TI 2015-085/VIII
Tinbergen Institute Discussion Paper

Congestion Pricing in Urban Polycentric Networks with Distorted Labor Markets: A Spatial General Equilibrium Model for the Area Randstad

Ioannis Tikoudis

Faculty of Economics, VU University Amsterdam, and Tinbergen Institute, the Netherlands.

Congestion pricing in urban polycentric networks with distorted labor markets: a spatial general equilibrium model for the area Randstad.

Ioannis Tikoudis

Department of Spatial Economics, VU University Amsterdam,
De Boelelaan 1105, 1081HV Amsterdam

and

Tinbergen Institute, Gustav Mahlerplein 117, 1082MS Amsterdam,
the Netherlands

Abstract

The paper presents a polycentric general equilibrium model with congestion externalities and distortionary labor taxation calibrated to fit the key empirical regularities of the regional economy and transport system of Randstad conglomeration. In line with more stylized models, marginal external cost pricing (i.e. a *quasi first-best* Pigouvian toll that ignores the pre-existing taxation in the labor market) is shown to generate considerable welfare losses. Surprisingly, the *quasi first-best* Pigouvian toll is welfare decreasing even when the road tax revenue is used to finance labor tax cuts. This is due to the large deviation of marginal external costs from the optimal toll levels, as the latter are found to be negative in many of the network links. Approximations of the key double-dividend effects show that, in those links, the tax interaction effect is strong enough to outweigh both the revenue-recycling and the Pigouvian effect.

Keywords: applied general equilibrium, network, road pricing, commuting, polycentricity, environmental taxation, double-dividend

JEL classification: D58, H21, H23, C63, R13, R40

1. Introduction

The taxation of externalities in road networks has received considerable attention in the literature of transport economics during the last decades. To a large extent, this is due to a series of second-best issues that arise from the use of pricing schemes that leave certain links, routes, or areas of the network untaxed. It has been shown in a variety of stylized settings that this *partial taxation* pushes the existing externality taxes below or above their Pigouvian levels (i.e. from marginal external costs).[1] Some earlier contributions in the first-best and second-best literature have derived rules for optimal road pricing in a generic static network. For instance, Verhoef (2002a; 2002b) offers a general analytical solution for the second-best problem where not all links of a congested network can be charged; an algorithm based on

[1] Throughout the entire paper the term *Pigouvian level* of a tax refers to a tax level that is equal to the marginal external cost.

this analytical solution is then tested on a medium size network. Also, van Dender (2004) shows that constraints in network pricing can cause the optimal toll to deviate in a complex way from the marginal external cost of congestion.[2]

The above partial taxation is *unintentional*, in the sense that it is always suboptimal to a Pigouvian tax rule, but the regulator cannot impose the latter because of some exogenous hampering factor (e.g. political acceptability, implementation costs etc.). Because no other distortionary tax or subsidy is considered elsewhere in the economy (even in the transport system), as soon as the hampering factor is removed, optimal taxes return to their Pigouvian levels.

The recent advances in the *double dividend* theory have highlighted a second reason for which second-best settings may emerge.[3] This regards the existence of at least one distortionary tax (or subsidy) somewhere in the economy, even inside the transport system (e.g. public transport subsidies). Like the case of partial taxation, the presence of the distortionary tax causes deviations of the optimal tolls from their Pigouvian levels. In fact, as in the partial taxation case (see for instance Verhoef et al., 1996) optimal externality taxes may turn out to be negative even when the entire network can be taxed. But the critical differentiator between the two streams is that the deviation of the externality tax from its Pigouvian level in the latter case may be welfare increasing, something that is not possible in the case of *partial taxation*. This is because the Pigouvian equilibrium in the presence of a distortionary tax is suboptimal (or *quasi first-best*) to begin with. Parry and Bento (2001) have highlighted the case of an optimal negative road tax for a single-link network with exogenous residential and working locations; this is to be juxtaposed against the *quasi first-best* Pigouvian toll which is welfare decreasing.

However, the use of a single-link network by Parry and Bento (2001) did not allow for a real merge of the two streams of literature. This merge has been recently explored by Tikoudis et al. (2015a), who show that in a serial monocentric network, the introduction of partial taxation in the form of a cordon toll can be welfare improving with a preexisting labor tax and the rest of determining factors set in accordance with the hypotheses made in Parry and Bento (2001).[4] On the other hand, the *quasi first-best* Pigouvian toll is shown to generate significant welfare losses. The key mechanisms that generate the above result are i) that the (positively priced) cordon toll imposed in a certain distance from CBD is affecting only the subgroup of the population that provides labor relatively inelastically, and ii) that the elasticity of labor supply falls with distance from the CBD.

This paper investigates further the welfare effects of *partial taxation* incorporated in a double dividend setting for a polycentric, mixed network setting. In a such setting, the (general equilibrium) elasticity of labor supply is determined in a fairly complex way: it does not only depend on household location, but also on the location of the job, the chosen route and the transport mode. We are interested in testing the earlier finding by Tikoudis et al. (2015a) in its strong form, i.e. with lump-sum revenue recycling, and in its weak form, i.e. with road tax revenue used to finance labor tax cuts.

The paper has a clear geographical reference. The area of Randstad is a polycentric urban conglomeration in western Netherlands, which comprises the country's four largest cities (Amsterdam, Rotterdam, the Hague and Utrecht). The region is of considerable economic significance; while it covers only 20 percent of the country's land area, at least 40 percent of the population resides there, and half of

[2] The above contributions assume that both the residence and job location, i.e. the OD pair of the commuter, is fixed.

[3] Goulder (1995) and Bovenberg (1999) provide excellent synopses of the existing literature in the field.

[4] In this case, the cordon toll is expressed as a series of consecutive links (extending from the edge of the serial network, i.e. the CBD to some given link) that remain untolled and a series of consecutive links that are subject to a uniform charge.

the national income is generated within its boundaries. Despite being a prosperous region, it has, for a series of years, experienced a lower productivity growth compared to other regions in the Netherlands and Europe.[5] It is characterized by large commuting flows between zones and severe congestion during the peak hours.

