A.1.2 Ageing of technologies

For the equilibrium inflow outflow equations, we need to compute the flow of ageing matched technologies beyond a. The infinitesimal flow of ageing technologies over a period dt gives

$$\begin{aligned} d\text{Ageing}(a) &= (1-u)\left[\int_0^{\overline{x}(a)} g(a, y)dy\right] da \\ \Rightarrow \frac{d\text{Ageing}(a)}{dt} &= (1-u)\frac{d\widetilde{G}(a)}{da} = (1-u)\widetilde{g}(a), \end{aligned} \tag{A.5}$$

where, going to the second equality we used that $da/dt = 1$ and the definition of $\widetilde{G}(a)$ in Equation (13). Similarly the rate of ageing vacancy technologies is given by $vm(a)$.

A.1.3 Endogenous destruction of matches

The outflow due to endogenous destruction occurs as technologies get older and thus less productive. The infinitesimal flow over a period dt is, similarly to (A.5), given by

$$\begin{aligned} d\text{Endogenous Destruction}(a) &= -(1-u)\int_0^a g(b, x(b))\frac{d\overline{x}(b)}{db} dbda \\ \Rightarrow E(a) = \frac{d\text{Endogenous Destruction}(a)}{dt}/(1-u) &= -\int_0^a g(b, x(b))\frac{d\overline{x}(b)}{db} db \\ \Rightarrow e(a) = \frac{dE(a)}{da} &= -\frac{\widetilde{g}(a)}{\overline{x}(a)}\frac{d\overline{x}(a)}{da}. \end{aligned} \tag{A.6}$$

A.1.4 Competitive equilibrium

We compare the frictional equilibrium with the benchmark frictionless competitive equilibrium (CE). In the absence of labour market frictions, perfect skill sorting occurs instantaneously, and firms match costlessly with their optimal worker. The profit function of a perfectly competitive firm is given by

$$\pi(w) = \int_0^{a^*_{CE}} e^{-\rho a}\left(e^{-\phi a} - w\right) da. \tag{A.7}$$

Similarly to Hornstein et al. (2007), all workers receive the same wage, $w = e^{-\phi a^*_{CE}}$, equal to the marginal product of labour when using the oldest in-use technology. The competitive equilibrium scrapping age, a^*_{CE}, is such that expected profits from purchasing a new technology equal the investment costs

$$I = \int_0^{a^*_{CE}} e^{-(\rho+\phi)a} \left[1 - e^{-\phi(a^*_{CE}-a)}\right] da. \tag{A.8}$$

