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Limit Pricing, Climate Policies, and Imperfect Substitution

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Limit pricing, climate policies, and imperfect substitution*

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Abstract

The effects of climate policies are often studied under the assumption of perfectly competitive markets for fossil fuels. In this paper, we allow for monopolistic fossil fuel supply. We show that, if fossil and renewable energy sources are perfect substitutes, a phase will exist during which the monopolist chooses a limit pricing strategy. If limit pricing occurs from the beginning, a renewables subsidy increases initial extraction, whereas a carbon tax leaves initial extraction unaffected. However, if initially fossil fuels are cheaper than renewables, a renewables subsidy and a carbon tax lower initial extraction, contrary to the case of perfect competition. Both policy instruments lower cumulative extraction. If fossil fuels and renewables are imperfect but good substitutes, the monopolist will exhibit 'limit pricing resembling' behavior, by keeping the effective price of fossil close to that of renewables for considerable time. The empirical question whether energy demand is elastic or inelastic has less drastic implications for the fossil price and extraction paths than under perfect substitutability.

JEL codes: Q31, Q42, Q54, Q58
Keywords: limit pricing, non-renewable resource, monopoly, climate policies

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1 Introduction

In the seventies and eighties of the previous century the literature on monopolistic supply of non-renewable resources was mainly inspired by the question how the development of fossil fuel use under monopoly (e.g., OPEC) would deviate from the perfectly competitive case. It famously turned out that under certain conditions "the monopolist is the conservationist's best friend" (e.g. Dasgupta and Heal, 1979, p. 329), which was especially interesting in the light of the sustainability debate following the publication of the Club of Rome's report on the 'Limits to Growth' (Meadows et al., 1972). The most recent sustainability debate, however, is no longer solely about resource scarcity, but rather about the impact of fossil fuel abundance on global warming. The problems associated with climate change have spawned a lot of research on the effects of climate policies like carbon taxation and renewables subsidies under different circumstances. Nevertheless, most of these analyses are carried out in a context of perfect competition. It is the aim of the present paper to fill this gap in the literature.

Our main objective is to study the effect of carbon taxes and renewables subsidies in a world where the demand for energy comes from a set of homogeneous price-takers, whereas supply of fossil fuels is managed by a monopoly. Hence, the analysis is restricted in several ways. We do not consider a strategic game between a monopolistic supplier and a monopsonistic group of demanders (cf. Liski and Tahvonen, 2004; Kagan et al., 2015), or a differential game between a resource monopolist and a competitive backstop producer (cf. Jaakkola, 2015). We also do not study the more realistic setting with a dominant supplier and a competitive fringe. See Groot et al. (1992) for the case without backstop. Moreover, we do not allow for heterogeneity of climate change policies across fossil fuel consuming countries. Hence, here we do not address issues like spatial carbon leakage under monopoly. This is considered in Van der Meijden et al. (2015). The central objectives are then, first, the characterization of the optimum, allowing for general profit functions, including stock-dependent extraction costs and second, the examination of the effects of environmental policies on the time profiles of fossil prices and extraction, and on cumulative fossil fuel use over the entire extraction horizon. We briefly discuss the implications of our results for the so-called Green Paradox, which refers to the idea that suboptimal climate policies, such as a renewables subsidy or a rapidly rising carbon tax, encourages fossil producers to extract more quickly and accelerates global warming (cf. Van der Ploeg and Withagen, 2015).

We show that, with perfect substitution between renewable and fossil energy, under general conditions a limit pricing phase will exist in which the fossil monopolist prevents suppliers of renewable energy from entering the market by just undercutting their price. If the initial fossil stock is small, or if energy demand is inelastic, limit pricing will even occur from the beginning until the end of fossil extraction. In this case, a constant renewables subsidy increases the extraction rate at any time, whereas a constant carbon tax leaves fossil use unaffected. If the initial fossil stock is large and if energy demand is elastic, the fossil energy price will be lower than the price of renewable energy initially. In this case, a constant renewables subsidy lowers initial extraction, but increases the extraction rate during the limit pricing phase. A constant carbon tax lowers initial extraction as well, but leaves the extraction rate during the limit pricing phase unaffected. We find that, with HARA utility and with constant marginal extraction costs, an increase in the carbon tax and the renewables subsidy increase the duration of the limit pricing phase. As long as fossil extraction costs are stock-dependent, both policy instruments induce the monopolist to leave a larger share of the initial stock untapped.

On a macroeconomic level, one could argue that renewable and fossil energy sources are not perfectly substitutable, e.g. because of problems due to intermittence. Therefore, we extend our framework to allow for imperfect substitution. We show that the price elasticity of fossil demand is a weighted average of the price elasticity of aggregate energy demand and the elasticity of substitution between fossil and renewables, with changing weights over time. Irrespective of the price elasticity of energy demand, the monopolist will always choose fossil supply such that the price elasticity of fossil demand exceeds unity. We show that, as a result, the empirical question whether energy demand is elastic or inelastic has less drastic implications for the resource price and extraction paths than under perfect substitutability (cf. Andrade de Sá and Daubanes, 2016). Still, as long as the elasticity of substitution between fossil and renewable energy is large enough, the fossil monopolist will exhibit 'limit pricing resembling' behavior, by keeping the effective price of fossil close to that of renewables for considerable time, to prevent too rapidly increasing marginal profits over time.

In the next section we introduce the model, derive the main results, and compare them to what other authors have found. We give a full characterization of the optimum for the monopolist and perform a policy analysis. Section 3 extends the model with imperfect substitutability. Section 4 concludes.

2 The model

We consider a single country that derives welfare from the use of energy. Energy comes from fossil fuel, that is supplied by a monopolist, or from a renewable resource that is supplied competitively. Production of renewable energy has constant marginal cost. We abstract from set up costs. For the time being we assume fossil fuel and renewables to be perfect substitutes. This assumption will be relaxed in Section 3. We suppose that the importing country's government imposes a constant carbon tax on the domestic consumption of fossil fuel. This can be justified by linear marginal climate damages and price taking behavior on the fossil fuel market by the consumers, or the consumers' country. We will also assume a constant unit subsidy on renewables. Since for the consumer country climate change constitutes the only externality, a carbon tax suffices to tackle the externality. However, in practice we more often observe second-best policies such as subsidizing solar or wind energy.

