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Luiz Félix¹

Roman Kräussl²

Philip Stork^{1,3}

¹*Faculty of Economics and Business Administration, VU University Amsterdam, the Netherlands;*

²*University of Luxembourg, Luxembourg;*

³*Tinbergen Institute, the Netherlands.*

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Tinbergen Institute Rotterdam
Burg. Oudlaan 50
3062 PA Rotterdam
The Netherlands
Tel.: +31(0)10 408 8900

Single stock call options as lottery tickets: overpricing and investor sentiment*

Luiz Félix

VU University Amsterdam
APG Asset Management

Roman Kräussl

Luxembourg School of Finance,
University of Luxembourg

Philip Stork[†]

VU University Amsterdam
Tinbergen Institute Amsterdam

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ABSTRACT

This paper investigates whether the overpricing of out-of-the money single stock calls can be explained by Tversky and Kahneman's (1992) cumulative prospect theory (CPT). We hypothesize that these options are overpriced because investors overweight small probability events and overpay for positively skewed securities that resemble lottery tickets. We match a set of subjective density functions derived from risk-neutral densities with the empirical probability distribution of U.S. equity returns. We find that overweighting of small probabilities embedded in the CPT explains the richness of out-of-the money single stock calls better than other utility functions. The overweighting of small probability events is less pronounced than suggested by the CPT, is strongly time-varying and most frequent in options of short maturity. We find that the fluctuations in overweighting of small probabilities are to a large degree explained by the sentiment factor of Baker and Wurgler (2007).

JEL classification: G02, G12.

Keywords: Cumulative prospect theory; investor sentiment; risk-neutral densities; call options.

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[†]Philip Stork (corresponding author) is Professor at VU University Amsterdam, FEWEB, School of Finance and Risk Management, de Boelelaan 1105, 1081 HV Amsterdam, The Netherlands. Tel.: + 31 20 59 83651. E-mail address: p.a.stork@vu.nl.

1 Introduction

Barberis and Huang (2008) hypothesize that Tversky and Kahneman’s (1992) cumulative prospect theory (CPT) explains a number of seemingly unrelated pricing puzzles. In contrast to previous literature, which concentrates on the CPT’s value function (see Benartzi and Thaler, 1995; Barberis et al., 2001; Barberis and Huang, 2001), Barberis and Huang (2008) focus on the probability weighting functions of the model. They conclude that the CPT’s over-weighting of small probability events explains why investors prefer positively skewed returns, or “lottery ticket” type of securities. Because of such preference, investors overpay for positively skewed securities, turning them expensive and causing them to yield low forward returns. The authors argue that this mechanism is the reason why IPO stocks, private equity, distressed stocks, single segment firms and deep out-of-the money (OTM) single stock calls are overpriced among other irrational pricing phenomena.

The proposition made by Barberis and Huang (2008) that deep OTM single stock calls resemble overpriced lottery-like securities due to investors’ overweight of tails has not yet been verified empirically¹. Empirical studies on probability weighting functions implied by option prices are offered by Dierkes (2009), Kliger and Levy (2009), and Polkovnichenko and Zhao (2013)². The evidence provided by these papers is, however, based on the index put options market, which behaves very differently from the single stock option market. The main buyers of OTM index puts are institutional investors, which use them for portfolio insurance (Bates, 2003; Bollen and Whaley, 2004; Lakonishok et al., 2007; Barberis and Huang, 2008). Because institutional investors comprise around two-thirds of the total equity market capitalization (Blume and Keim, 2012), their option trading activity strongly impacts the pricing of put options (Bollen and Whaley, 2004) by making them expensive. The results of Dierkes (2009) and Polkovnichenko and Zhao (2013) reiterate this evidence and suggest that overweighting of small probabilities partially explains the pricing puzzle present in the equity index option market.

Contrary to the index put market, trading activity in single stock calls is concentrated among individual investors (Bollen and Whaley, 2004; Lakonishok et al., 2007) and is speculative in nature (Lakonishok et al., 2007; Bauer et al., 2009; Choy, 2015). Beyond that, Mitton and Vorkink (2007); Bauer et al. (2009); Kumar (2009) provide important empirical support to the link between preference for skewness and individual investor trading activity. The fact that many individual investors have a substantial portion of their portfolios tied up in low risk investments, such as pensions, social security, 401(k)s, IRAs, or are averse (or constrained) to borrow (Frazzini and Pedersen, 2014) encourages them to buy financial instruments with implicit leverage such call options. Hence, given the very distinct clientele of these two option

¹Boyer and Vorkink (2014) provide evidence that lottery-like single stock options do deliver lower forward returns than options with lower ex-ante skewness. However, their paper does not test why these options are overvalued, nor does it analyse the potential time-variation in ex-ante skewness and forward returns.

²These studies focus on the rank-dependent expected utility (RDEU) rather than the CPT, as the RDEU is seamlessly effective in dealing with the overweighting of probability phenomena. The RDEU’s probability weighting functions are strictly monotonically increasing, whereas the CPT one is not. RDEU functions are also easier to estimate because they use one less parameter than the CPT.

markets (institutional investors vs. retail investors) and the different motivation for trading (portfolio insurance vs. speculation), we reason that the OTM single stock calls overpricing is a puzzle in itself, requiring an independent empirical proof from the index option market.

The first contribution of our study is to investigate whether the CPT can empirically explain the claimed overpricing of OTM single stock call options. To that purpose, we empirically test whether tails of the CPT density function outperform the risk-neutral density and rational subjective probability density functions on matching tails of the distribution of realized returns. We find that our estimates for the CPT probability weighting function parameter γ are qualitatively consistent with the ones predicated by Tversky and Kahneman (1992), particularly for short-term options. Our estimates do suggest that overweight of small probabilities is less pronounced than suggested by the CPT though. This analysis complements the results of Barberis and Huang (2008) and provides novel support to explain the overpricing of OTM single stock calls. Our empirical results extend the findings of Dierkes (2009), Kliger and Levy (2009), Polkovnichenko and Zhao (2013), because we show that investors' overweighting of small probabilities is not restricted to the pricing of index puts but also applies to single stock calls.

Secondly, we provide evidence that overweighting of small probabilities is strongly time-varying and connected to the Baker and Wurgler (2007) investor sentiment factor. These findings contrast the CPT model, where the probability weighting parameter for gains (γ) is constant at 0.61. In fact, our estimations suggest that the γ parameter fluctuates widely around that level, sometimes even reflecting underweighting of small probabilities. We show that overweighting of small probabilities was quite strong during the dot-com bubble, which coincided with a strong rise in investor sentiment. The strong time-variation in overweight of tails indicates that investors have either a "bias in beliefs" or time-varying (rather than static) skewness preferences, see Barberis (2013) for a discussion on the topic³.

Moreover, we find that overweighting of small probabilities is largely horizon-dependent, because this bias is mostly observed within short-term options prices (i.e., three- and six-months) rather than in long-term ones (i.e., twelve-months). We reason that such positive term structure of tails' overweighting exist because individual investors may speculate using the cheapest available call at their disposal. In other words, individual investors buy the cheapest lottery tickets that they can find. As three- and six-month options have much less time-value than twelve-month ones, more pronounced overweighting of small probabilities within short-term options seems sensible. This result is consistent with individual investors being the typical buyers of OTM single stock calls and the fact that they mostly use short-term instruments to speculate on the upside of equities (Lakonishok et al., 2007).

³We acknowledge that it is unclear whether overpricing of OTM calls is caused by overweighting of small probabilities (i.e., a matter of preferences), or rather by biased beliefs. Barberis (2013) eloquently discusses how both phenomena are distinctly different and how both (individually or jointly) may explain the overpricing in OTM options. In this paper we take a myopic view and use only the first explanation, for ease of exposition. Disentangling the two (beliefs and preferences) would potentially be very interesting, but we deem it to be outside the scope of this paper.

In our analysis of probability weighting functions, we focus on the outmost tails of RNDs⁴. We argue that, as distribution tails (mostly estimated from OTM options) are the sections of the distribution that reflect low probability events, we may analyze these locally, thus, isolated from the distribution’s body. To this purpose, we use extreme value theory (EVT) and Kupiec’s test (as a robustness check), which are especially suited for the analysis of tail probabilities and, so far, have not been employed yet to the evaluation of overpricing of OTM options. As an additional robustness check, we replace the CPT by the rank-dependent expected utility (RDEU) function of Prelec (1998). This alteration reconfirms the presence of overweighted small probabilities by investors within the OTM single stock call market and, at the same time, reiterates that such bias is less pronounced than suggested by the CPT model. Time-variation of the weighting function parameters is also observed when RDEU is applied.

The remainder of this paper is organized as follows. Section 2 describes the CPT model. Section 3 describes the data and methodology employed in our study. Section 4 presents our empirical analysis and Section 5 discusses our robustness tests. Section 6 concludes.

2 Cumulative Prospect Theory

The Prospect theory (PT) of Kahneman and Tversky (1979) incorporates behavioral biases into the standard utility theory (von Neumann and Morgenstern, 1947), which presumes that individuals are rational⁵. Such behavioral anomalies are i) loss aversion, ii) risk seeking behavior and iii) non-linear preferences⁶. The CPT is described in terms of a value function (v) and a probability distortion function (π). The value function is analogous to the utility function in the standard utility theory and it is defined relative to a reference point zero. Therefore, positive values within the value function are considered as gains and negative values are losses, which leads to:

$$v(x) = \begin{cases} x^\alpha, & \text{if } x \geq 0 \\ -\lambda(-x)^\beta, & \text{if } x < 0 \end{cases} \quad (1)$$

where $\lambda \geq 1$, $0 \leq \beta \leq 1$, $0 \leq \alpha \leq 1$, and x are gains or losses. Thus, along the dominium

⁴Per contrast, Dierkes (2009) and Polkovnichenko and Zhao (2013) explore the relation between overweighting of small probabilities and options prices by analyzing the *full* RND from options. Dierkes (2009) applies Berkowitz’s tests, whereas Polkovnichenko and Zhao (2013) estimate an empirical weighting function via polynomial regressions.

⁵The expected utility theory of von Neumann and Morgenstern (1947) is the standard economics framework on decision making under risk. Their theory assumes that decision-makers behave as if they maximize the expected value of some function defined over the potential (probabilistic) outcomes. Individuals are assumed to have stable and rational preferences; i.e., not influenced by the context or framing.

⁶Loss aversion is the property in which people are more sensitive to (or affected by) losses than gains. For details, see Kahneman and Tversky (1979), Tversky and Kahneman (1992) and Barberis and Huang (2001). Risk-seeking behavior happens when individuals are attracted by gambles with unfair prospects. In other words, the risk-seeking individual is the one that chooses for a gamble versus a sure thing even though the two outcomes have the same expected value. For details, see Kahneman and Tversky (1979). Non-linear preferences occur when preferences between risky prospects are not linear in the probabilities, thus, equally probable prospects are more heavily weighted by agents than others. For details, see Tversky and Kahneman (1992), Fox et al. (1996), Wu and Gonzalez (1996), Prelec (1998) and Hsu et al. (2009).

of x , the CPT's value function is asymmetrically S -shaped (see Figure 1A) with diminishing sensitivity as $x \rightarrow \pm\infty$.

The value function is, thus, concave over gains and convex over losses, differently from the traditional utility function used by standard utility theory. Such a shape of the value function implies diminishing marginal values as gains or losses increase, which means that any additional unit of gain (loss) becomes less relevant when wealth increases (decreases). As α and β increase, the effect of diminishing sensitivity decreases, and as λ increases the degree of loss aversion increases. We also note in Figure 1A that the value function has a kink at the reference point, which implies loss aversion, as the function is steeper for losses than for gains.

[Please insert Figure 1 about here]

The use of a probability distortion function or decision weight function is the adjustment made to the PT to address nonlinear preferences. This function takes probabilities and weights them nonlinearly, so that the difference between probabilities at high percentiles, e.g., between 99 percent and 100 percent, has more impact on preferences than the difference between probabilities at small percentiles, e.g., between 10 percent and 11 percent. This is the main advance of the CPT over the original PT. The CPT applies probability distortions to the cumulative probabilities (i.e., the CDF), whereas the PT applies them to individual probabilities (i.e., the PDF). The enhancement brought by this new formulation satisfies stochastic dominance conditions not achieved by the PT, which renders the CPT applicable to a wider number of experiments. The probability distortion functions suggested by Tversky and Kahneman (1992), respectively, for gains (π_n^+) and losses (π_{-m}^-) are:

$$\pi_n^+ = w^+(p_n) \tag{2a}$$

$$\pi_i^+ = w^+(p_i + \dots + p_n) - w^+(p_{i+1} + \dots + p_n) , \text{ for } 0 \leq i \leq n - 1 \tag{2b}$$

$$\pi_{-m}^- = w^-(p_{-m}) \tag{2c}$$

$$\pi_i^- = w^-(p_{-m} + \dots + p_i) - w^-(p_{-m} + \dots + p_{i-1}) , \text{ for } 1 - m \leq i \leq 0 \tag{2d}$$

where p are objective probabilities of outcomes, which are ranked for gains from the reference point $i = 0$ to $i = n$, the largest gain, and for losses from the largest loss $i = -m$ to $i = 0$, the reference point. Further, w^+ and w^- , the parametric form of the decision weighting functions, are given by:

$$w^+(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}} \tag{3a}$$

$$w^-(p) = \frac{p^\delta}{(p^\delta + (1-p)^\delta)^{1/\delta}} \tag{3b}$$

where parameters γ and δ define the curvature of the weighting function for gains and losses, which leads the probability distortion functions to assume inverse S -shapes. Figure 1B depicts how low probability events are overweighted at the cost of moderate and high probabilities

within the CPT probability distortion functions. Tversky and Kahneman (1992) indicate that the weighting functions for gains are slightly more curved than for losses (i.e., $\gamma < \delta$). The parameters estimated by the authors for the CPT model, which are discussed in our empirical analysis below, are $\lambda = 2.25$; $\beta = 0.88$; $\alpha = 0.88$; $\gamma = 0.61$; $\delta = 0.69$.

3 Data and Methodology

In this section, we first describe the theoretical background that allows us to relate empirical density functions (EDF), RND, and subjective density functions. This is a key step for testing the hypothesis that the CPT helps to explain overpricing of OTM options, because we build on the assumption that investors' subjective density estimates should correspond, on average⁷, to the distribution of realizations (see Bliss and Panigirtzoglou, 2004). Thus, testing whether the CPT's weighting function explains the overpricing of OTM options, ultimately, relates to how the subjective density function produced by CPT's preferences matches empirical returns. Because the representative agent is not observable, subjective density functions are not estimable like EDF and RND are. As such, we build on the following theory to derive subjective density functions from RNDs.

In our empirical exercise, we first derive subjective density functions for (a) the power and (b) exponential utility functions. Because the CPT model contains not only a utility function (the value function) but also a probability weighting scheme (the weighting function), we produce two density functions: (c) the hereafter called partial CPT density function (PCPT), where only the value function is taken into account, and (d) the CPT density function, where the value and the weighting functions are considered. Lastly, we also calibrate γ to market data and are, then, able to compute (e) the estimated CPT density (ECPT). We provide details on estimation methods for our five subjective density functions, (a) to (e), in Section 3.1, and for the RND and EDF in Section 3.4.

Once all five subjective density functions are obtained, we distinguish four analyses in our empirical analysis section: 1) the estimation of long-term CPT value and weighting function parameters (from which we can produce the ECPT density) (Section 4.1); 2) EVT-based tests of consistency between tails of the EDF, the RND and our five subjective probability distributions (Section 4.2); 3) the estimation of time-varying γ parameter (Section 4.3); and 4) a regression linking the CPT time-varying probability weighting parameter (γ) to sentiment measures as well as numerous control variables (Section 4.4).

We use single stock weighted average IV data used for the largest 100 stocks of the S&P 500 index within our RND estimations. Appendix A.2 shows how single stock weighted average IV are computed. Weights applied are the S&P 500 index weights normalized by the sum of weights of stocks for which IVs are available. Following the S&P 500 index methodology and the

⁷This implies that investors are somewhat rational. This assumption is not inconsistent with the CPT assumption that the representative agent is less than fully rational. The CPT suggests that investors are biased, not that decision makers are utterly irrational to the point that their subjective density forecast should not correspond, on average, to the realized return distribution.

unavailability of IV information for every stock in all days in our sample, stocks weights in this basket change on a daily basis. The sum of weights is, on average, 58 percent of the total S&P 500 index capitalization and it fluctuates between 46 and 65 percent. The IV data comes from closing mid-option prices from January 2, 1998 to March 19, 2013 for fixed maturities for five moneyness levels, i.e., 80, 90, 100, 110, and 120, at the three-, six- and twelve-month maturity. Continuously compounded stock market returns are calculated throughout our analysis from the basket of stocks weighted with the same daily-varying loadings used for aggregating the IV data. IV data and stock weights are kindly provided by Barclays⁸. Single stock returns are downloaded via Bloomberg.

We take the perspective of end-users of single stock OTM call options⁹. Hence, we assume that supply imbalances are minimal and do not impact implied volatilities. We think this assumption is reasonable because 1) option markets for the largest 100 U.S. stocks are liquid; 2) any un-hedged risk run by market makers can be easily hedged by purchasing the stock; and 3) unhedged risk by market makers is likely much smaller when supplying call options relative to put options. Market makers run little unhedged risk when supplying call options vis-à-vis supplying puts because stocks returns are negatively skewed, making gap and jump risk much lower on the upside than on the downside. Garleanu et al. (2009) have shown that this condition is different for the index option market, where market makers mostly provide put options for portfolio insurance programs. As the authors suggest, put sellers become more risk-sensitive following equity market declines, as their un-hedged risk increases, which makes them unwilling to write additional puts to the market. Our implied volatility data show no indication of an increase in the implied volatility skew from 120 percent moneyness options, nor from at-the-money options around moments of market stress (e.g., the 2008-09 global financial crisis). Hence, we find no evidence of the presence of supply imbalances in the OTM calls in our sample.