A.2 Additional figures

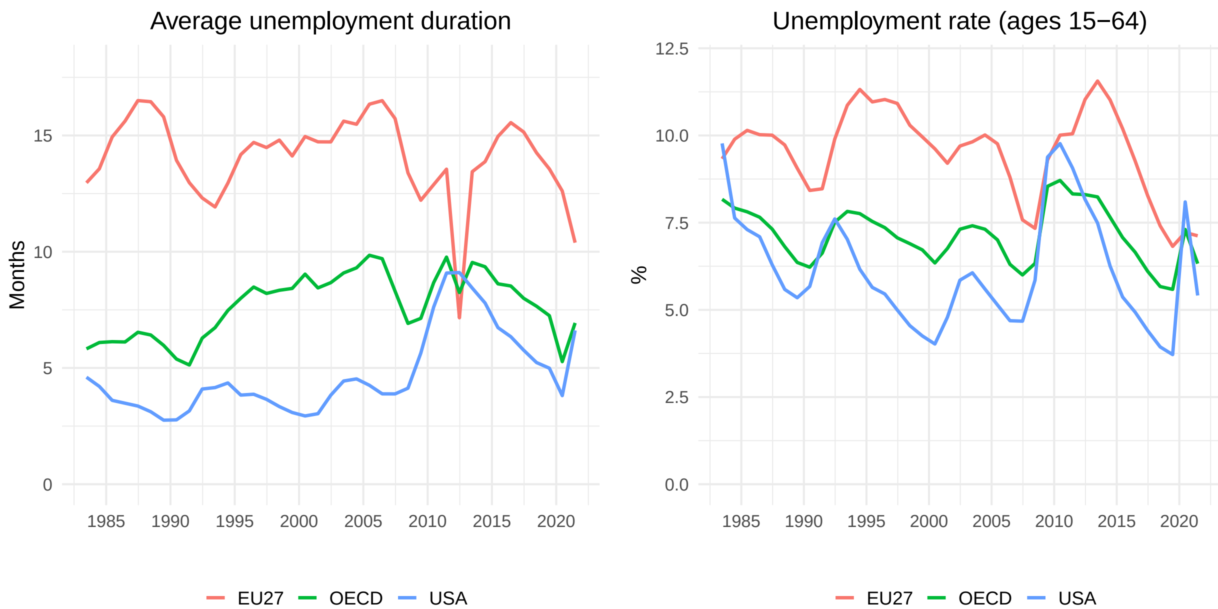

Figure A.1: The average unemployment duration for various groups of countries over time (left) and the unemployment rate for ages 15-64 over time (right), Source: OECD.

A.3 Solving the model numerically

First, from equation (5) we have that

$$\rho V^U = \rho V^E(a, \overline{x}(a)) = w(a, \overline{x}(a)), \tag{A.9}$$

where in going to the last equality equation (3) is used. This defines the reservation skills gap as a function of the technology age, $\overline{x}(a)$, and the reservation wage $\overline{w}(a) = w(a, \overline{x}(a)) = y(a, \overline{x}(a)) = y(\overline{a}(x), x)$.

We, therefore, have that

$$\begin{aligned} &\rho V^U = e^{-\phi \overline{a}(x)} \left[1 - \frac{1}{2}\gamma x^2\right] \\ \Rightarrow\ &\overline{x}(a) = \sqrt{\frac{2}{\gamma}} \left(\sqrt{1 - \rho V^U e^{\phi a}}\right) \quad \text{and} \quad \overline{a}(x) = -\frac{1}{\phi} \ln\left[\frac{\rho V^U}{1 - \frac{1}{2}\gamma x^2}\right]. \end{aligned} \tag{A.10}$$

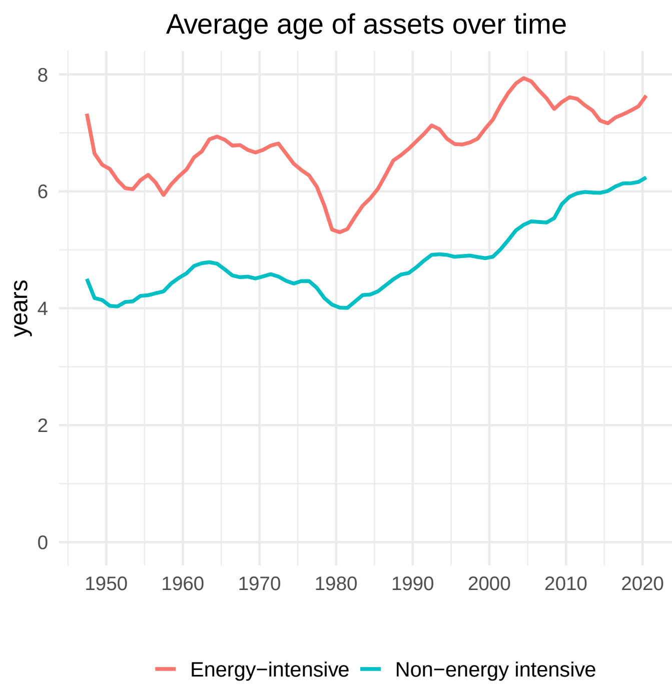

Figure A.2: The average age of assets, split up into non-energy intensive and energy-intensive sectors.