The constancy of the subsidy rate is questionable, although constant subsidies are frequently assumed in the existing literature (Van der Ploeg and Withagen, 2014). In principle one can calculate the optimal subsidy, absent carbon taxes. But it has to be taken into account then that different approaches yield different outcomes. For example, we could consider a game where the consuming country sets the subsidy conditional upon the size of the stock of fossil fuel and the fossil fuel exporter sets its price conditional on its remaining stock. Then the feedback Nash equilibrium concept applies, or possibly a feedback Stackelberg equilibrium concept with either the monopolist or the importing country acting as the leader. The analysis of such a game is beyond the purpose the present paper. Alternatively, we may consider the optimal open-loop subsidy. This can pose the problem of dynamic inconsistency, if at some instant of time the subsidy jumps to zero, namely when the stock of fossil fuel is depleted. The discontinuity will lead the fossil fuel supplier to leave fossil fuel in the ground in order to benefit from the price jump. In both cases it is unlikely that the optimal subsidy is constant. We have chosen to assume that the consumer country's government can commit itself to a constant subsidy from the beginning.

In the remainder of this section we will in turn discuss energy supply and demand and profit maximization by the fossil monopolist. Subsequently, we will characterize the equilibrium and describe the effects of policy changes on the use of fossil and renewable energy.

2.1 Energy demand and supply

The producer and consumer prices of energy at instant of time t are denoted by $p^p(t)$ and $p^c(t)$, respectively. The tax per unit of fossil fuel use is τ, the constant cost of producing energy from renewables is b, and the subsidy per unit of energy from the renewable source is σ. The limit consumer price for the monopolist is $b - \sigma$. As long as $p^c(t) = p^p(t) + \tau \leq b - \sigma$ all energy demand at instant of time t is met by fossil fuel, whereas demand for fossil fuel is zero if $p^c(t) > b - \sigma$. Hence, we assume that at the limit price the monopolist captures the entire market. The corresponding supply is denoted by $\hat{q}$. We define $\hat{b} = b - \sigma - \tau$ so that $\hat{q}$ is fossil fuel demand if $p^p(t) = \hat{b}$. With $S(t)$ denoting the resource stock at instant of time t net instantaneous profits of the monopolist are denoted by $\Pi(q(t), S(t))$. The remaining stock is included so as to allow for extraction costs that get higher with a lower remaining resource stock. We assume Π is well-defined and continuously differentiable. In addition we make the following assumption:

Assumption 1

(i) There exists $0 \leq \tilde{S} \leq S_0$ such that $\Pi(\hat{q}, \tilde{S}) > 0$.

(ii) There exists $M > 0$ such that $M \geq \Pi_S(q, S) \geq 0$ for all $q \geq \hat{q}, S > 0$.

(iii) $\Pi(q, S)$ is concave in q: $\Pi(q, S) > \Pi_q(q, S)q$ for all $q \geq \hat{q}, S > 0$.

Hence, to make the problem interesting, profits are positive at the limit price for some feasible resource stock (i), and although profits are non-decreasing in the existing resource stock, the benefits of larger resource stocks are bounded (ii). Finally, we impose concavity of the profit function in q to guarantee second order conditions (iii).[1]

From here on we will denote the producer price by p, thus omitting the producer index. It will be shown in the sequel that the equilibrium consists of three phases. Initially, from time 0 until time T_1, the monopolist supplies at a consumer price below the net renewables price $b - \sigma$, so that $p(t) + \tau < b - \sigma$ (or: $p(t) < \hat{b}$); then, from T_1 until T_2, the price is set marginally below $\hat{b}$. This is a phase with limit pricing. Finally, after T_2 there is no supply anymore. The first interval may be degenerate.

[1]Note that for $q = \hat{q}$ the derivative of Π with respect to q is the derivative for q approaching $\hat{q}$ from above (or from the right).

2.2 The monopolist's problem

The monopolist chooses an extraction path so as to maximize its profits, given the resource constraint and the condition that the extraction rate is high enough to keep renewables from the market. Hence, we consider the following problem.

$$\max_{T_2, q(t)} \int_0^{T_2} e^{-rt}\Pi(q(t), S(t))dt \tag{1}$$

subject to

$$\dot{S}(t) = -q(t),\ S(t) \geq 0,\ S(0) = S_0,\ q(t) \geq \hat{q}. \tag{2}$$

Here, r is the constant rate of interest. By λ we denote the shadow price of unextracted fossil fuel. The Hamiltonian $\mathcal{H}$ and the Lagrangian $\mathcal{L}$ of the problem read

$$\begin{aligned} \mathcal{H}(q, \lambda, t) &= e^{-rt}\Pi(q, S) + \lambda[-q], \\ \mathcal{L}(q, \lambda, \mu, t) &= e^{-rt}\Pi(q, S) + \lambda[-q] + \mu[q - \hat{q}]. \end{aligned}$$

According to the Maximum Principle, the necessary condition reads

$$e^{-rt}\Pi_q(q(t), S(t)) = \lambda(t) \text{ if } q(t) \geq \hat{q}, \tag{3a}$$

$$e^{-rt}\Pi_q(\hat{q}(t), S(t)) + \mu(t) = \lambda(t),\ \mu(t) \geq 0 \text{ if } q(t) = \hat{q}. \tag{3b}$$

Along the optimal path, the evolution of the shadow price satisfies

$$-\dot{\lambda}(t) = e^{-rt}\Pi_S(q(t), S(t)). \tag{4}$$

Furthermore, the transversality conditions are given by

$$\mathcal{H}(q(T_2), \lambda(T_2), T_2) = e^{-rT_2}\Pi(q(T_2), S(T_2)) + \lambda(T_2)[-q(T_2)] = 0, \tag{5a}$$

$$\lambda(T_2)S(T_2) = 0. \tag{5b}$$

2.3 Characterization of the equilibrium

We first show that there exists a non-degenerate interval of time with limit pricing.

Lemma 1 *There exists $0 \leq T_1 < T_2$ such that $q(t) = \hat{q}$ for all $T_1 \leq t \leq T_2$.*

Proof. Once extraction stops, extraction will be zero forever from that moment on. In view of Assumption 1 (i) and since $q(t) \geq \hat{q} > 0$ as long as extraction takes place, the moment where extraction stops is larger than zero and finite: $0 < T_2 < \infty$. If $q(T_2) > \hat{q}$, then it follows from (3a) and (5a) that

$$\lambda(T_2) = e^{-rT_2}\Pi(q(T_2), S(T_2))/q(T_2) = e^{-rT_2}\Pi_q(q(T_2), S(T_2)),$$

which would be a violation of Assumption 1 (iii). Hence, $q(T_2) = \hat{q}$. Suppose then that there exists $\varepsilon > 0$ such that $q(t) > \hat{q}$ for all $T_2 - \varepsilon \leq t < T_2$. Since λ is continuous and therefore also q is continuous up to T_2, we get a violation of Assumption 1 (iii) again. $\square$

The lemma suggests that there is a critical initial resource stock level such that at this level the phase of limit pricing starts immediately, whereas for higher initial stocks there is a phase with lower prices initially. This is shown in the following lemma.

