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Jonathan A. Attey* and Casper G. de Vries†

Abstract

Empirical estimations suggest heavy-tailed unconditional distributions for inflation, the output gap and the interest rate. However, standard NK models used in policy analysis imply normal distributions for these variables. In this study, we propose a model which replicates the above mentioned empirical features of inflation,the output gap and the interest rate and subsequently investigate the conduct of monetary policy in this model. The novelty of this study is the introduction of random wage indexation as a source of multiplicative shocks. The findings of this study include the following: Firstly, the unconditional distributions of inflation, the output gap and the interest rates exhibit heavy-tailed characteristics. Secondly, under an indexation to lagged inflation scheme, there exists a positive relationship between expected inflation and conditional variance of inflation. Finally, it is better to target current inflation rather than lagged inflation when conducting monetary policy under a Taylor rule.

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1 Introduction

The bulk of literature on wage indexation assumes a constant degree of indexation. While this assumption might describe the empirical reality during recent periods of low and stable inflation, the degree of wage indexation has been observed to exhibit a substantial amount of time-variation. For instance, the percentage of contracts in the US with cost-of-living-adjustment (COLA) clauses has been observed to rise from 31% in the mid 1960s to 61% in the mid 1970s (see Weiner (1996)). Given that percentage COLA coverage is a widely accepted proxy for the degree of wage indexation, one can conclude that the degree of wage indexation is not constant. Furthermore, results from recent studies provide evidence in support of the time variation in the degree of wage indexation. Analysis by Holland (1986) and Ascari et al. (2011) show that the degree of wage indexation is positively correlated to inflation uncertainty. Empirical studies documenting substantial time variation in inflation uncertainty imply a substantial time variation in the degree of wage indexation.

Even though the assumption of a constant degree of wage indexation may describe the behaviour of wage indexation only for recent times, the effects of wage indexation under this assumption have nevertheless been shown to be quite consequential. The seminal study on wage indexation, Gray (1976) examines the effect of wage indexation on the conduct of monetary policy. Results of this paper show that wage indexation insulates the real sector of the economy from nominal or monetary shocks, but tends to make the effects of real shocks worse. Jadresic (1998) also investigates the effects of constant wage indexation. The indexation rule employed in the aforementioned study differs from that of previous studies in that it assumes an indexation to lagged inflation scheme. It is shown that indexation to lagged inflation destabilizes output.

It is conceivable that the time variation in the degree of wage indexation adds another dimension to the implications of wage indexation for macroeconomic stability. The purpose of this study is to theoretically investigate the additional implications that come with time variation in wage indexation. In particular, we investigate the macroeconomic consequences of independent and identically distributed (iid) shocks to the degree of wage indexation. Empirical estimates either imply an autoregressive (AR) process or a near random walk process for the degree of wage indexation which contrasts with the iid assumption regarding wage indexation we make in this study. We nevertheless work with iid shocks to wage indexation in order to obtain preliminary insights into the effects of time variation in the degree of wage indexation. Attey (2015) estimates time varying degree of wage indexation for 12 OECD countries under the assumption. Figure 1 presents the country specific estimates and their 95% confidence bounds. The estimates reveal three main properties of the degree of wage indexation.

First, there is a substantial time variation in the degree of wage indexation in all countries. This observation provides further evidence for the time varying nature of wage indexation. Second, the empirical estimations do not give a conclusive view on whether the distribution of wage indexation is bounded or not. The process assumed for the degree of wage indexation implies an unbounded distribution for this variable. However, it can be seen from Figure 1 that estimates of the degree of wage indexation do not generally stray much from the unit interval. Thus, one cannot conclusively rule out the possibility of bounded distributions for wage indexation. Finally, the estimates of the degree of wage indexation can lie outside the unit interval. For instance, the estimates of the degree of wage indexation were significantly less than 0 for the Netherlands since the beginning of the 1980s. Also, the estimates show that the degree of wage indexation for Belgium were above 1 during the mid 1970s. It is also worth noting that wage moderation was sometimes agreed upon during periods of stagflation, thus resulting in the negative correlation between lagged inflation and wage inflation.[1] These two observations stand in contrast to conventional wisdom that wage indexation should be on the unit interval.

Key among the results of this study is that the unconditional distributions of inflation, the output gap and the interest rates can potentially exhibit heavy-tailed characteristics. This result relies on the assumption that wage indexation is random and can lie outside the unit interval. Thus it is implied that countries with full indexation schemes are more likely to have heavy-tailed distributions of variables than countries with the degree of wage indexation which lies within the unit interval. Also, a Taylor rule targeting current inflation outperforms a rule that targets past inflation regarding the minimization of the loss function. The analysis employed in deriving this study's results assumes that wage indexation is iid uniformly distributed. While this implies taking a definite stand on the boundedness of the distribution of wage indexation, assuming otherwise does not qualitatively alter the main results.

Recent empirical studies including Grier and Perry (1996), Chang (2012) and Caporale et al. (2012) employ the use of various versions of GARCH models to estimate inflation and inflation uncertainty. The relatively good fit of these models imply that the unconditional distribution of inflation exhibits tails heavier than that of a normal distribution. Furthermore, Fagiolo et al. (2008) concludes that in the majority of OECD countries, the distribution of output growth exhibits tails heavier than those of the Gaussian distribution. Contrary to this empirical evidence, the class of new Keynesian models commonly used for macroeconomic analysis typically imply that inflation and the output gap have normal unconditional distributions.

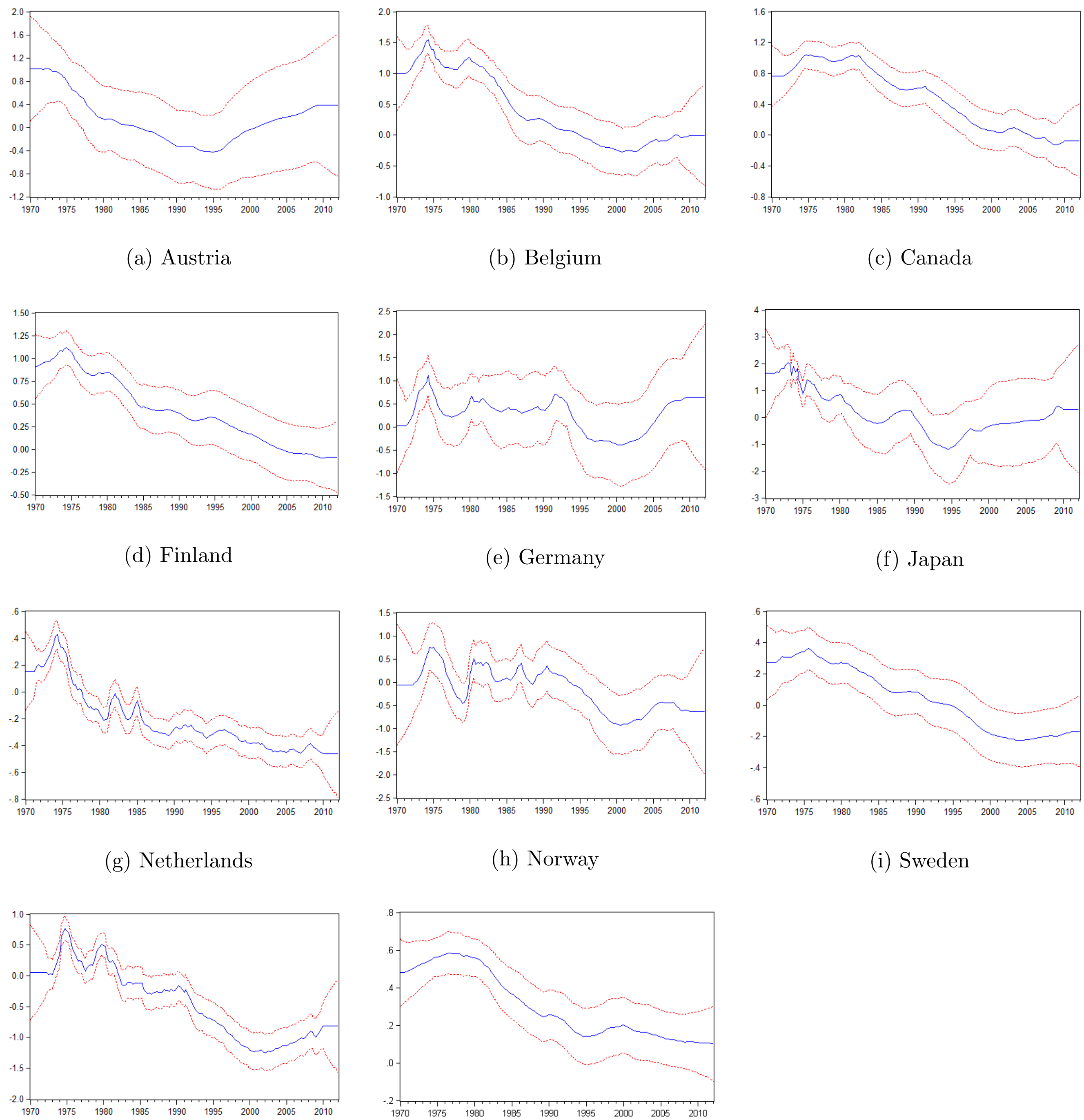
This study can be seen as an attempt to theoretically explain the source of the heavy tails in the aforementioned macroeconomic variables. Other approaches to explaining the presence of heavy tails involve the assumption of Student-t distributed error terms (see Curdia et al. (2012) and Chib and Ramamurthy (2011) for example). De Grauwe (2012) criticizes this exogenous approach of introducing the fat tail, maintaining that it does not shed light on how endogenous clustered volatility can be generated. The approach in this study involves a multiplicative shock similar to that first espoused by Brainard (1967) and later adopted by Attey and de Vries (2011). Following the latter study, it is assumed that the random degree of wage indexation is the source of the multiplicative shocks. The role of these multiplicative shocks is to amplify extreme realizations of the lag of inflation. This results in tails heavier in the unconditional distribution of inflation than would be expected under the normal distribution.[2] These heavy tails are passed on to the distribution of the output gap and ithe nterest rate.

The remainder of this study is organized as follows. Section 2 derives the Phillips curve under the assumption of random degree of wage indexation. Section 3 investigates optimal monetary policy. Section 4 investigates monetary policy under two alternative policy rules. The performances of these policy rules are subsequently compared to that of optimal monetary policy in the chapter. Finally, Section 5 concludes.

[1]The so-called Wassenaar Agreement in the Netherlands in 1982 is a widely known example of this case.

[2]The inflation process derived under random wage indexation exhibits a random AR coefficient.

Figure 1: Degree of wage indexation in selected OECD countries, Source: Attey (2015)



(a) Austria (b) Belgium (c) Canada

(d) Finland (e) Germany (f) Japan

(g) Netherlands (h) Norway (i) Sweden

(j) UK (k) US

2 Wage indexation and the Phillips curve

The representative firm has a fixed coefficient Ricardian production technology with labour as the sole input.[3] Assuming diminishing marginal returns to labour, the expression for output, Y_t, is:

$$Y_t = A_t N_t^{\alpha} \qquad\qquad 0 < \alpha < 1,$$

[3] McCallum and Nelson (1999) argue that modeling variations in capital stock as exogenous is largely consistent with empirical observation.

where N_t is the amount of labour employed in production. The logarithmic level of total factor productivity, A_t, follows a stationary AR process. The process is given below:

$$\log A_t = a_t = \rho_a a_{t-1} + \varepsilon_{at},$$

where iid random variable ε_{at}, has the following distribution: $N(0, \sigma_a^2)$. Firms maximize profit with respect to labour inputs. Thus, marginal productivity of labour should be equal to real wages. Let $\delta_0 = \log \alpha/(1-\alpha)$ and $\delta_1 = 1/(1-\alpha)$. The following equation gives the labour demand in log values:

$$n_t = \delta_0 - \delta_1(w_t - p_t) + \delta_1 a_t. \tag{1}$$

Labour supply in micro-founded models is typically derived from the optimization conditions of the representative household. Let the labour supply relation be given as follows:

$$n_t = \beta_0 + \beta_1(w_t - p_t) \qquad \beta_1 > 0,$$

where the parameters β_0 and β_1 are functions of the parameters governing household preferences. The market clearing wage implied by the labour supply and labour demand relations is:

$$w_t^* = \frac{\delta_0 - \beta_0}{\delta_1 + \beta_1} + p_t + \frac{\delta_1}{\delta_1 + \beta 1} a_t. \tag{2}$$

The corresponding market clearing output is obtained by substituting this expression into the labour demand (or the labour supply) relation and again substituting the resulting expression in the expression for aggregate output. This gives the log market clearing output (y^*) as:

$$y_t^* = \alpha \frac{\beta_1 \delta_0 + \beta_0 \delta_1}{\delta_1 + \beta_1} + \left(1 + \alpha \frac{\delta_1 \beta_1}{\delta_1 + \beta_1}\right) a_t.$$

Wage indexation

Fischer (1988) among others argues that informational lags make it impossible to index wages to current inflation. At a particular point in time, any available information concerning inflation relates to either inflation forecasts or lagged inflation, and not current inflation. In view of this, we consider an indexation scheme with indexation to a period's lagged inflation as follows:

$$w_t = w_t^{*e} + x_t(\pi_{t-1} - \hat{\pi}), \tag{3}$$

where w_t, w_t^*, and x_t are respectively the nominal wages, market clearing nominal wages and time varying wage indexation respectively. Inflation is denoted by the variable π_t. The superscript ^{e} in the model denotes the expectations of private agents. It is assumed that the inflation target announced by the policy maker ($\hat{\pi}$) effectively captures the expected inflation on which basis wage contracts are set a period in advance. While this assumption may come across as ad-hoc, the ECB's constant target of 2% can be cited as evidence in support of our assumption.

