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A Comment on Revealed Preference with a Subset of Goods

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Abstract

Varian (1988) introduced an important proposition regarding restrictions on consumption data if observations of the quantities of a good are missing. In this paper, a simple counterexample is presented to show that the original proof is incorrect, and a new proof is provided. The new proof is not based on choosing the missing quantities such that some bundles are revealed preferred to others, but rather on choosing the unknown quantities such that bundles are not revealed preferred to any bundle with a higher index.

JEL Classification: C14, D11

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1 Introduction

Since the work by Afriat (1967) and Varian (1982), the revealed preferences or nonparametric approach has proven to be extremely fruitful. The approach has been used in such different contexts as production (Hanoch & Rothschild, 1972; Varian, 1984), life-cycle rational expectations models (Browning, 1989), characteristics models (Blow, Browning & Crawford, 2007), consumption habits (Crawford, 2010), bandwidth pricing and Google advertisement auctions (Varian, 2012) and Cournot competition in the oil market (Carvajal et al., 2013).

Two fundamental results in the field are due to Varian (1988). The first result concerns missing prices for known quantities; the second theorem concerns cases where prices are known but the quantities of some good are unknown. This paper is concerned with the latter (the theorem is copied verbatim in Section 2). Informally, the seminal result states that it is always possible to find values for the missing quantities such that the data can be rationalised. Thus, without additional assumptions, the method of revealed preferences loses all its power if the quantities of at least one good are missing. Section 2 contains the original proof as well as a simple counterexample against it. In Section 3 a new proof is provided.

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First, some definitions are in order. Let there be T observations $(\boldsymbol{p}_t, \boldsymbol{x}_t)$, $t = 1, ..., T$, where $\boldsymbol{x}_t$ is a k-dimensional non-negative vector of consumer goods and $\boldsymbol{p}_t$ is a k-dimensional vector of corresponding non-negative prices. Under the hypothesis of utility maximisation, given some non-negative level of expenditure e_t, $t = 1, ..., T$,

$$\boldsymbol{x}_t = \arg\max_{\boldsymbol{x}} \{u(\boldsymbol{x}) : \boldsymbol{p}_t \cdot \boldsymbol{x} \leq e_t\} \tag{1}$$

Definition. A utility function $u(\boldsymbol{x})$ rationalises a set of observations $(\boldsymbol{p}_t, \boldsymbol{x}_t)$, $t = 1, ..., T$, if $u(\boldsymbol{x}_t) \geq u(\boldsymbol{x})$ for all $\boldsymbol{x}$ such that $\boldsymbol{p}_t \cdot \boldsymbol{x}_t \geq \boldsymbol{p}_t \cdot \boldsymbol{x}$.

This definition of rationalisability follows directly from (1) for a locally non-satiated utility function.

Definition. A bundle $(\boldsymbol{x}_t)$ is revealed preferred to $(\boldsymbol{x})$, written $(\boldsymbol{x}_t)R(\boldsymbol{x})$, if there exists some sequence $\boldsymbol{p}_t \cdot \boldsymbol{x}_t \geq \boldsymbol{p}_t \cdot \boldsymbol{x}_i, \boldsymbol{p}_i \cdot \boldsymbol{x}_i \geq \boldsymbol{p}_i \cdot \boldsymbol{x}_w, ..., \boldsymbol{p}_j \cdot \boldsymbol{x}_j \geq \boldsymbol{p}_j \cdot \boldsymbol{x}$. A bundle $(\boldsymbol{x}_t)$ is strictly directly revealed preferred to a bundle $(\boldsymbol{x})$, written $(\boldsymbol{x}_t)P^0(\boldsymbol{x})$, if $\boldsymbol{p}_t \cdot \boldsymbol{x}_t > \boldsymbol{p}_t \cdot \boldsymbol{x}$.

Definition. A set of observations $(\boldsymbol{p}_t, \boldsymbol{x}_t)_{t=1,...,T}$ satisfies the Generalised Axiom of Revealed Preferences (GARP) if $(\boldsymbol{x}_t)R(\boldsymbol{x}_m)$ implies not $(\boldsymbol{x}_m)P^0(\boldsymbol{x}_t)$.

Afriat's Theorem (see Afriat (1967) and Varian (1982), see also Diewert (1973) for comments on required properties of continuity and non-satiation) states that if a set of observations $(\boldsymbol{p}_t, \boldsymbol{x}_t)_{t=1,...,T}$ satisfies GARP, there exists a continuous, non-satiated utility function that rationalises the data. Varian (1988) considers a case where the quantities of one good, denoted for clarity by the non-negative scalar z_t, are unknown, but its corresponding prices denoted q_t are observed. The question is whether quantities (z_t), $t = 1, ..., T$ can be found such that the data $(\boldsymbol{p}_t, q_t, \boldsymbol{x}_t, z_t)$, $t = 1, ..., T$ are consistent with utility maximisation, that is, by Afriat's Theorem, such that the data $(\boldsymbol{p}_t, q_t, \boldsymbol{x}_t, z_t)$, $t = 1, ..., T$ satisfy GARP.