Lemma 2 *There exists $\hat{S}_0$ such that if $S_0 \leq \hat{S}_0$, it is optimal to have limit pricing from the start: $T_1 = 0$.*

Proof. First, suppose $S(T_2) > 0$ and $\hat{S}_0$ is finite. Then $\lambda(T_2) = 0$ (from (5b)) and $\Pi(\hat{q}, S(T_2)) = 0$ (from (5a)). This determines $S(T_2)$. Instant of time T_2 follows from $S(T_2) = S_0 - T_2\hat{q}$. If S_0 is going to be the threshold it must be the case that $0 < S(T_2) \leq S_0$. So, we restrict ourselves to initial stocks that satisfy this condition. Consider the differential equation for λ: $-\dot{\lambda}(t) = e^{-rt}\Pi_S(\hat{q}, S_0 - t\hat{q})$ with the boundary condition $\Pi_q(\hat{q}, S_0) = \lambda(0)$. The solution for $\lambda(t)$ is downward sloping in time because $\Pi_S > 0$. For every given initial S_0 the solution gives an instant of time t_2 at which $\lambda(t_2) = 0$. It also gives $S(t_2) \equiv S_0 - t_2\hat{q}$. The threshold $\hat{S}_0$ we are looking for is such that $T_2 = t_2$ and $S(T_2) = S(t_2)$. If $S_0 = S(T_2)$ then $T_2 = 0$ and $t_2 > 0$. For S_0 arbitrarily high $\Pi_S(\hat{q}, S_0)$ is bounded from above (Assumption 1 (ii)). T_2 is then arbitrarily large but it will take a finite time t_2 for λ to become zero. Hence $t_2 < T_2$. In view of continuity, there is an $\hat{S}_0$ such that for $S_0 = \hat{S}_0$ the resulting $S(t_2) = S(T_2)$.

Now suppose $S(T_2) = 0$ and $\hat{S}_0$ is finite. Then $\lambda(T_2) = e^{-rT_2}\Pi(\hat{q}, 0)/\hat{q}$ (from (5a)). The threshold level follows from $\Pi_q(\hat{q}, \hat{S}_0) = \lambda(0)$, where $\lambda(0)$ is determined by

$$-\dot{\lambda}(t) = e^{-rt}\Pi_S(\hat{q}, S_0 - t\hat{q}),\ \lambda(T_2) = e^{-rT_2}\Pi(\hat{q}, 0)/\hat{q},\ \text{and } T_2 = \hat{S}_0/\hat{q}.$$

Suppose $\Pi_q(\hat{q}, S_0) < 0$ for all $q \geq \hat{q}$ and $S > 0$. Then it follows from (3b) that $\mu(t) > 0$ for all t, yielding limit pricing throughout, which implies that $\hat{S}_0$ is infinitely large.

In all three cases the necessary conditions are satisfied and the tranversality condition holds. In view of our concavity assumption, these conditions are sufficient. Hence the optimum has been established. □

Note that the case with an infinitely large $\hat{S}_0$ in which there is limit pricing throughout, prevails if the producer price elasticity of demand is smaller than unity over the whole range of $q > \hat{q}$. To see this, write $\Pi(q, S) = p(q)q - G(q, S)$ where G gives the extraction costs. Let us define the consumer price elasticity of demand as

$$\varepsilon^c(q) = -\frac{dq}{d(p+\tau)} \frac{p+\tau}{q},$$

and the producer price elasticity of demand as

$$\varepsilon^p(q) = -\frac{dq}{d(p+\tau)} \frac{p}{q} = -\frac{dq}{d(p+\tau)} \frac{p+\tau}{q} \frac{p}{p+\tau} = \varepsilon^c(q) \frac{p}{p+\tau}.$$

The difference is in the carbon tax to be paid by the consumer. We can then rewrite (3b) as

$$e^{-rt}[p'(q)q + p(q) - G_q(q, S)] = e^{-rt} \left[\left(1 - \frac{1}{\varepsilon^p(q)} \right) p(q) - G_q(q, S) \right] = \lambda - \mu.$$

With inelastic demand (i.e., $\varepsilon^c(q) < 1$ and thus $\varepsilon^p(q) < 1$, for all $q \geq \hat{q}$) there is limit pricing throughout, because μ is necessarily strictly positive then. This result was derived by Andrade de Sá and Daubanes (2016) for the special case of linear extraction costs.

2.4 Policy analysis

The existing previous literature on monopoly and limit pricing is scarce and mainly addresses the effect of changes in the renewables price on limit pricing. No attention is paid to the policy instruments, subsidies and carbon taxes. For the renewables cost we have the following results.

Proposition 1 *Consider a marginal decrease in b.*

(i) Suppose $q(0) = \hat{q}$. Then $q(0)$ goes up.

(ii) Suppose $q(0) > \hat{q}$. Then $q(0)$ goes down.

Proof. Part (i) is trivial, because $\hat{q}$ is fossil fuel demand if $p^p(t) = \hat{b} = b - \sigma - \tau$ so that with a marginal decrease in b demand goes up. To prove part (ii), note that

$$\dot{\mathcal{H}} = \frac{\partial \mathcal{H}}{\partial t}. \tag{6}$$

Hence

$$\int_0^{T_2} e^{-rt}\Pi(q(t), S(t))dt = \frac{\mathcal{H}(0) - \mathcal{H}(T_2)}{r}. \tag{7}$$

Since $\mathcal{H}(T_2) = 0$ in an optimum, we find

$$\Lambda(S_0, b, \sigma, \tau) \equiv \int_0^{T_2} e^{-rt}\Pi(q(t), S(t))dt = \frac{\mathcal{H}(0)}{r}. \tag{8}$$

We have $\mu(0) = 0$ if $S_0 > \hat{S}_0$, because then $T_1 > 0$ and $q(0) > \hat{q}$. Hence, we substitute (3a) into the Hamiltonian to get

$$\mathcal{H}(0) = \Pi(q(0), S_0) - \Pi_q(q(0), S_0)q(0). \tag{9}$$

Due to concavity of Π in q, $\mathcal{H}(0)$ is increasing in $q(0)$. An increase in b lowers $\hat{q}$ and thus relaxes the constraint the monopolist faces. Hence $d\Lambda(S_0, b, \sigma, \tau)/db > 0$, implying $d\mathcal{H}(0)/db > 0$. Therefore, $dq(0)/db > 0$. □

