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Unconditional Aid and Green Growth

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Abstract

Environmentally motivated aid can help developing countries to achieve economic growth while mitigating the impact on emission levels. We argue that the usual practice of giving aid conditionally is not effective, and we therefore study aid that is given unconditionally. Our framework is a differential open-loop Stackelberg game between a developed country (leader) and a developing country (follower). The leader chooses the amount of mitigation aid given to the follower, which the follower either consumes or invests in costly nonpolluting capital or cheap high-emission capital. The leader gives unconditional mitigation aid only when sufficiently rich or caring sufficiently about the environmental quality, while the follower cares about environmental quality to some extent. If aid is given in steady state, it decreases the steady state level of high-emission capital and capital investments in the recipient country and the global pollution stock, but it has no effect on the levels of non-polluting capital and non-polluting investments. It accelerates the economic growth of the follower; this effect is however lower than what static growth theory predicts since most of the aid is consumed. Moreover, we find that the increase in growth takes place in the nonpolluting sector.

Keywords: Development aid, Green growth, Conditionality, Open loop Stackelberg equilibrium

JEL codes: Q56, O13

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1 Introduction and literature review

Through the 2015 Paris Climate Agreement all countries acknowledge the negative impact of climate change to each country regardless of its development level. Countries at a low level of development face the challenge to accomplish economic growth while preserving environmental resources at the same time. Growth is usually accompanied with high levels of pollution, especially in the early stages of economic development; as climate change is a global rather than national problem, it is therefore in the interest of all countries to direct the growth path of poor countries towards 'green' rather than 'brown' growth, that is, towards building nonpolluting instead of high-emission industrial capital.

The first best solution to solve the climate change problem proposed in the literature is to implement a unique carbon tax among all countries. In practice most developing and least developed countries have weak and underdeveloped institutions, making the enforcement of an efficient climate policy difficult (Dixit et al. 2012). Another solution is for developed countries to voluntarily donate environmentally motivated aid: this mechanism is envisaged by the Paris Agreement, under which each country specifies a 'Nationally Determined Contribution'. The agreement has been criticised precisely because these contributions are voluntary and there is no enforcing mechanism in place.

While we think that the agreement may possibly run into a common pool problem, we disagree with the statement that an agreement based on voluntary contributions cannot possibly help to mitigate the emissions problem. The present paper shows that a fully developed country can have an environmentally motivated incentive to provide a developing country with mitigation aid, making both countries better off.

Generally, aid can be donated either unconditionally or conditional on the recipient country investing in certain kinds of nonpolluting capital. It has been argued that aid conditionality may not work optimally because of institutional failures (Adam and O'Connell 1999). Using a noncooperative infinite horizon framework, in this article we therefore investigate situations in which it a donor may give unconditional aid, and we determine the optimal amount of aid to be granted. We also analyse the effects of this aid on the growth of the recipient country and the direction of the resulting growth. We find that in some configurations aid is given, either over a finite period of time or indefinitely. In our model, most of the aid is used to increase consumption, which relieves the recipient country from the need to invest deeply in a high-emissions 'brown' industry, and allows it to build up a non-polluting 'green' industry instead.

Analysis of the steady state shows that unconditional aid decreases the steady state levels of the brown capital in the recipient country, while it has no effect on the steady state levels of the green capital: effectively, it substitutes output from the fully developed country, which is assumed to be produced by fully green capital, for the output of brown capital of the developing country. Moreover, giving unconditional aid decreases the stock of global pollution. Our model shows that the fully developed country gives mitigation aid only if the sensitivity of the developing country on pollution damage is neither too low nor too high: if the weight is too low, aid will not change the behaviour of the recipient; if the weight is sufficiently high, the recipient will invest in its green capital stock by its own accord. We conclude that unconditional aid may Pareto-improve the situation of the two countries.

Our study is related to several strands of the literature: aid motivations, aid-growth, conditionality problems, climate finance, green investments, and dynamic games. We find it useful to give brief review of these strands in order to justify the framework we use and to highlight our contribution.

1.1 Aid motivations

Donors might be motivated to give aid by several incentives, such like: ethical international equity concerns, historical relations, political and strategic reasons, or poverty alleviation and growth promotion in the recipient country (Rajan and Subramanian 2008, Alesina and Dollar 2000). Sometimes the need to secure a global agreement might include transfers between countries. Other motives include strategic environmental concerns, donors caring about global environmental quality. The 2015 Paris Climate Agreement represents an example of these motives, where environmentally motivated transfers were an essential aspect to secure the agreement.

The literature distinguishes between two kinds of environmentally motivated donations: mitigation transfers and adaptation transfers (Eyckmans et al. 2013). Mitigation transfers aim at reducing emissions in the recipient country; therefore they can be considered as a public good benefiting all countries. Payoffs from these reductions are realised immediately. Adaptation transfers aim at boosting climate resilience in the recipient country. They can be considered as a private good that benefits only the implementing country, and the associated payoffs are realised in the future.[1]

1.2 Aid-growth literature

The literature on conditional aid focuses mainly on identifying the effectiveness of foreign aid on the economic growth of recipient countries. Hansen and Tarp (2000) mention two basic theoretical models in this literature: the Harrod and Domar growth model with a stable linear relationship between growth and investment in physical capital, and the two gaps model of Chenery and Strout (1966).

[1] For other kinds of donor's incentives we refer to Lahiri and Raimondos-Moller (2000).

Adam and O'Connell (1999) address the 'institutional failures' problem by examining the effect of foreign aid, focusing on the role and limitations of conditionality when the government may not work in the public interest. Boone (1996) stylises the importance of political regime in the recipient country for aid effectiveness.

No solid evidence about the effect of development aid on growth has been provided in empirical work. Many studies like Boone (1996) have found that there is no effect, or even if it does, it is lower than what the Harrod and Domar model predicts. Mosley (1986) highlights the fact that on the micro level there seems to be a positive effect of aid, while on the macro level it is hard to determine any systematic effects of aid on growth (macro-micro paradox). Hansen and Tarp (2000) propose a classification of the empirical cross-country work on aid effectiveness, concluding that the existing literature supports the proposition that aid improves economic performance, and that there is no macro-micro paradox to resolve. Rajan and Subramanian (2008) find no systematic effect of aid on growth regardless of the estimation approach, the time horizon, or the types or sources of aid. Doucouliagos and Paldam (2008) conducted a meta-analysis of the effectiveness of development aid on growth; they found no significant positive effects. Mekasha and Tarp (2013) re-examined key hypotheses of Doucouliagos and Paldam and concluded that the effect of aid on growth is positive and statistically significant, and that there is no evidence to suggest presence of publication bias. Rajan and Subramanian (2007) suggested that any beneficial effects of additional capital on growth might be offset by adverse spillovers effects; for example, aid in a foreign currency leads to the appreciation in the exchange rate which affect exports adversely — Dutch disease — which might explain the ineffectiveness of aid on economic growth.

1.3 Conditionality problems

When one country grants aid to another country for a specific purpose, moral hazard is an issue, as the actions of — typically — the recipient country may deviate from what is initially agreed on after the aid payment has taken place. Conditionality is the typical mechanism to deal with moral hazard situations between recipient and donor countries (Svensson 2000). Using conditionality, donors try to influence policy and to induce reforms in recipient countries; they also try to make sure that the promised aid flow will be used effectively, according to the donor's criteria (Azam and Laffont 2003).

Easterly (2003) mentions that aid agencies and intermediaries usually impose conditions on loans before they are granted, and evaluate their effect after they are completed. However, in practice minimal effort is made to ensure that aid conditions are fulfilled, or that the subsequent evaluation of aid effectiveness is conducted. He emphasises a fundamental problem related to conditionality: both success and failure of the recipient to satisfy conditions are used to justify giving more aid. He notes that one of the reasons to keep giving aid, even after conditions are not met, is when a new government takes over power: this government is usually given a clean record from aid agencies. Another issue, highlighted by Mosley (1996), is that aid agencies and multilateral aid institutions suffer from an agency problem, due to the internal delegation process. He argues that these institutions might be led to give more aid than the minimum amount needed to get a specific outcome, or, equivalently, to ask for less effort for a given level of aid, as large disbursements would enhance the career prospects of the officer in charge. An additional issue, studied by Svensson (2003), is budget pressure: in most donor organisations allocation and disbursement decisions are separated, which results in disbursing committed funds to a fixed, already designed, recipient. This, in turn, results in not shifting resources towards countries where they can be utilised most effectively. Mosley et al. (1995) highlight a problem that is faced by some World Bank country loans officers: when the enforcement of conditionality might be in conflict with some other goals of the bank, one way of proceeding is to disburse an urgent payment in order to avoid a potential default on outstanding loans; this is similar to what happened in the Greek crisis of 2015. Both Svensson (2003), and Mosley et al. (1995) argue that the current working system is biased towards disbursing aid regardless of the reform effort. Imposing general conditions on every recipient country irrespective of its specific economic and social characteristics tends moreover not to work. Finally, intermediate aid organisations sometimes push aid, or give loans, to countries where these resources are not effectively used, in order not to have an unallocated balance which could be used as a reason to lower future budgets from donor countries (Easterly 2003). In this way 'spending the budget' becomes a goal by itself (Paldam 1997, Edgren 1996).

As a result of these investigations, the credibility of aid conditionality can be questioned, and we conclude that conditionality to some extent fails to achieve its purpose in practice. This may serve as an explanation of the empirical evidence of the ineffectiveness of aid on economic growth on the macro level.

1.4 Climate finance literature

Under the terms of the 2009 Copenhagen Accord, which were later re-emphasised by the 2015 Paris Climate Agreement, developed countries engaged in providing climate finance up to $100 billion per year, starting from 2020 onwards, to help developing countries reduce their emissions and adapt to the consequences of climate change.

A literature on climate finance that relates to our study has been emerging. Eyckmans et al. (2013), in a two period Stackelberg game, use a framework in which a donor cares about the well-being of the recipient, while using a binding global emissions constraint to address the climate change externality. They study the interaction between climate finance and development aid by comparing three types of transfers from a donating developed country towards a receiving developing country: development, mitigation, and adaptation aid. They found that a large part of the intended effect of transfers dissipates as the follower reallocates its own resources to achieve the balance it prefers. Pittel and Rübbelke (2013) develop a two regions model to study the difference between transfers that subsidise mitigation efforts and financial adaptation transfers that are conditional on mitigation efforts. They conclude that the outcome depends strongly on the productivity of mitigation and adaptation technologies. Heuson et al. (2012) introduce a static two region model of mitigation and adaptation with different types of transfers from the developed region, and conclude that there are many instruments of climate funding that could yield Pareto improvements for donor and recipient countries. Therefore, transfers might induce an implicit cooperation between regions.[2]

1.5 Green investments literature

According to the 14 July 2009 Report by the United Nations Environment Programme (UNEP), green investments are not a luxury anymore; instead, they are a social and legal responsibility. The report argues that if investment consultants and other parties do not include environmental, social and governance (ESG) aspects into their services, they face "a very real risk that they will be sued for negligence". Eyraud et al. (2011) provide a definition of green investments from a macroeconomic perspective: "The investment necessary to reduce greenhouse gas and air pollutant emissions, without significantly reducing the production and consumption of non-energy goods". Rozenberg et al. (2014) compare two policies for the optimal switch towards green capital: having a climate tax or subsidising green investments in one economy. They found that the climate tax is optimal, but if the environmental conditions are not at a critical level, subsidising green capital is a good long term policy. Claude et al. (2012) use a dynamic model with two jurisdictions to discover the properties of price-based policies to control environmental externalities, introducing temporary heterogeneity between jurisdictions in the initial stocks of infrastructure which diminish over time. They conclude that the optimal policy scheme may require to simultaneously tax one jurisdiction and subsidise the other for a period of time. The policy chosen in each jurisdiction depends on the degree to which stocks are complements or substitutes.

