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Private Road Supply in Networks with Heterogeneous Users

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Abstract

We study different mixes of private and public supply of roads in a network with bottleneck congestion and heterogeneous users. In our setting, there are two parallel links for one origin and destination pair and two groups of travellers, where the group with higher value of time also has higher schedule delay value. Previously scholars have argued that as travellers become more heterogeneous, they benefit more from product differentiation, and thus private supply of roads becomes more efficient. However, we find that local monopoly power might also increase, i.e. the private supplier can increase the toll on his link without worrying that travellers will move to the other link. This can undermine the efficiency of private supply of roads. The problem is especially severe with flat tolls, where travellers in equilibrium tend to travel on separate roads. With queue-eliminating tolls, however, both types tend to travel on both roads in equilibrium, and the competition between road providers remains relatively intense.

Keywords: Congestion Pricing, Bottleneck Model, Heterogeneity, Private Supply

JEL codes: R41, R42, R48

1 Introduction

Private supply of roads is considered by many scholars as a viable complement to public supply of roads. Often mentioned reasons include a lack of public funds in some countries and commonly-believed higher operational efficiency of the private sector. However, the possibility of excessive pricing under market power calls for caution. The tradeoff is further complicated by user heterogeneity, where some people prefer a lower level of congestion at the cost of a higher toll, while others prefer a lower toll despite a higher level of congestion. When travellers' preferences become more heterogeneous, on the one hand, product differentiation offered by different road providers makes travellers better off; on the other hand, different products become less close substitutes, and the increasing local monopoly power of the operator makes travellers worse off. We study the impact of such tradeoffs in various private, public and mixed regimes, using the bottleneck congestion model.

The key assumption of the bottleneck congestion model is that congestion cost comes from time wasted waiting in traffic jams, and the inconvenience of arriving at the destination (usually workplace in the morning commuting setting) earlier or later than desired. People in general differ in their values of time and schedule delay costs, so we incorporate heterogeneity into the analysis. This paper considers two groups of commuters, where one group has both higher value of time and higher schedule delay value than the other. We thus use "proportional heterogeneity", as in Vickrey (1973) and Van den Berg and Verhoef (2011). This heterogeneity could for example stem from differences in income, affecting the marginal utility of income, and therefore all relevant values. In addition, we examine two kinds of tolls: a flat toll and a queue-eliminating toll. A flat toll is the same for all travellers entering the bottleneck at different times, while a queue-eliminating toll varies over time.

Our paper offers several new insights. On the one hand, with flat tolls, a pooling equilibrium, where both types of users travel on both roads, is neither socially nor commercially optimal. As a result, for competing suppliers on parallel links, local monopoly power increases with user heterogeneity and this can lower social welfare. On the other hand, with queue-eliminating tolls, a pooling equilibrium is optimal for both a social planner and for a profit-maximizing operator. Since providers on both roads compete for both types of travellers, competition remains relatively intense, and social welfare increases with user heterogeneity.

Our paper is related to three strands of literature. The first is on the combination of bottleneck congestion and user heterogeneity. Arnott et al. (1992) show that with fixed demand and a flat toll, a pooling equilibrium is better for the society under certain conditions, for example, when one group has both higher value of time and schedule delay value. Van den Berg and Verhoef (2011) assume continuous heterogeneity both in value of time and schedule delay value, and study the distributional effect of changing from flat toll to queue-eliminating toll. We focus on the comparison in social welfare and distributional effect between a variety of public, private and mixed regimes for both flat and queue-eliminating tolls.

The second strand is on private roads with bottleneck congestion and homogeneous users. Arnott et al. (1992) and De Palma and Lindsey (2002) demonstrate that private supply of roads generally enhances social welfare relative to free access when congestion is severe. The efficiency is higher when both routes are tolled with time-varying queue-eliminating tolls. Our study shows that in addition to the usual benefits of queue-eliminating tolls over flat tolls, with heterogeneity, a queue-eliminating toll is more likely than a flat toll to generate a pooling equilibrium and thus promotes competition between private sellers.

The third and final strand of literature is on static congestion and heterogeneous users. Small and Yan (2001) find out that heterogeneity improves the performance of second-best regimes, because product differentiation can better cater for the specific need of each group. Verhoef and Small (2004), by looking at a continuous distribution of heterogeneous users, confirm the benefits of product differentiation in both second-best pricing and revenue-maximizing pricing. Our study shows that heterogeneity also brings a side-effect: the increasing local monopoly power of a road operator can undermine the gains from product differentiation.

The remainder of the paper is organized as follows: section 2 briefly explains the model setup. Section 3 derives, for both tolls, the optimal equilibrium for a welfare-maximizing social planner and a profit-maximizing monopolist operator in a network of two parallel links. For more complex ownership structures on the network, theoretical results are not clear-cut. Hence section 4 shows the simulation results for various ownership structures in the network, namely public, private and mixed supply of roads. Section 5 discusses the main limitations and concludes.

2 Model Setup

We assume there are two groups of travellers, denoted by $i = h, l$, whose group size is N^i. They have a group-specific inverse demand function, denoted by $D^i(N^i)$, which is decreasing in N^i. Two parallel links, denoted by $j = 1, 2$, serve one origin and destination pair. A link has a capacity of s_j.

The bottleneck model assumes that people dislike waiting in traffic jams, and also dislike arriving either early or late for work. If a person's desired arrival time is t^*, but he arrives at work at time t, the congestion cost consists of two parts. The first part is the time costs associated with waiting at the bottleneck, which is the product of value of time, denoted by α, and time spent waiting, denoted by $T(t)$. The second part is the schedule delay costs. For people arriving early, the cost is the product of how early they arrive, measured by $t-t^*$, and the schedule delay value of arriving early, denoted by β. For people arriving late, the schedule delay cost is defined similarly, where γ denotes the schedule delay value of arriving late. In sum, the congestion cost for arriving at time t is:

$$C(t) = \alpha T(t) + \begin{cases} \beta(t^* - t) & \text{if } t \leq t^* \\ \gamma(t - t^*), & \text{if } t > t^* \end{cases}$$

For ease of analysis, we assume the two groups have proportional value of time and schedule delay value, i.e. $\frac{\alpha^h}{\alpha^l} = \frac{\beta^h}{\beta^l} = \frac{\gamma^h}{\gamma^l}$ and $\alpha^h > \alpha^l$. This implies people with higher value of time also has high schedule delay value and vice versa. Given that all values have the marginal utility of income in the denominator, this pattern could result from income differences (Van den Berg and Verhoef (2011)).