Hoel (1978) shows the existence of a limit pricing regime for linear cost functions. He also performs a sensitivity analysis with respect to the renewables price for the case without extraction cost ($G(q, S) \equiv 0$) and a constant price elasticity of demand. He proves that the initial price with limit pricing is higher than in the case of no backstop and argues that a marginally lower backstop price b increases the initial price the monopolist charges. Finally, he demonstrates that depletion of the resource then occurs sooner. To see this in our setting, note that from (3a) and (5a) with $\Pi(q, S) = p(q)q$ at the moment T_1 of the transition to limit pricing we have

$$e^{-rT_1}\left(1 - \frac{1}{\varepsilon^p(\hat{q})}\right) = e^{-rT_2}.$$

With a constant price elasticity the limit pricing interval is not affected by a change in the backstop price. But the initial price goes up. Then T_2 must go down. Salant (1977) was the first to show that limit pricing prevails for general convex cost functions, without stock-dependent extraction costs. However, he did not perform a sensitivity analysis and did not discuss policy issues.

The result in part (ii) of Proposition 1 could have been stated more generally as: Suppose $q(0) > \hat{q}$, then *any* change that lowers future profits without affecting current profits at the initial extraction level, lowers initial extraction. This result, that we have proved by using optimal control theory, was demonstrated before by Gilbert and Goldman (1978) for the case of constant marginal extraction costs. More specifically, they showed that a monopolist facing a threat of future entry sets a higher initial price than an unconstrained monopolist. In a similar vein, though in a more general setting allowing for stock-dependent extraction costs, Hoel (1978) proves that an inward shift of the demand function facing a monopolist that leaves demand unchanged near the original initial price, lowers initial extraction. In these two papers, however, limit pricing behavior is not explicitly discussed, and therefore, our case (i) in which we find the opposite effect on initial extraction, was ignored by Gilbert and Goldman (1978) and Hoel (1978). Furthermore, we are also interested in situations in which demand near the original price is affected as well, e.g. by taxation of fossil fuels, which is not discussed these earlier papers. The effects of fossil fuel taxes and renewables subsidies are discussed in the following proposition.

Proposition 2

(i) Consider a marginal increase in σ.

- *(a) Suppose $q(0) = \hat{q}$. Then $q(0)$ goes up.*
- *(b) Suppose $q(0) > \hat{q}$. Then $q(0)$ goes down.*
- *(c) Suppose $S(T_2) > 0$. Then $S(T_2)$ goes up.*

(ii) Consider a marginal increase in τ.

- *(a) Suppose $q(0) = \hat{q}$. Then $q(0)$ is unaffected.*
- *(b) Suppose $q(0) > \hat{q}$. Then $q(0)$ goes down.*
- *(c) Suppose $S(T_2) > 0$. Then $S(T_2)$ goes up.*

Proof. The proof of parts (ia) and (ib) follow the proof of Proposition 1. Concerning part (ic): with partial exhaustion the amount of fossil fuel left in the ground is determined by $\Pi(\hat{q}, S(T_2)) = 0$, which follows from (5a) and (5b). Since $\hat{q}$ goes up, the stock left in the ground is larger because $\Pi_q d\hat{q} + \Pi_S dS(T_2)) = 0$ and $\Pi_q < 0$ (which follow from (3b) with $\mu(T_2) > 0$ and $\lambda(T_2) = 0$ due to (5b)), $d\hat{q} > 0$ and $\Pi_S > 0$ implying $dS(T_2) > 0$.

The proof of part (iia) is immediate, because $\hat{q}$ does not depend on τ. To prove part (iib), note that we can rewrite (9) as

$$\mathcal{H}(0) = \Pi(q(0), S_0; \tau) - \Pi_q(q(0), S_0; \tau)q(0), \tag{10}$$

where we have explicitly included τ as a parameter in $\Pi(\cdot)$. Total differentiation gives

$$d\mathcal{H}(0) = -\Pi_{qq}dq(0) + (\Pi_\tau - \Pi_{q\tau}q(0))\, d\tau = -\Pi_{qq}dq(0),$$

where the second equality uses $\Pi_\tau = -\Pi_{q\tau}q = -q$, which follows from $\Pi(q, S) = (p^c(q) - \tau)q - G(q, S)$. Hence, $\mathcal{H}(0)$ is increasing in $q(0)$. An increase in τ lowers profits of the monopolist for all (S, q), implying $d\Lambda(S_0, b, \sigma, \tau)/d\tau < 0$, so that $d\mathcal{H}(0)/d\tau < 0$ (from (8)). Therefore, $dq(0)/d\tau < 0$. To prove part (iic), note again that with partial exhaustion we have $\Pi(\hat{q}, S(T_2); \tau) = 0$. Since $\hat{q}$ remains unaffected, the stock left in the ground is larger because $\Pi_S dS(T_2)) + \Pi_\tau d\tau = 0$ and $\Pi_S > 0$, and $\Pi_\tau < 0$ implying $dS(T_2) > 0$. $\square$

These results are of relevance for the incidence of the so called Green Paradox (Sinn, 2008, 2012). A *Weak* Green Paradox is said to occur if the initial emissions of carbon dioxide increase as a result of climate policies (e.g., the introduction of a subsidy for renewable energy), whereas a *Strong* Green Paradox materializes if the present discounted value of climate damages increases (Gerlagh, 2011). Green Paradoxes have been predominantly studied under perfect competition. In that framework, a higher subsidy for renewables results in more initial supply and consumption of fossil fuel, implying the occurrence of a Weak Green Paradox. Under monopoly, however, a Weak Green Paradox merely occurs if there is limit pricing from the start. If there is no limit pricing from the beginning, we have shown that a higher renewables subsidy will lead to lower short-term fossil fuel supply and consumption levels in the case of a resource market monopoly. Consequently, a Weak Green Paradox does not materialize. The intuition is simple: As the consumer price of renewables and fossil is lowered, more resources will be demanded during the future limit pricing phase. Therefore, fewer fossil fuels are available for extraction during the first phase, causing the initial supply of fossil fuels to fall. The effect of climate policies on the amount of fossil fuel that remains unextracted does not differ from the perfectly competitive case.