1.6 Dynamic games

There is a body of literature that uses dynamic games to study international transfers for different environmental motivations. Our study contributes to this literature. Van Soest and Lensink (2000) studied a differential game where aid contracts are introduced to preserve a forest stock in the recipient country. They conclude that conditioning aid only on the forest stock increases conservation in the long run but only slightly on the short run. A more active short run policy would be to condition transfers negatively on current deforestation. Martín-Herrán et al. (2006) use a Stackelberg differential game framework with financial

[2] Bowen et al. (2012) discuss development, climate vulnerability, and adaptation.

transfers to help developing countries to preserve their rainforests. They found that, under certain conditions, both the long and the short run size of the rainforest can be increased by adopting a feedback information type for both players. Fredj et al. (2004) study a differential Stackelberg game between two countries in order to design an aid program that aims at conserving rainforests located in the recipient country. They consider a transfer function which takes into account both the forest size and the deforestation rate; they conclude that making the transfer function dependent on the deforestation rate in addition to its dependency on the forest stock would induce slower deforestation. Cabo (2002) used a differential infinite horizon game to analyse the feasibility and optimality of sustainable economic growth in a North–South trade model. He studies the effect of this growth upon the dynamics of the natural resources stocks, where capital transfers of both physical and human capital from North to South are possible. He distinguishes between two scenarios depending on the effect of capital transfers upon South. In both cases, South is found to be able to produce the same amount of an intermediate good and to reduce the risk of resource depletion. Finally, Tornell and Velasco (1992) studied a differential game between parties who have access to a common technology. They found that under some configurations introducing a less productive private technology ameliorates the tragedy of the commons and improves welfare, as the inferior technology creates a lower bound on the rate of return of the common access asset, and therefore, a ceiling on the appropriation rate. Using this mechanism, the authors explained the problem of capital flight from poor towards rich countries as a consequence of the poorly defined property rights in the developing countries, where investing abroad represents a recourse to the tragedy of commons.

1.7 Our contribution

In our model, a possible donor is a developed sovereign country for which greenhouse gas emissions by another sovereign country constitute an externality, and which gives aid in order to induce the developing country to reduce these emissions. We therefore investigate aid given in a noncooperative differential game framework where the donor, North, is a Stackelberg leader and the recipient, South, a Stackelberg follower. We have argued that the practical effectiveness of conditionality is questionable. The leader, motivated by environmental considerations, therefore announces an aid programme, where aid is given unconditionally and independently of the actual actions of the recipient follower. This gives the recipient country the choice to use the aid in a way that achieves its best interests. If under these conditions, there is a positive aid flow towards the follower, it will be a Pareto-improvement. At the same time, it provides a lower bound on the effect aid can have on the growth of the recipient (Azam and Laffont 2003).

We consider open-loop Stackelberg equilibria. In order to achieve time consistency, we extend the game by introducing a binary state variable, *trust*, which starts at the value 1, and which switches to 0 whenever the leader deviates from the aid schedule announced at the beginning of the game. When *trust* has the value 0, South assumes that no aid will be forthcoming any more and optimises its investment strategy under this assumption. We show that a subset of open-loop Stackelberg equilibria is time consistent under this extension.

Among the papers mentioned above, Eyckmans et al. (2013) is closest to ours. We depart from existing literature by analysing unconditional aid in a theoretical dynamic model, using a noncooperative infinite horizon framework. Unlike Eyckmans et al. (2013), we employ a Ramsey framework to model South's growth, with endogenous capital investment processes for green and brown capital respectively. Damage flows from global pollution, affecting the welfare of both countries, address the pollution externality. We treat unconditional aid as a component of national income of the recipient country, rather than assume a direct relationship between aid and investments as in Rosenstein-Rodan (1961) and in the Harrod and Domar model. From the donor's perspective, in our model unconditional aid is a mitigation transfer. To the recipient, aid acts as development aid, influencing consumption and total investment, as well as mitigation aid, by influencing the relative investments in green and brown capital. We follow Eyckmans et al. (2013) by not considering global welfare: each country takes only its own welfare into account when taking its decisions. Our model also links the green investments literature and climate finance literature; to our knowledge only Claude et al. (2012) study a similar link.

The next section presents the model and the dynamic optimisation problem of each player. Then we give theoretical results about the steady state of the resulting Stackelberg equilibrium dynamics. To analyse the transient dynamics, numerical techniques are necessary. We discuss their methodology before turning to the results and the conclusions.

2 The model

2.1 The aid game

In our framework, all countries care about the consumption and the quality of the environment of their citizens, which translates into an intertemporal tradeoff between short term consumption benefits and long term environmental costs. A country can grow by either investing in costly, non-polluting, 'green' capital or cheap, high-emission, 'brown' capital. We assume that both kinds of capital are equally productive. Investment in brown capital contributes through emissions to degradation of the global environmental quality, which is a public good affecting both developed and developing countries alike. Developed countries trying to avoid future environmental degradation may be motivated to give aid to developing countries, helping these countries to achieve economic growth with minimal effect on the environment. As the adverse effects of climate change are felt over a long time, it is natural to study this problem in an infinite horizon framework.

We study a Stackelberg differential game between two countries: The leader, which will be called ‘North’, is a developed country; the follower, ‘South’, a developing country. North’s decision variable is the amount of aid that it gives to South; this lessens North’s consumption budget. North is assumed to be unable to observe how South uses the aid it receives; aid therefore automatically becomes unconditional. South’s decision is how to allocate its output and the aid it receives from North between consuming, investing in brown capital, or investing in green capital.

We solve for open-loop Stackelberg equilibria: this means that North’s aid schedule is fixed at the initial time, and that the amount of aid given depends merely on the date, but not on any other variable. A closed-loop approach, where the amount of aid would depend on the current state variables, would involve similar problems as discussed above in the context of conditionality: South would need strong institutions to measure and report the capital stocks correctly, and in practice the lowering of aid as a consequence of an adverse stock evolution might easily give rise to political tensions. The open-loop approach avoids this, as the aid schedule is fixed and known beforehand. Of course, for an announced aid schedule to be credible, it needs to be time consistent. This issue will be addressed in Section 5.3.

2.2 South’s decision problem

We begin by describing South’s decision problem, given North’s aid schedule a_t.

2.2.1 Consumption

South’s citizens are assumed to be identical and to be represented by an infinitely lived representative agent, who gains utility $u(c_t)$ from consuming c_t of a generic good and dis-utility $D(E_t)$ from environmental degradation represented by a damage function of a global pollution stock E_t. The discounted intertemporal welfare of the South’s representative consumer can be written as:

$$W = \int_0^\infty e^{-\rho t}(u(c_t) - D(E_t))dt. \tag{1}$$

Here ρ denotes the time preference rate. The utility function u and the damage function D are assumed to be, respectively, increasing and concave and increasing and convex, i.e. $u' \geq 0$ and $u'' \leq 0$; $D' \geq 0$ and $D'' \geq 0$.

2.2.2 Production

Output comes from production processes using either brown capital $K_{b,t}$ or green capital $K_{g,t}$ as factors of production. For our purpose, we focus on physical capital as the only variable input for production and leave the inclusion of labor as a second variable input for future research. We have two production functions F_b and F_g, for brown and green capital respectively. South's total output is:

$$Y_t = F_b(K_{b,t}) + F_g(K_{g,t})$$

The functions F_b and F_g are assumed to be increasing and concave: $F_b' \geq 0$, $F_g' \geq 0$, and $F_g'' \leq 0$, $F_b'' \leq 0$.

South's invests, per unit time, $I_{b,t}$ in brown capital and $I_{g,t}$ in green capital. The investment costs $C_i(I_{i,t})$ are assumed to be increasing and convex: $C_i' \geq 0$ and $C_i'' \geq 0$ for $i \in \{b, g\}$. Both types of investment are assumed to be irreversible: once an investment has been made, the resulting capital cannot be transformed to a different type of capital.

Along with its output from the production process, South may receive aid from North. At each point of time South allocates its output and the aid it receives between consuming, investing in green capital and investing in brown capital, taking into account its budget constraint

$$F_b(K_{b,t}) + F_g(K_{g,t}) + a_t \geq c_t + C_b(I_{b,t}) + C_g(I_{g,t}). \tag{2}$$

Investment costs are assumed to be quadratic:

$$C_i(I_{i,t}) = \frac{\beta_i}{2} I_{i,t}^2, \qquad i \in \{b, g\},$$

where $\beta_i > 0$ is the rate of increase of the marginal investment costs. We assume that brown investments are cheaper than green investments, i.e. $\beta_b \leq \beta_g$. The price of the generic good is normalised to 1.

Capital dynamics are assumed to take the same form for both kinds of capital

$$\dot{K}_{i,t} = I_{i,t} - \delta K_{i,t}, \quad K_{i,0} \text{ given}, \qquad i \in \{b, g\}. \tag{3}$$

Each type of capital increases with new investments and depreciates with a uniform capital depreciation rate δ.

Production processes involving brown capital emit greenhouse gases, which accumulate in the atmosphere. Pollution is therefore transboundary, affecting consumers in both countries. The dynamics of the pollution stock is given as:

$$\dot{E}_t = \alpha K_{b,t} - \vartheta E_t, \qquad E_0 \text{ given}. \tag{4}$$

That is, pollution emissions are proportional to the amount of installed brown capital, with an emission intensity α; without emissions, the pollution stock decreases at the natural decay (absorption) rate ϑ.

2.2.3 South's policy

South maximizes its intertemporal welfare, taking into account capital and pollution dynamics. That is, South maximizes the objective functional (1) subject to the budget constraint (2), the dynamic constraints (3) and (4).

The current value Hamiltonian of the intertemporal maximisation problem is given in Appendix A.1, together with the optimal policies. The necessary conditions for the co-states μ_t, $\nu_{b,t}$, and ν_{g_t} of, respectively, the pollution stock, South's brown capital, and South's green capital read as

$$\dot{\mu}_t = D'(E_t) + (\rho + \vartheta)\mu_t, \tag{5}$$

$$\dot{\nu}_{b,t} = -u'(c_t)F_b'(K_{b,t}) + (\rho + \delta)\nu_{b,t} - \alpha\mu_t, \tag{6}$$

$$\dot{\nu}_{g,t} = -u'(c_t)F_g'(K_{g,t}) + (\rho + \delta)\nu_{g,t}. \tag{7}$$

They are complemented by initial conditions for the states E_t, $K_{b,t}$ and $K_{g,t}$ and, since there are no terminal conditions on the states, by the transversality conditions

$$\lim_{t\to\infty} e^{-\rho t} E_t = 0, \quad \lim_{t\to\infty} e^{-\rho t} K_{i,t} = 0, \qquad i \in \{b, g\}, \tag{8}$$

which hold whenever the state variables are uniformly bounded away from 0.

