TI 2017-005/VI
Tinbergen Institute Discussion Paper

Debt overhang and the macroeconomics of carry trade

Sweder J.G. van Wijnbergen[1]
Egle Jakucionyte[1]

[1]Tinbergen Institute and the University of Amsterdam, the Netherlands.

Debt Overhang and the Macroeconomics of Carry Trade

Egle Jakucionyte* and Sweder J. G. van Wijnbergen†‡

December 30, 2016

Abstract

The depreciation of the Hungarian forint in 2009 left Hungarian borrowers with a skyrocketing value of foreign currency debt. The resulting losses worsened debt overhang in to debt-ridden firms and eroded bank capital. Therefore, although Hungarian banks had partially isolated their balance sheets from exchange rate risk by extending FX-denominated loans, the ensuing debt overhang in borrowing firms exposed the banks to elevated credit risk. Firms, households and banks had run up the open FX-positions hoping to profit from low foreign rates in the run-up to Euro adoption. This example of carry trade in emerging Europe motivates our analysis of currency mismatch losses in different sectors in the economy, and the macroconsequences of reallocating losses from the corporate to the banking sector *ex post*. We develop a small open economy New Keynesian DSGE model that accounts for the implications of domestic currency depreciation for corporate debt overhang and incorporates an active banking sector with financial frictions. The model, calibrated to the Hungarian economy, shows that, in periods of unanticipated depreciation, allocating currency mismatch losses to the banking sector generates a milder recession than if currency mismatch is placed at credit constrained firms. The government can intervene to reduce aggregate losses even further by recapitalizing banks and thus mitigating the effects of currency mismatch losses on credit supply.

Keywords: Debt overhang, foreign currency debt, leveraged banks, small open economy, Hungary;
JEL codes: E44, F41, P2

1 Introduction

In the period leading up to the crisis Hungarian households and businesses exploited a favourable interest rate differential and ran up massive foreign currency debt. This carry trade was in expectation of low exchange rate volatility in the run-up to the anticipated adoption of the Euro in the

*Tinbergen Institute and the University of Amsterdam, the Netherlands.
†Tinbergen Institute and the University of Amsterdam, the Netherlands.
‡Corresponding author. Address: Roetersstraat 11, 1018 WB Amsterdam. Email address: s.j.g.vanwijnbergen@uva.nl (S.J.G. van Wijnbergen)

near future. Both motives turned out to be wrong when in the first months in 2009 the Hungarian forint lost 26% of its value against the euro and even more against the Swiss franc [1]. The sharp depreciation of the forint considerably magnified the debt-to-GDP ratio; as a consequence the ratio of non-performing private loans increased sharply. Even those banks that shifted currency mismatch losses to borrowers by denominating loans in FX did not escape: while avoiding FX losses, they got increased credit risk in return.

We focus on Hungary as the most pronounced case of currency carry trade via corporate loans in emerging Europe, but unhedged foreign currency borrowing in the private non-financial sector and substantial bank foreign debt were ubiquitous in the region (IMF, 2012b). This motivates our focus on the macroeconomic implications of currency mismatch losses. In particular, what are the macroeconomic consequences of shifting exchange rate risk from borrowers to banks? Thus, besides the allocation of currency mismatch losses that reasonably resembles the Hungary's case before 2009, we also study a counterfactual case with bank lending denominated in domestic currency only. In contrast to foreign currency loans, domestic currency denomination relieves domestic firms of currency mismatch and thus reduces potential debt overhang in the corporate sector, but at the expense of leaving banks with substantial funding from abroad with increased currency mismatch on their balance sheets. Resulting bank losses may impair the credit transmission channel as much as losses from non-performing loans in the former scenario. This trade-off is the topic of this paper[2].

We explore the macroeconomic consequences of this trade-off by developing a quantitative model with corporate debt overhang and an active banking sector facing financial frictions. We confirm that avoiding direct exposure to exchange rate fluctuations does not save banks from losses in times of domestic currency depreciation but we do show that, after unanticipated depreciation, the economy bears smaller aggregate losses if firms' net worth is preserved by shifting currency mismatch losses to banks. Banks are in a better position to absorb currency mismatch losses because, in contrast to firms, they do not face default risk due to the various forms of insurance and bail-out provisions they are subject to. Even though banks are more leveraged than firms, unexpected bank losses affect borrowing conditions for firms and thus aggregate economic activity to a smaller extent than the investment distortion that can stem from a rising default probability in the firms' sector. This conclusion relies on the fact that banks may expect to be rescued by either the government or parent banks, while a large number of financially constrained firms cannot expect to be nationalized or receive other types of financial support to prevent them from going bankrupt. The second reason why allocating currency mismatch losses to firms generates larger real losses is that excessive corporate debt affects firms' decisions as they occur and thus inflicts output losses directly, while bank losses affect aggregate economic activity with a lag and only after a share of

[1] By March 2009, compared to September 2008.

[2] Corporate debt overhang in Hungary was as important as household debt overhang: in 2009 the share of corporate loans denominated in Swiss francs or euros was as high as the counterpart share in mortgages and amounted to more than fifty percent (Bank of Hungary, 2012). In this paper we choose to look at borrowing firms rather than indebted households to distinguish between the very different impact effects and transmission channels of non-performing corporate loans problems and the macroeconomic problems triggered by non-performing mortgages. We address household bankruptcies triggered by the deteriorating value of domestic currency in a companion paper (*reference*).

the effect is absorbed by bank equity.

Currency mismatch losses in Hungary

The currency mismatch situation in Hungary was unavoidably shaped by financial vulnerabilities developed prior to the forint depreciation. Our focus on debt overhang as triggered (or intensified) by the forint depreciation is supported by the data. In the run up to the crisis more than one half of private loans were taken in Swiss francs or euros (IMF, 2012b). Brown and Lane (2011) and Herzberg (2010) state that foreign currency borrowing in emerging Europe was not large-scale and concentrated among exporting firms, but studies with access to firm-level data in Hungary cast doubt on the firms' ability to hedge against the currency risk: Endrész et al. (2012) find that more than 82% of firms with foreign currency debt had no foreign currency revenue from exports, the survey of 698 Hungarian firms (Bodnár, 2009) discovers that also around 80% of foreign currency borrowers did not have a natural hedge. The weaker Hungarian forint resulted in significantly more bankruptcies among firms that borrowed in Swiss francs rather than Hungarian forints (Figure 2). Vonnák (2016) confirms that currency mismatch, and not the lending practices of Hungarian banks, contributed the most to the riskiness of foreign currency borrowers.

After 2008, foreign currency borrowers in Hungary were more likely to default and reduce investment (Endrész et al., 2012). Foreign currency borrowers were not only riskier, but, as data analysis in Endrész et al. (2012) shows, also had sizable shares in aggregate variables such as investment and debt in Hungary. We notice that at the macro level the gap between private investment and profit shares in Hungary kept increasing: after 2008 investment declined by more and took longer to recover than the measure for corporate profitability (Figure 1). Apparently, Hungarian firms were unwillling to invest retained earnings for several years which is a strong indic of worsening debt overhang. In contrast to monitoring costs based models (like Bernanke, Gertler and Gilchrist (1999)), Debt Overhang based approaches can explain prolonged under-investment in the recovery environment. If firms perceive their chances to default on accumulated debt as sufficiently high, their private benefits from investing diminish (Myers (1977)). Recessions with investment falling below the socially optimal level of investment tend to be deeper and longer.

Currency mismatch both in the corporate sector and in the banking sector is at the heart of the problem. Both businesses and banks in Hungary borrowed in foreign currency (Hungarian bank association, 2012). The banks' currency mismatch was reinforced by tight funding links between foreign parent banks and their subsidiaries in Hungary before the crisis. Moreover, isolation of currency mismatch losses in one sector is impossible due to the credit channel as banks are the main source of credit in the economy. This is common in all of emerging Europe, where they intermediate up to 80% of total credit (World Bank, 2015). Passing on FX mismatch to bank borrowers would not really isolate the banks given their predominant position as providers of debt to fnon-financial firms: even if only borrowers would have faced currency mismatch, domestic currency depreciation would deteriorate the quality of such loans and banks would shrink credit supply anyhow faced with rising Non Performing Loans (NPL) ratio's. Damage to the credit provision channel constituted the core of the ECB critique of the early repayment scheme of foreign currency mortgages with an artificially strong exchange rate instituted by the Hungarian Government for consumer mortgages, effectively shifting losses back to the lending banks: In 2011, against the advice of the ECB (ECB, 2011), the Hungarian government adopted such a scheme to aid debt-ridden households and forced banks to take massive losses[3]. In the authorities'view, losses that extensive might have posed a real threat of interrupting credit provision in Hungary and casted doubt on saving borrowers at the expense of lenders (even when lenders are foreign-owned). Even though this policy targeted households, we take it as evidence for the importance of credit channel.

For bank losses to impair credit provision, bank funding costs and loan supply have to depend on bank performance. Indeed, banks are frequently leverage-constrained themselves during crises as their own access to funding depends on the riskiness of their balance sheets (e.g. Diamond and Rajan, 2009). The banking system in Hungary was well-capitalized in 2008 (IMF, 2008), however, liquidity shocks at the outbreak of the crisis changed the situation dramatically (IMF, 2012a). The sudden dry-up of foreign funding caused a tightening of leverage constraints. To capture this channel, we introduce the second financial friction in the banking sector, namely a leverage constraint. We model it as an agency problem between banks and depositors following Gertler and Karadi (2011). The agency problem prevents banks from unlimited expansion of their balance sheets in good times. In bad times, non-performing loans in the corporate sector deplete bank equity so that the leverage constraint becomes tighter and leads to higher borrowing costs for banks. Eventually, the endogenous leverage constraint amplifies the drop in lending and economic activity. The feedback in bank lending is what makes the model structure complete and suitable to answer the research question formulated.

But what triggered the debt overhang situation to begin with? We look at the major shocks at the onset of the crisis in Hungary that could have led to domestic currency depreciation and so magnified the domestic currency value of foreign currency loans. The chronology of the pre-crisis events in emerging Europe points to external triggers instead of shocks of a local origin: despite severe domestic imbalances in emerging Europe, depreciation of local currencies followed spill-overs from the looming economic crash in advanced economies rather than happening at the same time. Based on anecdotal evidence and data (IMF, 2012a) we choose to look at three alternative (but not mutually exclusive) potential culprits: capital outflows, a drop in world demand for domestic exports and an increase in volatility in the markets.

We feed shocks into a small open economy New Keynesian model calibrated to Hungarian data. The international trade structure embedded in the model economy is similar to the set up used in Galí and Monacelli (2002), García-Cicco et al. (2014) and Adolfson et al. (2014). But the main new feature is the introduction of explicit debt overhang on the corporate level in the manner of Merton (1974)'s famous paper on pricing credit risk, where he shows that limited liability essentially implies a put option written by creditors to equity holders. Myers (1977) uses this approach to explore

[3]The estimated total bank losses from the early repayment scheme were around 1.1 billion euros or around 10% of total bank capital in Hungary (Reuters, 2012; authors' calculations).

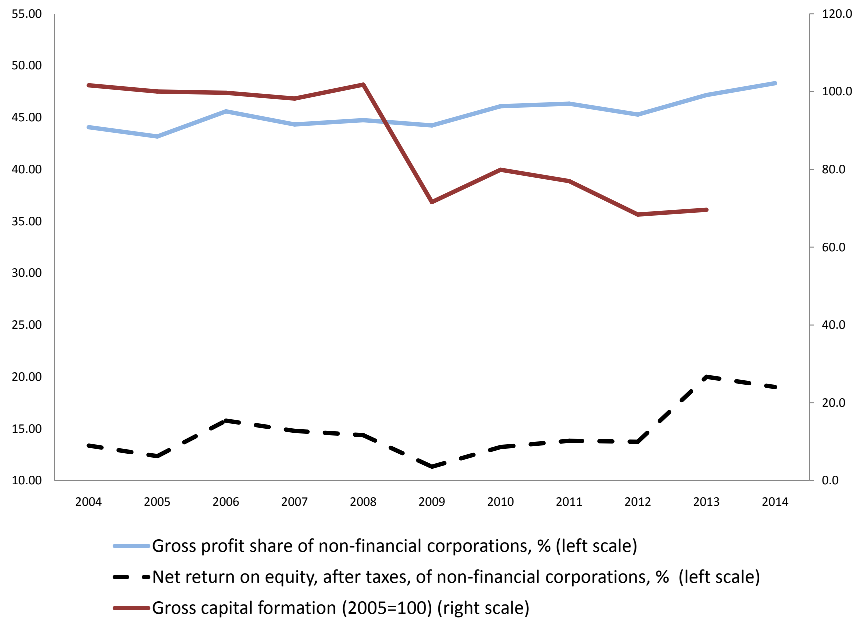


Figure 1: Profit share and private investment in Hungary.

Source: Eurostat.

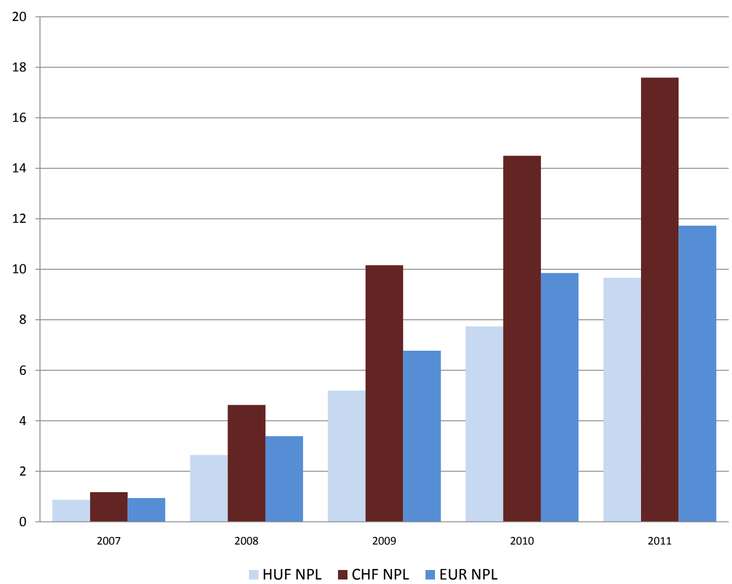


Figure 2: Ratio of non-performing corporate loans by currency in Hungary.

Source: Vonnák (2016).

the concept of debt overhang and its impact on investment, also a key element of our paper [4]. So we extend the endogenous leverage constraint model of Gertler and Karadi (2011) to include Merton (1974) like debt overhang on the corporate level with its associated moral hazard problems highlighted by Myers (1977).

The Merton put option approach to financial frictions between lender and borrower leads to another novelty in the paper. Despite using first-order approximation techniques to solve the resulting DSGE, volatility does have a first order impact on model outcomes because volatility shows up in the derivatives of that Merton put with respect to corporate investment and employment, in the same way volatility has an impact on general option derivatives (" the Greeks"), so we can use our model to study the impact of volatility shocks. The volatility related put option term in the financially constrained firms' optimization problem drives a wedge between social and private benefits from investing. Besides modeling a shock to volatility of firms' future profits, we endogenize volatility by incorporating uncertainty about prices: we simulate the model going back and forth between assumed and generated volatility until the two converge, thus endogenizing the overall volatility of corporate profits [5]. The obtained volatility value contains more information about the propagation of a particular shock in our model and thus is superior to an arbitrarily calibrated value.

The debt overhang friction stems from a particular limited contractibility feature of the debt contracts in the model. Borrowing firms are subject to limited liability which skews incentives towards taking too much risk and rules out a risk-free debt contract from the menu of optimal contracts. Second, banks cannot write a contract on how the loans they extend will be used: the quantities of capital and labour are determined unilaterally by the firm after it has received the loan. In the event of adverse shocks, these frictions may create debt overhang and distort the firms' choice of capital and labour demand.

The idea that risky debt makes firm forego valuable investment opportunities of course goes back to Myers (1977). Limited liability implies that debt is risky which may incentivize a sub-optimal investment strategy. Myers (1977) does not explore how the reduced value of the firm would affect firm's borrowing costs, the idea that default risk feeds into the credit spread is formalized in Merton (1974) who derives the credit spread as reflecting the unavoidable put option on the future assets of a debtor written out by the creditor to the equity holder. Our setup incorporates both seminal ideas: if debt is high enough, firms' incentives to invest diminish and a default spread goes up reinforcing the mechanism.

Out of several explanations how debt can reinforce business cycle fluctuations, only debt overhang is suitable for our research problem. The costly state verification framework famously introduced in macroeconomics by Gertler et alii (1999) just introduces an interest rate wedge, but because it allows lenders and borrowers to contract on investment and employment, avoids moral hazard and the associated debt overhang problems. A default wedge as in Gourio (2014) introduces corporate default effects on input providers instead of lenders and thus abstracts from the

[4] Occhino and Pescatori (2015) follow a similar approach.

[5] We thank Christian Stoltenberg for suggesting this numerical approach to endogenizing volatility.

credit channel which is crucial in the Hungarian story. This paper is the first attempt to use the non-contractible investment approach to explain the role of excessive debt and foreign currency debt in particular in business cycle analysis.

The structure of the paper is as follows. We discuss related literature in Section 2 and the model in detail in Section 3, and show simulations in Section 4. We discuss the results in Section 5, while Section 6 concludes.

2 Related literature

There is a lengthy corporate finance literature on debt overhang that starts with the seminal paper of Myers (1977). We contribute to the literature on macroeconomic consequences of debt overhang that were firstly examined in Lamont (1995). He argues that debt overhang can create strategic complementarities among investments of individual agents, thus potentially leading to multiple equilibria. Philippon (2010) studies the interaction between different indebted sectors in the model economy. The paper argues that debt overhang can create strategic complementarities between different economic sectors, namely, households and banks. In a closed economy, bailing out banks is efficient, while bailing out insolvent households means transferring funds to households that made inefficient saving decisions. In an open economy, countries have an incentive to free ride on foreign recapitalization programs, therefore, international coordination is required. Besides the shared focus on the credit transmission channel in an indebted economy, we go beyond the analysis in Philippon (2010) and study the business cycle properties of the model economy and apply the concept of debt overhang to excessive foreign currency debt.

Our set up comes closer to Gomes, Jermann and Schmid (2013) and Occhino and Pescatori (2014), who analyze the conduct of monetary policy in an environment with nominal debt. However, they focus on the effect of unanticipated inflation, while we focus on the debt overhang situation that arizes after domestic currency depreciation.

There is a vast literature that explores foreign currency debt effects in the costly state verification framework as implemented in Bernanke and Gertler (1989) and Bernanke, Gertler and Gilchrist (1999). Traditionally domestic currency depreciation invokes an expenditure switching effect that should stabilize demand for domestic goods. However, high foreign currency debt together with monitoring costs and sticky prices can potentially outweigh the expenditure switching effect and in turn make depreciations contractionary. Céspedes, Chang, and Vélasco (2004), Devereux, Lane and Xu (2006), and Gertler, Gilchrist, and Natalucci (2007) study the depreciation effects on firms in a small open economy setting. They incorporate a model of investment in which net worth affects the cost of capital and allow firms to borrow in foreign currency. They argue that even with high foreign currency debt depreciations remain expansionary. A similar model is considered in Cook (2004) where it leads to the opposite conclusion. Cook (2004) attributes this discrepancy to the type of price stickiness. If, as in Céspedes et al. (2004), input prices are sticky but output prices are not, domestic currency depreciation lowers real wages and increases revenues. Thus, the increase in firms' revenues might compensate for the soaring foreign currency debt and the depreciation remains expansionary. If, as in Cook (2004), output prices are sticky and input prices are not, revenues do not increase as fast as input costs and the depreciation can become contractionary. Despite the fact that these studies abstract from debt overhang, they emphasize the negative role of foreign currency debt and support our question too.

Empirical studies have established the relevance of financial frictions in explaining the macroeconomic outcomes. Without taking a stand on the prevalent financial friction, Towbin and Weber (2011) look at the data for 101 countries from 1974-2007 and show that high foreign currency debt increases the decline in investment in response to adverse external shocks. Kalemli-Özcan, Laeven and Moreno (2015) advance further by studying firm-bank-sovereign linkages in Europe to weigh the role of several financial frictions. They find that debt overhang is more important in explaining weak investment relative to explanations focusing on weak bank and other weak firm balance sheet channels. Therefore, debt overhang also has on average better chances in explaining poor investment performance in Hungary compared to other financial frictions.

Another branch of the literature that we relate to is centered upon volatility shocks. A recent contribution by Christiano, Motto and Rostagno (2014) attributes a significant share of business cycle fluctuations to idiosyncratic risk shocks fed through the time-varying idiosyncratic variance component. The variance component appears in the credit spread of entrepreneurs as in the costly state verification framework implemented in Bernanke et al. (1989). Thus the impact of the risk shock affects the credit spread rather than the default wedge in the firm's investment decision.

3 Model

Our focus is on the interaction between FX losses induced debt overhang, undercapitalized banks and corporate investment and employment decisions. To that end we introduce a Merton (1974)/Myers (1977) like debt overhang friction[6] in a model with leverage constrained banks in a small open economy context with foreign currency denominated private debt. The open sector with nominal rigidities generates realistic lending and output dynamics in the presence of foreign currency loans. We start the outline of the model by describing the more novel sections. We describe the more standard model blocks only briefly in the main text, all model details and associated derivations are in the supplementary appendix.

3.1 Financially constrained firms

Financially constrained firms live for two periods. Every period there is a new-born generation of firms and the total number of firms always constitute a continuum of mass one. In the first period

[6]See Ochino and Pescatori (2015) for a similar way of introducing corporate debt overhang, but in a closed economy model.

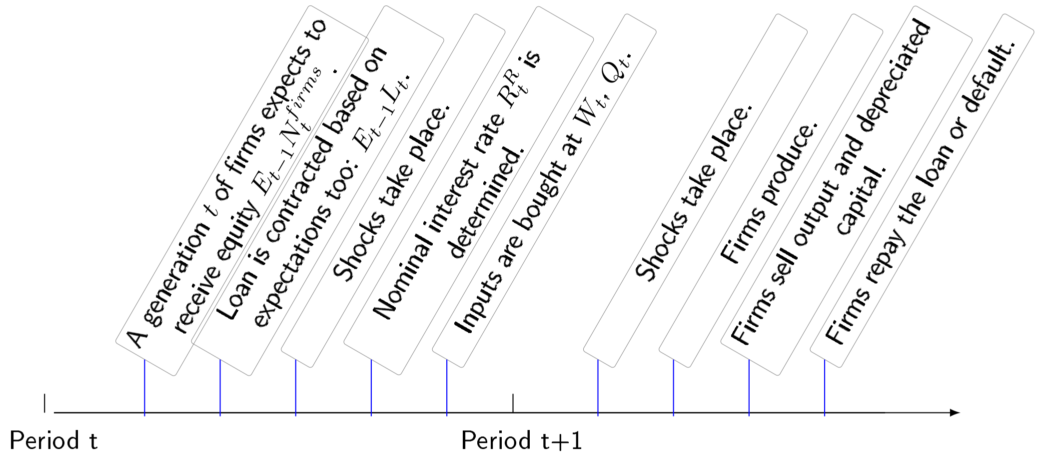


Figure 3: Timing for financially constrained firms.

firms buy two types of inputs, capital k and labour h, and have to pay for a fraction ρ in advance, which generates their demand for working capital. Production takes place in the next period.

To pay in advance, a financially constrained firm i uses two types of financing. First, it receives equity from households, $N_{i,t}^{firms}$. Second, it borrows from the bank an amount $L_{i,t}$ that consists of both domestic currency funds $L_{i,t}^D$ and foreign currency denominated funds $L_{i,t}^F$ such that $L_{i,t} = L_{i,t}^D + S_t L_{i,t}^F$ where S_t is the nominal exchange rate. We assume that the share of foreign currency denominated funds is fixed and denoted by α^F, so that the firm can choose the size of the total loan but not the denomination structure. This assumption allows us to calibrate the open position of banks and is innocuous enough, since we study the consequences of foreign currency borrowing rather than the choice of the borrowing currency.

To borrow, the firm has to pledge a share κ of future revenue as collateral where $0 < \kappa \leq 1$. We assume that the firm decides how much to borrow before shocks arrive and the prices of production inputs are revealed. Then the demanded size of the loan is equal to the expected expenditure for working capital minus the expected equity transfer from the household. It follows that in the beginning of period t the following condition holds:

$$E_{t-1}\left\{l_{i,t}\right\} + E_{t-1}\left\{n_{i,t}^{firms}\right\} = E_{t-1}\left\{\rho\left(q_t k_{i,t} + w_t h_{i,t}\right)\right\} \tag{1}$$

where q_t, w_t and rer_t denote the real price of capital, the real wage and the real exchange rate respectively. All three prices are expressed in units of composite goods. It follows that we define the real exchange rate as $S_t P_t^* / P_t$ where S_t is the nominal exchange rate, P_t is the price of composite goods and P_t^* defines the price level of foreign composite goods. $n_{i,t}^{firms}$ stands for the real equity transfer from the domestic household, where $n_{i,t}^{firms} \equiv N_{i,t}^{firms}/P_t$. $l_{i,t}$ stands for the size of the total loan expressed in units of composite goods and is defined as $l_{i,t} \equiv L_{i,t}/P_t$. After the loan is taken, shocks materialize, however, the predetermined size of the loan creates the debt overhang effect by distorting firm's private incentives to invest in production inputs.

The amount of corporate equity available is a factor in determining the firms' demand for funds and sets its "distance to default". In bad times, a higher fraction of firms default, which decreases the total value of corporate net worth. The household pools retained earnings and distributes them to new-born firms equally. So in bad times new generations of firms receive less equity from the household, therefore to produce the same amount of goods they have to leverage up more and thus will face a higher default risk. Note that firms die after two periods and thus do not take into account profits further out in the future, which mutes the macroeconomic net worth effect to some extent. The first generation of firms that enters the scene after the shock makes its borrowing decision based on expectations about the value of its net worth, so the net worth effect materializes for future generations of firms only.

Because of the timing of new information, the actual demand for working capital by the firm will in most cases not equal the loan amount received. We assume that in such cases the owner of the firm (the domestic household) steps in and transfers lump-sum funds $Z_{i,t}$ (where $z_{i,t} \equiv Z_{i,t}/P_t$) to cover the difference. Importantly, these funds constitute residual funding and firms cannot rely on them as the main source of finance. These funds enter the domestic household's budget constraint as a lump-sum transfer and have no effect on either the household's or the firm's incentives.

Let the matured loan in units of composite goods be $R^R_{i,t}\left(\frac{l^D_{i,t}}{\pi_{t+1}} + rer_{t+1}\frac{l^F_{i,t}}{\pi^*_{t+1}}\right)$, where $R^R_{i,t}$ is the nominal gross interest rate on the loan. The bank sets interest rates on loans after the shocks take place, therefore, the loan rate adjusts to clear the loan market. We define real loans in different currencies as $l^D_{i,t} \equiv L^D_{i,t}/P_t$ and $l^F_{i,t} \equiv L^F_{i,t}/P^*_t$. The contracted collateral is a fraction κ of firms' revenue from selling goods and depreciated capital in the next period, $p^R_{t+1}y^R_{i,t+1} + q_{t+1}(1-\delta)k_{i,t}$. p^R_{t+1} stands for the price of homogenous goods, expressed in units of composite goods ($p^R_{t+1} \equiv P^R_{t+1}/P_{t+1}$). Then the decision of the financially constrained firm i born in period t whether to default or not is determined by the lower value:

$$\min\left\{R^R_{i,t}\left(\frac{l^D_{i,t}}{\pi_{t+1}} + rer_{t+1}\frac{l^F_{i,t}}{\pi^*_{t+1}}\right), \quad \kappa\left(p^R_{t+1}y^R_{i,t+1} + q_{t+1}(1-\delta)k_{i,t}\right)\right\} \tag{2}$$

where $p^R_{t+1}y^R_{i,t+1} = p^R_{t+1}A_{t+1}\theta_{i,t+1}k^{\alpha}_{i,t}h^{1-\alpha}_{i,t}$.