The territorial review by OECD (2007) places *heavy congestion* and the *incoherency of public transport system* as the most important drivers of this sluggish growth. Regarding congestion, roughly 80 percent of the traffic jams in the Netherlands in 2005 occurred in Randstad. The problem is deteriorated because the regional public transport is relatively fragmented, i.e. the coherence between the multiple operators and facilities is limited, resulting in a suboptimal use of the public transport system. Other identified factors include distortions originating from the labor and, especially, the housing market, where a series of land-use practices (e.g. density regulations) add burden to the social cost of public transport provision. Given this complex reality, the natural question arising is whether there are road pricing policies which can simultaneously address congestion and mitigate distortions elsewhere in the economy.

To study this in a coherent framework, this paper presents a stylized *general equilibrium network* model that is strongly related to previous work of Anas and Kim (1996) and Anas and Liu (2007) and more loosely to relevant CGE models (see for instance Böhringer and Rutherford, 2007).[6] Anas and Liu (2007) introduced a polycentric model for the wider metropolitan area of Chicago, and a recent application of the model in order to evaluate the potential welfare effects of a cordon toll in the area (Anas and Hiramatsu, 2013).[7] The interaction of transport with the markets of housing and labor is captured in a detailed way, since residence and job locations are endogenous. However, the distortions generated in the respective markets are not considered. Subsequently, the marginal social benefit of road use may be understated or overstated, depending on the relative conditions in these markets (taxes, subsidies, regulations).

Our model accounts explicitly for a double dividend setting, with a realistic tax rate on labor income. The calibration is designed to fit a series of stylized facts characterizing the behavior of the average household (expenditures shares, allocation of time, etc.) and the characteristics of Randstad region: the general spatial lay-out and network, the population and employment share of the population in each zone, the average commuting speed of modes, modal split, and the relative land rents, housing prices, wages and floor-to-area ratios.

We use the model to explore various pricing schemes (systems of uniform and differentiated cordon tolls around the major cities, as well as Pigouvian taxes in the entire network) accompanied by two distinct revenue recycling programs: i) lump-sum transfers and ii) labor tax cuts. In line with more stylized models, a Pigouvian road tax is shown to generate considerable welfare losses. These are less severe in the case of a system of cordon toll around the largest employment zones of the conglomeration. Surprisingly, Pigouvian tolls are also welfare *decreasing* even when the road tax revenue is used to finance labor tax cuts. By computing the optimal road tax by road link in this case, we find that the latter may not only lie far below its Pigouvian level, but it may also be negative in a large part of the network. We establish a clear connection of the above result with the double-dividend theory by approximating the

---

[5] annually 1.7 % over the period 1995-2005.

[6] The model is able to capture the various distortions occurring simultaneously in the markets for transport, housing and labor in Randstad. In the years to come, the model will be further developed and used as a tool for the evaluation of drastic reforms in pricing (road pricing, parking), programs that grant fiscal autonomy to certain network operators (causing tax exporting behavior), and for the appraisal of investments in drastic commuting technologies (e.g. the automated highway).

[7] See also Rhee et al. (2014).

Pigouvian, *tax-interaction* and *revenue recycling* effect for each link. A system of multiple cordon tolls leaves most of these links untaxed and is found to produce significant welfare gains, provided that is accompanied by a small downward adjustment in the uniform labor tax rate.

The paper has the following structure: section 2 presents the behavior of the various agents and the mechanics of the *general, stochastic user equilibrium*. Section 3 discusses the solution algorithm employed to solve for it. Section 4 presents the data and the overview of the algorithm used in the calibration of the model. Section 5 provides the policy analysis and sensitivity checks. Section 6 concludes.

2. Model

The model proposed in this section is a network-based, polycentric extension of the general equilibrium monocentric city models by Verhoef (2005), Tikoudis et al. (2015a; 2015b), and is in line with the preexisting contributions in the field (Anas and Kim, 1996; Anas and Xu, 1999; Anas and Liu, 2007; Anas and Hiramatsu, 2012).[8]

2.1. Space, network representation and discrete choice

Economic activity takes place in an ordered set of J zones (each represented by a single node), $\mathcal{J}$.[9] Let the ordered subsets $\mathcal{J}_R$ and $\mathcal{J}_W$ denote the locations that host residences and jobs respectively, with $\mathcal{J} = \mathcal{J}_R \cup \mathcal{J}_W$. Throughout the text, the subscript i is used to denote an arbitrary zone in the ordered set $\mathcal{J}_R$ that serves as a residential node, i.e. $i \in \mathcal{J}_R$. Similarly, the subscript j is used to denote an arbitrary zone in the ordered set $\mathcal{J}$ that serves as an employment node, i.e. $j \in \mathcal{J}_W$. Every zone is characterized by mixed land-use, in this case $\mathcal{J}_R \cap \mathcal{J}_W = \mathcal{J}$. Let the set $\mathcal{C}_{OD} = \mathcal{J}_R \times \mathcal{J}_W$ denote the Cartesian product of sets $\mathcal{J}_R$ and $\mathcal{J}_W$, i.e. the set that contains all possible pairs of residential and employment locations. Each element $\mathrm{a}_{ij} \in \mathcal{C}_{OD}$ is an *origin-destination pair* (hereafter, *OD pair*).

Two arbitrary zones, s and e, are *neighboring* if there is at least one transport link $l_m^{(se)}$ starting at s and ending at e, where the subscript m denotes the type of transport network the link belongs to (e.g. road, rail, etc.). Links are directed, thus $l_m^{(se)} \neq l_m^{(es)}$. A route q is defined as a *sequence* (i.e. an ordered list) of links such that, for each pair of consecutive links in the sequence, $l_m^{(se)}$ and $l_{m'}^{(s'e')}$, it holds that $e = s'$, although it can be that $m \neq m'$, i.e. paths can be *multimodal*. However, an arbitrary path cannot reach the same node twice, i.e. *cyclical* paths that contain at least two links, $l_m^{(se)}$ and $l_{m'}^{(s'e')}$, for which it holds that $s = s'$ or $s = e'$ or $e = s'$ or $e = e'$ are excluded.

