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Premium Auctions in The Field

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Abstract. In a field experiment, we study the revenue-generating properties of premium auctions. In a premium auction, the runner-up obtains a premium for driving up the price paid by the winner. Previous research, both theoretical and in the lab, has shown that the relative performance of premium auctions compared to standard auction formats is context-specific. In the experiment, we compare two types of premium auctions with the standard Vickrey auction selling high-quality, limited-edition posters in an online auction. We observe that neither premium auction raises higher revenue than the Vickrey auction. The variance of the revenue in the Amsterdam auction, one of the premium auctions, is lower than that in the Vickrey auction.

Keywords: Premium auctions, field experiment

JEL classification: C93, D44

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1. Introduction

In premium auctions, the runner-up (the highest losing bidder) obtains a premium for driving up the price paid by the winner. Various versions of premium auctions have been used since at least 1529 to sell timber, wine, spices, tea, coffee, books, art, tulips, financial securities, and real estate (Van Bochove et al., 2016). Premium auctions are believed to outperform standard auctions like the first-price sealed-bid auction, the Dutch auction, the English auction, and the Vickrey auction because bidders compete fiercely for the premium, driving up the price paid by the winner. The additional competition may then compensate for the premium the seller has to pay to the runner-up.

Several studies confirm this intuition. Milgrom (2004) shows theoretically that a premium auction may attract more entry than the English auction and consequently may generate more revenue. Goeree and Offerman (2004) study the "Amsterdam auction," a premium auction in which both the winner and the runner-up obtain a premium. They find theoretically and in the lab that two variants of the Amsterdam auction raise higher average revenue than the first-price sealed-bid auction and the English auction in a setting with strong *ex ante* bidder asymmetries. Hu et al. (2011a) observe in a laboratory experiment that the Amsterdam auction is less conducive to cartel formation and raises more money than the English auction and the first-price sealed-bid auction in the case of strong *ex ante* bidder asymmetries.

In *ex ante* symmetric settings, the results are mixed. According to the celebrated revenue-equivalence theorem (Myerson, 1981), expected revenue is the same in premium auctions as in standard auctions in the case of risk-neutral bidders and separating equilibria. Experimental evidence in symmetric settings or settings with weak asymmetries is in line with this finding (Goeree and Offerman, 2004; Hu et al., 2011a). In theoretical work, Hu et al. (2011b, 2017) find that the Amsterdam auction generates lower revenue than the English auction in the case of risk-averse bidders. Data from Brunner et al.'s (2014) laboratory experiment are consistent with this prediction.[1] Only if the seller is sufficiently risk averse could the Amsterdam auction be considered a Pareto improvement compared to the English auction (Hu et al., 2017).

In this paper, we study the external validity of the above results for *ex ante* symmetric settings such as Internet auctions where bidders can submit bid anonymously. We do so in an online field experiment[2] in which we compare the revenue-generating properties of two kinds of premium auctions with the Vickrey auction. In the Vickrey auction, bidders independently submit a bid. The highest bidder wins and pays a price equal to the second-highest bid. The two premium auctions that we study are variations of the Vickrey auction and only differ in that some bidders obtain a premium. In the (sealed-bid) Amsterdam auction, both the winner and the runner-up get a premium equal to 50% of the difference between the second-highest and third-highest bid. In the Fischer auction, the runner-up receives a premium equal to 5% of the price paid by the winner (i.e., the runner-up's bid).[3]

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[1] In contrast to theory, the authors do not find the average revenue in the Amsterdam auction to be higher than in the English auction for risk-loving bidders.

[2] See Chen and Konstan (2015) for a discussion of design choices for online field experiments.

[3] The Fischer auction is modeled after the auction format carrying the same name used by art dealer Simonis & Buunk. The art dealer is located in the Dutch town of Ede and chose the auction's name to honor the Fischer family that delivered several generations of notaries in the 19th and 20th century in Ede.

In our online field experiment, we sell three identical copies of a high-quality, limited-edition print. We invited almost 10,000 members of a general-population panel to participate in an auction. The roughly 950 panel members that chose to enter were randomized over the three auction formats, resulting in over 300 participants per treatment. During the auction, the participants obtained no information on the number of other bidders or others' bids. As a consequence, our setting is arguably a symmetric one in that anonymity assures us that, from the viewpoint of the bidders, no subset of competing bidders could be identified as 'strong' or 'weak.' Participants also answered survey questions on their background demographics (age, gender, education, and marital status), their expectation of the highest bid, their risk attitude (à la Dohmen et al., 2011), and their bidding strategy.

Our data show that the premium auctions do not outperform the Vickrey auction in terms of average revenue. We also find that the Amsterdam auction's revenue dispersion is lower than in the other two auctions. The comparative statics between the Vickrey and Amsterdam auctions are in line with the laboratory results discussed above. We conclude that in settings that lack strong *ex ante* bidder asymmetries, premium auctions are unattractive for sellers. The Amsterdam auction might be an exception in that it might be an interesting format for strongly risk-averse sellers because of its relatively low revenue dispersion.