3.1 Subjective density functions

Standard utility theory tells us that since the representative agent does not have risk-neutral preferences, RNDs are inconsistent with subjective and EDF¹⁰, thus both “real-world” probabilities. Hence, if investors are risk-averse or risk seeking, their subjective probability function should differ from the one implied by option prices. The relation between the RND $f_Q(S_T)$,

⁸We thank Barclays for providing the implied volatility data. Barclays disclaimer: *“Any analysis that utilizes any data of Barclays, including all opinions and/or hypotheses therein, is solely the opinion of the author and not of Barclays. Barclays has not sponsored, approved or otherwise been involved in the making or preparation of this Report, nor in any analysis or conclusions presented herein. Any use of any data of Barclays used herein is pursuant to a license.”*

⁹We implicitly assume that end-users of single stock OTM call options have the same preferences across underlyings. This assumption is supported by the evidence provided by Bollen and Whaley (2004) and Lakonishok et al. (2007) that trading activity in equity calls is concentrated among individual investors and is speculative in nature.

¹⁰Anagnou et al. (2002) and Bliss and Panigirtzoglou (2004) have tested the consistency between RNDs and physical densities estimated from historical data and found that such distributions are inconsistent, i.e., RNDs are poor forecasters of the distribution of realizations.

and “real-world” probability distributions, $f_P(S_T)$, with S_T being wealth or consumption¹¹, is described by $\varsigma(S_T)$, the pricing kernel or the marginal rate of substitution (of consumption at time T for consumption at time t)¹²:

$$\frac{f_P(S_T)}{f_Q(S_T)} = \Lambda \frac{U'(S_T)}{U'(S_t)} \equiv \varsigma(S_T), \quad (4)$$

where Λ is the subjective discount factor (the time-preference constant) $U(\cdot)$ is the representative investor utility function. As $U(S_T)$ is a random variable, the pricing kernel is also called the stochastic discount factor. Thus, Eq. (4) tell us that the “real-world” distribution equates to the RND when adjusted by the pricing kernel. The intuition behind Eq. (4) is that a real-world or risk-adjusted probability distribution can be obtained from the RND, once the risk trade-off embedded in the representative investor utility function is considered.

Since CPT-biased investors price options as if the data-generating process has a cumulative distribution $F_{\tilde{P}}(S_T) = w(F_P(S_T))$ ¹³, where w is the weighting function, its density function becomes $f_{\tilde{P}}(S_T) = w'(F_P(S_T)) \cdot f_P(S_T)$ (see Dierkes, 2009; Polkovnichenko and Zhao, 2013). Thus, CPT-biased agents assess probability distributions as if their tails would contain more weight than in reality they do, i.e., they have a preference for skewness or “bias in beliefs”, as Barberis (2013) argues. Consequently, evaluating whether the CPT’s propositions apply is equivalent to testing whether Eq. (4) still holds if $f_P(S_T)$ is replaced by $f_{\tilde{P}}(S_T)$, leading to:

$$\frac{w'(F_P(S_T)) \cdot f_P(S_T)}{f_Q(S_T)} = \varsigma(S_T). \quad (5)$$

We, then, further manipulate Eq. (5) so to directly relate the original EDF to the CPT subjective density function, by “undoing” the effect of the CPT probability distortion functions within the PCPT density function. The relation between EDF and the CPT density function is given by Eq. (6) and its derivation, from Eq. (5), is provide in Appendix A.1:

$$\underbrace{f_P(S_T)}_{\text{EDF}} = \underbrace{\frac{f_Q(S_T)}{\nu'(S_T)} (w^{-1})'(F_P(S_T))}_{\text{CPT density function}} \int \frac{f_Q(x)}{\nu'(x)} dx \quad (6)$$

where $\nu'(S_T)$ is the CPT’s marginal utility function.

This result allows us to obtain a clear representation of the CPT subjective density function, thus, where the value and the weighting function are simultaneously taken into account. At this stage, as we can produce RND and the set of subjective densities of our interest, including

¹¹Note that, as the value function within the CPT measures utility versus a reference point, S_T is not strictly positive in this model. A negative S_T denotes a loss of wealth or consumption, whereas a positive S_T represents a gain.

¹²The condition necessary for Eq. (4) to hold is that markets are complete and frictionless and a single risky asset is traded.

¹³Similarly, if investors are rational, their subjective density functions should be consistent, on average, with the empirical density function. Bliss and Panigirtzoglou (2004) find that subjective density functions, produced from RND adjusted by two types of representative investors’ utility functions (power and exponential) with plausible relative risk aversion parameters, outperform RND on forecasting density functions.

the CPT density, one can evaluate how consistent with realizations their tails are.

3.2 Estimating CPT parameters

We start evaluating the empirical validity of the CPT for single stock call options by comparing EDF to the CPT density function parameterized by Tversky and Kahneman (1992). Subsequently, we estimate CPT weighting function parameters λ and γ with the same goal. We only estimate γ within the probability weighting function, and not δ , because we are interested in the gains-side of the distribution, which is extracted from call options. We estimate these parameters non-parametrically, by minimizing the weighted squared distance between physical distribution and the partial CPT density function for every bin above the median of the two distributions, as follows:

$$v(\lambda) = \text{Min} \sum_{b=1}^B W_b (EDF_{prob}^b - CPT_{prob}^b)^2, \quad (7)$$

where, EDF_{prob}^b and CPT_{prob}^b are, respectively, the probability within bin b in the empirical and CPT density functions and W_b are weights given by $\frac{1}{\frac{1}{\sqrt{2}} \int_{0.5}^{\infty} e^{-\frac{x^2}{2}} dx} = 1$, the reciprocal of the normalized normal probability distribution (above its median), split in the same total number of bins (B) used for the EDF and CPT. The loss aversion parameter, λ , in Eq. (7) is optimized using multiple constraint intervals: $[0,3]$, $[0,5]$ and $[0,10]$. Once the optimal λ is known, we minimize Eq. (8) using its estimate and the CPT λ :

$$w^+(\gamma, \delta = \gamma) = \text{Min} \sum_{b=1}^B W_b (EDF_{prob}^b - CPT_{prob}^b)^2, \quad (8)$$

where γ , the probability weighting parameter for gains, is constrained by the permutation of the following upper bounds (1.2, 1.35, 1.5, 1.75 and 2) and lower bounds (-0.25, 0 and 0.28). Weights applied in these optimizations are due to the higher importance of matching probabilities tails in our analysis than the body of the distributions.

Our non-linear bounded optimization is a single parameter one, where we first estimate optimal γ (which we impose to equal δ) across all permutations of upper and lower bounds to select the bounds that produce the lowest residual sum of square (RSS). Subsequently, we estimate λ and γ as suggested by the sequence of optimizations described by Eqs. (7) and (8). This method resembles the ones of Kliger and Levy (2009), Dierkes (2009), Chabi-Yo and Song (2013), and Polkovnichenko and Zhao (2013). Once optimal parameters λ and γ are estimated, we can produce another long-term subjective density function: the ECPT, which stands for estimated CPT, where we apply the optimal γ for the characterization of its probability weighting function. Finally, we also estimate time-varying γ using different assumptions of λ , so to evaluate the sensitivity of γ to changes in λ .

3.3 Density function tails' consistency test

We check for tail consistency of our set of five subjective density functions (CPT, PCPT, ECPT, power and exponential), RND, and the EDF by applying extreme value theory (EVT). EVT allows us to estimate the shape of the tails of these eight PDFs and to extract the returns implied by an extreme quantile within our PDFs. We estimate the tail shape estimator (φ) by means of the Hill (1975) estimator:

$$\hat{\varphi} = \frac{1}{\hat{\theta}} = \frac{1}{k} \sum_{j=1}^K \ln\left(\frac{x_j}{x_{k+1}}\right), \quad (9)$$

where k is the number of extreme returns used in the tail estimation, and x_{k+1} is the tail cut-off point. The tail shape estimator φ measures the curvature, i.e., the fatness of the tails of the return distribution: a high (low) φ indicates that the tail is fat (thin). The inverse of φ is the tail index (θ), which determine the tail probability's rate of decay. A high (low) θ indicates that the tail decays quickly (slowly) and, therefore, is thin (fat). Such tail shape estimator and tail index give us a good representation of the curvature of the tails, but since tails may have the same shape while estimating diverse extreme observations, we also employ the semi-parametric extreme quantile estimator from De Haan et al. (1994):

$$\hat{q}_p = x_{k+1} \left(\frac{k}{pn}\right)^{\frac{1}{\hat{\theta}}}, \quad (10)$$

where n the sample size, p is a corresponding exceedance probability, which means the likelihood that a return x_j exceeds the tail value q , and x_{k+1} is the tail cut-off point. We note that one of the input of \hat{q}_p is the tail shape estimator φ . Similar to value-at-risk (VaR) modeling, the \hat{q}_p^- statistic indicates the level of the worst return occurring with probability p , which is small. This is the reason why we call \hat{q}_p extreme quantile return (EQR). As we are interested only in the upside returns with a p probability estimated from calls, we only compute \hat{q}_p^+ by applying the same methodology to the right side of the RND obtained from the single stock option market¹⁴.

In addition to the EQR, we also evaluate the density function tails using expected shortfall (ES), which captures the average loss beyond the tail cut-off point. As we are interested in the upside of the distribution, we call such measure expected upside (EU) as the average gain beyond the tail cut-off point. We evaluate the EU following Danielsson et al. (2006) formulae for the ES, which relates the EQR (i.e., the VaR) to the ES (i.e., the CVaR) as described below:

$$\widehat{EU}_{q(p)} = \frac{\hat{\theta}}{\hat{\theta} - 1} \cdot x_{k+1} \left(\frac{k}{pn}\right)^{\frac{1}{\hat{\theta}}}, \quad (11)$$

where θ is the tail index.

De Haan et al. (1994) show that the tail shape estimator statistic $\sqrt{k}(\hat{\varphi}(k) - \varphi)$ and the tail quantile statistic $\frac{\sqrt{k}}{\ln(\frac{k}{pk})} [\ln \frac{\hat{q}(p)}{q(p)}]$ are asymptotically normally distributed. Hence, according

¹⁴Our EQR measure is closely connected to the risk-neutral tail loss measure of Vilkov and Xiao (2013).

to Hartmann et al. (2004) and Straetmans et al. (2008), the t -statistics for such estimators are given by:

$$T_\varphi = \frac{\hat{\varphi}_1 - \hat{\varphi}_2}{\sigma[\hat{\varphi}_1 - \hat{\varphi}_2]} \sim N(0, 1), \quad (12a)$$

and

$$T_q = \frac{\hat{q}_1 - \hat{q}_2}{\sigma[\hat{q}_1 - \hat{q}_2]} \sim N(0, 1), \quad (12b)$$

where the denominators are calculated as the bootstrapped difference between the estimated shape parameters φ and the quantile parameters \hat{q}_p using 1000 bootstraps. The null hypothesis of this test is that $\hat{\varphi}$ and \hat{q}_p parameters do not come from independent samples of normal distributions, therefore, $\hat{\varphi}_1 = \hat{\varphi}_2$ and $\hat{q}_1 = \hat{q}_2$. The alternative hypothesis is that $\hat{\varphi}$ and \hat{q}_p have unequal means. Such t -test is also applied to our EU analysis, as the distribution of EU follows the same distribution of the tail quantile statistic $\frac{\sqrt{k}}{\ln(\frac{k}{pk})}[\ln \frac{\hat{q}(p)}{q(p)}]$, given that EU is the extreme quantile estimator multiplied by a constant.

3.4 Estimating RND and EDF

For the estimation of the RND, the first step taken is the application of the Black-Scholes model to our IV data to obtain options prices (C) for the S&P 500 index. Once our data is normalized so strikes are expressed in terms of percentage moneyness, the instantaneous price level of the S&P 500 index (S_0) equals 100 for every period for which we would like to obtain implied returns. Contemporaneous dividend yields for the S&P 500 index are used for the calculation of P as well as the risk-free rate from three-, six-, and twelve-month T-bills. Because we have IV data for five levels of moneyness, we implement a modified Figlewski (2010) method for extracting our RND structure, as in Felix et al. (2016). The main advantage of this method over other techniques is that it extracts the body and tails of the distribution separately, thereby allowing for fat tails.

The Figlewski (2010) method is close to the one employed by Bliss and Panigirtzoglou (2004), where body and tails are also extracted separately. Bliss and Panigirtzoglou (2004) use a weighted natural spline algorithm for interpolation, which has the same decreasing-noise effect in RNDs of using splines in the absence of knots, as done in Figlewski (2010). The extrapolation in Bliss and Panigirtzoglou (2004) is done by the introduction of a pseudo-data point, which has the effect of pasting lognormal tails into the RND. One advantage of these two approaches is that the extrapolation does not result in negative probabilities, which is possible when splines is applied in such case. Nevertheless, we favor Figlewski's (2010) approach as the lognormal tails employed by Bliss and Panigirtzoglou (2004) assume that IV is constant beyond the observable strikes, resembling the Black-Scholes model. The modification made to the Figlewski (2010) method by Felix et al. (2016) entailed having flexible inner anchor points (as opposed to having fixed anchor points) for fitting tails to the risk neutral density. The aim of this modification is to prevent the method to estimate distribution density functions with

implausible shapes.

We estimate the EDF in two different ways. First, using the entire sample of realized returns (r), we estimate long-term EDFs non-parametrically, where $r = \ln(S_T/S_t)$ and S_t is the realized return index at time t and S_T is the forward level of the same index three-, six- or twelve-months forward, i.e., respectively 21, 63 and 252-days forward. Because of overlapping periods, we initially estimate our empirical distribution from non-overlapping returns for these three maturities by using distinct starting points. This methodology is also applied by Jackwerth (2000) and Ait-Sahalia and Lo (2000). However, because the length of the overlapping periods is relatively large compared to our total sample, especially for the twelve-month forward returns, we average the distribution with distinct starting points to smooth the shape of our multiple-horizon distributions¹⁵.

In a second step, we estimate time-varying EDFs built from an invariant component, the *standardized innovation density*, and a time-varying part, the conditional variance ($\sigma_{t|t-1}^2$) produced by an EGARCH model (see Nelson, 1991). We first define the *standardized innovation*, being the ratio of empirical returns and their conditional standard deviation ($\ln(S_t/S_{t-1})/\sigma_{t|t-1}$) produced by the EGARCH model. From the set of standardized innovations produced, we can then estimate a density shape, i.e., the *standardized innovation density*. The advantage of such a density shape versus a parametric one is that it may include, the typically observed, fat-tails and negative skewness, which are not incorporated in simple parametric models, e.g., the normal. As mentioned, such density shape is invariant and it is turned time-varying by multiplication of each standardized innovation by the EGARCH conditional standard deviation at time t , which is specified as follows:

$$\ln(S_t/S_{t-1}) = \mu + \epsilon_t, \epsilon \sim f(0, \sigma_{t|t-1}^2) \quad (13a)$$

and

$$\sigma_{t|t-1}^2 = \omega_1 + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1|t-2}^2 + \vartheta \text{Max}[0, -\epsilon_{t-1}]^2, \quad (13b)$$

where α captures the sensitivity of conditional variance to lagged squared innovations (ϵ_{t-1}^2), β captures the sensitivity of conditional variance to the conditional variance ($\sigma_{t-1|t-2}^2$), and ϑ allows for the asymmetric impact of lagged returns ($\vartheta \text{Max}[0, -\epsilon_{t-1}]^2$). The model is estimated using maximum log-likelihood where innovations are assumed to be normally distributed.

Up to this point, we managed to produce a one-day horizon EDF for every day in our sample but we still lack time-varying EDFs for the three-, six-, and twelve-month horizons. Thus, we use bootstrapping to draw 1,000 paths towards these desired horizons by randomly

¹⁵As a robustness check to this approach, we compare our three-, six- and twelve-month empirical distributions with the ones calculated from non-overlapping returns. We use data since 1871 for the US equity price index, made available by Welch and Goyal (2008), who use S&P 500 data since 1926, and data from Robert Shiller's website for the preceding period. Our empirical distributions are quite similar to the ones estimated from the longer data set, suggesting that they are, indeed suitable as long-term distributions. The use of overlapping returns is less problematic in our calculations than in regression estimation, where statistical inferences on parameter estimates can be strongly affected by overlapping returns' serial correlation.

selecting single innovations (ϵ_{t+1}) from the one-day horizon EDFs available for each day in our sample. We note that once the first return is drawn, the conditional variance is updated ($\sigma_{t-1|t-2}^2$) affecting the subsequent innovation drawings of a path. This sequential exercise continues through time until the desired horizon is reached. In order to account for drift in the simulated paths, we add the daily drift estimated from the long-term EDF plus the risk-free rate to drawn innovations, thus the one-period simulated returns is $\epsilon_{t+1} + \mu + Rf$. The density functions produced by the collection of returns implied by the terminal values of every path and their starting points are our three-, six-, and twelve-month EDFs. These simulated paths contain, respectively, 63, 126, and 252 daily returns. We note that by drawing returns from stylized distributions with fat-tails and excess skewness, our EDFs for the three relevant horizons also imbed such features. Finally, once these three time-varying EDFs are estimated for all days in our sample, we estimate γ for each of these days using Eq. (8)¹⁶.

Our approach for estimating both the long-term EDF and the time-varying EDF is closely connected to the method applied by Polkovnichenko and Zhao (2013). The time-varying method used by these authors is based on Rosenberg and Engle (2002). The choice for an EGARCH approach versus the standard GARCH model is due to the asymmetric feature of the former model that imbeds the “leverage effect”¹⁷.

4 Empirical analysis and results

In this section, we present our results of the empirical analysis described in section 3. We note that since we estimate EDF in the two ways described (the long-term and time-varying EDFs), we are able to estimate long-term γ 's and time-varying γ 's by minimizing Eq. (8). We use our long-term γ estimates to compute the ECPT with the aim to compare it to the other subjective density functions using the tests described in Section 3.3. The time-varying estimates of γ are analyzed in Sections 4.3 and 4.4 with the use of a regression model. We describe this regression together with its results in Section 4.4. Finally, in Section 5, we perform robustness tests on our results by using an alternative weighting function to the CPT, the one imbedded in the Prelec (1998) model, and we apply Kupiec's test to probability tails, among other checks.