3 Theoretical Result

In this section, we study social welfare maximisation and monopolistic profit maximisation. The equilibrium characteristics of these two benchmark regimes carry over to more complex ownership structures, as shown later in the simulation section. We find that with flat tolls a pooling equilibrium, where both types of travellers use both roads, is neither optimal for a social planner nor for a monopolist. But with queue-eliminating tolls a pooling equilibrium is optimal for both types of operators.

In the remainder of this section, we will discus flat tolls and queue-eliminating tolls respectively.

3.1 Flat Toll

A flat toll is constant over time. When the two types of travellers with proportional heterogeneity use the same link, they travel together in time, and cannot be distinguished by the time they enter. The reason is that with a flat toll, the equilibrium growth rates of the queueing time of group i are, $\frac{\beta^i}{\alpha^i}$ for early arrivals and $-\frac{\gamma^i}{\alpha^i}$ for late ones, and under the assumption of proportional heterogeneity, these ratios are the same for both types of users. There is, therefor, no temporal separation between travellers if they use the same link.

We show in Appendix I that the maximal social welfare can not be achieved by a pooling equilibrium, where both types use both links. We prove by contradiction: from the optimal pooling equilibrium, social welfare can be further increased in two steps. First, move an h type user from link 2 to link 1, and at the same time, move an l type user from link 1 to link 2. Social welfare does not change due to this switch, because at the optimal pooling equilibrium, the traffic flow capacity ratio on both roads are the same. Next remove one marginal h type from link 1 and keep her "home", and add one marginal l type to link 2, then social welfare will increase. This is because the marginal social congestion cost of an h type user on link 1 now exceeds the generalized price from the pooling equilibrium, due to more h types on link 1, while the opposite is true for an l type user on link 2.

The same reasoning can also be applied to a monopolist, who charges link-group-specific tolls on both links and maximises the joint profit. Appendix I shows that a pooling equilibrium is again not optimal.

The result, that with flat tolls a pooling equilibrium is neither socially nor commercially optimal, depends crucially on the assumption of proportional heterogeneity. For a more general heterogeneity structure, Arnott et al. (1992) derive conditions on the heterogeneity structure for a pooling equilibrium to be socially optimal.

3.2 Queue-Eliminating Tolls

A queue-eliminating toll increases with time before the desired time of arrival, and decreases with time after that. The time varying part of the toll is set so as to eliminate queueing, and the initial level of the toll (i.e. a flat component paid by all drivers on top of the queue-eliminating time-varying toll component) can be set by the road operator. If both groups use the same link, h types travel at the center of the traffic hour, while l types travel at the tails under time-varying tolling. The reason is that the toll rises at a rate β^i for early arrivals, and falls at a rate γ^i for late arrivals. A type h driver would find it attractive to move towards t^* at any moment where type l driver are in equilibrium. Reversely, a type l driver would move away from t^* when travelling with type h drivers. Hence, temporal separation will result under time-varying tolling.

Appendix II proves that a pooling equilibrium with a zero initial toll is the global optimum for social welfare maximization. This is because under a queue-eliminating toll with zero initial toll, the generalized price for a traveler equals the marginal social cost.

For a monopolist seller, Appendix II shows the same logic applies and the pooling equilibrium with a group-specific initial toll is profit maximizing, due to a monopoly markup based on group-specific demand elasticity.

The result, that with queue-eliminating tolls a pooling equilibrium is optimal for both the social planner and the monopolist, does not depend on the assumption of proportional cost parameters. The logic can be applied to a general heterogeneity structure, and it stems from the notion that with eliminated queues, the best ordering of travellers is such that those with the highest schedule delay value travel closest to the preferred arrival time on both bottlenecks.

In sum, for two parallel links and social welfare/monopolistic profit maximization, flat tolls and queue-eliminating tolls have different equilibrium characteristics, i.e. separating versus pooling. We will show in the next section that those differences remain for more complex ownership structure on the network. In addition, these differences lead to different market structure and performance, which are interesting for both policy makers and practitioners. For more complex ownership structure on the network, both pooling and separating equilibrium can be optimal for different operators. When different road operator interact with each other, sometimes Nash equilibrium does not exist and the results depend on the parameters used. So we will use simulation to examine more complex ownership structure on the network in the next section.

4 Simulation

To examine and compare more complex ownership structure on the network, such as public, private and mixed regimes, we use simulation. We consider the following seven regimes: free, public, monopoly, free-public, free-private, private-public and private-private. In the first three cases, one operator controls both links. The free regime, where both roads are free of tolling, is likely to be the status quo and a good starting point for comparison. The public regime aims to achieve the highest possible social welfare, and the efficiency of all other regimes can be evaluated against it. The monopoly regime aims to maximize joint profits, and can provide useful information for private supply of roads. However, in reality often only part of the network can be tolled, so it is important to also study the next two cases, where link 1 is left free of charge, but an operator on link 2 can charge a toll. We distinguish between a private and a public provider on link 2, because they have significantly different implications for travellers. Finally we analyse two competitive regimes, where the two roads are tolled by different operators, because competition is usually believed to increase efficiency. With competitive regimes, a private operator sets a toll on link 1, and either a public or private operator offers a competing service on link 2.

The parameters we use are: $\alpha^h = 10.59$, $\beta^h = 6.45$, $\gamma^h = 25.16$, and $\frac{\alpha^l}{\alpha^h} = 0.9$. In this way we make sure that the ratios between the cost parameters are as in Small (1982), and the average value of time in the free regime is 10. The inverse demand function is linear, and is constructed to ensure that in the free regime, the price elasticity of demand is -0.4 for each group, and the number of users is 1200 for h types and 1500 for l types. The capacity of link 1 is set as 4000 and that of link 2 is 8000.

In the simulation, we only consider link-specific tolls, τ_j, not link-group-specific toll. This is because when we cannot tell the groups apart, there can be an incentive compatibility problem of a link-group-specific toll, where a member of one group can pretend to belong to the other group. A link-specific toll is also easier to implement in real life than a link-group-specific toll, due to costly verification for user types, if this were technically possible.

4.1 Simulation: Flat Toll

4.1.1 Flat Toll: Base Equilibrium

Table 1 shows the equilibrium values of the key factors in all seven regimes. Similar to the theoretical results, we observe mostly separating equilibria, where at least one of the two links is only used by one type of travellers.