The wage indexation variable x_t effectively captures the elasticity of wages to lag of prices. Some of the country-specific estimates for x_t provided in Figure 1 suggest the possibility of overindexation (when $x_t > 1$). We therefore assume that $x_t \sim U(0, \kappa_a)$, where $\kappa_a > 1$, in order to allow for this possibility.

Wage indexation under a rule given by (3) may even exacerbate the destabilizing effects of monetary shocks. This runs contrary to the finding in Gray (1976) that wage indexation insulates the economy from monetary shocks. The reason behind the differing results lies in the way wages are indexed. Gray (1976) considers an indexation scheme under which wages are indexed to current inflation while we assume that wages are indexed to lag of inflation. This implies that real wages are flexible most of the time.

Take expectations of the market clearing wage in (2) and substitute the resulting expression into (3). This gives the following expression for real wages in the presence of wage indexation:

$$w_t - p_t = \frac{\delta_0 - \beta_0}{\delta_1 + \beta_1} + \frac{\delta_1}{\delta_1 + \beta_1} a_t^e - (p_t - p_t^e) + x_t(\pi_{t-1} - \hat{\pi}). \tag{4a}$$

It is worth to note that $(p_t - p_t^e) = (\pi_t - \pi_t^e)$. Following the assumption we earlier on made, expected inflation is equal to the target inflation, ie $\pi_t^e = \hat{\pi}$. This implies that (4a) can be rewritten as:

$$w_t - p_t = \frac{\delta_0 - \beta_0}{\delta_1 + \beta_1} + \frac{\delta_1}{\delta_1 + \beta_1} a_t^e - (\pi_t - \hat{\pi}) + x_t(\pi_{t-1} - \hat{\pi}). \tag{4b}$$

Aggregate supply or the Phillips curve

The aggregate supply is derived by substituting out real wages in (1) with (4b). This gives the log labor demand as a function of inflation, lagged inflation and productivity. The output is subsequently computed by noting that $y_t = \alpha n_t^d + a_t$. Section A.1 gives a more detailed derivation of the aggregate supply equation.

Let $\lambda_{1t} = \alpha\delta_1 x_t$ and $\lambda_2 = \alpha\delta_1$. Further assume that the output gap is defined as the deviation of log output under wage indexation from the log market clearing output level , ie $g_t = (y_t - y_t^*)$. The expression for the AS curve is:

$$g_t = -\lambda_{1t}\tilde{\pi}_{t-1} + \lambda_2\tilde{\pi}_t + u_t, \tag{5}$$

where $u_t = [\alpha\delta_1^2/(\delta_1 + \beta_1)]\varepsilon_{at}$. The variable $\tilde{\pi}_t$ is the deviation of inflation from target inflation, ie $\pi_t - \hat{\pi}$.

The aggregate supply relation on (5) implies a time-varying response of the output gap to lag of inflation. This is due to the assumption that wages are indexed to lag of inflation. If the degree of wage indexation is positive ($x_t > 0$), the lag of inflation has a negative effect on the output gap. In other words, indexation just increases the labour cost thereby decreasing output. A negative indexation resulting from a wage moderation response to a high inflation implies a positive effect of lag of inflation on output. This suggests that wage moderation as a response to high level of lagged inflation increases output beyond the level determined by total factor productivity shocks (u_t) and current inflation.

3 Monetary policy

This section derives inflation, the interest rate and the output gap under optimal monetary policy and two interest rate rate rules. The set-up adopted in solving for optimal monetary policy is similar to that of Clarke et al. (1999). A major distinction between our model and Clarke et al. (1999) lies in the slope parameter of the Phillips curve. The Phillips curve in our model has a random slope coefficient as opposed to the conventional constant slope Phillips curve used in Clarke et al. (1999).

We earlier on assumed that the the degree of wage indexation is iid random variable distributed as follows: $x \sim U[0, \kappa_a]$, where $\kappa_a > 1$. While the uniform distribution suggested as the distribution of the degree of wage indexation might seem adhoc, Attey and de Vries (2013) show that it can be a mixed equilibrium outcome of wage indexation bargaining under arbitration.

Optimal monetary policy

We now investigate the effect of random wage indexation to lagged inflation. The interest rate is introduced into the model by incorporating the aggregate demand or the IS curve. The aggregate demand relation is given as follows:

$$g_t = y_t - y^* = -\phi(i_t - \tilde{\pi}_t^e - r) + v_t \qquad v_t \sim iid\ \mathcal{N}(0, \sigma_v), \tag{6}$$

where r corresponds to the natural rate of interest which is assumed to be constant and v_t is a demand shock uncorrelated with productivity and the random wage indexation.

In deriving the optimal monetary policy, we make the following assumptions: the policy maker uses the interest rate (i_t) as an instrument, all bargaining with regards to wage indexation in the economy are concluded at the beginning of the current time period, and private agents do not observe the aggregate wage indexation outcome. For all purposes, the aggregate wage indexation outcome can also be viewed as a supply shock, uncorrelated to productivity shocks. The use of the interest rate as the instrument requires the policy maker to observe the supply and demand shocks in order to react before the private sector does. The expected inflation can be derived from the expressions (5) and (6) to obtain the following

$$\tilde{\pi}_t^e = -\frac{\phi}{\lambda_2 - \phi}(i_t^e - r) + \frac{\lambda_1}{\lambda_2 - \phi}\tilde{\pi}_{t-1} \qquad \lambda_1 = \lambda_{1t}^e. \tag{7}$$

Alternative forms of the expression (6) can be derived by expressing inflation and the output gap in terms of the control variable i_t and state variables (π_{t-1} and the random shocks), as well as substituting in (7) as follows:

$$g_t = -\phi(i_t - i_t^e) - \frac{\phi\lambda_2}{\lambda_2 - \phi}(i_t^e - r) + \frac{\phi\lambda_1}{\lambda_2 - \phi}\tilde{\pi}_{t-1} + v_t \tag{8a}$$

$$g_t = g_t^e - \phi(i_t - i_t^e) + v_t. \tag{8b}$$

Similarly, analogous expressions can be derived for the aggregate supply relation as follows:

$$\tilde{\pi}_t = -\frac{\phi}{\lambda_2}(i_t - i_t^e) - \frac{\phi}{\lambda_2 - \phi}(i_t^e - r) + \lambda_{3t}\tilde{\pi}_{t-1} + \frac{1}{\lambda_2}(v_t - u_t) \tag{9a}$$

$$\tilde{\pi}_t = \tilde{\pi}_t^e - \frac{\phi}{\lambda_2}(i_t - i_t^e) + \frac{\eta_t}{\lambda_2}\tilde{\pi}_{t-1} + \frac{1}{\lambda_2}(v_t - u_t), \tag{9b}$$

where $\lambda_{3t} = \phi\lambda_1/[\lambda_2(\lambda_2 - \phi)] + \lambda_{1t}/\lambda_2$ and $\eta_t = \lambda_{1t} - \lambda_1$.

Optimization problem of the policy maker

It is assumed that the policy maker targets both inflation and the output gap. In particular, they seek to stabilize both g_t and $\tilde{\pi}_t$ at 0, albeit without necessarily placing equal weights on both objectives. Let θ be the weight the policy maker places on inflation stabilization. The loss function of the policy maker is:

$$\mathcal{L}_t = g_t^2 + \theta\tilde{\pi}_t^2 \qquad \theta \geq 0.$$

We consider the case of optimal monetary policy under commitment, thus requiring the policy maker to take into account the effect of its policy on the expectations of agents in the economy. This requires the presence of another constraint in addition to (8a) and (9a) (or alternatively (8b) and (9b)) as follows:

$$i_t^e = E_{t-1} i_t. \tag{10}$$

The policy maker aims at minimizing all current and future losses stemming from deviations of the output gap and inflation from their respective targets. Let β be the discount factor and E_{t-1} be the expectation of the policy maker. The optimization problem of the policy maker is given as follows:

$$\begin{aligned} &\max_{i_t, i_t^e} \; E_{t-1} - \sum_{t=1}^{\infty} \beta^t \mathcal{L}_t \\ &s.t. \;\; \text{(8a), (9a) and (10).} \end{aligned} \tag{11}$$

The constraint (9a) is dynamic in $\tilde{\pi}$. In stabilizing current inflation and the output gap, one has to be mindful of the intertemporal effects of one's actions on the subsequent period's inflation. Thus, we can conclude that the optimization problem is a dynamic one with $\tilde{\pi}_t$ as the endogenous state variable. The Bellman formulation of the expression (11) is given as:

$$\begin{aligned} V(\pi_{t-1}) = &\max_{i_t, i_t^e} E_{t-1} \left[-g_t^2 - \theta \tilde{\pi}_t^2 + \beta V(\tilde{\pi}_t) \right] \\ &s.t. \; \text{(8a), (9a) and (10).} \end{aligned} \tag{12}$$

Following Clarke et al. (1999), we argue that since the problem is of linear-quadratic nature as far as the endogenous state variable is concerned, and owing to the independence of the exogenous state variables u_t and v_t, the value function must also be quadratic. Thus, we conjecture the following value function:

$$V(\tilde{\pi}_{t-1}) = \gamma_0 + 2\gamma_1 \tilde{\pi}_{t-1} + \gamma_2 \tilde{\pi}_{t-1}^2.$$

Let the variable Λ_{t-1} be the Lagrangian multiplier associated with the commitment constraint indicated by (10). We write down the first order conditions associated with the problem as follows:

$$\begin{aligned} 0 &= 2\phi[g_t + \tilde{\pi}_t(\theta/\lambda_2) - (\gamma_1 + \gamma_2 \tilde{\pi}_t)(\beta/\lambda_2)] - \Lambda_{t-1} \\ 0 &= -2\phi[g_t^e(1 - \lambda_2/(\lambda_2 - \phi)) + \tilde{\pi}_t^e(\theta/\lambda_2 - \theta/(\lambda_2 - \phi)) + (\gamma_1 + \gamma_2 \tilde{\pi}_t^e)(\beta/\lambda_2 - \beta/(\lambda_2 - \phi))] + \Lambda_{t-1}. \end{aligned}$$

The sum of the last two expressions derives the following expression:

$$0 = 2\phi[(g_t - g_t^e) + (\tilde{\pi}_t - \tilde{\pi}_t^e)(\theta - \beta\gamma_2)/\lambda_2] + 2\phi[\lambda_2 g_t^e + (\theta - \beta\gamma_2)\tilde{\pi}_t^e - \beta\gamma_1]/(\lambda_2 - \phi). \tag{13}$$

Taking expectation of the above expression yields

$$0 = 2\phi[\lambda_2 g_t^e + (\theta - \beta\gamma_2)\tilde{\pi}_t^e - \beta\gamma_1]/(\lambda_2 - \phi). \tag{14}$$

Substitute in the expressions (8b) and (9b) to get an expression in terms of the control variables. The derived optimal feedback rule after imposing (14) and after simplifying is given below:

$$0 = [-\phi(i_t - i_t^e) + v_t] \left(1 + \frac{\theta - \beta\gamma_2}{\lambda_2^2} \right) - \left(\frac{\theta - \beta\gamma_2}{\lambda_2^2} \right) (u_t - \eta_t \tilde{\pi}_{t-1}). \tag{15}$$

For the value function to be concave in $\tilde{\pi}$, we require that $\gamma_2 < 0$. Therefore, we know that $1 + (\theta - \beta\gamma_2)/\lambda_2^2 \neq 0$. This implies that under optimal control, $[-\phi(i_t - i_t^e) + v_t]$ is a function of u_t and $\eta_t\tilde{\pi}_{t-1}$. We also know that $i_t^e - r$ (and $\tilde{\pi}_t^e$) under optimal control must be a function of the endogenous state variable , $\tilde{\pi}_{t-1}$. Thus it follows from (9a) that under optimal policy, we can conjecture the following for the inflation process:

$$\tilde{\pi}_t = a + b\tilde{\pi}_{t-1} + \delta\eta_t\tilde{\pi}_{t-1} + cu_t, \tag{16}$$

where a, b, δ and c are parameters to be determined. For the value function to be concave, and thus, for the existence of a solution to the maximization problem, it is required that $\beta(b^2 + \delta^2\sigma_\eta^2) < 1$. Appendix A.2 derives the process for equilibrium inflation under optimal control, which is

$$\tilde{\pi}_t = b\left(1 + \frac{\eta_t}{\lambda_1}\right)\tilde{\pi}_{t-1} - \frac{b}{\lambda_1}u_t, \tag{17}$$

where

$$b = \frac{[(\lambda_2^2 + \theta) + \beta(1 + \sigma_\eta^2/\lambda_1^2)\lambda_1^2] - \sqrt{[(\lambda_2^2 + \theta) + \beta(1 + \sigma_\eta^2/\lambda_1^2)\lambda_1^2]^2 - 4\beta(\lambda_1\lambda_2)^2(1 + \sigma_\eta^2/\lambda_1^2)}}{2\beta\lambda_1\lambda_2(1 + \sigma_\eta^2/\lambda_1^2)}. \tag{18}$$

There are a few points worth noting about the behavior of the value representing the mean persistence of inflation (b) under optimal monetary policy. First, this parameter is always positive and it is bounded from above by $\bar{x}$. This implies that under random wage indexation to lagged inflation, the mean persistence in equilibrium inflation is at most the mean of the aggregate wage indexation. This maximum occurs when the weight of inflation stabilization in the policy maker's loss function is 0 ($\theta = 0$). To see this, define $a = (\lambda_2^2 + \theta)$ and $y = \beta(1 + \sigma_\eta^2/\lambda_1^2)\lambda_1^2$ thus permitting the mean persistence to be written down as follows:

$$b = \frac{(a + y) - \sqrt{(a + y)^2 - 4\lambda_2^2 y}}{4y}\kappa_a,$$

whereby we made the substitution $\lambda_2 = 2\lambda_1/\kappa_a$. Taking all other parameters as given, this function assumes its extremum value when the derivative with respect to the variable y equals zero ($\partial b/\partial y = 0$). The expression for this derivative is

$$\frac{\partial b}{\partial y} = \frac{-a\sqrt{(a + y)^2 - 4\lambda_2^2 y} + a(a + y) - 2\lambda_2^2 y}{4y\sqrt{(a + y)^2 - 4\lambda_2^2 y}}\kappa_a.$$

Imposing the first order maximization condition and simplifying the above expression further yields $y^2\lambda_2^2(\lambda_2^2 - a) = 0$. The necessary condition for maximization is therefore satisfied if any combination of the following expressions holds: $\lambda_2 = 0$ and $a = \lambda_2^2$. Reasonable estimates of the output elasticity to labour input in a Cobb-Douglass production function[4] imply that $\alpha > 0$, thus ruling out the condition $\lambda_2 = \alpha/(1 - \alpha) = 0$.[5] The remaining condition for maximization implies $\theta = 0$ at which b is at its maximum irrespective of the value of the variance of of wage indexation. We now show that the extremum value of b is indeed the maximum if $\theta = 0$. In order to do this, we show that for $\theta > 0$ the following must hold: $\partial b/\partial y < 0$.