The firm i born in period t and endowed with corporate equity $n^{firms}_{i,t}$ maximizes profits taking the loan as given. The firm maximizes the expected sum of future revenue from selling goods and depreciated capital subtracted by the second fraction of working capital expenditure together with expenses related to the debt payment. Financial flows received in period t also enter the maximization problem and can be summarized as the difference between the loan plus equity (both $n^{firms}_{i,t}$ and $z_{i,t}$) and working capital expenditure:

$$\max_{\{k_{i,t},h_{i,t}\}} E_t\beta\Lambda_{t,t+1}\left\{p^R_{t+1}y^R_{i,t+1}+q_{t+1}(1-\delta)k_{i,t}-(1-\rho)\frac{q_tk_{i,t}+w_th_{i,t}}{\pi_{t+1}}\right\}$$
$$-E_t\beta\Lambda_{t,t+1}\min\left\{R^R_{i,t}\left(\frac{l^D_{i,t}}{\pi_{t+1}}+rer_{t+1}\frac{l^F_{i,t}}{\pi^*_{t+1}}\right),\quad \kappa\left(p^R_{t+1}y^R_{i,t+1}+q_{t+1}(1-\delta)k_{i,t}\right)\right\}$$
$$+l_{i,t}+n^{firms}_{i,t}+z_{i,t}-\rho\left(q_tk_{i,t}+w_th_{i,t}\right)$$

s.t.

$$E_{t-1}\left\{l_{i,t}\right\}+E_{t-1}\left\{n^{firms}_{i,t}\right\}=E_{t-1}\left\{\rho\left(q_tk_{i,t}+w_th_{i,t}\right)\right\}$$

The resulting first-order conditions are[7]:

$$k_{i,t}:\quad E_t\beta\Lambda_{t,t+1}\left\{p^R_{t+1}\frac{\partial y^R_{i,t+1}}{\partial k_{i,t}}+q_{t+1}(1-\delta)-(1-\rho)\frac{q_t}{\pi_{t+1}}\right\}$$
$$-E_t\beta\Lambda_{t,t+1}\left\{(1-\Phi(d_{1,t}))\kappa\left(p^R_{t+1}\frac{\partial y^R_{i,t+1}}{\partial k_{i,t}}+q_{t+1}(1-\delta)\right)\right\}$$
$$=\frac{\partial cov\left(\beta\Lambda_{t,t+1},\quad \min\left\{R^R_{i,t}\left(\frac{l^D_{i,t}}{\pi_{t+1}}+rer_{t+1}\frac{l^F_{i,t}}{\pi^*_{t+1}}\right),\quad \kappa\left(p^R_{t+1}y^R_{i,t+1}+q_{t+1}(1-\delta)k_{i,t}\right)\right\}\right)}{\partial k_{i,t}}$$
$$+\rho q_t$$

$$h_{i,t}:\quad E_t\beta\Lambda_{t,t+1}\left\{p^R_{t+1}\frac{\partial y^R_{i,t+1}}{\partial h_{i,t}}-(1-\rho)\frac{w_t}{\pi_{t+1}}\right\}$$
$$-E_t\beta\Lambda_{t,t+1}\left\{(1-\Phi(d_{1,t}))\kappa\left(p^R_{t+1}\frac{\partial y^R_{i,t+1}}{\partial h_{i,t}}\right)\right\}$$
$$=\frac{\partial cov\left(\beta\Lambda_{t,t+1},\quad \min\left\{R^R_{i,t}\left(\frac{l^D_{i,t}}{\pi_{t+1}}+rer_{t+1}\frac{l^F_{i,t}}{\pi^*_{t+1}}\right),\quad \kappa\left(p^R_{t+1}y^R_{i,t+1}+q_{t+1}(1-\delta)k_{i,t}\right)\right\}\right)}{\partial h_{i,t}}$$
$$+\rho w_t$$

where

$$d_{2,t}\equiv\frac{E_t\ln\left(\kappa\left(p^R_{t+1}y^R_{i,t+1}+q_{t+1}(1-\delta)k_{i,t}\right)-R^R_t rer_{t+1}\frac{l^F_{i,t}}{\pi^*_{t+1}}\right)-E_t\ln\left(R^R_{i,t}\frac{l^D_{i,t}}{\pi_{t+1}}\right)}{\sigma_y},\qquad d_{1,t}=d_{2,t}+\sigma_y$$

[7]The derivation of the first-order conditions and the term $d_{2,t}$ in particular are provided in the supplementary appendix A1-A2.

Note the similarity to the credit risk approach pioneered by Merton (1974): because of limited liability firms effectively receive a put option from creditors, ex ante this is priced in (that is where the credit risk comes from) but because investment and employment are not contractible in the debt contract, a moral hazard problem persists. The debt overhang friction introduces an additional term in otherwise standard demand functions for capital and labour: conditions incorporate a proxy for the default probability, $(1 - \Phi(d_{1,t}))$, that reduces a marginal product of capital and a marginal product of labour. Thus in this problem the default probability is what drives the wedge between social benefits from investing and private benefits from investing. When the default probability increases, private benefits would diminish and demand for labour and capital would shrink resulting in a lower level of working capital than a socially optimal one. Under-investment in working capital has negative and prolonged implications on aggregate variables: we can distinguish between static debt overhang effects and dynamic debt overhang effects. Static debt overhang results from a decline in demand for working capital which depresses aggregate demand on impact. Dynamic debt overhang occurs, if the indebted sector uses capital as input. Then sub-optimally lower demand for capital shrinks demand for investment. Lower investment today decreases capital stock available for production tomorrow which prolongs the economic recovery.

The second implication of the first-order conditions relates to the option structure as reflected by the definition of the function argument $d_{2,t}$. The default probability directly depends on a volatility term σ_y^2 which captures the variance of future profits. σ_y^2 is given by

$$var\left(\pi_{t+1}\left(\kappa\left(p^R_{t+1}y^R_{i,t+1} + q_{t+1}(1-\delta)k_{i,t}\right) - R^R_t\frac{rer_{t+1}l^F_{i,t}}{\pi^*_{t+1}}\right)\right)$$

and depends on exogenous productivity shocks, working capital and endogenous volatility of prices and exchange rate value in the domestic economy. The first-order conditions imply that increased uncertainty about of future collateral value reduces firms' chances to repay. Looming uncertainty during the latest crisis[8] highlights the importance of the volatility term in explaining borrowing conditions for firms and firms' willingness to borrow and suggests that we cannot assume constant volatility without a loss of generality. Thus we model an exogenous shock to a volatility term to simulate increased uncertainty about financially constrained firms' performance in the future as one of possible triggers of debt overhang.

Noteworthy, the default probability varies not only with stochastic components such as technology but with expected prices and exchange rates as well. This motivates our simulation exercise in which we simulate the model until the endogenously implied volatility of firms' expected collateral value converges. This exercise allows us to incorporate the second-order characteristics of the economy and obtain a better estimate for the volatility term than an arbitrary calibrated value.

In the beginning of every period, after shocks take place and a fraction of firms default, the

[8]The implied volatility indexes for both European markets and Poland rocketed in the end of 2008, see the plot in the Appendix (Figure 10). We do not have a measure for Hungary, however, the implied volatility index for Polish markets should serve as a satisfactory proxy for the markets' risk perception for the Hungarian economy.

domestic household pools the remaining net worth from non-defaulted firms into aggregate net worth by following the aggregation rule:

$$\begin{aligned} n_t^{firms} = \;& \omega^{firms}\left(p_t^R y_t^R + q_t(1-\delta)k_{t-1} - (1-\rho)\frac{q_{t-1}k_{t-1} + w_{t-1}h_{t-1}}{\pi_t}\right) \\ & - \omega^{firms}\left((1-\Phi(d_{1,t-1}))\,\kappa\left(p_t^R y_t^R + q_t(1-\delta)k_{t-1}\right) + \Phi(d_{2,t-1})R_{t-1}^R\frac{l_{t-1}^D}{\pi_t} + \Phi(d_{1,t-1})rer_t\frac{l_{t-1}^F}{\pi_t^*}\right) \\ & + \iota^{firms}\cdot n^{firms} \end{aligned}$$

Recall that $(1-\Phi(d_{1,t-1}))$ proxies for the default rate (by the law of large numbers this is equal to the share of defaulted firms in the economy). Then the first term on the right hand side is aggregate firms' revenue from production and selling depreciated capital minus the rest of the expenditure for working capital. The second term is the firms' aggregate expenditure for repaying loans. The difference between the two gives financially constrained firms' profits. The third term is the injection of new equity. We assume that the domestic households acts as distributor and cannot divert pooled equity funds anywhere else. Also the existing equity can be increased only by the amount $\iota^{firms}\cdot n^{firms}$ that is fixed and proportional to aggregate net worth in the steady state. Thus, this equity transfer does not depend on the household's decision. ω^{firms} is a fraction that is close but lower than unity. We assume that this parameter proxies for the equity management costs incurred by the household and use this parameter to calibrate the steady state corporate leverage to the one observed in the data.

3.2 Banks

Domestic households own all banks that operate in the domestic economy and lend to financially constrained domestic firms. We assume that there is a continuum of these banks and every period there is a probability ω that a bank continues operating. Otherwise, the net worth is transferred to the owners of the bank, domestic households.

We assume that banks give loans to firms out of accumulated equity n_t, domestic deposits d_t and foreign debt d_t^*. A fraction of banks' liabilities (foreign debt) is denominated in foreign currency which exposes banks to currency mismatch. Lending in foreign currency hedges the open currency position for banks[9]. However, shifting exchange rate risk to the credit constrained corporate sector increases the credit risk for banks. We consider two lending scenarios which have different implications for bank currency mismatch. First, banks lend in domestic currency only which creates currency mismatch on their balance sheets. The second scenario is described by bank lending in both foreign currency and domestic currency so that banks are relieved from currency mismatch. We will consider these two cases in the following discussion on shifting currency mismatch. The

[9]We calibrate the share of loans denominated in foreign currency such that banks do not have a zero open currency position in that case. This allows us to distinguish between the credit risk effects and the exchange rate risk effects.

model with loans denominated in both currencies is described here, while the model with lending in domestic currency only is described in the supplementary appendix B2.

The balance sheet constraint of a bank j, expressed in units of composite goods, is given by

$$n_{j,t} + d_{j,t} + rer_t d^*_{j,t} = l_{j,t}$$

Banks pay a nominal domestic interest rate R_t on deposits and a nominal foreign interest rate $R^*_t \xi_t$ on foreign debt. R^*_t follows a stationary AR(1) process. ξ_t denotes a premium on bank foreign debt. To ensure stationarity in the model, we assume that the premium depends on the level of foreign bank debt (as in Schmitt-Grohé and Uribe, 2003):

$$\xi_t = \exp\left(\kappa_\xi \frac{(rer_t d^*_t - rer \cdot d^*)}{rer \cdot d^*} + \frac{\zeta_t - \zeta}{\zeta}\right) \tag{3}$$

where ζ_t is an exogenous shock that follows a stable AR(1) process.

Banks are subject to an agency problem as in Gertler and Karadi (2011). At the end of every period, bankers can divert a fraction λ^L of assets, but if that happens the bank goes bankrupt (i.e. cannot continue). Creditors take this possibility into account and lend only up to the point where the continuation value of the bank is equal to or higher than the value of what can be diverted. This condition acts as an incentive constraint for the bank and eventually limits expansion of the balance sheet of the bank for given amount of equity.

Loan performance directly affects bank profits, loans to domestic financially constrained firms are the only asset on the banks' balance sheet. When the default probability $(1 - \Phi(d_{2,t}))$ for financially constrained firms increases, banks expect lower returns. High corporate leverage has similar consequences as it increases the size of loans for the same level of production and reduces firms' chances to repay ceteris paribus. We define the expected return for the bank j as $R^L_{j,t}$. The definition makes use of the derivation of the expected loan payment (see the supplementary appendix A2) and in its final expression directly incorporates the default probability on corporate loans:

$$E_t\left\{\frac{R^L_{j,t}}{\pi_{t+1}} l_{j,t}\right\} \equiv E_t \min\left\{R^R_{j,t}\left(\frac{l^D_{j,t}}{\pi_{t+1}} + rer_{t+1}\frac{l^F_{j,t}}{\pi^*_{t+1}}\right), \quad \kappa\left(p^R_{t+1} y^R_{j,t+1} + q_{t+1}(1-\delta)k_{j,t}\right)\right\}$$

$$\Rightarrow E_t\left\{\frac{R^L_{j,t}}{\pi_{t+1}} l_{j,t}\right\} \equiv E_t\left\{(1-\Phi(d_{1,t}))\,\kappa\left(p^R_{t+1} y^R_{j,t+1} + (1-\delta) q_{t+1} k_{j,t}\right) + \Phi(d_{2,t}) R^R_{j,t}\frac{l^D_{j,t}}{\pi_{t+1}} + \Phi(d_{1,t}) R^R_{j,t} rer_{t+1}\frac{l^F_{j,t}}{\pi^*_{t+1}}\right\} \tag{4}$$

To facilitate further discussion, we define two components of the overall bank spread (actual rate charged to borrowers minus the cost of funds to the bank). The first is the default spread, measured as the difference in the actual interest rate charged on the loan and the expected return on the loan: $E_t\left(R^R_{j,t}-R^L_{j,t}\right)/\pi_{t+1}$. The higher is the spread, the more the bank charges to compensate for the default risk. Second, there is the component of the overall bank spread that depends on the banking friction: it captures the premium that arises due to the endogenous leverage constraint. This spread is given by the difference in the expected return on the loan to financially constrained firms and the expected funding costs to the bank: $E_t\left(R^L_{j,t}/\pi_{t+1}-R^*_t\xi_t/\pi^*_{t+1}\frac{rer_{t+1}}{rer_t}\right)$. Note the role of real exchange rate changes in determining the expected costs of funding. So the overall credit spread is the sum of the default spread and the bank spread and is given by $E_t\left(R^R_{j,t}/\pi_{t+1}-R^*_t\xi_t/\pi^*_{t+1}\frac{rer_{t+1}}{rer_t}\right)$. A higher credit spread reflects tighter borrowing conditions due to either one or both of the financial frictions.

Then the optimization problem of the bank j can be written as:

$$V_{j,t}=\max_{\left\{d_{j,t},d^*_{j,t},l_{j,t}\right\}} \quad E_t\left[\beta\Lambda_{t,t+1}\left\{(1-\omega)n_{j,t+1}+\omega V_{j,t+1}\right\}\right]$$

s.t.

$$V_{j,t}\geq\lambda^L l_{j,t}, \qquad \textit{(Incentive constraint)}$$

$$n_{j,t}+d_{j,t}+rer_t d^*_{j,t}=l_{j,t}, \qquad \textit{(Balance sheet constraint)}$$

$$n_{j,t}=\frac{R^L_{j,t-1}}{\pi_t}l_{j,t-1}-\frac{R_{t-1}}{\pi_t}d_{j,t-1}-\frac{R^*_{t-1}\xi_{t-1}}{\pi^*_t}rer_t d^*_{j,t-1} \qquad \textit{(LoM of net worth)}$$

The first-order conditions follow:

$$d_{j,t}: \quad (1+\nu_{1,t})\beta E_t\Lambda_{t,t+1}\left\{(1-\omega)+\omega\nu_{2,t+1}\right\}\left(\frac{R_t}{\pi_{t+1}}\right)=\nu_{2,t} \tag{5}$$

$$d^*_{j,t}: \quad (1+\nu_{1,t})\beta E_t\Lambda_{t,t+1}\left\{(1-\omega)+\omega\nu_{2,t+1}\right\}\left(\frac{R^*_t\xi_t}{\pi^*_{t+1}}\frac{rer_{t+1}}{rer_t}\right)=\nu_{2,t} \tag{6}$$

$$l_{j,t}: \quad (1+\nu_{1,t})\beta E_t\Lambda_{t,t+1}\left\{(1-\omega)+\omega\nu_{2,t+1}\right\}\left(\frac{R^L_{j,t}}{\pi_{t+1}}\right)=\lambda^L\nu_{1,t}+\nu_{2,t} \tag{7}$$

$\nu_{1,t}$ and $\nu_{2,t}$ are the Lagrangian multiplier to the incentive constraint and the Lagrangian multiplier to the balance sheet constraint combined with the law of motion for equity, respectively.

Equations (5) and (6) govern the bank debt portfolio choice. Equation (5) presents the marginal cost to the bank from issuing one additional unit of deposits (the left hand side) in relation to the marginal benefit from increasing equity by one unit, $\nu_{2,t}$ (the right hand side). The marginal cost from issuing one additional unit of foreign bank debt is compared to the marginal benefit from increasing equity on the right hand side of equation (6) and is adjusted for changes in the exchange rate value. The structure of these choice rules suggests that in equilibrium the bank has to be indifferent between taking deposits or issuing bank debt to foreign agents.

Equation (7) presents the relation between the marginal benefit to the bank from issuing one additional unit of loans (the left hand side) and the marginal cost (the right hand side). We see that in equilibrium one additional unit of loans earns the discounted risk adjusted return on loans. Firstly, this return has to increase in the marginal cost from issuing bank debt to finance the expansion of the balance sheet, $\nu_{2,t}$. Secondly, due to the endogenous bank leverage constraint, the risk adjusted bank return on loans also increases in the share of divertable assets λ^L and the marginal loss to the bank creditor in the case of asset diversion, $\nu_{1,t}$. Both terms proxy for the marginal cost associated with the tighter incentive constraint. Moreover, the tighter leverage constraint increases the bank spread as well which translates into more credit tightening.

The first-order conditions hold together with complementary slackness conditions:

$$\nu_{1,t}: \quad \nu_{1,t}\left(V_{j,t} - \lambda^L l_{j,t}\right) = 0$$

$$\nu_{2,t}: \quad \nu_{2,t}\left(\frac{R^L_{j,t-1}}{\pi_t} l_{j,t-1} - \frac{R_{t-1}}{\pi_t} d_{j,t-1} - \frac{R^*_{t-1}\xi_{t-1}}{\pi^*_t} rer_t d^*_{j,t-1} - rer_t l^*_{j,t} + d_{j,t} + rer_t d^*_{j,t}\right) = 0$$

The set of equilibrium conditions also includes the law of motion for aggregate net worth of banks and the bank incentive constraint. First, we formulate the law of motion for aggregate net worth. We assume that aggregate net worth consists of the net worth of non-bankrupted banks and the new worth of new banks. The new equity is injected by domestic households and is assumed to be of the size ιn. Then

$$n_t = \omega\left(\frac{R^L_{j,t-1}}{\pi_t} l_{t-1} - \frac{R_{t-1}}{\pi_t} d_{t-1} - \frac{R^*_{t-1}\xi_{t-1}}{\pi^*_t} rer_t d^*_{t-1}\right) + \iota n \tag{8}$$

3.3 Financial sector support

Financial sector support is modelled as in Kirchner and van Wijnbergen (2016), we assume that the government can intervene during the crisis by injecting capital τ_t^{FS} in the banks. We assign the following rule to the recapitalization of the financial intermediary j:

$$\tau_t^{FI} = \kappa_{FS}\left(shock_{t-l} - shock\right) n_{j,t-1}, \qquad \kappa_{FS} > 0, \quad l \geq 0$$

where $n_{j,t-1}$ is the net worth of the intermediary from the previous period. The recapitalization can be immediate ($l = 0$) or delayed ($l > 0$). The variable $shock_t$ equals the shock driving the crisis, e.g. the risk premium shock ($shock_t \equiv \xi_t$). We assume that the recapitalization is a gift from the government and does not have to be repaid (van der Kwaak and van Wijnbergen (2014) explore the consequences of different payback rules).

Now the bank equity increases in the equity injection from the government besides being a function of loan returns and borrowing costs:

$$n_{j,t} = \frac{R^L_{j,t-1}}{\pi_t} l_{j,t-1} - \frac{R_{t-1}}{\pi_t} d_{j,t-1} - \frac{R^*_{t-1}\xi_{t-1}}{\pi^*_t} rer_t d^*_{j,t-1} + \kappa_{FS}\left(shock_{t-l} - shock\right) n_{j,t-1}$$

Bank's optimization problem would yield different results now. We present modified first-order conditions in the supplementary appendix B3.

3.4 Households

We assume a representative household. The household has two alternatives to invest in: make deposits d_t in a bank or buy domestic bonds issued by the government, b_t. The household supplies labour to a competitive labour market. The household has Greenwood–Hercowitz–Huffman (henceforth, GHH) preferences as in Greenwood et al. (1988), so labour supply does not depend on wealth. The household chooses a level of real consumption c_t and working hours h_t such that the following lifetime utility function is maximized:

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} \left(c_t - \frac{\chi \left(h_t\right)^{1+\varphi}}{1+\varphi} \right)^{1-\gamma} \qquad \gamma, \chi, \varphi > 0 \tag{9}$$

subject to the household's budget constraint, expressed in units of composite goods:

$$c_t + b_t + d_t = w_t h_t + \frac{R_{t-1}}{\pi_t} b_{t-1} + \frac{R_{t-1}}{\pi_t} d_{t-1} + \Pi_t - t_t \tag{10}$$

π_t denotes the composite goods price inflation. We assume that the household is indifferent between buying domestic bonds and making deposits, thus, R_t is nominal gross interest rate of both domestic bonds and deposits. The household owns all banks in the model economy and thus receives lump-sum dividends, Π_t. Taxes t_t enter the household's budget constraint in a lump-sum way as well. Lump-sum dividends from financially constrained firms are included in total dividends Π_t. Lump-sum dividends from financially constrained firms consist of firms' profits that the household receives in the beginning in the period minus the equity that the household transfers in the beginning of the period.

3.5 Production and Pricing

There are several types of firms in the domestic economy. It takes three types of firms to produce domestic aggregate inputs for composite goods. First, there are the financially constrained firms that combine purchased capital with labour and produce homogenous goods. They were analyzed in Section 3.1. Their homogenous outputs are bought by retail firms who costlessly differentiate the products bought and sell them as (local) monopolists, in Dixit-Stiglitz (1977) fashion. A similar group of firms called importers differentiate foreign (imported) goods. A composite goods producer buys the differentiated home goods and aggregates them into an aggregate domestic good y_t^H with associated price p_t^H. The same composite goods producer also buys imported differentiated goods and aggregates them into a foreign aggregate good y_t^F. The corresponding aggregate price level of foreign goods is p_t^F. All details of the derivations of the various first order conditions optimization problems can be found in the supplementary appendix D. We discuss each step in more detail below.

3.5.1 Retail firms

Homogenous goods produced by financially constrained firms are sold to domestic retail firms. A domestic retail firm j differentiates purchased inputs at no cost and sells at a monopolistic price $p_t^H(j)$. We assume that only a fraction $(1-\omega^H)$ of domestic retail firms can adjust prices every period as in Calvo (1983). The fraction ω^H of remaining firms adjust past prices by the rate π_t^{adj}. The aggregate price level that prevails in the retail sector is denoted by p_t^H. Differentiated goods from the domestic retail sector, $y_t^H(j), \quad j \in (0,1)$, are purchased by the composite goods producer.

3.5.2 Importers

Imported foreign goods undergo a differentiation process that is similar to what happens with domestic goods. The retailers differentiating foreign composite goods are called importers. Importers also exercise (local) market power and set prices in a staggered way, again as in Calvo (1983), which allows for incomplete exchange rate pass-through. Thus, $(1-\omega^F)$ of importers change their past prices to the optimal price at period t. The fraction ω^F of remaining firms adjust past prices by the rate π_t^{adj}.

3.5.3 Composite goods producer

We assume that the composite goods producer has access to an aggregation technology and can assemble differentiated goods at no cost. First, the composite goods producer assembles differentiated domestic goods $y_t^H(j)\forall j$ into domestic aggregate goods y_t^H and differentiated imported goods $y_t^F(j)\forall j$ into foreign aggregate goods y_t^F. She uses the following assembling technologies:

$$y_t^H = \left(\int_0^1 y_t^H(j)^{1-\frac{1}{\epsilon_H}}\, dj\right)^{\frac{\epsilon_H}{\epsilon_H-1}},$$

$$y_t^F = \left(\int_0^1 y_t^F(j)^{1-\frac{1}{\epsilon_F}}\, dj\right)^{\frac{\epsilon_F}{\epsilon_F-1}}$$

Then she combines domestic aggregate goods and foreign aggregate goods into composite goods y_t^C with the aggregation technology that takes the taste parameter for foreign aggregate goods η as given:

$$y_t^C \equiv \left((1-\eta)^{\frac{1}{\epsilon}}(y_t^H - ex_t)^{\frac{\epsilon-1}{\epsilon}} + \eta^{\frac{1}{\epsilon}}(y_t^F)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{11}$$

Only a share of domestic aggregate goods is used to produce composite goods and the rest is exported, because we assume exports not to have imported content. Thus, exporters would export domestic aggregate goods rather than composite goods. ϵ stands for elasticity of substitution between domestic aggregate goods and foreign aggregate goods. The composite good y_t^C is sold to the domestic household, the government and capital goods producers. Its associated price is P_t.

3.5.4 Capital producers

Capital producers sell capital to financially constrained firms at the real competitive price q_t and buying the depreciated capital stock back next period. To restore the depreciated capital, capital producers add composite goods (investment) i_t as additional inputs to the depreciated capital stock by using a technology subject to investment adjustment costs $\Gamma\left(\frac{i_t}{i_{t-1}}\right)$:

$$k_t = (1-\delta)k_{t-1} + \left(1 - \Gamma\left(\frac{i_t}{i_{t-1}}\right)\right) i_t \tag{12}$$

where adjustment costs Γ equal:

$$\Gamma\left(\frac{i_t}{i_{t-1}}\right) = \frac{\gamma}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2$$

3.5.5 Exporters

We assume that perfectly competitive exporters demand ex_t units of the domestic aggregate good y_t^H, so the supply of the assembled production of domestic retailers has to satisfy both the demand of the composite goods producer and the demand of exporters. Exported goods consist of the domestic aggregate, so they do not use imported inputs.

Exports are sold at a price p_t^H/rer_t which is the price of domestic aggregate goods expressed in units of foreign composite goods. The foreign demand for domestic aggregate goods is price-sensitive:

$$ex_t = \eta^* \left(\frac{p_t^H}{rer_t}\right)^{-\epsilon_*} y_t^* \tag{13}$$

Consistent with the small open economy assumption, P_t^* and y_t^* are assumed to evolve exogenously.

3.6 Government

We abstract from normative analysis of government policies and take government spending as exogenous. We assume that to finance a stochastic stream of real government expenditure g_t and the bank recapitalization program τ_t^{FS}, the government collects lump-sum taxes t_t from the household and issues domestic bonds b_t. It has to satisfy the budget constraint (expressed in units of composite goods):

$$g_t + \tau_t^{FS} + \frac{R_{t-1}}{\pi_t} b_{t-1} = t_t + b_t$$

We assume that taxes follow this rule:

$$t_t = t + \kappa^B (b_{t-1} - b) + \kappa^{FS} \tau_t^{FS} + e_t, \qquad 0 < \kappa^B \leq 1, \quad 0 \leq \kappa^{FS} \leq 1$$

So a fraction κ^{FS} of the recapitalization expenditure is covered by increasing the lump-sum tax and the remaining fraction $(1 - \kappa^{FS})$ is financed by issuing new government debt.

3.7 Monetary policy

The central bank conducts monetary policy by following the Taylor rule:

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\gamma_R} \left(\frac{y_t^H}{\bar{y^H}}\right)^{(1-\gamma_R)\gamma_Y} \left(\frac{\pi_t^H}{\bar{\pi^H}}\right)^{(1-\gamma_R)\gamma_\pi} \exp(mp_t) \tag{14}$$

where mp_t is a monetary policy shock and the domestic aggregate goods price inflation π_t^H can be expressed as $\pi_t^H = p_t^H / p_{t-1}^H \pi_t$.

3.8 Market clearing

The domestic household, the government and capital producers buy composite goods. Therefore, the supply of composite goods y_t^C has to satisfy the aggregate demand of domestic agents:

$$y_t^C = c_t + i_t + g_t \tag{15}$$

3.9 Current account and its components

Trade balance expressed in units of composite goods is given by:

$$tb_t = p_t^H ex_t - m_t$$

where m_t denotes the value of imports and can be expressed as $m_t \equiv rer_t D_t^F y_t^F$ (see the supplementary appendix J for details).

So the current account is given by the sum of real trade balance and real net income from abroad. In units of composite goods the current account is given by:

$$ca_t = tb_t + ni_t \tag{16}$$

The domestic household owns banks that issue foreign debt d_t^*. Banks are the only agents to borrow from abroad. Also, we assume that nobody in the domestic economy lends to foreign agents.

As a result, real net income from abroad is negative and equal to minus payments of bank foreign debt. It follows that

$$ca_t = tb_t - \left(R^*_{t-1}\xi_{t-1} - 1\right) rer_t \frac{d^*_{t-1}}{\pi^*_t}$$

In equilibrium the current account has to equal the capital account balance which is given by the change in bank foreign debt. The equilibrium condition is follows, expressing the change in foreign debt in units of composite goods as well to get:

$$tb_t - \left(R^*_{t-1}\xi_{t-1} - 1\right) rer_t \frac{d^*_{t-1}}{\pi^*_t} = -\left(rer_t d^*_t - rer_t \frac{d^*_{t-1}}{\pi^*_t}\right)$$

4 Preliminaries to analyzing the model

4.1 Calibration

To employ the theoretical model for empirical simulation, all parameters are calibrated to Hungarian data. We list calibrated parameter values and targeted steady state values in Table 2 in the Appendix. Parameters that are endogenously determined in steady state are $\beta, \chi, \eta^*, \kappa, \omega$ and π^*. χ is chosen such that average working hours in the steady is 0.3 as it is common in the literature. η^* is chosen such that the ratio between the steady state foreign output and the domestic output is equal to the share of the Hungarian GDP in the EU GDP, namely 0.007. π^* follows from satisfying the UIP condition in the steady state given the foreign nominal interest rate of 4.5 p.p. in annual terms. The most important ones of the rest of endogenously determined and calibrated parameters are discussed below.