Table 1: Flat Toll

Pricing Regime	Free	Public	Monopoly	Free-Pub	Free-Priv	Priv-Pub	Priv-Priv
τ_1	0	8.736	19.216	0	0	10.389	10.903
τ_2	0	8.369	18.964	3.528	11.030	9.324	12.713
N_1^h	4000	6805	4370	0	818	6165	0
N_2^h	8000	2737	2117	11367	9000	3031	8520
N_1^l	5000	0	0	10493	12272	0	7189
N_2^l	10000	11444	7015	3512	0	10958	2727
Π_1	0	59446	83967	0	0	64046	78388
Π_2	0	118682	173175	52488	99265	130429	142976
CS^h	173321	109583	50650	155512	116025	101794	87367
CS^l	194986	113501	42642	169975	130528	104051	85216
SW	368307	401213	350434	377975	345818	400321	393947
ω	0	0.218	-0.118	0.064	-0.149	0.212	0.170

Toll(τ_j), traffic flow(N_j^i), seller's profit(Π_j), consumer surplus(CS^i), social welfare(SW) and efficiency($\omega = \frac{SW - SW^{Free}}{SW^{FirstBest} - SW^{Free}}$).

In the first three cases, at most one operator charges tolls on both roads. When both roads are free of charge, a pooling equilibrium results both in the temporal sense as between the roads, since both groups require the same growth rate of the queue to be in equilibrium, and hence have no group-specific preferences of one used arrival moment over the other. There is therefore actually a continuum of equilibria at the group-route level, as long as the ratio of aggregate traffic flow to capacity is the same on both roads.

When a social planner charges link-specific tolls on both roads, social welfare increases by 10% compared to the free regime. Due to the higher price and the lower congestion on link 1, the h types travel on both links, but the l types only travel on link 2. Consumer surplus decreases, but the increase in toll revenue more than compensates it, so that social welfare increases. When a private operator sets tolls to maximise the joint toll revenue of both roads, the tolls are more than twice those in the public regime. A sharp drop in consumer surplus leads to social welfare that is even lower than in the free regime.

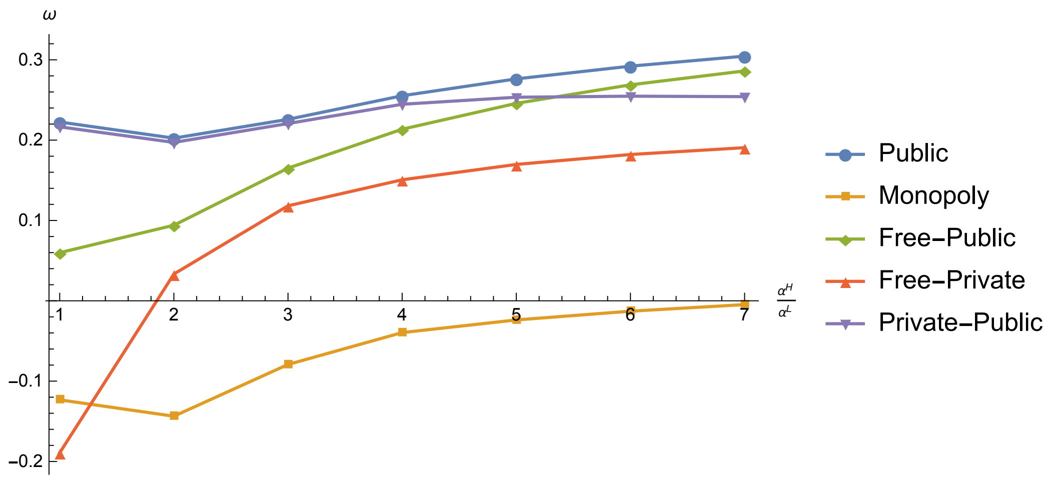
In the next two cases, link 1 is free but an operator can charge a toll on link 2. The increase in social welfare compared to the free regime is moderate, because only one third of the total road capacity is tolled. The consumer surplus is the second highest among all regimes. On the contrary, a private provider on link 2 decreases social welfare compared to the free regime. It is even worse than the monopoly regime, because the monopolist of both roads wants to reduce congestion on link 1 in order to raise total revenue, while the private operator on link 2 only cares about profit on link 2 and causes too much congestion on link 1, a result that was also found in a static single-type setting (Verhoef et al. (1996)).

The last two cases show the effect of competition between two operators. In all cases we assume Nash behaviour between operators. When a public operator on link 2 competes with a private operator on link 1, the efficiency and consumer surplus are nearly as high as in the public regime. When two private operators compete, the value for ω shows that we can achieve 17% of the welfare gain we get from moving from the free regime to the first best regime (public with queue-eliminating tolls). The private-private regime has even higher efficiency than the free-public regime, because the higher tolls greatly reduce congestion. On the other hand, the consumer surplus in the private-private regime is much less than in the free-public regime.

4.1.2 Flat Toll: Heterogeneity and Efficiency

When the two groups become more different in terms of value of time and schedule delay value -in other words, as the degree of heterogeneity(defined as $\frac{\alpha^h}{\alpha^l}$) increases- the efficiencies of all regimes change[1]. Figure 1 shows the result.

Figure 1: Flat Toll: Heterogeneity and Efficiency



Efficiency(ω), Heterogeneity($\frac{\alpha^H}{\alpha^L}$)

The free regime has an efficiency of zero. The competing private-private regime does not have a Nash equilibrium for some parameters, due to a discontinuity in the pay-off function that arises when a marginal change in the toll lead to a switch between a fully separated equilibrium (each group uses one bottleneck exclusively) and a partially separated equilibrium (one groups uses both bottlenecks, one group uses only one of the two bottlenecks).

[1]To make the results comparable when changing cost parameters, we also change the intercept of the inverse demand function, but keep the slope and the number of travellers at the free regime unchanged.

So theses two regimes are omitted (Appendix III). [2]. Figure 1 shows that for the remaining five regimes, the efficiency generally increases with the degree of heterogeneity. This is because, as argued by Small and Yan (2001), when the two groups' values of time and schedule delay value become more different, product differentiation is more appreciated by both groups. In other words, h types strictly prefer the link with a higher toll and lower congestion level, while l types strictly prefer the link with a lower toll and higher congestion level. This increases the efficiency of free-private regime. However, we also find that the efficiency of private-public regime decreases over a certain range with the degree of heterogeneity, mainly because of increasing local monopoly power when tastes become more diverse. More specifically, as the groups become more different, the optimal toll on link 2 has to be very high to correct for the congestion externality of the h group. This leaves the l group traveling on link 1 to the monopoly power of the private suppler. In addition, the efficiency does not change monotonically with the degree of heterogeneity, because there can be regime changes. When $\frac{\alpha^h}{\alpha^l} = 1$, both types are the same, so a pooling equilibrium results for all regimes. But as $\frac{\alpha^h}{\alpha^l}$ increases, separating equilibrium starts to emerge, hence the kinks. For