4.1 Estimated CPT long-term parameters

We report the estimated CPT parameters (λ and γ) extracted from long-term density functions in Table 1, Panel A. Our first finding is that λ , the parameter of loss aversion, which is 2.25 in the CPT, fluctuates around that number for six- and twelve-month options but shows a quite different outcome for three-month options. Our estimation of λ from three-month options is

¹⁶Due to drift, the model of time-varying EDF for the twelve-month horizon occasionally does not match the one of the PCPT model. This difference is challenging to estimation of γ (Eq. (8)), as a large amount of γ estimates produce unreasonable PDFs such as non-monotonic CDFs. Therefore, to perform the optimizations given by Eq. (8), we neutralize the impact of the drift by forcing the mode of the simulated EDF to match the one of the PCPT.

¹⁷The leverage effect is the negative correlation between an asset's returns and changes in its volatility. For a comparison between alternative GARCH approaches, see Bollerslev et al. (2009).

1.02, which indicates no loss aversion. For the six- and twelve-month options λ is 2.66 and 3.00, respectively. This finding suggests that loss aversion is more pronounced at longer maturities than suggested by the CPT. Apart from that, twelve-month λ estimates are highly variant across the different optimization upper bounds used (i.e. 3, 5 and 10), always matching the bound value, whereas estimates from three- and six option maturities are very stable across upper bounds.

[Please insert Table 1 about here]

The estimated probability weighting function parameter γ is slightly higher than the one suggested by the CPT (i.e., 0.61) at the three- and six-month horizons, respectively, at 0.75 and 0.81. For twelve-month options, γ is around 1.09. These results suggest that overweighting of small probabilities occurs in short-term options (up to six-months), while twelve-month options seem to behave more rationally. These findings support our hypothesis that individual investors are, on average, biased when purchasing single stock call options, as suggested by Barberis and Huang (2008).

4.2 Density functions tails' consistency test results

As specified in section 3.3, we test the empirical consistency of density function tails among a set of five subjective distributions (CPT, PCPT, ECPT, power, exponential), the RND, and the EDF. We perform these tests by employing EVT through the application of Eqs. (9) to (12b). For such purpose, we require return streams (x_j) , which are only available for the long-term EDF. Thus, we apply an inversion transform sampling technique to our other PDFs to obtain sampled returns for them. Such method, also known as the Smirnov method, entails drawing n random numbers from a uniformly distributed variable $U = (u_1, u_2, \dots, u_n)$ bounded at interval $[0, 1]$ and, subsequently, computing $x_j \leftarrow F^{-1}(u_j)$, where F are the CDFs of interest (see Devroye, 1986, p.28). Hence, the Smirnov method simulates returns that resemble the ones of the inverse CDF by randomly drawing probabilities along such function.

Once we obtain returns for all five PDFs, the next step is to set k as the optimal number of observations used for estimation of φ by Eq. (9), the Hill-estimator. For this purpose, we produce Hill-plots for the right tail of our distributions, which depict the relationship between k and φ as a curve (see Straetmans et al., 2008). Picking the optimal k is done by observing the interval in this curve where the value of φ stabilizes while k changes. This area suggests a stable trade-off between a good approximation of the tail shape by the Pareto distribution and the uncertainty of such approximation (by the use of fewer observations). The interval that corresponds to roughly four to seven percent of observations seems to be a stable region across the Hill-plots of the tails of the EDF and the CPT. As an increase in k increases the statistical power of the estimator but may distort the shape of the tail, we decide to set k as chosen from the Hill-plots for EDF and CPT tails equal to four percent.

We examine whether the tail shape parameter (φ), computed via the Hill (1975) estimator, for the RND and for our subjective density functions (i.e., power, exponential, PCPT, CPT and ECPT) matches the one for the EDF. The outcomes from the statistical tests performed

to compare tail shape parameters (Eq. (12a)) are provided in Table 1, Panel B. Results suggest that for the three-month maturity options, φ for the RND, CPT and ECPT (at 0.20) are the closest to the EDF parameter (at 0.29) but they are not statistically equal. The φ estimate for the power, exponential, and PCPT density functions do not match the one for the EDF, as they are all around 0.17 and, thus, exhibit fatter tails than the EDF.

We observe that the results for the six- and twelve-month options are very similar to the ones obtained for the three-month expiry. The parameter estimate φ of the EDF is statistically equal to the RND and CPT. Parameter φ ranges from 0.18 to 0.19 for the CPT, ECPT, and RND for the six- and twelve-month maturities, whereas it is 0.23 for the EDF. The estimate of φ for the RND (0.19 and 0.22 for the six- and twelve-month maturities, respectively) somewhat matches the one for the EDF at the six-month maturity but it is off at the twelve-month maturity. The parameter estimates φ for the power, exponential, and PCPT density functions match the EDF's φ at the twelve-month maturity only. Generally, the parameter estimates φ for these subjective density functions are too small in comparison to the one of the EDF. This means that these six- and twelve-month maturity subjective density functions have fatter tails than the EDF, the other subjective densities (CPT and ECPT), and the RND. These results suggest that the shape of the CPT density function is a good match to the shape of realized tails.

After k is chosen and the shape estimator φ for the EDF, RND, power, exponential, PCPT, CPT, and ECPT is computed, extreme quantile returns (EQR) can also be estimated via Eq. (10). Subsequently, the t -test in Eq. (12b) is applied using the one, five and ten percent statistical significance levels. This test evaluates whether the EQRs estimated from a set of two distributions (RND, power, exponential, PCPT, and CPT versus EDF) have equal means (the null hypothesis). The results of this test are shown in Table 2, Panel A.

[Please insert Table 2 about here]

Analyzing the density functions derived from the three-month option maturity, we find that the EQR implied by the CPT is the only one that matches the realized EQR and at the first quantile solely at 21 percent. The EQR implied by the ECPT is almost the same as implied by the CPT, thus, it also statistically matches the EDF. Per contrast, the EQRs for the RND, power, exponential, and PCPT densities always overshoot the one for the EDF. All comparisons between these distributions' EQR at the three-month maturity reject the null hypothesis that returns at the same quantile are equal. This pattern is observed across all quantiles analyzed, i.e., at the tenth, fifth, and first quantiles. This empirical finding indicates that the equity market upside implied in option markets (i.e., the RND) and the power, exponential and PCPT densities are always higher than the ones realized by the equity market. The results for the PCPT resemble the ones for the RND. The EQRs from the CPT and the ECPT are clearly the best matches for the EDF.

For the six-month maturity, upside returns priced by the RND and ECPT best match the EQR. The EQRs for the EDF are roughly 18, 22, and 32 percent for the tenth, fifth, and first quantile of returns, respectively, whereas the EQRs for the ECPT are 19, 21, and 28 percent.

For the RND, such extreme upside return estimates are 19, 22, and 30 percent. Thus, the ECPT statistically matches the realized EQR best at the tenth and fifth quantile, whereas the RND is the best match for the third quantile. No rational subjective density function consistently matches the EQR of the EDF. The power, exponential, and PCPT densities almost always overshoot the EQR of realized returns. Per contrast, the CPT density always undershoots the EDF's extreme returns. Despite always overshooting the EQR of the EDF, the PCPT is the only other subjective density (apart from the ECPT) that has EQR statistically equal to the EDF, which happens only at the first quantile EQR.

In contrast to the three- and six-month maturities, the EQRs from the RND for the twelve-month maturity all underestimate the EQRs from realized returns. The EQRs of realized returns are 32, 35, and 44 percent for the tenth, fifth and first quantiles, respectively, whereas for the RND these are 22, 26, and 37 percent, respectively. The same underestimation is documented for the densities linked to the CPT (i.e., PCPT, CPT and ECPT) as tail returns are largely out of sync with realized ones, especially for the CPT in which overweight of tails will force EQRs further away from EDF ones (*vis-à-vis* the PCPT EQRs). The EQRs of the exponential densities continue to largely overshoot the ones for the EDF. However, the power utility function density successfully matches the EQR returns across all EQR values and with strong statistical significance.

In line with these results for the EQR, Table 2, Panel B, shows that the expected upside (EU) for the EDF is more closely matched within the three-month horizon by the CPT and ECPT density functions for the tenth, fifth and first quantiles. The three-month horizon EUs estimated from the realized returns are 15, 19, and 30 percent for the mentioned quantiles. The ECPT EUs for the same horizon are 18, 21, and 28 percent, respectively. For the CPT, EUs are 16, 19 and 26 percent. Thus, estimates from these two density functions are mostly statistically equal to the realized returns. Similarly to our analysis on the EQR, for the other subjective densities, the EUs for all quantiles are also much larger than the EDF expected upside. The exponential density has the highest expected upside across the different quantiles, being the furthest away from the realized returns. The RND-implied expected upside is somewhat conservative and relatively closer to the realized ones but only statistically significant at the one percent quantile.

For the six-months maturity, the expected upsides for the CPT and ECPT density functions are no longer that close to each other nor to the realized ones. The EDF expected upside always exceeds the ones for the CPT and ECPT. Only at the tenth quantile, the expected upside of the ECPT density function equals the realized one. The densities which better match the expected upside of the EDFs are the PCPT and the RND.

For the twelve-month horizon, the expected upside for the realized returns is 37, 40, and 51 percent for the tenth, fifth and first quantiles. In line with the results from our EQR analysis, the power density again best matches realized EUs, as estimates are statistically equal across all maturities. Second best performers are the PCPT and ECPT densities, which match the realized EU at the one percent quantile level.

In summary, across the three EVT tests performed (i.e., on tail shape, EQR and EU), the

three option maturities and the three quantiles evaluated, we observe that the success rate of the CPT subjective density functions on matching the EDF tails is 57 percent. In contrast, this success rate is 38 percent for the power utility, 33 percent for the RND and only 10 percent for the exponential utility density function. These results suggest that CPT-related distributions, although not always matching the EQRs and ES of the EDF, seem to best match the EQR of the EDF, especially at the short maturities. More specifically, the ECPT seems to have some advantage over the other methods for the three- and six-month maturities. This result is not a surprise, because allowing the CPT weighting function to assume different shapes entails extra flexibility to match the data relative compared to traditional utility functions. Thus, if our findings suggest that the CPT does not fully explain single stock options pricing, its overweighting of small probabilities feature goes very far in explaining such market data, with the exception of twelve-month options. These findings reiterate our takeaway from Section 4.1, in which a positive term structure of overweight of tails appears to play a substantial role: twelve-month options are priced more rationally than shorter term ones, which seem to be priced as a result of lottery buying by individual investors. Figure 2 compares the CDFs from six of our equity return densities: the EDF, the RND, the CPT, the PCPT, the exponential- and the power-utility density¹⁸. We focus on the right tails of these distributions as we are interested in how closely the RND from call options and derived subjective density functions match the tails of the EDF. The plots display the cumulative probabilities on the y-axis and the terminal price levels on the x-axis, given an initial price level of 100.

[Please insert Figure 2 about here]

In Figure Figure 2, we see that the tails implied by option prices (RND, in red) seem fatter than the tails from the CPT (in dark blue) and EDF (in green) density functions over the three-month horizon. The tails for the CPT and the EDF are almost identical above the 120 terminal level, i.e., at the 20 percent return. The right tail of the RND distribution is clearly much fatter than the ones of the CPT and EDF, but it is still thinner than the ones of the PCPT, the exponential- and the power-utility densities. Thus, the upside risk implied from options is much higher than the one realized by the EDF, a sign of a potentially biased behavior by investors in such options. This observation is confirmed by the tail shape parameter (φ), the EQRs and the EU estimated across the different quantiles, which in all cases report higher upside in the RND than in the EDF and the CPT. Figure 2 also suggests that the upside risk of the RND is more consistent with the PCPT density, whereas the CPT tails seem very distinct from the PCPT, which is in line with our earlier findings.

The plot in column B, which depicts the CDF for our studied densities at the six-month horizon, suggests that the RND and the EDF are closer than at the three-month horizon. At the same time, the CPT density seems more disconnected from the EDF. This finding matches our results from the EQR and the expected upside comparisons. The PCPT tail is, at this horizon, higher than the EDF, CPT, and RND ones and closer to the EDF one than

¹⁸We omit the ECPT for better visualization as its CDFs are very similar to the CPT ones. The similarity is caused by the ECPT left tail weighting function parameter (δ) being the same for the CPT and because the estimated long-term γ for the three maturities are close to the Tversky and Kahneman (1992) one.

to the CPT one, especially at its very extreme. This finding is also confirmed by our EQR and expected upside tests, as the PCPT is statistically equal to the EDF at the one percent quantile. The exponential and power utility densities have right tails that are much fatter than the other densities, including the EDF.

Figure 2 shows that at the twelve-months horizon the CPT's CDF tails seem completely disconnected from the EDF. The EDF tails are much fatter than the CPT ones and slightly fatter than the RND ones. In fact, the RND seems to match the EDF for terminal levels above 120. This finding suggests that long-term options trade in a much less CPT-biased manner than short-term options.

Overall, the visual inspection of our density function CDFs confirms our hypothesis that end-users of OTM single stock calls are likely biased and behave as buying lottery tickets when trading short-term options. These results strengthen the evidence provided by Barberis and Huang (2008), Ilmanen (2012), Barberis (2013), Boyer and Vorkink (2014) and Choy (2015) that investors push single stock options prices to extreme valuation levels. Investors seem to overweight small probabilities especially at short-term horizons. Next, we analyze the time-variation in overweight of small probabilities to better understand the underlying reasons for our findings.

4.3 Estimated CPT time-varying parameters

To investigate time-variation in the CPT's overweighting of small probabilities in single stock options, we apply Eq.(8) to each day in the sample to estimate the empirical γ (weighting function) parameter. Lower and upper bounds of -0.25 and 1.75 were used in this optimization as they produced the lowest RSS across permutation of all bounds when γ was optimized using the CPT parameterization. We estimate γ under four different assumptions about λ , the loss aversion parameter: 1) λ equals 2.25, the CPT parameterization; 2) no loss aversion, λ equals 1; 3) augmented loss aversion, λ equals 3; and 4) optimal λ , as estimated by Eq.(7).

Table 3, Panel A reports the statistics when λ equals 2.25. We find that the median and the mean time-varying values of γ , estimated from the three-month options are above its CPT value of 0.61 but still reflect overweight of small probabilities. This suggests that overweighting of small probabilities is present within the pricing of three-month call options as suggested by the theory. The distribution of γ is skewed to the right and overweight of small probabilities is present 64 percent of times within three-month maturity. The 25th percentile of γ is 0.74, clearly suggesting a less pronounced overweight of small probabilities than suggested by the CPT. The estimates of γ range from 0 to 1.75 (i.e., an underweighting of small probabilities) and are volatile, with a standard deviation of 0.23. Interestingly, when we split the sample in three parts (as shown in Table 3), we observe that overweight of small probabilities is very present at the beginning of our sample, in 97 percent of the days from 1998-01-05 to 2003-01-30, but that has faded since 2003. During the period from 2003-01-31 to 2008-02-21, underweight of small probabilities is present in 65 percent of the days, whereas such condition is less pervasive from 2008-02-22 onwards, i.e., until 2013-03-19. This finding suggests that overpricing of single

stock options is sample specific and not structural. Even if sample specific, overweight of small probabilities seems, in general, much less pronounced than the 0.61 parameter offered by the CPT. These results seem to only partially confirm our hypothesis that the CPT can empirically explain the overpricing of OTM single stock call options.

[Please insert Table 3 about here]

At the six-month maturity, overweighting of small probabilities is less frequent than in three-month tenor. The median γ for such maturity is 0.99, implying roughly neutral probability weighting. The long-term γ equals 0.81 and is somewhat out-of-sync with the time-varying estimates. Similarly to the three-month maturity, the distribution of γ is also slightly skewed to the right. The 75th quantile of γ equals 1.14 and suggests an underweighting of tail probabilities. However, probability weighting is largely sample dependent as within the overall sample, 52 percent of all observations reflect overweight of small probabilities but, between 1998 and 2003, its occurrence is 92 percent.

Differently from the other maturities, γ estimates for the twelve-month maturity tend towards underweight of tail probabilities. The median γ is 1.03, whereas the mean γ is 1.01. Time variation and sample dependence are present as for the other maturities but, at the twelve-month maturity, the percentage of days with overweight of tails is smaller, 41 percent in the full sample but still 83 percent for the 1998-2003 sample.

In summary, the statistics in Table 3, Panel A, indicate that the weighting function parameters γ for the three maturities evaluated are time-varying and sample specific. Overweight of small probabilities holds for the three-month maturity, less convincingly so for the six-month maturity, and not at all for the twelve-month maturity, in which neutral probabilities and underweight of tails respectively prevails.

Because the loss-aversion parameter λ is of high importance in the CPT model, we estimate γ under different λ parameterizations, more specifically, for 1) λ equals 1, 2) λ equals 3 and optimal λ , as estimated from the long-term empirical distribution (see Table 1).

We report the summary statistics of the new γ estimates in Panel B of Table 3, when we assume λ equals 1. The new median and mean estimates for γ are 0.66 and 0.67 for the three-month maturity, respectively, and, thus, lower than when γ was estimated under the CPT loss aversion calibration ($\lambda=2.25$). The 75th percentile of γ also decreases, from 1.14 to 0.80. At the six-month horizon, the difference between γ with λ equals 2.25 and with λ equals 1 is also large. The median γ for the CPT λ is 0.99, whereas for when λ equals 1 it is 0.71. The means are 0.96 and 0.72, respectively. At the 75th percentile using λ equals 1, γ becomes 0.87. For the twelve-month maturity, we observe a similar effect. The median γ for when λ equals 1 is 0.83, whereas for when λ equals 2.25 it is 1.03. In brief, a lower loss aversion parameter consistently gives rise to higher γ estimates, across the different options' maturities and quantiles. The opposite effect is observed when the λ is increased from 2.25 to 3, as shown by Table 3, Panel C. The median and mean γ when λ equals 3 becomes 0.96 and 0.98 for the three-month maturity, in comparison to 0.91 and 0.89 when λ equals 2.25. Such rise in central tendency of γ estimates is also observed within the six- and twelve-month maturities and across the 25 and 75 percent

quantiles. Table 3, Panel D, which reports γ estimates when optimized λ parameters are used, shows distinct results for the three-month maturity versus the six- and twelve-month maturity. For the three-month maturity, we observe a downward shift to γ estimates, whereas for six- and twelve-month maturities, an upward movement in estimates occurs. However, this initially opposite effect in estimates is, in fact, qualitatively equal to the result just described when we use λ as 1 or 3, as the optimal λ parameters estimated for the three-, six and twelve-month maturities are, respectively, 1.02, 2.66 and 3.00 (i.e., it decreases for the three-month maturity and increases for the six- and twelve-month maturity vis-à-vis the CPT parameterization).