The financial frictions we introduce bring a few additional parameters to calibrate. The debt overhang friction depends on the corporate default rate value in the steady state, $1 - \Phi(d_2)$. Due to *de facto* non-existent corporate bond market in Hungary, we choose to calibrate the steady state default probability to an average default frequency of corporate loans in Hungary over the period 2002-2007 as reported by the Bank of Hungary. This makes $1 - \Phi(d_2) \approx 0.03$. We choose the bankruptcy loss parameter κ such that the steady state default probability in the model matches the data counterpart. The banking friction relies on the fraction of capital that can be diverted, λ^L, the proportional transfer to the entering bankers, ι, and bank leverage in the steady state. We calibrate ι to 0.002 following the original paper of Gertler and Karadi (2011). Bank leverage matches the average bank leverage in the OECD data for year 2007. We make an adjustment to the average bank leverage of 8.6 in Hungary as reported by Bank of Hungary: we adjust for the average fraction of loans in total assets and get $8.6 \cdot 0.65 \approx 5.6$. The remaining parameter, λ^L, is chosen such that the lending spread in the steady state match the observed difference between nominal corporate loan interest rate and nominal corporate deposit rate in Hungary in 2001:Q1-2008:Q3 (data from the Bank of Hungary). Our computations yield an annual lending spread of 2.7 p.p. It follows that

$\lambda^L = 0.45$.

We calibrate the share of foreign currency loans in total corporate loans to 0.6 to match the aggregate share of FX corporate loans in Hungary in 2007-2008 (Krekó and Endrész, 2010). For the model with loans of hybrid denomination we calibrate the steady state trade balance such that bank liabilities denominated in foreign currency would match foreign currency loans exactly.

We have also calibrated several steady state values using data from the Eurostat online database. The steady state annual inflation in Hungary over the period 2001:Q1-2008:Q3 was 5.9 p.p., we choose the discount factor β such that the steady state inflation in the model matches the data counterpart. The ratio of government spending to GDP, s^g, is set to 0.22. The ratio of imported goods in domestic consumption is computed in the following way. We take the share of imports to GDP in Hungary (72.7 percent) over the period 2002:Q1-2008:Q4 and adjust it given the average import share in the Hungarian exports (56 percent; OECD, 2015). Since in our model exports are assumed to be of domestic origin entirely, we lower the observed import share in GDP by the amount of imports used in export production and get that the import share in domestic demand should constitute around 37 percent in our model. Thus we calibrate η to 0.37 to achieve the desired steady state share. For simplicity we set the steady state level of the nominal exchange rate to unity.

4.2 Endogenizing volatility

As we pointed in the financially constrained firms' optimization problem, our model is capable of studying volatility effects. Besides modeling a shock to volatility of firms' future profits, we can endogenize the volatility term by incorporating uncertainty about prices. We obtain the endogenized volatility value for future profits of financially constrained firms by simulating the theoretical model as long as the value converges. In this section we explain why the obtained volatility value is a better choice than an arbitrary calibrated value. We shortly describe the simulation procedure as well.

First order conditions that govern financially constrained firms' behavior contain a proxy for the default probability. The default probability depends not only on expected values of future revenue and liabilities but on variances of future revenue and liabilities as well and, as a result of endogenous prices, it varies not only with stochastic components such as technology but with production prices and exchange rates as well. Therefore, we cannot postulate the variance of future output or future liabilities to be an exogenous process dependent on technology and current state variables only. The variance of endogenous variables is unknown, but we can obtain an estimate from simulated series. In the supplementary appendix A2 we derive what variance exactly we are interested in to be able to compute the default probability and simulate the model:

$$\sigma^2_{y,t+1} = var\left(ln\left(\pi_{t+1}\left(\kappa\left(p^R_{t+1}y^R_{t+1} + q_{t+1}(1-\delta)k_t\right) - R^R_t rer_{t+1}\frac{l^F_{i,t}}{\pi^*_{t+1}}\right)\right)\right)$$

Debt denomination	Banking friction	Shock	Value
FX & domestic currency	No	Risk premium	0.1428
FX & domestic currency	No	World demand	0.0568
FX & domestic currency	No	All shocks	0.1459
Domestic currency	No	Risk premium	0.0678
Domestic currency	No	World demand	0.0591
Domestic currency	No	All shocks	0.0848
FX & domestic currency	Endogenous leverage constraint	Risk premium	0.2148
FX & domestic currency	Endogenous leverage constraint	World demand	0.0785
FX & domestic currency	Endogenous leverage constraint	All shocks	0.2117
Domestic currency	Endogenous leverage constraint	Risk premium	0.1216
Domestic currency	Endogenous leverage constraint	World demand	0.0768
Domestic currency	Endogenous leverage constraint	All shocks	0.1294

Table 1: Simulated standard deviations of expected profits for firms (σ_y)

Hence to simulate the model we need a numerical value for $\sigma^2_{y,t+1}$ or, more precisely, $\sigma_{y,t+1}$, where $\sigma_{y,t+1} = \sqrt{\sigma^2_{y,t+1}}$. We assume $\sigma_{y,t+1}$ to be constant ($\sigma_{y,t+1} = \sigma_y$).

To find a value for $\hat{\sigma}_y$ as close to the true value as possible we follow several steps:

1. Set a threshold level for convergence of the calibrated $\hat{\sigma}_y$ to the value of $\tilde{\sigma}_y$ that follows from the simulated time series generated by the model.

2. Choose an initial value for $\hat{\sigma}_y$.

3. Simulate the model with the chosen value for $\hat{\sigma}_y$.

4. Compute volatility of $\bar{y}_{t+1}$ from simulated time series and denote it by $\tilde{\sigma}^2_y$.

5. Compute the difference between the chosen value $\hat{\sigma}_y$ and the simulated value $\tilde{\sigma}_y$. If the difference is larger than the threshold value, set $\hat{\sigma}_y = \tilde{\sigma}_y$ and repeat steps 3-5.

Converged values are presented in Table 1. We obtain estimates of the volatility value generated by capital outflows shocks and a drop in world demand only. The exogenous volatility shock sometimes prevents the simulation from converging because every new value shapes the results of the next simulation (the shock effect directly depends on the simulated value in the last period). So instead of simulating to obtain the volatility estimate generated by the exogenous volatility shock we use the average volatility retrieved after a set of shocks hit the economy: the productivity shock, the risk premium shock and the world demand shock.

5 Results

In the following section we dissect the interaction of financial distress in the firms' sector and losses in the banking sector. We begin by discussing the debt overhang friction in the firms' sector and its consequences in the periods of unanticipated depreciation. Next we add the banking friction to the setup to see how leverage-constrained banks can amplify the shocks even further. The relative importance of the frictions is analyzed by comparing two scenarios of allocating currency mismatch losses. Given immense foreign bank funding flows in emerging Europe, we assume that domestic banks issue debt denominated in foreign currency which creates currency mismatch unless banks match foreign currency liabilities with loans issued in foreign currency. In the latter case currency mismatch is shifted to domestic borrowers. We compare the model economy with bank lending in domestic currency and bank lending in both foreign currency and domestic currency to explore which currency mismatch situation generates larger macroeconomic losses.

More plots for every shock discussed in the following section can be found in the Appendix. Here we present graphs with the most important variables only.

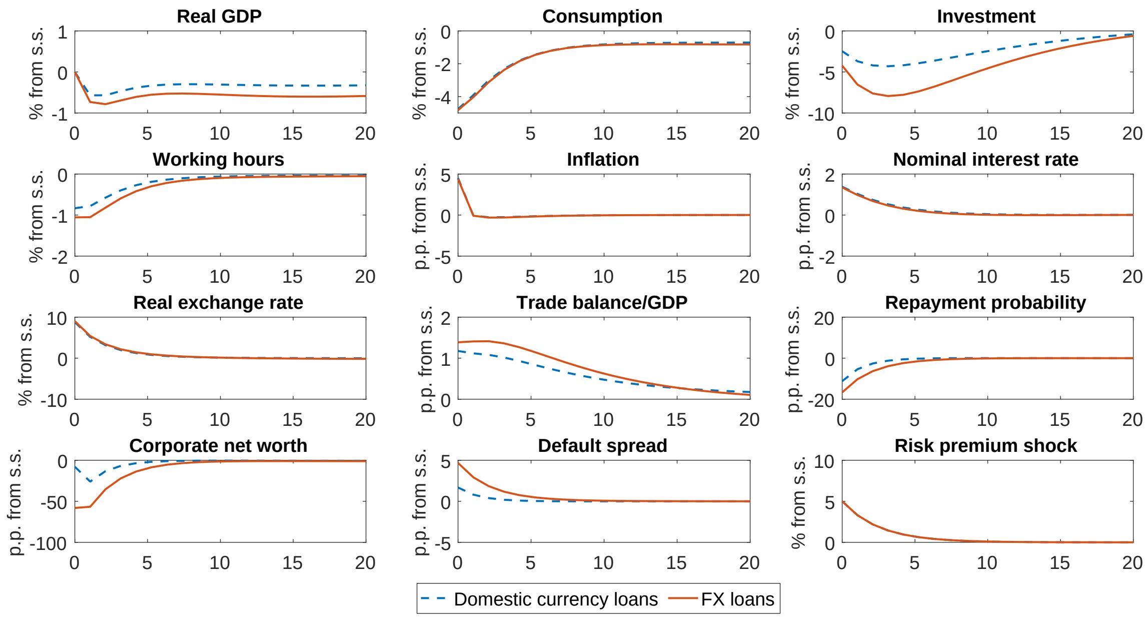
5.1 Debt overhang in the financially constrained firms' sector

Borrowing in foreign currency makes domestic financially constrained firms prone to debt overhang whenever the domestic currency depreciates. If the expected value of debt indeed exceeds the expected collateral value, the indebted firm faces a higher chance of losing its collateral (future revenue) to creditors. The firm's marginal benefits from investing diminish. In the setting with non-contractible investment, the rising possibility of default is enough to create a slump in output by decreasing investment. We consider exogenous events that may trigger domestic currency depreciation in a small open economy setup and thus increase the default probability: a country risk premium shock, a negative world demand shock and a shock to volatility of profits generated in the financially constrained firms' sector.

Regardless of the denomination of corporate debt, the listed shocks are expected to bring an economic downturn by either dampening aggregate demand or supply. Accumulated foreign currency debt makes the corporate default probability depend not only on the aggregate level of economic activity but the degree of currency mismatch as well. Thus, whenever the domestic currency depreciates, foreign currency debt opens an additional contractionary channel that operates through even higher default probabilities and thus more intense debt overhang in the financially constrained firms' sector.

Simulation results confirm our hypothesis that debt overhang amplifies adverse effects on aggregate variables more, if firms have their debt denominated in foreign currency rather than in domestic currency. In Figure 4 capital outflows, which we model by increasing a country risk premium on bank foreign debt, decrease demand for domestic currency and make it depreciate. To mute rising domestic inflation, the central bank responds by raising the domestic nominal interest rate and in turn creates a recession. The main driver behind it is the decline in consumption driven by the substitution effect. Currency mismatch for firms makes the recession deeper as investment in working capital decreases not only due to lower aggregate demand but also due to debt burden weighing on firms' marginal benefits from investing. We observe that, if financially constrained firms borrow in foreign currency, a repayment probability is substantially lower, an interest rate

Figure 4: Country risk premium shock of 5 p.p. in the model without leverage-constrained banks.

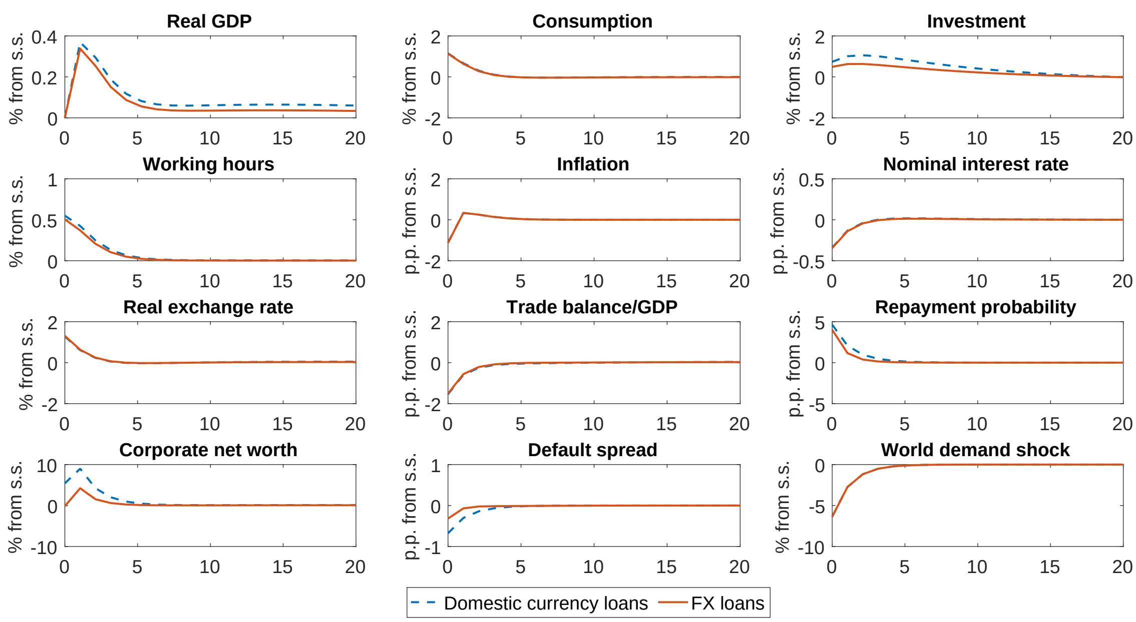


on their loans rises higher and they post lower demand for labour and capital goods. Noteworthy, domestic currency depreciation not only distorts decisions ex-ante, but deprives firms of available funds ex-post: lower firms' profits result in lower corporate worth and thus higher dependence on external funds which come at a now high default spread.

A decline in world demand for domestic exports, as exhibited in Figure 5, results in deflation. Domestic prices have to decrease so that the drop in external demand would be compensated by increased competitiveness. Domestic currency depreciates. Interesting enough, financially constrained firms face a lower default probability. They do not experience losses in corporate net worth which also contributes to higher demand for labour and capital. Consequently the effect on output is positive. The paradoxical result partially owes to the predetermined demand for working capital as posted by financially constrained firms. When external demand for domestic goods declines, the domestic demand has to increase to absorb the idle output produced out of predetermined production inputs. Therefore, domestic prices drop sufficiently to make domestic consumers capable of consuming more. The increased domestic demand effect does not die immediately and the next period output grows to catch up with higher demand. To see that this is indeed the reason we modify the model so that labour demand can adjust immediately and output responds to changes in aggregate demand on impact (see the supplementary appendix A4 for modeling details). Figure 6 shows how the drop in world demand becomes contractionary once labour demand can shrink in response to fewer orders for domestic goods from abroad.

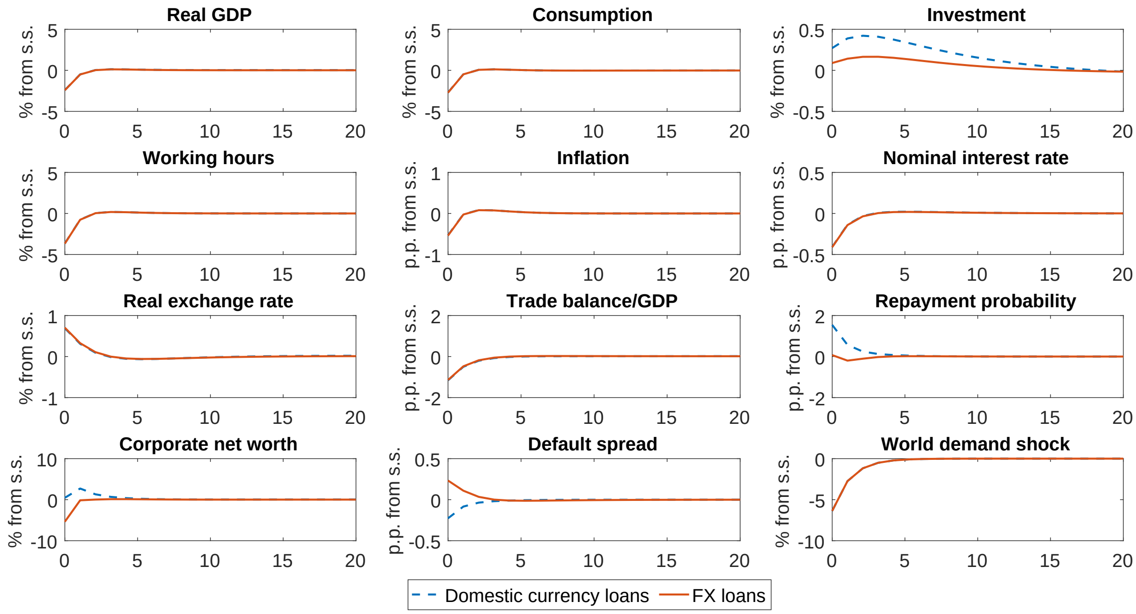
Currency mismatch brings in more negative effects, however, the difference is relatively small, see Figure 5. It turns out that the resulting domestic currency depreciation is too small to increase the wedge between the value of debt and the collateral value for financially constrained firms. A higher depreciation is needed due to a relatively restrictive version of the debt overhang model. First, we model short-term debt which, in contrast to long-term debt, makes debt overhang fade away after the first period. Second, the timing of the firm's optimization problem is such that firms learn about their net worth value after the borrowing amount is decided. Therefore, even though domestic currency depreciation triggers more defaults and thus reduces corporate net worth (Figure 5), the shock feedback through the corporate net worth comes with a delay. Third, firms die after two periods and do not take into account future profits which mutes the net worth effect to some extent as well. Shocks have to propagate through prices mostly and thus the exchange rate effect on firms' performance in the future is limited.

Figure 5: World demand shock of 6.4% in the model without leverage-constrained banks.



In contrast to other shocks, the volatility shock primarily affects not the demand side but the supply side of the economy by making the firms' future profits more uncertain. This has a direct effect on the default probability as the uncertainty magnifies the expected distance between the collateral value and the debt value. Then, for any debt burden and any productivity level, firms face lower chances to repay their debt and lenders respond by raising interest rates on corporate loans. Figure 7 depicts how in this case debt overhang weighs on the firms' incentives to invest and in turn the economy falls into a recession. The increased uncertainty of firms' future profits has an indirect effect on household consumption by lowering income: firms post lower demand for labour and wages decrease. The substitution effect stimulates consumption as the central bank

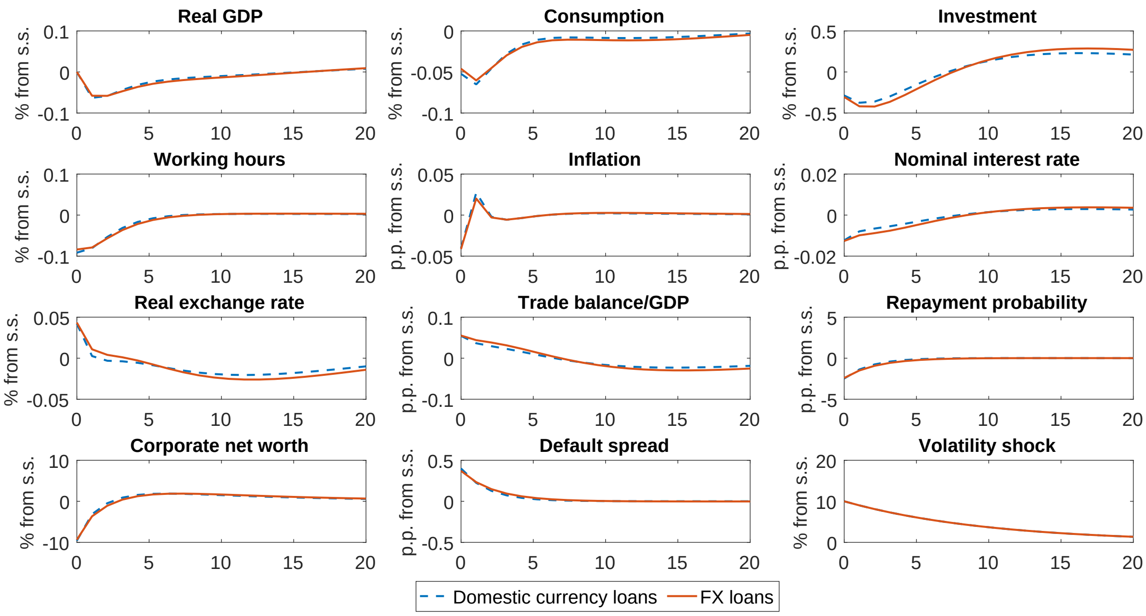
Figure 6: World demand shock of 6.4% in the model without leverage-constrained banks when labour demand is not predetermined.



copes with the slump and the corresponding deflation by cutting the policy rate, however, this effect appears to be negligible. Overall, the volatility shock generates responses of relatively large magnitude, changes in investment are particularly large. Initially, foreign currency debt generates more contraction than accumulated domestic currency debt, however, after two periods the real exchange rate depreciation in the former cases subsides and depreciation-driven debt overhang loses its influence completely. The difference between the case with borrower currency mismatch and without it is negligible. Besides the reasons mentioned before, the volatility shock directly hits firms' chances to repay and the depreciation effect becomes of the second order. In other words, the magnitude of the change in the default probability overshadows the risk related to the increased value of foreign currency debt.

Therefore, capital outflows can trigger domestic currency depreciation that increases currency mismatch in the corporate sector. Compared to firms borrowing in domestic currency only, the depreciation lands firms indebted in foreign currency in a more severe debt overhang situation. Under-investment and a deeper fall in output follow. The effects of the negative world demand shock and the increased exogenous uncertainty are less clear as they trigger an apparently insufficient loss in the domestic currency value. Also, the volatility shock increases firms' chances to default to an extent that depreciation effects get overshadowed and debt denomination loses its role in ranking the outcomes. The type of shocks appears to have important implications for the role of foreign currency debt and debt overhang.

Figure 7: Volatility shock of 10% in the model without leverage-constrained banks.



5.2 Introducing leverage-constrained banks

The agency problem between banks and depositors generates an endogenous credit spread which tightens or improves borrowing conditions for banks depending on bank leverage. Highly leveraged banks face larger credit spreads on their debt. It follows that the credit spread moves countercyclically: in bad times non-performing loans deplete bank capital and bank leverage goes up.

Financial distress in the banking sector translates into worse borrowing conditions for the borrowing firms: the tighter endogenous leverage constraint and thus higher borrowing costs for banks make banks charge higher interest rates on loans issued to financially constrained firms. In bad times the binding bank leverage constraint amplifies initial losses in the economy.

In our experiment bank losses are triggered by currency mismatch losses placed in either the firms' sector or the banking sector. Bank debt denominated in foreign currency exposes the banking sector to currency mismatch, so that domestic currency depreciation has an immediate negative effect on bank equity and leverage. If the bank lends in foreign currency as much as it borrows in foreign currency, the depreciation increases the value of both sides of the bank balance sheet and bank earnings do not deteriorate *ceteris paribus*. However, domestic currency depreciation triggers large losses for domestic firms that borrowed in foreign currency. Lower firms' profits result in a higher ratio of non-performing loans and bank profits decline. Therefore, even if lending in foreign currency insulates the bank balance sheet from the exchange rate risk, a potentially higher increase in non-performing loans can still impair the credit transmission channel and worsen the recession.

This paper shows that, even if the model is enriched with the endogenous bank leverage constraint, aggregate losses are still smaller when corporate loans are denominated in domestic currency than when a share of debt is foreign currency loans.

Banks are in a better position to absorb currency mismatch losses because, in contrast to firms, they do not internalize default risk. Consequently, even though banks are more leveraged than firms, unexpected bank losses affect borrowing conditions for firms and thus aggregate economic activity to a smaller extent than the investment distortion that stems from the rising default probability in the firms' sector. This assumption relies on that fact that banks may expect to be rescued by either the government or parent banks, while a large number of firms cannot expect to be nationalized or receive other types of financial support to prevent them from going bankrupt. The second reason why allocating currency mismatch losses to firms generates larger real losses is that firms burdened with debt decrease aggregate output and demand directly, while banks affect aggregate economic activity with a lag and only after a share of the effect is absorbed by bank equity.

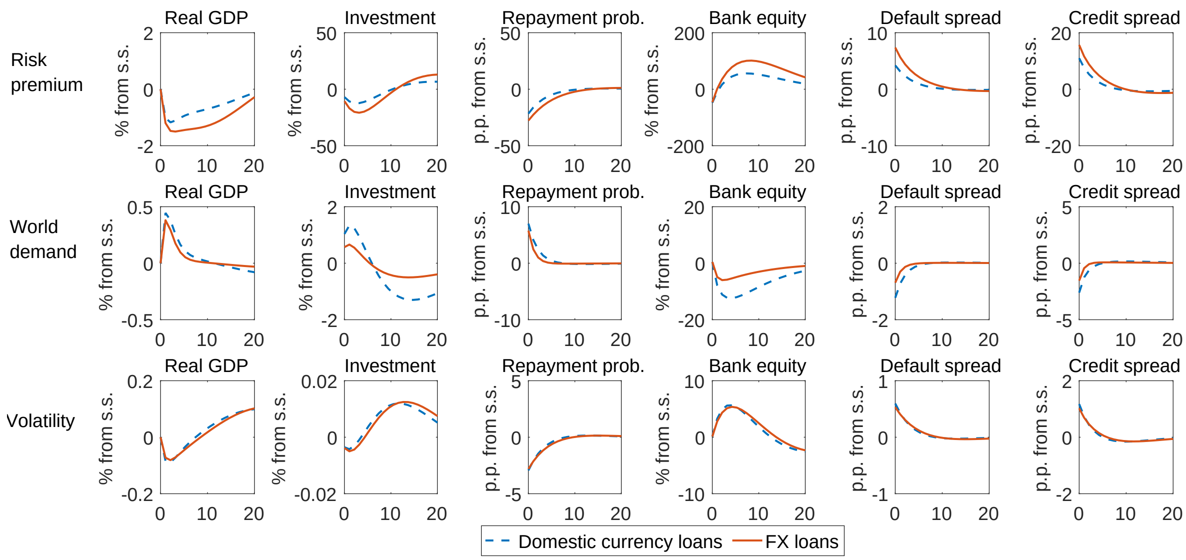
We arrive at the previously described conclusion after simulating the same set of shocks as before for the extended model. After the risk premium shock or the world demand shock, foreign currency debt worsens firms' chances to repay which generates larger output losses, see Figure 8. More non-performing loans deplete bank equity on impact and make banks ration credit for future borrowers. Over time, as the default frequency for firms goes down, banks replenish bank equity and the recession is contained. On the contrary, if banks face currency mismatch on their balance sheets, bank losses are smaller on impact but, since banks cannot switch to foreign currency lending later, the depreciation has a persistent negative effect on bank equity. Bank losses translate into persistent real losses for two additional reasons: bank cut lending to all firms rather than just troubled firms which constrains economic activity severely. Second, since banks accumulate equity out of retained earnings, even temporary bank losses can have a persistent effect on borrowing conditions in the economy. Nevertheless, we see that in the case of capital outflows magnified foreign currency debt and the related failures to repay offset bank gains from insulating their balance sheets from the exchange rate risk. Consequently foreign currency loans make domestic depreciation deepen the recession.

The drop in world demand for domestic goods also suggests that currency mismatch shifted to banks produce smaller aggregate losses in the short-run, however, it may generate a situation when currency mismatch in banking inflicts more recessionary outcomes in the future. However, this is not obvious. The volatility shock makes the default probability skyrocket and the role of foreign currency debt is limited in ranking the outcomes.

Simulations show that, in the first period after the shock, banks charge a substantially higher default spread. In subsequent periods changes in the default spread are approximately the same regardless of the allocation of currency mismatch losses. It follows that corporate default risk determines borrowing costs for firms in initial periods and the bank leverage constraint dominates the dynamics of costs further in the future.

In closing this section, it is important to note that the assumption of the financially constrained

Figure 8: IRFs in the model with leverage-constrained banks.