We are not the first to compare the relative performance of single-unit auction formats in a field experiment. Lucking-Reiley (1999) tests revenue equivalence between first-price sealed-bid and Dutch auctions, and between English and Vickrey auctions. Reiley (2006), Katkar and Reiley (2006), Brown and Morgan (2009), and Ostrovsky and Schwarz (2011) study the effect of reserve prices on auction revenue. Houser and Wooders (2005) and Brown and Morgan (2009) examine how auction ending rules affect auction revenue. Carpenter et al. (2008) compare the first-price, second-price, and all-pay sealed-bid auctions in terms of money raised for charity. As far as we are aware, we are the first to study the revenue-generating properties of premium auctions in a field experiment.

The structure of this paper is as follows. In Section 2, we derive the theoretical properties of the sealed-bid Amsterdam auction and the Fischer auction and compare those with the well-known properties of the Vickrey auction. In Section 3, we discuss our experimental design and hypotheses. Our data analysis is in Section 4. Section 5 contains a short conclusion. Lengthy proofs of lemmas and propositions are relegated to Appendix B.

2. Theory

In this section, we develop a theory of bidding in premium auctions. We analyze bidding in the independent private values framework. One indivisible item is sold in an auction to one out of $n \geq 3$ risk-neutral bidders, labeled $i = 1, \dots, n$. Let v_i denote bidder i's value for the item, $i = 1, \dots, n$. All values are drawn independently from the interval $[\underline{v}, \overline{v}]$, $0 \leq \underline{v} < \overline{v}$, according to the same differentiable distribution function F. We assume that F has no mass points and that it is strictly increasing over $[\underline{v}, \overline{v}]$. Let $f \equiv F'$ denote the corresponding density function.

The item is awarded through one of the following three auction formats:

- **The Vickrey auction:** Bidders independently submit a bid. The highest bidder wins the item and pays the second-highest bid.
- **The (sealed-bid) Amsterdam auction:** Bidders independently submit a bid. The highest bidder wins the item and pays the second-highest bid. Both the winner and the runner-up

receive a premium equal to a fraction $\alpha \in (0,1/2]$ of the difference between the second-highest and third-highest bid.

- **The Fischer auction:** Bidders independently submit a bid. The highest bidder wins the item and pays the second-highest bid. The runner-up receives a premium equal to a fraction $\varphi \in (0,1)$ of the second-highest bid.

The following properties of the Vickrey auction are well-known (see, e.g., Vickrey, 1961).

Lemma 1 *The Vickrey auction has a unique equilibrium in weakly dominant strategies in which all bidders bid value. In this equilibrium, the item is always awarded to the bidder with the highest value. A bidder with the lowest possible value $\underline{v}$ obtains zero expected utility.*

A dynamic version of the above Amsterdam auction has been studied extensively (see Goeree and Offerman, 2004, Hu et al., 2011ab, 2017, and Brunner et al. 2014). The dynamic version has the same rules as the English auction with the additional feature that both the winner and the runner-up obtain a premium proportional to the difference between the second-highest and third-highest bid. In the independent private values framework, equilibrium bids do not depend on the auction history (i.e., the prices at which other bidders stepped out of the auction). Therefore, it is straightforward to establish the following lemma on the basis of the unique perfect Bayesian equilibrium of the dynamic version of the Amsterdam auction.

Lemma 2 *The Amsterdam auction has a unique symmetric equilibrium which is given by*

$$b(v) = v + \int_{v}^{\overline{v}} \left(\frac{1-F(x)}{1-F(v)}\right)^{\frac{1}{\alpha}} dx\,, v \in [\underline{v}, \overline{v}].$$

In this equilibrium, the item is always awarded to the bidder with the highest value. A bidder with the lowest possible value $\underline{v}$ obtains zero expected utility.

The following result follows immediately from Myerson's (1981) revenue-equivalence theorem because in both the Vickrey auction and the Amsterdam auction, (1) the item is awarded to the bidder with the highest value, and (2) a bidder that has the lowest possible value obtains zero expected utility.

Proposition 1 *The Vickrey auction and Amsterdam auction yield the same expected revenue.*

The following result follows immediately from Hu et al.'s (2014) study of the dynamic version of the Amsterdam auction.

Proposition 2 *The Vickrey auction's revenue is a mean-preserving spread of that of the Amsterdam auction.*

This result implies that the variance of the revenue in the Vickrey auction is higher than that in the Amsterdam auction. The intuition is as follows. First of all, bids in the Vickrey auction are more spread out than in the Amsterdam auction. In the Vickrey auction, they span over the entire value range $[\underline{v}, \overline{v}]$ because all bidders bid value, while in the Amsterdam auction, the range is $\left[\underline{v} + \int_{\underline{v}}^{\overline{v}} \left(\frac{1-F(x)}{1-F(v)}\right)^{1/\alpha} dx\,, \overline{v}\right]$. Second, in the Vickrey auction, revenue equals the second-highest bid while in the Amsterdam auction, it is equal to the weighted average of the third-highest and second-highest bid, which yields a lower variance even if the bids were drawn from the same distribution.

Finally, we find that expected revenue in the Fischer auction is lower than in the other two auctions, as the results below show.

Lemma 3 *The Fischer auction does not have an efficient equilibrium.*

Proposition 3 *If F is log-concave, the expected revenue is strictly lower in the Fischer auction than in both the Vickrey auction and the Amsterdam auction.*

The following examples show that the Fischer auction's revenue loss compared to the other two auction formats may be non-negligible.