The reason why a lower (higher) loss aversion gives rise to a decreased (increased) γ is that it increases (decreases) the probability on the left side of distribution, influencing the probabilities and the shape of the right side of the CPT distribution. High values of λ push the CPT density to have more probability on the right side of the distribution, which is spread proportionally to the probabilities originally observed in the right-side bins (i.e., creating a bump into the center-right side of the distribution), all else equal. Thus, the impact of such probability shift fades as the tail approaches. Nevertheless, the right tail of the CPT density does turn fatter (and the γ parameter higher) as λ is made higher. The opposite occurs if low values of λ are assumed: the right tail of the CPT density becomes thinner, causing γ estimates to be low (which more forcefully can turn the RND right tail into such thin CPT tail). One important finding from our experimentation with different λ parameters is that the time variation observed when λ equals 2.25 is unchanged. The standard deviation and range of γ estimates across the use of the different λ values are somewhat the same. Though, the percentage of days that overweight of tails is observed in the different samples studied dramatically changes towards a more frequent presence of overweight of small probabilities, as low levels of λ are used (and vice-versa). The large difference in the presence of overweight of small probabilities across samples remains.

We interpret our finding that γ is strongly time-varying and sample dependent across all maturities and under different λ assumptions as a strong evidence that single stock options are not overvalued due to a structural skewness preference, as Barberis (2013) may suggest. We reckon that, if static skewness preferences would drive overweight of small probabilities, parameter γ would be relatively stable throughout our sample. Given that the γ is largely volatile, we support the view that investors experience (time-varying) “bias in beliefs” or, alternatively, time-varying preferences (see Barberis, 2013)¹⁹. Our results are in line with Green and Hwang (2011), Chen et al. (2015) and Jiao (2016), who report similar time-varying effects in the overpricing, skewness effects and returns for IPOs and lottery-like stocks. These papers also report that, beyond time-varying effects, stronger skewness preferences are associated with higher participation of individual investor (trading in IPOs, trading around earnings announcements and

¹⁹Barberis (2013) distinguishes investors’ time-varying beliefs from skewness preferences as he argues that investors with biased beliefs mistakenly *overestimate* tail events, whereas preference for skewness leads to overweight of tails, which is less likely to be a mistake. As an example, the author suggests that investors that overweight small probabilities events correctly anticipate the distribution of a stock’s future returns but overweight the state of the world in which a stock turns out to be “the next Google”. In the example, *overestimation* of tail events would occur when the investor attributes a higher chance to the stock being the next Google. As we do not attempt to distinguish between biased beliefs and time-varying preferences, we use the term *overweight* of small probabilities throughout our paper.

owning stocks) in detriment of institutional investors.

4.4 Time variation in probability weighting parameter and investors' sentiment

As observed in section 4.3, the probability weighting parameter γ is clearly time-varying. In the following, we investigate which factors may explain this time-variation of γ . Our main hypothesis is that it is linked to investor sentiment. The link between sentiment and overweighting of small probabilities or lottery buying in OTM single stock calls originates from the fact that individual investors are highly influenced by market sentiment and attention-grabbing stocks (Barberis et al., 1998; Barber and Odean, 2008), and that OTM single stock calls trading is speculative in nature and mostly done by individual investors (Lakonishok et al., 2007). For instance, Lakonishok et al. (2007) argue that the IT bubble of 2000, a period of high variation of γ , is linked to elevated investor sentiment, when the least sophisticated investors were the ones most inclined to purchase calls on growth and IT stocks. Figure 3 depicts time-varying γ 's and the Baker and Wurgler (2007) sentiment factor. It provides evidence that these measures move in tandem at times. For example, during the IT bubble, the level of γ seems quite connected with the level of sentiment, especially for the three- and six-month options.

[Please insert Figure 3 about here]

To formally test our hypothesis that time variation of γ is linked to investor sentiment, we design a regression model. In Eq. (14) the explained variables are γ for the three-, six-, and twelve-month horizons and the explanatory variables are the Baker and Wurgler (2007) sentiment measure²⁰; the percentage of bullish investors minus the percentage of bearish investors given by the survey of the American Association of Individual Investors (AAII), used as a proxy for individual investor sentiment by Han (2008); and a set of control variables among the ones tested by Welch and Goyal (2008)²¹ as potential forecasters of the equity market. The data frequency used in the regression is monthly as this is the highest frequency available from the sentiment data and from the Welch and Goyal (2008) data set²². Our regression sample starts in January 1998 and ends in December 2010²³. Our OLS regression model is specified as follows:

²⁰Available at <http://people.stern.nyu.edu/jwurgler/>.

²¹The complete set and description of variables suggested by Welch and Goyal (2008) is provided in Appendix B. From the complete set of variables used by Welch and Goyal (2008), we select a smaller set using the cross-correlation between them to avoid multicollinearity in our regression analysis. Because we run a multivariate model, using the full set of variables is undesirable as some of them correlate 80 percent with each other. We exclude variables that correlate more than 40 percent with each other.

²²Given the fact that γ is estimated on a daily basis, we average γ throughout each month.

²³This regression sample is only possible because Welch and Goyal (2008) updated their dataset after the paper publication. The regression sample however, could not be extended further than December 2010 because the sentiment measure of Baker and Wurgler (2007) is not available after that date.

$$\gamma_t = c + \psi_1 \cdot Sent_t + \psi_2 \cdot IISent_t + \psi_3 \cdot E12_t + \psi_4 \cdot B/m_t + \psi_5 \cdot Ntis_t + \psi_6 \cdot Rfree_t + \psi_7 \cdot Infl_t + \psi_8 \cdot Corpr_t + \psi_9 \cdot Svar_t + \psi_{10} \cdot CSP_t + \epsilon_t, \quad (14)$$

where *Sent* is the Baker and Wurgler (2007) sentiment measure, *IISent* is the AAI individual investor sentiment measure, *E12* is the twelve-month moving sum of earnings on the S&P5000 index, *B/m* is the book-to-market ratio, *Ntis* is the net equity expansion, *Rfree* is the risk-free rate, *Infl* is the annual inflation rate, *Corpr* is the corporate spread, *Svar* is the stock market variance and, *CSP* is the cross-sectional premium.

Additionally, we run univariate models for each explanatory factor to understand the individual relation between γ and the control variables:

$$\gamma_t = c_i + \psi_i \cdot x_{i,t} + \epsilon_t, \quad (15)$$

where x replaces the n explanatory variable earlier specified, given $i = 1 \dots n$.

[Please insert Table 4 about here]

Table 4, Panel A presents the estimates of Eq. (14). We note the high explanatory power of the multivariate regression, ranging from 68 to 71 percent. As expected, we observe that *Sent* is consistently negative and statistically significant across the three different horizons studied. On average, each one-unit difference in *Sent* is linked to roughly -0.1 difference in γ , all else being equal. The univariate regressions of *Sent* confirm the negative link between sentiment and γ . For all option maturities, a negative relation between the Baker and Wurgler (2007) sentiment measure and γ is clearly found. The explanatory power of the variable *Sent* in the univariate setting is also high, between 22 and 29 percent. These findings altogether support our hypothesis that overweighting of small probabilities increases at higher levels of sentiment and that sentiment strongly impacts the probability weighting bias of call option investors.

In contrast with the variable *Sent*, the coefficients for the individual investor sentiment (*IISent*) are positive but not statistically significant either on the multivariate setting or on the univariate one (see Table 4). The univariate regressions run on γ have rather low explanatory power. The positive relationship between *IISent* and γ at the three-month maturity may be attributed to potential capitulations in individual investor sentiment, as such indicator is strongly mean-reverting.

The nine Welch and Goyal (2008) control variables used in our multivariate regression are linked to γ in very distinct manners. First, it is fair to say that they add substantial explanatory power to our multivariate regressions. The three-, six-, and twelve-month multivariate models explain, respectively, 71, 68, and 67 percent of the level of γ . Most of these relations are stable, because the coefficient signs change only rarely. The control variables that are statistically significant in our multivariate setting are *E12*, *B/m*, *Rfree*, *Infl*, *Svar*, and *CSP* (Table 4). We observe that γ is positively linked to *E12*, the twelve-month moving sum of earnings on the S&P 500 index, as well as to *B/m*, the book to market ratio, in both multivariate and univariate

regressions. The positive relation between $E12$, B/m and γ could be explained by mean-reversion of earnings and valuation being linked to a greater overweighting of small probabilities, which could be justified by the higher investor sentiment outweighing earning downgrades and rising valuations in a rallying market. These two variables have high explanatory power of γ , respectively, 37 and 31 percent for the three-month horizon. The significance of $Rfree$ is, however, somewhat unstable. At the three- and six-month maturity at the multivariate regression $Rfree$ is significant but not at the univariate regression. Further, the stock market variance, $Svar$, is negatively linked to γ . Apparently, the higher the risk environment, the higher the overweighting of small probabilities is. In a univariate setting (at the three-month horizon), the explanatory power of such univariate regression is 18 percent, thus relatively high. Table 4, Panel B indicates that the cross-sectional premium CSP is positive and statistically significant in the univariate setting for the three-month horizon, despite being negative and not significant in the multivariate regressions.

To reiterate our results, we also apply the Least Absolute Shrinkage and Selection Operator (Lasso) methodology to our main multivariate regressions (see Tibshirani, 1996, and Appendix A.3). We apply Lasso to select the regressors that are most relevant for the overall fit of the γ by our sentiment and control variables. The coefficients that shrink to zero via the Lasso are identified in Table 4 (Panel A) with a dagger (\dagger). Model selection via the Lasso confirms that $Sent$ and $IISent$ are more relevant for the overall fit of γ than some of the fundamental factors used, namely, $Ntis$, $Infl$, $Corpr$ and CSP .

The results provided by our OLS regression and by the Lasso indicate that supportive fundamental data for equity markets do not necessarily intensify biased behavior of single stock call option investors. This is an interesting takeaway, especially considering the notion that sentiment does appear to affect such behavior: single stock option investors seem to overweight small probabilities when sentiment is exuberant, not necessarily when stock fundamentals are exuberant.

More importantly, these results support our earlier findings that overweight of small probabilities is strongly time-varying and linked to sentiment. Therefore, overweight of small probabilities is unlikely to result from (static) investor preferences but from investors' bias-in-beliefs or time-varying preferences, which seem conditional on sentiment levels. Furthermore, we also run our regression models (Eqs. (14) and (15)) using different assumptions about the value of λ , the loss aversion parameter. In this exercise we set λ to imply 1) no loss aversion ($\lambda=1$), 2) augmented loss aversion ($\lambda=3$) and 3) optimal loss aversion, where λ assumes the estimated value by Eq. (7) and reported in Table 1, Panel A.

Table 5 indicates that the results for $Sent$ are similar to the ones obtained in our main regressions: $Sent$ is negatively linked to γ and statistically significant at all horizons but with less statistical significance, explanatory power and magnitude at the twelve-month horizon. This result applies to the multivariate regression model only. Across all options maturities, the $Sent$ coefficients become larger when λ equals 3 and they shrink when λ equals 1. The relation between changes in λ and $Sent$ observed is intuitive. We argue that as λ increases, the

probabilities on the left side of the CPT distribution increase, favoring a thinner tail on the right side of the PCPT distribution, which, then, requires less overweight of tail adjustment (through a higher γ) for the PCPT to match the EDF. As a higher γ is obtained by such increase in λ , the coefficient of γ with the given sentiment factor also increases in magnitude. The explanatory power of these regressions are, once again, high, as R^2 ranges from 62 to 73 percent in the multivariate models. The explanatory power of $Sent$ ranges from 16 to 24 percent in the univariate setting. Table 5 reiterates the relation between $IISent$, the AAI individual investor sentiment measure, the Welch and Goyal (2008) control variables and γ in our main regressions. $IISent$ is rarely significantly linked to γ . The control variables that are robustly linked to γ in our main regression ($E12$, B/m and $Svar$) remain strongly connected to it within these auxiliary regressions. Applying the Lasso model selection technique to these regressions gives results that are analogous to these ones. $Sent$, $IISent$, $E12$, B/m , $Svar$ and $Rfree$ always survive the Lasso variable selection procedure, whereas $Ntis$, $Infl$, $Corp$ and CSP coefficient often shrink to zero (as in our main regression, these coefficients are identified with a dagger (\dagger) in Table 5, Panel A).

[Please insert Table 5 about here]

The robustness of the relation between γ and $Sent$ suggests that changes in the overweighting of tails are not conditional on the level of the loss aversion parameter. In other words, levels of loss aversion do not drive investors to overweight upside tail events, as one could hypothesize when associating upside speculation with a state of low loss aversion. Thus, our results suggest that overweighting of small probabilities is a phenomenon stably linked to sentiment, rather than positive fundamentals or loss aversion levels. Our results tie closely with the findings of Green and Hwang (2011), who investigate the relation between IPOs expected skewness and returns. They find that the skewness effect is stronger during period of high investor sentiment. In the same line, Chen et al. (2015) conclude that when gambling sentiment is high, stocks with lottery-like characteristics earn positive abnormal returns in the short-run followed by underperformance in the long run.

5 Robustness tests

5.1 Kupiec's test for tail comparison

We employ Kupiec's (1995) test to compare the tails of the EDF with the ones of the subjective density functions and of the RND as a robustness test to the EVT methods applied. Kupiec's test was originally designed to evaluate the accuracy of Value-at-risk (VaR) models, where the estimated VaR were compared with realized ones. Because the VaR is no different from the EQR on the downside, i.e., the \hat{q}_p^- statistic, we can also make use of Kupiec's method to test the accuracy of the \hat{q}_p^+ statistic for subjective densities and the RND on matching realized EQRs. Kupiec's method computes a proportion of failure (POF) statistic that evaluates how often a VaR level is violated over a specified time span. Thus, if the number of realized violations

is significantly higher than the number of violations implied by the level of confidence of the VaR, then such a risk model or consistency of tails is challenged. Kupiec's POF test, which is designed as a log-likelihood ratio test, is defined as:

$$LR_{POF} = -2\log[(1 - p^*)^{(n-v)}(p^*)^v] + 2\log[(1 - \frac{v}{n})^{(n-v)}(\frac{v}{n})^v] \sim \chi^2(1), \quad (16)$$

where p^* is the POF under the null hypothesis, n is the sample size, and v is the number of violations in the sample. The null hypothesis of such test is $\frac{v}{n} = p^*$, i.e., the realized probability of failure matches the predicted one. Thus if the LR exceeds the critical value, $\chi^2(1)=3.841$, the hypothesis is rejected at the five percent level. In our empirical problem, p^* equals the assumed probability that the EQR of the subjective and risk-neutral densities will violate the EQR of the realized returns, whereas $\frac{v}{n}$ is the realized number of violations. Because we apply Kupiec's test to upside returns, violations mean that returns are higher than a positive threshold.

The first step in applying Kupiec's test to our data set is outlining the expected percentage of failure (p^*) between the EQR from the EDF and from the subjective and risk-neutral densities. We pick p^* as being five and ten percent. The percentages can be seen as the expected frequency that the tails of the subjective and of the RND distributions overstate the tails of the distribution of the realized returns. As a fatter tail is a symptom of an overweighting of small probabilities, we expect that densities that do not adjust for the CPT weighting function will deliver a higher frequency of failures than the CPT density function. The Kupiec's test results are reported in Table 6.

[Please insert Table 6 about here]

Panel A in Table 6 suggests that the probability of failure for the RND, power, exponential, and PCPT densities is particularly high at the three-month horizon, with more than 99 percent for the EQR at 90 and 95 percent and for p^* equal to five and ten percent. These densities often contain fatter tails than the EDF. For the CPT density, the POF is much lower across the two values of p^* used and the 90 and 95 percent EQR. The POF for the 90 percent EQR is roughly 58 percent for the CPT, irrespective of p^* . At the 95 percent EQR, the POF is 46 percent for the CPT. These findings suggest that at the 90 and 95 percent EQR, the CPT densities overstate less frequently the EDF tails than other densities. The violations of the EDF tails are, however, still significant as they occur between 41 and 52 percent of times. Nevertheless, when we analyze the 99 percent EQR, we find that the POF for all densities decreases considerably and, for the CPT, it becomes 16 percent.

Panel B of Table 6 depicts a very similar pattern of the POF for the probability densities derived from the six-month options as we find for the three-month options. The POF is very close to 100 percent for all densities apart from the CPT at the 90 and 95 percent EQR, while at the 99 percent EQR violations fall substantially, even more than what we observed for the three-month options. Nevertheless, the CPT remains the best approximation for the EDF, as its POF is the lowest. The Kupiec's test result suggests that the CPT density is statistically equal to the EDF, whereas the RND also equals the empirical returns at the ten percent level. The results for p^* at the five or ten percent are very similar. Panel C presents the POF for

the twelve-month maturity. We find once again that the CPT tails are the ones that violate the EDF tails the least. The POF for these densities are about 29 percent for the 90 percent EQR, seven percent for the 95 percent EQR, and four percent for the 99 percent EQR. These findings suggest that the tails of the CPT closely match the EDF ones, especially far out in the tail, i.e., at the 95 and 99 percent EQR. The RND, power, exponential, and PCPT densities record POFs that are much smaller than for the three- and six-month maturities but that are still high in comparison to the CPT.

We note that results for the PCPT and the CPT are quite distinct, whereas results for the PCPT are somewhat closer to the ones of the RND. This suggests that the weighting function is the component within the CPT density function that more forcefully causes the RND to approximate the EDF, so not the value function. Overall, our analysis using Kupiec’s test leads to similar results as the ones reached within our EVT analysis and further evidences that the CPT model is superior in matching realized returns.