For each *OD pair* a_{ij} in $\mathcal{C}_{OD}$ there is a set of corresponding possible routes, which we denote by $\mathcal{Q}(\mathrm{a}_{ij})$. It is straightforward that, if origin zone, i, and destination zone, j, are neighboring, then it holds that any $l_m^{(ij)} \in \mathcal{Q}(\mathrm{a}_{ij})$. An *alternative*, a, is a set that contains the *OD pair* a_{ij} and a route $q \in \mathcal{Q}(\mathrm{a}_{ij})$, i.e. $\mathrm{a} = \{\mathrm{a}_{ij}, q\} = \{i, j, q\}$. The choice set, denoted by $\mathcal{C}$, contains all possible alternatives.

[8] The exposition in Section 2 draws, to some extent from the earlier contributions by Tikoudis et al. (2015a; 2015b). Literal citations from these sources are not marked as such for legibility; duplicating equations are provided in order to keep this paper self-contained.

[9] Therefore, we abstract from intra-zonal links.

2.2. Households

Households can locate in any zone i and supply labor in any zone j. For simplicity, we normalize the *exogenous population*, N, to one. For each feasible alternative, $\mathrm{a} = \{\mathrm{a}_{ij}, q\}$, the household maximizes the quasi-linear utility function:

$$U_{\mathrm{a}} = \pi_0\, y_{\mathrm{a}} + \pi_1 \Bigg(\underbrace{s_{\mathrm{a}}^{\alpha}\, T_{F\mathrm{a}}^{\beta}}_{x_{\mathrm{a}}} \Bigg)^{\gamma}, \tag{1}$$

where y_{a} corresponds to the general consumption of a composite good and x_{a} to the consumption of a good composed by housing consumption, s_{a}, and leisure, $T_{F\mathrm{a}}$ (hereafter, x_{a} is referred to as the *lifestyle choice*). The marginal utility of income is constant and equal to π_0. Given that the parameters of the Cobb-Douglas subutility function for the *lifestyle choice* are such that $\alpha < 1$, $\beta < 1$ and $\alpha + \beta = 1$, the marginal utility with respect to the residential space and leisure is diminishing for $\gamma < 1$. The total time endowment of the household, T, is spent on commuting from i to j, $T_{C\mathrm{a}}$, working, $T_{L\mathrm{a}}$, and leisure, $T_{F\mathrm{a}}$:

$$T = T_{C\mathrm{a}} + T_{L\mathrm{a}} + T_{F\mathrm{a}}. \tag{2}$$

Labor supply is inelastic throughout a working day, which is of fixed duration, t_L, independent of working location. The household anticipates every trip to work to require t_{a} units of time. This anticipated commuting time equals the endogenously determined expected commuting time, $\hat{t}_q$, in the stochastic user equilibrium (see below), where the subscript q refers to the route associated with alternative $\mathrm{a} = \{\mathrm{a}_{ij}, q\}$. For D_{Wa} working days the time constraint becomes:

$$T = D_{\mathrm{Wa}}\,(t_L + t_{\mathrm{a}}) + T_{F\mathrm{a}}\,. \tag{3}$$

Normalizing the duration of the working day, t_L, to 1, the above constraint becomes:

$$T = D_{\mathrm{Wa}}\,(1 + t_{\mathrm{a}}) + T_{F\mathrm{a}} \Leftrightarrow D_{\mathrm{Wa}} = \frac{T - T_{F\mathrm{a}}}{1 + t_{\mathrm{a}}}\,. \tag{4}$$

The net wage per working day is defined as the difference between wage in zone j, w_j, the labor tax, τ_L, and the expected pecuniary cost of commuting under the choice of alternative a, i.e. c_{a}. The full income, M_{a}, of the household that has chosen alternative $\mathrm{a} = \{\mathrm{a}_{ij}, q\}$ is the maximum income that can be realized when leisure time is zero. That is:

$$M_{\mathrm{a}} = B + B_{\ell} + \underbrace{\frac{\big(w_j(1 - \tau_L) - c_{\mathrm{a}}\big)}{1 + t_{\mathrm{a}}}}_{value\ of\ time\ if\ \mathbf{a}} T, \tag{5}$$

where B denotes a lump-sum transfer from the government to the household and B_{ℓ} is the income from land rents, which are returned to households lump-sum. Both B and B_{ℓ} are exogenous (from the viewpoint of the household) and independent of any element composing the alternative $\mathrm{a} = \{\mathrm{a}_{ij}, q\}$. We refer to this type of redistribution as *horizontal revenue recycling*. For simplicity, we assume that intra-zonal travel time and cost is zero, i.e. $c_{\mathrm{a}} = t_{\mathrm{a}} = 0$ if $\mathrm{a} = \{\mathrm{a}_{ij}, q\} = \{i, j, q\}$ is such that $i = j$. The full income can be used to buy back leisure at the shadow price of time, $\left(w_j(1-\tau_L) - c_{\mathrm{a}}\right)/(1+t_{\mathrm{a}})$, (hereafter the *value of time*, denoted by v_{a}) and for the consumption of the composite good and residential space. The budget constraint can then be written as:

$$B + B_\ell + v_{\mathrm{a}}T = v_{\mathrm{a}}T_{F\mathrm{a}} + p\, y_{\mathrm{a}} + p_{Hi}\, s_{\mathrm{a}}, \tag{6}$$

where p is the (uniform in the entire region) price of the composite good (hereafter normalized to one) and p_{Hi} the price of housing per unit of space at the zone indexed by i. By definition, both housing consumption and leisure are essential, thus $s_{\mathrm{a}} > 0$ and $T_{F\mathrm{a}} > 0$. Furthermore, leisure is upper-bounded by the total time endowment, therefore $T_{F\mathrm{a}} < T (= T$ if $D^*_{W\mathrm{a}} = 0)$, and consumption has to be non-negative, i.e. $y_{\mathrm{a}} \geq 0$. To maximize (1) subject to (6) and the above non-negativity constraints we set up the Lagrangian:

$$\mathcal{L} = \pi_0\, y_{\mathrm{a}} + \pi_1 \left(s_{\mathrm{a}}^{\alpha}\, T_{F\mathrm{a}}^{\beta}\right)^{\gamma} - \psi[v_{\mathrm{a}}T_{F\mathrm{a}} + p\, y_{\mathrm{a}} + p_{Hi}\, s_{\mathrm{a}} - (B + B_\ell + v_{\mathrm{a}}T)] + \vartheta_C\, y_{\mathrm{a}} + \vartheta_F^U (T - T_{F\mathrm{a}}), \tag{7}$$

and solve the system of the three first order conditions equations $\mathcal{L}'_y = 0$, $\mathcal{L}'_S = 0$, $\mathcal{L}'_F = 0$ and the budget constraint in (6). For an interior optimum the corresponding four unknowns are y_{a}, s_{a}, $T_{F\mathrm{a}}$ and the Lagrangian multiplier, ψ, since the complementary slackness conditions require that $\vartheta_C = \vartheta_F^U = 0$.[10] Solving the system yields the Marshallian demand functions for housing and leisure time respectively:

$$s_{\mathrm{a}}^* = \left(\frac{p_{Hi}\pi_0}{\alpha\gamma\pi_1}\right)^{\frac{1}{\gamma-1}} \left(\frac{\alpha v_{\mathrm{a}}}{\beta p_{Hi}}\right)^{\frac{\beta\gamma}{\gamma-1}}, \tag{8}$$

$$T_{F\mathrm{a}}^* = \left(\frac{p_{Hi}\pi_0}{\alpha\gamma\pi_1}\right)^{\frac{1}{\gamma-1}} \left(\frac{\alpha v_{\mathrm{a}}}{\beta p_{Hi}}\right)^{\frac{\beta\gamma}{\gamma-1}-1}. \tag{9}$$

Inserting (9) into (4) yields the optimal labor supply for alternative a:

$$D_{W\mathrm{a}}^* = \frac{T - \left(\frac{p_{Hi}\pi_0}{\alpha\gamma\pi_1}\right)^{\frac{1}{\gamma-1}} \left(\frac{\alpha v_{\mathrm{a}}}{\beta p_{Hi}}\right)^{\frac{\beta\gamma}{\gamma-1}-1}}{1 + t_{\mathrm{a}}}. \tag{10}$$

[10] Corner optima with zero consumption are found by setting $y_{\mathrm{a}} = 0$ and solving the same system for ϑ_C, s_{a}, $T_{F\mathrm{a}}$ and ψ. Similarly, corner optima with zero labor supply are found by setting $T_{F\mathrm{a}} = 0$ and solving the system for y_{a}, ϑ_F^U, s_{a}, and ψ. The requirements for an admissible solution are that the remaining endogenous variables lie in the interior and that the respective multiplier (ϑ_C in the first case and ϑ_F^U in the second) is positive. Zero consumption is an artifact of the quasi-linear preference relation in (1). However, the chosen parameters rule out the possibility of a corner solution (in which at least one of the conditions $D^*_{W\mathrm{a}} = 0$ and $y^*_{\mathrm{a}} = 0$ holds) for any given alternative in the choice set. Later on we discuss the rationale behind the choice of a preference relation characterized by constant marginal utility of income.

Optimal consumption, y_{a}^{*}, can be computed by inserting (8) and (9) into (6). Substituting y_{a}^{*}, s_{a}^{*} and $T_{F\mathrm{a}}^{*}$ into the objective function and allowing an *alternative-specific constant*, $\mathrm{z_a}$, yields the *indirect utility* of alternative a:

$$V_{\mathrm{a}}^{*}=V\left(w_j,p_{Hi},\tau_L,c_{\mathrm{a}},t_{\mathrm{a}},B,B_{\ell}\right)=\mathrm{z_a}+\pi_0 y_{\mathrm{a}}^{*}+\pi_1\left(s_{\mathrm{a}}^{*\alpha}\,T_{F\mathrm{a}}^{*\beta}\right)^{\gamma}. \tag{11}$$

The alternative-specific constant is the sum of: i) a *residential-specific constant*, z_{Ii}, that captures the average utility of locational characteristics (e.g. amenities, ambient pollution) not modeled specifically in zone i, ii) an *employment-specific constant*, z_{Jj}, that captures the average utility of non-pecuniary or time characteristics of the average job offered in zone *j* (e.g. prospects for a better future job arrangement due to spatial concentration of jobs), iii) a *mode-specific constant*, z_M, (discussed below) and iv) a sum of link-specific constants, z_q, capturing the average utility of non-pecuniary or time characteristics of the links (e.g. the presence of a gas station or other facility) that form the route *q* involved in alternative $\mathrm{a}=\{\mathrm{a}_{ij},q\}$. Therefore:

$$\mathrm{z_a}=\mathrm{z}_{Ii}+\mathrm{z}_{Jj}+\mathrm{z}_M+\underbrace{\sum_{l_m^{(se)}\in q} z_m^{(se)}}_{z_q}. \tag{12}$$

The *mode-specific constant*, z_M, captures the average (dis)utility of commuting stemming from factors that are not modelled explicitly: waiting times, changing from a private to a public mode and vice versa, in-vehicle-comfort, cruising time, etc. More specifically, it is assumed that:

$$\mathrm{z}_M=\sum_m I(q,m)\cdot\mathrm{z}_m+I_t(q)\cdot\mathrm{z}_t, \tag{13}$$

where the indicator function $I(q,m)$ equals one if route q makes use of mode m (zero otherwise), the indicator function $I_t(q)$ equals one if route q involves a transit from a private to a public mode (or vice versa, otherwise zero), z_m is the average disutility inflicted to the individual by the use of mode m (compared to those that do not have to commute, i.e. individuals which choose an $\mathrm{a}=\{\mathrm{a}_{ij},q\}$ such that $i=j$), and z_t the average disutility of a mode change. Finally, a stochastic term, ε_{a}, which is *i.i.d. extreme value type I* across alternatives in $\mathcal{C}$, is added to (11) in order to capture the rest of the factors that are omitted in the model and may determine the choice of $\mathrm{a}=\{\mathrm{a}_{ij},q\}$. Total utility is, thus:

$$U_{\mathrm{a}}^{*}=\underbrace{\mathrm{z}_{Ii}+\mathrm{z}_{Jj}+\sum_m I(q,m)\mathrm{z}_m+I_t(q)\mathrm{z}_t+\sum_{l_m^{(se)}\in q} z_m^{(se)}+\pi_0 y_{\mathrm{a}}^{*}+\pi_1\left(s_{\mathrm{a}}^{*\alpha}\,T_{F\mathrm{a}}^{*\beta}\right)^{\gamma}}_{V_{\mathrm{a}}^{*}} \tag{14}$$
$$+\,\varepsilon_{\mathrm{a}}.$$