Because the effectiveness of climate policies might be severely reduced by limit pricing behavior (cf. Andrade de Sá and Daubanes, 2016), it is of policy relevance to determine the effects of different types of policies on the duration of the limit pricing phase. In the general case described so far, the effects of policy changes on the duration of the different regimes in the model are ambiguous. However, by imposing a bit more structure on extraction costs, we obtain the following univocal results.

Proposition 3 *Suppose marginal extraction costs k are stock-independent and constant. Define the super-elasticity as $\eta(q) \equiv \varepsilon_q^c(q)q/\varepsilon^c(q)$.*

(i) Consider a marginal increase in σ.

(a) Suppose $q(0) = \hat{q}$. Then the duration of the limit pricing phase decreases.

(b) Suppose $q(0) > \hat{q}$. Then the duration of the limit pricing phase increases (decreases) if $(\hat{b} - k)\eta(\hat{q})\varepsilon^c(\hat{q}) > (<) - (\tau + k)$.

(c) T_2 decreases.

(ii) Consider a marginal increase in τ.

(a) Suppose $q(0) = \hat{q}$. Then the duration of the limit pricing phase remains unchanged.

(b) Suppose $q(0) > \hat{q}$. Then the duration of the limit pricing phase increases.

Proof. In case of limit pricing from the start, i.e., $q(0) = \hat{q}$, an increase in σ lowers $p^c = b - \sigma$ and therefore increases $\hat{q}$, whereas an increase in τ leaves $p^c = b - \sigma$ and $\hat{q}$ unchanged. Together with $T_2 = S_0/\hat{q}$, this proves parts (ia) and (iia). To prove part (ib), combine (3a) and (5a) and use $\Pi(\hat{q}, S) = (\hat{b} - k)q$ to get

$$T_2 - T_1 = \frac{1}{r} \ln \left(\frac{\hat{b} - k}{(b - \sigma)\left(1 - 1/\varepsilon^c(\hat{q})\right) - k} \right)$$

By taking the derivative with respect to σ and τ, respectively, we find

$$\text{sign}\frac{d(T_2 - T_1)}{d\sigma} = \text{sign}\left[(\hat{b} - k)\eta(\hat{q}) + \frac{1}{\varepsilon^c(\hat{q})}(\tau + k)\right], \tag{11a}$$

$$\text{sign}\frac{d(T_2 - T_1)}{d\tau} = \text{sign}\frac{b - \sigma}{\varepsilon^c(\hat{q})} > 0. \tag{11b}$$

This proves parts (ib) and (iib). To prove part (ic), suppose first that there is limit pricing from the start, i.e., $q(0) = \hat{q}$. The result then immediately follows, as $d\hat{q}/d\sigma > 0$. Next, suppose $q(0) > \hat{q}$. Note from (4) that λ is constant in case of stock-independent extraction costs. From (3a) we get that $d\lambda = \Pi_{qq}dq(0) + \Pi_S dS(0) + \Pi_\tau d\tau = \Pi_{qq}dq(0)$, where the second equality uses $dS(0) = d\tau = 0$. By using $\Pi_{qq} < 0$ and result (ib) from Proposition 2, we get $d\lambda/d\sigma > 0$. From (5a) we obtain $e^{-rT_2}(\hat{b} - k)\lambda$, implying that T_2 must go down as λ increases and $\hat{b}$ falls. $\square$

Unambiguous results for the effect of the renewables subsidy on the duration of the limit pricing regime (i.e., result (ib) in Proposition 3) can be found by imposing additional structure on the demand side. Appendix A.1 shows that we get $d(T_2 - T_1)/d\sigma > 0$ for the class of HARA utility functions,

$$U(E) = \frac{1-\varphi}{\varphi}\left[\left(\frac{\psi E}{1-\varphi} + \chi\right)^{\varphi} - \chi^{\varphi}\right], \tag{12}$$

where $E = x + q$ denotes a composite energy good, x is consumption of renewables, $\varphi > 0$, $\psi > 0$, $\chi \geq 0$, and $\zeta \geq 0$. Hence, for HARA utility functions the duration of the limit pricing phase increases upon an increase in the renewables subsidy. A counter example can be obtained by using the following non-HARA utility function:

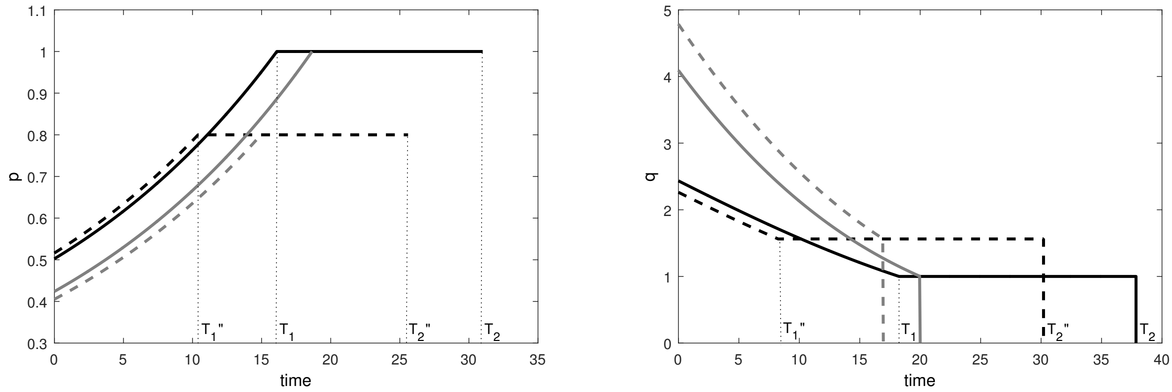
$$U(E) = \chi E + \frac{E^{1-\varphi}}{1-\varphi}. \tag{13}$$

Appendix A.1 shows that, with this utility function, the sign of the derivative of the length of the limit pricing phase is given by the sign of $\varphi(\chi - \tau - k)$, which is only positive for small enough $\tau + k$. Hence, with this non-HARA utility function, an increase in the renewables subsidy may cause the duration of the limit pricing phase to go down.