Example 1 *Suppose $(n-1)\overline{v} \leq (n-1+\varphi)\underline{v}$. Then the Fischer auction has an equilibrium in which all bidders bid $B = \underline{v}$ independently of their value. To see that this is an equilibrium, first note that none of the bidders have a reason to bid lower than $\underline{v}$: by bidding $\underline{v}$ they obtain positive utility because they have a strictly positive probability both of winning the item and paying a price lower than their value and of obtaining the premium amounting to $\varphi\underline{v}$. Second, bidders do not have a reason to bid higher than $\underline{v}$. Suppose they did, then their payoff is less than their expected payoff when bidding $\underline{v}$:*

$$v_i - \underline{v} = \frac{v_i - \underline{v}}{n} + \frac{\varphi\underline{v}}{n} + \frac{(n-1)v_i}{n} - \frac{n-1+\varphi}{n}\underline{v}$$

$$\leq \frac{v_i - \underline{v}}{n} + \frac{\varphi\underline{v}}{n} + \frac{(n-1)\overline{v}}{n} - \frac{(n-1+\varphi)\underline{v}}{n} \leq \frac{v_i - \underline{v}}{n} + \frac{\varphi\underline{v}}{n}.$$

The first [second] inequality is implied by $v_i \leq \overline{v}$ [the assumption that $(n-1)\overline{v} \leq (n-1+\varphi)\underline{v}$]. The auction's revenue equals $(1-\varphi)\underline{v}$ which is more than a factor $(1-\varphi)$ less than the expected revenue in the Vickrey and Amsterdam auctions (i.e., the expected value of the second highest value).

Example 2 *If $\underline{v} \geq (1-\varphi)\overline{v}$, the Fischer auction has an equilibrium in which all bidders bid $B = \overline{v}$ independently of their value. Notice that for a bidder, deviating to a higher bid is not interesting because then they are guaranteed to win the auction and pay more than their value for the item without getting the premium. For bidder i, bids below $\overline{v}$ yield a payoff of zero while the expected payoff when bidding $\overline{v}$ equals*

$$\frac{1}{n}(v_i - \overline{v}) + \frac{\varphi\overline{v}}{n} \geq \frac{1}{n}\left(\underline{v} - \overline{v}\right) + \frac{\varphi\overline{v}}{n} = \frac{\underline{v}}{n} - \frac{(1-\varphi)\overline{v}}{n} \geq 0.$$

The first [second] inequality follows from $v_i \geq \underline{v}$ [the assumption that $\underline{v} \geq (1-\varphi)\overline{v}$]. So, deviating to a lower bid is not interesting either. The auction's revenue equals $(1-\varphi)\overline{v} \leq \underline{v}$ which is strictly lower than the expected revenue in the Vickrey and Amsterdam auctions (i.e., the expected value of the second-highest value).

3. Experimental design and hypotheses

3.1 Experimental design

The experiment was conducted in cooperation with Veylinx, a Dutch online experimental auction platform that is mainly used for marketing research. We compare three treatments in a between-subjects design: the Vickrey auction, the Amsterdam auction, and the Fischer auction. The rules of the auctions are described in the theory section above. For the Amsterdam auction, we set $\alpha = 1/2$, i.e., both the winner and the runner-up received a 50% share of the difference between the second-

highest and the third-highest bid. For the Fischer auction, we set $\varphi = 1/20$, i.e., the runner-up received 5% of the own bid. Table 1 summarizes the experimental design, including the number of bidders per treatment.

Table 1: Experimental design

	Vickrey	Amsterdam	Fischer
Premium winners	-	Auction winner and runner-up	Runner-up
Premium size	-	$\left(b^{[2]} - b^{[3]}\right)/2$	$b^{[2]}/20$
# bidders	317	305	342

Notes: For the treatments presented in the first row, we indicate which bidders obtain a premium (row 2), the premium size (row 3), and the number of bidders (row 4). In row 3, $b^{[2]}$ ($b^{[3]}$) represents the second-highest (third-highest) bid.

The experimental procedures are as follows. On May 5, 2016, we sent out an e-mail to 9,763 members of a general-population panel inviting them to "participate in an auction." In the e-mail, we did not specify either the item that was up for auction or the auction type that bidders would participate in. Panel members could enter the auction once by clicking on a button in the e-mail. The auction was open until noon, May 11, 2016. The 949 panel members (9.72%) that chose to enter were randomized over the three auction formats. The item sold in each auction was a copy of a framed, high-quality, limited-edition print that was available at the Amsterdam-based photo gallery YellowKorner at a retail price of €590.

Upon arrival at Veylinx' online platform, the participants browsed through several pages. The first page informed the participants that (1) they could submit only one bid, (2) they could not withdraw their bid, (3) the bid included VAT and shipment, (4) the winner paid less than their own bid, (5) the winner must pay within 48 hours, (6) the auction closed at noon, May 11, 2016, and (7) the object would be shipped to the winner within seven days after payment. On the next page, the participants were given information about the item being auctioned along with the rules of the auction. At the bottom of this page, the participants were given six minutes to enter their bid. After entering a bid, participants were asked to fill out a multiple-choice survey. The survey contained five questions about (1) the participant's education, (2) their expectation of the highest bid, (3) their risk attitude (à la Dohmen et al., 2011), (4) their bidding strategy, and (5) their marital status. Appendix A contains a translation of the instructions (including a picture of the print).

3.2 Hypotheses

We aim at testing the following hypotheses using the data from our experiment.

Hypothesis 1 The Vickrey auction and the Amsterdam auction raise the same average revenue.