[4]Our production function can considered as a Cobb-Douglass function with capital normalized to 1.

[5]Estimates from Christiano and Eichenbaum (1992) give the value of $1 - \alpha$ to be between 0.339 and 0.35 while values widely used in literature on Real Business Cycle range from 1/3 to 0.4.

This requires that either any or all of the following expressions must hold: $\lambda_2 < 0$, $y < 0$, and $\lambda_2^2 < a$. As indicated earlier, all reasonable estimates in earlier studies imply that $\lambda_2 > 0$. This rules out the first condition. We know that the second condition is also ruled out since $y = \beta(1 + \sigma_\eta^2/\lambda_1^2)\lambda_1^2 > 0$. This leaves us with the condition $\lambda_2^2 < a$ which implies $\theta > 0$.

Second, b is strictly decreasing in θ. This can be seen from the partial derivative of b with respect to θ. The derivative is

$$\partial b/\partial\theta = (\partial b/\partial a)(\partial a/\partial\theta) = \frac{\sqrt{(a+y)^2 - 4\lambda_2^2 y} - (a+y)}{4y\sqrt{(a+y)^2 - 4\lambda_2^2 y}} < 0.$$

The average persistence of inflation is therefore smaller when the policy maker attaches more weight to inflation stabilization in their loss function and it is zero in the extreme case when the policy maker targets only inflation (ie $\theta = \infty$).

Third, there are two cases in which the mean persistence of inflation assumes the highest value: when the production function exhibits constant marginal returns to labour ($\alpha = 1$) and when the policy maker does not put any weight on stabilizing inflation ($\theta = 0$). In the former case, the effects of productivity and the output gap on inflation are zero, implying that the persistence in inflation is solely determined by wage indexation. With regards to the latter case, an intuitive explanation can be given as follows: in the absence of any commitment to inflation stabilization, the expected persistence in equilibrium inflation is solely determined by how much, on the average, economic agents index to past inflation. Therefore, in order to disinflate an economy characterized by high persistent inflation, monetary authorities need to be committed to an inflation stabilization policy. This is in line with the empirical observation that inflation is less persistent under inflation targeting than under the absence of any form of commitment to stabilizing inflation.[6]

Equilibrium inflation under optimal monetary policy

That inflation is a persistent phenomenon is a well known observation. Most new Keynesian models incorporate inflation persistence by assuming that prices are indexed to lagged inflation. Jadresic (1998) and Perez (2003), among others, introduce inflation persistence by indexing wages to lagged inflation. The latter study concludes that persistence in inflation is higher, the higher the proportion of labour contracts that include indexation clauses. There is one fundamental difference between our study and the last two studies cited: wage indexation in our model is a random outcome rather than a given constant. The variance of the aggregate wage indexation outcome also affects the mean persistence of inflation in the economy. Consider the expression for expected the equilibrium inflation:

$$\tilde{\pi}_t^e = b\tilde{\pi}_{t-1}.$$

The expression for $\partial b/\partial y$ in the preceding section implies that the average persistence of inflation (b) is a decreasing function of the variance of wage indexation, $\sigma_x^2 = \sigma_\eta^2/\lambda_2^2$. In other words, on the average, past inflation is less important in explaining current inflation the higher the variance of aggregate wage indexation. The conditional variance of inflation under optimal control can be derived from (17) as follows:

$$\sigma_\pi^2 = \frac{b^2}{\lambda_1^2}(\sigma_\eta^2\tilde{\pi}_{t-1}^2 + \sigma_u^2). \tag{19}$$

[6]Literature that report this finding include Gerlach and Tillman (2012) and Kuttner and Posen (2001)

The expression above reveals that the conditional variance in inflation depends on three variables: the variance of wage indexation, lagged inflation and the variance of productivity shocks. The effects of lagged inflation and the variance of productivity shocks are unambiguous: they increase the conditional variance of inflation. However, no concrete conclusion can be drawn with regards to the effect of the variance of wage indexation on the conditional variance of inflation under general conditions. Under the rather specific assumption that the lagged inflation is at its target (ie. $\tilde{\pi}_{t-1} = 0$), it can then be concluded that the variance of wage indexation has a decreasing effect on the variance of inflation. To see this, one must first note that the higher the variance in wage indexation (captured by the variable σ_η^2), the lower the average persistence in inflation (b), and thus the lower the variance of inflation holding all other variables constant.

Interest rate under optimal monetary policy

The expression (A.21) substituted into (A.22) (both found in Appendix A.2) gives an interest rate rule to which a policy maker has to adhere when conducting optimal monetary policy. After making the substitution $a = 0$ and further simplifications, the interest rate rule under optimal monetary policy is given below:

$$i_t = r + b\tilde{\pi}_{t-1} + \frac{\lambda_1 - b\lambda_2}{\lambda_1 \phi}(\lambda_{1t}\tilde{\pi}_{t-1} - u_t) + \frac{1}{\phi}v_t. \tag{20}$$

As will be shown later, the expression above is reminiscent of the Taylor rule in that it contains a sort of reaction function to inflation and the output gap. The expression for inflation under optimal control as given in (17) and the implied the output gap derived from (5) can be written as follows:

$$\begin{aligned} \tilde{\pi}_t &= (b/\lambda_1)[\lambda_{1t}\tilde{\pi}_{t-1} - u_t] \\ g_t &= -(1 - b\lambda_2/\lambda_1)[\lambda_{1t}\tilde{\pi}_{t-1} - u_t]. \end{aligned}$$

A substitution of the former of the above two expressions into (20) permits the rendition of the interest rate rule under optimal monetary policy into a more recognizable form as follows:

$$i_t = r + b\tilde{\pi}_{t-1} + \frac{\lambda_1 - b\lambda_2}{b\phi}\tilde{\pi}_t + \frac{1}{\phi}v_t. \tag{21}$$

The last expression indicates a reaction function of the interest rate to lagged inflation and current inflation. In addition to the variables just mentioned, it also reacts to demand shocks v_t as per the assumptions made when solving the optimal control problem in Appendix A.2. Given reasonable values for the model's structural parameters, the coefficients of $\tilde{\pi}_{t-1}$, $\tilde{\pi}_t$, and v_t are all greater than 1. This suggests an aggressive reaction to deviation of these variables from 0, thus ensuring determinacy of the model under this rule.

Two simple interest rate rules

In what follows in this part, we examine monetary policy under two types of Taylor rules: one that targets current inflation (hereafter denoted by CTR) and the backward looking Taylor rule (hereafter denoted by BTR). These rules are given in the following expressions:

$$i_t = r + \omega_c\tilde{\pi}_t \tag{22}$$

$$i_t = r + \omega_b\tilde{\pi}_{t-1}. \tag{23}$$

The rules considered above are similar to those considered in Gali and Monacelli (2005). Besides, as can be seen in the preceding paragraphs, the equilibrium output gap (g_t) under optimal control is a linear function inflation thus permitting the interest rate rule to be expressed in terms of shocks and inflation only. In the case of the BTR, the policy maker reacts to lagged inflation. A motivation for considering this version of the Taylor rule can be drawn from the same reasoning as to why one should consider wage indexation to lagged inflation: policy makers may not have information on current shocks during policy formulation and implementation. The other expressions needed for the analysis are the aggregate demand or the IS curve and the aggregate supply or the Phillips curve equations. They are repeated here below.

$$\begin{aligned} \lambda_2 \tilde{\pi}_t &= \lambda_{1t} \tilde{\pi}_{t-1} + g_t - u_t \\ g_t &= -\phi(i_t - \tilde{\pi}_t^e - r) + v_t. \end{aligned}$$

After substituting the Taylor rule (22) into the IS equation, a compact representation of the linear system is given below as follows:

$$\mathbf{A_c} \begin{pmatrix} \tilde{\pi}_t \\ g_t \end{pmatrix} = \mathbf{B_{c,t}} \begin{pmatrix} \tilde{\pi}_{t-1} \\ g_{t-1} \end{pmatrix} + \mathbf{C_c} \begin{pmatrix} \tilde{\pi}_t^e \\ g_t^e \end{pmatrix} + \mathbf{D_c} \begin{pmatrix} u_t \\ v_t \end{pmatrix}, \tag{24}$$

where $\mathbf{A_c} = \begin{pmatrix} \lambda_2 & -1 \\ \phi\omega_c & 1 \end{pmatrix}$; $\mathbf{B_{c,t}} = \begin{pmatrix} \lambda_{1t} & 0 \\ 0 & 0 \end{pmatrix}$; $\mathbf{C_c} = \begin{pmatrix} 0 & 0 \\ \phi & 0 \end{pmatrix}$ and $\mathbf{D_c} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$.
Let the vector X be defined as $[\tilde{\pi}_t \quad g_t]'$ and vector ϵ_t be defined as $[u_t \quad v_t]'$. The above representation can further be simplified to get the following

$$X_t = \mathbf{F_{c,t}} X_{t-1} + \mathbf{G_c} E_{t-1} X_t + \mathbf{H_c} \epsilon_t,$$

where $\mathbf{F_{c,t}} = \mathbf{A_c^{-1}} \mathbf{B_{c,t}}$, $\mathbf{G_c} = \mathbf{A_c^{-1}} \mathbf{C_c}$ and $\mathbf{H_c} = \mathbf{A_c^{-1}} \mathbf{D_c}$. It is shown in Appendix B that the solution to the above system of equations is

$$X_t = \mathbf{P_{c,t}} X_{t-1} + \mathbf{H_c} \epsilon_t, \tag{25}$$

where $\mathbf{P_{c,t}} = [\mathbf{F_{c,t}} + \mathbf{G_c}(\mathbf{I} - \mathbf{G_c})^{-1} \mathbf{F_c}]$. The solution is basically an autoregressive system with time varying coefficients. Unlike its counterpart in extant literature investigating determinacy under a Taylor rule, the eigen value criterion for determinacy is not applicable. If the coefficient matrix $P_{c,t}$ were constant, then the obvious requirement for such a system to be determinate will be that both eigen values of the matrix must lie within the unit circle. Given the random nature of the coefficient matrix, the Kesten conditions are used to verify the existence of a stable asymptotic unconditional distribution of both the output gap and inflation. Algebraic verification of the Kesten conditions in the case of monetary policy under the two Taylor rules are rather tedious. We therefore resort to numerical computations using the the MATLAB programme to verify them.

The solution derived in (25) implies the following expressions for equilibrium inflation and the output gap under the CTR:

$$\tilde{\pi}_t = [\lambda_{1t}/\Delta + \phi\lambda_1/(\Delta^2 - \phi\Delta)]\tilde{\pi}_{t-1} + [1/\Delta]v_t - [1/\Delta]u_t \tag{26}$$

$$g_t = [-\lambda_{1t}(\phi\omega_c)/\Delta + (\phi\lambda_1\lambda_2)/(\Delta^2 - \phi\Delta)]\tilde{\pi}_{t-1} + [\lambda_2/\Delta]v_t + [\phi\omega_c/\Delta]u_t, \tag{27}$$

where $\Delta = \lambda_2 + \phi\omega_c$. Let $\Lambda = \phi(\lambda_1 - \phi\omega_b)/(\lambda_2 - \phi)$. A similar derivation procedure in the case of the BTR permits us to derive the equilibrium process for inflation and the output gap as follows:

$$\tilde{\pi}_t = [\lambda_{1t}/\lambda_2 + \Lambda/\lambda_2]\tilde{\pi}_{t-1} + [1/\lambda_2]v_t - [1/\lambda_2]u_t \tag{28}$$

$$g_t = \Lambda\tilde{\pi}_{t-1} + v_t. \tag{29}$$

Comparing the equilibrium the output gap under the CTR (27) with its counterpart under the BTR (29) reveals a difference in the conditional distributions of the output gap under the two rules: while the conditional distribution under the CTR is not normal, that under the BTR is normally distributed if one assumes a normal distribution for v_t.[7] The CTR therefore comes closer to mimicking the optimal monetary policy as far as conditional distribution of the output gap is concerned. If the Kesten conditions are satisfied, the unconditional distribution of all variables are heavy tailed under both Taylor rules.