2.3 North

2.3.1 Consumption

North's citizens are, analogously to South's, represented by an infinitely lived agent who gains utility from consumption and dis-utility from environmental degradation. We use a superscript n to denote North's variables. The discounted intertemporal welfare of North's representative agent is

$$W^n = \int_0^\infty e^{-\rho t}(u^n(c_t^n) - D^n(E_t))dt, \tag{9}$$

where u^n and D^n are respectively assumed to be increasing and concave and increasing and convex: that is, $(u^n)' \geq 0$, $(u^n)'' \leq 0$; $(D^n)' \geq 0$, $(D^n)'' \geq 0$. Moreover, we assume that the Inada conditions hold, that is, $(u^n)'(c) \to \infty$ as $c \downarrow 0$ and $(u^n)'(c) \to 0$ as $c \to \infty$.

North can only affect the pollution stock through influencing the investment decision of South by giving it aid. As motivated in the introduction, we assume that North is unable to observe South's state variables, and it can therefore condition aid only on time.

2.3.2 Production

We assume further that North is a fully developed country having only green capital, which is moreover at the steady state level. This level is at least equal to the sum of the steady state levels of South's brown and green capitals. North's production processes use only green capital K_g^n; therefore, North's total output is:

$$Y_t^n = F_g^n(K_{g,t}^n);$$

here F_g^n denotes North's production function, which is assumed to be increasing and concave, that is $(F_g^n)' \geq 0, (F_g^n)'' \leq 0$. North has to decide at each point of time how to allocate its output between consumption and unconditional aid to South. Its budget constraint takes the form:

$$Y_t^n = c_t^n + a_t. \tag{10}$$

Moreover, North can choose whether or not to give aid to South, but it cannot force South to pay aid back; hence there is a positivity constraint on aid:

$$a_t \geq 0. \tag{11}$$

As Stackelberg leader, North is assumed to be able to credibly commit to the aid profile it announces.

2.3.3 North's dynamic optimisation problem

Since we have a Stackelberg open-loop game, North will choose the amount of aid that maximizes the intertemporal welfare of its representative consumer, subject to its budget constraint (10), the aid positivity constraint (11), as well as South's first order conditions (2), (3), (4), (5)–(7), along with the transversality conditions from South's problem (8).

The Lagrangian associated to the maximisation of North's social welfare can be found in Appendix B.1. We indicate by $\kappa_{b,t}$ and $\kappa_{g,t}$ North's shadow prices of South's brown and green capital, while ψ_t represents North's shadow cost of global pollution. The Lagrange multiplier attributed to the aid positivity constraint is denoted ξ_t; North's shadow price of South's marginal valuation of the pollution stock is denoted τ_t, and North's shadow prices of South's marginal valuation of brown and green capital are denoted $\lambda_{b,t}$ and $\lambda_{g,t}$ respectively.

The maximum principle yields equations (2)–(7), (10) and (11), as well as:

$$\begin{aligned}
\dot\kappa_{b,t} &= (\rho+\delta)\kappa_{b,t} - F_b'(K_{b,t})\left((u^n)'(c_t^n) - \xi_t\right) + \lambda_{b,t}u'(c_t)F_b''(K_{b,t}) - \alpha\psi_t, && (12)\\
\dot\kappa_{g,t} &= (\rho+\delta)\kappa_{g,t} - F_g'(K_{g,t})\left((u^n)'(c_t^n) - \xi_t\right) + \lambda_{g,t}u'(c_t)F_g''(K_{g,t}), && (13)\\
\dot\psi_t &= (\rho+\vartheta)\psi_t + (D^n)'(E_t) - \tau_t D''(E_t), && (14)\\
\dot\lambda_{b,t} &= -\frac{\kappa_{b,t}}{\beta_b u'(c_t)} - \lambda_{b,t}\delta, && (15)\\
\dot\lambda_{g,t} &= -\frac{\kappa_{g,t}}{\beta_g u'(c_t)} - \lambda_{g,t}\delta, && (16)\\
\dot\tau_t &= -\tau_t\vartheta + \alpha\lambda_{b,t}, && (17)\\
0 &= \xi_t a_t, && (18)\\
\xi_t &= (u^n)'(c_t^n) + \frac{u''(c_t)}{u'(c_t)^2}\left(\frac{\kappa_{b,t}\nu_{b,t}}{\beta_b} + \frac{\kappa_{g,t}\nu_{g,t}}{\beta_g}\right) && (19)\\
&\qquad + u''(c_t)\left(\lambda_{b,t}F_b'(K_{b,t}) + \lambda_{g,t}F_g'(K_{g,t})\right).
\end{aligned}$$

Equations (12)–(17) are differential equations for North's co-states. Equation (18) is the complementary slackness condition, and (19) the expression for the Lagrange multiplier ξ_t, which also has to satisfy the positivity condition $\xi_t \geq 0$ for all $t \geq 0$.

These conditions have to be complemented by initial and terminal conditions. We already have the initial conditions for the states E_t, $K_{b,t}$ and $K_{g,t}$ and the terminal conditions (8) on the co-states of South's problem. Moreover, both South's states and South's co-states are states of North's problem. Since there is no terminal condition on South's states and no initial condition on South's co-states, there will be a terminal transversality condition on North's co-states of South's states, that is, on $\kappa_{i,t}$ and ψ_t, and an initial transversality condition on North's co-states of South's co-states, that is, on $\lambda_{i,t}$ and τ_t. These conditions read as

$$\lim_{t\to\infty} e^{-\rho t}\psi_t = 0, \quad \lim_{t\to\infty} e^{-\rho t}\kappa_{b,t} = 0, \quad \lim_{t\to\infty} e^{-\rho t}\kappa_{g,t} = 0, \tag{20}$$

again assuming that the corresponding states are uniformly bounded away from 0, and

$$\lambda_{b,0} = 0, \quad \lambda_{g,0} = 0, \quad \tau_0 = 0. \tag{21}$$

3 Steady state analysis

In this section we present a comparative statics analysis for the steady state levels of South's capital and consumption, and we give sufficient conditions for a positive aid flow to occur at steady state.

To analyse the steady state levels of South's capital and consumption, it is sufficient to study a rest point of the evolution equations (3) – (7) of South's states and shadow prices. The solution procedure for the steady state can be found in Appendix A.2. One of the results is the relation

$$F_g'(K_g) = (\rho + \delta)\delta\beta_g K_g, \tag{22}$$

which determines the steady state levels of South's green capital. It states that the ratio of the steady state marginal productivity of green capital $F_g'(K_g)$ over the steady state marginal cost of green investments $\beta_g K_g$ equals the product $(\rho + \delta)\delta$increases with the capital depreciation rate δ and the time discount rate ρ. In particular the steady state level of green capital, and consequently that of green investments, is not affected by the aid received from North.

It follows directly from (4) that the steady state level of emissions E is a function of the steady state level of brown capital

$$E = \frac{\alpha}{\vartheta} K_b. \tag{23}$$

The steady state levels of consumption and brown capital are determined jointly by the two equations

$$F_b'(K_b) = (\rho + \delta)\delta\beta_b K_b + \frac{\alpha}{\rho + \vartheta} \frac{D'(\frac{\alpha}{\vartheta} K_b)}{u'(c)} \tag{24}$$

and

$$c = F_b(K_b) + F_g(K_g) + a - \frac{\beta_b \delta^2}{2} K_b^2 - \frac{\beta_g \delta^2}{2} K_g^2. \tag{25}$$

We note first that if the pollution emission intensity $\alpha = 0$, or if there is no marginal damage from global pollution, that is, if $D'(E) = 0$ for all E, there is no distinction between green and brown capital, and equation (24) has same form as equation (22). Since we have assumed that $\beta_g \geq \beta_b$ and that brown and green capital have the same productivity, we conclude that the steady state level of brown capital with no pollution is at least equal to the steady state level of green capital. This is natural, as green investments are more expensive than brown ones.

Equations (24) and (25) readily furnish information about the effects of parameter changes on the steady state levels c and K_b of consumption and brown capital. We begin with the effect of an increase in the aid flow a.

Theorem 1. *If the aid flow a rises, the steady state level K_b of brown capital falls, the steady state consumption level c rises, while the steady state level K_g of green capital is unaffected.*

Consequently, the steady state levels I_b of brown investment and E of the pollution stock fall as well, whereas green investments I_g are also unaffected.

Finally, South's total welfare rises.

The next result investigates the effects of changing the investment cost parameters β_g and β_b and the capital depreciation rate δ.

Theorem 2. *If the cost of green investments β_g rises, the consumption level c and the level of green capital K_g fall, while the level K_b of brown capital rises.*

If the costs of brown investments β_b rises, the consumption level falls and the green capital level K_g is unaffected.

If the capital depreciation rate δ rises, the green capital level and the consumption level fall.

Finally, for small positive values of the emission intensity α, the brown capital level falls if either the cost of brown investments or the capital depreciation rate rise.

Finally, we have a result on parameters affecting the pollution stock.

Theorem 3. *Assume that $D(E) = (\eta/2)E^2$. If either the natural decay rate ϑ falls, the emission intensity α rises, or the weight η of environmental quality rises, then both the consumption level c and the brown capital level K_b fall. The green capital level K_g is unaffected.*

Moreover, for small positive values of the emission intensity α, the pollution level rises with increasing values of α, while it falls with increasing values of the natural decay rate ϑ.

These theorems are proved in Appendix A.3, except the last statement of theorem 3, which we shall discuss now.

The effect on the global steady state pollution depends on the elasticity of brown capital at steady state with respect to emission intensity, for

$$\frac{\partial E}{\partial \alpha} = \frac{K_b}{\vartheta}\left(\frac{\alpha}{K_b}\frac{\partial K_b}{\partial \alpha} + 1\right).$$

The elasticity $\epsilon_\alpha = \frac{\alpha}{K_b}\frac{\partial K_b}{\partial \alpha}$ is negative, therefore the effect of α on E is positive if and only if $\epsilon_\alpha > -1$. Clearly this elasticity is 0 if $\alpha = 0$, yielding that E rises with α for small values of α.

The dependence of the steady state level of pollution on the natural decay rate can be written as

$$\frac{\partial E}{\partial \vartheta} = \frac{\alpha}{\vartheta^2}K_b\left(\frac{\vartheta}{K_b}\frac{\partial K_b}{\partial \vartheta} - 1\right);$$

the effect of ϑ on E is positive if and only if the elasticity $\epsilon_\vartheta = \frac{\vartheta}{K_b}\frac{\partial K_b}{\partial \vartheta} > 1$. If the emission intensity α is zero, industrial production does not affect the pollution level. Conversely the natural decay rate cannot affect the steady state level of brown capital: this results in the fact that $\partial K_b/\partial \vartheta = 0$, and hence that $\epsilon_\vartheta = 0$ if $\alpha = 0$. By continuity, for small but positive values of α, we have that ϵ_ϑ is close to zero, which results in the steady state pollution level decreasing as ϑ increases.