Due to the inclusion of alternative specific constants, the error term has a mean equal to zero by construction and standard deviation equal to $\lambda\left(\pi/\sqrt{6}\right)$, where λ is the scale parameter of the distribution. Because exogenous income, $B+B_{\ell}$, does not appear in (8) or (9) it is straightforward that the marginal (systematic) utility of income is constant and equal to π_0.[11] Because the error component ε_{a} is additive to the systematic utility and follows a *Generalized Extreme Value* distribution, the expectation of the maximum utility (hereafter, $Emax$) that can be derived when facing the choice set $\mathcal{C}$ is the well-known *logsum* expression:

$$Emax = \lambda \left[\mathcal{E} + log \sum_{\mathrm{a} \in \mathcal{C}} \left\{ \frac{exp(V_{\mathrm{a}}^*)}{\lambda} \right\} \right], \tag{15}$$

where $\mathcal{E} \approx 0.5772$ is the Euler constant. The resulting logit choice probability for each alternative a in the choice set $\mathcal{C}$ is:

$$P_{\mathrm{a}} = \frac{(exp(V_{\mathrm{a}}^*)/\lambda)}{\sum_{\mathrm{a} \in \mathcal{C}} \{exp(V_{\mathrm{a}}^*)/\lambda\}}. \tag{16}$$

2.3 Firms

A competitive, representative firm is located in each zone $j \in \mathcal{J}$ and produces a zone-specific intermediate output, Q_j, under constant returns to scale, using capital (K), and labor (L):

$$Q_j^S = A_j\, K^{\delta} L^{1-\delta}, \tag{17}$$

where A_j denotes the zone-specific total factor productivity. The zero profit condition, implies that the price of the good produced in zone j, p_j, is equal to the unit cost:

$$p_j = \frac{1}{A_j} \underbrace{\left\{ \left(\frac{\delta}{1-\delta}\right)^{1-\delta} + \left(\frac{1-\delta}{\delta}\right)^{\delta} \right\}}_{\Phi} \; R^{\delta} w_j{}^{1-\delta}, \tag{18}$$

where w_j is the local equilibrium wage, and R the exogenous price of capital. The conditional factor demands for labor, capital and land can be computed using the Shephard's lemma, i.e. by differentiating (18) with respect to the corresponding price of the input and multiplying with the level of output, Q_j. Thus, labor demand is:

$$L_j^D = \frac{1}{A_j} \Phi(1-\delta)\, R^{\delta} w_j{}^{-\delta} Q_j^S, \tag{19}$$

and capital demand is:

$$K_j^{D_f} = \frac{1}{A_j} \Phi\delta\, R^{\delta-1} w_j{}^{1-\delta} Q_j^S. \tag{20}$$

An *assembly industry* combines the J distinct intermediate goods (which are bought from the local firms, each at price p_j) to produce the composite good demanded by the consumers (see section 2.1) and by the rest of the world. The amount of the composite good produced is given by the Cobb-Douglas production function:

[11] Solving (6) for y_{a}, replacing s_{a} and $T_{F\mathrm{a}}$ by their optimal values in (8) and (9) respectively, plugging the resulting y_{a}^* in (14) and differentiating V_{a}^* with respect to B gives π_0.

$$Y = \prod_{j \in \mathcal{J}} Q_j^{\zeta_j}, \tag{21}$$

where ζ_j is the share of intermediate good produced in zone j in the total cost of Y, and $\sum_{j \in \mathcal{J}} \zeta_j = 1$. The associated minimum cost function is:

$$c(Y) = Y \underbrace{\left(\prod_{j \in \mathcal{J}} p_j^{\zeta_j}\right)\left(\sum_{j \in \mathcal{J}} \omega_j\right)}_{marginal\ cost}, \tag{22}$$

where the auxiliary parameter ω is:

$$\omega_j = \frac{\zeta_j^{(\sum_{k \neq j} \zeta_k)}}{\prod_{k \neq j} \zeta_k{}^{\zeta_k}}. \tag{23}$$

The conditional factor demand for each intermediate can be derived using Shephard's lemma. This is:

$$Q_j^D = \frac{\partial c(Y)}{\partial p_j} = \zeta_j\, Y\, p_j^{\zeta_j - 1}\left(\prod_{k \neq j} p_k^{\zeta_k}\right)\left(\sum_{j \in \mathcal{J}} \omega_j\right). \tag{24}$$

Note that neither capital nor labor is used in the combining process.

2.4 Developers

A competitive, representative developer produces homogenous residential space, s, using capital (K) and land (X). We assume a Cobb-Douglas production function:

$$s_i^S = K^{\theta} X^{1-\theta}. \tag{25}$$

Just like the ordinary firms, the firms in the construction sector make zero profits in equilibrium. This implies the following housing price per unit of floor space:

$$p_{Hi} = \underbrace{\left\{\left(\frac{\theta}{1-\theta}\right)^{1-\theta} + \left(\frac{1-\theta}{\theta}\right)^{\theta}\right\}}_{\widehat{\Phi}} R^{\theta} p_{Li}{}^{1-\theta}\ . \tag{26}$$

Again, Shephard's lemma can be used to derive the conditional factor demands for capital:

$$K_i^{D_d} = \widehat{\Phi}\theta\, R^{\theta-1} p_{Li}^{1-\theta} s_i^S, \tag{27}$$

and land:

$$X_i^D = \widehat{\Phi}(1-\theta) R^{\theta} p_{Li}^{-\theta} s_i^S, \tag{28}$$

where the supply of land in each zone is exogenous and equal to $\bar{X}_i$.