Hypothesis 1 is based on Proposition 1 that, in turn, relies on the assumption that all bidders are risk-neutral. In the case of risk-averse bidders, the expected revenue is lower in the Amsterdam auction than in the English auction (Hu et al., 2017) that is strategically equivalent to a Vickrey auction in a private-values setting. Brunner et al. (2014) find support for that prediction using lab data.

Hypothesis 2 The expected revenue of the Fischer auction is lower than that of the Vickrey auction.

Hypothesis 3 The expected revenue of the Fischer auction is lower than that of the Amsterdam auction.

Hypotheses 2 and 3 mirror Proposition 3.

Hypothesis 4 The variance of the revenue in the Vickrey auction is greater than that of the Amsterdam auction.

Hypothesis 4 is implied by the finding in Proposition 2 that the Vickrey auction's revenue is a mean-preserving spread of that of the Amsterdam auction. For the Fischer auction, we do not have clear hypotheses regarding its revenue variance. Example 1 suggests that the variance may be as low as zero in equilibrium because all bidders submit the same bid. At the same time, it is hard to find equilibria for more general settings than the one presented in the example because the Fischer auction does not have an efficient equilibrium, according to Lemma 3.

4. Experimental results

This section contains our experimental results. In section 4.1, we present summary statistics and initial results on the basis of our raw data. In section 4.2, we compare the estimates for the auctions' revenue. In section 4.3, we look in more detail at bidding behavior at the individual level.

Table 2: Summary statistics

	Vickrey	**Amsterdam**	**Fischer**	**F/χ^2 tests on joint significance**
Female (dummy)	0.514 (0.501)	0.489 (0.501)	0.506 (0.501)	0.8083
Age	40.9 (13.7)	40.6 (13.9)	41.1 (14.6)	0.8927
Married (dummy)	0.397 (0.489)	0.403 (0.491)	0.421 (0.494)	0.7813
University degree (dummy)	0.662 (0.474)	0.636 (0.482)	0.664 (0.473)	0.7132
Risk aversion	3.73 (1.27)	3.72 (1.26)	3.70 (1.32)	0.9502
Bid	€14.20 (26.52)	€15.21 (28.80)	€13.04 (15.87)	0.9066
Highest bid	€200.00	€222.22	€200.00	
Second-highest bid	€160.00	€150.00	€150.00	
Third-highest bid	€153.00	€125.00	€150.00	
Strictly positive bid (dummy)	0.539 (0.499)	0.521 (0.500)	0.547 (0.499)	0.8031
Bid value (dummy)	0.552 (0.498)	0.538 (0.499)	0.550 (0.498)	0.9284
Bid more than value (dummy)	0.098 (0.298)	0.105 (0.307)	0.073 (0.261)	0.3346
Bid less than value (dummy)	0.350 (0.478)	0.357 (0.480)	0.377 (0.485)	0.7549
Expected highest bid				
lower than €20	5.40%	4.30%	8.58%	0.0704
€20 – €49.99	10.48%	13.58%	8.58%	0.1258
€50 – €99.99	23.81%	17.55%	23.37%	0.1087
€100 – €199.99	33.33%	37.42%	28.70%	0.0623
€200 – €499.99	23.49%	23.18%	26.33%	0.6049
higher than €500	3.49%	3.97%	4.44%	0.8339
Observations	317	305	342	964

Notes: Statistics shown are averages, with standard deviations between brackets. The final column presents F/χ^2 tests on jointly significant effects of the treatments on the outcome variable. For Bid the test is based on a tobit regression. For Age and Risk the tests are based on OLS regressions. For the remaining, dummy, variables the tests are based on logit regressions. The statistics for Expected highest bid, Married, and Risk aversion are based on 955 observations (315, 302, and 338 for the Vickrey, Amsterdam, and Fischer auctions, respectively) because nine participants failed to answer the corresponding questions in the post-auction questionnaire.

4.1 Summary statistics

Table 2 contains summary statistics of the experiment. First of all, it demonstrates that treatments were balanced across treatments for observable bidder characteristics like gender, age, marital

status, education, and risk attitude. Table 2 also gives a first impression of the revenue-generating properties of the auctions. The realized revenue for the Vickrey auction (€160) is higher than for both the Amsterdam auction (€125) and the Fischer auction (€142.50). The average submitted bid was the highest in the Amsterdam auction and the lowest in the Fischer auction, although the differences between the auctions are not statistically significant. This observation suggests that the Fischer auction performs relatively poorly compared to the Vickrey auction because for the Fischer auction revenue has to be discounted with the 5% premium.

The auctions also do not differ significantly in terms of the fraction of bidders submitting a strictly positive bid. This is remarkable in the sense of the attractive property of premium auctions in that entering such auctions might be interesting for bidders without a genuine interest in the object because it gives them the opportunity to compete for the premium (Milgrom, 2004). Our field experiment tests this claim in a setting where entry costs are negligible. Zero-value bidders may decide to 'enter' the auction by submitting a strictly positive bid. However, we do not observe more strictly positive bids in the premium auctions than in the Vickrey auction.