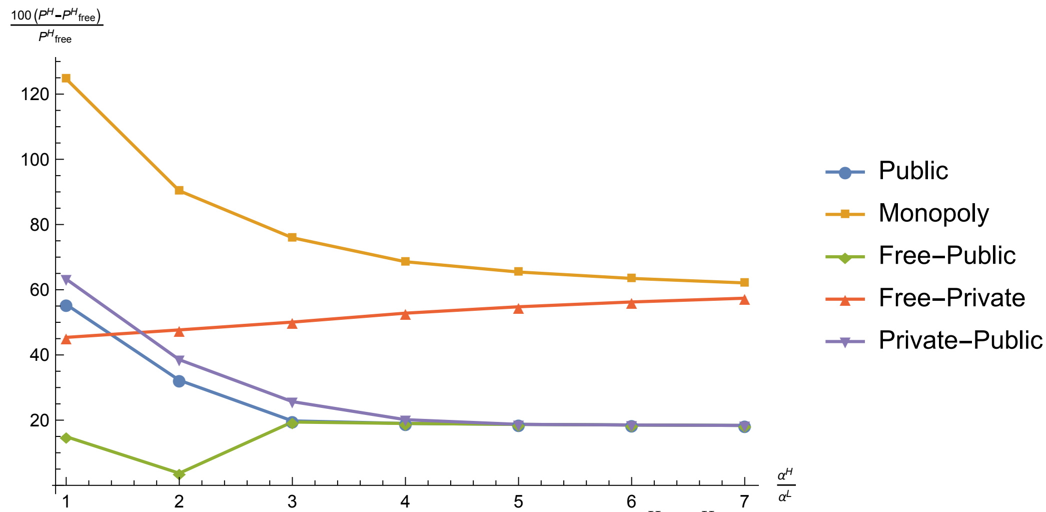
[2] If a Nash equilibrium exists, its efficiency is bounded above by the private-public case.

the monopoly case, a regime change happens again at $\frac{\alpha^h}{\alpha^l} = 1.83$, where the most profitable equilibrium changes from partial separating to full separating (see Appendix III). As a result, efficiency drops at that point because full separating equilibrium means more local monopoly power. Similar is true for public and private-public regimes. The free-public and free-private regime do not show efficiency decrease, because the free road offers relatively strong competition.

4.1.3 Flat Toll: Heterogeneity and Distributional Effects

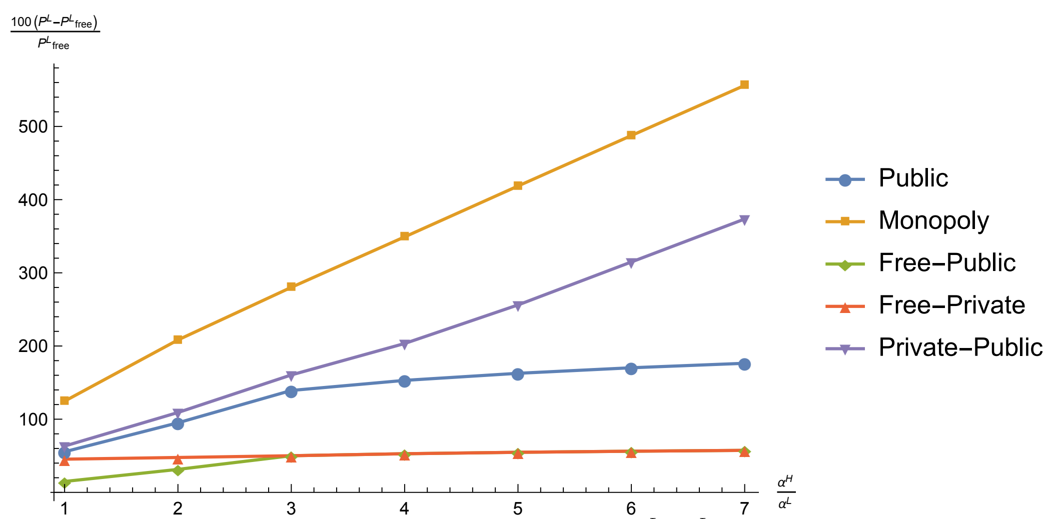
Apart from social welfare concerns, policy makers and practitioners are also interested in the distributional effects of pricing, not in the least place because it has a strong impact on the social and political feasibility. A project is likely to meet resistance from travellers if they are made worse off. The percentage change in generalized price for both types are summarized in Figure 2 and Figure 3 respectively. A higher generalized price means less travellers are willing to use the road, and the consumer surplus drops.

Figure 2: Flat Toll: Heterogeneity and Percentage Price Change(H Types)



Percentage Change in Generalized Price $\left(\frac{100(P^H - P^H_{free})}{P^H_{free}}\right)$, Heterogeneity $\left(\frac{\alpha^H}{\alpha^L}\right)$

Figure 3: Flat Toll: Heterogeneity and Percentage Price Change(L Types)



Percentage Change in Generalized Price $\left(\frac{100(P^L - P^L_{free})}{P^L_{free}}\right)$, Heterogeneity $\left(\frac{\alpha^H}{\alpha^L}\right)$

First of all, for both types the percentage changes in generalized price are positive, which means a higher price with flat toll for all degrees of heterogeneity, so all projects are likely to meet resistance from travellers (unless the allocation of toll revenues convinces them otherwise). Secondly, as the degree of heterogeneity increases($\frac{\alpha^h}{\alpha^l} > 3$ in the both figures), each link becomes more specialized in serving only one group of travellers: h types travel on link 2 only and l types travel on link 1 only. That's why the price for h types is the same for the public, free-public and private-public regimes, and that for l types is the same for the free-public and free-private regimes. As a result, the percentage price change for h types increases with the degree of heterogeneity in the free-private regime, because the private operator on link 2 has increasingly more market power over h types. Similarly, the percentage price change for l types increases most significantly with monopoly and private-public regimes. Lastly, the percentage change in generalized prices are mostly decreasing for h types, but increasing for l types. This is because the prices in free regime(P^i_{free}) increases for h types but decreases for l types when the degree of heterogeneity increases. We observe some a kink for h types in the free-public regime, because the increase in price in the free-public regime is smaller compared with that in the free regime between regime changes for h types.

In sum, for the parameters used in the simulation for flat tolls, monopoly performs consistently worse than the free regime, while the free-private regime performs better than the free regime when the degree of heterogeneity is sufficiently high, due to the benefit of product differentiation. Private-public is better than free-regime, because of competition between the two links. However, the competition effect can be undermined by local monopoly power when the two types become so different that they travel on separate links. If the most efficient public regime is not available, the free-public regime is the best in terms of social welfare when heterogeneity is large and private-public regime is the best when heterogeneity is small. Finally, the generalized price for both types are lowest with the free regime.