2.5 Transport

The volume delay function is assumed to be linear in road links. That is, the time required to move from node s to node e, using a private mode, i.e. the road (R) link $l_R^{(se)}$ that connects them, is:

$$t_R^{se} = \ell_R^{se}(\xi_{0R}^{se} + \xi_{1R}^{se} d_R^{se}), \tag{29}$$

where ℓ_R^{se} and d_R^{se} are the length and demand (see below) of link $l_R^{(se)}$ respectively, ξ_{0R}^{se} is the free-flow travel time per unit of distance (i.e. the inverse of the free-flow speed) and ξ_{1R}^{se} is the marginal delay caused by an additional unit of (traffic) demand in the link. Public transport (P) links are not subject to congestion, but free flow travel time is longer:[12]

$$t_P^{se} = \ell_P^{se} \xi_{0P}^{se}. \tag{30}$$

We now turn to *network loading*. Total demand for road link $l_R^{(se)}$ is:

$$d_R^{se} = N \sum_{\mathrm{a} \in \mathcal{C}} \left\{ I\left(l_R^{(se)} | q\right) P_{\mathrm{a}} D_{W\mathrm{a}}^{*} \right\}, \tag{31}$$

where the indicator function $I\left(l_R^{(se)} | q\right)$ equals one if the route q of alternative $\mathrm{a} = \{\mathrm{a}_{ij}, q\}$ contains link $l_R^{(se)}$ (zero if not or if a does not imply any commuting). The aggregation of (29) across all links of a feasible route q yields the *resulting* route-specific (as opposed to the anticipated travel time t_{a} by households) travel time:

$$\hat{t}_q = \sum_{(\ell_R^{se}, \ell_P^{se}) \in q} t_m^{se} \quad \text{with } m = (R, P). \tag{32}$$

Adding up the lengths of the road links involved in route q yields the total distance generated by car in this route:

$$L_{Rq} = \sum_{\ell_R^{se} \in q} \ell_R^{se}. \tag{33}$$

Similarly, the distance covered by the public transport mode in the arbitrary route q is:

[12] Therefore, for two links connecting an identical pair of nodes, it is generally assumed that $\xi_{0R}^{se} < \xi_{0P}^{se}$.

$$L_{Pq} = \sum_{\ell_P^{se} \in q} \ell_P^{se}. \tag{34}$$

The pecuniary cost of route q is:

$$\hat{c}_q = \underbrace{(p_g L_{Rq}) + (p_P L_{Pq})}_{commuting\ expenditure} + \underbrace{\sum_{\ell_R^{se} \in q} \tau_R^{se}}_{total\ road\ toll}, \tag{35}$$

where p_g and p_P is the monetary cost per unit of distance when commuting by car and public transport respectively, and τ_R^{se} is the road toll imposed to the commuter that uses the road link $l_R^{(se)}$.[13] Both prices, p_g and p_P, are equal to their marginal costs, denoted by $\hat{p}_g$ and $\hat{p}_P$ respectively. Weighting commuting expenditure in (35) across alternatives yields the total transport expenditure of households in the economy, i.e.:

$$E_T = N \cdot \underbrace{\sum_{\mathrm{a} \in \mathcal{C}} \{ P_\mathrm{a} D_{W\mathrm{a}}^* [(p_g L_{Rq}) + (p_P L_{Pq})] \}}_{total\ household\ transport\ expenditure}, \tag{36}$$

where q refers to the commuting route of alternative $\mathrm{a} = \{\mathrm{a}_{ij}, q\}$. The total cost of transport provision is assumed to be:

$$C_T = F + N \cdot \underbrace{\sum_{\mathrm{a} \in \mathcal{C}} \{ P_\mathrm{a} D_{W\mathrm{a}}^* [(\hat{p}_g L_{Rq}) + (\hat{p}_P L_{Pq})] \}}_{variable\ cost\ per\ household}, \tag{37}$$

where F is the fixed cost of public transport provision. From (36) and (37) it can be seen that the transport provision deficit is:

$$D_T = \underbrace{F + N \cdot \sum_{\mathrm{a} \in \mathcal{C}} \{ P_\mathrm{a} D_{W\mathrm{a}}^* (\hat{p}_P - p_P) L_{Pq} \}}_{public\ transport\ deficit} + \underbrace{N \cdot \sum_{\mathrm{a} \in \mathcal{C}} \{ P_\mathrm{a} D_{W\mathrm{a}}^* (\hat{p}_g - p_g) L_{Pq} \}}_{private\ transport\ subsidies}. \tag{38}$$

2.6 Government and public budget

The federal government functions as a benevolent planning authority who controls the tax instruments τ_L and τ_R^{se}, and the redistribution instruments B and B_ℓ in order to maximize the expected maximum utility in (15). It must be stressed that welfare analysis based on the maximization of (15) might be *invalid* if the marginal utility of income is not constant, i.e. if (1) is replaced by a utility function in which income effects are switched on. In such a case, it has been shown by Anas (2012) that, despite expected maximum utility still coincides with the logsum expression, the maximization of (15) subject to the equilibrium conditions (as described in sections 2.1-2.5) cannot be achieved without *alternative-specific*

[13] Thus, equation (35) gives the monetary cost of all alternatives $\mathrm{a} = \{i, j, q\}$ that make use of route q.

redistribution instruments that aim to equalize the marginal utility of income across alternatives. Consequently, with *horizontal revenue recycling*, optimal externality-correcting taxes (in the context of this model τ_R^{se}) deviate from their Pigouvian levels (see equation (56) below) even in the absence of any other failure in the model.[14] Furthermore, welfare analysis through the computation of compensating variations from a policy change (e.g. a tax reform) might prove a cumbersome task (see Herriges and Kling, 1999; Dagsvik and Karlström, 2005 for a complete discussion of the issue).