The next result gives a sufficient condition for a positive aid flow to occur in steady state

Theorem 4. *Assume that $D^n(E) = (\eta^n/2)E^2$. If either North's output Y^n or North's weight η^n of environmental damage are sufficiently large, then there is a positive aid flow from North to South in steady state.*

This theorem is a direct consequence of Theorem 5, proved in appendix (B.2).

To conclude, aid decreases the steady state level brown capital, brown investments, and the stock of global pollution, it increases South's consumption and total welfare, and it has no effect on the steady state level of green capital or green investments. Moreover, in certain circumstances it is in North's interest to provide South with mitigation aid, which effectively amounts to North buying off the need to build brown capital, and by that, buying off the resulting pollution.

4 Methodology

Next to the steady state, we are also interested in the growth path towards it, and its dependence on the parameter change, its 'comparative dynamics'. If there are to be any aid transfers, we expect the bulk to be effected during the growth phase of South, which is, by definition, not in steady state. Solving the model analytically is however beyond our capabilities; we have therefore resorted to numerical simulations.

In this section we present the numerical methods which we used to determine the Stackelberg open loop equilibria of the dynamic game, and we motivate our calibration of the model parameters.

4.1 Numerical Solution

Section 2.2 formulated the necessary conditions of South's decision problem in the form of a boundary value problem over an infinite time interval, involving six nonlinear differential equations, together with initial and terminal conditions; North's boundary value problem features twelve nonlinear differential equations. We adapt a numerical approach taken from Grass (2012).

In general, boundary value problems deriving from infinite horizon optimisation problems with m state variables are characterised by the following elements: a $2m$-dimensional system of differential equations, determining solution paths $z_t = (x_t, y_t) \in \mathbb{R}^m \times \mathbb{R}^m$, where x_t is the state evolution and y_t the co-state evolution; a specification of the initial states x_0, which yields m initial conditions; a specification of m asymptotic transversality conditions, which are typically satisfied by a solution of the system that tends to steady state values $\hat{z} = (\hat{x}, \hat{y})$.

In order to solve for such solution paths numerically, we approximate the asymptotic conditions by conditions that hold for a large, but finite, time T. Following Grass (2012), we impose the following 'asymptotic transversality condition'

$$M^T \begin{pmatrix} x_T - \hat{x} \\ y_T - \hat{y} \end{pmatrix} = 0; \tag{26}$$

here the columns of the matrix M form a basis spanning the orthogonal complement to the stable eigenspace at steady state, M^T denoting the transpose of M. The geometrical content of (26) is that the vector $z_T = (x_T, y_T)$ is contained in the stable eigenspace of the steady state $\hat{z}$, and therefore approximately in the stable manifold of the steady state. Note that (26) consists of m scalar conditions on the $2m$-dimensional vector z_T. The $2m$ differential equations, together with m initial state conditions and m asymptotic transversality condition then form a boundary value problem over the finite time interval $[0, T]$. As $T \to \infty$, the solution curves of the approximate problem tend uniformly to solution curves of the original problem.

Specifically, South's boundary value problem consists of equations (3)–(7), together with initial conditions at $t = 0$ for the three states $K_{b,t}$, $K_{g,t}$, and E_t, and the transversality conditions (8). The initial conditions are South's initial capital stocks $K_{b,0}$ and $K_{g,0}$, and the initial pollution stock E_0.

North's problem involves twelve differential equations: the state equations (3)–(7) and the co-state equations (12)–(17), as well as twelve boundary conditions. The first six of these are equal to South's boundary conditions, the initial conditions for the states and the transversality conditions (8) for the co-states. In addition, boundary conditions on North's co-states are furnished by the transversality conditions (20) and the initial conditions (21).

4.2 Functional forms

We assume that both South's and North's representative agent have a constant intertemporal elasticity of substitution utility

$$u(c) = u^n(c) = \frac{c^{1-\sigma}}{1-\sigma}.$$

In computations, we take $\sigma = 0.5$. We take Cobb–Douglas production functions with the factor labour taken constant; we assume moreover that green and brown technology are equally productive, yielding

$$F_b(K) = F_g(K) = \frac{\Omega}{1-\gamma} K^{1-\gamma} \qquad \text{for all } K.$$

In computations we set $\Omega = 0.6$ and $\gamma = 0.75$.

The damage functions are assumed to be quadratic:

$$D(E) = \frac{\eta}{2} E^2, \qquad D^n(E) = \frac{\eta^n}{2} E^2, \qquad \text{for all } E.$$

The parameters η and η^n govern the weight of the environmental quality in the welfare of each country.

4.3 Calibration

To calibrate the parameters in our model, we take a wind energy plant as a model for green industrial capital, and a traditional coal or gas energy plant as a model for brown capital.

The relative cost β_g/β_b of green investments with respect to brown is calibrated as the ratio between investment costs of a wind plant to that of a coal/gas plant. Salvadore and Keppler (2010) estimate that the specific overnight construction costs of most coal-fired plants range between 1000 and 1500 USD/kWe, while those of a gas-fired plants range between 400 and 800 USD/kWe. In contrast, for nuclear and wind generating technologies overnight construction costs range between 1000 and 2000 USD/kWe. Accordingly, we calibrate β_g/β_b to range between 1 and 2.5.

For the emission intensity α of brown capital we use the average emission intensity of a coal energy plant, which is estimated to be 0.888 tonnes CO_2/MWh, while for a gas plant those estimates average at 0.499 tonnes CO_2/MWh, as reported by WNA (2011). Salvadore and Keppler (2010) reported an investment cost between 9–18 USD/MWh at a 5% discount rate, while at a 10% discount rate the investment costs range between 17.5 and 30 USD/MWh. Therefore, at a 5% discount rate we get an emission intensity of 5% – 10% per unit of capital invested in a coal plant, while at a 10% discount rate, the emission intensity ranges from 3% to 6%. For a gas plant, investment costs range between 5.5 – 9 USD/MWh at 5% discount rate, and therefore, the emission intensity ranges between 5% – 9% of each unit of capital invested in a gas plant.

Damage from global pollution stock is likely to be a persistent problem for a long time, and small values, between 1.5% (Stern) and 4.5% (Nordhuas), are usually used for the time discount rate ρ. However, in order to be consistent with the calibration of other parameters we use ρ between 5% - 10%. This does not greatly affect the results obtained.

The investment cost parameter β_b represents the rate of increase of the marginal investment cost in brown capital per unit of investment. We use values of β_b ranging between 2% and 9%.

Depending on the estimated life time for a wind energy plant (around 40 years), we use the same depreciation rate for both types of capital, resulting in a range for δ between 2.5%–5%.

Higher values of the parameters η and η^n imply that governments care more about the environmental quality of their consumers when taking decisions. We choose different values of these parameters to test different assumptions about the weight of environmental quality between North and South.

Annual carbon emissions from burning fossil fuels in the United States are about 1.6 gigatons (billion metric tons), whereas annual uptake is only about 0.5 gigatons, resulting in a net release of about 1.1 gigatons per year. This implies that only 31% of the U.S carbon emissions are absorbed naturally (Sundquist et al. 2008). Using this, and an estimated emission rate between 5% and 9% of installed capital at a 5% discount rate, we arrive at a natural absorption rate of installed capital between 1.55% and 2.8% at a 5% discount rate. The resulting benchmark values for parameters can be found in Appendix C.

5 Results

For the analysis of the growth dynamics, we set low initial values for brown and green capital as well for the pollution stock, as we are interested in the situation that South initially falls in the 'least developed' class of countries.

5.1 North's allocation of aid

We start the analysis with investigating the aid allocation of North in equilibrium, and how it is affected by parameter changes.

We know from the steady state analysis that North will give aid in steady state if either its output is sufficiently high, or if it values environmental quality highly enough. In the simulations, we therefore choose η^n sufficiently low, such that there is no aid flow in steady state. If South does not care at all about the environment, that is if $\eta = 0$, there will be no incentive for North to give any aid to South, as South will never make green investments. On the other hand, we find that if South cares a lot, that is, if η is sufficiently large, then again there will be no incentive for North to give aid, as South will make sufficient green investments on its own accord. The benchmark parametrisation describes therefore an intermediate situation. Moreover, the benchmark parameters are chosen such that North gives aid to South for a finite time period.

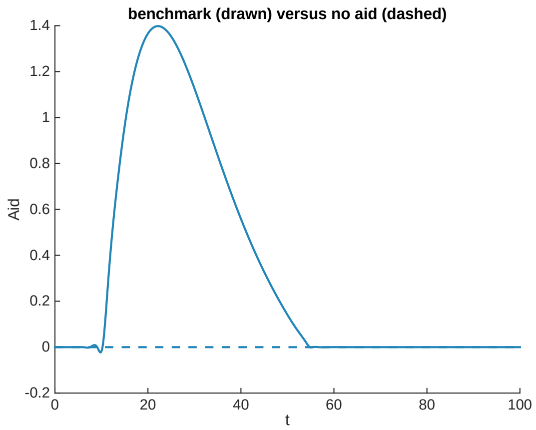


Figure 1: Aid profile over time (benchmark)

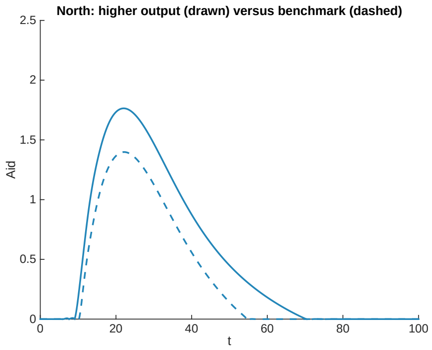
Figure 1 shows the benchmark aid profile over time. There is an initial time interval where no aid is given: this is when South's stocks of brown capital and global pollution are still at low levels. It is only when South's brown capital stock is sufficiently large that North starts giving aid. Although most of the aid is consumed, a part of it enables South to invest in green capital and thereby to lessen its emissions. North's decision to give aid is motivated only by environmental reasons — there is no 'warm glow' term in its utility function — and therefore it should be considered as mitigation aid.

Figures 2 and 3 show changes in the aid profile with respect to changes to different parameters, compared to the benchmark profile. In these figures, a dashed line represents the benchmark aid profile, while the solid line indicates the aid profile after the change. In all cases, the parameter has been increased or decreased by 20% with respect to its benchmark value.