To illustrate the effects of the introduction of a renewables subsidy, Figure 1 shows the time profiles of the price and use of fossil fuels for the HARA utility case with $\psi = 2$, $\phi = 0.5$, $\chi = 0$. Furthermore, we have imposed $b = 1$, $k = 0.2$, $r = 0.05$,

Figure 1: Effect of a renewables subsidy on fossil time profiles

Panel (a) - Fossil price Panel (b) - Fossil use



Notes: The solid (dashed) lines represent the pre(post)-subsidy situation. We have picked the following parameter values: $\psi = 2$, $\phi = 0.5$, $\chi = 0$, $b = 1$, $k = 0.2$, $r = 0.05$, and $S_0 = 50$. The black (gray) lines show the time profiles under monopoly (perfect competition).

and $S_0 = 50$. The solid lines correspond to the scenario with $\sigma = 0$ and the dashed lines with $\sigma = 0.2$. The black lines represent the situation with monopolistic supply of fossil fuels, whereas the gray lines depict the time profiles under perfect competition. The figure clearly shows that initial extraction goes down upon the introduction of a subsidy for renewables under monopoly, whereas initial extraction goes up under perfect competition. In both cases, depletion will occur sooner. In the monopolistic case, the length of the limit pricing phase increases upon the introduction of the subsidy.

3 Imperfect substitution

Contrary to our assumption (in line with most of the literature about the transition from fossil fuels to renewables), in reality fossil fuels and renewables are not perfect substitutes. In this section, we first relax the assumption of perfect substitutability. Subsequently, we will pay attention to consequences for the price the elasticity of fossil fuel demand.

3.1 Example with CES functions

To illustrate the consequences of imperfect substitutability, we consider an example in which utility from energy is given by the following CES specification:

$$U(E) = \frac{E^{1-\frac{1}{\gamma}} - 1}{1 - \frac{1}{\gamma}},$$

where energy is a CES aggregate of fossil fuel q and renewables x:

$$E(q,x) = \left(\delta q^{\frac{\varepsilon-1}{\varepsilon}} + (1-\delta)x^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{14}$$

The elasticity of substitution between fossil fuels and renewables is equal to ε. Assuming quasilinear utility and denoting the composite energy price by p_E, consumers maximize $U(E) - p_E E$, implying that energy demand is given by

$$E = p_E^{-\gamma}, \tag{15}$$

from which it can be seen that the (positively defined) price elasticity of energy demand equals γ. The first-order conditions for fossil and renewables use read

$$p_E \frac{\partial E(q,x)}{\partial q} \leq p + \tau, \tag{16a}$$

$$p_E \frac{\partial E(q,x)}{\partial x} \leq b - \sigma, \tag{16b}$$

with equalities holding if $q > 0$ and $x > 0$, respectively. If there is positive demand for both energy sources, fossil fuel demand can be solved from (14)-(16b), yielding

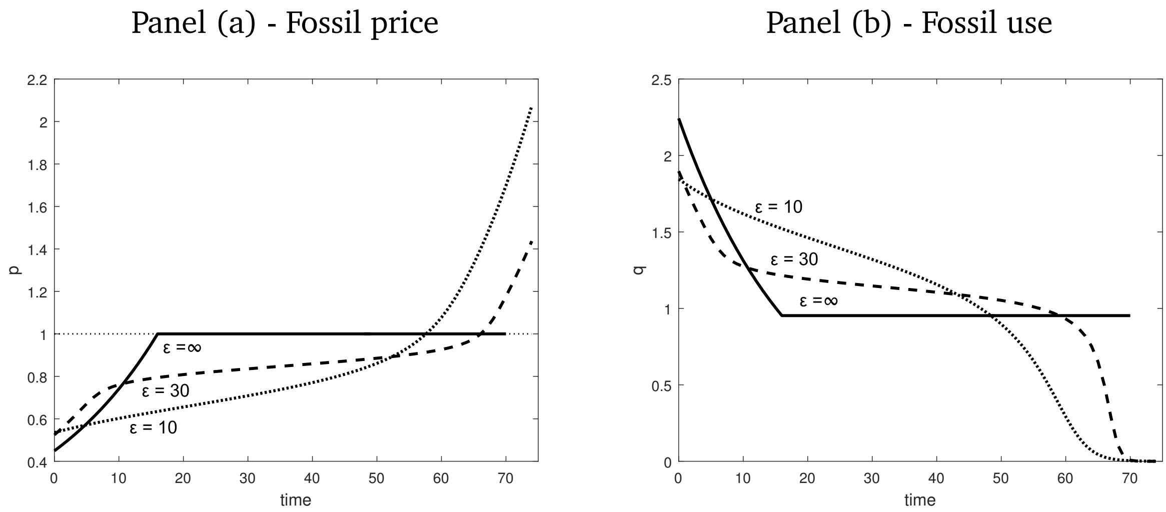
$$q(p) = \left(\frac{p+\tau}{\delta}\right)^{-\gamma} \left(\delta + (1-\delta)\left(\frac{(p+\tau)/\delta}{(b-\sigma)/(1-\delta)}\right)^{\varepsilon-1}\right)^{\frac{\gamma-\varepsilon}{1-\varepsilon}}. \tag{17}$$

With constant marginal extraction costs k, the problem of the monopolist is to

$$\max_{q(t)} \int_0^\infty e^{-rt}(p(q(t)) - k)q(t)dt \text{ subject to } \dot{S}(t) = -q(t),\ S(t) \geq 0,\ S(0) = S_0, \tag{18}$$

where $p(q)$ is the inverse function of (17). The solution to the monopolist's problem is provided in Appendix A.2.

Figure 2: Time profiles: the role of the elasticity of substitution



Notes: The solid, dashed and dotted line correspond to the scenarios with $\varepsilon = \infty$, $\varepsilon = 30$, and $\varepsilon = 10$, respectively. We have used $\gamma = 1.07$, $\sigma = 0$, $\tau = 0$, $b = 1$, $k = 0$, $r = 0.05$, and $S_0 = 76.5$.

In order to show the effects of imperfect substitutability, we simulate the model for different values of the elasticity of substitution between fossil fuels and renewables. Figure 2 shows the time profile of the fossil fuel price in Panel (a) and of fossil fuel use in Panel (b). We have imposed parameter values $\gamma = 1.07$, $\sigma = 0$, $\tau = 0$, $b = 1$, $k = 0$, $r = 0.05$, and $S_0 = 76.5$. The solid line represents the case in which fossil fuels and renewables are perfect substitutes (i.e., $\varepsilon = \infty$). For the dashed line, we have used $\varepsilon = 30$ and for the dotted line $\varepsilon = 10$. The figure shows that the time profiles of the price and use of fossil fuels converge to those under perfect substitutability if the elasticity of substitution between fossil and renewables is increased. This illustrates the robustness of our earlier results in which we have assumed perfect substitutability: the results of the generalized model with good, but imperfect substitution resemble those of the model with perfect substitution as long as the elasticity of substitution is large enough.