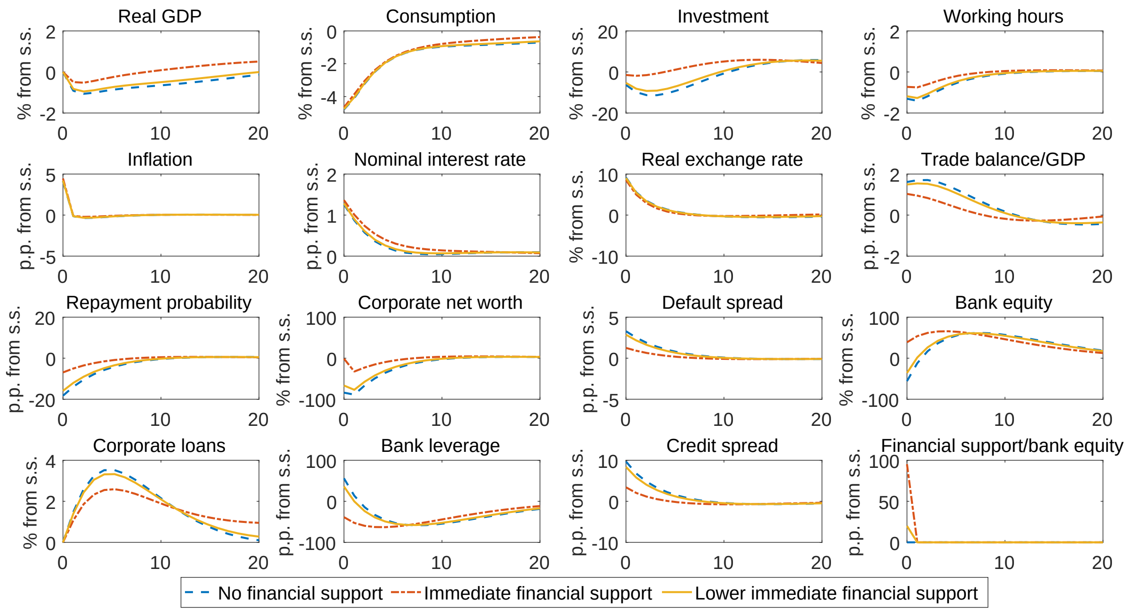
firms' exit after two-periods makes the effect of the debt overhang friction rather suspended in time. In contrast, banks incorporate their net worth dynamics in their optimization problem which makes bank losses have a prolonged effect on the economy. This can be considered as a bias towards the banking friction. The result that debt overhang nevertheless governs the dynamics of aggregate variables in the extended model lends more support to the importance of currency mismatch losses in the corporate sector in amplifying negative shocks than our model could offer. Even though the government should not underestimate the effects of bank losses derived from currency mismatch on the bank balance sheets, our simulations show that increasing currency mismatch for banks at the expense of lowering currency mismatch for borrowers is likely to result in lower macroeconomic losses.

5.3 Bank recapitalization

Shifting currency mismatch losses to banks reduces debt overhang and, as we showed before, leads to most likely less recessionary macroeconomic outcomes. However, this implies saving financially constrained firms at the expense of the banking sector. Further we study the efficiency of a government intervention that aims at compensating for bank losses. We study the scenario where bank losses stem from bearing the exchange rate risk while financially constrained firms avoid currency mismatch altogether.

Financial sector support is modeled as a gift from the government to banks given in the form of an equity injection. Consider the case of capital outflows which generated the largest economic downturn in the series of our experiments. Figure 9 shows how full recapitalization of the banking

Figure 9: No government intervention vs. bank recapitalization.



sector after the increase in the country risk premium immediately relaxes the endogenous bank leverage constraint and improves bank borrowing conditions. Banks cut credit supply by less and the economy undergoes a smaller recession than otherwise. Corporate loans increase by less in response to this policy because corporate net worth is replenished faster than the investment demand increases. The reason is the following. Financially constrained firms take the size of their net worth as given, therefore, higher net worth makes them demand fewer loans. However, investment demand is late to catch up with the increase in corporate net worth, because firms make borrowing decisions given their expectations of net worth value rather than the actual value. This assumption creates a lag in the net worth feedback to firms' working capital expenditure. Nevertheless, banks cut lending spreads as loans become less risky. Financial support of 20% bank equity would yield similarly positive but smaller changes in aggregate outcomes.

Therefore, currency mismatch in the banking sector can be efficiently alleviated ex-post. Noteworthy here we abstract from the potential negative implications of government interventions such as increasing public debt during times of fiscal distress (van der Kwaak and van Wijnbergen, 2014).

6 Conclusions

Hungary's experience after the fall in the domestic currency value in 2009 raised questions about the macroeconomic implications of allocating currency mismatch losses. We attempt to evaluate the consequences of shifting exchange rate risk from borrowers to banks: we weigh losses triggered by increased currency mismatch in the financially constrained firms' sector against losses for banks, if banks bear currency mismatch instead. As almost everywhere in emerging Europe banks heavily rely on foreign currency debt. This borrowing pattern exposes banks to currency mismatch, unless they lend in foreign currency and thus shift exchange rate risk to borrowers. Empirical evidence suggests that the forint depreciation amplified debt overhang in the private sector in Hungary and banks operating in Hungary were leverage-constrained. Therefore, to answer the research question, we develop a small open economy New Keynesian DSGE model with debt overhang in the corporate sector and the banking sector that operates under the endogenous leverage constraint.

The model, calibrated to the Hungarian economy, suggests that debt overhang in the corporate sector and losses at leverage-constrained banks are closely related and reinforce each other through the channel of credit provision. Nevertheless, we determine that capital outflows can trigger domestic currency depreciation that is large enough to strengthen debt overhang in the corporate sector and generate a large recession. Debt overhang and the related real losses dominate alternative losses from placing currency mismatch on the bank balance sheets. The result stems from the high power of the debt overhang distortion which, if strengthened, affects private investment to a larger extent than tighter borrowing conditions for firms that would alternatively result from currency mismatch losses attributed to highly leveraged banks. Besides this, firms burdened with debt decrease aggregate output and demand directly, while banks affect aggregate economic activity with a lag and only after a share of the effect is absorbed by bank equity. The results suggest that shifting exchange rate risk from borrowers to banks is most likely to have a positive effect on the depth and length of a recession.

To contain currency mismatch losses in the banking sector, the government can resort to bank recapitalization. We show that currency mismatch in the banking sector can be efficiently alleviated ex-post by injecting bank equity.

Our model abstracts from long-term debt and fully-fledged effects of corporate net worth which would potentially make the effects of adverse shocks more persistent and strengthen debt overhang in the corporate sector. Nevertheless, we still find macroeconomic outcomes to be in favour of placing currency mismatch in the banking sector rather than shifting to credit constrained firms. This context offers more support for our conclusions.

Our result should serve as an additional argument for why bank should bear currency risk besides such advantages as easier coordination of a few troubled banks than thousands of insolvent borrowers and the fact that, in contrast to firms in emerging Europe, banks can access foreign exchange markets for hedging purposes.

References

Adolfson, M., Laséen, S., Lindé, J., Villani, M., 2007. Bayesian estimation of an open economy DSGE model with incomplete pass-through. Journal of International Economics, vol. 72(2), 481-

511.

Bank of Hungary, 2012. Report on Financial Stability (November 2012).

Bernanke, B., Gertler, M., 1989. Agency costs, net worth, and business fluctuations. American Economic Review vol. 79, p. 14–31.

Bernanke, B., Gertler, M., Gilchrist, S., 1999. The financial accelerator in a quantitative business cycle framework. In: Taylor, J., Woodford, M. (Eds.), Handbook of Macroeconomics. North-Holland, Amsterdam, vol. 1, part C, 1341–1393.

Bloom, N., 2009. The Impact of Uncertainty Shocks, Econometrica, Econometric Society, vol. 77(3), p. 623-685, 05.

Brown, M., Lane, P., 2011. Debt Overhang in Emerging Europe? The World Bank, Policy Research Paper No. 5784, (Washington, August).

Bodnár, K., 2009. Exchange rate exposure of Hungarian enterprises – results of a survey. MNB Occasional Papers, 2009/80, Magyar Nemzeti Bank (Central Bank of Hungary).

Calvo, G. A., 1983. Staggered prices in a utility-maximizing framework. Journal of Monetary Economics, vol. 12, 383–398.

Céspedes, L. F., Chang, R., Vélasco, A., 2004. Balance sheets and exchange rate policy. American Economic Review, vol. 94(4), 1183-1193.

Chen, H., Manso, G., 2014. Macroeconomic risk and debt overhang. Unpublished manuscript.

Cook, D., 2004. Monetary policy in emerging markets: Can liability dollarization explain contractionary devaluations? Journal of Monetary Economics, vol. 51, p. 1155–1181.

Christiano, L. J., Motto, R., Rostagno, M., 2014. Risk Shocks. American Economic Review, vol. 104(1), p. 27-65.

Devereux, M. B., Lane, P. R., Xu, J., 2006. Exchange Rates and Monetary Policy in Emerging Market Economies. Economic Journal, Royal Economic Society, vol. 116 (April), p. 478–506.

Diamond, D. W., Rajan, R. G., 2009. The Credit Crisis: Conjectures about Causes and Remedies, American Economic Review, American Economic Association, vol. 99(2), p. 606-10, May.

Eichengreen, B., Hausmann, R., 1999. Exchange rates and financial fragility. In: New Challenges for Monetary Policy. Federal Reserve Bank of Kansas City, Kansas City, 329–368.

Endrész, M., Gyöngyösi, G., Harasztosi, P., 2012. Currency mismatch and the sub-prime crisis: firm-level stylised facts from Hungary, MNB Working Papers 2012/8, Magyar Nemzeti Bank (Central Bank of Hungary).

García-Cicco, J., Kirchner, M., Justel, S., 2014. Financial Frictions and the Transmission of Foreign Shocks in Chile. Working Papers Central Bank of Chile 722, Central Bank of Chile.

Gali, J., Monacelli, T., 2005. Monetary Policy and Exchange Rate Volatility in a Small Open Economy. Review of Economic Studies, vol. 72, p. 707–734.

Gertler, M., Gilchrist, S., Natalucci, F., 2007. External constraints on monetary policy and the financial accelerator. Journal of Money, Credit and Banking, vol. 39, No. 2-3 (March-April 2007), 295-330.

Gertler, M., Karadi, P., 2011. A model of unconventional monetary policy, Journal of Monetary Economics, vol. 58(1), p. 17-34.

Gomes, J., Jermann, U., Schmid, L., 2013. Sticky Leverage. The Wharton School Research Paper No. 52.

Gourio, F., 2014. Financial distress and endogenous uncertainty. Unpublished manuscript.

Gertler, M., Karadi, P., 2011. A model of unconventional monetary policy. Journal of Monetary Economics, vol. 58(1), p. 17–34.

Herzberg, V., 2010. Assessing the Risk of Private Sector Debt Overhang in the Baltic Countries. IMF Working Papers 10/250, International Monetary Fund.

Hungarian Banking Association, 2012. Report on 2011 activities of the Hungarian banking association. Budapest, March 2012.

IMF, 2008. Hungary: 2008 Article IV Consultation—Staff Report.

IMF, 2012a. How Emerging Europe Came Through the 2008/09 Crisis. An Account by the Staff of the IMF's European Department, 2012. Eds: Bakker, B. B., Klingen, C.

IMF, 2012b. 2011 Article IV consultations and second post-program monitoring discussions, IMF Country Report No. 12/13.

Jakab, Z. M., Világi, B., 2008. An estimated DSGE model of the Hungarian economy. Magyar Nemzeti Bank working paper 2008/9.

Kalemli-Özcan, Ş., Laeven, L., Moreno, D., 2015. Debt Overhang in Europe: Evidence from Firm-Bank-Sovereign Linkages. Mimeo.

Kirchner, M., van Wijnbergen, S., 2012. Fiscal Deficits, Financial Fragility, and the Effectiveness of Government Policies, Tinbergen Institute Discussion Papers 12-044/2, Tinbergen Institute.

Konya, I. Jakab, Z., 2016. An estimated open-economy DSGE model with search-and-matching frictions: the case of Hungary. Mimeo.

Krekó, J., Endrész, M., 2010. The role of foreign currency lending in the impact of the exchange rate on the real economy, MNB Bulletin (discontinued), Magyar Nemzeti Bank (Central Bank of Hungary), vol. 5(1), p. 29-38, March.

Lamont, O., 1995. Corporate-debt overhang and macroeconomic expectations. The American Economic Review, vol. 85, no. 5 (Dec., 1995), p. 1106-1117.

Myers, S. C., 1977. Determinants of Corporate Borrowing. Journal of Financial Economics, vol. 5, p. 147-175.

Occhino, F., Pescatori, A., 2014. Leverage, investment and optimal monetary policy. B.E. Journal of Macroeconomics, vol. 14 (1),p. 511–531.

Occhino, F., Pescatori, A., 2015. Debt Overhang in a Business Cycle Model. European Economic Review, vol. 73 (2015), p. 58-84.

Philippon, T., 2010. Debt Overhang and Recapitalization in Closed and Open Economies, IMF Economic Review, Palgrave Macmillan, vol. 58(1), 157-178.

Reuters, 2012. Hungarian banks see 20 pct of FX borrowers opting for early repayment, January 3, 2012. Available at http://uk.reuters.com/article/hungary-banks-idUKL6E8C316K20120103.

Schmitt-Grohé, S., Uribe, M., 2003. Closing Small Open Economy Models, Journal of International Economics, vol. 61(1), p. 163-185.

Towbin, P., Weber, S., 2011. Limits of Floating Exchange Rates: the Role of Foreign Currency Debt and Import Structure. IMF working paper 11/42.

van der Kwaak, C., van Wijnbergen, S., 2014. Financial fragility, sovereign default risk and the limits to commercial bank bail-outs. Journal of Economic Dynamics and Control 43 (C), 218–240.

Vonnák, D., 2016. Decomposing the Riskiness of Corporate Foreign Currency Lending: the Case of Hungary. Mimeo.

Appendix

Tables and figures

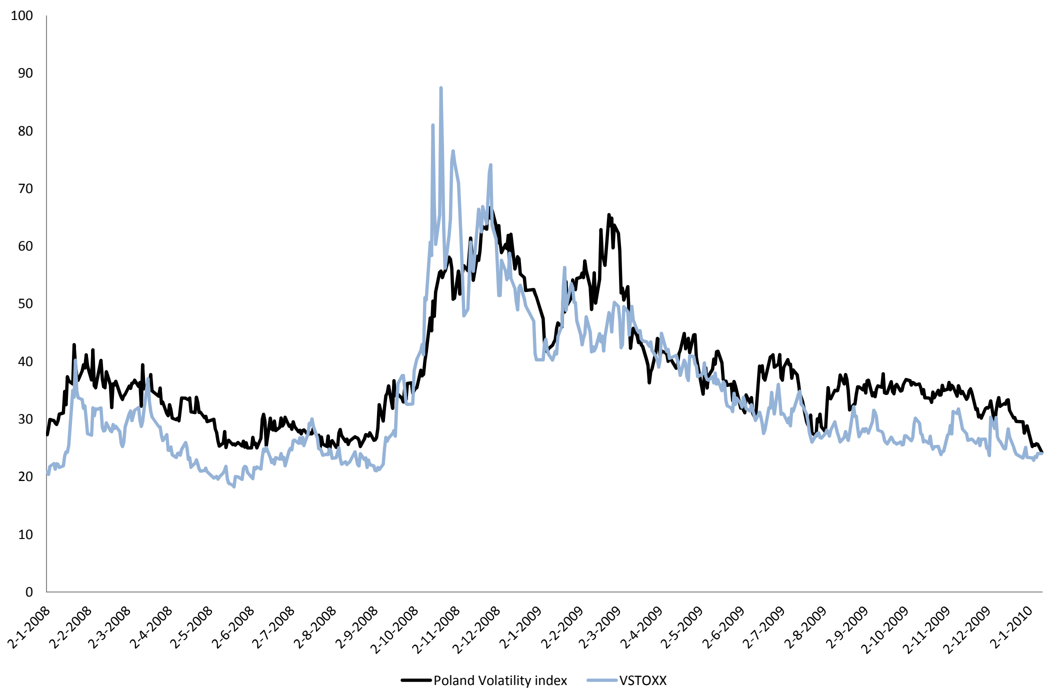


Figure 10: Implied volatility indexes.

Sources: EURO STOXX 50 Volatility Indices database and the courtesy of the blog 'Volatility Futures & Options', available at http://onlyvix.blogspot.nl/2013/03/polands-volatility-index.html.

Parameter	Description	Value	Source
	Calibrated parameters		
β	Household's discount factor	0.9970	to match $\pi = 1.059$
γ	Coefficient in GHH preferences	1.6	Jakab and Világi (2008)
ϕ	Labour supply elasticity	8	Jakab and Világi (2008)
α	Capital share in production	0.34	calibrated
δ	Capital depreciation rate	0.025	Jakab and Világi (2008)
ϵ	E.o.S. between domestic and imported goods	1.5	Gali and Monacelli (2002)
ϵ^H	E.o.S. between varieties of domestic goods	6	Jakab and Világi (2008)
ϵ^F	E.o.S. between varieties of imported goods	6	Jakab and Világi (2008)
ϵ^*	E.o.S. for exports	1.5	Gali and Monacelli (2002)
θ^H	Calvo parameter, domestic goods	0.75	Gali and Monacelli (2002)
θ^F	Calvo parameter, imported goods	0.75	calibrated
η	Share of x^F in y^C	0.37	to match average imports share of 37%
η^*	Share of ex in y^*	0.0033	calibrated
κ	Investment adjustment cost parameter	13	Jakab and Világi (2008)
κ_b	Tax feedback parameter for government debt	0.05	calibrated
z	Technology in SS	1	calibrated
π	Inflation in SS	1.059	average in the data in annual terms
p^H	Relative price of x^H in SS	1	calibrated
n	Working hours in SS	0.3	calibrated
S	Nominal exchange rate in SS	1	calibrated
y^*	Total foreign output in SS	104	calibrated
R	Risk-free rate in SS	1.073	average in the data in annual terms
R^*	Foreign interest rate in SS	1.045	calibrated
s^g	Gov. consumption/ GDP in SS	0.22	average in the data
π^*	Foreign inflation rate	1	from RER definition in SS
ξ	Risk premium on international bonds in SS	1.01	calibrated
κ_ξ	Elasticity of country risk to net asset position	0.001	Jakab and Világi (2008)
ζ	Exogenous shock to the bond premium in SS	1	calibrated
ρ_R	Interest rate smoothing	0.766	Jakab and Világi (2008)
α_π	Interest policy rule (inflation)	1.375	Jakab and Világi (2008)
α_y	Interest policy rule (output)	0.2	calibrated
ρ_σ	Volatility shock autoregr. coeff.	0.9	Occhino and Pescatori (2015)
ρ_{y^*}	World demand shock autoregr. coeff.	0.43	Konya and Jakab (2016)
ρ_ζ	Risk premium shock autoregr. coeff.	0.66	Konya and Jakab (2016)
	Financially constrained firms' parameters		
$1-\Phi(d_2)$	Corporate default rate in SS	0.03	average in the data
ρ	Fraction of working capital to be paid in advance	0.8	-
α^F	Share of FX loans	0.6	average in the data
ι^{firms}	Proportional transfer to the entering firms	0.002	calibrated
lev^{firms}	Bank leverage in SS	3.3	average in the data
	Banking sector parameters		
λ^L	Fraction of capital that can be diverted	0.45	calibrated
ι	Proportional transfer to the entering bankers	0.002	Gertler and Karadi (2011)
lev	Bank leverage in SS	5.6	average in the data

Table 2: Parameters

Figure 11: Country risk premium shock in the model without leverage-constrained banks.

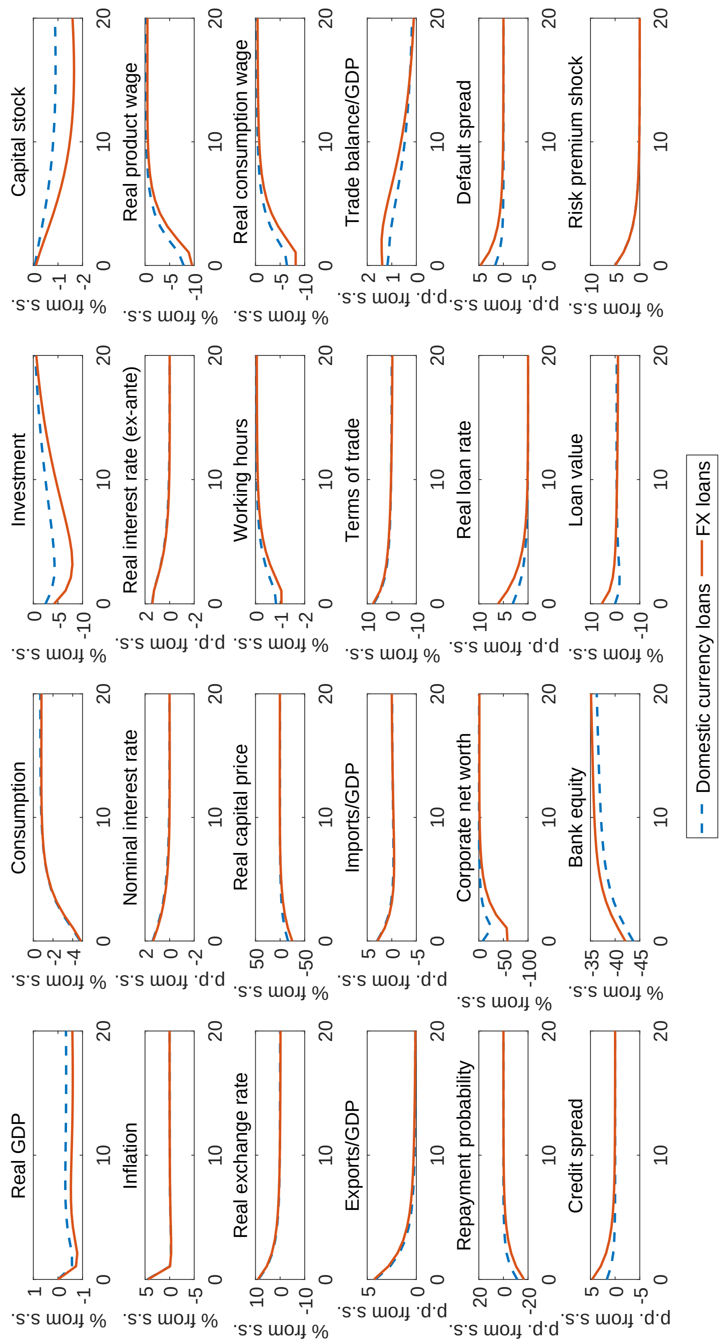


Figure 12: World demand shock in the model without leverage-constrained banks.

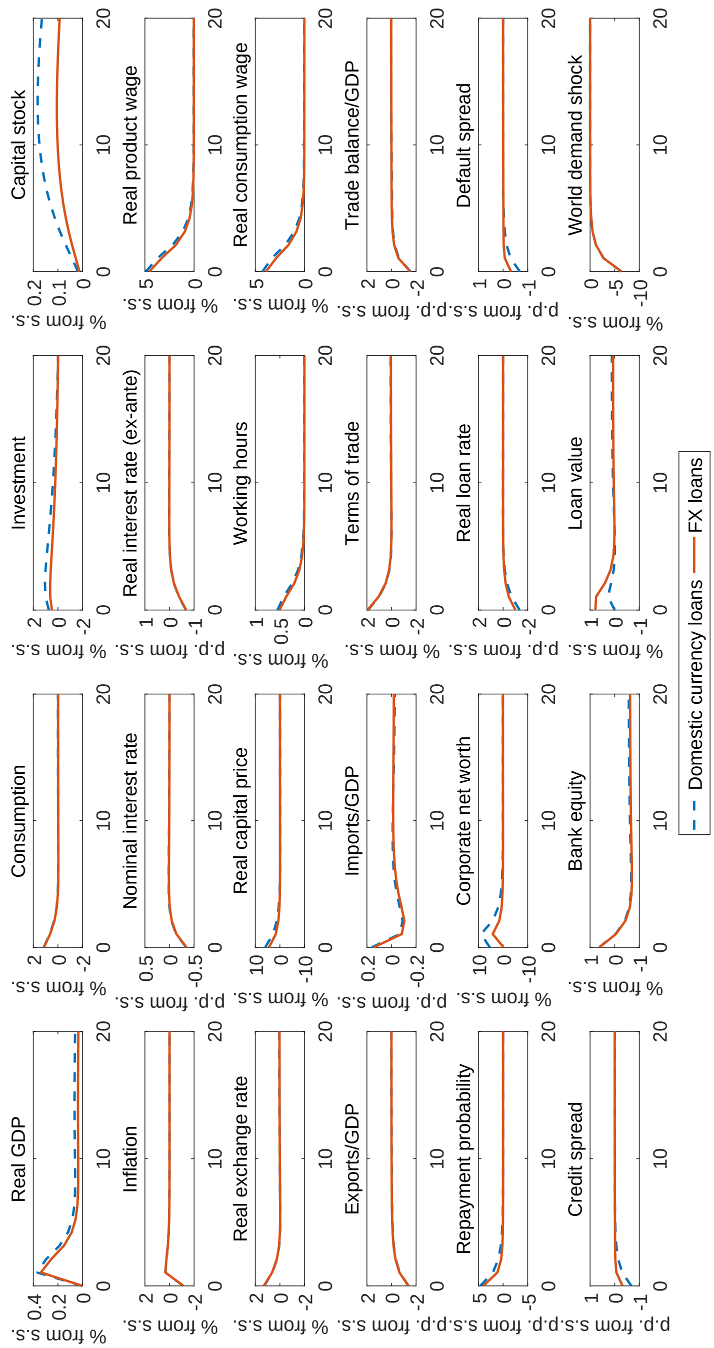


Figure 13: World demand shock in the model without leverage-constrained banks when labour demand is not predetermined.

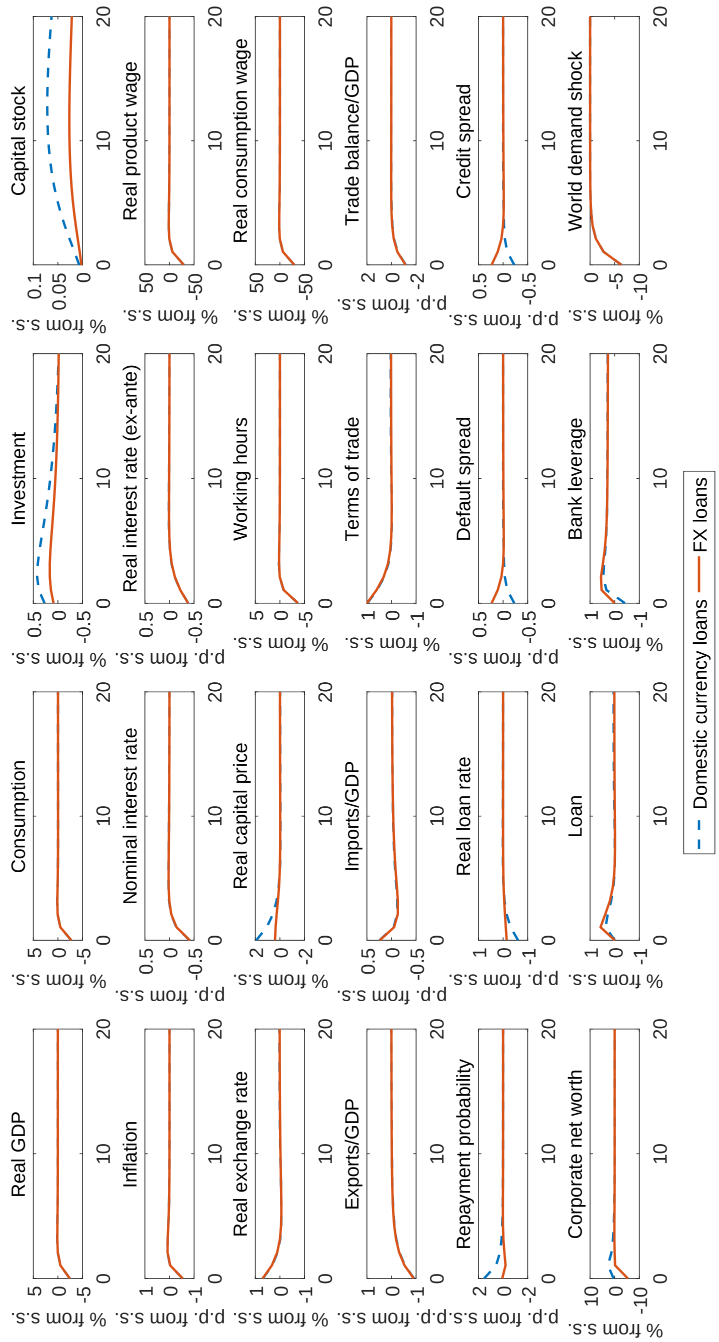


Figure 14: Volatility shock in the model without leverage-constrained banks.