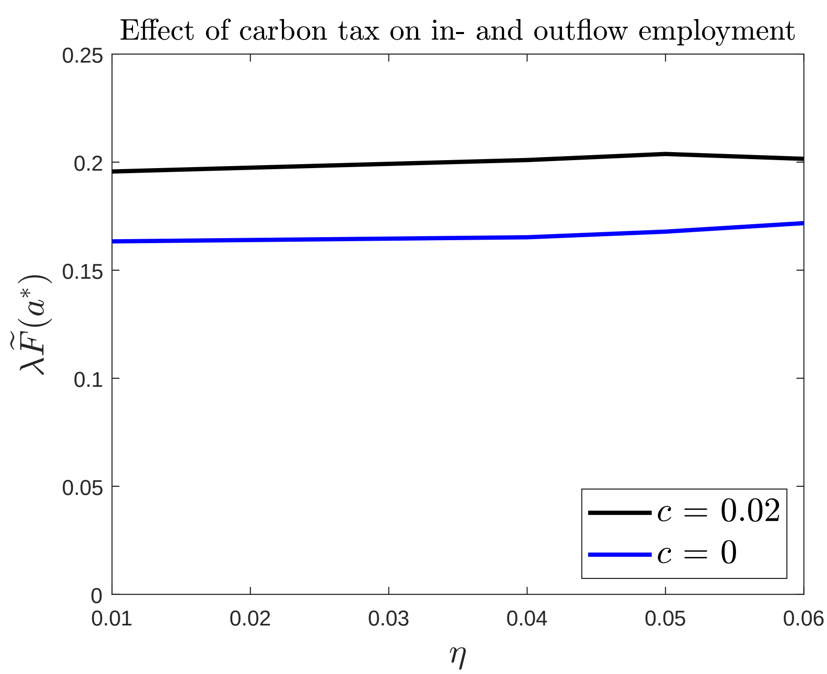

Figure A.3: In- and outflow of employment along BGP for the main calibration and the carbon tax calibration, for various values of η.

If for some a, $\overline{x}(a) > 1/2$, it is replaced by $\overline{x}(a) = 1/2$.

Second, from the free entry equation (8) we have that

$$v\rho I = 2\lambda \int_0^{x^*} \left[V^J(0,y) - I\right] dy + V_a^V(0). \tag{A.11}$$

Next, the value functions can be re-written in terms of the surplus function

$$\begin{aligned} S(a,x) :&= V^J(a,x) + V^E(a,x) - V^V(a) - V^U \\ &= \frac{1}{1-\beta}\left[V^J(a,x) - V^V(a)\right] = \frac{1}{\beta}\left[V^E(a,x) - V^U\right], \end{aligned} \tag{A.12}$$

where the second line follows from Equation (10). Subtracting Equation (7) from Equation (6) and Equation (4) from Equation (3), then adding them, and using Equation (A.12) gives

$$(\rho+\sigma)S(a,x) = y(a,x) - \frac{2\lambda}{v}(1-\beta)\int_0^{\overline{x}(a)} S(a,y)dy + S_a(a,x) - \rho V^U. \tag{A.13}$$

Next, we use the fact that at the oldest technology age in production, the match surplus is zero

$$S(\bar{a}(0), 0) = 0, \tag{A.14}$$

and numerical backward approximation, to obtain a solution for $S(a,x)$. Figure A.4 depicts the numerical approximation of the surplus.

Stationary distributions Evaluating Equation (17) at a^* gives the unemployment rate at the BGP

$$u = 1 - \frac{\lambda \widetilde{F}(a^*)}{\sigma + \widetilde{g}(a^*) + E(a^*)}. \tag{A.15}$$

On the BGP, the distributions are constant. We solve for $\tilde{g}(a)$ by combining Equations 15 and 17

$$\frac{d\widetilde{g}(a)}{da} = -\left[\frac{2\lambda\overline{x}(a)}{v} + \sigma - \frac{1}{\overline{x}(a)}\frac{d\overline{x}(a)}{da}\right]\widetilde{g}(a) + \frac{2\lambda\overline{x}(a)}{1-u}m(0) \tag{A.16}$$

This $\widetilde{g}(a)$ solution is estimated numerically and is used to also obtain $m(a)$, $\widetilde{F}(a)$, and $E(a)$.

A.4 Extensions

In this section we extend the model presented in Section 2 to account for more ways in which green technology adoption can interact with frictions: worker retainment, worker retraining, aggregate skill shortages and skill-biased technical change. As such, the model can also describe the interaction between technological transformations and skill mismatches in general, and not only for the green transition.