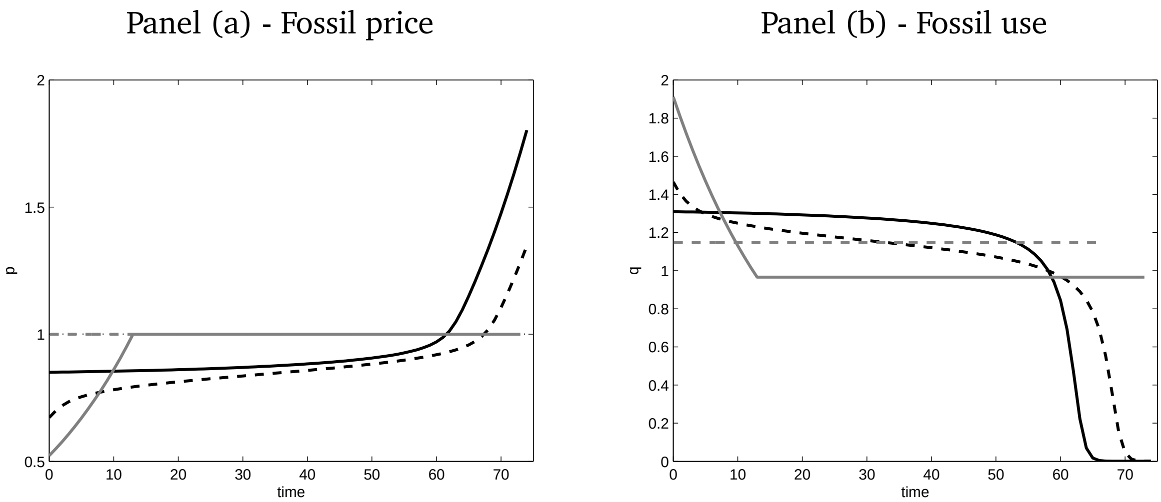
3.2 Price elasticity of fossil fuel demand

As noted by Andrade de Sá and Daubanes (2016), if fossil and renewable energy are perfect substitutes, there is a crucial role for the price elasticity of energy demand. If demand is inelastic, the monopolist will optimally choose a strategy of limit pricing throughout, which effectively implies choosing the point on the demand curve where the price elasticity of demand for fossil is infinitely large. In case of elastic energy demand, the price elasticity of demand for fossil fuels is constant and equal to the price elasticity of energy demand until the limit pricing phase starts, when it jumps to infinity. With imperfect substitutability, however, the elasticity $\Phi(p) \equiv -(dq/dp)(p+\tau)/q$ gradually changes over time. By using (17) we find

$$\Phi(p) = \frac{\Omega(p)}{1+\Omega(p)}\gamma + \frac{1}{1+\Omega(p)}\varepsilon, \text{ with } \Omega(p) = \frac{\delta}{1-\delta}\left(\frac{(p+\tau)/\delta}{(b-\sigma)/(1-\delta)}\right)^{1-\varepsilon}. \tag{19}$$

Hence, the price elasticity of fossil demand can be written as a weighted average of the elasticity of energy demand, γ, and the elasticity of substitution between fossil fuels and renewables, ε. Moreover, if fossil and renewable energy are close substitutes, which we assume to be the case, the relative weight of the elasticity of substitution increases over time as the fossil price rises. To see this, note that by imposing $1 < \varepsilon \ll \infty$ we ensure that fossil fuels and renewables are good, but imperfect substitutes. As a result, $\Omega(p)$ tends to zero if p becomes infinitely large. Therefore, (19) implies that the elasticity of fossil demand tends to ε.

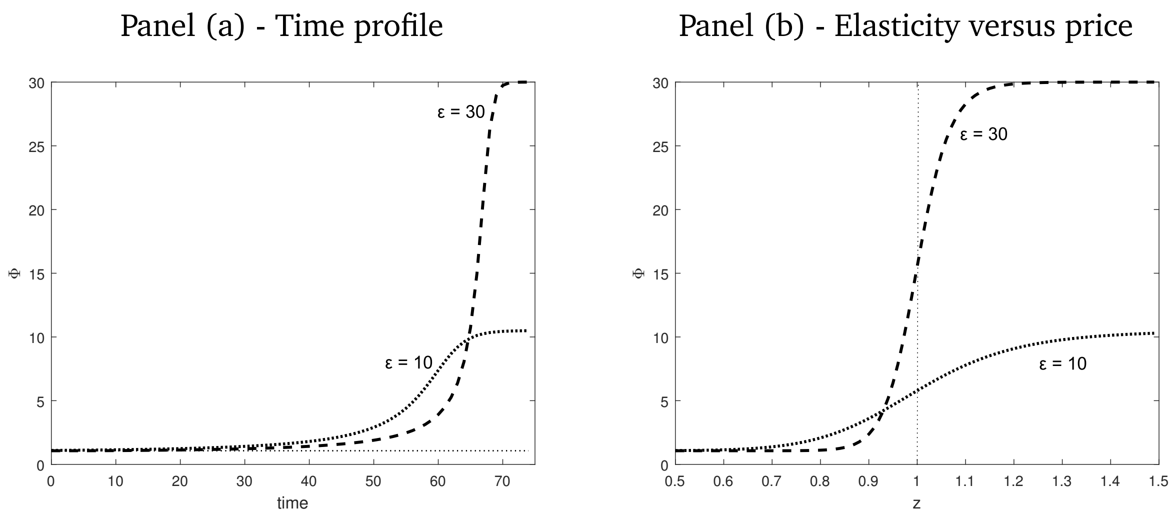
Figure 3: Time profiles: the role of the energy demand elasticity



Notes: The solid and dashed lines correspond to the scenarios with $\gamma = 1.05$ and $\gamma = 0.8$, respectively. The black lines represent the case with good, but imperfect substitution ($\varepsilon = 30$). The gray lines represent the case with perfect substitution ($\varepsilon = \infty$). We have used $\sigma = 0$, $\tau = 0$, $b = 1$, $k = 0$, $r = 0.05$, and $S_0 = 76.5$.

Note that, irrespective of the price elasticity of energy demand (which may well be chosen smaller than unity on empirical grounds), the monopolist always chooses extraction such that the price elasticity of fossil demand exceeds one. As a result, the difference between the case with inelastic and elastic energy demand is less sharp than it is under perfect substitutability. Figure 3 shows that by moving from elastic demand (gray black lines, $\gamma = 1.05$) to inelastic energy demand (dashed gray lines, $\gamma = 0.8$) under perfect substitutability, the price and extraction paths in panel (a) and (b), respectively, change considerably, because in the case with $\gamma = 0.8$ there will be limit pricing throughout. Under imperfect substitution, however, the solid black lines ($\gamma = 1.05$) do not differ drastically from the dashed black lines ($\gamma = 0.8$).