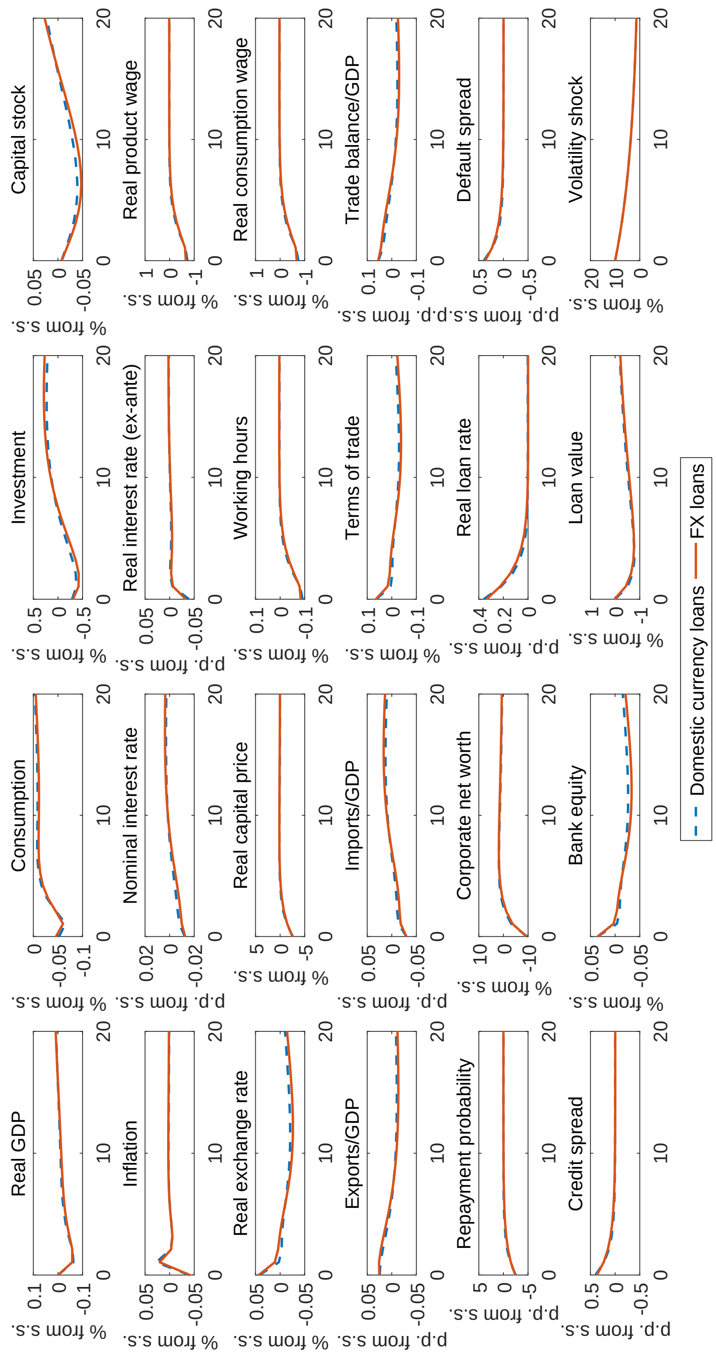


Figure 15: Country risk premium shock in the model with leverage-constrained banks.

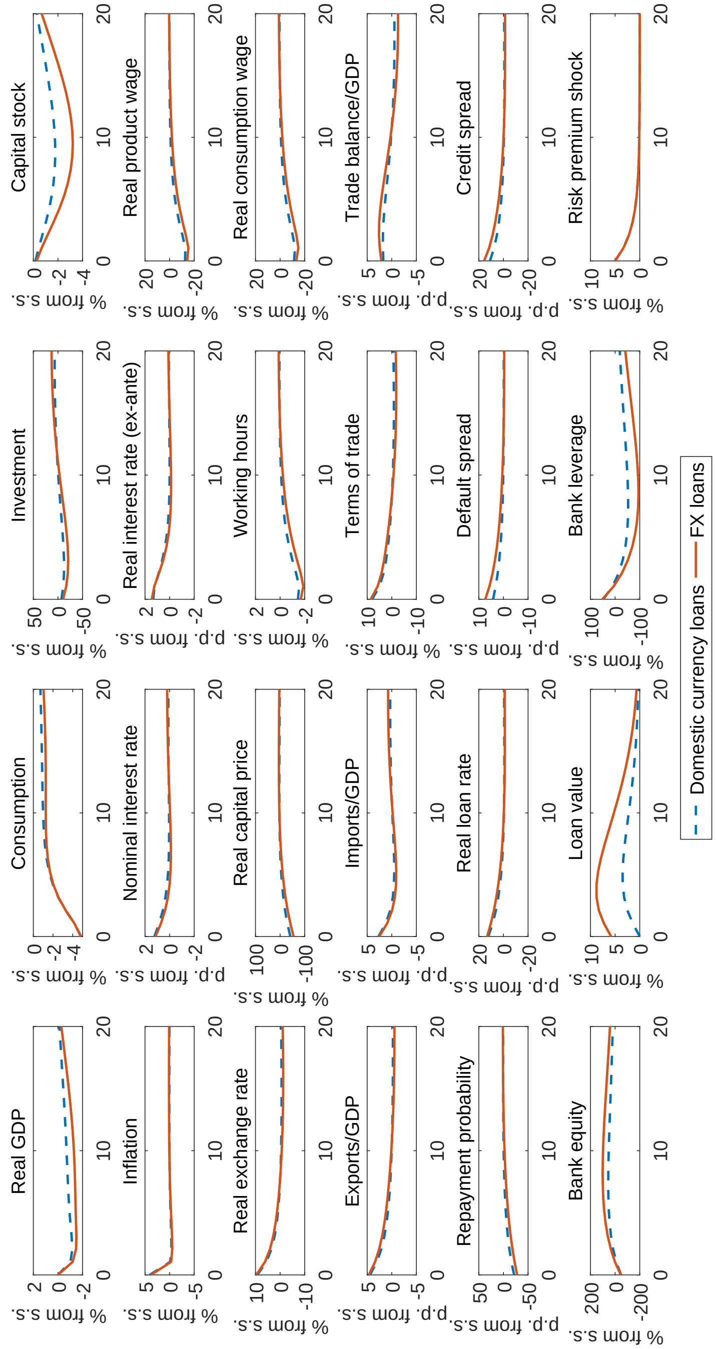


Figure 16: World demand shock in the model with leverage-constrained banks.

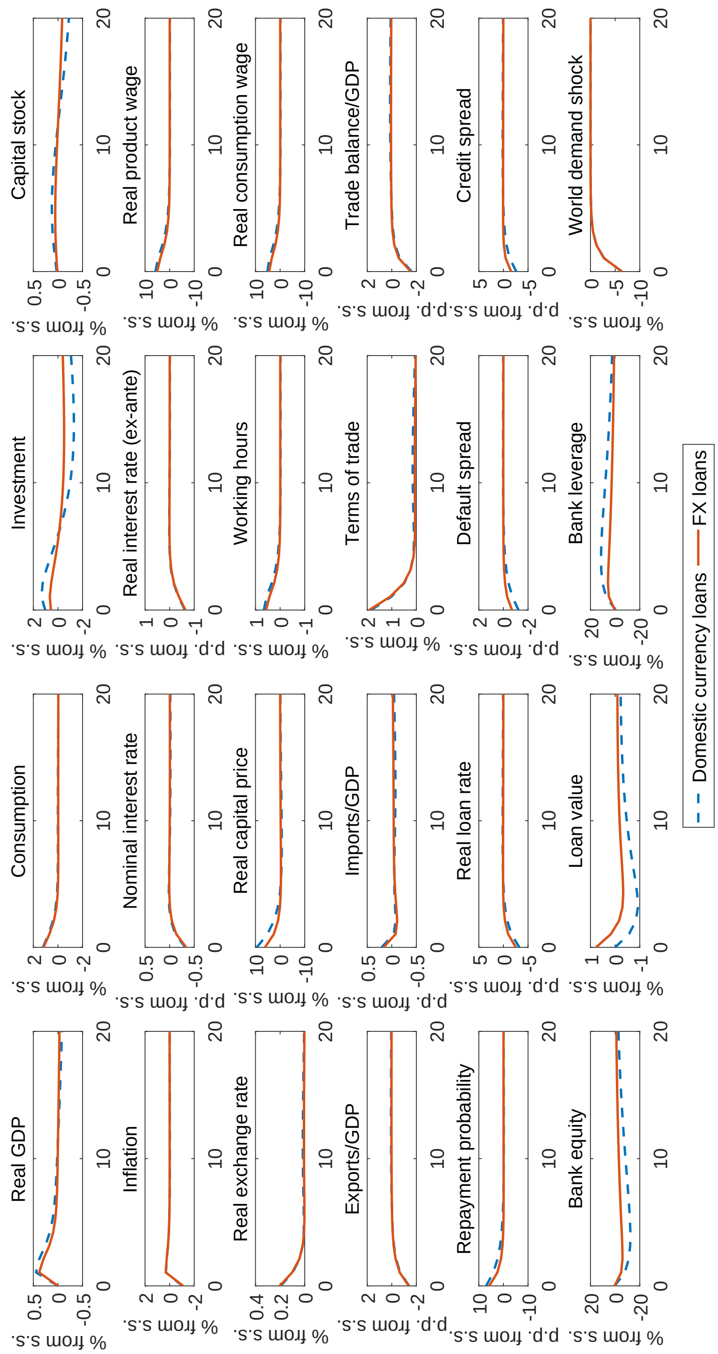
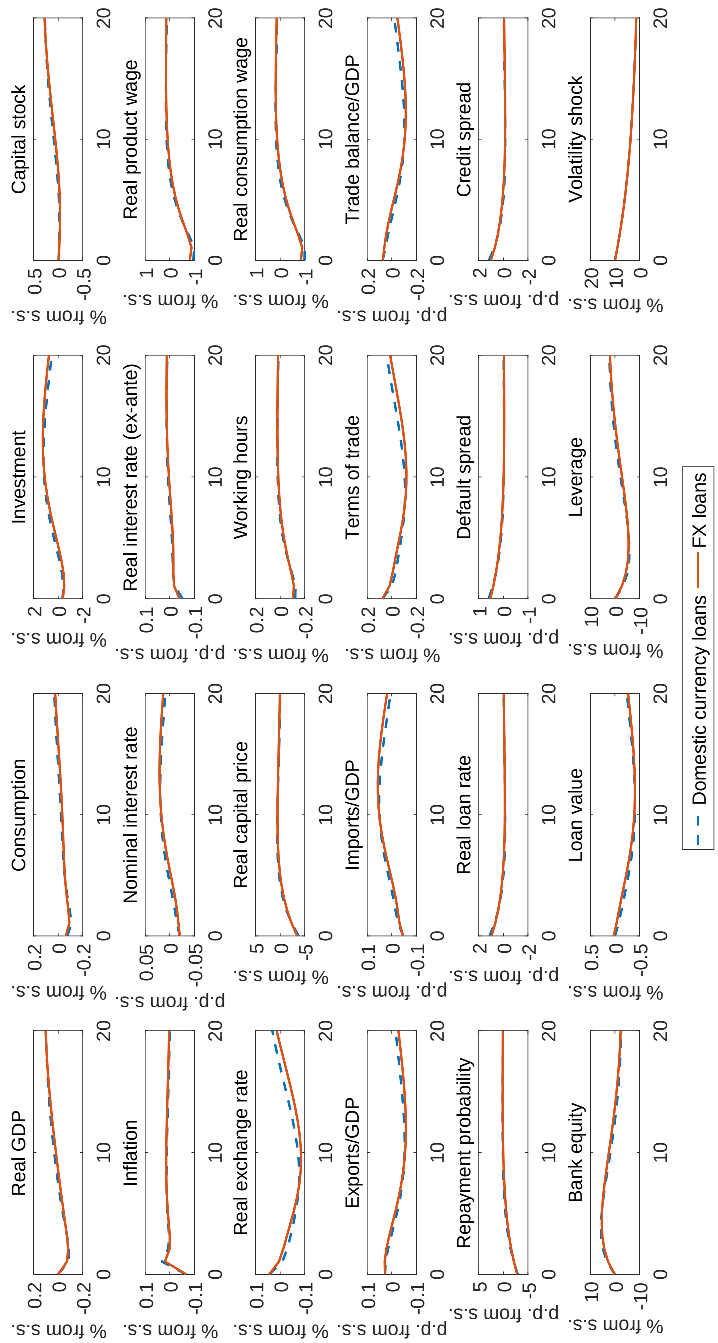


Figure 17: Volatility shock in the model with leverage-constrained banks.



Mathematical derivations

A: Financially constrained firms

A1: Solving the financially constrained firms' profit maximization problem with FX loans

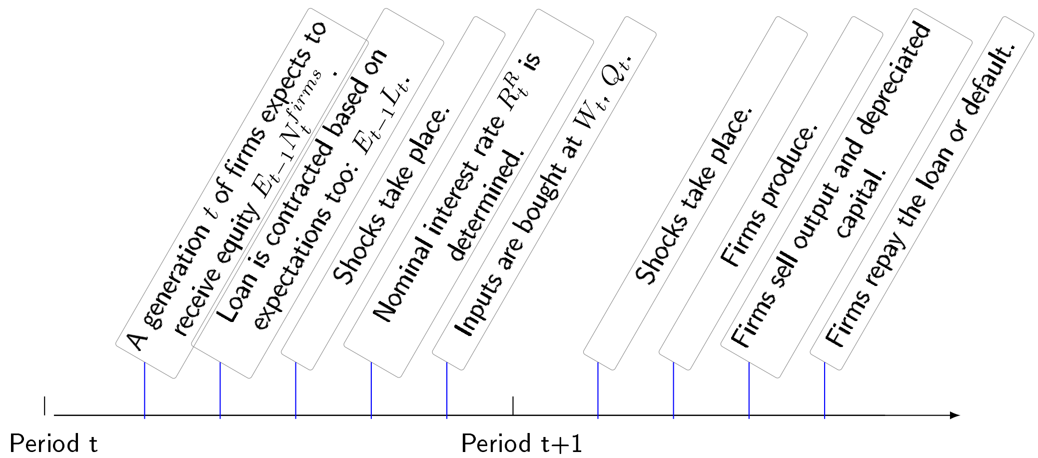


Figure 18: Timing for financially constrained firms.

Financially constrained firms live for two periods. Every period there is a new-born generation of firms and the total number of firms always constitute a continuum of mass one. In the first period firms buy two types of inputs, capital k and labour h, and have to pay for a fraction ρ in advance, which generates their demand for working capital. Production takes place in the next period.

To pay in advance, a financially constrained firm i uses two types of financing. First, it receives equity from households, $N_{i,t}^{firms}$. Second, it borrows from the bank an amount $L_{i,t}$ that consists of both domestic currency funds $L_{i,t}^D$ and foreign currency denominated funds $L_{i,t}^F$ such that $L_{i,t} = L_{i,t}^D + S_t L_{i,t}^F$ where S_t is the nominal exchange rate. We assume that the share of foreign currency denominated funds is fixed and denoted by α^F, so that the firm can choose the size of the total loan but not the denomination structure. This assumption allows us to calibrate the open position of banks and is innocuous enough, since we study the consequences of foreign currency borrowing rather than the choice of the borrowing currency.

To borrow, the firm has to pledge a share κ of future revenue as collateral where $0 < \kappa \leq 1$. We assume that the firm decides how much to borrow before shocks arrive and the prices of production inputs are revealed. Then the demanded size of the loan is equal to the expected expenditure for working capital minus the expected equity transfer from the household. It follows that in the beginning of period t the following condition holds:

$$E_{t-1}\left\{L_{i,t}\right\} + E_{t-1}\left\{N_{i,t}^{firms}\right\} = E_{t-1}\left\{\rho\left(Q_t k_{i,t} + W_t h_{i,t}\right)\right\} \tag{17}$$

Or, in units of composite goods associated with price P_t,

$$E_{t-1}\left\{l_{i,t}\right\} + E_{t-1}\left\{n_{i,t}^{firms}\right\} = E_{t-1}\left\{\rho\left(q_t k_{i,t} + w_t h_{i,t}\right)\right\} \tag{18}$$

q_t, w_t and rer_t denote the real price of capital, the real wage and the real exchange rate respectively. We express all three prices are expressed in units of composite goods. It follows that we define q_t as Q_t/P_t, w_t as W_t/P_t and the real exchange rate as $S_t P_t^*/P_t$ where S_t is the nominal exchange rate, P_t is the price of composite goods and P_t^* defines the price level of foreign composite goods. $n_{i,t}^{firms}$ stands for the equity transfer from the domestic household, where $n_{i,t}^{firms} \equiv N_{i,t}^{firms}/P_t$. $l_{i,t}$ stands for the size of the total loan expressed in units of composite goods and is defined as $l_{i,t} \equiv L_{i,t}/P_t$. After the loan is taken, shocks materialize, however, the predetermined size of the loan creates the debt overhang effect by distorting firm's private incentives to invest in production inputs.

Because of the timing of new information, the actual demand for working capital by the firm will in most cases not equal the loan amount received. We assume that in such cases the owner of the firm (the domestic household) steps in and transfers lump-sum funds $Z_{i,t}$ (where $z_{i,t} \equiv Z_{i,t}/P_t$) to cover the difference. Importantly, these funds constitute residual funding and firms cannot rely on them as the main source of finance. These funds enter the domestic household's budget constraint as a lump-sum transfer and have no effect on either the household's or the firm's incentives.

Let the matured loan in units of composite goods be $R_{i,t}^R \left(\frac{l_{i,t}^D}{\pi_{t+1}} + rer_{t+1} \frac{l_{i,t}^F}{\pi_{t+1}^*} \right)$, where $R_{i,t}^R$ is the nominal gross interest rate on the loan. The bank sets interest rates on loans after the shocks take place, therefore, the loan rate adjusts to clear the loan market. We define real loans in different currencies as $l_{i,t}^D \equiv L_{i,t}^D/P_t$ and $l_{i,t}^F \equiv L_{i,t}^F/P_t^*$. The contracted collateral is a fraction κ of firms' revenue from selling goods and depreciated capital in the next period, $p_{t+1}^R y_{i,t+1}^R + q_{t+1}(1-\delta)k_{i,t}$. p_{t+1}^R stands for the price of homogenous goods, expressed in units of composite goods ($p_{t+1}^R \equiv P_{t+1}^R/P_{t+1}$). Then the decision of the financially constrained firm i born in period t whether to default or not is determined by the lower value:

$$\min \left\{ R_{i,t}^R \left(L_{i,t}^D + S_{t+1} L_{i,t}^F \right), \quad \kappa \left(P_{t+1}^R y_{i,t+1}^R + Q_{t+1}(1-\delta)k_{i,t} \right) \right\} \tag{19}$$

Deflating by P_{t+1} gives the expression in units of composite goods:

$$\min \left\{ R_{i,t}^R \left(\frac{l_{i,t}^D}{\pi_{t+1}} + rer_{t+1} \frac{l_{i,t}^F}{\pi_{t+1}^*} \right), \quad \kappa \left(p_{t+1}^R y_{i,t+1}^R + q_{t+1}(1-\delta)k_{i,t} \right) \right\} \tag{20}$$

where $p_{t+1}^R y_{i,t+1}^R = p_{t+1}^R A_{t+1} \theta_{i,t+1} k_{i,t}^{\alpha} h_{i,t}^{1-\alpha}$.

The firm i born in period t and endowed with corporate equity $N_{i,t}^{firms}$ maximizes profits taking the loan as given. The firm maximizes expected profits given by future revenue from selling goods and depreciated capital minus the second fraction of working capital expenditure together with expenses related to the debt payment. Financial flows received in period t also enter the maximization problem and can be summarized as the difference between the loan plus equity and working capital expenditure:

$$\begin{aligned} \max_{\{k_{i,t}, h_{i,t}\}} & E_t \beta \Lambda_{t,t+1} \frac{\left\{ P_{t+1}^R y_{i,t+1}^R + Q_{t+1}(1-\delta)k_{i,t} - (1-\rho)\left(Q_t k_{i,t} + W_t h_{i,t}\right) \right\}}{P_{t+1}} \\ & - E_t \beta \Lambda_{t,t+1} \min \left\{ \frac{R_{i,t}^R \left(L_{i,t}^D + S_{t+1} L_{i,t}^F \right)}{P_{t+1}}, \quad \frac{\kappa \left(P_{t+1}^R y_{i,t+1}^R + Q_{t+1}(1-\delta)k_{i,t} \right)}{P_{t+1}} \right\} \\ & + \frac{L_{i,t} + N_{i,t}^{firms} + Z_{i,t}}{P_t} - \frac{\rho \left(Q_t k_{i,t} + W_t h_{i,t} \right)}{P_t} \end{aligned}$$

s.t.

$$\frac{E_{t-1}\left\{L_{i,t}+N_{i,t}^{firms}\right\}}{P_t}=\frac{E_{t-1}\left\{\rho\left(Q_t k_{i,t}+W_t h_{i,t}\right)\right\}}{P_t}$$

Using the previously introduced definitions yields

$$\begin{aligned}&\max_{\{k_{i,t},h_{i,t}\}} E_t\beta\Lambda_{t,t+1}\left\{p_{t+1}^R y_{i,t+1}^R+q_{t+1}(1-\delta)k_{i,t}-(1-\rho)\frac{q_t k_{i,t}+w_t h_{i,t}}{\pi_{t+1}}\right\}\\&\quad-E_t\beta\Lambda_{t,t+1}\min\left\{R_{i,t}^R\left(\frac{l_{i,t}^D}{\pi_{t+1}}+rer_{t+1}\frac{l_{i,t}^F}{\pi_{t+1}^*}\right),\quad\kappa\left(p_{t+1}^R y_{i,t+1}^R+q_{t+1}(1-\delta)k_{i,t}\right)\right\}\\&\quad+l_{i,t}+n_{i,t}^{firms}+z_{i,t}-\rho\left(q_t k_{i,t}+w_t h_{i,t}\right)\end{aligned}$$

s.t.

$$E_{t-1}\left\{l_{i,t}\right\}+E_{t-1}\left\{n_{i,t}^{firms}\right\}=E_{t-1}\left\{\rho\left(q_t k_{i,t}+w_t h_{i,t}\right)\right\}$$

The resulting first-order conditions are:

$$\begin{aligned}k_{i,t}:\quad&E_t\beta\Lambda_{t,t+1}\left\{p_{t+1}^R\frac{\partial y_{i,t+1}^R}{\partial k_{i,t}}+q_{t+1}(1-\delta)-(1-\rho)\frac{q_t}{\pi_{t+1}}\right\}\\&-E_t\beta\Lambda_{t,t+1}\left\{(1-\Phi(d_{1,t}))\kappa\left(p_{t+1}^R\frac{\partial y_{i,t+1}^R}{\partial k_{i,t}}+q_{t+1}(1-\delta)\right)\right\}\\&=\frac{\partial cov\left(\beta\Lambda_{t,t+1},\quad\min\left\{R_{i,t}^R\left(\frac{l_{i,t}^D}{\pi_{t+1}}+rer_{t+1}\frac{l_{i,t}^F}{\pi_{t+1}^*}\right),\quad\kappa\left(p_{t+1}^R y_{i,t+1}^R+q_{t+1}(1-\delta)k_{i,t}\right)\right\}\right)}{\partial k_{i,t}}\\&+\rho q_t\end{aligned}$$

$$\begin{aligned}h_{i,t}:\quad&E_t\beta\Lambda_{t,t+1}\left\{p_{t+1}^R\frac{\partial y_{i,t+1}^R}{\partial h_{i,t}}-(1-\rho)\frac{w_t}{\pi_{t+1}}\right\}\\&-E_t\beta\Lambda_{t,t+1}\left\{(1-\Phi(d_{1,t}))\kappa\left(p_{t+1}^R\frac{\partial y_{i,t+1}^R}{\partial h_{i,t}}\right)\right\}\\&=\frac{\partial cov\left(\beta\Lambda_{t,t+1},\quad\min\left\{R_{i,t}^R\left(\frac{l_{i,t}^D}{\pi_{t+1}}+rer_{t+1}\frac{l_{i,t}^F}{\pi_{t+1}^*}\right),\quad\kappa\left(p_{t+1}^R y_{i,t+1}^R+q_{t+1}(1-\delta)k_{i,t}\right)\right\}\right)}{\partial h_{i,t}}\\&+\rho w_t\end{aligned}$$

where

$$d_{2,t}\equiv\frac{E_t\ln\left(\kappa\left(p_{t+1}^R y_{i,t+1}^R+q_{t+1}(1-\delta)k_{i,t}\right)-R_t^R rer_{t+1}\frac{l_{i,t}^F}{\pi_{t+1}^*}\right)-E_t\ln\left(R_{i,t}^R\frac{l_{i,t}^D}{\pi_{t+1}}\right)}{\sigma_y},\qquad d_{1,t}=d_{2,t}+\sigma_y$$

The derivation of $d_{2,t}$ is given in the next subsection and results for the first-order conditions are given by equations (A2.1) and (A2.2).

The first-order conditions hold together with the ex-ante budget constraint:

$$E_{t-1}\left\{l_{i,t}\right\}+E_{t-1}\left\{n_{i,t}^{firms}\right\}=E_{t-1}\left\{\rho\left(q_t k_{i,t}+w_t h_{i,t}\right)\right\}$$

In the beginning of the next period, after shocks take place and a fraction of firms default, the domestic household pools the remaining net worth from non-defaulted firms into aggregate net worth by the following aggregation rule:

$$\begin{aligned} n_t^{firms} = \; & \omega^{firms}\left(p_t^R y_t^R + q_t(1-\delta)k_{t-1} - (1-\rho)\tfrac{q_{t-1}k_{t-1}+w_{t-1}h_{t-1}}{\pi_t}\right) \\ & -\omega^{firms}\left(\left(1-\Phi(d_{1,t-1})\right)\kappa\left(p_t^R y_t^R + q_t(1-\delta)k_{t-1}\right) + \Phi(d_{2,t-1})R_{t-1}^R\tfrac{l_{t-1}^D}{\pi_t} + \Phi(d_{1,t-1})rer_t\tfrac{l_{t-1}^F}{\pi_t^*}\right) \\ & +\iota^{firms}\cdot n^{firms} \end{aligned}$$

Recall that $(1-\Phi(d_{1,t-1}))$ proxies for the default rate (by the law of large numbers this is equal to the share of defaulted firms in the economy). Then the first term on the right hand side is aggregate firms' revenue from production and selling depreciated capital minus the rest of the expenditure for working capital. The second term is the firms' aggregate expenditure for repaying loans. The difference between the two gives financially constrained firms' profits. The third term is the injection of new equity. We assume that the domestic household acts as distributor and cannot divert pooled equity funds anywhere else. Also the existing equity can be increased only by the amount $\iota^{firms}\cdot n^{firms}$ that is fixed and proportional to aggregate net worth in the steady state. Thus, this equity transfer does not depend on the household's decision. ω^{firms} is a fraction that is close but lower than unity. We assume that this parameter proxies for the equity management costs incurred by the household and use this parameter to calibrate the steady state corporate leverage to the one observed in the data.