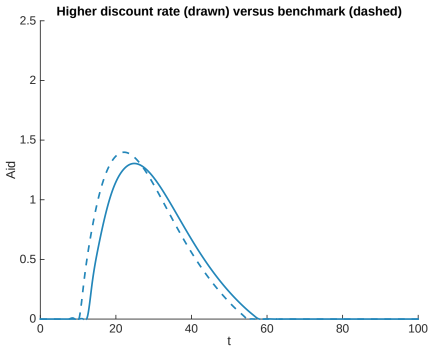
5.1.1 Effects of changing capital parameters

Figure 2 illustrates the effects of changing parameters that affect the industrial output of North or South. Figure 2a shows the effect of increasing North's output: the level of aid is higher and aid is given over a longer period of time. The first finding is in line with the result of Theorem 4 on steady state aid. However, aid starts at almost the same time as in the benchmark case, which suggests that even if North is richer, it is not interested in giving aid if South's emissions are still low and do not cause North much damage.

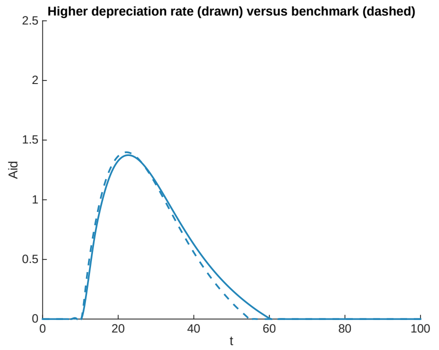
Figure 2b increases the discount rate, which both decreases aid and shifts the aid profile to the future, because the long term effects of environmental pollution impact North's welfare less. Increasing the depreciation rate δ, as in Figure 2c, has a similar but smaller effect,



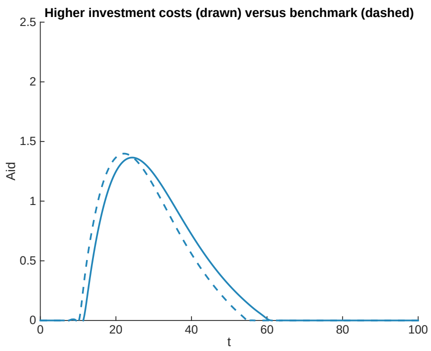
(a) Increasing North's output Y^n



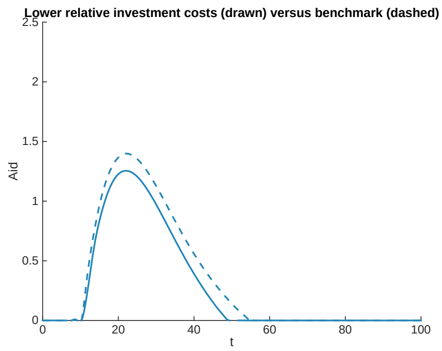
(b) Increasing time discount rate ρ



(c) Increasing depreciation rate δ

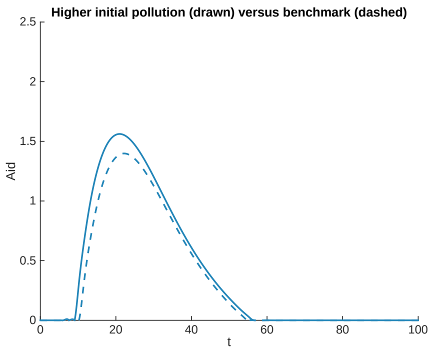


(d) Increasing investment cost parameters β_b and β_g while keeping β_g/β_b constant

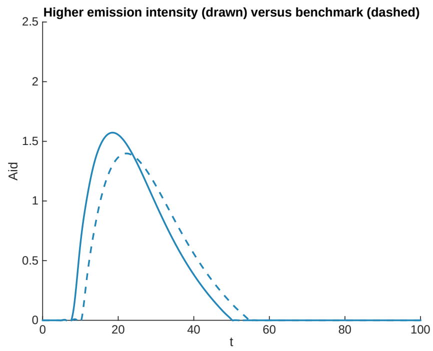


(e) Decreasing β_g while keeping β_b constant

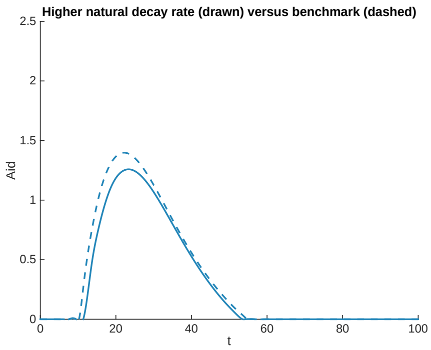
Figure 2: Influence of capital-related parameter changes on the aid profile



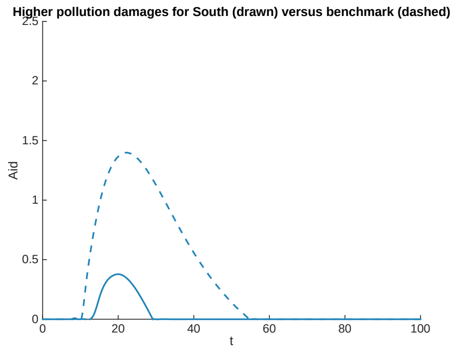
(a) Increasing initial pollution E_0



(b) Increasing emission intensity α



(c) Increasing natural decay rate ϑ



(d) Increasing South's weight of environmental quality η

Figure 3: Influence of environmental parameter changes on the aid profile

although the explanation is different: if capital depreciates quickly, brown capital is less quickly at a critical level. Moreover, it is inefficient to start to enable South too early to invest in green capital.

Higher values of the rate of increase in the marginal cost of investments β_g and β_b, while keeping their ratio constant, imply again that South needs more time to build up capital towards critical levels, implying a shift of the aid profile into the future, as seen in Figure 2d.

If the cost β_g/β_b of green investments relative to brown investments falls, aid goes down and is provided over a shorter time horizon, for South is less constrained when building up its green capital.

5.1.2 Effects of changing environmental parameters

Figure 3 documents the consequences of changes to environmental parameters. The first panel, Figure 3a, shows the effect of an increase in the initial pollution stock: this aggravates the environmental conditions and leads North to start giving more aid more quickly, as already a smaller stock of green capital build by South improves the situation.

Higher emission intensity of brown capital makes the aid programme start sooner, Figure 3b: as brown capital emits more pollution, more damages from pollution are realised sooner by North.

If the natural decay rate of pollution decreases, Figure 3c, the pollution stock decreases faster and South's emissions take longer to reach critical levels. Together this makes the problem less urgent for North, whose aid programme is reduced.

Finally, Figure 3d shows that if South's consumers put more weight on environmental quality, their incentive to build green capital increases, which in turn lowers North's incentive to give aid dramatically.

5.2 South's use of the aid

We turn to South consumption and investment decisions. First we analyse these as function of the model parameters. Then we study the how South allocates the aid it receives from North between consumption and total investments, and how it allocates investment aid between brown and green investments.

5.2.1 Aid increases consumption and green growth

The decisions of South how to allocate aid show how efficiently aid promotes economic growth of the recipient country as well as the effect of aid on the direction of growth.

In order to identify the choice of South for both decision processes we compare the time paths of South's controls when it receives aid to those when it does not, holding all parameters constant.

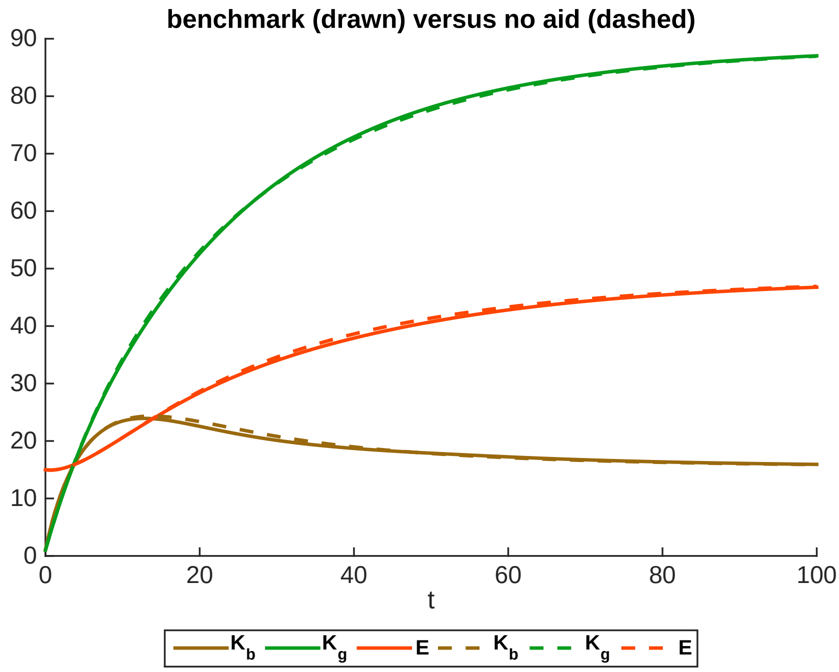
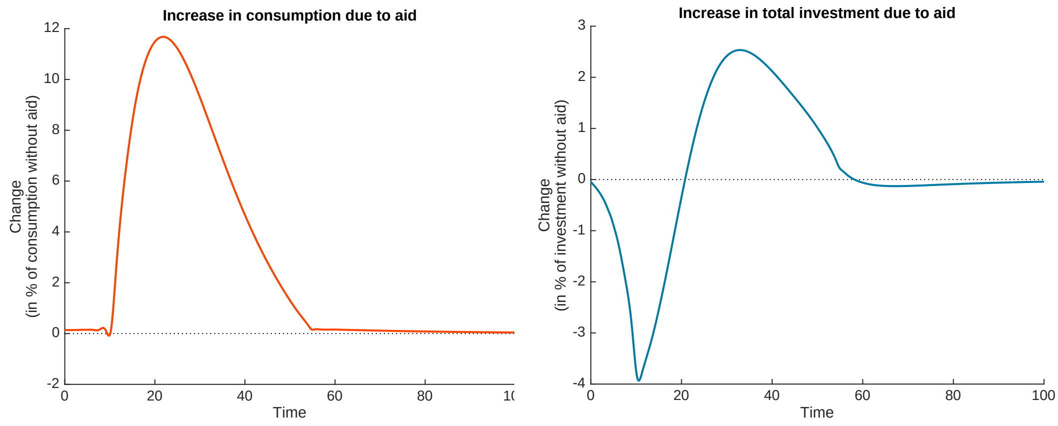


Figure 4: Evolution of South's green and brown capital as well as the pollution level, for the benchmark parametrisation, when North gives aid (solid curves) compared to when no aid is given (dashed curves).

As mentioned in Section 5.1, in the benchmark situation North gives aid only during a certain time interval. Figure 4 shows the evolution of the pollution and both capital levels. Note that the pollution level is depressed temporarily by the aid, mainly by shifting the initial part of the brown capital curve downwards.

Figure 5a depicts the relative increase of South's consumption when receiving aid compared to the situation where no aid is received; Figure 5b gives the corresponding increase in total investments.