Result 1 *The fraction of strictly positive bids is not greater for the premium auctions than for the Vickrey auction.*

Table 2 also gives an impression of the used bidding strategies. In all three auctions, the majority of the bidders report having bid value, a substantial fraction of the bidders report below-value bidding, and only a small minority were shown to have bid above value. The auctions do not differ significantly in these respects. While bidding value is consistent with equilibrium for the Vickrey auction, in the Amsterdam auction, all bidders bid more than the value in equilibrium in a symmetric setting (Hu et al., 2017), which is clearly not what we observe. At the same time, the bidding behavior in the Amsterdam auction is qualitatively in line with the observed behavior in Brunner et al.'s (2014) laboratory experiment in that they also observe many bids close to value as well as weakly-dominated bids below value. For the Fischer auction, we do not have a clear benchmark to compare bidding behavior with. Both bids above and below value might be compatible with the equilibrium as Examples 1 and 2 show.

Result 2 *The auctions do not differ significantly in the number of bidders reported as bidding below or above value.*

Remarkably, the auctions differ substantially in terms of the bids submitted by those who overbid. While in the Vickrey auction, those who overbid bid only €0.14 higher than those who did not ($p=0.977$, t-test). For the Amsterdam and Fischer auctions, those numbers are €9.10 ($p=0.091$) and €26.09 ($p<0.001$), respectively. Among the 31 (36) [31] bidders in the Vickrey (Amsterdam) [Fischer] auction who bid at least €50, 3 (8) [9] reported having bid more than their value. These observations suggest that overbidding in the premium auctions is partly driven by some bidders competing for the premium.

Result 3 *In the Vickrey auction, bidders who reported bidding more than their value, submitted only marginally higher bids than other bidders. In contrast, in both premium auctions, those who reported having overbid submitted significantly higher bids that those who did not.*

4.2 Revenue

In this section, we present estimates for the first two moments of the auction revenues. Our empirical analysis relies on the joint assumption that (1) submitted bids are independent of each other, and (2) bidders are not aware of the number of competing bidders. The first assumption is arguably a reasonable one in our settings because our participants are drawn from a large general-population panel and because they do not interact with each other on the auction platform before submitting a bid. To make sure that the second assumption is satisfied, we did not inform the bidders about how many others would participate (in fact, we did not know this ourselves). We use two methods to estimate the auctions' revenue's mean and variance for settings where the number of bidders equals 10 to 400. For low bidder numbers (up to 50), we make use of Mullin and Reiley's (2006) recombinant estimation. For higher bidder numbers (between 50 and 400), we use estimates of the cumulative distributions of the bids. Table 3 contains the resulting estimates for the auction revenue and its standard deviation for the three auctions based on recombinant estimation.

Table 3: Estimates for the mean and the standard deviation of auction revenue for $n = 10, 20, 30, 40, 50$.

	$n = 10$		$n = 20$		$n = 30$		$n = 40$		$n = 50$	
Vickrey	33.40	*21.80*	53.46	*25.29*	66.55	*27.48*	76.94	*28.86*	85.93	*29.86*
	(3.15)		(4.15)		(4.09)		(3.65)		(3.25)	
Fischer	30.60	*22.71*	48.19	*26.12*	61.72	*28.19*	72.42	*28.44*	80.92	*28.60*
	(2.91)		(4.39)		(3.69)		(3.70)		(2.81)	
Amsterdam	20.49	*17.85*	44.04	*21.86*	59.56	*20.80*	69.52	*18.88*	76.88	*17.27*
	(2.60)		(4.01)		(2.77)		(2.40)		(2.15)	

Notes: The first [second] entry in a cell displays the estimate for average revenue [the standard deviation (in italics)]. Standard errors are between brackets.

Mullin and Reiley's (2006) recombinant estimation is based on averages over a large range of group outcomes that have resulted from randomly drawn combinations from subsamples of the submitted bids. Recombinant estimation is not very useful for pools of bidders containing more than 50 bidders: the randomly drawn group observations are too highly correlated because they are drawn from only about 300 observations per auction. For 50 or more bidders, we rely on the following two-step procedure to estimate the auctions' revenue's mean and variance. First, for each auction, we estimate the probability distribution the bids are drawn from. More specifically, we assume that bids for all auctions are drawn from a Weibull distribution, i.e., a distribution with cdf $G(b) = max\left\{0, 1 - e^{-(b/\lambda)^k}\right\}$ with $\lambda, k > 0$, $b \in \mathbb{R}$. The parameter estimates for each are based on OLS regressions using the 20% highest bids only. The P-P plots in Figure 1 indicate that the estimates are reasonably accurate for all three auctions, in particular for the top 5% of the bids. The second step is to use the parameter estimates to calculate the first and second moments of the second and third highest-order statistics to obtain the auctions' expected revenue and its variance. The results are in Table 4.

The estimates in tables 3 and 4 indicate that, if anything, the Vickrey auction raises higher revenue than both premium auctions. As a consequence, if we have reason to reject Hypothesis 1 that states that the Vickrey and the Amsterdam are revenue equivalent, it is in favor of the hypothesis that the

Vickrey auction outperforms the Amsterdam auction. We also have no reason to reject Hypothesis 2 in favor of the hypothesis that the Fischer auction raises more money than the Vickrey auction. For all estimates, apart from $n = 400$ bidders, the average revenue is estimated to be higher for the Vickrey auction than for the Fischer auction. For low numbers of bidders ($n \leq 50$), the estimated revenue difference exceeds 5%, which is more than the premium paid to the runner-up. Tables 3 and 4 also indicate that the variance of the revenue is substantially lower in the Amsterdam auction than in the Vickrey and Fischer auctions for any number of bidders used in the simulations.