A2: Derivation of the default probability

We need to compute the expected value of the firm's payment function (we abstract from indices i for the sake of brevity):

$$E_t \min\left\{R_t^R\left(\frac{l_t^D}{\pi_{t+1}}+rer_{t+1}\frac{l_t^F}{\pi_{t+1}^*}\right),\quad \kappa\left(p_{t+1}^R y_{t+1}^R + q_{t+1}(1-\delta)k_t\right)\right\}$$

To simplify, we re-order the terms in the following way:

$$E_t \min\left\{R_t^R\frac{l_t^D}{\pi_{t+1}},\quad \kappa\left(p_{t+1}^R y_{t+1}^R + q_{t+1}(1-\delta)k_t\right) - R_t^R rer_{t+1}\frac{l_t^F}{\pi_{t+1}^*}\right\} + E_t R_t^R rer_{t+1}\frac{l_t^F}{\pi_{t+1}^*}$$

Further we focus on the first term only, since it defines the default decision and contains all necessary prices too:

$$E_t \min\left\{R_t^R\frac{l_t^D}{\pi_{t+1}},\quad \kappa\left(p_{t+1}^R y_{t+1}^R + q_{t+1}(1-\delta)k_t\right) - R_t^R rer_{t+1}\frac{l_t^F}{\pi_{t+1}^*}\right\}$$

Define $\bar{y}_{t+1}\equiv\pi_{t+1}\left(\kappa\left(p_{t+1}^R y_{t+1}^R + q_{t+1}(1-\delta)k_t\right) - R_t^R rer_{t+1}\frac{l_t^F}{\pi_{t+1}^*}\right)$, where

$$\bar{y}_{t+1}\sim \text{log-normal}\left(\mu_{\bar{y}_{t+1}},\sigma_y^2\right)$$

Then the modified minimum function can be re-written as

$$E_t \min\left\{R_t^R l_t^D, \quad \bar{y}_{t+1}\right\}$$

Further,

$$\begin{aligned}
E_t \min\left\{R_t^R l_t^D, \quad \bar{y}_{t+1}\right\} &= R_t^R l_t^D Pr\left(R_t^R l_t^D < \bar{y}_{t+1}\right) + \left(1 - Pr\left(R_t^R l_t^D < \bar{y}_{t+1}\right)\right) E_t\left(\bar{y}_{t+1} \mid \bar{y}_{t+1} < R_t^R l_t^D\right) \\
&= R_t^R l_t^D Pr\left(R_t^R l_t^D < \bar{y}_{t+1}\right) + \left(1 - Pr\left(R_t^R l_t^D < \bar{y}_{t+1}\right)\right) \int_0^{R_t^R l_t^D} \frac{\bar{y}_{t+1} dF\left(\bar{y}_{t+1}\right)}{1 - Pr\left(R_t^R l_t^D < \bar{y}_{t+1}\right)} \\
&= R_t^R l_t^D Pr\left(R_t^R l_t^D < \bar{y}_{t+1}\right) + \int_0^{R_t^R l_t^D} \bar{y}_{t+1} dF\left(\bar{y}_{t+1}\right) \\
&= R_t^R l_t^D \int_{R_t^R l_t^D}^{\infty} dF\left(\bar{y}_{t+1}\right) + \int_0^{R_t^R l_t^D} \bar{y}_{t+1} dF\left(\bar{y}_{t+1}\right) \\
&= R_t^R l_t^D \int_{R_t^R l_t^D}^{\infty} \frac{1}{\bar{y}_{t+1}\sigma_y\sqrt{2\pi}} e^{\frac{-\left(ln\left(\bar{y}_{t+1}\right) - \mu_y\right)^2}{2\sigma_y^2}} d\left(\bar{y}_{t+1}\right) \\
&+ \int_0^{R_t^R l_t^D} \frac{\bar{y}_{t+1}}{\bar{y}_{t+1}\sigma_y\sqrt{2\pi}} e^{\frac{-\left(ln\left(\bar{y}_{t+1}\right) - \mu_y\right)^2}{2\sigma_y^2}} d\left(\bar{y}_{t+1}\right) \\
&= R_t^R l_t^D \Phi\left(\frac{ln\left(\bar{y}_{t+1}\right) - \mu_y}{\sigma_y}\right) |_{R_t^R l_t^D}^{\infty} + \int_0^{R_t^R l_t^D} \frac{1}{\sigma_y\sqrt{2\pi}} e^{\frac{-\left(ln\left(\bar{y}_{t+1}\right) - \mu_y\right)^2}{2\sigma_y^2}} d\left(\bar{y}_{t+1}\right) \\
&= R_t^R l_t^D \left(1 - \Phi\left(\frac{ln\left(R_t^R l_t^D\right) - \mu_y}{\sigma_y}\right)\right) - \frac{1}{2} e^{\mu_y + \frac{\sigma_y^2}{2}} erf\left(\frac{-ln\left(\bar{y}_{t+1}\right) + \mu_y + \sigma_y^2}{\sqrt{2}\sigma_y}\right) |_0^{R_t^R l_t^D} \\
&= R_t^R l_t^D \Phi\left(\frac{\mu_y - ln\left(R_t^R l_t^D\right)}{\sigma_y}\right) + \frac{1}{2} E_t(\bar{y}_{t+1}) \left(erf\left(\frac{ln\left(R_t^R l_t^D\right) - \mu_y - \sigma_y^2}{\sqrt{2}\sigma_y}\right) + 1\right) \\
&= R_t^R l_t^D \Phi\left(\frac{\mu_y - ln\left(R_t^R l_t^D\right)}{\sigma_y}\right) + E_t(\bar{y}_{t+1}) \Phi\left(\frac{ln\left(R_t^R l_t^D\right) - \mu_y - \sigma_y^2}{\sigma_y}\right) \\
&= R_t^R l_t^D \Phi\left(\frac{\mu_y - ln\left(R_t^R l_t^D\right)}{\sigma_y}\right) + E_t(\bar{y}_{t+1}) \left(1 - \Phi\left(\frac{\mu_y - ln\left(R_t^R l_t^D\right)}{\sigma_y} + \sigma_y\right)\right)
\end{aligned}$$

The expression can be simplified as

$$E_t \min\left\{R_t^R l_t^D, \quad \bar{y}_{t+1}\right\} = \left(1 - \Phi(d_{1,t})\right) E_t\left(\bar{y}_{t+1}\right) + \Phi(d_{2,t}) R_t^R l_t^D$$

where

$$d_{2,t} \equiv \frac{\mu_y - \ln\left(R_t^R l_t^D\right)}{\sigma_y}, \qquad d_{1,t} \equiv d_{2,t} + \sigma_y$$

where

$$\mu_y \equiv E_t \ln\left(\bar{y}_{t+1}\right)$$

or

$$d_{2,t} \equiv \frac{E_t \ln\left(\bar{y}_{t+1}/\pi_{t+1}\right) - \ln\left(R_t^R/\pi_{t+1} l_t^D\right)}{\sigma_y}, \qquad d_{1,t} \equiv d_{2,t} + \sigma_y$$

Recall that $\bar{y}_{t+1} \equiv \pi_{t+1}\left(\kappa\left(p^R_{t+1}y^R_{t+1} + q_{t+1}(1-\delta)k_t\right) - R^R_t rer_{t+1}\frac{l^F_t}{\pi^*_{t+1}}\right)$ so it can be substituted back to get complete expressions. Then $\sigma^2_y = var\left(\bar{y}_{t+1}\right) = var\left(\pi_{t+1}\left(\kappa\left(p^R_{t+1}y^R_{t+1} + q_{t+1}(1-\delta)k_t\right) - R^R_t rer_{t+1}\frac{l^F_t}{\pi^*_{t+1}}\right)\right)$.

To solve for the first-order conditions, we differentiate the expected loan payment w.r.t. k_t:

$$
\begin{aligned}
\frac{\partial E_t \min\left\{R^R_t l^D_t, \quad \bar{y}_{t+1}\right\}}{\partial k_t} &= \left(1-\Phi(d_{1,t})\right)\frac{\partial E_t\bar{y}_{t+1}}{\partial k_t} \\
&- E_t\bar{y}_{t+1}\frac{\partial\Phi(d_{1,t})}{\partial d_{1,t}}\frac{\partial d_{1,t}}{\partial k_t} + R^R_t l^D_t\frac{\partial\Phi(d_{2,t})}{\partial d_{2,t}}\frac{\partial d_{2,t}}{\partial k_t} \\
&= \left(1-\Phi(d_{1,t})\right)\frac{\partial E_t\bar{y}_{t+1}}{\partial k_t}
\end{aligned}
$$

where the proof of the last expression comes from by using $\frac{\partial d_{1,t}}{\partial k_t} = \frac{\partial d_{2,t}}{\partial k_t}$ and computing the following:

$$
\begin{aligned}
& -E_t\left(\bar{y}_{t+1}\right)\Phi'(d_{1,t}) + R^R_t l^D_t \Phi'(d_{2,t}) \\
= & -e^{\ln(E_t\bar{y}_{t+1})}\Phi'(d_{1,t}) + e^{\ln(R^R_t l^D_t)}\Phi'(d_{2,t}) \\
= & -e^{\ln(E_t\bar{y}_{t+1})}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d^2_{1,t}} + e^{\ln(R^R_t l^D_t)}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d^2_{2,t}} \\
= & -e^{\ln(E_t\bar{y}_{t+1})}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(d^2_{2,t}+2d_{2,t}\sigma_y+\sigma^2_y\right)} + e^{\ln(R^R_t l^D_t)}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d^2_{2,t}} \\
= & -e^{\ln(E_t\bar{y}_{t+1})}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d^2_{2,t}}e^{-\left(d_{2,t}\sigma_y+\frac{1}{2}\sigma^2_y\right)} + e^{\ln(R^R_t l^D_t)}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d^2_{2,t}} \\
= & -e^{\ln(E_t\bar{y}_{t+1})}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d^2_{2,t}}e^{-E_t\left(\ln\bar{y}_{t+1}\right)-\ln(R^R_t l^D_t)+\frac{1}{2}\sigma^2_y} + e^{\ln(R^R_t l^D_t)}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d^2_{2,t}} \\
= & \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d^2_{2,t}}\left[-e^{\ln(E_t\bar{y}_{t+1})}e^{-\left(\ln(E_t\bar{y}_{t+1})-\frac{1}{2}\sigma^2_y-\ln(R^R_t l^D_t)+\frac{1}{2}\sigma^2_y\right)} + e^{\ln(R^R_t l^D_t)}\right] \\
= & -\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d^2_{2,t}}e^{\ln(R^R_t l^D_t)} + e^{\ln(R^R_t l^D_t)}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d^2_{2,t}} \\
= & \; 0,
\end{aligned}
$$

where such expressions are used as

$$
E_t\ln\left(\bar{y}_{t+1}\right) = \ln\left(E_t\bar{y}_{t+1}\right) - \frac{1}{2}\sigma^2_y
$$

and the definition of the variable $d_{1,t}$. Substituting a definition for $\bar{y}_{t+1}$ back gives

$$
\frac{\partial E_t \min\left\{\frac{R^R_t}{\pi_{t+1}}l^D_t, \quad \kappa\left(p^R_{t+1}y^R_{t+1} + q_{t+1}(1-\delta)k_t\right) - R^R_t rer_{t+1}\frac{l^F_t}{\pi^*_{t+1}}\right\}}{\partial k_t} = \left(1-\Phi(d_{1,t})\right)\frac{\partial E_t\kappa\left(p^R_{t+1}y^R_{t+1} + q_{t+1}(1-\delta)k_t\right)}{\partial k_t} \tag{A2.1}
$$

Similarly it can be showed that

$$
\frac{\partial E_t \min\left\{\frac{R^R_t}{\pi_{t+1}}l^D_t, \quad \kappa\left(p^R_{t+1}y^R_{t+1} + q_{t+1}(1-\delta)k_t\right) - R^R_t rer_{t+1}\frac{l^F_t}{\pi^*_{t+1}}\right\}}{\partial h_t} = \left(1-\Phi(d_{1,t})\right)\frac{\partial E_t\kappa\left(p^R_{t+1}y^R_{t+1}\right)}{\partial h_t} \tag{A2.2}
$$

A3: Solving the financially constrained firms' profit maximization problem with domestic currency loans

Now the matured loan in units of composite goods is $R^R_{i,t}\frac{L_{i,t}}{P_{t+1}} \equiv R^R_{i,t}\frac{l_{i,t}}{\pi_{t+1}}$. The loan is denominated in domestic currency and $R^R_{i,t}$ is the nominal gross interest rate on the loan. The contracted collateral is a fraction κ of firms' revenue from selling goods and depreciated capital in the next period. In units of composite goods the contracted collateral can be expressed as $p^R_{t+1}y^R_{i,t+1} + q_{t+1}(1-\delta)k_{i,t}$. Then the decision of the financially constrained firm i born in period t whether to default or not is determined by the lower value:

$$\min\left\{R^R_{i,t}\frac{l_{i,t}}{\pi_{t+1}},\quad \kappa\left(p^R_{t+1}y^R_{i,t+1} + q_{t+1}(1-\delta)k_{i,t}\right)\right\}$$

As previously, $p^R_{t+1}y^R_{i,t+1} = p^R_{t+1}A_{t+1}\theta_{i,t+1}k^{\alpha}_{i,t}h^{1-\alpha}_{i,t}$, $p^R_{t+1} \equiv P^R_{t+1}/P_{t+1}$ and $q_{t+1} \equiv Q_{t+1}/P_{t+1}$.

Financial flows received in period t also enter the maximization problem and can be summarized as the difference between the loan plus equity (both $N^{firms}_{i,t}$ and $Z_{i,t}$) and working capital expenditure expressed in units of composite goods:

$$\begin{aligned}\max_{\{k_{i,t},h_{i,t}\}} & E_t\beta\Lambda_{t,t+1}\left\{\frac{P^R_{t+1}y^R_{i,t+1} + Q_{t+1}(1-\delta)k_{i,t} - (1-\rho)\left(Q_tk_{i,t} + W_th_{i,t}\right)}{P_{t+1}}\right\} \\ & - E_t\beta\Lambda_{t,t+1}\min\left\{\frac{R^R_{i,t}L_{i,t}}{P_{t+1}},\quad \frac{\kappa\left(P^R_{t+1}y^R_{i,t+1} + Q_{t+1}(1-\delta)k_{i,t}\right)}{P_{t+1}}\right\} \\ & + \frac{L_{i,t} + N^{firms}_{i,t} + Z_{i,t}}{P_t} - \frac{\rho\left(Q_tk_{i,t} + W_th_{i,t}\right)}{P_t}\end{aligned}$$

s.t.

$$\frac{E_{t-1}\left\{L_{i,t} + N^{firms}_{i,t}\right\}}{P_t} = \frac{E_{t-1}\left\{\rho\left(Q_tk_{i,t} + W_th_{i,t}\right)\right\}}{P_t}$$

Using the previously introduced definitions yields

$$\begin{aligned}\max_{\{k_{i,t},h_{i,t}\}} & E_t\beta\Lambda_{t,t+1}\left\{p^R_{t+1}y^R_{i,t+1} + q_{t+1}(1-\delta)k_{i,t} - (1-\rho)\frac{q_tk_{i,t} + w_th_{i,t}}{\pi_{t+1}}\right\} \\ & - E_t\beta\Lambda_{t,t+1}\min\left\{R^R_{i,t}\frac{l_{i,t}}{\pi_{t+1}},\quad \kappa\left(p^R_{t+1}y^R_{i,t+1} + q_{t+1}(1-\delta)k_{i,t}\right)\right\} \\ & + l_{i,t} + n^{firms}_{i,t} + z_{i,t} - \rho\left(q_tk_{i,t} + w_th_{i,t}\right)\end{aligned}$$

s.t.

$$E_{t-1}\left\{l_{i,t}\right\} + E_{t-1}\left\{n^{firms}_{i,t}\right\} = E_{t-1}\left\{\rho\left(q_tk_{i,t} + w_th_{i,t}\right)\right\}$$

The resulting first-order conditions are:

$$
\begin{aligned}
k_{i,t}: \quad & E_t\beta\Lambda_{t,t+1}\left\{p^R_{t+1}\frac{\partial y^R_{i,t+1}}{\partial k_{i,t}} + q_{t+1}(1-\delta) - (1-\rho)\frac{q_t}{\pi_{t+1}}\right\} \\
& - E_t\beta\Lambda_{t,t+1}\left\{(1-\Phi(d_{1,t}))\kappa\left(p^R_{t+1}\frac{\partial y^R_{i,t+1}}{\partial k_{i,t}} + q_{t+1}(1-\delta)\right)\right\} \\
& = \frac{\partial cov\left(\beta\Lambda_{t,t+1}, \quad \min\left\{R^R_{i,t}\frac{l_{i,t}}{\pi_{t+1}}, \quad \kappa\left(p^R_{t+1}y^R_{i,t+1} + q_{t+1}(1-\delta)k_{i,t}\right)\right\}\right)}{\partial k_{i,t}} \\
& + \rho q_t
\end{aligned}
$$

$$
\begin{aligned}
h_{i,t}: \quad & E_t\beta\Lambda_{t,t+1}\left\{p^R_{t+1}\frac{\partial y^R_{i,t+1}}{\partial h_{i,t}} - (1-\rho)\frac{w_t}{\pi_{t+1}}\right\} \\
& - E_t\beta\Lambda_{t,t+1}\left\{(1-\Phi(d_{1,t}))\kappa\left(p^R_{t+1}\frac{\partial y^R_{i,t+1}}{\partial h_{i,t}}\right)\right\} \\
& = \frac{\partial cov\left(\beta\Lambda_{t,t+1}, \quad \min\left\{R^R_{i,t}\frac{l_{i,t}}{\pi_{t+1}}, \quad \kappa\left(p^R_{t+1}y^R_{i,t+1} + q_{t+1}(1-\delta)k_{i,t}\right)\right\}\right)}{\partial h_{i,t}} \\
& + \rho w_t
\end{aligned}
$$

where

$$
d_{2,t} \equiv \frac{E_t \ln\left(\kappa\left(p^R_{t+1}y^R_{i,t+1} + q_{t+1}(1-\delta)k_{i,t}\right)\right) - E_t \ln\left(R^R_{i,t}\frac{l_{i,t}}{\pi_{t+1}}\right)}{\sigma_y}, \qquad d_{1,t} = d_{2,t} + \sigma_y
$$

and $\sigma_y^2 = var\left(\pi_{t+1}\kappa\left(p^R_{t+1}y^R_{t+1} + q_{t+1}(1-\delta)k_t\right)\right)$.

The first-order conditions hold together with the ex-ante budget constraint:

$$
E_{t-1}\left\{l_{i,t}\right\} + E_{t-1}\left\{n^{firms}_{i,t}\right\} = E_{t-1}\left\{\rho\left(q_t k_{i,t} + w_t h_{i,t}\right)\right\}
$$

A4: Model with flexible labour demand

In simulation exercises, when we relax the assumption of predetermined labour supply, we make the following modifications to the model. Firstly, we assume that the only input for financially constrained firms' production is capital. Second, we introduce a new layer of production firms and call them intermediate firms. These firms combine financially constrained firms' production with labour and sell homogenous goods to domestic retail firms. The novel type of firms is not subject to financial frictions.

Then the financially constrained firm's problem changes accordingly. The firm's borrowing decision depends on the firm's expected working capital needs such that in the beginning of period t the following condition holds:

$$
E_{t-1}\left\{L_{i,t}\right\} + E_{t-1}\left\{N^{firms}_{i,t}\right\} = E_{t-1}\left\{\rho\left(Q_t k_{i,t}\right)\right\}
$$

Or, units of composite goods,

$$E_{t-1}\left\{l_{i,t}\right\}+E_{t-1}\left\{n_{i,t}^{firms}\right\}=E_{t-1}\left\{\rho\left(q_t k_{i,t}\right)\right\}$$

Definition of $P_{t+1}^{R}y_{i,t+1}^{R}$ changes in the following way: $P_{t+1}^{R}y_{i,t+1}^{R}=P_{t+1}^{R}A_{t+1}\theta_{i,t+1}k_{i,t}$.

After shocks take place, the generation of firms t will solve the profit maximization problem taking the loan as given. They will sell goods at the competitive price P_{t+1}^{R} which is defined p_{t+1}^{R}, if expressed in units of composite goods. The profit optimization problem of a financially constrained firm i will be the following (the numeraire is the composite good):

$$\begin{aligned}\max_{\left\{k_{i,t},h_{i,t}\right\}} & E_t\beta\Lambda_{t,t+1}\left\{p_{t+1}^{R}y_{i,t+1}^{R}+q_{t+1}(1-\delta)k_{i,t}-(1-\rho)\frac{q_t k_{i,t}}{\pi_{t+1}}\right\}\\ & -E_t\beta\Lambda_{t,t+1}\min\left\{\frac{R_{i,t}^{R}}{\pi_{t+1}}l_{i,t},\quad \kappa\left(p_{t+1}^{R}y_{i,t+1}^{R}+q_{t+1}(1-\delta)k_{i,t}\right)\right\}\\ & +l_{i,t}+n_{i,t}^{firms}+z_{i,t}-\rho\left(q_t k_{i,t}\right)\end{aligned}$$

s.t.

$$E_{t-1}\left\{l_{i,t}\right\}+E_{t-1}\left\{n_{i,t}^{firms}\right\}=E_{t-1}\left\{\rho\left(q_t k_{i,t}\right)\right\}$$

The corresponding first-order condition is:

$$\begin{aligned}k_{i,t}:\quad & E_t\beta\Lambda_{t,t+1}\left\{p_{t+1}^{R}\frac{\partial y_{i,t+1}^{R}}{\partial k_{i,t}}+(1-\delta)q_{t+1}-(1-\rho)\frac{q_t}{\pi_{t+1}}\right\}\\ & -\frac{\partial E_t\beta\Lambda_{t,t+1}E_t\min\left\{\frac{R_{i,t}^{R}}{\pi_{t+1}}l_{i,t},\quad \kappa\left(p_{t+1}^{R}y_{i,t+1}^{R}+q_{t+1}(1-\delta)k_{i,t}\right)\right\}}{\partial k_{i,t}}\\ & =\frac{\partial cov\left(\beta\Lambda_{t,t+1},\quad \min\left\{\frac{R_{i,t}^{R}l_{i,t}}{\pi_{t+1}},\quad \kappa\left(p_{t+1}^{R}y_{i,t+1}^{R}+q_{t+1}(1-\delta)k_{i,t}\right)\right\}\right)}{\partial k_{i,t}}\\ & +\rho q_t\end{aligned}$$

If we substitute the expression for the expected value of loan repayment, we get:

$$\begin{aligned}k_{i,t}:\quad & E_t\beta\Lambda_{t,t+1}\left\{p_{t+1}^{R}\frac{\partial y_{i,t+1}^{R}}{\partial k_{i,t}}+q_{t+1}(1-\delta)-(1-\rho)\frac{q_t}{\pi_{t+1}}\right\}\\ & -E_t\beta\Lambda_{t,t+1}\left\{(1-\Phi(d_{1,t}))\kappa\left(p_{t+1}^{R}\frac{\partial y_{i,t+1}^{R}}{\partial k_{i,t}}+q_{t+1}(1-\delta)\right)\right\}\\ & =\frac{\partial cov\left(\beta\Lambda_{t,t+1},\quad \min\left\{\frac{R_{i,t}^{R}l_{i,t}}{\pi_{t+1}},\quad \kappa\left(p_{t+1}^{R}y_{i,t+1}^{R}+q_{t+1}(1-\delta)k_{i,t}\right)\right\}\right)}{\partial k_{i,t}}\\ & +\rho q_t\end{aligned}$$

where

$$d_{2,t} \equiv \frac{E_t \ln\left(\kappa\left(p^R_{t+1} y^R_{i,t+1} + q_{t+1}(1-\delta)k_{i,t}\right)\right) - E_t \ln\left(\frac{R^R_{i,t}}{\pi_{t+1}} l_{i,t}\right)}{\sigma_y}, \qquad d_{1,t} = d_{2,t} + \sigma_y$$

σ_y^2 is given by $var(\pi_{t+1}\kappa\left(p^R_{t+1} y^R_{i,t+1} + q_{t+1}(1-\delta)k_{i,t}\right))$.

Homogenous goods produced by financially constrained firms are purchased as inputs by the new layer of competitive producers, intermediate producers. Intermediate producers hire labour and combine it with homogenous goods produced by financially constrained firms by using the following technology:

$$y^I_t = \left(y^R_t\right)^{\alpha} h_t^{1-\alpha}$$

Recall that financially constrained firms' aggregate production function now is given by: $y^R_t = A_t k_{t-1}$. Produced goods are sold to retail firms at the nominal price P^I_t immediately after production takes place. This gives two equilibrium conditions that can be derived from profit maximization with respect to inputs:

$$y^R_t: \quad p^R_t = p^I_t \alpha \left(y^R_t\right)^{\alpha-1} h_t^{1-\alpha}$$

$$h_t: \quad w_t = p^I_t (1-\alpha) \left(y^R_t\right)^{\alpha} h_t^{-\alpha}$$

In derivations we defined the following relative prices: $p^I_t \equiv P^I_t/P_t$, $p^R_t \equiv P^R_t/P_t$ and $w_t \equiv W_t/P_t$.

Marginal costs of the retail firms changes from being the price of financially constrained firms' goods to the price of intermediate goods.

B: Solving the banks' optimization problem

B1: Lending in foreign currency and domestic currency with a fixed denomination structure

The domestic household owns all banks that operate in the domestic economy and lend to financially constrained firms. We assume that there is a continuum of these banks and every period there is a probability ω that a bank continues operating. Otherwise, the net worth is transferred to the owner of the bank, the domestic household. We assume that banks give loans out of accumulated equity N_t, deposits D_t and foreign debt D^*_t. The balance sheet constraint of a bank j, expressed in units of composite goods, is given by

$$\frac{N_{j,t} + D_{j,t} + S_t D^*_{j,t}}{P_t} = \frac{L_{j,t}}{P_t}$$

$L_{j,t}$ consists of both domestic currency funds $L^D_{j,t}$ and foreign currency denominated funds $L^F_{j,t}$ such that $L_{j,t} = L^D_{j,t} + S_t L^F_{j,t}$ where S_t is the nominal exchange rate.

Banks pay a nominal domestic interest rate R_t on deposits and a nominal foreign interest rate $R^*_t \xi_t$ on foreign debt. R^*_t follows a stationary AR(1) process. ξ_t denotes a premium on bank foreign debt. To ensure stationarity in the model, we assume that the premium depends on the level of bank foreign debt (as in Schmitt-Grohé and Uribe, 2003):

$$\xi_t = \exp\left(\kappa_\xi \frac{(S_t D^*_t - S \cdot D^*)}{S \cdot D^*} + \frac{\zeta_t - \zeta}{\zeta}\right) \tag{21}$$

where ζ_t is an exogenous shock that follows a stable AR(1) process.

Banks are subject to an agency problem as in Gertler and Karadi (2011). At the end of every period, bankers can divert a fraction λ^L of assets, but if that happens the bank goes bankrupt (i.e. cannot continue). Creditors take this possibility into account and lend only up to the point where the continuation value of the bank is equal to or higher than the value of what can be diverted. This condition acts as an incentive constraint for the bank and eventually limits expansion of the balance sheet of the bank for given amount of equity.

The only asset on the banks' balance sheet is loans to financially constrained firms, thus, the expected nominal return of the bank j is defined as $R^L_{j,t}$ and given by:

$$E_t\left\{R^L_{j,t}L_{j,t}\right\} \equiv E_t \min\left\{R^R_{j,t}\left(L^D_{j,t}+S_{t+1}L^F_{j,t}\right), \quad \kappa\left(P^R_{t+1}y^R_{j,t+1}+Q_{t+1}(1-\delta)k_{j,t}\right)\right\}$$

Or, units of composite goods,

$$E_t\left\{\frac{R^L_{j,t}}{\pi_{t+1}}l_{j,t}\right\} \equiv E_t \min\left\{R^R_{j,t}\left(\frac{l^D_{j,t}}{\pi_{t+1}}+rer_{t+1}\frac{l^F_{j,t}}{\pi^*_{t+1}}\right), \quad \kappa\left(p^R_{t+1}y^R_{j,t+1}+q_{t+1}(1-\delta)k_{j,t}\right)\right\}$$

$$\Rightarrow E_t\left\{\frac{R^L_{j,t}}{\pi_{t+1}}l_{j,t}\right\} \equiv E_t\left\{(1-\Phi(d_{1,t}))\,\kappa\left(p^R_{t+1}y^R_{j,t+1}+(1-\delta)q_{t+1}k_{j,t}\right)+\Phi(d_{2,t})R^R_{j,t}\frac{l^D_{j,t}}{\pi_{t+1}}+\Phi(d_{1,t})R^R_{j,t}rer_{t+1}\frac{l^F_{j,t}}{\pi^*_{t+1}}\right\} \tag{22}$$

Then the optimization problem of the bank j can be written as:

$$V_{j,t} = \max_{\left\{D_{j,t},D^*_{j,t},L_{j,t}\right\}} \quad E_t\left[\beta\Lambda_{t,t+1}\left\{(1-\omega)\frac{N_{j,t+1}}{P_{t+1}}+\omega V_{j,t+1}\right\}\right]$$

s.t.

$$V_{j,t} \geq \lambda^L\frac{L_{j,t}}{P_t}, \qquad \textit{(Incentive constraint)}$$

$$\frac{N_{j,t}+D_{j,t}+S_tD^*_{j,t}}{P_t} = \frac{L_{j,t}}{P_t}, \qquad \textit{(Balance sheet constraint)}$$

$$\frac{N_{j,t}}{P_t} = \frac{R^L_{j,t-1}}{P_t}L_{j,t-1}-\frac{R_{t-1}}{P_t}D_{j,t-1}-\frac{R^*_{t-1}\xi_{t-1}}{P_t}S_tD^*_{j,t-1} \qquad \textit{(LoM of net worth)}$$

We define $rer_t \equiv P^*_tS_t/P_t$, $d^*_{j,t} \equiv D^*_{j,t}/P^*_t$, $d_{j,t} \equiv D_{j,t}/P_t$, $l_{j,t} \equiv L_{j,t}/P_t$, and $n_{j,t} \equiv N_{j,t}/P_t$. It follows that

$$V_{j,t} = \max_{\left\{d_{j,t},d^*_{j,t},l_{j,t}\right\}} \quad E_t\left[\beta\Lambda_{t,t+1}\left\{(1-\omega)n_{j,t+1}+\omega V_{j,t+1}\right\}\right]$$

s.t.

$$V_{j,t} \geq \lambda^L l_{j,t}, \qquad \textit{(Incentive constraint)}$$

$$n_{j,t}+d_{j,t}+rer_td^*_{j,t} = l_{j,t}, \qquad \textit{(Balance sheet constraint)}$$

$$n_{j,t} = \frac{R^L_{j,t-1}}{\pi_t}l_{j,t-1}-\frac{R_{t-1}}{\pi_t}d_{j,t-1}-\frac{R^*_{t-1}\xi_{t-1}}{\pi^*_t}rer_td^*_{j,t-1} \qquad \textit{(LoM of net worth)}$$

Lagrangian of the problem can be formulated as:

$$L = \begin{aligned}[t] & (1+\nu_{1,t})E_t\beta\Lambda_{t,t+1}\left\{(1-\omega)\left(\frac{R^L_{j,t}}{\pi_{t+1}}l_{j,t} - \frac{R_t}{\pi_{t+1}}d_{j,t} - \frac{R^*_t\xi_t}{\pi^*_{t+1}}rer_{t+1}d^*_{j,t}\right) + \omega V_{j,t+1}\right\} \\ & - \nu_{1,t}\lambda^L l_{j,t} \\ & + \nu_{2,t}\left(\frac{R^L_{j,t-1}}{\pi_t}l_{j,t-1} - \frac{R_{t-1}}{\pi_t}d_{j,t-1} - \frac{R^*_{t-1}\xi_{t-1}}{\pi^*_t}rer_t d^*_{j,t-1} - l_{j,t} + d_{j,t} + rer_t d^*_{j,t}\right)\end{aligned}$$