4.2 Simulation: Queue Eliminating Toll

4.2.1 Queue Eliminating Toll: Base Equilibrium

We now turn to the case where there is queue-eliminating, time-varying tolling on both roads. Table 1 shows the equilibrium values of the key variables. In line with the theoretical discussion, we observe mostly pooling equilibria, where both groups travel on both links. The free-public and free-private regimes have separating equilibrium, because link 1 is free and does not have a queue-eliminating toll component.

Table 2: Queue-Eliminating Toll

	Free	Public	Monopoly	Free-Pub	Free-Priv	Priv-Pub	Priv-Priv
τ_1	0	0	15.924	0	0	2.832	3.550
τ_2	0	0	15.924	-3.989	9.706	1.324	7.100
n_1^h	4000	4077	2507	0	0	3897	3494
n_2^h	8000	8155	5014	12919	10080	7794	6988
n_1^l	5000	4986	2766	7368	12600	3862	6209
n_2^l	10000	9972	5532	8721	0	10332	6274
Π_1	0	48525	100425	0	0	57724	89624
Π_2	0	97049	200850	54308	130448	120869	146526
CS^h	173321	180092	68075	200885	122295	164528	132230
CS^l	194986	193890	59665	224304	137582	174588	135039
SW	368307	519555	429015	479497	390325	517708	503419
ω	0	1	0.401	0.735	0.146	0.988	0.893

Toll (τ_j), traffic flow (N_j^i), seller's profit (Π_j), consumer surplus (CS^i), social welfare (SW) and the efficiency measure ($\omega = \frac{SW - SW^{Free}}{SW^{FirstBest} - SW^{Free}}$).

As a benchmark, the free regime is the same as with the flat toll. If both roads are tolled by a social planner, the increase in social welfare is 41%. This is because first queueing is eliminated, and second the users with higher schedule delay value now travel closer to the desired arrival time. The consumer surplus is higher with queue-eliminating toll. If both roads are tolled by a monopolist, the social welfare is still higher than in the free regime, but consumer surplus naturally drops due to the private monopolist's pricing behaviour.

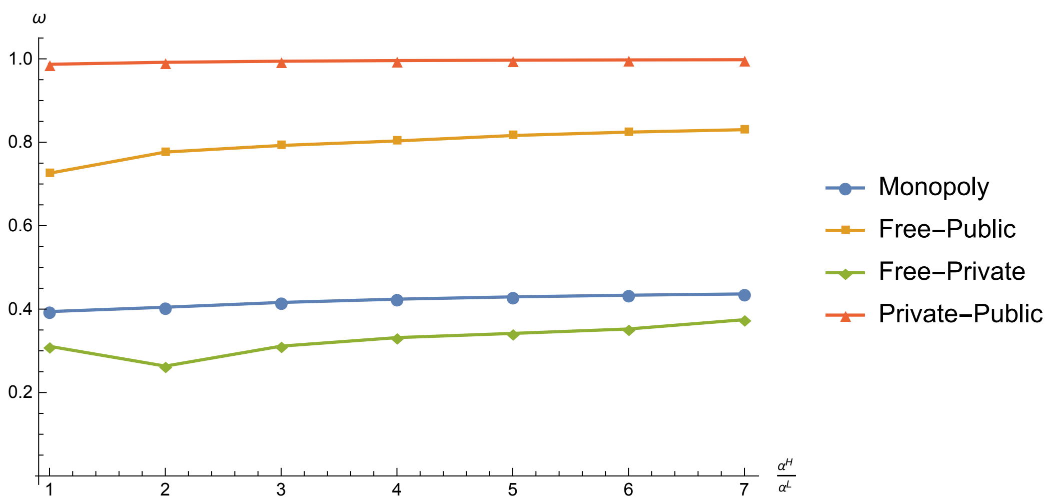
The next two regimes concerns cases where link 1 is free. When a public supplier on link 2 hands out a fixed time-independent subsidy to attract travellers (Braid (1996)), the consumer surpluses of both groups are higher, and the public supplier earns a positive return, so it is a Pareto improvement compared to the free regime. When a private supplier charges a toll on link 2, the increase in social welfare is still positive but much smaller.

Both regimes with competition again perform quite well. The private-public and private-private regimes have a pooling equilibrium. Similar to the case of flat tolls, the social welfare of the private-private regime is higher than that of free-public regime, because congestion is greatly reduced due to having less travellers.

4.2.2 Queue Eliminating Toll: Heterogeneity and Efficiency

As the degree of heterogeneity increases, the ranking of the regimes in terms of social welfare remains the same, as is shown by Figure 4.

Figure 4: Queue-Eliminating Toll: Heterogeneity and Efficiency



Efficiency (ω), Heterogeneity ($\frac{\alpha^H}{\alpha^L}$)

The free and public regimes have an efficiency of zero and one respectively, and the competing private-private regime does not have a Nash equilibrium for some parameters for the same reason as for flat tolls (see Appendix III), so these are omitted[3]. For the remaining four regimes, the efficiency of queue-eliminating tolling increases with the degree of heterogeneity, because product differentiation is appreciated more by a more diverse population of trav-

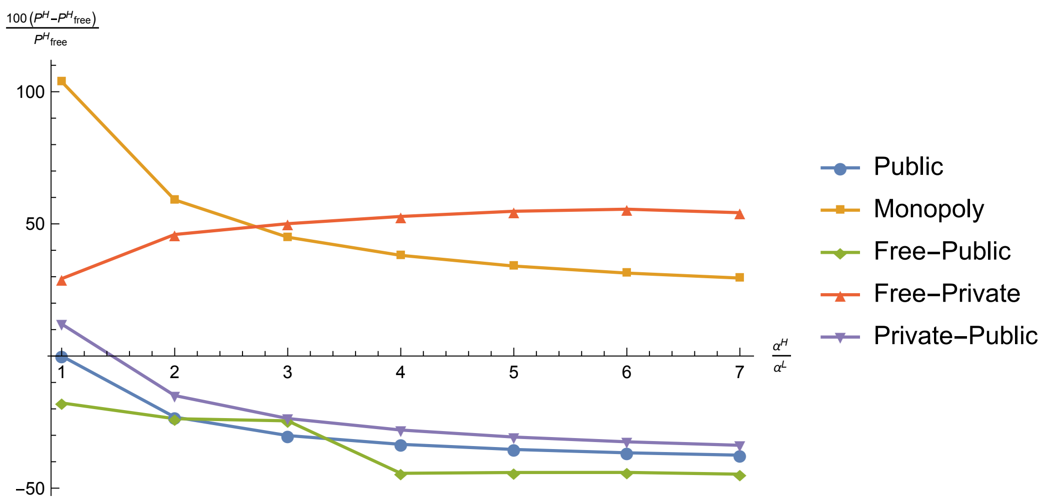
[3] If a Nash equilibrium exists, its efficiency is bounded above by the private-public case.