Result 4 *The Vickrey auction generates weakly more revenue than the Amsterdam and Fischer auctions. The Amsterdam auction's revenue dispersion is lower than in the other two auctions*

Figure 1: P-P plots estimated quantiles vs. sample quantiles

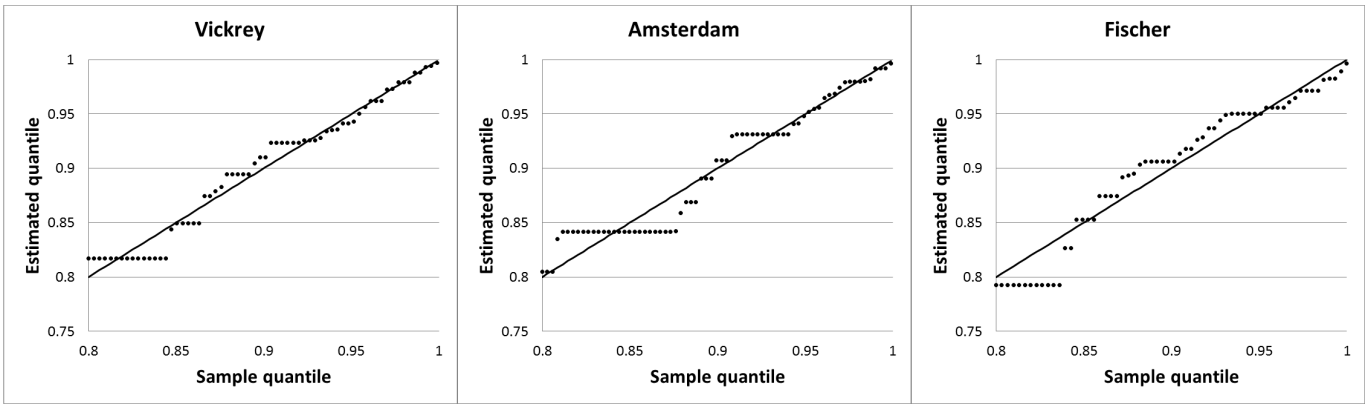

Table 4: Estimates for the mean and the standard deviation of auction revenue for $n = 50, 100, 200, 300, 400$.

	$n = 50$		$n = 100$		$n = 200$		$n = 300$		$n = 400$	
Vickrey	86.05	*33.93*	115.6	*38.24*	148.7	*42.27*	169.7	*44.54*	185.3	*46.11*
Fischer	80.25	*35.16*	110.7	*40.67*	146.0	*46.00*	168.7	*49.05*	185.8	*51.19*
Amsterdam	76.07	*27.13*	107.4	*31.43*	143.2	*35.43*	166.1	*37.66*	183.2	*39.19*

Notes: The first [second] entry in a cell displays the estimate for average revenue [the standard deviation (in italics)].

4.3 Individual bidding behavior

In this section, we zoom in on individual bidding behavior. First, we consider decisions to submit 'serious' bids, i.e., non-zero bids. Table 5 contains the results of a logit regression of strictly positive bids on gender, age, education, marital status, and risk attitude. Correcting for bidder characteristics does not alter the conclusion that the decision to enter a strictly positive bid is independent of auction format (see section 4.1). Men, young, low-educated, and unmarried people are significantly more likely to submit a non-zero bid. Bidders are less likely to submit a strictly positive bid the more risk averse they are, although this effect is only statistically significant for the Amsterdam auction in the regression without controls for other bidder characteristics.

Result 5 *A bidder's decision to enter a strictly positive bid does not significantly depend on the auction format. Risk-averse participants are less likely to submit a strictly positive bid but only statistically significantly so for the Amsterdam auction.*

Table 5: Submitting a strictly positive bid – Logit regressions

Constant	1.64***	.664*
	(.450)	(.364)
Vickrey	–.599	–.460
	(.507)	(.505)
Fischer	–.440	–.435
	(.495)	(.488)
Gender (female=1)	–.480***	
	(.135)	
Age	–.011**	
	(.006)	
Education (university=1)	–.373***	
	(.146)	
Married	–.328**	
	(.150)	
(Risk aversion)*Vickrey	.048	–.015
	(.088)	(.089)
(Risk aversion)*Amsterdam	–.141	–.161*
	(.095)	(.092)
(Risk aversion)*Fischer	.016	–.014
	(.084)	(.083)
Number of observations	955	955
Log pseudolikelihood	–643	–658
Pseudo R^2	0.0260	0.0027

Notes: Robust standard errors in parentheses. Significance levels: * 10%; ** 5%; *** 1%.

Table 6 includes regression results for the bids submitted. In each auction format, women submit significantly lower bids than men. Bids are decreasing in age but only statistically significantly so for the Vickrey and Fischer auctions. Bids do not depend significantly on the level of education or marital status in any of the three auctions. For all auctions, bids are decreasing in bidders' risk attitude albeit not significantly so. The effect for the Amsterdam auction is greater than for the Vickrey auction in both regressions. This is in line with the theoretical results that bids are independent of risk attitude for the Vickrey auction and decreasing in risk aversion for the Amsterdam auction (Brunner et al., 2014).