5.2 Prelec’s weighting function parameter

As another robustness check, we estimate the weighting function parameter ω of the RDEU model suggested by Prelec (1998) in order to test whether our conclusions are robust to other weighted functions formulations²⁴. The Prelec weighting function w_p^{+-} is given by Eq. 17

$$w_p^{+-}(p) = \exp(-(-\log(p))^\omega), \quad (17)$$

where the parameter ω defines the curvature of the weighting function for both gains and losses, which also leads to S -shaped probability distortion functions. We note that according to Prelec (1998) the standard ω parameter value equals 0.65. Our time-varying and long-term (LT) estimates for ω are presented in Table 7, Panel A.

The long-term estimates of ω are somewhat in line with the one suggested by the RDEU but less so for the twelve-month horizon: ω estimated from the three-, six-, and twelve-months are 0.46, 0.67, and 1.11, respectively. These parameters are somewhat consistent with our long-term estimates for γ being, 0.75, 0.81, and 1.09 (see Table 1), as they suggest overweighting of small probabilities that fades with the increase in the option horizon. Similarly, time-varying estimates of ω also indicate more overweight of small probabilities than suggested by γ estimations. We find the mean (0.95) and median (0.93) for time-varying estimates of ω from three-month options to be higher than the ones suggested by Prelec (1998). This outcome means that overweighting of small probabilities within the single stock option markets is less than suggested by RDEU (similar to our conclusion concerning CPT parameters) and that estimated Prelec parameters imply a less pronounced overweight of tails than suggested by our CPT parameter estimations. In line with our results for the CPT, for the six- and twelve-month maturities, underweight of small probabilities is, however, more frequent than an overweight. The average ω for the six-month options is 1.02 (median being 0.99), and for the

²⁴A major advance of Prelec’s (1998) weighting function vis-à-vis the CPT is that it is monotonic for any value of ω , whereas the CPT can have a non-monotonic probability weighting for low levels of γ .

twelve-months options is 1.05 (median being 1.07). The fact that investors tend to overweight small probabilities to a much lesser extent in the short-term and that estimates are higher than suggested by their respective lab-based estimates confirms our main findings.

[Please insert Table 7 about here]

The sample dependence observed in our main results is confirmed by the usage of Prelec’s weighting function as overweight of tails is pervasive mostly in the 1998-2003 sample. Overall, the robustness checks following Prelec (1998) confirm our main findings regarding time-variation and sample dependence of overweighting of small probabilities, and reiterate our conclusion.

5.3 Estimating time-varying γ under different assumptions for δ , α and β

As an additional robustness test to our time-varying estimates of γ , we also run optimizations where we fix parameter δ instead of jointly optimizing it with γ . We impose $\delta = 1$ (no overweight of small probabilities on the left-side of the distribution) or 0.69, the value of δ within the CPT. In line with our previous robustness test, Table 7, Panels B and C, suggests that results from optimizations with different values for δ are qualitatively the same to our main results, i.e., a positive term structure and sample dependency of overweight of small probabilities. Unreported results also indicate a negative correlation between γ and sentiment and high explanatory power of regressions. R^2 is between 13 and 21 percent for three- and six-month options and between 0 to 3 percent for twelve-month options. Though, neutral probability weighting on the left side of the distribution ($\delta=1$) adjusts γ downwards when compared to our main results. Conversely, when δ is 0.69, an upwards adjustment to γ estimates occurs.

Similarly, we also estimate γ under different assumptions for α and β . We assumed $\alpha=\beta=1$ (no diminishing sensitivity to gains and losses) and $\alpha=\beta=0.75$ (more pronounced diminishing sensitivity to gains and losses) instead of the CPT parameterization $\alpha=\beta=0.88$. Our results, reported in Table 7, Panels D and E, suggest that lower sensitivity to gains and losses (higher α and β) leads to a decrease in overweight of small probabilities (higher γ estimates), whereas higher sensitivity to gains and losses (lower α and β) leads to an increase in overweight of tails (lower γ estimates). This effect is similar to the one observed by changes in λ (described in Section 4.3), which also magnifies the sensitivity for losses when increased.

As indicated in Section 3.2, we have also estimated time-varying γ using different lower (-0.25, 0 and 0.28) and upper bounds (1.2, 1.35, 1.5, 1.75 and 2). Results across bounds used differ to the extent that higher bounds produce upward shifts in the estimated γ across all quantiles, median and averages to the extent that overweight of small probabilities becomes less pronounced but remain present. The time-variation pattern observed in Figure 3 and, more importantly, the strong negative relationship with sentiment reported in Table 4 are, though, extremely robust to changes in lower and upper optimization bounds. This result strengthens our conclusion that overweight of small probabilities is largely time varying and reflects investor sentiment.

5.4 Overweight of (right) tails driven by IV of single stock options

Finally, given that overweight of small probabilities by single stock call investors was most evident during the IT bubble period (as Table 3 suggests), we hereby evaluate whether this finding may have been driven by movements in the IV of index options rather than changes in the IV of single stock options. We perform such analysis because our methodology for calculation of average weighted stock IV volatilities partly relies on the IV on index options (as it depends on implied correlations), as Eqs. A.8i and A.8k in Appendix A.2 suggest. Essentially, we want to ensure that the overweight of small probabilities observed from our single stock options data is not caused by a rise in index options' IV. As overweight of small probabilities is a corollary of high IV skew²⁵, we examine the IV skews (120 percent moneyness versus at-the-money, ATM) from both index options and from single stock options within our sample using a k-Nearest-Neighbors (KNN) algorithm (see Appendix A.4 for detail). Figure 4 depicts a scatter plot that relates single stock IV skews (on the y-axis) with index option IV skew (on the x-axis) overlaid with the decision boundary between overweight of tails (in red) and its absence (in blue), produced by the application of the KNN algorithm to our full data sample. The picture suggests that that overweight of small probabilities is almost never caused by positive index IV skews, whereas positive single stock IV skews very often produce overweight of tails rather than underweights. Overweight of tails are mostly caused by situation where single stock IV skew are higher than index IV skew, which suggest that either high single stock IV skews or low implied correlation are responsible for overweight of tails, not index options' IV. These conditions can be anecdotally confirmed by our observation of IV skews during the 2000's IT bubble. During that period, when overweight of tails was pervasive, IV skew from single options was quite high, close to +10 volatility points, whereas the same IV skew from index options reached extreme low levels such as -15 volatility points. This disconnect between the two IV markets, which drove the implied correlation to 2.8 percent (an extreme low level), suggests that the index options' IV was not the driver for overweight of tails during the IT bubble. These findings reiterate our suggestions that overweight of small probabilities observed in our sample is caused by trading in single stock options by retail investors, rather than activity in the index option market.

6 Conclusion

Single stock OTM call options are deemed overpriced because investors overpay for positively skewed securities, resembling lottery tickets. The CPT's probability weighting function of Tversky and Kahneman (1992) theoretical model provides an appealing explanation why these options are expensive: investors' preferences for positively skewed securities. In our empirical analysis, we find that the CPT subjective density function implied by single stock options outperforms the RND and two rational densities functions (from the power and exponential

²⁵While this relation is widely acknowledged, Jarrow and Rudd (1982), Corrado and Su (1997) and Longstaff (1995) provide a formal theorem for the link between IV skew and risk-neutral.

utilities) in matching the tails of realized equity returns. We estimate the CPT probability weighting function parameter γ and find that they are qualitatively consistent with the one predicated by Tversky and Kahneman (1992), particularly for short-term options. This outcome endorses our hypothesis that investors in single stock call options are biased.

Our analysis provides detailed insights into the behavior of single stock option investors. Our empirical findings suggest that overweight of small probabilities is less pronounced than proposed by the CPT. We find the presence of a positive term structure of overweighting of tails, because it becomes less pronounced as the option maturity increases. Investors in single stock calls are more biased when trading short-term contracts, whereas they seem to be more rational (less biased) when trading long-term calls. This result is consistent with individual investors being the typical buyers of OTM single stock calls and the fact that they mostly use short-term options to speculate on the upside of equities.

We also find that investors overweighting of small probabilities is largely time-varying and sample dependent. Time-variation in γ 's remains strong even when we account for different levels of loss aversion, different diminishing sensitivities to gains and losses, different degrees of overweighting of the left tail and an alternative (Prelec's) weighting function. The strong time-variation and sample dependency of γ suggest that investors do not have a static preference for skewness, but rather time-varying preferences or "bias in beliefs" (see Barberis, 2013).

Such time-variation in γ is also confirmed by overweighting of tails to be pronounced in periods in which sentiment is high, for instance, the IT bubble period. This finding is consistent with the Baker and Wurgler (2007) sentiment measure being the main explanatory variable of overweighting of small probabilities. Our results challenge the view that single stock call options are structurally overpriced and offer the insight that overweight of tail events implied in these options are conditional on sentiment levels and option maturity rather than positive stock fundamentals, loss aversion levels or investor preferences for skewness.

Our findings have several important practical implications. First, the understanding of time-variation in investors' overweighting of small probabilities could be used in the development of behavioral option pricing models, which remains in its infancy. To the extent that overweighting of small probabilities is a latent variable or, simply, not trivial to estimate, we contemplate that future option pricing models should be more sentiment-aware than current ones. Second, of importance for such next generation option-pricing models is the inclusion of a positive term structure of tails' overweighting. Such potential modifications on options' pricing have large and direct consequences to risk-management, hedging and arbitrage activities. Third, from a financial stability point of view, investors' overweighting of small probabilities in single stock options could be of use to regulators for triangulating the presence of speculative equity markets bubbles.

A Appendix

A.1 Subject density function estimation

We hereby present the derivations required to achieve Eq. (6) in the main text, Eq. (A.7) here, from Eq. (5), called here Eq. (A.1):

$$\frac{w'(F_P(S_T)) \cdot f_P(S_T)}{f_Q(S_T)} = \varsigma(S_T). \quad (\text{A.1})$$

where $f_P(S_T)$ is the “real-world” probability distribution, $f_Q(S_T)$ is the RND, $\varsigma(S_T)$ is the pricing kernel, w is the weighting function and $F_P(S_T)$ is the “real-world” cumulative density function.

The first step of our derivation entails re-arranging Eq. (A.1) into (A.2b) via Eq. (A.2a), which demonstrates that for the CPT to hold, the subjective density function should be consistent with the probability weighted EDF:

$$\underbrace{f_Q(S_T)}_{\text{RND}} = \underbrace{w'(F_P(S_T))}_{\text{probability weighing}} \cdot \underbrace{f_P(S_T)}_{\text{EDF}} \cdot \underbrace{\varsigma(S_T)}_{\text{pricing kernel}} \quad (\text{A.2a})$$

$$\underbrace{f_Q(S_T)}_{\text{RND}} = \underbrace{f_{\tilde{P}}(S_T)}_{\text{probability weighted EDF}} \cdot \underbrace{\varsigma(S_T)}_{\text{pricing kernel}} \quad (\text{A.2b})$$

$$\underbrace{\frac{f_Q(S_T)}{\lambda \frac{U'(S_T)}{U'(S_t)}}}_{\text{Subjective density}} = \underbrace{\frac{f_Q(S_T)}{\varsigma(S_T)}}_{\text{probability weighted EDF}} = \underbrace{f_{\tilde{P}}(S_T)}_{\text{probability weighted EDF}} \quad (\text{A.3})$$

Following Ait-Sahalia and Lo (2000) and Bliss and Panigirtzoglou (2004), Eq. (A.3) can be manipulated so that the time-preference constant λ of the pricing kernel vanishes, producing Eq. (A.4), which directly relates the probability weighted EDF, the RND, and the marginal utility, $U'(S_T)$:

$$\underbrace{f_{\tilde{P}}(S_T)}_{\text{probability weighted EDF}} = \frac{\lambda \frac{U'(S_T)}{U'(S_t)} Q(S_T)}{\int \frac{U'(S_t)}{U'(x)} Q(x) dx} = \frac{\frac{f_Q(S_T)}{U'(S_T)}}{\int \frac{f_Q(x)}{U'(x)} dx} \quad (\text{A.4})$$

Generic subjective density function

where $\int \frac{Q(x)}{U'(x)} dx$ normalizes the resulting subjective density function to integrate to one. Once the utility function is estimated, Eq. (A.4) allows us to convert RND into the probability weighted EDF. Eq. (A.4) can also be used to estimate the subjective density function for an (rational) investor that has power or exponential utility function, by disregarding the weighting function $W(\cdot)$, so the left-hand side of the equation becomes $f_p(S_T)$. In the remainder of the paper we call these subjective distributions power and exponential density functions. As we hypothesize that the representative investor has a CPT utility function, its marginal utility function is $U'(S_T) = v'(S_T)$, and, thus, $v'(S_T) = \alpha S_T^{\alpha-1}$ for $S_T \geq 0$, and $v'(S_T) = -\lambda\beta(-S_T)^{\beta-1}$ for $S_T < 0$, leading to Eq. (A.5):

$$f_{\tilde{P}}(S_T) = \frac{\frac{f_Q(S_T)}{\alpha S_T^{\alpha-1}}}{\int \frac{f_Q(x)}{\alpha x^{\alpha-1}} dx} \quad \text{for } S_T \geq 0, \quad \text{and} \quad (\text{A.5})$$

$$\underbrace{f_{\tilde{P}}(S_T)}_{\text{probability weighted EDF}} = \frac{\frac{f_Q(S_T)}{-\lambda\beta(-S_T)^{\beta-1}}}{\int \frac{f_Q(x)}{-\lambda\beta(-x)^{\beta-1}} dx} \quad \text{for } S_T < 0, \quad \text{and} \quad (\text{A.6})$$

Partial CPT density function

Eqs. (A.5) and (A.6), hence, relate the EDF where probabilities are weighted according to the CPT probability distortion functions, on the LHS, to the subjective density function derived from the CPT value function, on the RHS, separately for gains and losses, i.e., the PCPT density function. The relationships specified by Eqs. (A.5) and (A.6) fully state the relation we would like to depict, although one additional manipulation is convenient for our argumentation. Assuming that the function $w(F_P(S_T))$ is strictly increasing over the domain $[0,1]$, there is a one-to-one relationship between $w(F_P(S_T))$ and a unique inverse $w^{-1}(F_P(S_T))$. So, result $f_{\tilde{P}}(S_T) = w'(F_P(S_T))f_P(S_T)$ also implies $f_{\tilde{P}}(S_T).(w^{-1})'(F_P(S_T)) = f_P(S_T)$ ²⁶. This outcome allows us to directly relate the original EDF to the CPT subjective density function, by “undoing” the effect of the CPT probability distortion functions within the PCPT density function:

$$\underbrace{f_P(S_T)}_{\text{EDF}} = \frac{\frac{f_Q(S_T)}{v'(S_T)}}{\int \frac{f_Q(x)}{v'(x)} dx} (w^{-1})'(F_P(S_T)) \quad (\text{A.7})$$

CPT density function

Thus, once the relation between the probability weighting function of EDF and the PCPT density is established, as in Eqs. (A.5) and (A.6), one can eliminate the weighting scheme affecting returns by applying the inverse of such weightings to the subjective density function without endangering such equalities, as in Eq. (A.7), numbered Eq. (6) in the main text.

A.2 Single stock weighted average implied volatilities

Starting from the portfolio variance formula, Eq. A.8a, we hereby provide the derivation of our single stock weighted-average implied volatility, given in Eq. A.8k:

$$\sigma_I^2 = \sum_{i,j=1}^n w_i w_j \rho_{ij} \sigma_i \sigma_j, \quad (\text{A.8a})$$

where,

$$\rho_{ij}(x) = \begin{cases} \bar{\rho}, & \text{if } i \neq j \\ 1, & \text{if } i = j \end{cases} \quad (\text{A.8b})$$

²⁶A drawback of the CPT model is that it allows for non-strictly increasing functions, which would not allow invertibility. This is the reason why the newer literature on probability distortions functions favors other strictly monotonic functions, such as Prelec’s (1998) $w(p) = e^{-(-\ln(p))^\delta}$, as the weighting functions. Nevertheless, because the CPT parameters of our interest ($\gamma = 0.61$; $\delta = 0.69$) impose strict monotonicity, we can obtain the inverse of the probability function, $w^{-1}(p)$ numerically.

and where σ_I^2 is the equity index option implied variance and i and j are indexes for the constituents of such equity index, then:

$$\sigma_I^2 = \bar{\rho} \sum_{i \neq j}^n w_i w_j \rho_{ij} \sigma_i \sigma_j + \sum_{i=1}^n w_i^2 \sigma_i^2, \quad (\text{A.8c})$$

$$= \bar{\rho} \sum_{i,j=1}^n w_i w_j \rho_{ij} \sigma_i \sigma_j + (1 - \bar{\rho}) \sum_{i=1}^n w_i^2 \sigma_i^2, \quad (\text{A.8d})$$

$$= \bar{\rho} \left(\sum_{i=1}^n w_i \sigma_i \right)^2 + (1 - \bar{\rho}) \sum_{i=1}^n w_i^2 \sigma_i^2, \quad (\text{A.8e})$$

$$= \bar{\rho} \left(\sum_{i=1}^n w_i \sigma_i \right)^2 + \sum_{i=1}^n w_i^2 \sigma_i^2 - \bar{\rho} \sum_{i=1}^n w_i^2 \sigma_i^2, \quad (\text{A.8f})$$

$$= \bar{\rho} \left(\left(\sum_{i=1}^n w_i \sigma_i \right)^2 - \sum_{i=1}^n w_i^2 \sigma_i^2 \right) + \sum_{i=1}^n w_i^2 \sigma_i^2, \quad (\text{A.8g})$$

$$\bar{\rho} \approx \frac{\sigma_I^2 - \sum_{i,j=1}^n w_i^2 \sigma_i^2}{\left(\sum_{i=1}^n w_i \sigma_i \right)^2 - \sum_{i,j=1}^n w_i^2 \sigma_i^2}. \quad (\text{A.8h})$$

As $\sum_{i,j=1}^n w_i^2 \sigma_i^2$ is relatively small, we can simplify A.8h into A.8i, the implied correlation:

$$\bar{\rho} \approx \frac{\sigma_I^2}{\left(\sum_{i=1}^n w_i \sigma_i \right)^2} \quad (\text{A.8i})$$

To obtain the single stock weighted average implied volatility (Eq. A.8k, we then square root both sides of the approximation and re-arrange its terms:

$$\sqrt{\bar{\rho}} \approx \frac{\sigma_I}{\left(\sum_{i=1}^n w_i \sigma_i \right)} \quad (\text{A.8j})$$

$$\sum_{i=1}^n w_i \sigma_i \approx \frac{\sigma_I}{\sqrt{\bar{\rho}}} \quad (\text{A.8k})$$

A.3 Least Absolute Shrinkage and Selection Operator (Lasso)

The regression coefficients obtained by the Lasso methodology applied (β_θ^L) are estimated by minimizing the quantity:

$$\sum_{i=1}^n (y_i - \beta_0 - \sum_{j=1}^p \beta_j x_{ij})^2 + \kappa \sum_{j=1}^p |\beta_j| = RSS + \kappa \sum_{j=1}^p |\beta_j| \quad (\text{A.9})$$

where κ is the tuning parameter, which is estimated via cross-validation. The cross-validation applied by us uses ten equal-size splits of our overall data set.