4 Evaluation of alternative policy rules

This section carries out a quantitative analysis of the two policy rules and compares the equilibrium dynamics of inflation, the output gap and the interest rate obtained under these rules to those obtained under optimal monetary policy in the previous section. The loss function used in deriving the optimal monetary policy in Section 3 reveals a hybrid stabilization policy that targets both inflation and the output gap. In the new Keynesian literature, a similar welfare function is derived as a second order approximation of the representative consumer's utility function.[8] In such a case, the relative weight placed on inflation stabilization is a function of structural parameters in the new Keynesian model. We restate the objective function of the policy maker below:

$$\mathcal{W} = -\sum_{t=0}^{\infty} \beta^t E_0 \left(g_t^2 + \theta\tilde{\pi}_t^2\right). \tag{30}$$

In Gali and Monacelli (2005), it is noted that for $\beta \to 1$, the loss function can be rewritten in terms of the unconditional variances of the output gap and inflation. The logic behind the expression of (30) in terms of these variances differs from that of Gali and Monacelli (2005). Assume $\beta \to 1$ and that the loss function can be approximated as a sum of instantaneous losses over a finite time horizon. A step by step approximation of (30) is given below:

$$\begin{aligned} \mathcal{W} &\approx -E_0 \sum_{t=0}^{T} \beta^t \left(g_t^2 + \theta\tilde{\pi}_t^2\right) \\ &= -TE_0 \sum_{t=0}^{T} \beta^t \frac{1}{T} \left(g_t^2 + \theta\tilde{\pi}_t^2\right) \\ &\approx -T[var(g_t) + \theta var(\pi_t)], \end{aligned}$$

where T is an arbitrarily large number. Given the ordinal nature of the measurement of the loss of the policy maker, any monotonic transformation of the last expression should be an adequate measure for the

[7] To see this, note that Λ is a function of constant parameters. Thus the conditional distribution of g_t under the BTR depends only on the distribution of v_t.

[8] Woodford (2003) contains such derivations

loss of the policy maker. We therefore express the loss function as follows:

$$\mathcal{V} = -[var(g_t) + \theta var(\pi_t)]. \tag{31}$$

The existence of stationary distributions for the output gap and inflation once Kesten conditions are satisfied guarantees a finite variance. The version of the loss function contained in (31) will be used to rank the rules and the performance of optimal monetary policy. The calibration in this section is carried out with respect to the dynamics of the output gap, inflation and the interest rate in the economy of the Euro area.

Recap of the calibrated models

We compare the dynamics of inflation, the output gap and interest rate under the optimal monetary policy and the two Taylor rules. In order to get a lucid comparison of the distributions, we include a version of the model under which inflation under optimal policy has a non random persistence (OCW). In other words, we assume that $\lambda_{1t} = \lambda_1$ in the OCW model. Each of the calibrated models can be summarized by the following three expressions: the aggregate supply or the Phillips curve, the aggregate demand curve and the interest rate rule. Let $\varphi = (\lambda_1 - b\lambda_2)/(b\phi)$. Table 1 gives a summary of the models employed in the calibrations.

Table 1: Summary of models

Optimal policy	
Constant index (OCW)	Random index (ORW)
$\begin{cases} \lambda_2\tilde{\pi}_t &= g_t + \lambda_1\tilde{\pi}_{t-1} - u_t \\ g_t &= -\phi(i_t - \tilde{\pi}_t^e - r) + v_t \\ i_t &= r + \bar{b}\tilde{\pi}_{t-1} + \varphi\tilde{\pi}_t + [1/\phi]v_t \end{cases}$	$\begin{cases} \lambda_2\tilde{\pi}_t &= g_t + \lambda_{1t}\tilde{\pi}_{t-1} - u_t \\ g_t &= -\phi(i_t - \tilde{\pi}_t^e - r) + v_t \\ i_t &= r + b\tilde{\pi}_{t-1} + \varphi\tilde{\pi}_t + [1/\phi]v_t \end{cases}$
Taylor rules	
Current inflation (CTR)	Lagged inflation (BTR)
$\begin{cases} \lambda_2\tilde{\pi}_t &= g_t + \lambda_{1t}\tilde{\pi}_{t-1} - u_t \\ g_t &= -\phi(i_t - \tilde{\pi}_t^e - r) + v_t \\ i_t &= r + \omega_c\tilde{\pi}_t \end{cases}$	$\begin{cases} \lambda_2\tilde{\pi}_t &= g_t + \lambda_{1t}\tilde{\pi}_{t-1} - u_t \\ g_t &= -\phi(i_t - \tilde{\pi}_t^e - r) + v_t \\ i_t &= r + \omega_b\tilde{\pi}_{t-1} \end{cases}$

It should be noted that the definition of the coefficient b given in (18) changes under the OCW. Recall that wage indexation is assumed constant at its mean under the OCW. Thus, the variance of wage indexation and by implication σ_η^2 are both 0. The average persistence under the OCW ($\bar{b}$) and the corresponding value under the ORW (b) are stated below:

$$\bar{b} = \frac{[(\lambda_2^2 + \theta + \beta\lambda_1^2] - \sqrt{(\lambda_2^2 + \theta + \beta\lambda_1^2)^2 - 4\beta(\lambda_1\lambda_2)^2}}{2\beta\lambda_1\lambda_2}$$

$$b = \frac{[(\lambda_2^2 + \theta) + \beta(1 + \sigma_\eta^2/\lambda_1^2)\lambda_1^2] - \sqrt{[(\lambda_2^2 + \theta) + \beta(1 + \sigma_\eta^2/\lambda_1^2)\lambda_1^2]^2 - 4\beta(\lambda_1\lambda_2)^2(1 + \sigma_\eta^2/\lambda_1^2)}}{2\beta\lambda_1\lambda_2(1 + \sigma_\eta^2/\lambda_1^2)}.$$

Parameter values

The parameter values used in the calibration exercise derive from the following sources: Amisano and Tristiani (2010), Gali and Monacelli (2005), and our own estimates. There are some cases in which directly corresponding values of certain parameters in the source literature are not available. In these cases, we construct values based on a set of related parameters obtained from the literature. The next three paragraphs give a more detailed explanation on how some parameter values are set for the calibration.

The constant in the labour supply equation (β_0) is set to 0. Using a different value does not change our results in any significant way. Besides, there is no constant term in most micro founded derivation of the labour supply curve found in literature.[9] It is assumed that the policy maker places twice as much weight on output stabilization as they place on inflation stabilization . Thus, we assume that $\theta = 0.5$. Following Gali and Monacelli (2005), we set the coefficients of inflation in both Taylor rules at 1.5 ($\omega_c = \omega_b = 1.5$).

The values of the interest rate elasticity of aggregate demand (ϕ), the standard deviation of the aggregate-demand shocks (σ_v), and the wage elasticity of labour supply (β_1) are not directly available from the estimates in Amisano and Tristiani (2010). We express these parameters as functions of available estimates under some plausible assumptions. Assume a power utility function which is separable in both consumption and labour (or leisure). A micro founded derivation of the aggregate demand (or the IS curve) implies that the interest rate elasticity is the inverse of the constant relative risk aversion (CRRA) parameter. We therefore set $\phi = 1/\gamma$, where γ is the CRRA estimate from Amisano and Tristiani (2010). Under the same assumption, it can be shown that the real wage elasticity of labour supply is a function of the labour share of production (α), the disutility of labour , and the constant relative risk aversion parameter when one assumes a power utility function. In particular, $\beta_1 = 1/(\phi + \alpha\gamma)$, where ϕ captures the disutiltiy of labour in the model of the study just cited. Finally, we assume that demand shock is the sum of the inflation target shock and the interest rate shock found in the literature. This permits us to set $\sigma_v = \sqrt{\sigma_{\bar{\pi}}^2 + \sigma_i^2}$ where $\sigma_{\bar{\pi}}^2$, and σ_i^2 are respectively the variances of inflation target shocks and the interest rate shocks.

The wage parameter indicating the extent of overindexation (κ_a) is fixed at 1.5. This value is motivated by the estimates obtained from Attey (2015) for the case of Belgium. While there are estimates found in other literature, those estimates are derived under the rather restrictive assumption of a time-invariant degree of wage indexation. We carry out our own estimations to estimate the parameters α, σ_a and ρ_a. Details concerning the estimation procedure are given in section C of the appendix. Table 2 gives a summary on the parameters and their corresponding values used in the calibration exercise.

[9] In these models, labour supply is typically given by the following $(w_t - p_t) = \log(MRS_t) = \log(-(U_n)) - \log(U_c)$. Assuming a power utility function then implies that real wages are increasing in labour hours and productivity after imposing equilibrium conditions. The log of the latter variable is typically assumed to be a stationary AR(1) process around a 0 unconditional mean.

Table 2: Parameter values

Parameter	Description	Source	Source parameters	Value
β_0	labour supply constant			0
θ	relative weight of inflation			0.5
β	time preference	Amisano and Tristiani (2010)	β	0.99
r	steady-state interest rate	Amisano and Tristiani (2010)	$1/\beta - 1$	0.01
ϕ	interest elasticity of agg demand	Amisano and Tristiani (2010)	$1/\gamma$	0.421
σ_v	std dev agg demand shock	Amisano and Tristiani (2010)	$\sqrt{\sigma_{\pi}^2 + \sigma_i^2}$	0.003
β_1	labour supply elasticity	Amisano and Tristiani (2010)	$1/(\phi + \alpha\gamma)$	0.201
κ_a	mean wage indexation	Attey (2015)		1.5
α	labour share	own estimates: Appendix (C.1)		0.64
ρ_a	persistence in productivity	own estimates: Appendix (C.1)		0.9
σ_a	std dev productivity shock	own estimates: Appendix (C.1)		0.013
ω_c	Taylor rule conventional	Gali and Monacelli (2005)	ϕ_π	1.5
ω_b	Taylor rule backwards	Gali and Monacelli (2005)	ϕ_π	1.5

The existence of a stationary unconditional distribution

We conduct tests on the inflation process presented in (17), (27), and (29) for the existence of heavy-tailed distributions. We do not need to conduct tests on the processes of the output gap and the interest rate since they are functions of inflation. Any heavy-tailed property of the unconditional distribution of inflation is automatically passed on to the other variables The expressions for equilibrium inflation obtained under optimal monetary policy (17), under the CTR (26), and the BTR (28) imply that inflation can generally be represented by the following univariate AR(1) process:

$$X_t = V_t + B_t X_{t-1}, \tag{32}$$

where (V_t, B_t) are iid with absolutely continuous distribution functions. Equation (32) is an AR process with random coefficient B_t. The Kesten conditions give the general conditions for such a process under which the unconditional distributions of inflation, the output gap and the interest rate under optimal monetary policy and the two interest rate rules considered are stationary.

Kesten Conditions: Consider a time varying autoregressive process as in (32) above. If there exists $\kappa > 0$ such that the following conditions are satisfied:

- $E \log |B_1| < 0$
- $E|B_1|^\kappa = 1$
- $E|B_1|^\kappa \log^+ |B_1| < \infty$
- $0 < E|V_1|^\kappa < \infty$

then a stationary distribution exists for the process X irrespective of how this process is initialized and it is heavy tailed. For an AR(1) univariate process to have a heavy-tailed unconditional distribution, it suffices that only the second condition is satisfied. In what follows in this section, we investigate the conditions for the existence stationary distribution of the inflation process under optimal monetary policy and the two Taylor rules.

Optimal monetary policy

As already mentioned in Section 2 of this chapter, we assume a uniform distribution for the degree of wage indexation. In particular, we assumed that $x_t \sim U(0, \kappa_a)$ where $\kappa_a > 1$. The implied process for inflation under both the OCW and the ORW are:

$$\tilde{\pi}_t = \bar{b}\tilde{\pi}_{t-1} - (\bar{b}/\lambda_1)u_t \tag{33a}$$

$$\tilde{\pi}_t = 2bA_t\tilde{\pi}_{t-1} - (b/\lambda_1)u_t, \tag{33b}$$

where $A_t \sim U(0, 1)$.

The inflation process under the OCW is an AR(1) process with a constant coefficient $\bar{b}$. The existence of a stationary unconditional distribution hinges on the following assumption: $|\bar{b}| < 1$. Given the parameters in Table 2, this condition is satisfied since $\bar{b} = 0.5859$.[10] Earlier on we assume that the productivity shock

[10] The computations for the calibration exercise were carried out in MATLAB.

term (u_t) is normally distributed. This implies that the unconditional distribution of inflation under the OCW (33a) is normal.

Concerning inflation under the ORW, the first Kesten condition requires that the following holds: $b < e/2$. If b is at its maximum ($\theta = 0$), this condition translates to $\kappa_a < \sqrt{e}$. The second condition implies solving for a κ which satisfies $(2b)^\kappa = \kappa + 1$. A solution exists for any $b \in (1/2, e/2)$. The last two conditions can easily be verified, given that there exists a κ that satisfies the second condition. For our set of parameters, $b = 0.5551$. This implies that the Kesten conditions are satisfied since $b = 0.5551 \in (1/2, e/2)$. This guarantees the existence of a stationary heavy-tailed distribution for inflation, the output gap and the interest rate.