Result 6 *Risk-averse bidders tend to bid lower in all three auctions, but in none of the auctions is the effect statistically significant.*

Table 6: Submitted bids – Tobit regression

	Vickrey		Amsterdam		Fischer	
Constant	23.8**	10. 8	21.8*	10.0	31.2***	4.07
	(11.4)	(7.93)	(13.1)	(9.16)	(10.9)	(7.14)
Gender (female=1)	−14.0***		−11.4*		−11.6**	
	(5.28)		(5.88)		(4.72)	
Age	−.133*		−.062		−.467**	
	(.210)		(.239)		(.200)	
Education	−6.19		−3.86		−5.27	
(university=1)	(5.48)		(6.17)		(5.05)	
Married	−5.23		−8.02		−6.80	
	(5.71)		(6.39)		(5.59)	
Risk aversion	−1.78	−3.22	−3.08	−3.61	−.446	−1.53
	(2.06)	(2.00)	(2.39)	(2.36)	(1.80)	(1.81)
Number of observations	315	315	302	302	338	338
Log likelihood	−963	−968	−912	−915	−1034	−1043
Pseudo R^2	0.0066	0.0013	0.0046	0.0013	0.0093	0.0003

Notes: Standard errors in parentheses. Significance levels: * 10%; ** 5%; *** 1%.

5. Conclusions

We have reported the results of an online field experiment in which we compared the revenue-generating properties of two premium auctions, the Amsterdam auction and the Fischer auction, with those of the standard Vickrey auction. We conclude that both premium auctions do not raise higher revenue than the Vickrey auction. The variance of the revenue in the Amsterdam auction is lower than that in the Vickrey and Fischer auctions. The auctions do not differ in terms of fraction of strictly positive bids despite the greater incentive to do so for the premium auctions than for the Vickrey auction.

The environment that we have studied is arguably an *ex ante* symmetric one in that anonymity assures that from the viewpoint of the bidders no subset of competing bidders could be identified as 'strong' or 'weak.' Our finding that the Amsterdam auction performs relatively poorly in such settings resonates with findings in laboratory experiments and theoretical results for risk-averse bidders. Our regression results indicate that risk aversion might partly drive the Amsterdam auction's poor relative performance in that risk aversion has a relatively large downward pressure on both entry and bids in the Amsterdam auction compared to the other auctions. The differences are statistically weak, though.

All in all, our results suggest that in *ex ante* symmetric settings (like anonymous online auctions) there is not a strong case for using premium auctions over standard auctions. For very risk-averse sellers, the Amsterdam auction might be attractive because of its relatively low revenue dispersion. Future research may reveal to what extent our findings extrapolate to *ex ante asymmetric* field settings, e.g., settings in which a 'strong' bidder competes against a set of 'weak' bidders. In such settings, premium auctions may perform relatively well as previous theoretical research and laboratory studies show. Future research may also highlight the effect of learning. In our field experiment, many bidders tended to bid value or below in all auctions, which is a weakly-dominated strategy for the Amsterdam auction. In the case of repeated interaction, bidders may learn that it

might make sense for them to bid above value in this auction so that the auction would perform better than in our one-shot setting. Finally, our finding that the auctions do not differ significantly in terms of strictly positive bids suggests that the incentive to enter the auction does not differ between auctions. Future research may test this hypothesis in an environment where the auction format is revealed to the bidders before they decide to enter.

Appendix A: Instructions

In this appendix, we present an English translation of the instructions on the Veylinx website for each of the three auctions. The instructions were presented to participants when they entered the website. Between treatments, the instructions only differed as far as the description of the auction rules is concerned. [We indicate the relevant auction format between square brackets.]

Screen 1

One bid
You can only submit a single bid.

Legally binding
It is not possible to withdraw your bid.

Everything included
Your bid includes VAT and shipping.

Payment
The winner pays less than his own bid. The winner must pay within 48 hours.

Closing
This auction closes on Wednesday, May 11, at 12 noon.

Shipment
The object will be shipped within a week once the payment is received.

Screen 2

Framed print, shop price €590

Description:

- Exclusive print, limited edition of 500 copies
- Including black wooden frame, 60 cm x 90 cm
- Acrylic glass premium sheet

Auction rules [Vickrey]:

- The highest bidder wins the framed print
- The winner does not pay his own bid but the second-highest bid

Auction rules [Amsterdam]:

- The highest bidder wins the framed print
- The winner does not pay his own bid but the second-highest bid
- Both the winner and the runner-up receive a bonus
- Both get 50% of the difference between the second-highest and the third-highest bid

Auction rules [Fischer]:

- The highest bidder wins the framed print
- The winner does not pay his own bid but the second-highest bid
- The runner-up receives a bonus equal to 5% of his or her bid

05:53

€ Place bid

Screens 3–7

What is your highest level of completed education?

Primary school
High school
Vocational training (*MBO*)
University of applied sciences (*HBO*)
Research university

In what range do you expect the highest bid to be?

Lower than €20
€20 – €49.99
€50 – €99.99
€100 – €199.99
€200 – €499.99
Higher than €500

Are you generally a person who is fully prepared to take risks or do you try to avoid taking risks?

(1) Very much prepared to take risks
(2)
(3)
(4) Neutral
(5)
(6)
(7) Not at all prepared to take risks

Did you submit a bid equal to what you are maximally willing to pay for the print?

Yes, I bid exactly my value for the print
No, I bid at most €10 more
No, I bid in between €10 and €20 more
No, I bid at least €20 more
No, I bid at most €10 less
No, I bid in between €10 and €20 less
No, I bid at least €20 less

Are you married?