A.4 k-Nearest-Neighbor classifier

The k-Nearest-Neighbor (KNN) classifier is one of the approaches in machine learning that attempts to estimate the conditional distribution of the explained variable (Y) given the explanatory variables (X) and, subsequently, classify new observations to the class with highest estimated probability. The KNN classifier uses the Euclidean distance to first identify the closest k^{th} observations within the training data (in-sample data) to a new test (out-of-sample) observation provided (x_0). Such neighborhood of points around the test observation x_0 is defined as \mathbb{N}_0 . KNN, then, estimates the conditional probability of x_0 to belong to a class j as the percentage of old observations (y_i) in the neighborhood \mathbb{N}_0 whose class is also j :

$$Pr(Y = j|X = x_0) = \frac{1}{k} \sum_{i \in \mathbb{N}_0} I(y_i = j) \quad (\text{A.10})$$

In a third step, KNN applies the Bayes rule to perform out-of-sample classification (in test data) of x_0 to the class with the largest probability. For further details, see Hastie et al. (2008).

B Appendix

B.1 Welch and Goyal (2008) equity market predictors

The complete set and summarized descriptions of variables provided by Welch and Goyal (2008)²⁷ that are used in our study is given as:

1. **Dividendprice ratio (log), D/P**: Difference between the log of dividends paid on the S&P 500 index and the log of stock prices (S&P 500 index).
2. **Dividend yield (log), D/Y**: Difference between the log of dividends and the log of lagged stock prices.
3. **Earnings, E12**: 12-month moving sum of earnings on the S&P500 index.
4. **Earnings-price ratio (log), E/P**: Difference between the log of earnings on the S&P 500 index and the log of stock prices.
5. **Dividend-payout ratio (log), D/E**: Difference between the log of dividends and the log of earnings.
6. **Stock variance, SVAR**: Sum of squared daily returns on the S&P 500 index.
7. **Book-to-market ratio, B/M**: Ratio of book value to market value for the Dow Jones Industrial Average.
8. **Net equity expansion, NTIS**: Ratio of twelve-month moving sums of net issues by NYSE-listed stocks to total end-of-year market capitalization of NYSE stocks.
9. **Treasury bill rate, TBL**: Interest rate on a three-month Treasury bill.
10. **Long-term yield, LTY**: Long-term government bond yield.
11. **Long-term return, LTR**: Return on long-term government bonds.
12. **Term spread, TMS**: Difference between the long-term yield and the Treasury bill rate.
13. **Default yield spread, DFY**: Difference between BAA- and AAA-rated corporate bond yields.
14. **Default return spread, DFR**: Difference between returns of long-term corporate and government bonds.
15. **Cross-sectional premium, CSP**: measures the relative valuation of high- and low-beta stocks.
16. **Inflation, INFL**: Calculated from the CPI (all urban consumers) using $t - 1$ information due to the publication lag of inflation numbers.

²⁷Available at <http://www.hec.unil.ch/agoyal/>.

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Table 1: Long-term CPT parameters and consistency test on tail shape

Panel A of this table reports the estimated long-term CPT parameters gamma (γ), lambda (λ), and γ conditional on optimal λ ($\gamma|\hat{\lambda}$) from the single stock options as well as residual sum of squares (RSS) of Eqs. (7) and (8). The parameter γ defines the curvature of the weighting function for gains, which leads the probability distortion functions to assume inverse S-shapes. Estimated parameters close to unity lead to weighting functions that are close to un-weighted probabilities, whereas parameters close to zero denotes a larger overweighting of small probabilities. The parameter λ is the loss aversion parameter. These parameters are long-term since their estimates are obtained by setting the average CPT density functions to match the return distribution realized within our full sample. These parameters are estimated using Eqs. (7) and (8). Panel B reports the results from the statistical test of the tail shape parameter phi (φ), according to Eqs. (9) and (12b) applied to the averaged probability density functions. As densities compared here are averaged, for the RND and subjective densities or estimated using our full sample for realized returns, such test aims to test for the long-term consistency between distribution tail shapes. The null hypothesis of these two tests is that φ from the two distributions being compared have equal means and, therefore, tail shapes are consistent. The null hypothesis of these two tests is that φ from the two distributions being compared have equal means and, therefore, tail shapes are consistent. The rejection of the null hypothesis is tested by t -tests of Eq. (12a) at the ten, five, and one percent statistical levels, respectively, shown by superscripts *, **, ***, assigned to φ for the RND, displayed in column (1), and φ for the EDF, shown in column (2).

Panel A - long-term CPT parameters						
Maturity	Gamma (γ)			Lambda (λ) ($\gamma \hat{\lambda}$)		
	Estimate	RSS	Estimate	RSS	Estimate	RSS
3 months	0.75	0.02	1.02	0.12	0.54	0.01
6 months	0.81	0.02	2.66	0.3	0.87	0.02
12 months	1.09	0.06	3	1.64	1.12	0.07

Panel B - statistical test on tail shape parameters						
Maturity	(1) vs (2)	Phi		(2)EDF	p-value	t-stat
		(1)	(2)			
3months	<i>RNDvsEDF</i>	0.20*	0.29	0.1%	0.1%	-3.6
	<i>PowerusEDF</i>	0.17***	0.29	0.0%	0.0%	-4.9
	<i>ExpousEDF</i>	0.18***	0.29	0.0%	0.0%	-4.6
	<i>PCPTvsEDF</i>	0.17***	0.29	0.0%	0.0%	-4.9
	<i>CPTvsEDF</i>	0.20	0.29	0.0%	0.0%	-4.6
6months	<i>ECPTvsEDF</i>	0.20	0.29	0.0%	0.0%	-3.7
	<i>RNDvsEDF</i>	0.19**	0.23	2.8%	2.8%	-2.3
	<i>PowerusEDF</i>	0.16***	0.23	0.0%	0.0%	-4.0
	<i>ExpousEDF</i>	0.16***	0.23	0.0%	0.0%	-3.9
	<i>PCPTvsEDF</i>	0.17***	0.23	0.0%	0.0%	-4.0
12months	<i>CPTvsEDF</i>	0.19**	0.23	0.0%	0.0%	-3.9
	<i>ECPTvsEDF</i>	0.18***	0.23	0.0%	0.0%	-2.8
	<i>RNDvsEDF</i>	0.22	0.14	0.0%	0.0%	4.0
	<i>PowerusEDF</i>	0.14***	0.14	37.7%	37.7%	0.3
	<i>ExpousEDF</i>	0.14***	0.14	39.7%	39.7%	-0.1
	<i>PCPTvsEDF</i>	0.18**	0.14	1.9%	1.9%	2.5
	<i>CPTvsEDF</i>	0.22***	0.14	0.0%	0.0%	4.3
	<i>ECPTvsEDF</i>	0.18**	0.14	1.7%	1.7%	2.5

Table 2: EVT consistency tests on tail returns

This table reports the results from statistical tests of the extreme quantile return, EQR (in Panel A) and tail expected upside returns, EU (in Panel B), performed according to Eqs. (10), (11) and (12b) applied to averaged density functions. Since the densities compared here are averaged for the RND and for the subjective densities or estimated using our full sample for realized returns, these tests aim to investigate the long-term consistency between the distribution tails. The null hypothesis of these tests is that the EQR and the tail expected upside returns from the distributions being compared have equal means and, therefore, tails are consistent. The rejection of the null hypothesis is tested by t -tests of Eq. (12b) at the ten, five, and one percent statistical levels, respectively, shown by superscripts *, **, ***, assigned to the EQR or the EU for the RND, displayed in column (1), and for the same statistics for the EDF, shown in column (2).

Panel A - Tails extreme quantile returns (EQR)													
Maturity	(1) vs. (2)	10% quantile			5% quantile			1% quantile					
		(1)	(2)	EDF	p-value	t-stat	(1)	(2)	EDF	p-value	t-stat		
3 months	RND vs. EDF	0.16***	0.11	0.19***	0.00%	-7.7	0.13	0.26***	0.00%	-8.2	0.21	0.0%	-5
	Power vs. EDF	0.21***	0.11	0.23***	0.00%	-13.9	0.13	0.31***	0.00%	-15.4	0.21	0.0%	-10.4
	Expo vs. EDF	0.21***	0.11	0.24***	0.00%	-13.9	0.13	0.32***	0.00%	-15.7	0.21	0.0%	-11.5
	PCPT vs. EDF	0.18***	0.11	0.21***	0.00%	-10.8	0.13	0.27***	0.00%	-11.6	0.21	0.0%	-7.3
	CPT vs. EDF	0.13***	0.11	0.15***	0.2%	-3.3	0.13	0.21	0.7%	-2.9	0.21	36.3%	0.4
ECPT vs. EDF	0.15***	0.11	0.17***	0.00%	-5.4	0.13	0.23*	0.00%	-5.3	0.21	8.1%	-1.8	
6 months	RND vs. EDF	0.19	0.18	0.22	18.5%	-1.2	0.22	0.3*	36.5%	-0.4	0.32	8.9%	1.7
	Power vs. EDF	0.25***	0.18	0.28***	0.00%	-9.6	0.22	0.36***	0.00%	-10.2	0.32	0.0%	-4.3
	Expo vs. EDF	0.26***	0.18	0.29***	0.00%	-10.6	0.22	0.37***	0.00%	-11.5	0.32	0.0%	-5.5
	PCPT vs. EDF	0.22***	0.18	0.24***	0.00%	-5.1	0.22	0.32	0.0%	-4.6	0.32	36.4%	-0.4
	CPT vs. EDF	0.16***	0.18	0.18***	0.00%	4.2	0.22	0.25***	0.00%	6.5	0.32	0.0%	6.6
ECPT vs. EDF	0.19	0.18	0.21	28.4%	-0.8	0.22	0.28***	35.2%	0.5	0.32	0.5%	2.9	
12 months	RND vs. EDF	0.22***	0.32	0.26***	0.00%	10.5	0.35	0.44	0.0%	11.5	0.44	0.0%	6
	Power vs. EDF	0.33	0.32	0.36*	19.8%	-1.2	0.35	0.45*	9.9%	-1.7	0.44	9.2%	-1.7
	Expo vs. EDF	0.34***	0.32	0.38***	0.2%	-3.2	0.35	0.47***	0.0%	-4	0.44	0.1%	-3.5
	PCPT vs. EDF	0.26***	0.32	0.31***	0.00%	6.4	0.35	0.39***	0.0%	7	0.44	0.0%	3.7
	CPT vs. EDF	0.18***	0.32	0.21***	0.00%	23.3	0.35	0.3*	0.0%	26.9	0.44	0.0%	12.2
ECPT vs. EDF	0.26***	0.32	0.29***	0.00%	7.1	0.35	0.39***	0.0%	7.8	0.44	0.0%	4.1	
Panel B - Tails expected upside returns (EU)													
Maturity	(1) vs. (2)	10% quantile			5% quantile			1% quantile					
		(1)	(2)	EDF	p-value	t-stat	(1)	(2)	EDF	p-value	t-stat		
3 months	RND vs. EDF	0.2***	0.15	0.23***	0.00%	-5.3	0.19	0.32*	0.0%	-5.1	0.3	8.9%	-1.7
	Power vs. EDF	0.25***	0.15	0.28***	0.00%	-10.3	0.19	0.37***	0.0%	-10.9	0.3	0.0%	-5.9
	Expo vs. EDF	0.26***	0.15	0.29***	0.00%	-10.6	0.19	0.39***	0.0%	-11.4	0.3	0.0%	-7.2
	PCPT vs. EDF	0.22***	0.15	0.25***	0.00%	-7.5	0.19	0.33***	0.0%	-7.4	0.3	0.6%	-2.9
	CPT vs. EDF	0.16	0.15	0.19	18.6%	-1.2	0.19	0.26***	39.0%	-0.2	0.3	0.1%	3.4
ECPT vs. EDF	0.18***	0.15	0.21**	0.5%	-3	0.19	0.28	3.2%	-2.2	0.3	13.3%	1.5	
6 months	RND vs. EDF	0.24	0.24	0.27	36.9%	0.4	0.28	0.37***	10.9%	1.6	0.41	0.2%	3.3
	Power vs. EDF	0.3***	0.24	0.33***	0.00%	-6.8	0.28	0.43	0.0%	-6.6	0.41	14.4%	-1.4
	Expo vs. EDF	0.31***	0.24	0.34***	0.00%	-7.9	0.28	0.45**	0.0%	-8	0.41	1.4%	-2.6
	PCPT vs. EDF	0.26**	0.24	0.29	1.4%	-2.6	0.28	0.38*	13.3%	-1.5	0.41	5.9%	2
	CPT vs. EDF	0.2***	0.24	0.22***	0.00%	6	0.28	0.3***	0.0%	8.8	0.41	0.0%	8.1
ECPT vs. EDF	0.23	0.24	0.26***	15.3%	1.4	0.28	0.35***	0.2%	3.3	0.41	0.0%	4.9	
12 months	RND vs. EDF	0.28***	0.37	0.33***	0.00%	7.6	0.4	0.51	0.0%	7.7	0.51	0.7%	2.8
	Power vs. EDF	0.38	0.37	0.42*	14.7%	-1.4	0.4	0.53*	5.8%	-2	0.51	5.8%	-2
	Expo vs. EDF	0.4***	0.37	0.44***	0.2%	-3.2	0.4	0.55***	0.0%	-4	0.51	0.1%	-3.4
	PCPT vs. EDF	0.32***	0.37	0.36***	0.00%	4.8	0.4	0.48*	0.0%	4.9	0.51	6.4%	1.9
	CPT vs. EDF	0.23***	0.37	0.27***	0.00%	19.2	0.4	0.38***	0.0%	21.5	0.51	0.0%	9.1
ECPT vs. EDF	0.31***	0.37	0.35***	0.00%	5.3	0.4	0.47**	0.0%	5.6	0.51	3.0%	2.3	

Table 3: Time-varying gamma parameter

This table reports the summary statistics of the estimated CPT time-varying parameter gamma (γ) from the single stock options market for each day in our full sample across different values of lambda (λ). The parameter λ is the loss aversion parameter and the parameter γ defines the curvature of the weighting function for gains, which leads the probability distortion functions to assume inverse S-shapes. An estimated γ parameter close to unity leads to a weighting function that is close to the unweighted probabilities, whereas values close to zero denote a larger overweighting of small probabilities. The column with heading $\% \gamma < 1$ reports the percentage of observations in which $\gamma < 1$, thus, the proportion of the sample in which overweight of small probabilities is observed. We report this metric for the full sample as well as for three equal-sized splits of our full sample, namely: (98-03), from 1998-01-05 to 2003-01-30; (03-08), from 2003-01-31 to 2008-02-21 and; (08-13) from 2008-02-22 to 2013-03-19. Panel A reports the summary statistics of γ when we assume the CPT parameterization, where λ equals 2.25. Panel B reports the summary statistics of γ when we assume the loss aversion parameter λ equals 1 (no loss aversion). Panel C reports the summary statistics of γ when we assume λ equals to 3 (augmented loss aversion). Panel D reports the summary statistics of γ when we assume λ to equal its estimated (optimal) values, as reported in Table 1, Panel A.

Panel A - Gamma with CPT loss aversion ($\lambda = 2.25$)

Maturity	Min	25% Qtile	Median	Mean	75% Qtile	Max	StDev	$\% \gamma < 1$	$\% \gamma < 1$ (98-03)	$\% \gamma < 1$ (03-08)	$\% \gamma < 1$ (08-13)	RSS
3 months	-	0.74	0.91	0.89	1.04	1.75	0.23	64%	97%	35%	59%	0.0209
6 months	-	0.81	0.99	0.96	1.14	1.75	0.28	52%	92%	18%	46%	0.017
12 months	0.04	0.91	1.03	1.01	1.14	1.75	0.22	41%	83%	11%	29%	0.0225

Panel B - Gamma with no loss aversion ($\lambda = 1$)

Maturity	Min	25% Qtile	Median	Mean	75% Qtile	Max	StDev	$\% \gamma < 1$	$\% \gamma < 1$ (98-03)	$\% \gamma < 1$ (03-08)	$\% \gamma < 1$ (08-13)	RSS
3 months	0.32	0.52	0.66	0.67	0.8	1.27	0.18	97%	100%	94%	97%	0.0253
6 months	0.32	0.55	0.71	0.72	0.87	1.75	0.21	90%	98%	79%	92%	0.0198
12 months	0.29	0.62	0.83	0.8	0.98	1.75	0.22	81%	98%	63%	83%	0.0169

Panel C - Gamma with augmented loss aversion ($\lambda = 3$)

Maturity	Min	25% Qtile	Median	Mean	75% Qtile	Max	StDev	$\% \gamma < 1$	$\% \gamma < 1$ (98-03)	$\% \gamma < 1$ (03-08)	$\% \gamma < 1$ (08-13)	RSS
3 months	0.45	0.81	0.96	0.98	1.11	1.75	0.25	58%	93%	27%	53%	0.023
6 months	0.38	0.89	1.02	1.06	1.25	1.75	0.25	45%	83%	13%	38%	0.0196
12 months	0.37	0.98	1.07	1.09	1.19	1.75	0.21	31%	66%	6%	22%	0.0265

Panel D - Gamma with optimized loss aversion

Maturity	Min	25% Qtile	Median	Mean	75% Qtile	Max	StDev	$\% \gamma < 1$	$\% \gamma < 1$ (98-03)	$\% \gamma < 1$ (03-08)	$\% \gamma < 1$ (08-13)	RSS
3 months	0.33	0.52	0.66	0.67	0.81	1.75	0.18	97%	100%	93%	97%	0.0249
6 months	0.37	0.86	1.01	1.02	1.17	1.75	0.24	47%	86%	15%	40%	0.0187
12 months	0.34	0.96	1.05	1.06	1.17	1.75	0.2	35%	73%	8%	24%	0.025

Table 4: Regression results: CPT parametrization

This table reports the regression results for Eq. (14) in a multivariate setting in Panel A. The dependent variable in this regression is gamma (γ), where the explanatory variables are 1) the Baker and Wurgler (2007) sentiment measure, 2) the AAI individual investor sentiment measure, and 3) the explanatory variables used by Welch and Goyal (2008), excluding the factors that correlate with each other in excess of 40 percent. The regressors identified with a dagger (†) are the ones shrank to zero by the application the Lasso. Panel B reports the regression results for Eq.(15), which regresses γ and the same explanatory variables mentioned before in the univariate setting.