ellers. The slopes appear somewhat flatter for private-public and monopoly regimes and steeper for free-public and free-private regimes. This is because the first two regimes have queue-eliminating toll on both roads and the degree of heterogeneity matters less. Unlike with flat tolls, both types travel on both links with queue-eliminating toll in the private-public regime, so there is enough competition between the two links to prevent the efficiency from dropping. The free-private regime is less efficient than monopoly, because the monopoly operator charges a much more efficient time-varying toll on link 1 (that remains untolled under free-private). We still observe a kink for the free-private regime, because the equilibrium changes from pooling when the two groups are the same to separating when the degree of heterogeneity increases, and the local monopoly power of link 2 increases. Overall, it seems that the relative efficiency from different pricing regimes is rather robust, and independent of the degree of heterogeneity. This has to do with the fact that time-varying pricing always eliminate all delays on the priced roads. The remaining inefficiency due to mark-up pricing depends on demand elasticities, is therefore largely independent on the degree of heterogeneity.

4.2.3 Queue Eliminating Toll: Heterogeneity and Distributional Effects

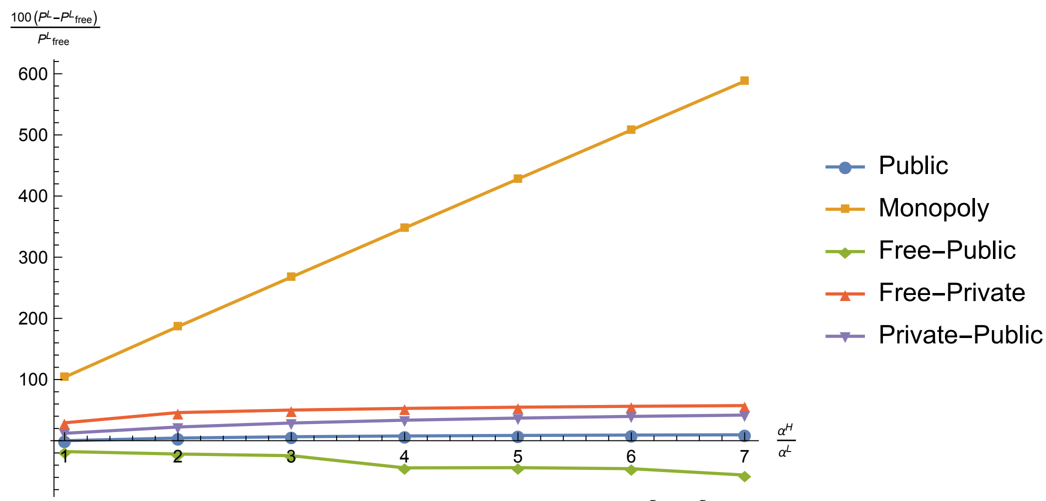
The distributional effects are summarized in Figures 5 and 6.

Figure 5: Queue-Eliminating Toll: Heterogeneity and Percentage Price Change (H Types)



Percentage Change in Generalized Price ($\frac{100(P^H - P^H_{free})}{P^H_{free}}$), Heterogeneity ($\frac{\alpha^H}{\alpha^L}$)

Figure 6: Queue-Eliminating Toll: Heterogeneity and Percentage Price Change (L Types)



Percentage Change in Generalized Price ($\frac{100(P^L-P^L_{free})}{P^L_{free}}$), Heterogeneity ($\frac{\alpha^H}{\alpha^L}$)

Compared with flat tolls, the generalized prices with queue-eliminating tolls are lower for all regimes, because queue-eliminating tolls are more efficient and more travellers can use the roads now. Similar to flat tolls, the percentage price change with queue-eliminating toll is in general decreasing for h types and increasing for l types. However, the percentage price change in the free-private regime is increasing for h types, because h types travel on link 2 only and are subject to the local monopoly power of the operator on link 2. Similarly, the percentage price changes increases most steeply in the monopoly regime for l types, because in other regimes with tolling on link 1, the operator is either public or faces competition from link 2. The free-public regime has in general lower prices for both types than the public regime, because link 2 offers a subsidy. However, there are kinks in the free-public regime for h types, which is caused again by a regime change. For example, at first when both types are the same, they travel on both roads. Then as the subsidy increases with the degree of heterogeneity, h types only travel on link 1 and l types on both roads. Finally as the subsidy increases further, all l types move to link 2 and h types travel on both links instead. In addition, the free-public regime generates a lower price than the free regime for both types, and public and private-public regime generate a lower price for h types.

In sum, for the parameters used in the simulation for queue-eliminating tolls, all regimes perform better in terms of social welfare than the free regime. The free-public regime also offers a higher consumer surplus for both types due to the flat subsidy it entails. Unlike in the flat toll case, the private-public regime remains more efficient than the free-public regime, even as the degree of heterogeneity increases, because the pooling equilibrium ensures relatively intense competition between the two links.

We have done robustness checks with respect to several supply and demand parameters: total capacity, capacity share of link 1 and elasticity of demand. The main conclusions just reported remain robust.

5 Conclusion

Our study shows that with operators on parallel links, as travellers become more different in values of time and of schedule delay, product differentiation offered by different operators is more appreciated by both groups, but each operator might also have more local monopoly power. With flat tolls, separating equilibria are more likely to arise and competition between different suppliers softens, while with queue-eliminating toll, pooling equilibria are more commonly observed so that the competitive force remains relatively strong.

There are at least two main limitations of our study. The first one is the discrete setting with only two types. This results in the non-existence of Nash equilibrium in some cases. When the heterogeneity is represented by a continuous distribution, this problem should be solved. However, the basic intuition still holds, with flat tolls each user strictly prefers one link over the other, while with queue-eliminating tolls all travellers use both roads.

As a result, the implications for local monopoly power and competition still remain. Another limitation is our assumption of proportional heterogeneity. For queue-eliminating tolls, this is not restricting, as we have shown in the theory part that a pooling equilibrium is socially optimal for a general cost structure. For flat tolls, this can be seen as an extreme case, where a pooling equilibrium is never optimal. However, with more general structures, it remains intuitive that there will be parameter ranges where pooling equilibria are inferior to the separating ones. So the results obtained in this paper still applies to those cases.