Remarkably, the mean persistence of inflation under the OCW is larger than that under the ORW (ie $\bar{b} > b$). However, inflation under the latter model rather exhibits heavy-tailed properties. This observation proves the importance of multiplicative shocks such as random wage indexation in generating heavy-tailed distributions.

Monetary policy under CTR and BTR

We derive bounds for the coefficients under the CTR and BTR that satisy the Kesten conditions using MATLAB. The process for inflation under CTR as found in (26) in the main text can respectively be expressed as follows:

$$\tilde{\pi}_t = [c_{cmin} + c_{cext}A_t]\tilde{\pi}_{t-1} + [1/\Delta]v_t - [1/\Delta]u_t, \tag{34a}$$

where $c_{cmin} = (\phi\lambda_1)/(\Delta^2 - \phi\Delta)$ and $c_{cext} = \lambda_2\kappa_a/\Delta$. As in (26), the parameter $\Delta = \lambda_2 + \phi\omega_c$. Here again, A_t is a random variable uniformly distributed on the unit interval. Similarly, the process for inflation under a BTR as found in (28) can be expressed as follows:

$$\tilde{\pi}_t = [c_{bmin} + c_{bext}A_t]\tilde{\pi}_{t-1} + [1/\lambda_2]v_t - [1/\lambda_2]u_t, \tag{34b}$$

where $c_{bmin} = \Lambda/\lambda_2$ and $c_{bext} = \kappa_a$.

From (33) and (34), the inflation processes can be given the following generic representation:

$$\tilde{\pi}_t = [c_{min} + c_{ext}A_t]\tilde{\pi}_{t-1} + \mu_t. \tag{35}$$

Earlier on, we asserted that one needs to only check the second of the Kesten conditions for the existence of a heavy-tailed unconditional distribution. We nevertheless check both the first and second of the conditions in our computations. As we will explain later, the first condition reveals information about the average persistence in the inflation process. Given the inflation process (35), the first two conditions can be derived using the following:

$$\int_{c_{min}}^{c_{max}} \frac{\log(|x|)}{c_{ext}} dx < 0$$

$$\int_{c_{min}}^{c_{max}} \frac{|x|^\kappa}{c_{ext}} dx = 1,$$

where $c_{max} = c_{min} + c_{ext}$.

The Table 3 gives the result of the tests regarding the Kesten conditions. From the table, the value of κ in the case of the CTR is 12.307 while that for the case of the BTR is 2.313. Thus, it can be concluded that the unconditional distributions of inflation under both types of Taylor rules are stationary and heavy tailed.

Table 3: Results from the Kesten tests

Condition	OCW	ORW	CTR	BTR
$E \log \lvert B_1 \rvert$	N/A	-0.896	-0.549	-0.305
κ	N/A	34.038	12.307	2.314
Distribution	normal	heavy tailed	heavy tailed	heavy tailed

[1] Table gives results of the Kesten tests from calibrations N/A denotes that the test is not applicable. Test conducted on the process $X_t = B_t X_{t-1} + V_t$.

[2] Calibrations were conducted in MATLAB

Interpreting $E \log |B_1|$ and the parameter κ

The first Kesten condition requires $E \log |B_1| < 0$ for the existence of a stationary unconditional distribution of inflation. This condition can be seen as analogous to the condition that $|\bar{b}| < 1$ under the OCW. We can therefore deduce preliminary insights into the persistence of the inflation process from the condition. It can be concluded from Table (3) that a stationary distribution exists for the inflation process under each of the models. Also, one can again conclude that the persistence in the inflation process is the highest under the BTR and the lowest under the ORW. The persistence of this process under the CTR falls between those of the BTR and the ORW.

The presence of a heavy-tailed unconditional distribution of inflation depends on the existence of a $\kappa > 0$ that satisfies the second condition. The magnitude of this parameter indicates how heavy the tails of the distribution are. In particular, κ is inversely related to the heaviness of the tail: a higher κ denotes that the tails of the particular distribution in question are relatively less heavy. The intuition behind this inverse relationship is as follows. A distribution which does not meet the second Kesten condition may have $\kappa = \infty$. Thus the further away κ is, the more likely it is that that distribution will fail the requirements for the presence of heavy tails.

From Table 3, it can be concluded that the distribution of inflation under the BTR has the highest persistence and the heaviest tails. We may prematurely conclude that variations of variables under that model are most extreme and most undesirable. The inflation rate, output gap, and interest rate have the least variances under the ORW (not counting the OCW). One may therefore give the following ranking of the models based on loss minimization: ORW, CTR, and BTR.

Time paths of variables

Figure 2 gives the time path of the various macroeconomic variables in this section based on a simulation of $10,000$ observations. Noting that the unconditional distributions of variables under OCW are normally distributed, the figure shows that extreme observations are more frequent under the ORW model than those under the OCW model. It can therefore be concluded that inflation, the output gap and the interest rate under the ORW model have heavy-tailed unconditional distributions.

Figure 2 also shows the dynamics of the three variables under the two Taylor rules. The unconditional distributions of both variables are heavy tailed since the processes of inflation under these two Taylor rules satisfy the Kesten conditions. The figure also shows that the distributions under the BTR are more heavy tailed than those under the CTR. This result was already implied by the κ values.

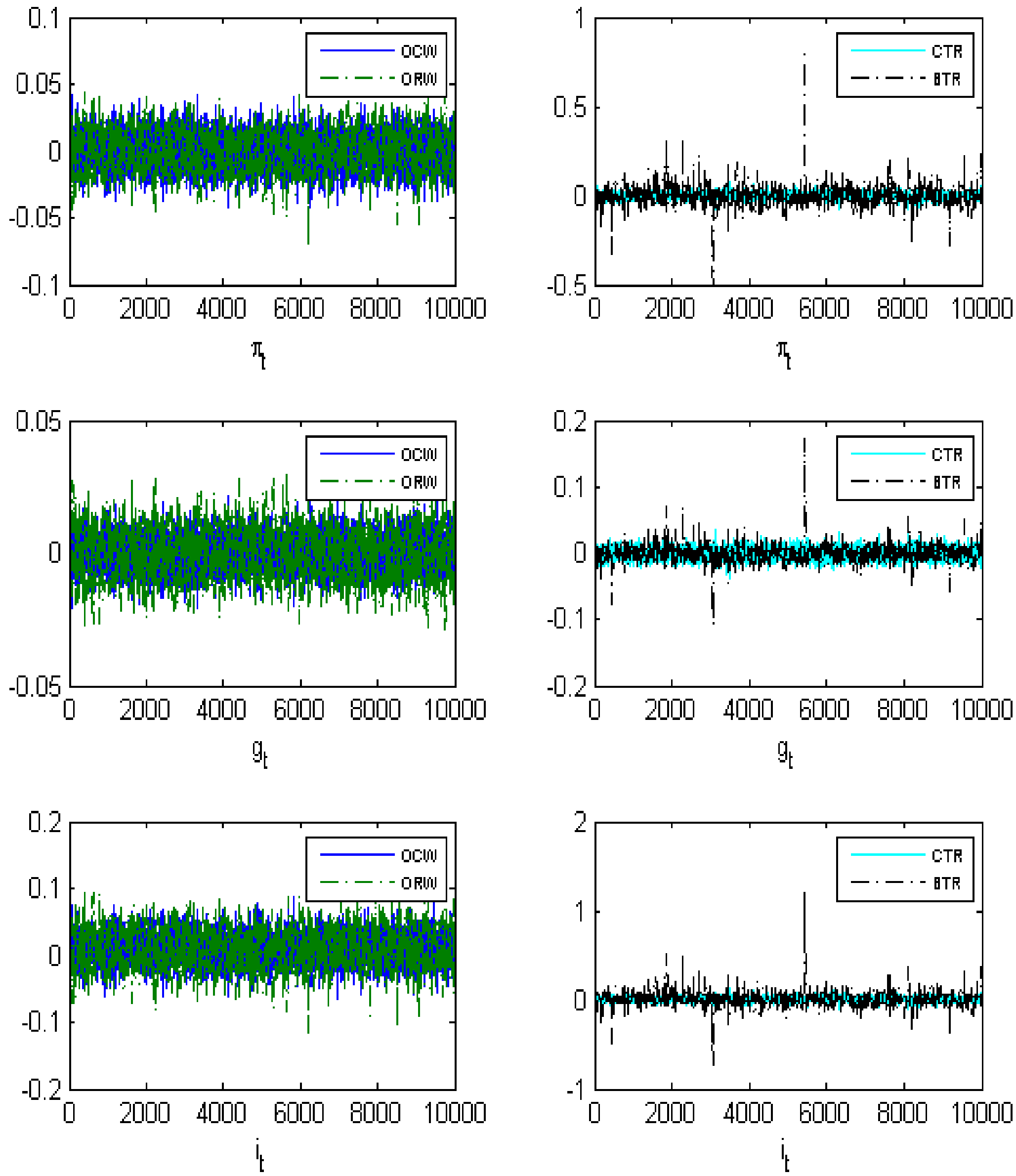


Figure 2: Time paths of variables

Impulse responses to productivity shocks

The dynamic effects of productivity shocks are displayed in Figure 3 in the appendix. The figure suggests that the three variables converge back to their steady states faster under the ORW model than the OCW model. However, this is the case only because of the particular set of draws of the random wage indexation parameter λ_{1t}. For other sets of draws, the variables converge back to their steady states faster under the OCW than the ORW. Repeated simulations show that on the average, it takes 10 periods to converge back to the steady state after an initial productivity shock. This is the same number of periods it takes for variables to converge back to their respective steady states under the OCW. Thus, random wage indexation induces uncertainty in the amount of time it takes for the three variables to converge back to their steady states.

Inflation has a higher initial response to productivity shocks under the CTR than under the BTR. Given the same draws of λ_{1t}, all variables converge faster to their respective steady states under the CTR than under the BTR. The reason for this result lies in the implied processes of inflation under the two models. From the expressions (26) and (28), the mean persistence parameters prevailing under these models are:

$$E[\lambda_{1t}/\Delta + \phi\lambda_1/(\Delta^2 - \phi\Delta)] = \lambda_1/(\Delta - \phi) \qquad = 0.6706 \qquad (CTR)$$
$$E[\lambda_{1t}/\lambda_2 + \Lambda/\lambda_2] = (\lambda_1 + \Lambda)/\lambda_2 \qquad = 0.8725 \qquad (BTR).$$

The computations above imply that on the average, the inflation process is more persistent under the BTR than under the CTR.[11] conditions We therefore expect inflation to converge back to its steady state faster under the CTR. Since the output gap and the interest rate are functions of the inflation rate, they also converge back to their steady states quicker under the latter model.

Impulse responses to demand shocks

Table 4 presents the responses of the various variables to a one standard deviation shock in demand v_t. Under optimal monetary policy, inflation and the output gap are not impacted by demand shocks. This is due to the assumption that the policy maker observes the demand shocks and moves to offset their likely effects. The interest rate initially rises in response to demand shocks, but converges back to its steady state in the subsequent period.

The responses of the output gap and inflation to a one-time demand shock are larger under the BTR than under the CTR. The interest rate has a delayed response to demand shocks under the BTR. All three variables converge back to their respective steady states faster under the CTR than under the BTR. The reason for this is identical to the one provided for the case of productivity shocks.

One can therefore conclude that compared to optimal monetary policy, a Taylor rule targeting only the inflation rate performs poorly when the economy is subject to demand shocks. This conclusion hinges on the assumption that a policy maker can observe demand shocks immediately in order to react to them.

Losses from alternative policy rules

Table 4 presents the standard deviations and the implied loss under each type of monetary policy considered in this work. Inflation is less volatile under the ORW than under the CTR while the output gap is less

[11]The results from the tests of the first of the Kesten already implied this.

volatile under the latter than the former. This contrast concerning the volatility of the output gap and inflation under these two policies stems from their respective interest rate rules. The ORW interest rate in (21) reacts to demand shocks and current inflation, while the CTR interest rate in (22) targets only current inflation. Therefore, the excessive volatility in the interest rate under ORW is transferred to the output gap under this policy regime.

However, it should be noted that the volatility of inflation increases when the interest rate does not respond to current shocks. The following observations can be made about the various interest rate policy rules. The BTR interest rate targets none of the current shocks, the CTR interest rate targets only productivity shocks (embedded in current inflation), and the ORW interest rate targets both productivity and demand shocks. As a result from the nature of the interest rate rules, inflation is most volatile under the BTR and least volatile under the ORW.

Table 4: Standard deviations and loss

	OCW	ORW	CTR	BTR
	sd%	*sd%*	*sd%*	*sd%*
Inflation (π_t)	1.16	1.16	1.34	4.67
Output gap (g_t)	0.58	0.73	0.69	1.06
Interest rate (i_t)	1.99	2.27	2.01	7
	Variance of variables in %			
Inflation	0.0135	0.0135	0.0180	0.2472
Output gap	0.0033	0.0052	0.0047	0.0126
Loss ($\mathcal{V}$)	0.0101	0.0120	0.0137	0.1362

Not surprisingly, the optimal monetary policy generates the lowest loss among the three types of monetary policy considered, although the losses from the ORW and the CTR do not differ that much in magnitude. This means that given the parameters, a Taylor rule targeting current inflation almost replicates optimal monetary policy. Of the two types of Taylor rules considered, the one targeting current inflation (CTR) outperforms the lagged inflation targeting Taylor rule (BTR). This comes as no surprise as it is already known that the CTR comes closest to mimicking the interest rate rule under optimal monetary policy (see Woodford (2001)).