Panel A - Multivariate analysis		Panel B - Univariate analysis																							
Maturity	3m	6m	12m	3m	6m	12m	3m	6m	12m	3m	6m	3m	6m	12m	3m	6m	3m	6m	12m	3m	6m	3m	6m	3m	
<i>Intercept</i>	0.564*** (0.068)	0.646*** (0.090)	0.769*** (0.074)	0.891*** (0.020)	0.969*** (0.023)	1.016*** (0.019)	0.864*** (0.022)	0.939*** (0.026)	0.996*** (0.022)	0.573*** (0.031)	0.513*** (0.061)	0.865*** (0.022)	0.917*** (0.031)	0.855*** (0.023)	0.869*** (0.022)	0.917*** (0.026)	0.869*** (0.022)	0.855*** (0.023)	0.869*** (0.022)	0.917*** (0.026)	0.869*** (0.022)	0.855*** (0.023)	0.869*** (0.022)	0.917*** (0.026)	0.864*** (0.021)
<i>Sent</i>	-0.078*** (0.024)	-0.121*** (0.032)	-0.114*** (0.024)	-0.145*** (0.020)	-0.206*** (0.028)	-0.157*** (0.020)																			
<i>AAIISent</i>	0.094 (0.059)	0.030 (0.085)	-0.003 (0.058)				0.031 (0.083)	-0.040 (0.107)	-0.067 (0.079)																
<i>E12</i>	0.049*** (0.008)	0.052*** (0.010)	0.044*** (0.011)							0.059*** (0.005)															
<i>B/m</i>	0.617** (0.268)	0.698* (0.344)	0.387 (0.331)								1.428*** (0.266)														
<i>Ntis</i>	0.357 (0.553)	0.201† (0.676)	-0.869 (0.567)									0.617 (0.811)													
<i>R/free</i>	-0.022** (0.010)	-0.030** (0.014)	-0.016 (0.011)										-0.022 (0.013)												
<i>lnfl</i>	-0.580† (2.539)	-3.111† (3.493)	0.370† (2.916)											5.867 (4.778)											
<i>Corpr</i>	0.100† (0.275)	-0.174† (0.373)	-0.037† (0.349)												-0.314 (0.554)										
<i>Suar</i>	-9.762*** (3.179)	-11.792*** (4.156)	-8.679** (3.686)																						
<i>CSP</i>	-0.219† (0.197)	-0.219† (0.233)	-0.116† (0.238)																					0.626* (0.313)	
<i>R2</i>	71%	68%	67%	22%	29%	27%	0%	0%	1%	37%	31%	0%	3%	1%	0%	18%	2%								
<i>F-stat</i>	36.5	31.7	30.2	43.7	64.3	57.7	0.2	0.2	0.8	90.3	70.0	0.7	5.4	2.2	0.3	33.9	3.8								
<i>AIC</i>	-247.1	-166.0	-233.7	-326.1	-186.0	34.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0								
<i>BIC</i>	-213.4	-132.3	-200.0	-320.0	-179.9	40.3	0.1	0.1	0.1	0.0	0.2	0.7	0.0	3.9	0.5	2.3	0.3								

Table 5: Regression results: alternative loss aversion parameterization

This table reports the regression results for Eq. (14), in a multivariate setting, in Panel A and for Eq. (15), in a univariate setting, in Panel B. Across columns, the parameterization of lambda (λ) differs so Eqs. (14) and (15) are run under the assumption that 1) loss aversion is absent ($\lambda = 1$); 2) loss aversion is augmented ($\lambda = 3$) and 3) λ is optimal (as given by Table 1 Panel A). The dependent variable in regression of Eq. (14) is gamma (γ), where the explanatory variables are 1) the Baker and Wurgler (2007) sentiment measure; 2) the AAI individual investor sentiment measure and 3) the explanatory variables used by Welch and Goyal (2008) excluding the factors that correlate with each other in excess of 40 percent. The regressors identified with a dagger (†) are the ones shrank to zero by the application of the Lasso. Panel B reports the regression results for Eq. (15), which regresses γ and the same explanatory variables mentioned before in the univariate setting.

		$\lambda = 1$						$\lambda = 3$						Optimum λ							
		3m	6m	12m	3m	6m	12m	3m	6m	12m	3m	6m	12m	3m	6m	12m	3m	6m	12m		
Panel A - Multivariate analysis																					
Maturity		3m	6m	12m	3m	6m	12m	3m	6m	12m	3m	6m	12m	3m	6m	12m	3m	6m	12m		
<i>Intercept</i>		0.348***	0.347***	0.413***	0.576***	0.661***	0.798***	0.576***	0.661***	0.798***	0.354***	0.663***	0.817***	0.354***	0.663***	0.817***	0.354***	0.663***	0.817***		
<i>Sent</i>		-0.056	-0.07	-0.065	-0.084	-0.094	-0.074	-0.084	-0.094	-0.074	-0.056	-0.085	-0.069	-0.056	-0.085	-0.069	-0.056	-0.085	-0.069		
		-0.050***	-0.061***	-0.050**	-0.077***	-0.094***	-0.091***	-0.077***	-0.094***	-0.091***	-0.051***	-0.089***	-0.093***	-0.051***	-0.089***	-0.093***	-0.051***	-0.089***	-0.093***		
		-0.018	-0.022	-0.02	-0.027	-0.03	-0.022	-0.027	-0.03	-0.022	-0.018	-0.027	-0.021	-0.018	-0.027	-0.021	-0.018	-0.027	-0.021		
<i>AAISent</i>		0.087*	0.082	0.110**	0.108	0.092	-0.008	0.108	0.092	-0.008	0.086*	0.074	-0.003	0.086*	0.074	-0.003	0.086*	0.074	-0.003		
		-0.05	-0.061	-0.049	-0.074	-0.083	-0.057	-0.049	-0.083	-0.057	-0.05	-0.076	-0.054	-0.05	-0.076	-0.054	-0.05	-0.076	-0.054		
<i>E12</i>		0.043***	0.045***	0.054***	0.049***	0.043***	0.036***	0.049***	0.043***	0.036***	0.043***	0.042***	0.036***	0.043***	0.042***	0.036***	0.043***	0.042***	0.036***		
		-0.007	-0.008	-0.008	-0.01	-0.01	-0.01	-0.008	-0.01	-0.01	-0.007	-0.01	-0.009	-0.007	-0.01	-0.009	-0.007	-0.01	-0.009		
<i>B/m</i>		0.563**	0.754***	0.647**	0.795**	0.885**	0.527*	0.795**	0.885**	0.527*	0.571**	0.823**	0.43	0.571**	0.823**	0.43	0.571**	0.823**	0.43		
		-0.227	-0.267	-0.269	-0.334	-0.377	-0.316	-0.269	-0.334	-0.377	-0.227	-0.336	-0.292	-0.227	-0.336	-0.292	-0.227	-0.336	-0.292		
<i>Ntis</i>		0.258	0.029	-0.357	0.266	0.057†	-0.884†	0.266	0.057†	-0.884†	0.253	0.223†	-0.900*†	0.253	0.223†	-0.900*†	0.253	0.223†	-0.900*†		
		-0.486	-0.593	-0.556	-0.686	-0.705	-0.558	-0.593	-0.686	-0.558	-0.486	-0.643	-0.525	-0.486	-0.643	-0.525	-0.486	-0.643	-0.525		
<i>Rfree</i>		-0.013*	-0.011	-0.021**	-0.01	-0.008	0.008	-0.011	-0.008	0.008	-0.013*	-0.014	-0.003	-0.013*	-0.014	-0.003	-0.013*	-0.014	-0.003		
		-0.008	-0.01	-0.009	-0.012	-0.012	-0.012	-0.009	-0.012	-0.012	-0.008	-0.012	-0.001	-0.008	-0.012	-0.001	-0.008	-0.012	-0.001		
<i>Imfl</i>		-0.558†	-1.626†	2.157†	0.468†	-0.3†	0.941†	0.468†	-0.3†	0.941†	-0.59†	-1.034†	0.259†	-0.59†	-1.034†	0.259†	-0.59†	-1.034†	0.259†		
		-2.029	-2.678	-2.313	-3.265	-3.836	-2.981	-2.678	-3.265	-2.981	-2.029	-3.496	-2.72	-2.029	-3.496	-2.72	-2.029	-3.496	-2.72		
<i>Compr</i>		0.086†	-0.038†	0.097†	0.094†	-0.112†	-0.153†	0.094†	-0.112†	-0.153†	0.089†	-0.168†	-0.133†	0.089†	-0.168†	-0.133†	0.089†	-0.168†	-0.133†		
		-0.206	-0.263	-0.262	-0.319	-0.38	-0.323	-0.262	-0.319	-0.38	-0.208	-0.347	-0.312	-0.208	-0.347	-0.312	-0.208	-0.347	-0.312		
<i>Svar</i>		-6.371**	-7.192**	-5.469*	-8.742**	-9.835**	-8.126**	-5.469*	-8.742**	-9.835**	-6.634**	-9.520**	-8.018**	-6.634**	-9.520**	-8.018**	-6.634**	-9.520**	-8.018**		
		-2.533	-3.188	-2.899	-3.484	-3.845	-3.373	-2.899	-3.484	-3.373	-2.542	-3.681	-3.142	-2.542	-3.681	-3.142	-2.542	-3.681	-3.142		
<i>CSP</i>		-0.076	-0.099†	-0.024†	-0.186†	-0.196†	-0.059†	-0.099†	-0.186†	-0.059†	-0.073	-0.152†	-0.066†	-0.073	-0.152†	-0.066†	-0.073	-0.152†	-0.066†		
		-0.153	-0.182	-0.178	-0.22	-0.235	-0.196	-0.182	-0.22	-0.235	-0.154	-0.215	-0.189	-0.154	-0.215	-0.189	-0.154	-0.215	-0.189		
R^2		70%	68%	73%	63%	62%	64%	73%	68%	64%	70%	65%	65%	70%	65%	65%	70%	65%	65%		
<i>F - stats</i>		35	32	40	25.3	24	25.7	40	25.3	24	34.9	26.7	27.6	34.9	26.7	27.6	34.9	26.7	27.6		
<i>AIC</i>		-309.4	-254.6	-278.5	-184.8	-166.5	-247.6	-278.5	-184.8	-166.5	-307.1	-193.4	-268.4	-307.1	-193.4	-268.4	-307.1	-193.4	-268.4		
<i>BIC</i>		-275.7	-220.9	-244.8	-151.1	-132.9	-213.9	-244.8	-151.1	-132.9	-273.4	-159.7	-234.8	-273.4	-159.7	-234.8	-273.4	-159.7	-234.8		
Panel B - Univariate analysis																					
		$\lambda = 1$						$\lambda = 3$						Optimum λ							
Maturity		3m	6m	12m	3m	6m	12m	3m	6m	12m	3m	6m	12m	3m	6m	12m	3m	6m	12m		
<i>Intercept</i>		0.662***	0.722***	0.797***	0.980***	1.055***	1.095***	0.980***	1.055***	1.095***	0.668***	1.023***	1.065***	0.668***	1.023***	1.065***	0.668***	1.023***	1.065***		
<i>Sent</i>		-0.017	-0.019	-0.02	-0.022	-0.023	-0.018	-0.022	-0.023	-0.018	-0.017	-0.021	-0.017	-0.017	-0.021	-0.017	-0.017	-0.021	-0.017		
		-0.105***	-0.126***	-0.117***	-0.141***	-0.163***	-0.117***	-0.141***	-0.163***	-0.117***	-0.106***	-0.162***	-0.125***	-0.106***	-0.162***	-0.125***	-0.106***	-0.162***	-0.125***		
		-0.015	-0.017	-0.017	-0.021	-0.023	-0.016	-0.021	-0.023	-0.016	-0.015	-0.022	-0.016	-0.015	-0.022	-0.016	-0.015	-0.022	-0.016		
R^2		18%	19%	16%	18%	22%	18%	18%	22%	18%	18%	24%	23%	18%	24%	23%	18%	24%	23%		
<i>F - stats</i>		33.2	36.6	30.4	34.3	44.2	34.9	34.3	44.2	34.9	33.6	49.6	45.4	33.6	49.6	45.4	33.6	49.6	45.4		
<i>AIC</i>		-326.1	-186	-34.1	-34.1	-34.1	-34.1	-34.1	-34.1	-34.1	-34.1	-34.1	-34.1	-34.1	-34.1	-34.1	-34.1	-34.1	-34.1		
<i>BIC</i>		-320	-179.9	40.3	40.3	40.3	40.3	40.3	40.3	40.3	40.3	40.3	40.3	40.3	40.3	40.3	40.3	40.3	40.3		

Table 6: Robustness checks: Kupiec's test

This table reports the results from Kupiec's (1995) percentage of failure (POF) test for violations of the extreme quantile returns (EQR) from the empirical density function (EDF) by the EQR of a set of RND and subjective density functions. The test is performed as a robustness check to the extreme value theory (EVT)-based tests performed on the EQR and on the expected upside returns. The null hypothesis, which is designed as a log-likelihood ratio test (Eq. (16)), is that the realized probability of failure ($\frac{v}{n}$) matches the predicted one p^* . Thus if the LR exceeds the critical value, $\chi^2(1)=3.841$, such a hypothesis is rejected at the five percent level. Translating the methodology to our empirical problem, (p^*) becomes the assumed probability that the EQR of the subjective and of the risk-neutral densities will violate the EQR of the realized returns, where $\frac{v}{n}$ is the realized number of violations. We note that because we apply Kupiec's test to the upside returns, violations mean that returns are higher than a positive threshold.

Panel A - Three-month calls									
	EQR 90%			EQR 95%			EQR 99%		
p = 10%	POF	p-value	LR-stat	POF	p-value	LR-stat	POF	p-value	LR-stat
RND vs EDF	99.9%	0.0000	∞	99.2%	0.0000	∞	50.5%	0.0000	414.8
Power vs EDF	100.0%	0.0000	∞	100.0%	0.0000	∞	84.7%	0.0000	∞
Expo vs EDF	100.0%	0.0000	∞	100.0%	0.0000	∞	86.8%	0.0000	∞
PCPT vs EDF	100.0%	0.0000	∞	100.0%	0.0000	∞	67.2%	0.0000	752.0
CPT vs EDF	58.2%	0.0000	559.6	45.7%	0.0000	333.3	16.0%	0.0002	13.6
p = 5%	POF	p-value	LR-stat	POF	p-value	LR-stat	POF	p-value	LR-stat
RND vs EDF	99.9%	0.0000	∞	99.2%	0.0000	∞	50.5%	0.0000	671.5
Power vs EDF	100.0%	0.0000	∞	100.0%	0.0000	∞	84.7%	0.0000	∞
Expo vs EDF	100.0%	0.0000	∞	100.0%	0.0000	∞	86.8%	0.0000	∞
PCPT vs EDF	100.0%	0.0000	∞	100.0%	0.0000	∞	67.2%	0.0000	∞
CPT vs EDF	58.2%	0.0000	861.9	45.7%	0.0000	561.3	16.0%	0.0000	65.5
Panel B - Six-month calls									
	EQR 90%			EQR 95%			EQR 99%		
p = 10%	POF	p-value	LR-stat	POF	p-value	LR-stat	POF	p-value	LR-stat
RND vs EDF	99.9%	0.0000	∞	93.3%	0.0000	∞	13.8%	0.0160	5.8
Power vs EDF	99.9%	0.0000	∞	97.7%	0.0000	∞	22.1%	0.0000	49.7
Expo vs EDF	99.9%	0.0000	∞	97.8%	0.0000	∞	23.0%	0.0000	56.4
PCPT vs EDF	99.9%	0.0000	∞	97.3%	0.0000	∞	17.0%	0.0000	18.2
CPT vs EDF	62.4%	0.0000	647.0	36.3%	0.0000	197.3	5.7%	0.0019	9.6
p = 5%	POF	p-value	LR-stat	POF	p-value	LR-stat	POF	p-value	LR-stat
RND vs EDF	99.9%	0.0000	∞	93.3%	0.0000	∞	13.8%	0.0000	44.8
Power vs EDF	99.9%	0.0000	∞	97.7%	0.0000	∞	22.1%	0.0000	137.7
Expo vs EDF	99.9%	0.0000	∞	97.8%	0.0000	∞	23.0%	0.0000	149.7
PCPT vs EDF	99.9%	0.0000	∞	97.3%	0.0000	∞	17.0%	0.0000	76.0
CPT vs EDF	62.4%	0.0000	∞	36.3%	0.0000	369.9	5.7%	0.5474	0.4
Panel C - Twelve-month calls									
	EQR 90%			EQR 95%			EQR 99%		
p = 10%	POF	p-value	LR-stat	POF	p-value	LR-stat	POF	p-value	LR-stat
RND vs EDF	62.8%	0.0000	655.1	25.0%	0.0000	72.9	20.3%	0.0000	37.0
Power vs EDF	93.5%	0.0000	∞	42.5%	0.0000	283.5	29.3%	0.0000	114.7
Expo vs EDF	94.6%	0.0000	∞	43.1%	0.0000	292.7	30.4%	0.0000	126.2
PCPT vs EDF	79.5%	0.0000	1067.2	36.1%	0.0000	194.7	24.4%	0.0000	68.3
CPT vs EDF	29.4%	0.0000	115.2	7.2%	0.0480	3.9	8.4%	0.2666	1.2
p = 5%	POF	p-value	LR-stat	POF	p-value	LR-stat	POF	p-value	LR-stat
RND vs EDF	62.8%	0.0000	∞	25.0%	0.0000	177.9	20.3%	0.0000	114.2
Power vs EDF	93.5%	0.0000	∞	42.5%	0.0000	492.6	29.3%	0.0000	245.6
Expo vs EDF	94.6%	0.0000	∞	43.1%	0.0000	505.3	30.4%	0.0000	263.6
PCPT vs EDF	79.5%	0.0000	∞	36.1%	0.0000	366.1	24.4%	0.0000	170.2
CPT vs EDF	29.4%	0.0000	246.4	7.2%	0.0631	3.5	8.4%	0.0048	8.0