Figure 4: Price elasticity of fossil demand



Notes: The solid, dashed and dotted line correspond to the scenarios with $\varepsilon = \infty$, $\varepsilon = 30$, and $\varepsilon = 10$, respectively. We have used $\gamma = 1.07$, $\sigma = 0$, $\tau = 0$, $b = 1$, $k = 0$, $r = 0.05$, and $S_0 = 76.5$.

Hence, when allowing for imperfect substitution between fossil fuels and renewables, the empirical question whether energy demand is elastic or inelastic becomes less important than in the case of perfect substitution studied by Andrade de Sá and Daubanes (2016). Still, the case with monopolistic supply differs considerably from the case with competitive resource supply. If fossil fuels and renewables are close substitutes, i.e., if ε is large, $\Phi(p)$ will rapidly change with p if the relative effective price of these energy sources deviates from $z \equiv \delta/(1-\delta)(p+\tau)/(b-\sigma)$. This gives rise to 'limit pricing resembling' behavior by the monopolistic fossil fuel supplier: if z rises above unity, marginal profits will rapidly rise with increases in p. Therefore, once z comes close to unity, it is profitable for the supplier to keep it close to unity until most of the stock is exhausted. Afterwards, the price will increase, fossil demand will tend to zero, the elasticity of fossil demand will rapidly increase and marginal profits will converge to average profits, as in the extreme case of perfect substitutability.

Figure 4 illustrates the development of the fossil demand elasticity over time in panel (a) and its dependence on the effective relative price z in panel (b), for two different values of the elasticity of substitution between fossil and renewable energy. The dashed line corresponds with $\varepsilon = 30$ and the dotted line with $\varepsilon = 10$. In both cases, the fossil demand elasticity starts out just above one (indicated by the flat dotted line in panel (a)) and tends towards ε in the long run.

4 Conclusion

In a general model of non-renewable resource supply by a monopolist we have shown that, if fossil fuels and renewables are perfect substitutes, the equilibrium necessarily contains a limit pricing regime. It has been shown that the effects of environmental policies, such as a carbon tax or a renewables subsidy, can be the opposite of what they would be in the case of perfect competition. In particular, the initial use of fossil fuels can decrease instead of increase as a consequence of more stringent climate change policy. This is not to say that such policies are less harmful from a social welfare perspective than in the case of perfect competition: whether or not this is the case depends on the acuteness of climate change damages.

We have demonstrated that our results are robust to introducing imperfect but good substitutability between fossil and renewable resources: the monopolist will choose a 'limit pricing resembling' strategy by keeping the effective fossil price just below the effective renewables price for a considerable period of time. Nevertheless, abrupt regime shifts from 'Hotelling pricing' to 'limit pricing' disappear and the empirical question whether energy demand is elastic or inelastic has less drastic implications for the fossil price and extraction paths than under perfect substitutability.

In future research, a strategic game in which the fossil importing country sets a renewables subsidy and the fossil fuel exporter sets its price—both conditional on the remaining stock—could be introduced. Another promising way to proceed is by generalizing the analysis to the case of oligopolistic fossil supply. This is an interesting field of research because of the possibility of strategic interaction among supplying firms, which is absent in the cases of monopoly and perfect competition.

Appendix

A.1 Comparative statics for HARA and non-HARA utility functions

The inverse demand function corresponding to the HARA utility function (12) is given by:

$$p(q+x) = \psi\left(\frac{\psi(q+x)}{1-\varphi} + \chi\right)^{\varphi-1} - \tau.$$

Use $p(\hat{q}) = \hat{b}$ and set $x = 0$. Then,

$$e^{rT_2 - rT_1} = \frac{\hat{b} - k}{\hat{b} - k - (1-\varphi)(b - \sigma - \psi\chi(\frac{b-\sigma}{\psi})^{\frac{\varphi-2}{\varphi-1}})}.$$

The sign of the derivative of the right-hand side with respect to $b-\sigma$ equals the sign of:

$$-\psi\chi\left(\frac{b-\sigma}{\psi}\right)^{\frac{\varphi-2}{\varphi-1}} - (\tau+k)\left(1 - \varphi + (\varphi-2)\chi(\frac{b-\sigma}{\psi})^{\frac{-1}{\varphi-1}}\right).$$

The first term is negative. If $(1 - \varphi + (\varphi-2)\chi\left(\frac{b-\sigma}{\psi}\right)^{\frac{-1}{\varphi-1}}) \geq 0$ the entire expression is negative. Otherwise, the second term is positive. But $\tau + k \leq b - \sigma$ in view of assumption 1, so as to make supply at the limit price profitable.For $\tau + k = b - \sigma$ the expression boils down to:

$$(1-\varphi)\psi\left(\frac{b-\sigma}{\psi}\right)\left\{\chi\left(\frac{b-\sigma}{\psi}\right)^{\frac{-1}{\varphi-1}} - 1\right\},$$

which is definitely negative since $\hat{q} = \frac{1-\varphi}{\psi}\left\{\left(\frac{b-\sigma}{\psi}\right)^{\frac{1}{\varphi-1}} - \chi\right\} > 0$. Hence, for HARA utility functions the limit pricing phase becomes longer upon a decrease in the cost of renewables b or an increase in the subsidy σ.

By taking the non-HARA utility function (13) and following the same approach as before we can show that the sign of the derivative of the length of the limit pricing phase is given by the sign of $\varphi(\chi - \tau - k)$, which is equal to the expression given in the main text.

A.2 Imperfect substitution

The Hamiltonian $\mathcal{H}$ associated with the profit maximization problem of the monopolist reads

$$\mathcal{H}(q, \lambda, t) = e^{-rt}(p(q) - k)q + \lambda[-q],$$

As before, λ denotes the shadow price of unextracted fossil fuel. According to the Maximum Principle, the necessary condition reads

$$e^{-rt}\left(p(q) + p'(q)q - k\right) = \lambda(t). \tag{A.1}$$

Along the optimal path, the evolution of the shadow price satisfies

$$-\dot{\lambda}(t) = 0. \tag{A.2}$$

Furthermore, the transversality condition is given by

$$\lim_{t\to\infty} \lambda(t)S(t) = 0. \tag{A.3a}$$

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