It is important for policymakers to take heterogeneity into account in evaluating the efficiency of various road pricing policies, both in terms of social welfare and distributional effect. With parallel links, we have shown queue-eliminating tolls can induce more competition when multiple road operators are active in a network. If a flat toll is used, special care should be taken to make sure the lack of competition within the network does not become too harmful for social welfare.

Appendix I Flat Toll

For social welfare and monopolistic profit maximization, we show why the pooling equilibrium, where both types of travellers use both links, is not optimal with a flat toll. We prove by contradiction: starting with an optimal pooling equilibrium, we prove that the value of the objective, i.e. social welfare or total revenue, can be further increased.

Following conventions, we denote the generalized congestion cost parameter by $\delta^i = \frac{\beta^i \gamma^i}{\beta^i + \gamma^i}$. The congestion cost for group i on link j, as derived in Van den Berg and Verhoef (2011), is thus the following:

$$\begin{aligned} C_j^h &= \delta^h \frac{N_j^h + N_j^l}{s_j} \\ C_j^l &= \delta^l \frac{N_j^h + N_j^l}{s_j} \end{aligned} \tag{1}$$

More travellers on a link naturally leads to higher congestion costs. Both types experience the same delays over time, but bear different congestions costs depending on the type-specific cost parameter.

Given the above cost function and the group-specific inverse demand functions, denoted by $D^i(N^h)$, a social planner aims to maximize social welfare, which is the total consumer willingness to pay minus the congestion costs. Denote the link-group-specific toll as τ_j^i, when both types travel on both roads, i.e. $N_j^i > 0$, the social welfare is as follows:

$$\begin{aligned} SW = &\int_0^{N_1^h+N_2^h} D^h(n)dn + \int_0^{N_1^l+N_2^l} D^l(n)dn \\ &- C_1^h N_1^h - C_1^l N_1^l - C_2^h N_2^h - C_2^l N_2^l \end{aligned} \tag{2}$$

The local first-order conditions for *a pooling equilibrium* are:

$$\begin{aligned} \frac{\partial SW}{\partial N_1^h} =& D^h(N_1^h + N_2^h) - \frac{\delta^h}{s1}(N_1^h + N_1^l) - \frac{\delta^h}{s1}N_1^h - \frac{\delta^l}{s1}N_1^l = 0 \\ \frac{\partial SW}{\partial N_1^l} =& D^l(N_1^l + N_2^l) - \frac{\delta^l}{s1}(N_1^h + N_1^l) - \frac{\delta^h}{s1}N_1^h - \frac{\delta^l}{s1}N_1^l = 0 \\ \frac{\partial SW}{\partial N_2^h} =& D^h(N_1^h + N_2^h) - \frac{\delta^h}{s2}(N_2^h + N_2^l) - \frac{\delta^h}{s2}N_2^h - \frac{\delta^l}{s2}N_2^l = 0 \\ \frac{\partial SW}{\partial N_2^l} =& D^l(N_1^l + N_2^l) - \frac{\delta^l}{s2}(N_2^h + N_2^l) - \frac{\delta^h}{s2}N_2^h - \frac{\delta^l}{s2}N_2^l = 0 \end{aligned} \tag{3}$$

It follows immediately that at the optimum, the traffic flow ratio of each group is equal to the link-capacity ratio, i.e. $\frac{N_1^i}{N_2^i} = \frac{s1}{s2}$, so for each group the congestion cost on both links are the same.

Starting from the above pooling equilibrium, we can improve social welfare further in two steps.

First we move a h type user from link 2 to link 1, and at the same time move a l type from link 1 to link 2. As a result, the social welfare remains the same. Keep moving travellers in this way, until we arrive at either a separating equilibrium with all h types on link 1 when initially $N_2^h < N_1^l$, or another separating equilibrium with all l types on link 2 when initially $N_2^h > N_1^l$.

Next we can remove one marginal h types from link 1 and keep her "at home", and add one l marginal types to link 2, then social welfare will increase. This is because, the marginal social congestion cost of an extra h types on link 1 now exceeds the generalized price from the pooling equilibrium, while the opposite is true on link 2. Mathematically, the partial derivatives of the social welfare function with respect to the new traffic flows have the following properties now: $\frac{\partial SW}{\partial N_1^h} < 0, \frac{\partial SW}{\partial N_1^l} < 0, \frac{\partial SW}{\partial N_2^h} > 0, \frac{\partial SW}{\partial N_2^l} > 0$.

So we have shown by way of contradiction that a pooling equilibrium would not be optimal for a social planner.

The same logic applies to profit maximisation. Staring from a pooling equilibrium, the monopolistic profit can always be improved in two similar steps as in the social welfare maximisation problem. So a pooling equilibrium would also not be optimal for monopolistic profit maximisation.

Appendix II Queue-Eliminating Toll

For social welfare and monopolistic profit maximisation, we show why a pooling equilibrium is optimal with a queue-eliminating toll. We only assume $\frac{\beta^h}{\gamma^h} = \frac{\beta^l}{\gamma^l}$, $\alpha^i > \beta^i$ and $\beta^h > \beta^l$, so the following result not only applies to proportional heterogeneity, but also holds for general heterogeneity structure.

The average congestion cost is (Van den Berg and Verhoef (2011)):

$$\begin{aligned} C_j^h &= \delta^h \frac{N_j^h}{2s_j} \\ C_j^l &= \delta^l \frac{N_j^h}{s_j} + \delta^l \frac{N_j^l}{2s_j} \end{aligned} \tag{4}$$

The factor $\frac{1}{2}$ stems from the fact that the travel delay cost is fully replaced by tolls: not a social cost, but a transfer. The term containing N_j^h for l types reflects the additional schedule delay costs that l types face because the h types occupy the central peak.