The figures show that it is optimal for South to use most of the aid to smooth out its consumption schedule. This seem at first sight to agree to the findings of Boone (1996), who concludes that aid primarily goes to consumption and that there is no relationship between aid and growth. Figure 5b depicts how South's total investments change over time with aid: it shows that investments fall steadily relative to the situation where no aid is expected, until the moment aid starts to arrive. Investments increase again and are then for a substan-

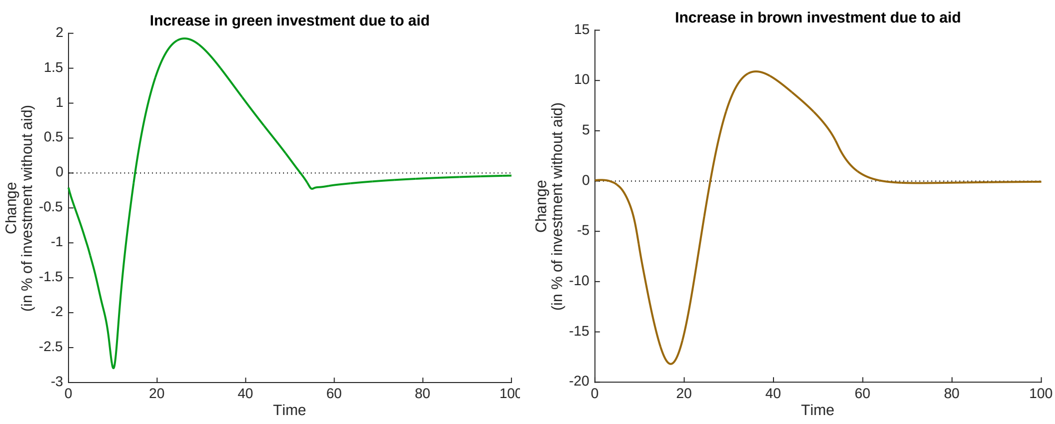


(a) Effect of aid on consumption

(b) Effect of aid on investments

Figure 5: South's allocation of aid between consumption and investments

tial period of time over the no-aid levels. Therefore we argue that the conclusion of Boone (1996) about the relationship between aid and growth is imprecise: in our situation aid has a positive effect on growth, but this is modest and lower than what the Harrod and Domar model would predict. These findings are in line with Chatterjee et al. (2003) who find that a temporary pure transfer has only modest short-run growth effects compared to a transfer tied to investment in public infrastructure. We note that a second effect of aid is to push investments into the future.



(a) Effect of aid on green investments

(b) Effect of aid on brown investments

Figure 6: South's allocation of aid between brown and green investments

Figure 6 depicts the change of South's investment schedule due to aid for, respectively, brown and green capital. There is a decrease of investments before the aid period begins. The maximal decrease of green investments respective to the case that no aid is received tops out at about 2.8%, before it starts to increase again and ends up at its highest about 2% higher than in the no-aid situation. Investments in brown capital fall much more strongly, to a minimum of 18% of investments in the no-aid regime. Also here we see that later on, investments in brown capital pick up again, topping out at an increase of 10% over the no-aid levels. Note however that these effects are small in absolute terms, as the brown capital level is much lower than the green capital levels. Moreover, brown capital growth is mainly redistributed over time by the aid, not much lessened.

We summarise these findings by noting that aid has two effects on investments: it modestly increases total eventual growth, in the benchmark situation mainly for green capital, and it pushes growth farther into the future, by enabling South to increase consumption earlier.

5.2.2 Effect of parameter changes

We have investigated how the model parameters affect South's consumption and investment decisions relative to the benchmark parametrization. For reasons of space, we do not give graphs of the results, but only report the effects.

An increase in the time discount rate ρ has the usual effect of raising consumption initially, while lowering both kinds of investments. Increasing the capital depreciation δ has the inverse effect: consumption decreases, while total investments increase, leading to more investments over time overall. The level of brown investment hardly changes, while that of green investments decreases.

If the rates of increase of the marginal investment costs β_g and β_b increase, while keeping their relative value β_g/β_b fixed, consumption and total investments fall. The same effect can be observed if β_g increases relative to β_b.

Increasing the emission intensity α is found to lead to initially increased consumption when aid starts being received, but this changes afterwards to a lower level of consumption and total investments. At the same time, green investments are hardly affected byα; it is mainly brown investments that fall. A higher natural decay rate ϑ has an exactly opposite effect: it lowers consumption initially, but raises it in the long run. Green investments are again virtually not affected, and the growth in investments is mainly driven by brown investments.

5.3 Time consistency

The Stackelberg equilibria we have investigated so far are open-loop equilibria: that is, at time $t = 0$ North announces an aid schedule a_t, and South subsequently makes its plans taking this schedule for granted. At any given point in time, North may reconsider its decision, which then can result in a change in the announced aid policy.

To model South's reaction to such a policy change, we extend the original differential game by introducing a binary state variable, *trust*, which can take the values 0 and 1. At the beginning of the game, *trust* is assumed to take the value 1, which is interpreted as South trusting North to stick to its announced aid schedule. When, at some time $t > 0$, North deviates from the announced schedule — this can be observed by South — *trust* switches from 1 to 0, and South falls back to that growth policy which is optimal if it will receive no aid from North. North will then switch to giving no aid at all, as in the 'no trust' regime giving aid will not alter South's behaviour.

In order to find out whether North will stick to its original aid schedule for all time, we have to compare, for each time $t > 0$, North's payoff over the time interval $[t, \infty)$ when sticking to the announced aid schedule versus its payoff when cutting aid at time t. More precisely, let $(E_t, K_{b,t}, K_{g,t})$ be the evolution of pollution level, brown and green capital stock, under the aid schedule a_t announced by North at time $t = 0$, and let

$$W^n(t_0) = \int_{t_0}^{\infty} \mathrm{e}^{-\rho(s-t_0)} \big(u^n(Y^n - a_s) - D^n(E_s)\big) \,\mathrm{d}s$$

the corresponding present value of North's welfare at time t_0. If North changes its aid payment at time t_0, South falls back to its optimal growth policy starting at time t_0, with initial values $(E_{t_0}, K_{b,t_0}, K_{g,t_0})$, under the assumption that it will receive no aid. This results, amongst other things, in a different evolution E_t^0 of the pollution stock and a different present value

$$W^{n,0}(t_0) = \int_{t_0}^{\infty} \mathrm{e}^{-\rho(s-t_0)} \big(u^n(Y^n) - D^n(E_s^0)\big) \,\mathrm{d}s$$

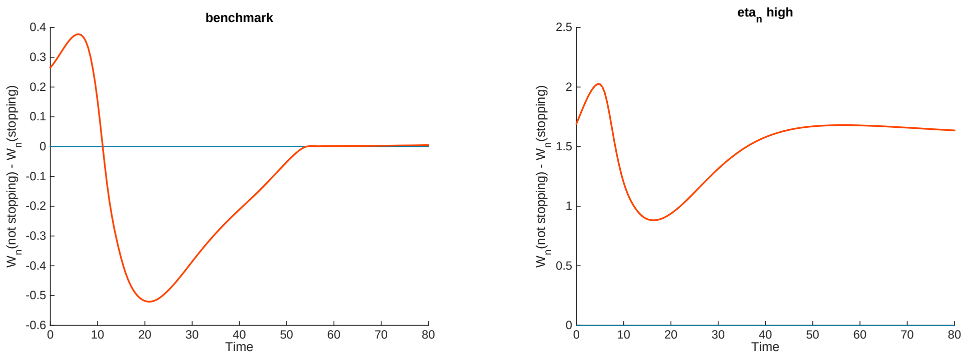
of North's welfare. If the difference

$$\Delta_t = W^n(t) - W^{n,0}(t)$$

is negative for some $t > 0$, North has an incentive to reconsider its aid policy at that date, and the announced policy is not time-consistent.

Figure 7a shows the evolution of Δ_t for the benchmark parametrisation. It is clear that this quantity starts taking negative values at the moment where North should be starting making aid payments. In Section 5.2 we saw that in anticipation of the aid transfers, South reduces production, resulting in lower emissions which benefits North. We conclude that in the benchmark parametrisation, giving aid is not a time-consistent policy.

Figure 7b illustrates a contrasting situation: if North is more sensitive to pollution damages than in the benchmark parametrisation, giving aid is time-consistent. Unlike the benchmark parametrisation, here aid is given also in steady state; in this situation, the long term gains in pollution reduction in steady state are always more important to North than the short time savings by not sticking to the announced aid transfers.



(a) Benchmark: not time consistent

(b) Higher η^n: time consistent

Figure 7: Time consistency of the Stackelberg equilibrium

5.4 Main effects of giving aid

The discussion in the previous section highlights some of the effects of aid on South's decision over time. We distinguish four effects: the first one is that South chooses to postpone a small amount of its investments until it starts receiving aid, increasing consumption instead. This is coherent with the life-cycle theory of consumption: what is however remarkable is that the intertemporal substitution of consumption is effectively small.

The second effect is that South consumes most of the aid received. This squares with much anecdotal evidence of development aid 'leaking away'. The present analysis shows however that apart from corruption and mismanagement, which undoubtedly play a role in practice and which are not addressed by our model, there is also the purely economic motivation that the aid is simply better employed elsewhere from South's point of view.

Thirdly, South still develops its brown capital when receiving, but it will do so later than in the no-aid scenario. Effectively, giving aid results in emissions being pushed into the future.

The fourth effect is that South uses the part of the aid which it allocates to capital investment mainly to increase green growth. We think this to be a remarkable finding, the more so as Theorem 1 shows that the steady state level of green capital is never affected by aid.

6 Conclusion

This study theoretically identifies the dynamic effects of unconditional aid on the growth and the direction of the growth of a recipient country. We studied a differential Stackelberg game between a leading donor country and a following recipient country. The decision of the donor to give aid in our model is motivated by environmental concerns, and should be classified as mitigation aid. Our model identifies circumstances under which the donor is motivated to give unconditional mitigation aid.

We conclude that if the recipient is sufficiently concerned about environmental quality, there is no incentive for the donor to give aid, as the recipient takes its decisions in a way that preserves the environment whether it receives assistance or not. If the recipient is not concerned about environmental quality at all, again there is no incentive for the donor to give aid, as the behaviour of the recipient will not be influenced by it. In between these two extreme situations, when the recipient is weakly concerned about environmental quality, the donor has an incentive to give aid.

In particular, since we argue that most 'conditional' aid is in practice given virtually unconditionally, our study provides an explanation for the empirical evidence that indicates the relative ineffectiveness of aid on growth of the recipient country: our model indicates that it is optimal for the recipient to consume most of the aid and only to allocate a minor part to investments. Still, even giving unconditional aid can be a Pareto-improvement over giving no aid at all.

Our model also shows that unconditional development aid has a modest positive effect on growth. This effect seems however much lower than what the Harrod and Domar model predicts. At least for our benchmark case we investigated, we found that most of the increase of growth caused by aid takes place in the green sector.

We propose two possible extensions to our model. The first is to include demographical changes in the recipient country by adding labour as a second input for production. This would help to complete the analysis, to study whether high population growth rate in these countries necessitates a higher growth rate to meet the demographical changes: the possible effect would be that aid is more effectively used to increase growth. The second extension would be to introduce a parameter that captures aid being given under the condition that it is used only for green investments. We expect then to find an intertemporal trade-off between consumption and investments, resulting in a higher consumption ex-ante and consequently a *de facto* failure of conditionality.