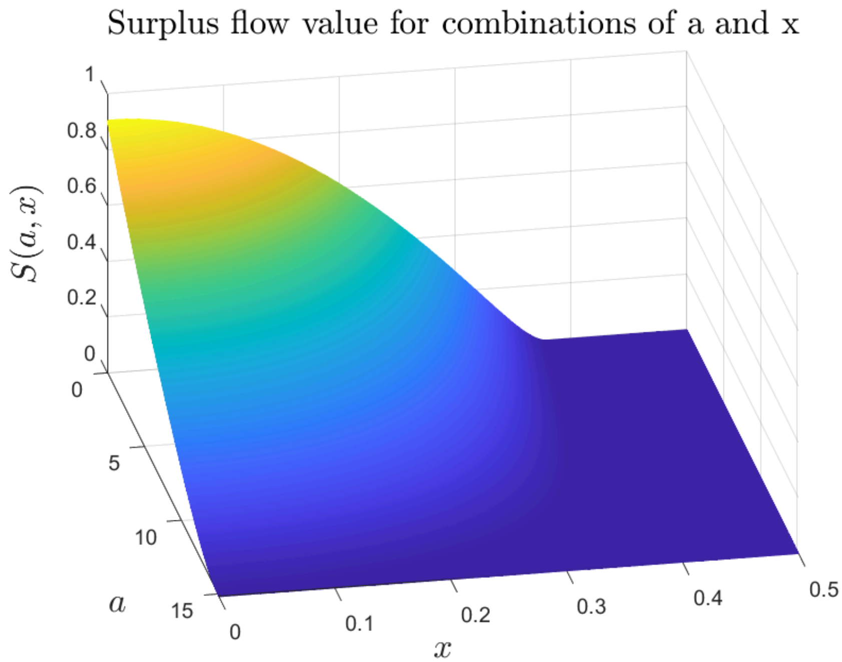

Figure A.4: Numerical solution of the surplus flow value function $S(a, x)$.

A.4.1 Worker Retainment

In the baseline model of Section 2, firms that scrap their technology let their worker go and exit the market. New firms, then, enter the market by investing in the new technology. Thus, a feature of reality that the baseline model does not capture is that firms often retain their workers when investing in new technologies and, sometimes, retrain them to be able to operate it. Hence, in this extension, we allow firms to retain their incumbent worker, observe the skills gap between them and the new technology, and then decide whether they should keep them or let them go and look for a new one.

Timing Given that the retainment of a worker reduces the cost of updating their technology, firms can opt for updating earlier than in our baseline model, namely at age $\widetilde{a}(x)$. The timing of this process for the firm is as follows:

1. Scrap old technology at age $\widetilde{a}(x)$.
2. Invest I for the new available technology.
3. Observe skills gap, x, between retained worker and new technology.
4. If skills gap is not too large, $x < \overline{x}(0) = x^*$, start producing. Otherwise, search for a new worker.

Value functions The value of a firm that has updated its technology but has not retained a worker equals $V_O^F = V^V(0)$. The value of a firm that updated their technology while retaining their worker is given by

$$
\begin{aligned}
V_W^F &= \int_0^{1/2} \max\left\{V^J(x,0), V^V(0)\right\} 2dx \\
&= \int_0^{x^*} V^J(x,0)2dx + (1-2x^*)V^V(0) > V^V(0).
\end{aligned} \tag{A.17}
$$

The first line indicates that the value of such a firm is the expected value of a filled job or a vacancy, depending on the skills gap of the incumbent worker with the new technology. This is larger that the value of a new vacancy, indicating that firms have an incentive to retain their worker.

Given that on the BGP not all firms retain a worker (some scrap their technology while not matched with a worker), there are new entrant firms in the market. Hence, the free entry condition still reads $V^V(0) = I$. Therefore, updating firms that retain their worker have positive expected gains $V_W^F - I > 0$. This means that they will choose to update their technology at an age earlier than a^*, at which the job value is zero, $V^V(a^*) = 0$.[7] Namely, they will update their technology, while retaining a worker, at the age $\widetilde{a}(x)$ when the value of the filled job equals the expected gains from updating

$$
V^J(\widetilde{a}(x), x) = V_W^F - I. \tag{A.18}
$$

Given that updating does not yield additional gains (as the value of the filled job destroyed equals the expected profits) the value function of a filled job and of employment remain unchanged from those in Section 2.