This gives the first-order conditions:

$$l_{j,t}: \quad (1+\nu_{1,t})\beta E_t\Lambda_{t,t+1}\left\{(1-\omega)\left(\frac{R^L_{j,t}}{\pi_{t+1}}\right) + \omega\frac{\partial V(.)}{\partial l_{j,t}}\right\} = \lambda^L\nu_{1,t} + \nu_{2,t}$$

$$d_{j,t}: \quad (1+\nu_{1,t})\beta E_t\Lambda_{t,t+1}\left\{(1-\omega)\left(\frac{R_t}{\pi_{t+1}}\right) - \omega\frac{\partial V(.)}{\partial d_{j,t}}\right\} = \nu_{2,t}$$

$$d^*_{j,t}: \quad (1+\nu_{1,t})\beta E_t\Lambda_{t,t+1}\left\{(1-\omega)\left(\frac{R^*_t\xi_t}{\pi^*_{t+1}}rer_{t+1}\right) - \omega\frac{\partial V(.)}{\partial d^*_{j,t}}\right\} = \nu_{2,t}rer_t$$

with complementary slackness conditions:

$$\nu_{1,t}: \quad \nu_{1,t}\left(V_{j,t} - \lambda^L l_{j,t}\right) = 0$$

$$\nu_{2,t}: \quad \nu_{2,t}\left(\frac{R^L_{j,t-1}}{\pi_t}l_{j,t-1} - \frac{R_{t-1}}{\pi_t}d_{j,t-1} - \frac{R^*_{t-1}\xi_{t-1}}{\pi^*_t}rer_t d^*_{j,t-1} - l_{j,t} + d_{j,t} + rer_t d^*_{j,t}\right) = 0$$

Further, the first-order conditions can be expressed as

$$l_{j,t}: \quad (1+\nu_{1,t})\beta E_t\Lambda_{t,t+1}\left\{(1-\omega) + \omega\nu_{2,t+1}\right\}\left(\frac{R^L_{j,t}}{\pi_{t+1}}\right) = \lambda^L\nu_{1,t} + \nu_{2,t}$$

$$d_{j,t}: \quad (1+\nu_{1,t})\beta E_t\Lambda_{t,t+1}\left\{(1-\omega) + \omega\nu_{2,t+1}\right\}\left(\frac{R_t}{\pi_{t+1}}\right) = \nu_{2,t}$$

$$d^*_{j,t}: \quad (1+\nu_{1,t})\beta E_t\Lambda_{t,t+1}\left\{(1-\omega) + \omega\nu_{2,t+1}\right\}\left(\frac{R^*_t\xi_t}{\pi^*_{t+1}}\frac{rer_{t+1}}{rer_t}\right) = \nu_{2,t}$$

Besides these first-order conditions, the set of equilibrium conditions includes the law of motion for aggregate net worth of banks and the bank incentive constraint. First, we formulate the law of motion for aggregate net worth. We assume that aggregate net worth consists of the net worth of non-bankrupted banks and the new worth of new banks. The new equity is injected by the domestic household and is assumed to be of the size

ιn. Then

$$n_t = \omega \left(\frac{R^L_{j,t-1}}{\pi_t} l_{t-1} - \frac{R_{t-1}}{\pi_t} d_{t-1} - \frac{R^*_{t-1}\xi_{t-1}}{\pi^*_t} rer_t d^*_{t-1} \right) + \iota n$$

To include the incentive constraint in the equilibrium conditions, we have to redefine it by using the value of marginal utility from increasing assets by one unit and the value of marginal disutility from increasing debt by one unit. It follows from the previously derived results that the value of the bank j can also be defined as:

$$\begin{aligned} V_{j,t} &= \left(\lambda^L \tfrac{\nu_{1,t}}{1+\nu_{1,t}} + \tfrac{\nu_{2,t}}{1+\nu_{1,t}} \right) l_{j,t} - \tfrac{\nu_{2,t}}{1+\nu_{1,t}} d_{j,t} - \tfrac{\nu_{2,t}}{1+\nu_{1,t}} rer_t d^*_{j,t} \\ &= \tfrac{\nu_{2,t}}{1+\nu_{1,t}} \left(l_{j,t} - d_{j,t} - rer_t d^*_{j,t} \right) + \lambda^L \tfrac{\nu_{1,t}}{1+\nu_{1,t}} l_{j,t} \end{aligned}$$

$$\Rightarrow \quad V_{j,t} = \frac{\nu_{2,t}}{1+\nu_{1,t}} n_{j,t} + \lambda^L \frac{\nu_{1,t}}{1+\nu_{1,t}} l_{j,t}$$

Then we can modify the incentive constraint as

$$\frac{\nu_{2,t}}{1+\nu_{1,t}} n_{j,t} + \lambda^L \frac{\nu_{1,t}}{1+\nu_{1,t}} l_{j,t} \geq \lambda^L l_{j,t}$$

$$\Rightarrow \quad \nu_{2,t} n_{j,t} \geq \lambda^L l_{j,t}$$

B2: Lending in domestic currency only

Now the only asset on the banks' balance sheet is domestic currency loans extended to financially constrained firms, thus, the expected nominal return of the bank j is defined as $R^L_{j,t}$ and given by:

$$E_t \left\{ R^L_{j,t} L_{j,t} \right\} \equiv E_t \min \left\{ R^R_{j,t} L_{j,t}, \quad \kappa \left(P^R_{t+1} y^R_{j,t+1} + Q_{t+1}(1-\delta) k_{j,t} \right) \right\}$$

Or, in units of composite goods,

$$E_t \left\{ \frac{R^L_{j,t}}{\pi_{t+1}} l_{j,t} \right\} \equiv E_t \min \left\{ R^R_{j,t} \frac{l_{j,t}}{\pi_{t+1}}, \quad \kappa \left(p^R_{t+1} y^R_{j,t+1} + q_{t+1}(1-\delta) k_{j,t} \right) \right\}$$

$$\Rightarrow E_t \left\{ \frac{R^L_{j,t}}{\pi_{t+1}} l_{j,t} \right\} \equiv E_t \left\{ (1 - \Phi(d_{1,t}))\, \kappa \left(p^R_{t+1} y^R_{j,t+1} + (1-\delta) q_{t+1} k_{j,t} \right) + \Phi(d_{2,t}) R^R_{j,t} \frac{l_{j,t}}{\pi_{t+1}} \right\} \tag{23}$$

The rest of derivations for the bank's optimization problem remain the same.

B3: Financial sector support

This segment of the model closely follows Kirchner and van Wijnbergen (2011). We assume that the government can intervene during the crisis by injecting capital τ^{FS}_t to the financial sector. We assign the following rule to the recap of the financial intermediary j:

$$\tau^{FI}_t = \kappa_{FS} \left(shock_{t-l} - shock \right) n_{j,t-1}, \qquad \kappa_{FS} > 0, \quad l \geq 0$$

where $n_{j,t-1}$ is the net worth of the intermediary from the previous period. The recap can be immediate ($l = 0$) or delayed ($l > 0$). We introduce a new variable $shock_t$ that coincides with the variable driving the crisis, e.g. the risk premium shock ($shock_t \equiv \xi_t$). We assume that the recap is a gift from the government and does not have to be repaid.

Then the optimization problem of the financial intermediary j as defined in subsection B1 can be modified to

$$V_{j,t} = \max_{l_{j,t},d_{j,t},d^*_{j,t}} \quad E_t\left[\beta\Lambda_{t,t+1}\left\{(1-\omega)n_{j,t+1} + \omega V_{j,t+1}\right\}\right]$$

s.t.

$$V_{j,t} \geq \lambda^L l_{j,t}, \qquad \textit{(Incentive constraint)}$$

$$n_{j,t} + d_{j,t} + rer_t d^*_{j,t} = l_{j,t}, \qquad \textit{(Balance sheet constraint)}$$

$$n_{j,t} = \frac{R^L_{j,t-1}}{\pi_t} l_{j,t-1} - \frac{R_{t-1}}{\pi_t} d_{j,t-1} - \frac{R^*_{t-1}\xi_{t-1}}{\pi^*_t} rer_t d^*_{j,t-1} + \kappa_{FS}\left(shock_{t-l} - shock\right) n_{j,t-1} \quad \textit{(LoM of net worth)}$$

Lagrangian of the problem can be formulated as:

$$\begin{aligned} L = \quad & (1+\nu_{1,t})E_t\beta\Lambda_{t,t+1}\left\{(1-\omega)\left(\frac{R^L_{j,t}}{\pi_{t+1}} l_{j,t} - \frac{R_t}{\pi_{t+1}} d_{j,t} - \frac{R^*_t\xi_t}{\pi^*_{t+1}} rer_{t+1} d^*_{j,t} + \kappa_{FS}\left(shock_{t-l+1} - shock\right) n_{j,t}\right) + \omega V_{j,t+1}\right\} \\ & - \nu_{1,t}\lambda^L l_{j,t} \\ & + \nu_{2,t}\left(\frac{R^L_{j,t-1}}{\pi_t} l_{j,t-1} - \frac{R_{t-1}}{\pi_t} d_{j,t-1} - \frac{R^*_{t-1}\xi_{t-1}}{\pi^*_t} rer_t d^*_{j,t-1} + \kappa_{FS}\left(shock_{t-l} - shock\right) n_{j,t-1} - l_{j,t} + d_{j,t} + rer_t d^*_{j,t}\right) \end{aligned}$$

This gives the first-order conditions:

$$l_{j,t}: \quad (1+\nu_{1,t})\beta E_t\Lambda_{t,t+1}\left\{(1-\omega)\left(\frac{R^L_{j,t}}{\pi_{t+1}} + \kappa_{FS}\left(shock_{t-l+1} - shock\right)\right) + \omega\frac{\partial V(.)}{\partial l_{j,t}}\right\} = \lambda^L\nu_{1,t} + \nu_{2,t}$$

$$d_{j,t}: \quad (1+\nu_{1,t})\beta E_t\Lambda_{t,t+1}\left\{(1-\omega)\left(\frac{R_t}{\pi_{t+1}} + \kappa_{FS}\left(shock_{t-l+1} - shock\right)\right) - \omega\frac{\partial V(.)}{\partial d_{j,t}}\right\} = \nu_{2,t}$$

$$d^*_{j,t}: \quad (1+\nu_{1,t})\beta E_t\Lambda_{t,t+1}\left\{(1-\omega)\left(\frac{R^*_t\xi_t}{\pi^*_{t+1}}\frac{rer_{t+1}}{rer_t} + \kappa_{FS}\left(shock_{t-l+1} - shock\right)\right) - \omega\frac{\partial V(.)}{\partial d^*_{j,t}}\right\} = \nu_{2,t}$$

with complementary slackness conditions:

$$\nu_{1,t} : \nu_{1,t}\left(V_{j,t} - \lambda^L l_{j,t}\right) = 0$$

$$\nu_{2,t} : \nu_{2,t}\left(\frac{R^L_{j,t-1}}{\pi_t} l_{j,t-1} - \frac{R_{t-1}}{\pi_t} d_{j,t-1} - \frac{R^*_{t-1}\xi_{t-1}}{\pi^*_t} rer_t d^*_{j,t-1} + \kappa_{FS}\left(shock_{t-l} - shock\right) n_{j,t-1} - l_{j,t} + d_{j,t} + rer_t d^*_{j,t}\right) = 0$$

Further, the first-order conditions can be expressed as

$$l_{j,t}: \quad (1+\nu_{1,t})\beta E_t \Lambda_{t,t+1}\left\{(1-\omega)+\omega\nu_{2,t+1}\right\}\frac{R^L_{j,t}}{\pi_{t+1}} + (1+\nu_{1,t})\beta E_t\Lambda_{t,t+1}(1-\omega)\kappa_{FS}\left(shock_{t-l+1} - shock\right)$$
$$= \nu_{1,t}\lambda^L + \nu_{2,t}$$

$$d_{j,t}: \quad (1+\nu_{1,t})\beta E_t \Lambda_{t,t+1}\left\{(1-\omega)+\omega\nu_{2,t+1}\right\}\frac{R_t}{\pi_{t+1}} + (1+\nu_{1,t})\beta E_t\Lambda_{t,t+1}(1-\omega)\kappa_{FS}\left(shock_{t-l+1} - shock\right)$$
$$= \nu_{2,t}$$

$$d^*_{j,t}: \quad (1+\nu_{1,t})\beta E_t \Lambda_{t,t+1}\left\{(1-\omega)+\omega\nu_{2,t+1}\right\}\frac{R^*_t\xi_t}{\pi^*_{t+1}}\frac{rer_{t+1}}{rer_t} + (1+\nu_{1,t})\beta E_t\Lambda_{t,t+1}(1-\omega)\kappa_{FS}\left(shock_{t-l+1} - shock\right)$$
$$= \nu_{2,t}$$

Aggregate net worth evolves as

$$n_t = \omega\left[\frac{R^L_{j,t-1}}{\pi_t}l_{t-1} - \frac{R_{t-1}}{\pi_t}d_{t-1} - \frac{R^*_{t-1}\xi_{t-1}}{\pi^*_t}rer_t d^*_{t-1} + \kappa_{FS}\left(shock_{t-l} - shock\right)n_{t-1}\right] + \iota n$$

C: Household's problem

We assume a representative household. The household has two alternatives to invest in: make deposits D_t in a bank or buy bonds issued by the government, B_t. The household supplies labour to a competitive labour market. The household has Greenwood–Hercowitz–Huffman (henceforth, GHH) preferences as in Greenwood et al. (1988), so labour supply does not depend on wealth. The household chooses a level of real consumption c_t and working hours h_t such that the following lifetime utility function is maximized:

$$E_0\sum_{t=0}^{\infty}\beta^t\frac{1}{1-\gamma}\left(c_t - \frac{\chi\left(h_t\right)^{1+\varphi}}{1+\varphi}\right)^{1-\gamma} \qquad \gamma,\chi,\varphi > 0 \tag{24}$$

subject to the household's budget constraint:

$$C_t + B_t + D_t = W_t h_t + R_{t-1}B_{t-1} + R_{t-1}D_{t-1} + P_t\Pi_t - T_t$$

The budget constraint, expressed in units of composite goods, is given by

$$c_t + b_t + d_t = w_t h_t + \frac{R_{t-1}}{\pi_t}b_{t-1} + \frac{R_{t-1}}{\pi_t}d_{t-1} + \Pi_t - t_t \tag{25}$$

π_t denotes the composite goods price inflation, $c_t \equiv C_t/P_t$, $w_t \equiv W_t/P_t$, $b_t \equiv B_t/P_t$, $d_t \equiv D_t/P_t$, $t_t \equiv T_t/P_t$. We assume that the household is indifferent between buying bonds and making deposits, thus, R_t is nominal gross interest rate of both bonds and deposits. The household owns all banks in the model economy and thus receives lump-sum dividends, Π_t. Taxes t_t enter the household's budget constraint in a lump-sum way as well. Lump-sum dividends from financially constrained firms are included in total dividends Π_t. Lump-sum dividends from financially constrained firms consist of firms' profits that the household receives in the beginning in the period minus the equity that the household transfers to the firms in the beginning of the period (in response to liquidity shortage, if there is any):

$$\begin{aligned}\Pi_t^{firms} = \quad & \omega^{firms}\left(p_t^R y_t^R + q_t(1-\delta)k_{t-1} - (1-\rho)\frac{q_{t-1}k_{t-1}+w_{t-1}h_{t-1}}{\pi_t}\right) \\ & -\omega^{firms}\left(\kappa\left(1-\Phi(d_{1,t-1})\right)\left(p_t^R y_t^R + q_t(1-\delta)k_{t-1}\right) + \Phi(d_{2,t-1})R_{t-1}^R\frac{l_{t-1}^D}{\pi_t} + \Phi(d_{1,t-1})rer_t\frac{l_{t-1}^F}{\pi_t^*}\right) \\ & -n_t^{firms} - z_t \\ & = -\iota^{firms}\cdot n^{firms} - z_t\end{aligned}$$

The final result follows from the definition of aggregate corporate net worth given in the financially constrained firms' problem in section A1.

The household's optimization problem gives first-order conditions:

$$\lambda_t = \left(c_t - \frac{\chi\,(h_t)^{1+\varphi}}{1+\varphi}\right)^{-\gamma}$$

$$w_t = \chi\,(h_t)^{\varphi}$$

$$E_t\beta\Lambda_{t,t+1}\frac{R_t}{\pi_{t+1}} = 1$$

We denote $\Lambda_{t,t+1} \equiv \frac{\lambda_{t+1}}{\lambda_t}$ where λ_t is the Lagrangian multiplier to the household's budget constraint.

D: Production and Pricing

There are several types of firms in the domestic economy. It takes three types of firms to produce domestic aggregate inputs for composite goods. First, there are the financially constrained firms that combine purchased capital with labour and produce homogenous goods. They were analyzed in Section 3.1. Their homogenous outputs are bought by retail firms who costlessly differentiate the products bought and sell them as (local) monopolists, in Dixit-Stiglitz (1977) fashion. A similar group of firms called importers differentiate foreign (imported) goods. A composite goods producer buys the differentiated home goods and aggregates them into an aggregate domestic good y_t^H with associated price p_t^H. The same composite goods producer also buys imported differentiated goods and aggregates them into a foreign aggregate good y_t^F. The corresponding aggregate price level of foreign goods is p_t^F. All details of the derivations of the various first order conditions optimization problems can be found in the supplementary appendix D. We discuss each step in more detail below.

The structure of the production sector is exhibited in Figure 19.

D1: Retail firms

Homogenous goods produced by financially constrained firms are sold to domestic retail firms. We assume that there is a continuum of domestic retail firms. A domestic retail firm j differentiates purchased inputs at p_t^R and sells at a monopolistic price $p_t^H(j)$. Differentiated goods from the domestic retail sector, $y_t^H(j), j \in (0,1)$, are purchased by the composite goods producer.

Retail firms are subject to sticky prices as in Calvo (1983), so every period $(1-\omega^H)$ of them adjust prices to the optimal reset price $P_t^{\#}(j)$. Then the profit of a retail firm j that is allowed to adjust its price in period t

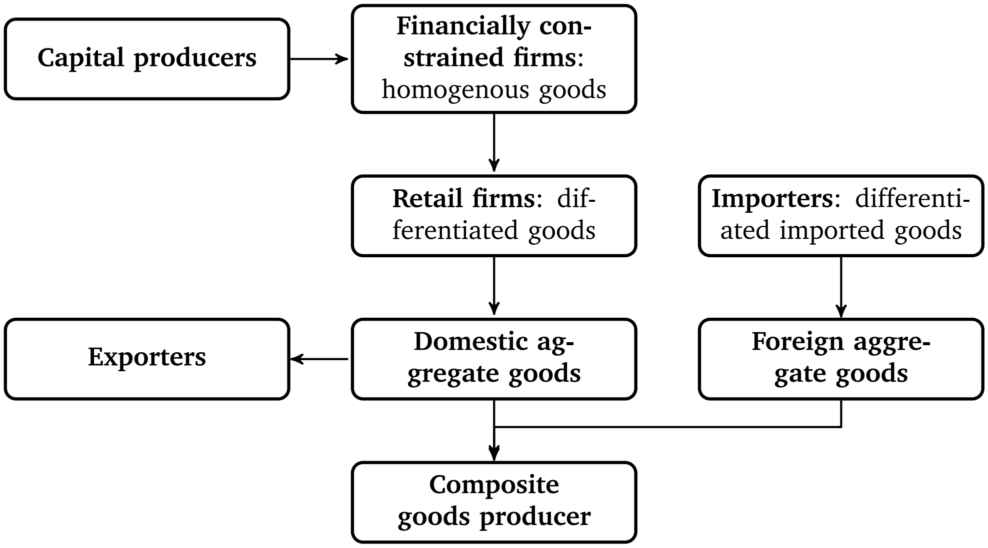


Figure 19: Structure of the production sector.

is thus given by $\left(P_t^{\#}(j) - P_t^R\right) y_t^H(j)$. The rest of retail firms adjust their past prices by the rate $\pi_t^{adj} = \pi$.

Then the aggregate price level of retail goods P_t^H is defined as

$$P_t^H = \left((1-\omega^H)\left(P_t^{\#}\right)^{1-\epsilon_H} + \omega^H \left(P_{t-1}^H \pi_t^{adj}\right)^{1-\epsilon_H}\right)^{1/(1-\epsilon_H)}$$

Define

$$\tilde{p}_t^H \equiv \frac{P_t^{\#}}{P_t^H} \tag{B.1}$$

It follows that

$$\Rightarrow \quad 1 = (1-\omega^H)\left(\tilde{p}_t^H\right)^{1-\epsilon_H} + \omega^H \left(\frac{P_{t-1}^H \pi_t^{adj}}{P_t^H}\right)^{1-\epsilon_H}$$

Re-writing in terms of relative prices with respect to the price level of composite goods P_t such that $p_t^H \equiv P_t^H / P_t$ gives

$$\Rightarrow \quad 1 = (1-\omega^H)\left(\tilde{p}_t^H\right)^{1-\epsilon_H} + \omega^H \left(\frac{p_{t-1}^H \pi_t^{adj}}{\pi_t p_t^H}\right)^{1-\epsilon_H}$$

As a result, a retail firm j solves the optimization problem how to set the optimal price $P_t^{\#}(j)$ conditional on not changing it in the future that be be formalized as:

$$\max_{P_t^{\#}(j)} E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \frac{\left(P_t^{\#}(j)\left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj}\right) - P_{t+s}^R\right)}{P_{t+s}} y_{t+s}^H(j)$$

s.t. demand for retail goods (equation (27))

$$y_t^H(j) = \left(\frac{P_t^{\#}(j) \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)}{P_t^H} \right)^{-\epsilon_H} y_t^H$$

Define $p_t^R \equiv \frac{P_t^R}{P_t}$:

$$\max_{P_t^{\#}(j)} E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left(\frac{P_t^{\#}(j)}{P_{t+s}} \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right) - p_{t+s}^R \right) y_{t+s}^H(j)$$

s.t. demand for retail goods

$$y_t^H(j) = \left(\frac{P_t^{\#}(j) \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)}{P_t^H} \right)^{-\epsilon_H} y_t^H$$

$$\Rightarrow \max_{P_t^{\#}(j)} E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left(\frac{P_t^{\#}(j) \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)}{P_{t+s}} \left(\frac{P_t^{\#}(j) \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)}{P_{t+s}^H} \right)^{-\epsilon_H} y_{t+s}^H - p_{t+s}^R \left(\frac{P_t^{\#}(j) \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)}{P_{t+s}^H} \right)^{-\epsilon_H} y_{t+s}^H \right)$$

$$\Rightarrow \max_{P_t^{\#}(j)} E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left(p_{t+s}^H \left(\frac{P_t^{\#}(j) \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)}{P_{t+s}^H} \right)^{1-\epsilon_H} y_{t+s}^H - p_{t+s}^R \left(\frac{P_t^{\#}(j) \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)}{P_{t+s}^H} \right)^{-\epsilon_H} y_{t+s}^H \right)$$

We take a derivative w.r.t. $P_t^{\#}(j)$ and rearrange terms:

$$E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left((1-\epsilon_H) p_{t+s}^H \left(P_t^{\#}(j)\right)^{-\epsilon_H} \left(\frac{\prod_{j=1}^{j=s} \pi_{t+j}^{adj}}{P_{t+s}^H} \right)^{1-\epsilon_H} - \epsilon_H p_{t+s}^R \left(P_t^{\#}(j)\right)^{-\epsilon_H - 1} \left(\frac{\prod_{j=1}^{j=s} \pi_{t+j}^{adj}}{P_{t+s}^H} \right)^{-\epsilon_H} \right) y_{t+s}^H$$

$$\Rightarrow E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left((1-\epsilon_H) p_{t+s}^H P_t^{\#}(j) \left(P_{t+s}^H\right)^{\epsilon_H - 1} \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)^{1-\epsilon_H} - \epsilon_H p_{t+s}^R \left(P_{t+s}^H\right)^{\epsilon_H} \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)^{-\epsilon_H} \right) y_{t+s}^H = 0$$

$$\Rightarrow P_t^{\#}(j) = \frac{\epsilon_H}{(\epsilon_H - 1)} \frac{E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left(p_{t+s}^R \left(P_{t+s}^H\right)^{\epsilon_H} \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)^{-\epsilon_H} y_{t+s}^H \right)}{E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left(p_{t+s}^H \left(P_{t+s}^H\right)^{\epsilon_H - 1} \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)^{1-\epsilon_H} y_{t+s}^H \right)} = 0$$

$$\Rightarrow \frac{P_t^{\#}(j)}{P_t^H} = \frac{\epsilon_H}{(\epsilon_H - 1)} \frac{E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left(p_{t+s}^R \left(\frac{P_{t+s}^H}{P_t^H} \right)^{\epsilon_H} \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)^{-\epsilon_H} y_{t+s}^H \right)}{E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left(p_{t+s}^H \left(\frac{P_{t+s}^H}{P_t^H} \right)^{\epsilon_H - 1} \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)^{1-\epsilon_H} y_{t+s}^H \right)}$$

Since $\tilde{p}_t^H \equiv P_t^{\#}/P_t^H$,

$$\tilde{p}_t^H = \frac{\epsilon_H}{(\epsilon_H - 1)} \frac{E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left(p_{t+s}^R \left(\frac{P_{t+s}^H}{P_t^H} \right)^{\epsilon_H} \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)^{-\epsilon_H} y_{t+s}^H \right)}{E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left(p_{t+s}^H \left(\frac{P_{t+s}^H}{P_t^H} \right)^{\epsilon_H - 1} \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)^{1-\epsilon_H} y_{t+s}^H \right)}$$

$$\Rightarrow \tilde{p}_t^H = \frac{\epsilon_H}{(\epsilon_H - 1)} \frac{E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left(p_{t+s}^R \left(\frac{p_{t+s}^H \pi_{t+s}}{p_t^H} \right)^{\epsilon_H} \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)^{-\epsilon_H} y_{t+s}^H \right)}{E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left(p_{t+s}^H \left(\frac{p_{t+s}^H \pi_{t+s}}{p_t^H} \right)^{\epsilon_H - 1} \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)^{1-\epsilon_H} y_{t+s}^H \right)}$$

$$\Rightarrow \tilde{p}_t^H = \frac{\epsilon_H}{(\epsilon_H - 1)} \frac{F_{1,t}^H}{F_{2,t}^H}$$

where

$$F_{1,t}^H = p_t^R y_t^H + E_t \omega^H \beta \Lambda_{t,t+1} \left(\frac{p_{t+1}^H \pi_{t+1}}{p_t^H \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)} \right)^{\epsilon_H} F_{1,t+1}^H$$

and

$$F_{2,t}^H = p_t^H y_t^H + E_t \omega^H \beta \Lambda_{t,t+1} \left(\frac{p_{t+1}^H \pi_{t+1}}{p_t^H \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)} \right)^{\epsilon_H - 1} F_{2,t+1}^H$$

D2: Importers

We assume that there is a continuum of monopolistically competitive importers. They buy a variety j of foreign goods $y_t^F(j)$ at price P_t^* and sell it to the composite goods producer at a nominal price $P_t^F(j)$, expressed in domestic currency.

Every period there is a fraction $(1 - \omega^F)$ of importers who can adjust their prices, in Calvo (1983) fashion. The set of importers who can adjust the price choose it such that their profits are maximized. The rest of importers adjust their past prices by the rate $\pi_t^{adj} = \pi$. As a result, an importer j solves the optimization problem how to set the optimal price $P_t^{\#F}(j)$ conditional on not changing it in the future:

$$\max_{P_t^{\#F}(j)} E_t \sum_{s=0}^{\infty} \left(\omega^F\right)^s \beta^s \Lambda_{t,t+s} \frac{\left(P_t^{\#F}(j) \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right) - S_{t+s} P_{t+s}^* \right)}{P_{t+s}} y_{t+s}^F(j) \tag{26}$$

s.t.

$$y_t^F(j) = \eta \left(\frac{P_t^{\#F}(j) \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)}{P_t^F} \right)^{-\epsilon} y_t^F$$

Since $rer_t \equiv S_t P_t^* / P_t$,

$$\max_{P_t^{\#F}(j)} E_t \sum_{s=0}^{\infty} \left(\omega^F\right)^s \beta^s \Lambda_{t,t+s} \left(\frac{P_t^{\#F}(j) \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)}{P_{t+s}} - rer_{t+s} \right) y_{t+s}^F(j)$$

s.t.

$$y_t^F(j) = \eta \left(\frac{P_t^{\#F}(j) \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)}{P_t^F} \right)^{-\epsilon} y_t^F$$

In analogy to the problem of retail firms, we maximize expected profits and rearrange terms. Since all importers who can adjust their price set the same optimal price, $P_t^{\#F}(j) = P_t^{\#F} \forall j$. After introducing a variable $\tilde{p}_t^F$, which is defined as

$$\tilde{p}_t^F \equiv P_t^{\#F} / P_t^F, \tag{B.2}$$

we can show that the optimal price-setting equation follows as

$$\Rightarrow \tilde{p}_t^F = \frac{\epsilon_F}{(\epsilon_F - 1)} \frac{E_t \sum_{s=0}^{\infty} \left(\omega^F\right)^s \beta^s \Lambda_{t,t+s} \left(rer_{t+s} \left(\frac{p_{t+s}^F \pi_{t+s}}{p_t^F} \right)^{\epsilon_F} \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)^{-\epsilon_F} y_{t+s}^F \right)}{E_t \sum_{s=0}^{\infty} \left(\omega^F\right)^s \beta^s \Lambda_{t,t+s} \left(p_{t+s}^F \left(\frac{p_{t+s}^F \pi_{t+s}}{p_t^F} \right)^{\epsilon_F - 1} \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)^{1-\epsilon_F} y_{t+s}^F \right)}$$

$$\Rightarrow \tilde{p}_t^F = \frac{\epsilon_F}{(\epsilon_F - 1)} \frac{F_{1,t}^F}{F_{2,t}^F}$$

where

$$F_{1,t}^F = rer_t y_t^F + E_t \omega^F \beta \Lambda_{t,t+1} \left(\frac{p_{t+1}^F \pi_{t+1}}{p_t^F \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)} \right)^{\epsilon_F} F_{1,t+1}^F$$

and

$$F_{2,t}^F = p_t^F y_t^F + E_t \omega^F \beta \Lambda_{t,t+1} \left(\frac{p_{t+1}^F \pi_{t+1}}{p_t^F \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)} \right)^{\epsilon_F - 1} F_{2,t+1}^F$$

Deriving an aggregate price level of imported goods' produces the following expression: $1 = (1 - \omega^F) \left(\tilde{p}_t^F\right)^{1-\epsilon_F} + \omega^F \left(\frac{p_{t-1}^F \pi_t^{adj}}{p_t^F \pi_t} \right)^{1-\epsilon_F}$.