2 Varian's Theorem

The original theorem and proof of Varian (1988) are stated below, copied verbatim barring some insignificant notational changes.

THEOREM 2, Varian (1988, p. 182):

Let $(\boldsymbol{p}_t, \boldsymbol{x}_t)$, $t = 1, .., T$, be a set of data and let (q_t), $t = 1, ..., T$ be a set of positive prices. Then there always exists a set of quantities (z_t), $t = 1, .., T$ such that the data $(\boldsymbol{p}_t, q_t, \boldsymbol{x}_t, z_t)$ satisfy GARP.

Proof of Theorem 2 in Varian (1988, p. 182): Choose $z_1 = 0$ and successively define

$$z_{t+1} > \max\left\{\frac{\boldsymbol{p}_{t+1}\cdot\boldsymbol{x}_t + q_{t+1}z_t - \boldsymbol{p}_{t+1}\cdot\boldsymbol{x}_{t+1}}{q_{t+1}}, 1\right\} \quad \text{for } t = 1, .., T-1 \tag{2}$$

Then, for all $t = 1, ..., T-1$ we have

$$\boldsymbol{p}_{t+1}\cdot\boldsymbol{x}_{t+1} + q_{t+1}z_{t+1} > \boldsymbol{p}_{t+1}\cdot\boldsymbol{x}_t + q_{t+1}z_t$$

so that each observation $t+1$ is revealed preferred to observation t. Thus the data must satisfy GARP. □

The problem is in the last line of the proof: that each observation $t+1$ is revealed preferred to observation t does not imply the data must satisfy GARP. The easiest way to verify this is through a counterexample. All we need to show is that i) we can find z_t, $t = 1, ..., T$ that satisfies (2) and ii) violates GARP.

In a two-dimensional case with $T = 3$, (2) is simply:

$$z_2 > \max\left\{\frac{p_2x_1 + q_2z_1 - p_2x_2}{q_2}, 1\right\}$$

$$z_3 > \max\left\{\frac{p_3x_2 + q_3z_2 - p_3x_3}{q_3}, 1\right\}$$

Through this construction,

$$p_2x_2 + q_2z_2 > p_2x_1 + q_2z_1$$

$$p_3x_3 + q_3z_3 > p_3x_2 + q_3z_2$$

That is, $(x_2, z_2)P^0(x_1, z_1)$ and $(x_3, z_3)P^0(x_2, z_2)$. If, for example, also $(x_2, z_2)P^0(x_3, z_3)$, GARP is violated. Indeed, we can find numbers that lead to exactly such a violation; an example is provided in Table 1.

Using the numbers from Table 1, z_2 and z_3 satisfy (2):

$$4 = z_2 > \max\left\{\frac{2\times 16 + 1\times 0 - 2\times 15}{1}, 1\right\} = 2$$

$$10 = z_3 > \max\left\{\frac{1\times 15 + 1\times 4 - 1\times 10}{1}, 1\right\} = 9$$

which means $(x_3, z_3)P^0(x_2, z_2)$. However, for the numbers provided it also holds that $(x_2, z_2)P^0(x_3, z_3)$:

$t:$	1	2	3
p_t	1	2	1
q_t	1	1	1
x_t	16	15	10
z_t	0	4	10

Table 1: Numerical counterexample

$$34 = 2 \times 15 + 1 \times 4 > 2 \times 10 + 1 \times 10 = 30$$

Thus there is a direct violation of GARP. This example shows that data constructed using (2) can violate GARP, and hence that the proof is incorrect.

3 A new proof of Varian's Theorem

All violations in GARP involve two different preference relations between the same two bundles. In order to ensure that the data do not violate GARP, we need to modify construction (2) so that at least one direction of the preference relation between any two bundles is broken, for example by ensuring lower index bundles are never directly revealed preferred to higher index bundles. Through such a construction no bundle with a lower index can be either directly or indirectly revealed preferred to a bundle with a higher index. This is the approach used in the proof of Theorem 2 proposed below.

Proof of Theorem 2: Choose $z_1 = 1$ and successively define

$$z_t > \max_{j<t} \left\{ \frac{\boldsymbol{p}_j \cdot \boldsymbol{x}_j + q_j z_j - \boldsymbol{p}_j \cdot \boldsymbol{x}_t}{q_j} \right\} \quad \text{for } j = 1, ..., T-1,\ t = 2, .., T \tag{3}$$

Then, for all $j = 1, ..., T-1,\ t = 2, .., T,\ j < t$ we have

$$\boldsymbol{p}_j \cdot \boldsymbol{x}_j + q_j z_j < \boldsymbol{p}_j \cdot \boldsymbol{x}_t + q_j z_t$$

so that for all $j < t$, no bundle (x_j, z_j) is revealed preferred to any bundle (x_t, z_t). Thus the data must satisfy GARP. □

This proof is different in two ways: First, instead of ensuring that some bundles are directly revealed preferred to others, we choose z_t such that bundles are not revealed preferred to certain other bundles. Secondly, instead of doing this only for every pair of successive bundles, we choose z_t such that all bundles with a lower index are not revealed preferred to any bundle t.

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