Table 7: Robustness checks: time-varying weighting function parameters

This table reports robustness checks of our time-varying estimates of overweight of small probabilities. Panel A reports the summary statistics of the estimated omega (ω) parameter, which is the parameter used in the Prelec's (1998) probability weighting function (see Eq. (17)). Similarly to the CPT, the parameter ω defines the curvature of the weighting function for gains and losses, which leads the probability weighting functions to assume inverse S-shapes. An ω parameter equal to one means a weighting function with un-weighted (neutral) probabilities, whereas $\omega < 1$ denotes overweighting of small probabilities. Similarly to γ , we estimate long-term ω 's (reported for γ in Table 1, Panel A) as well as time-varying ω 's (reported for γ in Table 4, Panel C). Panels B and C report γ estimates when the CPT's probability weighting parameter for left side of the distribution (δ) is assumed to be, respectively, 0.69 (the CPT parameterization) and 1 (neutral probability weighting). Panel C and D report γ estimates when the CPT's value weighting parameters α and β for diminishing sensitivity to gains and losses are assumed to be, respectively, 0.75 (increased diminishing sensitivity) and 1 (no diminishing sensitivity). We assume in these robustness tests that the loss aversion parameter λ equals 2.25.

Panel A - Prelec omega (ω)

Maturity	Min	25% Qtile	Median	Mean	75% Qtile	Max	StDev	% $\omega < 1$ (98-03)	% $\omega < 1$ (03-08)	% $\omega < 1$ (08-13)	RSS	LT
3 months	0.42	0.76	0.93	0.95	1.07	1.75	0.27	95%	36%	61%	0.0204	0.46
6 months	0.37	0.84	0.99	1.02	1.17	1.75	0.26	88%	21%	45%	0.017	0.68
12 months	0.44	0.94	1.07	1.05	1.18	1.75	0.21	79%	10%	28%	0.0201	1.14

Panel B - Gamma with overweight of small probabilities on the right tail ($\delta = 0.69$)

Maturity	Min	25% Qtile	Median	Mean	75% Qtile	Max	StDev	% $\gamma < 1$ (98-03)	% $\gamma < 1$ (03-08)	% $\gamma < 1$ (08-13)	RSS
3 months	0.44	0.7	0.86	0.97	1.21	1.75	0.34	99%	23%	53%	0.0231
6 months	0.4	0.75	1.01	1.04	1.27	1.75	0.31	93%	13%	43%	0.0198
12 months	0.4	0.83	1.05	1.04	1.24	1.75	0.25	87%	11%	32%	0.0238

Panel C - Gamma with neutral probability weighting on the right tail ($\delta = 1$)

Maturity	Min	25% Qtile	Median	Mean	75% Qtile	Max	StDev	% $\gamma < 1$ (98-03)	% $\gamma < 1$ (03-08)	% $\gamma < 1$ (08-13)	RSS
3 months	0.48	0.73	0.88	0.97	1.15	1.75	0.3	98%	30%	58%	0.023
6 months	0.43	0.8	0.99	1.03	1.24	1.75	0.3	92%	16%	45%	0.0191
12 months	0.5	0.87	1.03	1.02	1.13	1.75	0.22	84%	12%	35%	0.0233

Panel D - Gamma with pronounced diminishing sensitivities to gains and losses ($\alpha and \beta = 0.75$)

Maturity	Min	25% Qtile	Median	Mean	75% Qtile	Max	StDev	% $\gamma < 1$ (98-03)	% $\gamma < 1$ (03-08)	% $\gamma < 1$ (08-13)	RSS
3 months	0.45	0.81	0.96	0.98	1.09	1.75	0.25	93%	27%	52%	0.023
6 months	0.38	0.9	1.02	1.06	1.23	1.75	0.25	82%	13%	36%	0.0196
12 months	0.32	0.99	1.07	1.09	1.18	1.75	0.2	65%	5%	21%	0.0276

Panel E - Gamma with no diminishing sensitivities to gains and losses ($\alpha and \beta = 1$)

Maturity	Min	25% Qtile	Median	Mean	75% Qtile	Max	StDev	% $\gamma < 1$ (98-03)	% $\gamma < 1$ (03-08)	% $\gamma < 1$ (08-13)	RSS
3 months	0	0.72	0.88	0.87	1.03	1.75	0.24	98%	40%	64%	0.0204
6 months	0	0.78	0.98	0.93	1.14	1.75	0.3	94%	21%	49%	0.0163
12 months	0.04	0.86	1.02	0.98	1.14	1.75	0.25	88%	11%	33%	0.0207

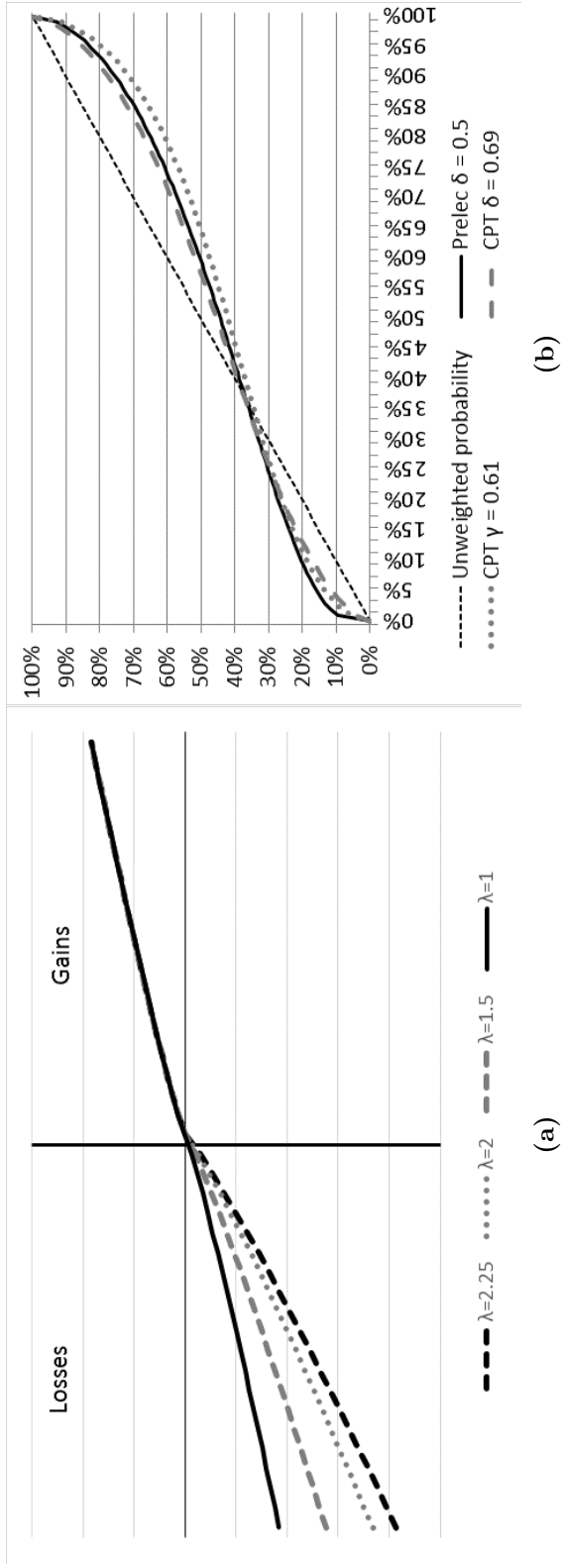
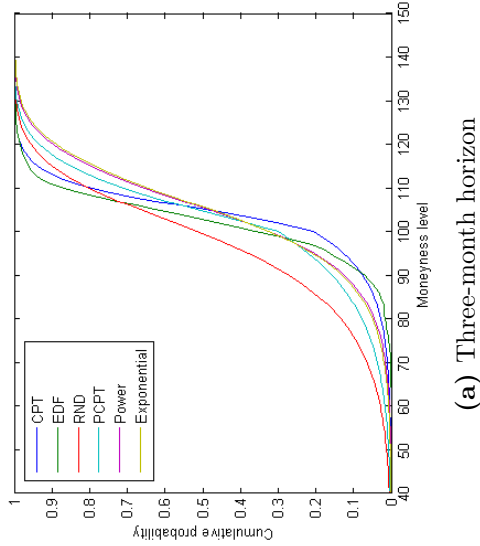
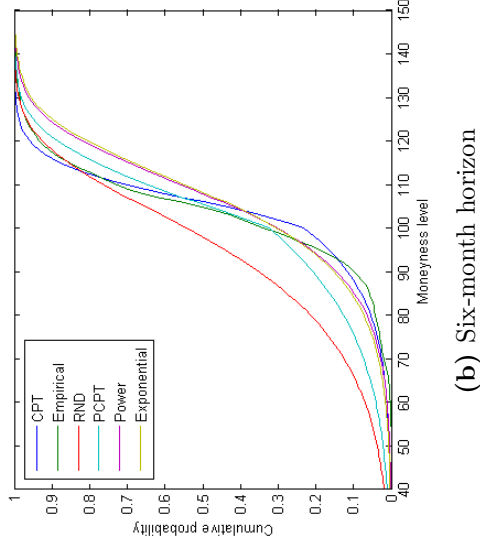


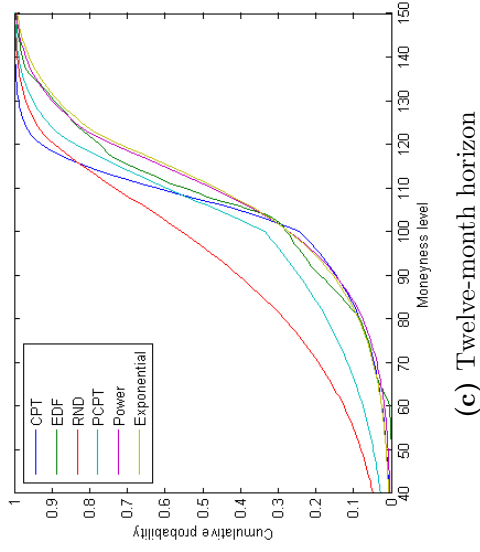
Figure 1: Impact of changes in λ , γ and δ . Plot A in this figure shows the impact of changes in the loss aversion parameter λ on the cumulative prospect theory (CPT) value function $v(x)$. Plot B depicts the CPT weighting function $w(x)$ for the probability weighting parameters γ and δ , respectively, for gains and losses, as well as the weighting function of Prelec (1998) for the probability weighting parameter δ (defined as ω in our study).



(a) Three-month horizon

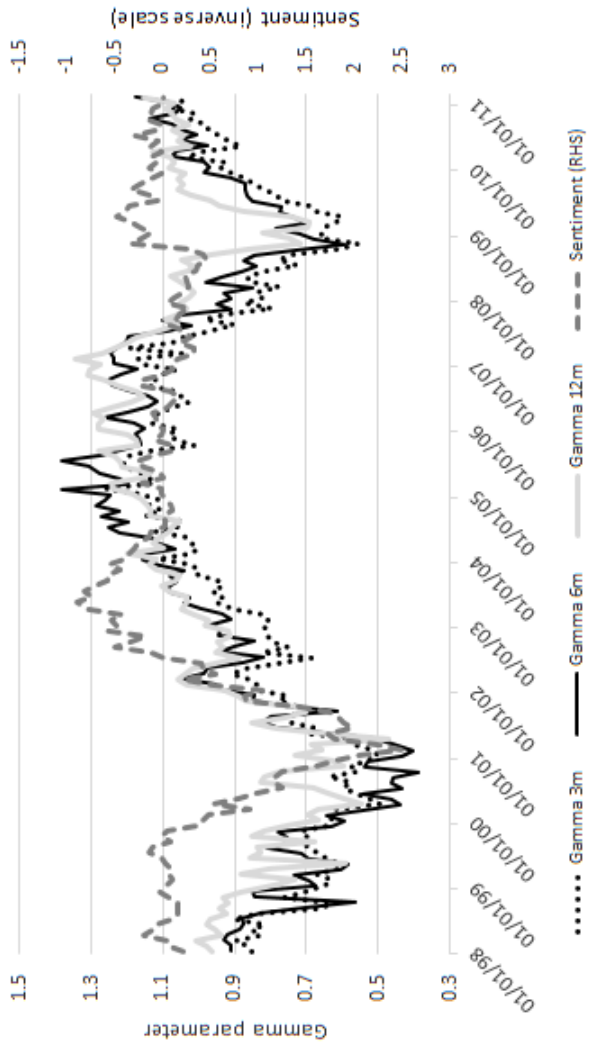


(b) Six-month horizon

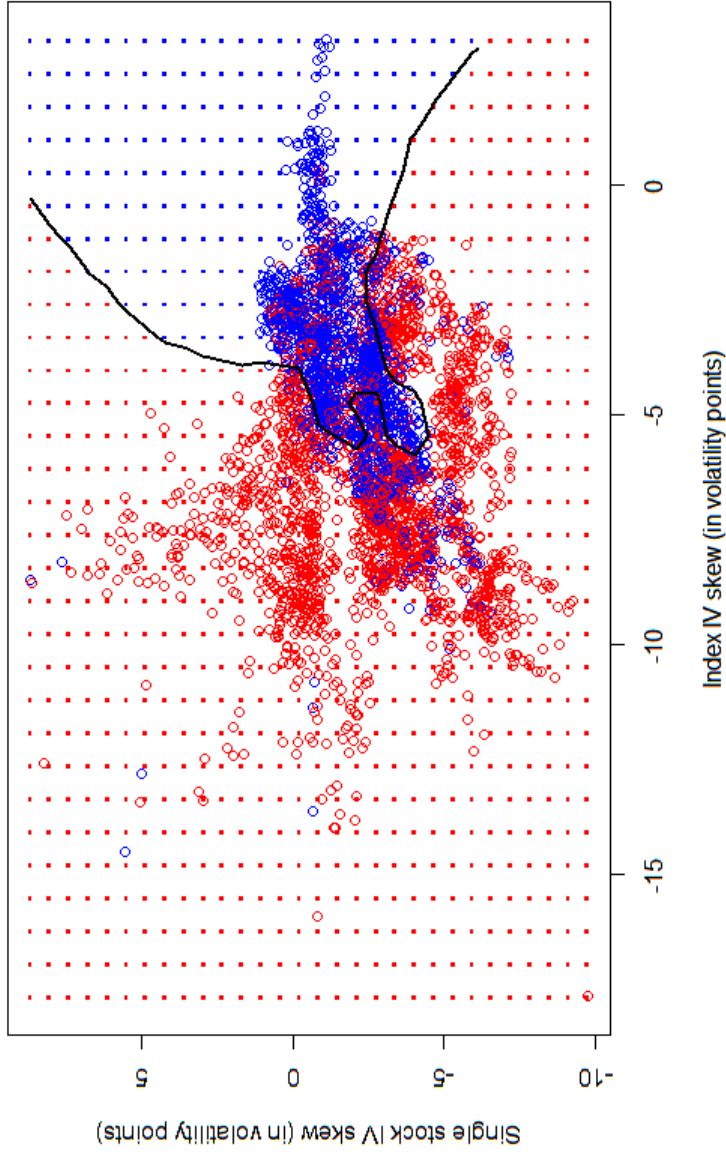


(c) Twelve-month horizon

Figure 2: Cumulative density functions. This figure shows three plots that depict the cumulative density function (CDF) for equity returns obtained from the empirical density function (EDF), the risk-neutral density (RND), and the four subjective density functions: 1) the power utility density, 2) the exponential utility density, 3) the cumulative prospective theory density (CPT), and the partial CPT (PCPT). The equity returns' CDFs from these six sources are presented for three-, six-, and twelve-month horizons.



47 **Figure 3: Time varying nature of gamma parameter in CPT.** This figure depicts the time-varying nature of the gamma (γ) parameter from three-, six-, and twelve-month single stock options estimated using the CPT parameterization as well as the sentiment factor of Baker and Wurgler (2007).



(a) 3 months

Figure 4: k -Nearest-Neighbors plot for IV skews. This figure shows a scatter plots depicting the relation between single stock (120 percent minus ATM) IV skews (on the y-axis) and index (120 percent minus ATM) IV skews (on the x-axis). Observations colored in red imply the presence of overweight of small probabilities on the right side of the distribution ($\gamma < 1$), whereas observations colored in blue imply either neutral probability weighting ($\gamma = 1$) or underweight of small probabilities ($\gamma > 1$). The decision boundary is produced by a k -Nearest-Neighbors algorithm ($k=41$, estimated via cross-validation) and delimits the region in which a new observation (of paired IV skews, such as the solid IV skews, will be assigned to the overweight of small probabilities class (in red) or the alternative class (in blue).