Distributions Given that firms retain workers that have already worked in the firm for a while, we model such updates to occur only for firms that have an existing match.[8] Therefore, new matches are only formed within $\Omega = \{x < \overline{x}(a)\} \cap \{x < \widetilde{x}(a)\}$, i.e. it must be both profitable to produce and to not have the incentive to update the technology straight away.

Figure A.5 presents schematically the area Ω in which matches can exist, where $\widetilde{a} = \widetilde{a}(0)$ is the maximum age of a technology in use. $a^\dagger$ is the age at which updating with retainment starts occurring, defined by $\overline{x}(a^\dagger) = \widetilde{x}(a^\dagger)$.[9]

As in Section 2, $f(a,x)$ and $g(a,x)$ are the distributions of the per-period meetings and the existing worker-firm matches. As meetings are random in this extension as well, we have that

$$
f(a,x) = m(a) \cdot 2, \quad Sup_f = \{0 \le a \le \widetilde{a}\} \times \{0 \le x \le 1/2\}. \tag{A.19}
$$

Similarly, in a meeting, workers still accept all offers below the maximum skills gap. Retained workers are also matched with a new technology with random skills requirements. Hence,

[7] At that point they update their technology because this equals their expected gains from updating, $V^V(a^*) = V^V(0) - I = 0$

[8] I.e. firms with a vacancy cannot meet a worker and, then, directly retain them to update their technology. Intuitively, even though not modelled, firms retain workers also because of firm-specific human capital, which the worker acquires after working for some time in the firm.

[9] Similarly to $\overline{x}(a)$, $\widetilde{x}(a)$ is the inverse of $\widetilde{a}(x)$.

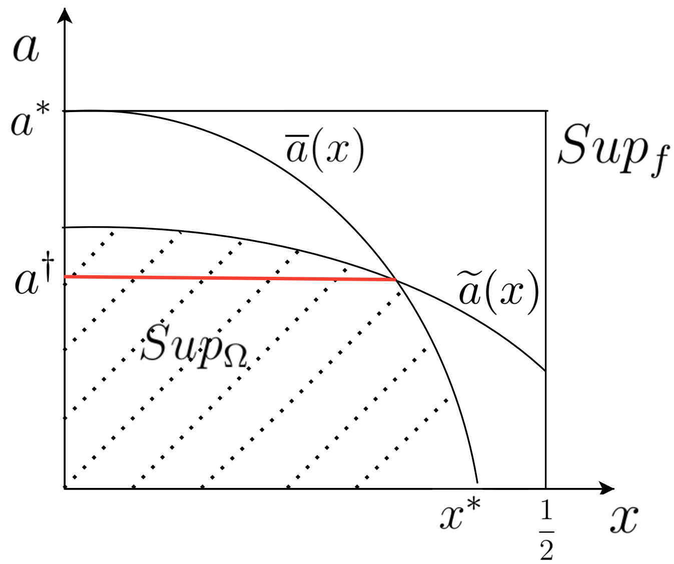

Figure A.5: The area of the $x-a$ phase space that matches can exist.

for a given age the distribution of matches is uniform:

$$g(a,x)=\begin{cases}\widetilde{g}(a)\cdot\frac{1}{\overline{x}(a)}, & \{0\le a\le a^\dagger, 0\le x\le \overline{x}(a)\}\\ \widetilde{g}(a)\cdot\frac{1}{\widetilde{x}(a)}, & \{a^\dagger\le a\le \widetilde{a}, 0\le x\le \widetilde{x}(a)\},\end{cases}\tag{A.20}$$

where $\widetilde{g}(a)$ is the technology age distribution of matches.

As before, $\widetilde{F}(a)$ denotes the share of meetings each period that lead to a match with technology age below a, and $\widetilde{G}(a)$ the share of existing matches with technology age below a

$$\begin{aligned}\widetilde{F}(a)&=\begin{cases}\int_0^a\left[\int_0^{\overline{x}(b)}f(b,x)dx\right]db, & 0\le a\le a^\dagger\\ \widetilde{F}(a^\dagger)+\int_{a^\dagger}^a\left[\int_0^{\widetilde{x}(b)}f(b,x)dx\right]db, & a^\dagger<a\le\widetilde{a}\end{cases},\\ \widetilde{G}(a)&=\begin{cases}\int_0^a\left[\int_0^{\overline{x}(b)}g(b,x)dx\right]db, & 0\le a\le a^\dagger\\ \widetilde{G}(a^\dagger)+\int_{a^\dagger}^a\left[\int_0^{\widetilde{x}(b)}g(b,x)dx\right]db, & a^\dagger<a\le\widetilde{a}\end{cases}.\end{aligned}\tag{A.21}$$