When both types travel on both roads ($N_j^i > 0$), the social welfare is thus the following:

$$SW = \int_0^{N_1^h + N_2^h} D^h(n)dn + \int_0^{N_1^l + N_2^l} D^l(n)dn \\ - C_1^h N_1^h - C_1^l N_1^l - C_2^h N_2^h - C_2^l N_2^l \tag{5}$$

The first order conditions for the pooling equilibrium is:

$$\begin{aligned} \frac{\partial SW}{\partial N_1^h} =& D^h(N_1^h + N_2^h) - \frac{\delta^h}{s_1} N_1^h - \frac{\delta^l}{s_1} N_1^l = 0 \\ \frac{\partial SW}{\partial N_1^l} =& D^l(N_1^l + N_2^l) - \frac{\delta^l}{s_1} N_1^h - \frac{\delta^l}{s_1} N_1^l = 0 \\ \frac{\partial SW}{\partial N_2^h} =& D^h(N_1^h + N_2^h) - \frac{\delta^h}{s_2} N_2^h - \frac{\delta^l}{s_2} N_2^l = 0 \\ \frac{\partial SW}{\partial N_2^l} =& D^l(N_1^l + N_2^l) - \frac{\delta^l}{s_2} N_2^h - \frac{\delta^l}{s_2} N_2^l = 0 \end{aligned} \tag{6}$$

The solution is $\tau_j^i = 0$ and the Hessian matrix is negative definite, so the pooling equilibrium is the global maximum.

Similarly, we can prove that with queue-eliminating tolling, the profit maximising equilibrium is also pooling with a group-specific $\tau^i = \frac{\partial D^i(N^i)}{\partial N^i} N^i$.

Appendix III Discontinuity

There are two types of separating equilibria: a fully separating equilibrium, where one group travels on one link and the other group on the other link; and a semi-separating equilibrium, where one group travels on one link, but the other group on both links. The optimal toll is different for these two equilibria. As a result, we have observed discontinuity in best response functions.

For a numerical example, consider a private supplier on link 1, who chooses τ_1 to maximize his profit, given τ_2 is around 8.229. If he charges a high toll, $\tau_1 = 14.546$, a full separating equilibrium results: h types will travel on link 1 and l types on link 2. If he charges a lower toll, $\tau_1 = 10.837$, a semi-separating equilibrium results: h types will travel on link 1 but l types on both links. The optimal toll for the full separating equilibrium is different from the one for the semi-separating equilibrium, as shown in Table 3. So when τ_2 changes from 8.228 to 8.230, the optimal τ_1 jumps from 10.837 to 14.546.

Table 3: Private Supplier on Link 1

Type	τ_1	τ_2	N_1^h	N_2^h	N_1^l	N_2^l	Π_1
Separating	14.546	8.228	8087	0	0	9254	117628
Semi-Separating	10.837	8.228	9143	0	1711	8116	117623
Separating	14.546	8.230	8087	0	0	9253	117628
Semi-Separating	10.837	8.230	9143	0	1712	8115	117634

Figure 7 shows how profit on link 1 changes with τ^1, given different τ_2. When τ_2 is small, i.e. $\tau_2 = 0$, it's better to have full separating equilibrium, so the highest profit for operator 1 is achieved on the left part of the green dots. But when τ_2 is large, i.e. $\tau^2 = 15$, it's better to have semi-separating equilibrium, and the highest profit is achieved on the right part of the blue dots. The discontinuity arises when τ^2 is some value in between, i.e. $\tau^2 = 8.228$, both left or right parts of the orange dots can generate the highest profit and a small change in τ^2 will change disdcontinually the optimal value of τ^1. As a result, the best response function in toll is not continuous in the opponent's toll, and sometimes a Nash equilibrium does not exist. It is likely that this feature disappears if the more realistic case of a continuous distribution were considered.

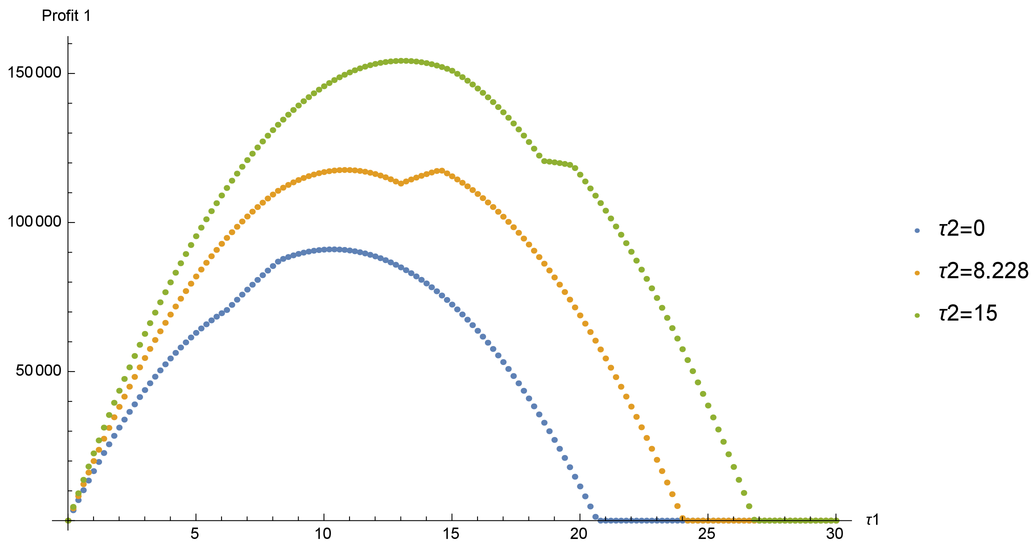


Figure 7: Discontinuity in Optimal Toll

References

Arnott, R., A. de Palma, and R. Lindsey (1992). Route choice with heterogeneous drivers and group-specific congestion costs. *Regional Science and Urban Economics 22*(1), 71–102.

Braid, R. M. (1996). Peak-Load Pricing of a Transportation Route with an Unpriced Substitute. *Journal of Urban Economics 40*(2), 179–197.

De Palma, A. and R. Lindsey (2002). Private roads, competition, and incentives to adopt time-based congestion tolling. *Journal of Urban Economics 52*(2), 217–241.

Small, K. A. (1982). The Scheduling of Consumer Activities: Work Trips. *American Economic Review 72*(3), 467–479.

Small, K. a. and J. Yan (2001). The Value of 'Value Pricing' of Roads: Second-Best Pricing and Product Differentiation. *Journal of Urban Economics 49*(2), 310–336.

Van den Berg, V. and E. T. Verhoef (2011). Winning or losing from dynamic bottleneck congestion pricing?. The distributional effects of road pricing with heterogeneity in values of time and schedule delay. *Journal of Public Economics 95*(7-8), 983–992.

Verhoef, E., P. Nijkamp, and P. Rietveld (1996). Second-Best Congestion Pricing: The Case of an Untolled Alternative. *Journal of Urban Economics 40*(3), 279–302.

Verhoef, E. T. and K. a. Small (2004). Product differentiation on roads: Constrained congestion pricing with heterogeneous users. *Journal of Transport Economics and Policy 38*(1), 127–156.

Vickrey, W. (1973). Pricing, metering, and efficiently using urban transportation facilities. *Highway Research Record 476*, 36–48.