The government is responsible for the recycling of the expected revenue from road tolls and public transport fares, by controlling the public transport. Inserting (9) into (4) and weighting across the alternatives of the choice set $\mathcal{C}$ yields the expected total labor supply. The expected government revenue from the labor tax is, therefore:

$$R_L = \tau_L \cdot \underbrace{N \cdot \sum_{\text{a}\in\mathcal{C}} \{P_{\text{a}} D_{W\text{a}}^* w_j\}}_{total\ labor\ supply\ (LS)} . \tag{39}$$

The total revenue from road taxes is:

$$R_R = N \sum_{\text{a}\in\mathcal{C}} \left\{ P_{\text{a}} \left(\sum_{\ell_R^{se}\in q} \tau_R^{se} \right) D_{W\text{a}}^* \right\}. \tag{40}$$

The total (equilibrium) revenue from the J land markets is:[15]

$$B_\ell = \sum_{i\in\mathcal{J}} p_{Li}\, X_i^D . \tag{41}$$

Public budget is balanced, therefore:

$$B = \frac{1}{N}(R_L + R_R - D_T). \tag{42}$$

2.5 General, stochastic user equilibrium

In equilibrium, labor, housing, land markets at each zone clear, together with the output market. For each of the J labor markets, the clearing condition is:

$$\underbrace{N \sum_{\text{a}\in\mathcal{C}} \{I(j|\text{a})\ P_{\text{a}}\ D_{W\text{a}}^*\}}_{labor\ supply\ to\ zone\ j} - L_j^D = 0, \tag{43}$$

[14] This finding by Anas (2012) has been confirmed using alternative specifications of the model in which income effects were on.

[15] Unlike monocentric city models, in which the opportunity cost of land is determined by the return to agriculture, in polycentric general equilibrium models there is generally no land-use alternative to development; thus all land is characterized by zero opportunity cost and is being developed in the equilibrium.

where $I(j|\mathrm{a})$ is an indicator function that takes the value one if the employment zone of alternative a is zone j (zero otherwise), and L_j^D is the labor demand from (19). Similarly, for each of the J housing markets, the clearing condition is:

$$\underbrace{N\sum_{\mathrm{a}\in\mathcal{C}}\{I(i|\mathrm{a})\,P_{\mathrm{a}}s_{\mathrm{a}}^{*}\}}_{housing\ demand\ in\ zone\ i} \quad - H_i^S = 0\,, \tag{44}$$

where $I(i|\mathrm{a})$ is an indicator function that takes the value one if the residential zone of alternative $\mathrm{a} = \{\mathrm{a}_{ij}, q\}$ is zone i (zero otherwise) and H_i^S is the housing supply in the same zone. Land markets also clear, therefore from (28):

$$\underbrace{\widehat{\Phi}(1-\theta)R^{\theta}p_{Li}^{-\theta}s_i}_{land\ demand\ in\ zone\ i} - \bar{X}_i = 0, \tag{45}$$

where the parametric function $\widehat{\Phi}$ has been defined in (26) and $\bar{X}_i$ denotes the total surface available for development in zone i. Clearing of the intermediate J markets implies that:

$$\underbrace{\zeta_j\,Y\,p_j^{\zeta_j-1}\left(\prod_{k\neq j}p_k^{\zeta_k}\right)\left(\sum_{j\,\in\,\mathcal{J}}\omega_j\right)}_{Q_j^D} - Q_j^S = 0\,, \tag{46}$$

The aggregate demand for the composite good in the entire region is given by:

$$Y^D = N\sum_{\mathrm{a}\in\mathcal{C}}\{\,P_{\mathrm{a}}y_{\mathrm{a}}^{*}\}. \tag{47}$$

In order for the model to close properly, a part of the composite output must be used to import the required capital, and to cover the costs associated with the use of private modes and the public transport system. This implies the closure condition:

$$\underbrace{p(Y^S - Y^D)}_{value\ of\ exports} \;=\; \underbrace{R\left(\sum_{j\,\in\,\mathcal{J}}K_j^{D_f} + \sum_{i\,\in\,\mathcal{J}}K_i^{D_d}\right)}_{total\ value\ of\ imported\ capital} \;+ C_T, \tag{48}$$

where the term on the left hand side is the value of the composite good (with price, p, normalized to one) that is not consumed inside the region but bought by a virtual trader that exports it to the rest of the world (*ROW*). The trader then buys capital and transport services (demanded by individuals, firms and developers in the region) of equal value from *ROW* and sells them back to the region.[16]

Because the equilibrium is competitive, the prices of all final and intermediate goods produced in the region equal their marginal cost. For each of the J intermediates this implies one zero profit condition

[16] See Chapter 4 of the dissertation for an elaborate discussion about closure conditions.

as in (18). Similarly, for each of the J housing markets this implies a zero profit condition as in (26). Because the price of the composite is normalized to one, the corresponding condition for this good is:

$$\left(\prod_{j\,\epsilon\,\mathcal{J}} p_j^{\zeta_j}\right)\left(\sum_{j\,\epsilon\,\mathcal{J}} \omega_j\right) = 1\,, \tag{49}$$

where ω has been defined in (23).

Finally, the disaggregate labor supply in (10), housing demand in (8) and consumption (computed from (6)-(9)) are based on a belief over the travel time and cost attached to each alternative a. Because these underlie the aggregate labor supply, housing demand and consumption, in equilibrium the above belief has to be correct. This implies that, for each alternative a, it holds that the commuting time and pecuniary cost used to derive optimal household behavior is equal to the resulting commuting time and cost given by (32) and (35) respectively. This is:

$$\hat{t}_q = t_q, \tag{50}$$

and

$$\hat{c}_q = c_q. \tag{51}$$

However, it is easy to see that the above holds if the expected travel times and pecuniary costs of links in the network are equal to the resulting ones.