The corresponding density function of $\widetilde{G}$ is $\widetilde{g}$ and of $\widetilde{F}$ is given by

$$\widetilde{f}(a)=\frac{d\widetilde{F}(a)}{da}=\begin{cases}2m(a)\overline{x}(a), & 0\le a<a^\dagger\\ 2m(a)\widetilde{x}(a), & a^\dagger<a<\widetilde{a}\end{cases}.\tag{A.22}$$

Inflow-outflow equations The inflow-ouflow equation of technologies-in-use in the market, with age below a, now reads

$$\begin{aligned}vm(0)+(1-u)\widetilde{g}(0)&=vm(a)+(1-u)\left[\widetilde{g}(a)+OU(a)\right],\quad 0<a<\widetilde{a},\\ \text{where}\quad OU(a)&=\begin{cases}0, & 0\le a<a^\dagger\\ -\int_{a^\dagger}^a\frac{\widetilde{g}(a)}{\widetilde{x}(a)}\frac{d\widetilde{x}(a)}{da}, & a^\dagger<a<\widetilde{a}\end{cases}.\end{aligned}\tag{A.23}$$

This includes two extra terms compared to Equation (15). The first is the second term of the left-hand-side, which denotes that at time zero there are some existing matches, namely the firms that retained their worker, updated their technology, and the worker had the right skills to operate it. The updated technologies where the retained worker did not have the required skills are included in the first term of the left-hand-side.

The second is the last term of the right-hand-side, which denotes the outflow of technologies in use below the age of a due to technology updating. Algebraically, it is similar to the endogenous destruction term in Equation (17), as it is given by the number of firms reaching a boundary line, this time $\widetilde{a}(x)$. Given that the left hand side includes all updated technologies from retained workers, this term denotes those that had a technology in use below a, so that the total inflow of new technologies is only given by those that update and retain at an age above a.

Given that the matches at $a = 0$ come from firms updating their technology, retaining their workers, and realising that the skills gap between the two is not too large, we have that

$$\widetilde{g}(0) = OU(\widetilde{a})2x^*. \tag{A.24}$$

Similarly, the inflow-outflow equation for matches with a technology age below a reads

$$\lambda\widetilde{F}(a) + (1-u)\widetilde{g}(0) = \begin{cases} (1-u)\left[\sigma\widetilde{G}(a) + \widetilde{g}(a) + E(a)\right], & 0 \le a < a^\dagger \\ (1-u)\left[\sigma\widetilde{G}(a) + \widetilde{g}(a) + E(a^\dagger) + OU(a)\right], & a^\dagger < a < \widetilde{a} \end{cases}, \tag{A.25}$$

where, again, the second term of the left-hand-side denotes the inflow of matches due to updating and retaining a worker with the right skills for the new technology. The last term of the right-hand-side is also different compared to Equation (17). For $a < a^\dagger$ there is only outflow of matches due to endogenous destructions, similarly to the baseline model. For $a > a^\dagger$, though, there is outflow due to updating.

A.4.2 Retraining all workers

Worker retraining is a policy that can directly affect the mismatch in production. For simplicity of exposition we consider here the retraining of all workers in new matches. Within our framework, this can be represented as a percent decrease of the skill mismatch, quantified by the retraining parameter ζ

$$y(a, x) = e^{-\phi a}\left[1 - \frac{1}{2}\zeta\gamma x^2\right].$$

This is equivalent to a reduction of the specialization of jobs by a factor ζ. As seen in Figure 4 and Figure 5 , a reduction in the level of specialization shrinks the matching effect on the scrapping age, reducing the frontier distance. At the same time, it reduces labour market transitions, as the matching process becomes more efficient. A more realistic treatment of retraining would consider the retraining only of workers with marginal skills gap around the reservation skill gap. That would also, though, be equivalent to a reduction in the specialization of the job.