5 Conclusion

This study investigates the effect of random wage indexation on monetary policy. Most of the extant literature on wage indexation and its role in monetary policy is based on the assumption that the degree of wage indexation is constant. However, recent empirical estimates suggest a time varying process for the degree of wage indexation. Drawing on the empirical properties of the degree of wage indexation, this study investigates the conduct of monetary policy in the presence of random wage indexation. In particular, we investigate the conduct of monetary policy under three interest rate rules: the rule implied by optimal monetary policy under commitment, a current inflation targeting Taylor rule and a lagged or expected inflation targeting Taylor rule.

Our findings reveal that under the plausible scenario of wages being overly indexed to inflation, the unconditional stationary distribution of inflation, the interest rate and the output gap do exhibit heavy-tailed characteristics under all of the three types of monetary policies considered. This implies that extreme observations in these variables are more likely to occur than as would be predicted under current standard theoretical models. Also, inflation exhibits volatility clustering with expected or lagged inflation having a positive effect on the conditional variance of inflation. Finally, it is better to commit to a Taylor rule targeting current inflation rather than one targeting lagged inflation.

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A Aggregate supply and optimal monetary policy

A.1 Deriving the aggregate supply (Phillips) curve

It is assumed that the representative firm is perfectly competitive. The real wage is therefore equal to the marginal productivity of labour. With the production technology assumed in the main text, the expression for real wages is:[12]

$$\frac{W_t}{P_t} = \alpha A_t N_t^{\alpha-1}.$$

Let $\delta_0 = (\ln \alpha))(1-\alpha)$ and $\delta_1 = 1/(1-\alpha)$. The labour demand expression can be derived by taking the log of the real wage expression just previously given. This is given below:

$$n_t^d = \delta_0 - \delta_1(w_t - p_t) + \delta_1 a_t. \tag{A.1}$$

The expression for labour supply can be derived from a representative household's optimising behaviour. For the purposes of this study, we make use of the following adhoc labour supply relation:

$$n_t^s = \beta_0 + \beta_1(w_t - p_t). \tag{A.2}$$

By equating (A.1) to (A.2) , one derives the following expressions for equilibrium nominal wage rate (w_t^*) and equilibrium labour (n_t^*). These expressions are as follows:

$$w_t^* = \frac{\delta_0 - \beta_0}{\delta_1 + \beta_1} + p_t + \frac{\delta_1}{\delta_1 + \beta_1} a_t \tag{A.3}$$

$$n_t^* = \frac{\beta_1\delta_0 + \beta_0\delta_1}{\delta_1 + \beta_1} + \frac{\beta_1\delta_1}{\delta_1 + \beta_1} a_t. \tag{A.4}$$

The production function was already given in the main part of this study as follows: $Y_t = A_t N_t^{\alpha}$. Taking the log of this function permits us to derive an expression in terms of log variables as follows:

$$y_t = \alpha n_t + a_t. \tag{A.5}$$

The expression for equilibrium output is then derived by substituting (A.4) into (A.5). We give the equation for equilibrium output below:

$$y_t^* = \alpha\frac{\beta_1\delta_0 + \beta_0\delta_1}{\delta_1 + \beta_1} + \left(\frac{\alpha\beta_1\delta_1}{\delta_1 + \beta_1} + 1\right) a_t. \tag{A.6}$$

The (log) productivity shock term a_t is assumed in the main part of this text to follow the stationary AR(1) process given below:

$$a_t = \rho_a a_{t-1} + \varepsilon_{at},$$

where ε_{at} is iid normal with a zero mean. The AR coefficient ρ_a is assumed to lie within the unit internal to ensure stationary of the AR process.

[12]Lower cases of variables denote their log values. In discussing these variables, we omit the word' 'log' for convenience

The wage indexation rule given in Equation (3) stipulates for wages to be adjusted if previously observed inflation deviates from the target inflation. The rule is repeated below:

$$w_t = w_t^{*e} + x_t(\pi_{t-1} - \hat{\pi}).$$

From the expression (A.3), we can derive the expression for the expectation of the wage rate prevailing at the competitive equilibrium. Let $a_t^e = a_t \equiv \rho_a a_{t-1}$. The expectation of the equilibrium wage rate is:

$$w_t^{*e} = \frac{\delta_0 - \beta_0}{\delta_1 + \beta_1} + p_t^e + \frac{\delta_1}{\delta_1 + \beta_1} a_t^e.$$

Substituting the expression above into the expression for wage indexation we get the following:

$$w_t = \frac{\delta_0 - \beta_0}{\delta_1 + \beta_1} + \frac{\delta_1}{\delta_1 + \beta_1} a_t^e + p_t^e + x_t(\pi_{t-1} - \hat{\pi}).$$

The presence of indexation introduces nominal rigidity into the model. A trade-off between inflation and the output gap can therefore be realized in the presence of wage indexation. Subtracting prices from both sides of the equation, one derives the following expression for real wages under wage indexation:

$$w_t - p_t = \frac{\delta_0 - \beta_0}{\delta_1 + \beta_1} + \frac{\delta_1}{\delta_1 + \beta_1} a_t^e - (p_t - p_t^e) + x_t(\pi_{t-1} - \hat{\pi}).$$

We note that $(p_t - p_t^e) = \pi_t - \pi_t^e$, where $\pi_t = p_t - p_{t-1}$. Substitute the expression for the real wage under wage indexation into the labour demand expression (A.1) to obtain the following:

$$n_t = \frac{\beta_1\delta_0 + \beta_0\delta_1}{\delta_1 + \beta_1} + \delta_1(\pi_t - \pi_t^e) - \delta_1 x_t(\pi_{t-1} - \hat{\pi}) + \delta_1 a_t - \frac{\delta_1^2}{\delta_1 + \beta_1} a_t^e.$$

We note that $\delta_1 a_t = \delta_1 a_t^e + \delta_1 \varepsilon_{at}$ and also that $\pi_t^e = \hat{\pi}$ as per the assumption made in the main text. Thus making this substitution into the labour demand equation previously written down results in the following equation:

$$n_t = \frac{\beta_1\delta_0 + \beta_0\delta_1}{\delta_1 + \beta_1} + \delta_1(\pi_t - \hat{\pi}) - \delta_1 x_t(\pi_{t-1} - \hat{\pi}) + \frac{\delta_1\beta_1}{\delta_1 + \beta_1} a_t^e + \delta_1 \varepsilon_{at}.$$

We can derive the output under wage indexation by using the log form of the production technology: $y_t = \alpha n_t + a_t$. The output is given as follows:

$$y_t = \alpha\frac{\beta_1\delta_0 + \beta_0\delta_1}{\delta_1 + \beta_1} + \alpha\delta_1(\pi_t - \hat{\pi}) - \alpha\delta_1 x_t(\pi_{t-1} - \hat{\pi}) + \alpha\frac{\delta_1\beta_1}{\delta_1 + \beta_1} a_t^e + \alpha\delta_1\varepsilon_{at} + a_t.$$

With the help of equation (A.6), we express the output under wage indexation as a function of equilibrium output prevailing under flexible wages (y_t^*). The resulting expression is as follows:

$$y_t = y_t^* + \alpha\delta_1(\pi_t - \hat{\pi}) - \alpha\delta_1 x_t(\pi_{t-1} - \hat{\pi}) + \frac{\alpha\delta^2}{\delta_1 + \beta_1}\varepsilon_{at}.$$

Let the output gap(g_t) be defined as the deviation of output from the output prevailing under flexible wage wage equilibrium. Further assume the following: $(\pi_t - \hat{\pi}) = \tilde{\pi}_t$, $\alpha\delta_1 = \lambda_2$, and $\lambda_2 x_t = \lambda_{1t}$. The aggregate supply relation is given as follows:

$$g_t = -\lambda_{1t}\tilde{\pi}_{t-1} + \lambda_2\tilde{\pi}_t + u_t,$$

where $u_t = \alpha\delta_1^2/(\delta_1 + \beta_1)\varepsilon_{at}$.

A.2 Optimal monetary policy

We assume that wages are indexed to lagged inflation. We assume that agents in the economy fix their expectations equal to a target inflation which does not necessarily need to be 0. We give the IS and the Phillips Curve as follows:

$$g_t = -\lambda_{1t}\tilde{\pi}_{t-1} + \lambda_2\tilde{\pi}_t + u_t$$
$$g_t = -\phi(i_t - \tilde{\pi}_t^e - r) + v_t.$$

We again assume that in conducting optimal monetary policy, the central bank uses the interest rate and the expected interest rate (i_t and i_t^e) as instruments. Alternative ways of expressing the Phillips and the IS expressions which will be useful for our optimization purposes are given below:

$$\tilde{\pi}_t = -\frac{\phi}{\lambda_2}(i_t - i_t^e) - \frac{\phi}{\lambda_2 - \phi}(i_t^e - r) + \lambda_{3,t}\tilde{\pi}_{t-1} + \frac{1}{\lambda_2}(v_t - u_t) \tag{A.7a}$$

$$\tilde{\pi}_t = \tilde{\pi}_t^e - \frac{\phi}{\lambda_2}(i_t - i_t^e) + \frac{\eta_t}{\lambda_2}\tilde{\pi}_{t-1} + \frac{1}{\lambda_2}(v_t - u_t), \tag{A.7b}$$

where $\lambda_{3,t} = \frac{\phi\bar{\lambda_1}}{\lambda_2(\lambda_2-\phi)} + \frac{\lambda_{1t}}{\lambda_2}$ and $\eta_t = \lambda_{1t} - \lambda_1$. The expected inflation can easily be obtained by taking expectation of the equation (A.7a). The expected inflation is:

$$\tilde{\pi}_t^e = -\frac{\phi}{\lambda_2 - \phi}(i_t^e - r) + \frac{\lambda_1}{\lambda_2 - \phi}\tilde{\pi}_{t-1} \qquad \lambda_1 = E[\lambda_{1t}] \tag{A.8}$$

$$g_t = -\phi(i_t - i_t^e) - \frac{\phi\lambda_2}{\lambda_2 - \phi}(i_t^e - r) + \frac{\phi\lambda_1}{\lambda_2 - \phi}\tilde{\pi}_{t-1} + v_t \tag{A.9a}$$

$$g_t = g_t^e - \phi(i_t - i_t^e) + v_t, \tag{A.9b}$$

The (endogenous) state variable in this model is inflation is $\tilde{\pi}_t$. Thus, we can write the value function, assuming a zero output gap target as follows:

$$V(\tilde{\pi}_{t-1}) = \max_{i_t, i_t^e} E_{t-1}\left[-g_t^2 - \theta\tilde{\pi}_t^2 + \beta V(\tilde{\pi}_t)\right]. \tag{A.10}$$

This is maximized subject to the constraints in (A.7a) and (A.9a) in addition to the expression which must hold under commitment:

$$i_t^e = E_{t-1}[i_t]. \tag{A.11}$$

Since the loss function is quadratic, the value function must be quadratic in the state variable. We therefore conjecture the following expression for the value function:

$$V(\tilde{\pi}_t) = \gamma_0 + 2\gamma_1\tilde{\pi}_t + \gamma_2\tilde{\pi}_t^2. \tag{A.12}$$

where the parameters γ_0, γ_1 and γ_2 are parameters assumed to be functions of the parameters in (A.7a) and (A.9a). Let Λ_{t-1} be the Lagrangian multiplier associated with the commitment constraint (A.11). By the chain rule of differentiation, we can write down the first order conditions as follows:

$$0 = -2g_t\frac{\partial g_t}{\partial i_t} - 2\theta\tilde{\pi}_t\frac{\partial\tilde{\pi}_t}{\partial i_t} + \beta\frac{\partial V(\tilde{\pi}_t)}{\partial\tilde{\pi}_t}\frac{\partial\tilde{\pi}_t}{\partial i_t} - \Lambda_{t-1} \tag{A.13}$$

$$0 = E_{t-1}\left[-2g_t\frac{\partial g_t}{\partial i_t^e} - 2\theta\tilde{\pi}_t\frac{\partial\tilde{\pi}_t}{\partial i_t^e} + \beta\frac{\partial V(\tilde{\pi}_t)}{\partial\tilde{\pi}_t}\frac{\partial\tilde{\pi}_t}{\partial i_t^e} + \Lambda_{t-1}\right]. \tag{A.14}$$

The expectation sign appears in the second of the first order conditions because the policy maker does not directly control i_t^e, but rather influences it through policy instrument i_t. From (A.7a) and (A.9a), we derive the following:

$$\partial g_t/\partial i_t = -\phi$$
$$\partial \tilde{\pi}_t/\partial i_t = -\phi/\lambda_2.$$

The conjectured value function in (A.12) implies that the derivative of the value function with respect to inflation is:

$$\partial V(\tilde{\pi}_t)/\partial \tilde{\pi}_t = 2(\gamma_1 + \gamma_2 \tilde{\pi}_t).$$

We obtain the following derivatives of g_t and $\tilde{\pi}_t^e$ with regards to i_t^e:

$$\partial g_t/\partial i_t^e = \phi - \phi\lambda_2/(\lambda_2 - \phi)$$
$$\partial \tilde{\pi}_t/\partial i_t^e = \phi/\lambda_2 - \phi/(\lambda_2 - \phi).$$

Substitute these expressions into the FOCs derived with respect to i_t and i_t^e as given by (A.13) and (A.14) to obtain the following equations.