D3: Price dispersion

We define the price dispersion for retail goods as

$$D_t^H \equiv \int_0^1 \left(\frac{P_t^H(j)}{P_t^H} \right)^{-\epsilon_H} dj$$

$(1-\omega^H)$ of firms update prices to the same optimal price $P_t^{\#}$ and ω^H of firms adjust the last period's price with the adjustment term π_t^{adj}. This gives

$$
\begin{aligned}
D_t^H &= \int_0^{1-\omega^H} \left(\frac{P_t^{\#}}{P_t^H}\right)^{-\epsilon_H} dj + \int_{1-\omega^H}^1 \left(\frac{P_{t-1}^H(j)\left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right)}{P_t^H}\right)^{-\epsilon_H} dj \\
&= \int_0^{1-\omega^H} \left(\frac{P_t^{\#}}{P_t^H}\right)^{-\epsilon_H} dj + \int_{1-\omega^H}^1 \left(\frac{P_{t-1}^H(j)}{P_{t-1}^H}\right)^{-\epsilon_H} \left(\frac{P_{t-1}^H\left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right)}{P_t^H}\right)^{-\epsilon_H} dj \\
&= (1-\omega^H)\left(\tilde{p}_t^H\right)^{-\epsilon_H} + \int_{1-\omega^H}^1 D_{t-1}^H \left(\frac{P_{t-1}^H\left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right)}{P_t^H}\right)^{-\epsilon_H} dj \\
&= (1-\omega^H)\left(\tilde{p}_t^H\right)^{-\epsilon_H} + \omega^H \left(\frac{p_{t-1}^H\left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right)}{\pi_t p_t^H}\right)^{-\epsilon_H} D_{t-1}^H
\end{aligned}
$$

In analogy, the price dispersion of importers' goods is given by

$$D_t^F \equiv \int_0^1 \left(\frac{P_t^F(j)}{P_t^F}\right)^{-\epsilon_F} dj$$

and it follows a rule

$$D_t^F = (1-\omega^F)\left(\tilde{p}_t^F\right)^{-\epsilon_F} + \omega^F \left(\frac{p_{t-1}^F\left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right)}{\pi_t p_t^F}\right)^{-\epsilon_F} D_{t-1}^F$$

D4: Composite goods producer

The composite goods producer combines domestic aggregate goods and foreign aggregate goods into composite goods and sells them to the household, the government and capital goods producers. We define the supply of composite goods as y_t^C. Its associated price is P_t. The demanded amount of production inputs, namely, domestic aggregate goods and foreign aggregate goods, is denoted as x_t^H and x_t^F respectively.

Domestic aggregate goods. Domestic aggregate goods y_t^H result from assembling retailers' production $y_t^H(j)$ for $j \in [0,1]$, each bought at price $P_t^H(j)$, expressed in domestic currency, and with no additional costs incurred. Let the aggregate price level of retail goods be $P_t^H \equiv \left(\int_0^1 \left(P_t^H(j)\right)^{1-\epsilon_H} dj\right)^{1/(1-\epsilon_H)}$, expressed in domestic currency. Then it follows that the demand for retail goods is given as a solution to the problem

$$\max_{y_t^H(j)} \left\{P_t^H y_t^H - \int_0^1 P_t^H(j) y_t^H(j) dj\right\}$$

subject to the assembling technology

$$y_t^H = \left(\int_0^1 y_t^H(j)^{1-\frac{1}{\epsilon_H}} dj\right)^{\frac{\epsilon_H}{\epsilon_H - 1}}$$

and to the market clearing constraint that says that domestic aggregate goods are used as input by the composite goods producer and face foreign demand ex_t:

$$y_t^H = x_t^H + ex_t$$

As a result, optimal demand for retail goods of variety j is given by

$$y_t^H(j) = \left(\frac{P_t^H(j)}{P_t^H}\right)^{-\epsilon_H} y_t^H \tag{27}$$

Foreign aggregate goods. Foreign aggregate goods y_t^F result from assembling importers' production $y_t^F(j)$ for $j \in [0,1]$, each bought at price $P_t^F(j)$, expressed in domestic currency, and with no additional costs incurred. Let the aggregate price level of importers' goods be $P_t^F \equiv \left(\int_0^1 \left(P_t^F(j)\right)^{1-\epsilon_F} dj\right)^{1/(1-\epsilon_F)}$, expressed in domestic currency. Then it follows that the demand for importers' goods is given as a solution to the problem

$$\max_{y_t^F(j)} \left\{ P_t^F y_t^F - \int_0^1 P_t^F(j) y_t^F(j) dj \right\}$$

subject to the assembling technology

$$y_t^F = \left(\int_0^1 y_t^F(j)^{1-\frac{1}{\epsilon_F}} dj\right)^{\frac{\epsilon_F}{\epsilon_F - 1}}$$

and to the market clearing constraint that says that all foreign aggregate goods are used to satisfy the demand of the composite goods producer:

$$y_t^F = x_t^F$$

As a result, optimal demand for importers' production of variety j is given by

$$y_t^F(j) = \left(\frac{P_t^F(j)}{P_t^F}\right)^{-\epsilon_F} y_t^F \tag{28}$$

and demand for foreign aggregate goods clears $x_t^F = y_t^F$.

Composite goods. Given inputs x_t^H and x_t^F, composite goods are assembled with the aggregation technology

$$y_t^C \equiv \left((1-\eta)^{\frac{1}{\epsilon}} (x_t^H)^{\frac{\epsilon-1}{\epsilon}} + \eta^{\frac{1}{\epsilon}} (x_t^F)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{29}$$

where ϵ stands for elasticity of substitution between domestically produced goods and imported goods. A parameter η proxies for openness of the home economy.

The composite goods producer operates in a perfectly competitive market, so she maximizes profits $P_t y_t^C - P_t^H x_t^H - P_t^F x_t^F$ subject to the technology (29). This boils down to two demand conditions:

$$x_t^H = (1-\eta)\left(\frac{P_t^H}{P_t}\right)^{-\epsilon} y_t^C$$

and

$$x_t^F = \eta \left(\frac{P_t^F}{P_t}\right)^{-\epsilon} y_t^C$$

Further, we introduce relative prices $p_t^H \equiv P_t^H/P_t$ and $p_t^F \equiv P_t^F/P_t$ and get

$$x_t^H = (1-\eta)\left(p_t^H\right)^{-\epsilon} y_t^C \tag{30}$$

and

$$x_t^F = \eta\left(p_t^F\right)^{-\epsilon} y_t^C \tag{31}$$

D5: Capital producers

Capital producers participate in the domestic economy by selling capital to financially constrained firms at the real competitive price q_t and buying the depreciated capital stock back next period. To restore the depreciated capital, capital producers add composite goods (investment) i_t as additional inputs to the depreciated capital stock by using the technology subject to investment adjustment costs $\Gamma\left(\frac{i_t}{i_{t-1}}\right)$:

$$k_t = (1-\delta)k_{t-1} + \left(1 - \Gamma\left(\frac{i_t}{i_{t-1}}\right)\right) i_t \tag{32}$$

where adjustment costs Γ equal:

$$\Gamma\left(\frac{i_t}{i_{t-1}}\right) = \frac{\gamma}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2$$

Capital producers maximize profits, expressed in units of composite goods, subject to the production technology by choosing an optimal level of investment:

$$\max_{i_t} \beta E_t \Lambda_{t,t+1}\left\{(1-\rho)\frac{q_t}{\pi_{t+1}} k_t\right\} + \rho q_t k_t - q_t(1-\delta)k_{t-1} - i_t \tag{33}$$

s.t.

$$k_t = (1-\delta)k_{t-1} + \left(1 - \Gamma\left(\frac{i_t}{i_{t-1}}\right)\right) i_t \tag{34}$$

The optimization problem takes into account the share of capital purchases paid immediately ρ as opposed to the share of the payment $(1-\rho)$ delayed to the next period which makes it slightly different from a standard optimization problem solved by competitive capital producers.

Optimizing gives the demand function for investment:

$$\frac{1}{q_t} = \rho\left(1 - \frac{\gamma}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2\right) - \rho\gamma\left(\frac{i_t}{i_{t-1}} - 1\right)\frac{i_t}{i_{t-1}} + \rho\gamma\beta E_t\Lambda_{t,t+1}\frac{q_{t+1}}{q_t}\left(\frac{i_{t+1}}{i_t} - 1\right)\left(\frac{i_{t+1}}{i_t}\right)^2 \tag{35}$$

$$+(1-\rho)\beta E_t\Lambda_{t,t+1}\frac{q_{t+1}}{q_t}\left(1 - \frac{\gamma}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2 - \gamma\left(\frac{i_t}{i_{t-1}} - 1\right)\frac{i_t}{i_{t-1}}\right) + (1-\rho)\gamma\beta^2 E_t\Lambda_{t,t+2}\frac{q_{t+1}}{q_t}\left(\frac{i_{t+1}}{i_t} - 1\right)\left(\frac{i_{t+1}}{i_t}\right)^2 \tag{36}$$

D6: Exporters

We assume that perfectly competitive exporters demand ex_t units of the domestic aggregate good y_t^H, so the supply of the assembled production of domestic retailers has to satisfy both the demand of the composite goods producer and the demand of exporters. Exported goods consist of the domestic aggregate, so they do not use imported inputs.

We abstract from modelling trade barriers. Hence, the rest of the world demands ex_t units of domestic aggregate goods at a price $P_t^{H*} = P_t^H/S_t$, which is the price of domestic aggregate goods expressed in units of foreign composite goods. We assume that all economies in the world are identical and their demand for domestic aggregate goods can be aggregated and expressed relative to world output y_t^*. The foreign demand for domestic aggregate goods is price-sensitive:

$$ex_t = \eta^* \left(\frac{p_t^H}{rer_t}\right)^{-\epsilon_*} y_t^* \tag{37}$$

Consistent with the small open economy assumption, P_t^* and y_t^* are assumed to evolve exogenously. η^* is the foreign households' taste parameter for domestic aggregate goods. ϵ^* defines the elasticity of substitution between domestic aggregate goods and goods produced in other economies.

E: Government

The government collects lump-sum taxes T_t from the household and issues domestic bonds B_t to finance a stochastic stream of nominal government expenditure, G_t, and the bank recap $P_t\tau_t^{FS}$. Therefore, it satisfies the budget constraint:

$$G_t + P_t\tau_t^{FS} + R_{t-1}B_{t-1} = T_t + B_t$$

Given $g_t \equiv G_t/P_t$, $b_t \equiv B_t/P_t$ and $t_t \equiv T_t/P_t$, the budget constraint can be expressed in units of composite goods as

$$g_t + \tau_t^{FS} + \frac{R_{t-1}}{\pi_t} b_{t-1} = t_t + b_t$$

Taxes in units of composite goods follow this tax rule:

$$t_t = t + \kappa^B \left(b_{t-1} - b\right) + \kappa^{FS}\tau_t^{FS} + e_t, \qquad 0 < \kappa^B \leq 1, \quad 0 \leq \kappa^{FS} \leq 1$$

The rule tells that a share κ^{FS} of the recap expenditure is covered by increasing the lump-sum tax and the rest (a share $(1-\kappa^{FS})$) is financed with new government debt.

F: Central bank

The central bank conducts monetary policy by following the Taylor rule:

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\gamma_R} \left(\frac{y_t^H}{\bar{y^H}}\right)^{(1-\gamma_R)\gamma_Y} \left(\frac{\pi_t^H}{\bar{\pi^H}}\right)^{(1-\gamma_R)\gamma_\pi} \exp(mp_t) \tag{38}$$

mp_t is a monetary policy shock and the domestic aggregate goods price inflation π_t^H can be expressed as $\pi_t^H = p_t^H/p_{t-1}^H\pi_t$.

G: Market clearing

The domestic household, the government and capital producers buy composite goods. Therefore, the supply of composite goods y_t^C has to satisfy the aggregate demand of domestic agents:

$$y_t^C = c_t + i_t + g_t \tag{39}$$

H: Current account and its components

First, we derive an expression for aggregate nominal imports M_t in units of domestic currency.

We aggregate importers' demand for foreign composite $y_t^F(j)\forall j \in (0,1)$ that is priced at P_t^* and use the nominal exchange rate S_t to convert to domestic currency:

$$M_t = \int_0^1 S_t P_t^* y_t^F(j) dj$$

Further we use the derived demand function (28) to get

$$M_t = \int_0^1 S_t P_t^* \left(\frac{P_t^F(j)}{P_t^F}\right)^{-\epsilon_F} y_t^F$$

Define the price dispersion of importers' goods as $D_t^F \equiv \int_0^1 \left(\frac{P_t^F(j)}{P_t^F}\right)^{-\epsilon_F} dj$ (more details on the price dispersion are in subsection D3). Then

$$M_t = S_t P_t^* D_t^F y_t^F \tag{40}$$

which in units of composite goods is given by

$$m_t \equiv \frac{M_t}{P_t} = rer_t D_t^F y_t^F \tag{41}$$

Second, we define nominal exports EX_t, expressed in units of domestic currency. Since exports are purchased at the price P_t^{H*}, expressed in foreign currency, nominal exports EX_t, expressed in units of domestic currency, is given by

$$EX_t = S_t P_t^{H*} ex_t = P_t^H ex_t \tag{42}$$

Finally, the trade balance TB_t evolves as

$$TB_t = EX_t - M_t$$

Recall definitions for nominal exports and nominal imports in units of domestic currency (equations (42) and (40)). Then the trade balance in units of composite goods can be expressed as

$$tb_t \equiv \frac{TB_t}{P_t} = \frac{P_t^H ex_t}{P_t} - \frac{S_t P_t^* D_t^F y_t^F}{P_t}$$

$$\Rightarrow \qquad tb_t = p_t^H ex_t - rer_t D_t^F y_t^F$$

Since $m_t \equiv rer_t D_t^F y_t^F$,

$$tb_t = p_t^H ex_t - m_t$$

A current account is given by the sum of nominal trade balance and nominal net income from abroad. The domestic household owns banks that borrow from the foreign household, so, as a result, net income from abroad is negative and equal to minus payments on bank foreign debt:

$$CA_t = TB_t - (R_{t-1}^* \xi_{t-1} - 1) S_t D_{t-1}^*$$

Further, we express the current account in units of composite goods as ca_t ($ca_t \equiv CA_t/P_t$):

$$ca_t = tb_t - \left(R^*_{t-1}\xi_{t-1} - 1\right)\frac{S_t D^*_{t-1}}{P_t}$$

$$\Rightarrow \quad ca_t = tb_t - \left(R^*_{t-1}\xi_{t-1} - 1\right) rer_t \frac{d^*_{t-1}}{\pi^*_t} \tag{43}$$

In equilibrium the current account has to equal the capital account balance CP_t. In our case the capital account balance is given by the change in stocks of bank foreign debt:

$$CP_t = -\left(S_t D^*_t - S_t D^*_{t-1}\right)$$

We express the capital account balance in units of composite goods as cp_t ($cp_t \equiv CP_t/P_t$):

$$cp_t = -\left(rer_t d^*_t - rer_t \frac{d^*_{t-1}}{\pi^*_t}\right)$$

Then, next to the current account definition (43), we impose an additional restriction that enters the set of equilibrium equations:

$$ca_t = -\left(rer_t d^*_t - rer_t \frac{d^*_{t-1}}{\pi^*_t}\right) \tag{44}$$

I: Equilibrium equations of the model with foreign currency debt and leverage-constrained banks

The model is described by 48 endogenous variables:

$$\Big\{\lambda_t, c_t, h_t, w_t, R_t, d_{1,t}, d_{2,t}, R^R_t, l_t, l^D_t, l^F_t, n^{firms}_t, \pi_t, \Lambda_{t,t+1}, p^R_t, k_t, i_t, q_t, p^H_t, \tilde{p}^H_t, D^H_t, y^H_t, x^H_t, F^H_{1,t},$$
$$F^H_{2,t}, y^C_t, p^F_t, y^F_t, x^F_t, m_t, ex_t, \tilde{p}^F_t, D^F_t, F^F_{1,t}, F^F_{2,t}, R^L_t, d^*_t, d_t, n_t, \nu_{1,t}, \nu_{2,t}, t_t, b_t, rer_t, S_t, tb_t, ca_t, \xi_t\Big\}$$

They are given by 48 equilibrium equations below.

Households

$$\lambda_t = \left(c_t - \frac{\chi\left(h_t\right)^{1+\varphi}}{1+\varphi}\right)^{-\gamma} \tag{I.1}$$

$$w_t = \chi\left(h_t\right)^{\varphi} \tag{I.2}$$

$$\Lambda_{t,t+1} \equiv \frac{\lambda_{t+1}}{\lambda_t} \tag{I.3}$$

$$E_t \beta \Lambda_{t,t+1} \frac{R_t}{\pi_{t+1}} = 1 \tag{I.4}$$

Financially constrained firms

$$E_t \beta \Lambda_{t,t+1} \left\{\left(1 - (1 - \Phi(d_{1,t}))\kappa\right)\left(\alpha p^R_{t+1} A_{t+1} k_t^{\alpha-1} h_t^{1-\alpha} + q_{t+1}(1-\delta)\right) - (1-\rho)\frac{q_t}{\pi_{t+1}}\right\} = \rho q_t \tag{I.5}$$

$$E_t \beta \Lambda_{t,t+1} \left\{\left(1 - (1 - \Phi(d_{1,t}))\kappa\right)(1-\alpha) p^R_{t+1} A_{t+1} k_t^{\alpha} h_t^{-\alpha} - (1-\rho)\frac{w_t}{\pi_{t+1}}\right\} = \rho w_t \tag{I.6}$$

$$E_{t-1}\left\{l_t\right\}+E_{t-1}\left\{n_t^{firms}\right\}=E_{t-1}\left\{\rho\left(q_t k_t+w_t h_t\right)\right\} \tag{I.7}$$

$$d_{2,t}\equiv\frac{E_t\ln\left(\kappa\left(p_{t+1}^R y_{i,t+1}^R+q_{t+1}(1-\delta)k_{i,t}\right)-R_t^R rer_{t+1}\frac{l_{i,t}^F}{\pi_{t+1}^*}\right)-E_t\ln\left(R_{i,t}^R\frac{l_{i,t}^D}{\pi_{t+1}}\right)}{\sigma_y} \tag{I.8}$$

$$d_{1,t}\equiv d_{2,t}+\sigma_y \tag{I.9}$$

$$\begin{aligned}n_t^{firms}=&\omega^{firms}\left(p_t^R y_t^R+q_t(1-\delta)k_{t-1}-(1-\rho)\frac{q_{t-1}k_{t-1}+w_{t-1}h_{t-1}}{\pi_t}\right)\\&-\omega^{firms}\left((1-\Phi(d_{1,t-1}))\,\kappa\left(p_t^R y_t^R+q_t(1-\delta)k_{t-1}\right)+\Phi(d_{2,t-1})R_{t-1}^R\frac{l_{t-1}^D}{\pi_t}+\Phi(d_{1,t-1})rer_t\frac{l_{t-1}^F}{\pi_t^*}\right)\\&+\iota^{firms}\cdot n^{firms}\end{aligned} \tag{I.10}$$

$$l_t=l_t^D+rer_t l_t^F \tag{I.11}$$

$$l_t^D=(1-\alpha^F)l_t \tag{I.12}$$

Capital producers

$$k_t=(1-\delta)k_{t-1}+\left(1-\Gamma\left(\frac{i_t}{i_{t-1}}\right)\right)i_t \tag{I.13}$$

$$\begin{aligned}\frac{1}{q_t}=&\rho\left(1-\frac{\gamma}{2}\left(\frac{i_t}{i_{t-1}}-1\right)^2\right)-\rho\gamma\left(\frac{i_t}{i_{t-1}}-1\right)\frac{i_t}{i_{t-1}}+\rho\gamma\beta E_t\Lambda_{t,t+1}\frac{q_{t+1}}{q_t}\left(\frac{i_{t+1}}{i_t}-1\right)\left(\frac{i_{t+1}}{i_t}\right)^2\\&+(1-\rho)\gamma\beta E_t\Lambda_{t,t+1}\frac{q_{t+1}}{q_t}\left(1-\frac{\gamma}{2}\left(\frac{i_t}{i_{t-1}}-1\right)^2-\gamma\left(\frac{i_t}{i_{t-1}}-1\right)\frac{i_t}{i_{t-1}}\right)+(1-\rho)\gamma\beta^2E_t\Lambda_{t,t+2}\frac{q_{t+1}}{q_t}\left(\frac{i_{t+1}}{i_t}-1\right)\left(\frac{i_{t+1}}{i_t}\right)^2\end{aligned}$$

Retail firms

$$1=(1-\omega^H)\left(\tilde{p}_t^H\right)^{1-\epsilon_H}+\omega^H\left(\frac{p_{t-1}^H\left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right)}{p_t^H\pi_t}\right)^{1-\epsilon_H} \tag{I.15}$$

$$D_t^H=(1-\omega^H)\left(\tilde{p}_t^H\right)^{-\epsilon_H}+\omega^H\left(\frac{p_{t-1}^H\left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right)}{p_t^H\pi_t}\right)^{-\epsilon_H}D_{t-1}^H \tag{I.16}$$

$$\tilde{p}_t^H=\frac{\epsilon_H}{(\epsilon_H-1)}\frac{F_{1,t}^H}{F_{2,t}^H} \tag{I.17}$$

$$F_{1,t}^H=p_t^R y_t^H+E_t\omega^H\beta\Lambda_{t,t+1}\left(\frac{p_{t+1}^H\pi_{t+1}}{p_t^H\left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right)}\right)^{\epsilon_H}F_{1,t+1}^H \tag{I.18}$$

$$F_{2,t}^H = p_t^H y_t^H + E_t \omega^H \beta \Lambda_{t,t+1} \left(\frac{p_{t+1}^H \pi_{t+1}}{p_t^H \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)} \right)^{\epsilon_H - 1} F_{2,t+1}^H \tag{I.19}$$

$$D_t^H y_t^H = A_t \theta_t F\left(k_{t-1}, n_{t-1}\right) \tag{I.20}$$

Composite goods producer

$$y_t^C \equiv \left((1-\eta)^{\frac{1}{\epsilon}} \left(x_t^H\right)^{\frac{\epsilon-1}{\epsilon}} + \eta^{\frac{1}{\epsilon}} \left(x_t^F\right)^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} \tag{I.21}$$

$$x_t^H = (1-\eta) \left(p_t^H\right)^{-\epsilon} y_t^C \tag{I.22}$$

$$x_t^F = \eta \left(p_t^F\right)^{-\epsilon} y_t^C \tag{I.23}$$

Exporters

$$ex_t = \eta^* \left(\frac{p_t^H}{rer_t} \right)^{-\epsilon_*} y_t^* \tag{I.24}$$

Definition of the real exchange rate

$$\frac{rer_t}{rer_{t-1}} = \frac{S_t}{S_{t-1}} \frac{\pi_t^*}{\pi_t} \tag{I.25}$$

Importers

$$1 = (1-\omega^F) \left(\tilde{p}_t^F\right)^{1-\epsilon_F} + \omega^F \left(\frac{p_{t-1}^F \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)}{p_t^F \pi_t} \right)^{1-\epsilon_F} \tag{I.26}$$

$$D_t^F = (1-\omega^F) \left(\tilde{p}_t^F\right)^{-\epsilon_F} + \omega^F \left(\frac{p_{t-1}^F \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)}{\pi_t p_t^F} \right)^{-\epsilon_F} D_{t-1}^F \tag{I.27}$$

$$\tilde{p}_t^F = \frac{\epsilon_F}{(\epsilon_F - 1)} \frac{F_{1,t}^F}{F_{2,t}^F} \tag{I.28}$$

$$F_{1,t}^F = rer_t y_t^F + E_t \omega^F \beta \Lambda_{t,t+1} \left(\frac{p_{t+1}^F \pi_{t+1}}{p_t^F \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)} \right)^{\epsilon_F} F_{1,t+1}^F \tag{I.29}$$

$$F_{2,t}^F = p_t^F y_t^F + E_t \omega^F \beta \Lambda_{t,t+1} \left(\frac{p_{t+1}^F \pi_{t+1}}{p_t^F \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)} \right)^{\epsilon_F - 1} F_{2,t+1}^F \tag{I.30}$$

$$m_t = rer_t D_t^F y_t^F \tag{I.31}$$

Banks

$$E_t \left\{ \frac{R_t^L}{\pi_{t+1}} l_t \right\} \equiv E_t \left\{ (1 - \Phi(d_{1,t})) \kappa \left(p_{t+1}^R y_{t+1}^R + (1-\delta) q_{t+1} k_t \right) + \Phi(d_{2,t}) R_t^R \frac{l_t^D}{\pi_{t+1}} + \Phi(d_{1,t}) R_t^R rer_{t+1} \frac{l_t^F}{\pi_{t+1}^*} \right\} \tag{I.32}$$

$$(1+\nu_{1,t})\beta E_t\Lambda_{t,t+1}\left\{(1-\omega)+\omega\nu_{2,t+1}\right\}\left(\frac{R_t^L}{\pi_{t+1}}\right)=\lambda^L\nu_{1,t}+\nu_{2,t} \tag{I.33}$$

$$(1+\nu_{1,t})\beta E_t\Lambda_{t,t+1}\left\{(1-\omega)+\omega\nu_{2,t+1}\right\}\left(\frac{R_t}{\pi_{t+1}}\right)=\nu_{2,t} \tag{I.34}$$

$$(1+\nu_{1,t})\beta E_t\Lambda_{t,t+1}\left\{(1-\omega)+\omega\nu_{2,t+1}\right\}\left(\frac{R_t^*\xi_t}{\pi_{t+1}^*}\frac{rer_{t+1}}{rer_t}\right)=\nu_{2,t} \tag{I.35}$$

$$n_t=\omega\left(\frac{R_{j,t-1}^L}{\pi_t}l_{t-1}-\frac{R_{t-1}}{\pi_t}d_{t-1}-\frac{R_{t-1}^*\xi_{t-1}}{\pi_t^*}rer_t d_{t-1}^*\right)+\iota n \tag{I.36}$$

$$\nu_{2,t}n_t\geq\lambda^L l_t \tag{I.37}$$

$$n_t+d_t+rer_t d_t^*=l_t \tag{I.38}$$

Monetary policy

$$\frac{R_t}{\bar{R}}=\left(\frac{R_{t-1}}{\bar{R}}\right)^{\gamma_R}\left(\frac{y_t^H}{\bar{y^H}}\right)^{(1-\gamma_R)\gamma_Y}\left(\frac{p_t^H/p_{t-1}^H\pi_t}{\bar{\pi}}\right)^{(1-\gamma_R)\gamma_\pi}\exp(mp_t) \tag{I.39}$$

Government

$$g_t+\frac{R_{t-1}}{\pi_t}b_{t-1}=t_t+b_t \tag{I.40}$$

$$t_t=\bar{t}+\kappa_b\left(b_{t-1}-\bar{b}\right)+\tau_t \tag{I.41}$$

Aggregate demand of domestic agents has to equal aggregate supply of composite goods

$$y_t^C=c_t+i_t+g_t \tag{I.42}$$

Aggregate demand for domestic aggregate goods and demand for exports clears with production of domestic aggregate goods

$$y_t^H=x_t^H+ex_t \tag{I.43}$$

Aggregate domestic demand for foreign aggregate goods clears with imports

$$y_t^F=x_t^F \tag{I.44}$$

Trade balance

$$tb_t=p_t^H ex_t-m_t \tag{I.45}$$

Current account

$$ca_t=tb_t-(R_{t-1}^*\xi_{t-1}-1)\,rer_t\frac{d_{t-1}^*}{\pi_t^*} \tag{I.46}$$

$$ca_t = -\left(rer_t d_t^* - rer_t \frac{d_{t-1}^*}{\pi_t^*}\right) \tag{I.47}$$

$$\xi_t = \exp\left(\phi \frac{(rer_t d_t^* - rer \cdot d^*)}{rer \cdot d^*} + \frac{\zeta_t - \zeta}{\zeta}\right) \tag{I.48}$$

There are 10 exogenous variables:

$$\{A_t, \theta_t, \pi_t^*, R_t^*, \zeta_t, y_t^*, mp_t, g_t, \tau_t\}$$