A.4.3 Aggregate Skill Shortage & Skill-Biased Technical Change

As discussed in 1, skill sorting is expected to play a major role in the green transition. Aggregate skill shortage, although ignored in the baseline model, can give rise to a larger total mismatch effect. We, thus, generalise our baseline model to be able to describe such a process. The model allows the interpretation of x to include not only gaps in skills per se, but also other mismatches that reduce production, such as spatial mismatch. This extension, therefore can also account for spatial mismatches of skill supply and demand.

In the baseline model skills are uniformly distributed over the unit circle and therefore the skills gap $x \sim U[0, 1/2]$. In the presence of aggregate skill shortage the distributions of workers' skills and firms' skill requirements differ. To capture this, we assume that

$$x \ \sim X_\kappa, \quad E[X_\kappa] > 1/4. \tag{A.26}$$

where κ is a parameter that controls the skewness of the distribution. The requirement $E[X_\kappa] > 1/4$ indicated that the average mismatch is larger than in the uniform baseline case, due to the presence of aggregate skill shortage.

The rest of the equations from Section 2 remain the same, with the only difference that the distributions are given by

$$\begin{aligned} f(a, x) &= m(a) \cdot f_{X_\kappa}(x), \\ g(a, x) &= \widetilde{g}(a) \cdot f_{X_\kappa}(x|x < \overline{x}(a)), \end{aligned} \tag{A.27}$$

where $f_{X_\kappa}(x)$ is the distribution of X_κ and $f_{X_\kappa}(x|x < \overline{x}(a))$ its truncated distribution.

The presence of skill shortage increases the effect of skill mismatches in delaying technology diffusion. It adds one more effect, the shortage effect, to the decomposition in Figure 4, by splitting the sunk cost effect in two parts. That is because our calibration identifies the sunk cost effect as the delay in the technology adoption in the absence of labour market frictions. Skill shortage, though, has an effect independently of frictions as well. Figure A.6 is a schematic representation of the four effects in the presence of aggregate skill shortage. The search and mismatch effect remain the same, whereas the total effect of skill heterogeneity increases when the effect of shortage is also taken into account.

In order to calibrate such a model one needs to separately identify skill shortage and skill sorting. Hence, letting Y be aggregate production, we define them as follows:

- Aggregate skills shortage is the output loss due to mismatch in the absence of labour market frictions: $\lim_{\lambda_0 \to \infty} [Y_\kappa - Y_{\kappa \to 0}]$
- Imperfect Skill Sorting is the output loss due to mismatch in the absence of aggregate skill shortage: $\lim_{\kappa \to 0} [Y_\gamma - Y_{\gamma \to 0}]$.

Finally, allowing the parameter κ to vary with time indicates a scenario where aggregate skill shortage changes over time, meaning that new technologies require more and more different skills than the old ones. Therefore, skill-biased technical change is described by an increasing κ_t.

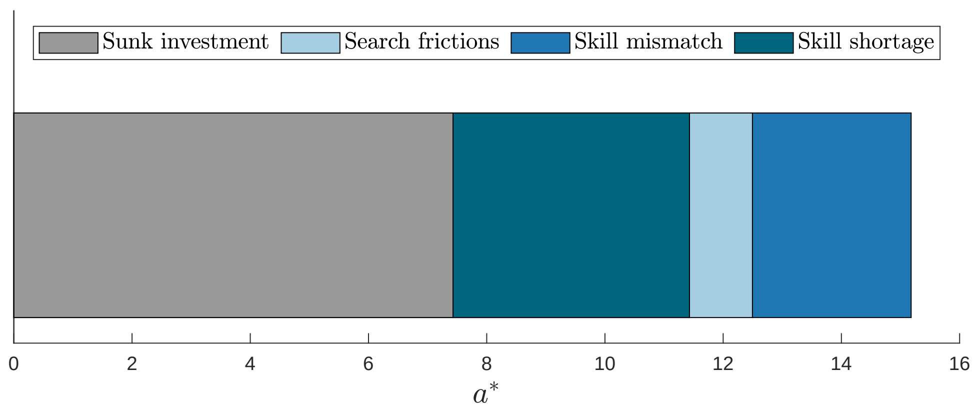

Figure A.6: A schematic representation of the decomposition of the scrapping age of technologies in the presence of aggregate skill shortage.