$$0 = 2\phi[g_t + \tilde{\pi}_t(\theta/\lambda_2) - (\gamma_1 + \gamma_2\tilde{\pi}_t)(\beta/\lambda_2)] - \Lambda_{t-1}$$
$$0 = -2\phi[g_t^e(1 - \lambda_2/(\lambda_2 - \phi)) + \tilde{\pi}_t^e(\theta/\lambda_2 - \theta/(\lambda_2 - \phi)) + (\gamma_1 + \gamma_2\tilde{\pi}_t^e)(\beta/\lambda_2 - \beta/(\lambda_2 - \phi))] + \Lambda_{t-1}.$$

Adding the two equations just listed above derives an intermediate version of the optimal feedback rule. This expression and a version derived by taking expectations are given below:

$$0 = 2\phi[(g_t - g_t^e) + (\tilde{\pi}_t - \tilde{\pi}_t^e)(\theta - \beta\gamma_2)/\lambda_2] + 2\phi[\lambda_2 g_t^e + (\theta - \beta\gamma_2)\tilde{\pi}_t^e - \beta\gamma_1]/(\lambda_2 - \phi) \tag{A.15}$$
$$0 = 2\phi[\lambda_2 g_t^e + (\theta - \beta\gamma_2)\tilde{\pi}_t^e - \beta\gamma_1]/(\lambda_2 - \phi). \tag{A.16}$$

We substitute the expressions (A.7b) and (A.9b) into (A.15) to obtain an expression in terms of the control variables. The derived optimal feedback rule after imposing (A.16) and some simplifications is given below as follows:

$$0 = [-\phi(i_t - i_t^e) + v_t]\left(1 + \frac{\theta - \beta\gamma_2}{\lambda_2^2}\right) - \left(\frac{\theta - \beta\gamma_2}{\lambda_2^2}\right)(u_t - \eta_t\tilde{\pi}_{t-1}). \tag{A.17}$$

The value function needs to concave in the state variable to ensure the existence of a solution to the dynamic optimization problem. It will later be shown that a necessary condition for the value function to be concave in the state variable is the following:

$$\beta(b^2 + \delta^2\sigma_\eta^2) < 1,$$

where b and δ are coefficients (to be later determined) governing the process of inflation under optimal control. The other variables, β and σ_η^2 are the discount rate and the variance of η_t respectively. Given that the necessary conditions for concavity are satisfied, we know that $1 + (\theta - \beta\gamma_2)/\lambda_2^2 \neq 0$. This implies that under optimal control, $[-\phi(i_t - i_t^e) + v_t]$ is a function of u_t and $\eta_t\tilde{\pi}_{t-1}$.[13]

[13]It is assumed that the policy maker observes and reacts to the shocks in an interim period within which private agents can neither observe those shocks nor react to them. The shocks are not observed by both parties ex-ante. See Clarke et al. (1999) for detailed discussion on the implication of this assumption

Thus g_t is a function of u_t and $\eta_t\tilde{\pi}_{t-1}$. This observation coupled with the Phillips curve expression $g_t = -\lambda_{1t}\tilde{\pi}_{t-1} + \lambda_2\tilde{\pi}_t + u_t$, implies that inflation under optimal control assumes the following general form

$$\tilde{\pi}_t = a + b\tilde{\pi}_{t-1} + \delta\eta_t\tilde{\pi}_{t-1} + cu_t, \tag{A.18}$$

where a, b, δ and c are parameters to be determined. Noting that $\eta_t = \lambda_{1t} - \lambda_1$ is a zero mean iid random variable, the expected inflation under this guess can easily be derived to yield as follows:

$$\tilde{\pi}^e = a + b\tilde{\pi}_{t-1}.$$

However, noting that the original specification of the AS (Phillips curve) relation implies $g_t^e = \lambda_1\tilde{\pi}_{t-1} + \lambda_2\tilde{\pi}_t^e$ and substituting this expression into (A.16), we get the following expression for expected inflation:

$$\tilde{\pi}_t^e = \frac{\beta\gamma_1}{\lambda_2^2 + \theta - \beta\gamma_2} + \frac{\lambda_1\lambda_2}{\lambda_2^2 + \theta - \beta\gamma_2}\tilde{\pi}_{t-1}. \tag{A.19}$$

We can identify the parameters a and b in terms of value function parameters and the structural parameters after comparing (A.19) to the expectation of (A.18) as follows.

$$a = \frac{\beta\gamma_1}{\lambda_2^2 + \theta - \beta\gamma_2} \qquad b = \frac{\lambda_1\lambda_2}{\lambda_2^2 + \theta - \beta\gamma_2}. \tag{A.20}$$

From the expression given for b, we can rule out that $\lambda_2^2[1 + (\theta - \beta\gamma_2)/\lambda_2^2] = 0$ as earlier on claimed.[14] This is a necessary condition for a stable inflation under optimal control process since b is an AR coefficient. We substitute the expression for expected inflation into the expression (A.8) to obtain the expected interest rate expression:

$$i_t^e = r + [(\lambda_1 - b(\lambda_2 - \phi))\tilde{\pi}_{t-1} - a(\lambda_2 - \phi)]/\phi. \tag{A.21}$$

The guess we made for equilibrium inflation under (A.18) implies that $\tilde{\pi}_t - \tilde{\pi}_t^e = \delta\eta_t\tilde{\pi}_{t-1} + cu_t$. Substituting this into the expression (A.7b) implies the following expression for the interest rate rule under optimal control.

$$i_t = i_t^e - \frac{1 + c\lambda_2}{\phi}u_t + \frac{1 - \delta\lambda_2}{\phi}\eta_t\tilde{\pi}_{t-1} + \frac{1}{\phi}v_t. \tag{A.22}$$

This expression substituted into (A.17) implies that the parameters δ and c can be identified as follows:

$$\delta = \frac{\lambda_2}{\lambda_2^2 + \theta - \beta\gamma_2} \qquad c = \frac{-\lambda_2}{\lambda_2^2 + \theta - \beta\gamma_2}. \tag{A.23}$$

To proceed further, we note once again that $\tilde{\pi}_t^e = a + b\tilde{\pi}_{t-1}$. One can derive the following expression for the deviation of expected real interest rate from the natural rate of interest as follows:

$$i_t^e - \tilde{\pi}_t^e - r = [(\lambda_1 b - \lambda_2)\tilde{\pi}_{t-1} - a\lambda_2]/\phi. \tag{A.24}$$

The interest rate equation in (A.22) implies the following expression for the deviation of the interest rate from its expected value $i_t - i_t^e = [-(1 + c\lambda_2)u_t + (1 - \delta\lambda_e)\eta_t\tilde{\pi}_{t-1} + v_t]/\phi$. Add $(i_t - i_t^e)$ to both sides of Equation (A.24). Using (A.22), make the necessary substitution at the RHS of the resulting equation, to obtain the following:

$$i_t - \tilde{\pi}_t^e - r = [(\lambda_1 - \lambda_2 b)\tilde{\pi}_{t-1} - a\lambda_2 - (1 + c\lambda_2)u_t + (1 - \delta\lambda_2)\eta_t\tilde{\pi}_{t-1} + v_t]/\phi. \tag{A.25}$$

The last expression implies that the output gap can then be expressed as a function of only the state variables. The output gap given by (6) in the main part of this text can then be rewritten as follows:

$$g_t = -\phi(i_t - \tilde{\pi}_t^e - r) + v_t = a\lambda_2 - (\lambda_1 - \lambda_2 b)\tilde{\pi}_{t-1} + (1 + c\lambda_2)u_t - (1 - \delta\lambda_2)\eta_t\tilde{\pi}_{t-1}. \tag{A.26}$$

[14] The fact that $\lambda_1, \lambda_2 \neq 0$ reinforces this claim.

Deriving parameters of the value function

We have expressed both inflation and the output gap in terms of the state variables. These are contained in equations (A.18) and (A.26) respectively. We now proceed to express the various components of the value function in terms of the state variables. From (A.18) and (A.26), we make the following derivations:

$$\begin{aligned}
E_{t-1}g_t^2 &= a^2\lambda_2^2 - 2a\lambda_2(\lambda_1-\lambda_2 b)\tilde{\pi}_{t-1} + (\lambda_1-\lambda_2 b)^2\tilde{\pi}_{t-1}^2 + (1+c\lambda_2)^2\sigma_u^2 + (1-\delta\lambda_2)^2\sigma_\eta^2\tilde{\pi}_{t-1}^2 \\
E_{t-1}\tilde{\pi}^2 &= a^2 + 2ab\tilde{\pi}_{t-1} + b^2\tilde{\pi}_{t-1}^2 + \delta^2\sigma_\eta^2\tilde{\pi}_{t-1}^2 + c^2\sigma_u^2 \\
E_{t-1}[V(\tilde{\pi}_t)] &= \gamma_0 + 2\gamma_1 a + \gamma_2(a^2+c^2\sigma_u^2) + 2b(\gamma_1 + a\gamma_2)\tilde{\pi}_{t-1} + \gamma_2(b^2+\delta^2\sigma_\eta^2)\tilde{\pi}_{t-1}^2.
\end{aligned}$$

Substitute the three expressions above into (A.10) to obtain the following:

$$\begin{aligned}
V(\tilde{\pi}_{t-1}) = {} & \beta\gamma_0 + 2\beta\gamma_1 a + (\beta\gamma_2-\theta)(a^2+c^2\sigma_u^2) - a^2\lambda_2^2 - (1+c\lambda_2)^2\sigma_u^2 \\
& + 2[\beta b\gamma_1 + (\beta\gamma_2-\theta)ab + a\lambda_2(\lambda_1-b\lambda_2)]\tilde{\pi}_{t-1} \\
& + \left[(\beta\gamma_2-\theta)(b^2+\delta^2\sigma_\eta^2) - [(\lambda_1-b\lambda_2)^2 + (1-\delta\lambda_2)^2\sigma_\eta^2]\right]\tilde{\pi}_{t-1}^2.
\end{aligned}$$

Equating the coefficients to the ones in the expressions $V(\tilde{\pi}_{t-1}) = \gamma_0 + 2\gamma_1\tilde{\pi}_{t-1} + \gamma_2\tilde{\pi}_{t-1}^2$, we obtain the following systems of equations :

$$\gamma_2 = -\left[\frac{\theta(b^2+\delta^2\sigma_\eta^2) + \lambda_1^2(1-\delta\lambda_2)^2 + (1-\delta\lambda_2)^2\sigma_\eta^2}{1-\beta(b^2+\delta^2\sigma_\eta^2)}\right] \tag{A.27}$$

$$\gamma_1 = a\left[\frac{\lambda_2\lambda_1 - (\lambda_2^2+\theta-\beta\gamma_2)b}{1-\beta b}\right] \tag{A.28}$$

$$\gamma_0 = \left[\frac{2\beta\gamma_1 a + (\beta\gamma_2-\theta)(a^2+c^2\sigma_u^2) - a^2\lambda_2^2 - (1+c\lambda_2)^2\sigma_u^2}{1-\beta}\right]. \tag{A.29}$$

Since the loss function, $\mathcal{L} = -g_t^2 - \theta\tilde{\pi}_t^2$, is concave in $\tilde{\pi}_{t-1}$, it holds that the value function must necessarily be concave in that state variable. This implies that $\gamma_2 < 0$, which holds only if $\beta(b^2+\delta^2\sigma_\eta^2) < 1$.

Solving for policy function parameters

The value for b as given by (A.20) implies that the numerator of (A.28) is 0. We can therefore conclude that $\gamma_1 = 0$. This implies the following:

$$a = \frac{\beta\gamma_1}{\lambda_2^2+\theta-\beta\gamma_2} = 0. \tag{A.30}$$

In order to solve for b, we begin by noting that $\delta = b/\lambda_1$ from (A.20) and (A.23). Substituting out the δ in (A.27) and substituting (A.20) into (A.28) gives a quadratic equation for b. In order to perform a step by step derivation of this quadratic equation, we begin by noting that an alternative rendition of (A.27) is the following:

$$\begin{aligned}
\gamma_2 &= -\left[\frac{[\theta b^2 + (\lambda_1-b\lambda_2)^2](1+\sigma_\eta^2/\lambda_1^2)}{1-\beta b^2(1+\sigma_\eta^2/\lambda_1^2)}\right] \\
&= -\left[\frac{[b^2(\lambda_2^2+\theta) - 2\lambda_2\lambda_1 b + \lambda_1^2](1+\sigma_\eta^2/\lambda_1^2)}{1-\beta b^2(1+\sigma_\eta^2/\lambda_1^2)}\right].
\end{aligned}$$

The next step is to derive an expression for $(\lambda_2^2 + \theta - \beta\gamma_2)$ and note (A.20) implies that $(\lambda_2^2 + \theta - \beta\gamma_2) = (\lambda_1\lambda_2)/b$. The derivations corresponding to this step are given below:

$$
\begin{aligned}
-\beta\gamma_2 &= \left[\frac{[\beta b^2(\lambda_2^2+\theta) - 2\beta\lambda_2\lambda_1 b + \beta\lambda_1^2](1+\sigma_\eta^2/\lambda_1^2)}{1-\beta b^2(1+\sigma_\eta^2/\lambda_1^2)}\right] \\
\lambda_2^2 + \theta - \beta\gamma_2 &= \left[\frac{[\beta b^2(\lambda_2^2+\theta) - 2\beta\lambda_2\lambda_1 b + \beta\lambda_1^2](1+\sigma_\eta^2/\lambda_1^2)}{1-\beta b^2(1+\sigma_\eta^2/\lambda_1^2)}\right] + \lambda_2^2 + \theta \\
\frac{\lambda_1\lambda_2}{b} &= \left[\frac{(\lambda_2^2+\theta) + \beta(1+\sigma_\eta^2/\lambda_1^2)\lambda_1^2 - 2\beta\lambda_1\lambda_2(1+\sigma_\eta^2/\lambda_1^2)b}{1-\beta b^2(1+\sigma_\eta^2/\lambda_1^2)}\right].
\end{aligned}
$$