Yes
No

Appendix B: Proofs of Lemmas and Propositions

Proof of Lemma 2. Hu et al. (2011) study a dynamic version of the Amsterdam auction. In this auction, the auctioneer raises the price. Bidders can signal at any price that they wish to leave the auction. All bidders are informed of other bidders leaving the auction at any price. The auction stops at the price at which only one bidder remains. This bidder wins the auctioned item and pays the price at which the auction stops. Both the winner and the runner-up receive a premium equal to α times the difference between the final price and the price where the third-last bidder left the auction. Hu et al. (2011) show that this auction has a unique perfect Bayesian equilibrium in which a bidder with value v leaves the auction at $b(v)$ independent of the observed history, i.e., the prices at which the other bidders left the auction. As a consequence, $b(v)$ is also the unique symmetric equilibrium of the sealed-bid Amsterdam auction. Because b is a strictly increasing function on $[\underline{v}, \overline{v}]$, the item is always awarded to the bidder with the highest value. Moreover, a bidder value $\underline{v}$ never obtains the item or the premium so her expected utility equals zero in equilibrium.

Proof of Lemma 3. Suppose the Fischer auction had an efficient equilibrium. Then all bidders bid according to the same, strictly increasing, bidding function. We call this function B. Let $F^{[1]}$ denote the distribution of the highest value among $n-1$ independent draws from F and $f^{[1]}$ the corresponding density function. Provided that all other bidders stick to the equilibrium bidding strategy, a bidder that has value v solves

$$max_w \varphi B(w) P^{[2]}(w) + \int_{\underline{v}}^{w} \big(v - B(x)\big) dF^{[1]}(x)$$

where $P^{[2]}(w)$ represents the probability that the bidder will submit the second-highest bid, i.e., the probability that exactly one bidder has a value greater than w. Observe that for all $v \in [\underline{v}, \overline{v}]$

$$P^{[2]}(v) = (n-1)F(v)^{n-2}\big(1 - F(v)\big) = \frac{f^{[1]}(v)}{f(v)}\big(1 - F(v)\big).$$

Let $p^{[2]}(v) \equiv P^{[2]\prime}(v)$ for $v \in [\underline{v}, \overline{v}]$.

The equilibrium first-order condition is given by

$$\varphi B'(v) P^{[2]}(v) + \varphi B(v) p^{[2]}(v) + (v - B(v)) f^{[1]}(v) = 0$$

or equivalently,

$$\varphi B'(v) P^{[2]}(v)\big(1 - F(v)\big)^{\frac{1}{\varphi}} + \varphi B(v) p^{[2]}(v)\big(1 - F(v)\big)^{\frac{1}{\varphi}} - \frac{B(v) P^{[2]}(v) f(v)}{\big(1 - F(v)\big)^{1-\frac{1}{\varphi}}} = -\frac{v P^{[2]}(v) f(v)}{\big(1 - F(v)\big)^{1-\frac{1}{\varphi}}}$$

The resulting differential equation is solved by

$$\varphi B(v) P^{[2]}(v)\big(1 - F(v)\big)^{\frac{1}{\varphi}} = \int_{v}^{\overline{v}} \frac{x P^{[2]}(x) f(x)}{\big(1 - F(x)\big)^{1-\frac{1}{\varphi}}} dx + C$$

where C is a constant. $C = 0$ because at the boundary $v = \bar{v}$, it holds true that $P^{[2]}(\bar{v}) = 1 - F(\bar{v}) = lim_{v \uparrow \bar{v}} \int_{v}^{\bar{v}} \frac{xP^{[2]}(x)f(x)}{(1-F(x))^{1-\frac{1}{\varphi}}} dx = 0$. Now, an inconsistency arises because at $v = \underline{v}$, the left-hand side is zero (as $P^{[2]}(\underline{v}) = 0$ for $n \geq 3$) while the right-hand side is strictly positive. Therefore, the assumption that B is a strictly increasing function must be violated. As a consequence, the Fischer auction does not have an efficient equilibrium.

Proof of Proposition 3. Myerson (1981) shows that in the case of risk-neutral bidders, a mechanism's expected revenue R is given by

$$R = \sum_{i=1}^{n} \left(v_i - \frac{1 - F(v_i)}{f(v_i)} \right) p_i(v_1, \dots, v_n) - \sum_{i=1}^{n} \underline{U_i}$$

where $p_i(v_1, \dots, v_n)$ denotes bidder i's probability of winning the auctioned item in equilibrium conditional on the bidders' values $v_1, \dots, v_n$. $\underline{U_i}$ represents bidder i's expected utility in equilibrium when she has the lowest possible value $\underline{v}$. Because F is log-concave, $v_i - \frac{1-F(v_i)}{f(v_i)}$ is strictly increasing in v_i. As a consequence, under the condition that the item is always awarded, it is optimal to award it to the bidder with the highest 'marginal value' $v_i - \frac{1-F(v_i)}{f(v_i)}$ and letting the bidder with the lowest possible value obtain zero expected utility. Expected revenue is lower in the Fischer auction than in both the Vickrey auction and the Amsterdam auction because according to the auction rules and Lemmas 1–3, (1) the item is always awarded in all three auctions, (2) the latter two auctions are efficient in contrast to the Fischer auction, and (3) bidders that have the lowest possible value obtain zero expected utility in the latter two auctions.

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