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A South

A.1 South's decision problem

South has to maximize its welfare (1), subject to its budget constraint (2) and the capital and pollution stock evolution equations (3) and (4). From the binding budget constraint, we solve

$$c_t = F_b(K_{b,t}) + F_g(K_{g,t}) + a_t - \frac{\beta_b}{2} I_{b,t}^2 - \frac{\beta_g}{2} I_{g,t}^2. \tag{27}$$

The current value Hamiltonian for maximization problem is

$$\begin{aligned} H = u\left(F_b(K_{b,t}) + F_g(K_{g,t}) + a_t - \frac{\beta_b}{2} I_{b,t}^2 - \frac{\beta_g}{2} I_{g,t}^2\right) - D(E_t) \\ + \mu_t(\alpha K_{b,t} - \vartheta E_t) + \nu_{b,t}(I_{b,t} - \delta K_{b,t}) + \nu_{g,t}(I_{g,t} - \delta K_{g,t}). \end{aligned}$$

Note that we have written out the argument c_t of u. Maximizing over the remaining decision variables $I_{b,t}$ and $I_{g,t}$ yields

$$\nu_{i,t} = \beta_i I_{i,t} u'(c_t), \qquad i \in \{b, g\}. \tag{28}$$

The equations for the shadow prices are given in the main text (equations (6)–(5)). We have moreover the initial states $K_{b,0}, K_{g,0}, E_0$ and the transversality conditions

$$\lim_{t\to\infty} e^{-\rho t} E_t = 0, \quad \lim_{t\to\infty} e^{-\rho t} K_{i,t} = 0, \qquad i \in \{b, g\}, \tag{29}$$

which hold for paths that are uniformly bounded away from 0.

A.2 South's steady state

Here, we compute and analyse the steady state of South's decision problem under the assumption that the aid schedule $a_t = a$ is constant in time. To denote a steady state value of a dynamic quantity, we drop the subscript t.

A.2.1 Green capital

First, we derive the steady state level of green capital. At steady state, we obtain from (3)

$$I_i = \delta K_i, \qquad \text{for} \quad i \in \{b, g\}. \tag{30}$$

Equation (28) implies at steady state

$$u'(c) = \frac{\nu_g}{\beta_g I_g}$$

Substituting in (7) yields

$$\left(\rho + \delta - \frac{1}{\beta_g I_g} F_g'(K_g)\right) \nu_g = 0.$$

The alternative $\nu_g = 0$ implies, by the previous expression, that $u'(c) = 0$, which is impossible. Hence the term in brackets vanishes, which yields

$$F_g'(K_g) = (\rho + \delta)\beta_g I_g.$$

Eliminating I_g using (30) yields finally

$$F_g'(K_g) = (\rho + \delta)\delta\beta_g K_g. \tag{31}$$

This equation determines the steady state level K_g of green capital as a function of the system's parameters; K_g in turn determines the steady state level I_g of green investments. Note in particular that green capital and green investments at steady state do not depend on aid.

A.2.2 Brown capital

We turn to brown capital. From the budget constraint (2), we write steady state consumption c as a function of aid a and brown capital K_b

$$c = F_b(K_b) + F_g(K_g) + a - \frac{\beta_b \delta^2}{2} K_b^2 - \frac{\beta_g \delta^2}{2} K_g^2. \tag{32}$$

From (4) and (5) it follows that

$$E = \frac{\alpha}{\vartheta} K_b \tag{33}$$

and

$$\mu = -\frac{D'(E)}{(\rho + \vartheta)} = -\frac{D'(\frac{\alpha}{\vartheta} K_b)}{(\rho + \vartheta)}. \tag{34}$$

This yields E and μ as functions of K_b.

Eliminating μ from (6) using (34) yields

$$(\rho + \delta)\nu_b = u'(c) F_b'(K_b) - \frac{\alpha}{\rho + \vartheta} D'\left(\frac{\alpha}{\vartheta} K_b\right).$$

Using (28) and (30), we obtain a second expression

$$\nu_b = \beta_b \delta u'(c) K_b$$

for ν_b. After elimination, we finally obtain

$$F_b'(K_b) = (\rho + \delta)\delta\beta_b K_b + \frac{\alpha}{(\rho + \vartheta)} \frac{D'\left(\frac{\alpha}{\vartheta} K_b\right)}{u'(c)}. \tag{35}$$

This equation determines the steady state brown capital level K_b, which in turn determines the steady state brown investments level I_b.

A.3 Proofs of theorem 1–3

We first prove theorem 1.

Proof.
It follows from (31) that aid does not affect the steady state level of green capital, and hence that $\frac{\partial K_g}{\partial a} = 0$. Consumption c and brown capital K_b are jointly determined by (32) and (35), which can be written as

$$\begin{aligned} G_1 &= c + \frac{\beta_b \delta^2}{2} K_b^2 + \frac{\beta_g \delta^2}{2} K_g^2 - F_b(K_b) - F_g(K_g) - a = 0, \\ G_2 &= (\rho + \delta)\delta\beta_b K_b - F_b'(K_b) + \frac{\alpha}{(\rho + \vartheta)} \frac{D'\left(\frac{\alpha}{\vartheta} K_b\right)}{u'(c)} = 0. \end{aligned}$$

Introduce the vector-valued functions $G = (G_1, G_2)$ and $X = (c, K_b)$, and the derivative

$$D_X G = \begin{pmatrix} \frac{\partial G_1}{\partial c} & \frac{\partial G_1}{\partial K_b} \\ \frac{\partial G_2}{\partial c} & \frac{\partial G_2}{\partial K_b} \end{pmatrix}.$$

We shall need the elements of the matrix $D_X G$ and its inverse. Compute first

$$\begin{aligned} \frac{\partial G_1}{\partial c} &= 1, \\ \frac{\partial G_1}{\partial K_b} &= \beta_b \delta^2 K_b - F_b'(K_b), \\ \frac{\partial G_2}{\partial c} &= \frac{\alpha}{\rho + \vartheta} D'\left(\frac{\alpha}{\vartheta} K_b\right) \frac{(-u''(c))}{u'(c)^2} \\ \frac{\partial G_2}{\partial K_b} &= (\rho + \delta)\delta\beta_b - F_b''(K_b) + \frac{\alpha^2}{(\rho + \vartheta)\vartheta} \frac{D''(\frac{\alpha}{\vartheta} K_b)}{u'(c)} \end{aligned}$$

It follows from the assumptions of F_b, u and D that $\frac{\partial G_1}{\partial c} > 0$, $\frac{\partial G_2}{\partial c} > 0$ and $\frac{\partial G_2}{\partial K_b} > 0$. Using (35) to eliminate $F_b'(K_b)$, we find that

$$\frac{\partial G_1}{\partial K_b} = \beta_b \delta^2 K_b - F_b'(K_b) = -\rho\delta\beta_b K_b - \frac{\alpha}{(\rho+\vartheta)} \frac{D'\left(\frac{\alpha}{\vartheta} K_b\right)}{u'(c)} < 0.$$

This implies that the determinant $\Delta = \det D_X G$ is positive. Setting

$$-(D_X G)^{-1} = B = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}$$

these results imply that $B_{11} < 0$, $B_{12} < 0$ and $B_{22} < 0$, while $B_{21} > 0$.

Since

$$D_a X = -\left(D_X G\right)^{-1} D_a G, \tag{36}$$

we also have to compute the elements $D_a G$:

$$\frac{\partial G_1}{\partial a} = -1, \qquad \frac{\partial G_2}{\partial a} = 0.$$

Equation (36) implies that

$$\frac{\partial c}{\partial a} = -B_{11} > 0, \quad \frac{\partial K_b}{\partial a} = -B_{21} < 0.$$

This shows the results about consumption and dirty and green capital. The results about investments and the pollution stock follow from equations (30) and (33). □

The proof of the other theorems is now straightforward. We continue with the proof of theorem 2.

Proof.
Retaining the notations from the previous proof, we note that

$$\begin{pmatrix} \frac{\partial c}{\partial \beta_i} \\ \frac{\partial K_b}{\partial \beta_i} \end{pmatrix} = B\, D_{\beta_i} G = B \begin{pmatrix} \frac{\partial G_1}{\partial \beta_i} \\ \frac{\partial G_2}{\partial \beta_i} \end{pmatrix} \qquad \text{for} \quad i \in \{b, g\}.$$

For green investment costs, we have

$$\frac{\partial G_1}{\partial \beta_g} = \frac{\delta^2}{2} K_g^2 > 0, \quad \frac{\partial G_2}{\partial \beta_g} = 0,$$

whence

$$\frac{\partial c}{\partial \beta_g} = B_{11} \frac{\delta^2}{2} K_g^2 < 0, \qquad \frac{\partial K_b}{\partial \beta_g} = B_{21} \frac{\delta^2}{2} K_g^2 > 0.$$

Then, for brown investment costs

$$\frac{\partial G_1}{\partial \beta_b} = \frac{\delta^2}{2} K_b^2 > 0, \quad \frac{\partial G_2}{\partial \beta_b} = (\rho + \delta)\delta K_b > 0,$$

which implies

$$\frac{\partial c}{\partial \beta_b} = B_{11} \frac{\delta^2}{2} K_b^2 + B_{21}(\rho + \delta)\delta K_b < 0,$$

and

$$\frac{\partial K_b}{\partial \beta_b} = B_{21} \frac{\delta^2}{2} K_g^2 + B_{22}(\rho + \delta)\delta K_b.$$

In the last expression, the two terms on the right hand side have opposite signs. However, if the emission intensity $\alpha = 0$, then $B_{21} = 0$ and

$$\frac{\partial K_b}{\partial \beta_b} \Big|_{\alpha=0} < 0,$$

which implies, by continuity, that $\frac{\partial K_b}{\partial \beta_b} < 0$ for values of α close to 0.

Finally, for the capital depreciation rate

$$\frac{\partial G_1}{\partial \delta} = \beta_g \delta K_g^2 + \beta_b \delta K_b^2 > 0, \quad \frac{\partial G_2}{\partial \delta} = (\rho + 2\delta)\beta_b K_b > 0.$$

Analogously to the situation of brown investment costs, this implies

$$\frac{\partial c}{\partial \delta} < 0$$

whereas the sign of $\frac{\partial K_b}{\partial \delta}$ is undetermined in general, but for α taking values close to 0, we have that $\frac{\partial K_b}{\partial \delta} < 0$.

□

Next, the proof of theorem 3.