The final of the previous expressions can be rearranged to obtain the following equation which is quadratic in b:

$$0 = [\beta(\lambda_1\lambda_2)(1+\sigma_\eta^2/\lambda_1^2)]b^2 - [(\lambda_2^2+\theta) + \beta(1+\sigma_\eta^2/\lambda_1^2)\lambda_1^2]b + (\lambda_1\lambda_2).$$

This equation has two roots on whose values the stability of the system of equations depends. The root that satisfies the condition $\beta(b^2 + \delta^2\sigma_\eta^2) \leq 1$ is

$$b = \frac{[(\lambda_2^2+\theta) + \beta(1+\sigma_\eta^2/\lambda_1^2)\lambda_1^2] - \sqrt{[(\lambda_2^2+\theta) + \beta(1+\sigma_\eta^2/\lambda_1^2)\lambda_1^2]^2 - 4\beta(\lambda_1\lambda_2)^2(1+\sigma_\eta^2/\lambda_1^2)}}{2\beta\lambda_1\lambda_2(1+\sigma_\eta^2/\lambda_1^2)}. \tag{A.31}$$

In what follows, we show that $0 \leq b \leq \bar{x}$. It is clear from (A.31) that $b \geq 0$ since $[(\lambda_2^2+\theta) + \beta(1+\sigma_\eta^2/\lambda_1^2)\lambda_1^2] > \sqrt{[(\lambda_2^2+\theta) + \beta(1+\sigma_\eta^2/\lambda_1^2)\lambda_1^2]^2 - 4\beta(\lambda_1\lambda_2)^2(1+\sigma_\eta^2/\lambda_1^2)}$. The derivation of the upper bound on this parameter is given below:

$$
\begin{aligned}
b &= \frac{[(\lambda_2^2+\theta) + \beta(1+\sigma_\eta^2/\lambda_1^2)\lambda_1^2] - \sqrt{[(\lambda_2^2+\theta) + \beta(1+\sigma_\eta^2/\lambda_1^2)\lambda_1^2]^2 - 4\beta(\lambda_1\lambda_2)^2(1+\sigma_\eta^2/\lambda_1^2)}}{2\beta\lambda_1\lambda_2(1+\sigma_\eta^2/\lambda_1^2)} \\
&= \frac{[(\lambda_2^2+\theta) + \beta(1+\sigma_\eta^2/\lambda_1^2)\lambda_1^2] - \sqrt{[(\lambda_2^2+\theta) - \beta(1+\sigma_\eta^2/\lambda_1^2)\lambda_1^2]^2 + 4\beta\theta(\lambda_1^2)(1+\sigma_\eta^2/\lambda_1^2)}}{2\beta\lambda_1\lambda_2(1+\sigma_\eta^2/\lambda_1^2)} \\
&\leq \frac{[(\lambda_2^2+\theta) + \beta(1+\sigma_\eta^2/\lambda_1^2)\lambda_1^2] - \sqrt{[(\lambda_2^2+\theta) - \beta(1+\sigma_\eta^2/\lambda_1^2)\lambda_1^2]^2}}{2\beta\lambda_1\lambda_2(1+\sigma_\eta^2/\lambda_1^2)} \\
&= \frac{\lambda_1}{\lambda_2} \\
&= \bar{x}.
\end{aligned}
$$

From (A.23) it is obvious that $c = -\delta$. We are therefore able to solve for the remaining policy function parameters as follows:

$$\delta = \frac{b}{\lambda_1} \tag{A.32}$$

$$c = -\frac{b}{\lambda_1}. \tag{A.33}$$

B Solution to a linear system with rational expectations

Consider the linear system given below as given in Section 3 of the main text:

$$X_t = \mathbf{F_t} X_{t-1} + \mathbf{G} E_{t-1} X_t + \mathbf{H}\epsilon_t. \tag{B.1}$$

We guess the solution is of the form $X_t = \mathbf{P_t} X_{t-1} + \mathbf{Q}\epsilon_t$ which implies that $\mathbf{G}E_{t-1}X_t = \mathbf{GP}X_{t-1}$ where $E_{t-1}\mathbf{P_t} = \mathbf{P}$. A substitution of this guess into B.1 allows us to solve the system by the method of undetermined coefficients. This is illustrated in a step by step manner below:

$$\begin{aligned}\mathbf{P_t}X_{t-1} + \mathbf{Q} &= \mathbf{F_t}X_{t-1} + \mathbf{GP}X_{t-1} + \mathbf{H}\epsilon_t \\ &= (\mathbf{F_t} + \mathbf{GP})X_{t-1} + \mathbf{H}\epsilon_t.\end{aligned}$$

By the comparing coefficients, we know that that the following should hold true:

$$\mathbf{Q} = \mathbf{H} \tag{B.2}$$

$$\mathbf{P_t} = \mathbf{F_t} + \mathbf{GP}. \tag{B.3}$$

Let $E_{t-1}\mathbf{F_t} = \mathbf{F}$. Taking expectation of the second equation, we obtain the following:

$$\begin{aligned}\mathbf{P} &= \mathbf{F} + \mathbf{GP} \\ (\mathbf{I} - \mathbf{G})\mathbf{P} &= \mathbf{F} \\ \mathbf{P} &= (\mathbf{I} - \mathbf{G})^{-1}\mathbf{F}.\end{aligned}$$

Substitute the last expression into B.3 to obtain the following expression for $\mathbf{P_t}$:

$$\mathbf{P_t} = \mathbf{F_t} + \mathbf{G}(\mathbf{I} - \mathbf{G})^{-1}\mathbf{F}. \tag{B.4}$$

C Productivity parameters

In this section, we derive alternative values for the parameters regarding productivity. We do this by first computing the Solow residual for 3 countries, namely, France, Germany and the UK. We then estimate the AR coefficient of productivity and the standard deviation of the productivity shocks.

We obtained the real income growth and growth in labour hours data from the OECD data base. Data on capital stock was obtained from the Federal Reserve Economic Data on FRED St.Louis website. We now proceed to discuss our estimations in detail. Consider the following Cobb-Douglass function

$$Y_t = Z_t N_t^{\alpha} K_t^{1-\alpha}. \tag{C.1}$$

where Z_t is productivity, N_t is capital and K_t is labour supplied. To allow for growth in the long-run, we assume that productivity has two components: one that follows a deterministic trend and the other which is stationary. In order words,

$$Z_t = A_t^{\tau} A_t \qquad\qquad A_t^{\tau} = A_0 \exp^{\upsilon t}. \tag{C.2}$$

Let log values of the variables be represented by small case versions of the relevant letters. Take the natural log of (C.1) to obtain the following:

$$y_t = a_0 + \upsilon t + \alpha n_t + (1-\alpha)k_t + a_t. \tag{C.3}$$

We proceed by first noting that a differenced version of (C.3) gives the growth version of (C.1). The difference version of (C.3) is

$$\Delta y_t = \upsilon + \alpha \Delta n_t + (1-\alpha)\Delta k_t + \Delta a_t. \tag{C.4}$$

The equation (C.4) can be easily estimated by a constrained OLS if one assumes Δa_t is the error term. This error term should be stationary, albeit, possibly serially correlated.

The next procedure is to obtain the estimated residual $\hat{\epsilon}_t = \Delta \hat{a}_t$ from the first estimation. Now, assume the following AR(1) structure for a_t:

$$a_t = \rho_a a_{t-1} + \varepsilon_{at}, \tag{C.5}$$

where $\varepsilon_{at} \sim \mathcal{N}(0, \sigma_a^2)$. It follows that both the AR coefficient ρ_a and the variance of the productivity shock σ_a^2 can be estimated using the following state-space specification:

$$\begin{cases} \hat{\epsilon}_t & = a_t - a_{t-1} \\ a_t & = \rho_a a_{t-1} + \varepsilon_{at} \\ \varepsilon_{at} & \sim \mathcal{N}(0, \sigma_a^2). \end{cases} \tag{C.6}$$

We used the version 8 of the EVIEWS statistical package to estimate equation (C.6). Table 5 provides the estimates of (C.4) for the three countries and Table (6) estimates for ρ_a and σ_a^2 for the same countries.

The estimates of α for Germany and the UK are similar to estimates obtained from other literature. That of the UK however is outside the generally accepted range for α. In the main part of this study, we set $\alpha = 0.64$ for the calibration exercise to reflect a notional average of the estimates for α. It can be seen from Table (6) that the country specific estimates for both ρ_a and σ_a^2 do not differ that much. The estimates suggest that productivity shocks are highly persistent, albeit stationary. We will therefore set ρ_a at 0.9 for the calibration. Finally, from the estimates, the country specific standard deviation of productivity shocks σ_a lies between 0.0121 and 0.0151. We will set $\sigma_a = 0.013$ for the calibration.

C.1 Tables

Table 5: Cobb-Douglass (C.4)

Country	$\hat{\upsilon}$	$\hat{\alpha}$	R^2
France	0.005	0.366**	0.50
	(0.004)	(0.098)	
Germany	0.013**	0.675**	0.57
	(0.003)	(0.107)	
UK	0.015**	0.64**	0.50
	(0.004)	(0.124)	

[1] Standard errors in parenthesis
[2] * $p > 0.05$, **$p > 0.01$

Table 6: Productivity (C.6)

Country	$\hat{\rho}_a$	$\ln(\hat{\sigma}_a^2)$	AIC
France	0.922**	-8.818**	-5.86
	(0.12)	(0.194)	
Germany	0.978**	-8.713**	-5.77
	(0.099)	(0.134)	
Uk	0.945**	-8.379**	-5.42
	(0.129)	(0.221)	

[1] Standard errors in parenthesis
[2] * $p > 0.05$, **$p > 0.01$

D Figures

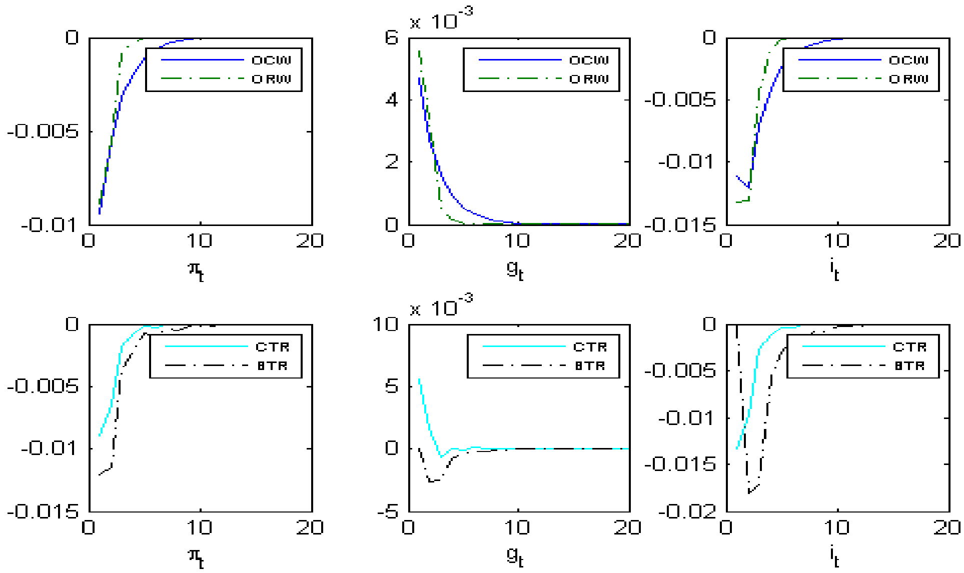

Figure 3: Impulse response to productivity shocks

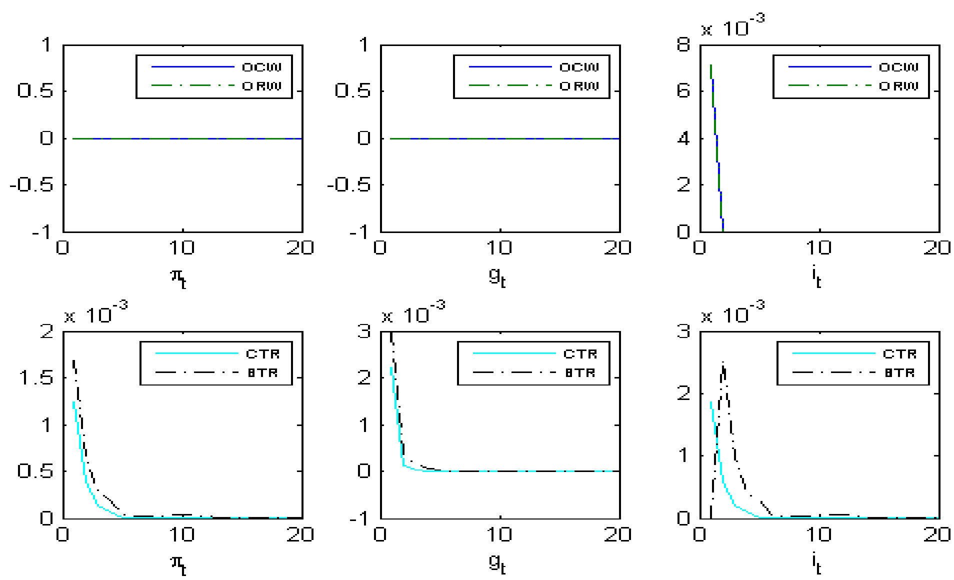

Figure 4: Impulse response to demand shocks