Proof.
Again retaining the notations of the proof of theorem 1, we note that

$$D_\vartheta G = -\left(\frac{\alpha}{(\rho + \vartheta)^2} \frac{D'(\frac{\alpha}{\vartheta} K_b)}{u'(c)} + \frac{\alpha^2}{(\rho + \vartheta)\vartheta} \frac{D''(\frac{\alpha}{\vartheta} K_b)}{u'(c)} K_b \right) \begin{pmatrix} 0 \\ 1 \end{pmatrix} = -C \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

with $C > 0$. It follows that

$$\frac{\partial c}{\partial \vartheta} = -B_{12}C > 0, \qquad \frac{\partial K_b}{\partial \vartheta} = -B_{22}C > 0.$$

From

$$D_\alpha G = \left(\frac{1}{\rho + \vartheta} \frac{D'(\frac{\alpha}{\vartheta} K_b)}{u'(c)} + \frac{\alpha}{(\rho + \vartheta)\vartheta} \frac{D''(\frac{\alpha}{\vartheta} K_b)}{u'(c)} K_b \right) \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

the factor in brackets being positive, it follows in the same manner that

$$\frac{\partial c}{\partial \alpha} < 0, \qquad \frac{\partial K_b}{\partial \alpha} < 0.$$

Using the functional form $D(E) = \eta E^2/2$, we find

$$D_\eta G = \left(\frac{\alpha}{\rho + \vartheta} \frac{\frac{\alpha}{\vartheta} K_b}{u'(c)} \right) \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

In the same manner as before, we obtain

$$\frac{\partial c}{\partial \eta} < 0, \qquad \frac{\partial K_b}{\partial \eta} < 0.$$

□

B North

B.1 North's decision problem

North maximizes its welfare (9) subject to: its budget constraint (10); the aid positivity constraint (11); the evolution equations of South's capital stocks (3) and that of the global pollution stock (4); the evolution equations of South's shadow prices for capital and pollution (5)–(7); and South's transversality conditions (29).

Since we need take into account of aid positivity $a_t \geq 0$, we need to compute a Lagrangian for North, with ξ_t as the multiplier of the positivity constraint:

$$
\begin{aligned}
L = &\, u^n(F_g^n(K_{g,t}^n) - a_t) - D^n(E_t) + \xi_t a_t \\
&+ \kappa_{b,t}\left(\frac{\nu_{b,t}}{\beta_b u'(c_t)} - \delta K_{b,t}\right) + \kappa_{g,t}\left(\frac{\nu_{g,t}}{\beta_g u'(c_t)} - \delta K_{g,t}\right) \\
&+ \lambda_{b,t}\Big((\rho+\delta)\nu_{b,t} - u'(c_t){F_b}'(K_{b,t}) - \alpha\mu_t\Big) \\
&+ \lambda_{g,t}\Big((\rho+\delta)\nu_{g,t} - u'(c_t){F_g}'(K_{g,t})\Big) \\
&+ \psi_t(\alpha K_{b,t} - \vartheta E_t) + \tau_t\big(D'(E_t) + (\rho+\vartheta)\mu_t\big).
\end{aligned}
$$

Here c_t is given by (27).

The multiplier ξ_t has to satisfy the positivity condition $\xi_t \geq 0$, the complementary slackness condition

$$\xi_t a_t = 0,$$

as well as the condition

$$
\begin{aligned}
\xi_t = (u^n)'(c_t^n) &+ \frac{u''(c_t)}{u'(c_t)^2}\Big(\frac{\kappa_{b,t}\nu_{b,t}}{\beta_b} + \frac{\kappa_{g,t}\nu_{g,t}}{\beta_g}\Big) \\
&+ u''(c_t)\Big(\lambda_{b,t}F_b'(K_{b,t}) + \lambda_{g,t}F_g'(K_{g,t})\Big).
\end{aligned}
$$

The equations for North's shadow prices read as

$$
\begin{aligned}
\dot{\kappa}_{b,t} = &\,(\rho+\delta)\kappa_{b,t} + \lambda_{b,t}u'(c_t)F_b''(K_{b,t}) - \alpha\psi_t \\
&+ F_b'(K_{b,t})\left[\frac{u''(c_t)}{u'(c_t)^2}\Big(\frac{\kappa_{b,t}\nu_{b,t}}{\beta_b} + \frac{\kappa_{g,t}\nu_{g,t}}{\beta_g}\Big)\right. \\
&\qquad\left. + u''(c_t)\Big(\lambda_{b,t}F_b'(K_{b,t}) + \lambda_{g,t}F_g'(K_{g,t})\Big)\right],
\end{aligned}
$$

$$
\begin{aligned}
\dot{\kappa}_{g,t} = &\,(\rho+\delta)\kappa_{g,t} + \lambda_{g,t}u'(c_t)F_g''(K_{g,t}) \\
&+ F_g'(K_{g,t})\left[\frac{u''(c_t)}{u'(c_t)^2}\left(\frac{\kappa_{b,t}\nu_{b,t}}{\beta_b} + \frac{\kappa_{g,t}\nu_{g,t}}{\beta_g}\right)\right. \\
&\qquad\left. + u''(c_t)\Big(\lambda_{b,t}F_b'(K_{b,t}) + \lambda_{g,t}F_g'(K_{g,t})\Big)\right],
\end{aligned}
$$

$$\dot{\lambda}_{b,t} = -\delta\lambda_{b,t} - \frac{\kappa_{b,t}}{\beta_b u'(c_t)}, \qquad \dot{\lambda}_{g,t} = -\delta\lambda_{g,t} - \frac{\kappa_{g,t}}{\beta_g u'(c_t)},$$

$$\dot{\psi}_t = (\rho + \vartheta)\psi_t + (D^n)'(E_t) - \tau_t D''(E_t),$$
$$\dot{\tau}_t = -\vartheta\tau_t + \alpha\lambda_{b,t}.$$

The transversality conditions are:

$$\lim_{t\to\infty} e^{-\rho t} K_{b,t} = 0, \quad \lim_{t\to\infty} e^{-\rho t} K_{g,t} = 0, \quad \lim_{t\to\infty} e^{-\rho t} E_t = 0,$$
$$\lim_{t\to\infty} e^{-\rho t} \nu_{b,t} = 0, \quad \lim_{t\to\infty} e^{-\rho t} \nu_{g,t} = 0, \quad \lim_{t\to\infty} e^{-\rho t} \mu_t = 0,$$

as usual holding for solution paths where the associated variable is bounded away from 0. These are complemented by the initial conditions for the states $K_{b,0}$, $K_{g,0}$, and E_0, and the following initial conditions for North's co-states: $\tau_0 = 0$, $\lambda_{b,0} = 0$, $\lambda_{g,0} = 0$.

B.2 Solving for North's steady state

In the analysis of South's steady state, aid a was treated as an external parameter. From the steady state conditions of North's co-state equations, we derive an equation that links North's steady state aid level to South's consumption level c and South's brown capital level K_b.

From equation (17), we obtain

$$\tau = \frac{\alpha}{\vartheta}\lambda.$$

Equation (14) then yields

$$\psi = \frac{\frac{\alpha}{\vartheta}\lambda_b D''(E) - (D^n)'(E)}{\rho + \vartheta};$$

we write here and later E instead of $(\alpha/\vartheta)K_b$ for the sake of legibility. Equations (15) and (16) allow us to eliminate κ_b and κ_g, as

$$\kappa_i = -\delta\beta_i u'(c)\lambda_i, \qquad i \in \{b, g\}.$$

Using this and equations (30) and (28), we obtain an expression for the multiplier

$$\xi = (u^n)'(c^n) + u''(c)\Big(\lambda_g(F_g'(K_g) - \delta^2\beta_g K_g) + \lambda_b(F_b'(K_b) - \delta^2\beta_b K_b)\Big). \tag{37}$$

We investigate the situation that aid is given, which occurs if $\xi = 0$. Equations (12) and (13) then yield

$$0 = \lambda_b u'(c)\left(F_b''(K_b) - (\rho + \delta)\delta\beta_b - \frac{\alpha^2}{(\rho + \vartheta)\vartheta}\frac{D''(E)}{u'(c)}\right)$$
$$- F_b'(K_b)(u^n)'(c^n) + \frac{\alpha}{\rho + \vartheta}(D^n)'(E)$$

and

$$0 = \lambda_g u'(c)\Big(F_g''(K_g) - (\rho+\delta)\delta\beta_g\Big) - F_g'(K_g)(u^n)'(c^n).$$

From these, we obtain

$$\lambda_b = -\frac{\frac{(u^n)'(c^n)}{u'(c)}F_b'(K_b) - \frac{\alpha}{\rho+\vartheta}\frac{(D^n)'(E)}{u'(c)}}{(\rho+\delta)\delta\beta_b - F_b''(K_b) + \frac{\alpha^2}{(\rho+\vartheta)\vartheta}\frac{D''(E)}{u'(c)}} \tag{38}$$

and

$$\lambda_g = -\frac{(u^n)'(c^n)}{u'(c)}\frac{F_g'(K_g)}{(\rho+\delta)\delta\beta_g - F_g''(K_g)}. \tag{39}$$

Substituting (38) and (39) in (37), for $\xi = 0$, and recalling North's budget constraint (10), yields

$$(u^n)'(Y^n - a) = \frac{A_2}{A_1}, \tag{40}$$

where

$$A_1 = 1 - \frac{u''(c)}{u'(c)}\left(\frac{(F_g'(K_g) - \delta^2\beta_g K_g)F_g'(K_g)}{(\rho+\delta)\delta\beta_g - F_g''(K_g)} + \frac{(F_b'(K_b) - \delta^2\beta_b K_b)F_b'(K_b)}{(\rho+\delta)\delta\beta_b - F_b''(K_b) + \frac{\alpha^2}{(\rho+\vartheta)\vartheta}\frac{D''(E)}{u'(c)}}\right),$$

and

$$A_2 = \frac{\alpha}{\rho+\vartheta}\frac{\big(-u''(c)\big)}{u'(c)}\frac{F_b'(K_b) - \delta^2\beta_b K_b}{(\rho+\delta)\delta\beta_b - F_b''(K_b) + \frac{\alpha^2}{(\rho+\vartheta)\vartheta}\frac{D''(E)}{u'(c)}}(D^n)'(E).$$

It follows from equations (31) and (35) that the factors $F_i'(K_i) - \delta^2\beta_i K_i$ are positive for $i \in \{b, g\}$. This, together with the standard assumptions put on F_i, u and D, implies that $A_1, A_2 > 0$.

Note that the right hand side of equation (40) depends only on K_g, K_b and c, which are differentiable functions of a, determined by equations (31), (32) and (35); hence $A_1 = A_1(a)$ and $A_2 = A_2(a)$ are also differentiable functions of a. Note moreover that $A_2(a)$ is bounded away from 0.

Theorem 5. *If $(u^n)'(Y^n) < A_2(0)/A_1(0)$, then there is a steady state with positive aid flow a.*

Proof.

This follows immediately from the intermediate value theorem, as

$$\lim_{a \uparrow Y^n} (u^n)'(Y^n - a) = \infty$$

by the Inada condition on u^n, whereas the expression $A_2(a)/A_1(a)$ is continuous for all $a > 0$. □

C Benchmark parametrization

Parameter	**Value**	**Parameter**	**Value**	**Parameter**	**Value**	**Parameter**	**Value**
$K_{b,0}$	1	δ	0.025	γ	0.75	α	0.05
$K_{g,0}$	1	T	300	ρ	0.05	ϑ	0.016
E_0	15	Ω	0.6	β_b	0.05	η	0.0006
ε	0.5	σ	0.5	β_g	0.125	η^n	0.02
Y^n	12.123						