Section 2 describes a system of 35 types of equations in 35 vectors of unknowns. These are equations: (6), (8), (9), (10), (11), (15), (16), (18), (19), (20), (24), (26), (27), (28), (29), (30), (31), (32), (33), (34), (35), (38), (39), (40), (41), (42), (43), (44), (45), (46), (47), (48), (49), (50) and (51) corresponding to the unknown vectors: y_{a}^*, s_{a}^*, $D_{W\mathrm{a}}^*$, $T_{F\mathrm{a}}^*$, V_{a}^*, P_{a} (each vector of size equal to the number of elements in the choice set, denoted by $N_{\mathcal{C}}$), p_j, w_j, L_j^D, $K_j^{D_f}$, Q_j^D, p_{Hi}, $K_i^{D_d}$, X_i^D, H_i^S, p_{Li} and Q_j^S (each vector of size equal to J), L_{Rq}, L_{Pq} , t_q, c_q, $\hat{t}_q$ and $\hat{c}_q$ (each vector of size equal to the number of feasible routes, i.e. N_Q), t_R^{se} and d_R^{se} (of size equal to the number of road links in the network, N_R), t_P^{se} (of size equal to the number of public transport, i.e. rail, links, N_P), D_T, R_L, R_R, B_ℓ, B, Y^D, Y, E_{max} and p (each of size one).

The model uses a network with $N_R = 52$ road links, $N_P = 50$ rail links, N_Q = 1738 feasible routes, J = 18 zones and $N_{\mathcal{C}} = N_Q + J$ = 1756 alternatives.[17] This implies a non-linear square system of N_e = 21325 equations in 21325 unknowns.[18] Section 3 describes how to solve this system for the general, stochastic user equilibrium with a computationally efficient approach.

3. Solution algorithm and OOP architecture

Figure 1 summarizes the pseudocode behind the solution of the $N_e \cdot N_e$ system described in section 2.5. The algorithm separates the system in several subsystems, the solution of which is allocated to different

[17] We consider 12386 non-cyclical routes of which 10648 are excluded: i) due to abnormal travel time/distance compared to the shortest path route or ii) because they violate the rule of a logical mode use. For example, routes that imply car use at two different, non-subsequent trip components: from *a* to *b* by car, from *b* to *c* by public mode and from *c* to *d* by car.

[18] That is: $N_e = (6 \cdot N_{\mathcal{C}}) + (6 \cdot N_Q) + (11 \cdot J) + (2 \cdot N_R) + N_P + 9 = 21325$ equations.

methods (functions) of various classes. Below, the classes used in the program are enumerated (together with the information each class stores), prior to the discussion of the functions each class utilizes.

The class *Alternative* stores the *current* values y_{a}^{*}, s_{a}^{*}, $D_{W\mathrm{a}}^{*}$, $T_{F\mathrm{a}}^{*}$, V_{a}^{*} and P_{a} for an arbitrary alternative a. The classes *Firm*, *Developer* and *Assembly industry* store the *current* values of L_j^D and $K_j^{D_f}$, $K_i^{D_d}$ and X_i^D, and Q_j^D respectively. The class *Network* stores L_{Rq}, L_{Pq}, $\hat{t}_q$, $\hat{c}_q$, t_R^{se}, d_R^{se} and t_P^{se}.

The *core class* of the program is *World*, which stores a list of *Alternative* instances (i.e. the choice set), a list of *Firm* instances, a list of *Developer* instances, an *Assembly industry* instance and a *Network* instance. This class stores the current values of the endogenous vector $\wp = (w_j, p_{Hi}, p_{Li}, p_j, Q_j^S, H_i^S, D_T, R_L, R_R, R_\ell, B, Y, t_q, c_q)$, where $\wp$ is a concatenation of its elements. Different classes have different access to different subvectors of $\wp$, as explained below. Whenever called, method *W1* of class *World* returns the vector $\mathbf{v}(\wp)$ with the equation values of the subsystem that is made up by the zero profit conditions in (18) and (26), market clearing conditions in (43), (44), (45), (46), and (48), revenue equations (38), (39), (40), (41) and (42), and the stochastic user equilibrium equations (50) and (51). This subsystem is made up by $N_1 = 6J + 6 + 2N_Q$ equations.[19]

That is, class *Alternative* receives the necessary information from the subvector $\wp_A = (w_j, p_{Hi}, \tau_L, c_{\mathrm{a}}, t_{\mathrm{a}}, B)$ and uses its method *A1* to compute (6), (8), (9), (10) and (11), and update its values y_{a}^{*}, s_{a}^{*}, $D_{W\mathrm{a}}^{*}$, $T_{F\mathrm{a}}^{*}$ and V_{a}^{*} to the optimal levels. Method *A1* is called more often than any other (see below), with frequency increasing as the size of the choice set, $N_{\mathcal{C}}$, becomes larger. It is therefore important to formulate the utility maximization problem in a way that (6), (8), (9) and (10) have closed form expressions.

Class *Firm* accesses $\wp_F = (w_j, Q_j^S)$ and the values R and A_j (as well as the necessary parameter values stored in *World*) and contains method *F1*, which computes the closed-form expressions in (19) and (20) to update the endogenous variables L_j^D and $K_j^{D_f}$. Similarly, *Developer* accesses $\wp_D = (p_{Li}, H_i^S)$ and the value of R and contains method *D1*, which computes the closed-form expressions in (27) and (28) to update the endogenous variables $K_i^{D_d}$ and X_i^D. Class *Assembly industry* accesses $\wp_I = (p_j, Y)$ and contains method *I1*, which computes the closed-form expressions of J intermediate demand equations, as in (24), to update vector Q_j^D (of size J).[20]Finally, *Network*'s method *N1* accesses $D_{W\mathrm{a}}^{*}$ and P_{a} in order to compute (29), (30), (31), (32), (33), (34) and (35) in order to update its fields, i.e. L_{Rq}, L_{Pq}, $\hat{t}_q$, $\hat{c}_q$, t_R^{se}, d_R^{se} and t_P^{se}.

World also uses method *W2* to compute the E_{max} and update the $N_{\mathcal{C}}$ choice probabilities P_{a} in (16), and method *W3* which uses P_{a} together with y_{a}^{*}, s_{a}^{*}, $D_{W\mathrm{a}}^{*}$, $T_{F\mathrm{a}}^{*}$ to compute the aggregate values of labor supply, housing demand and consumption (as they are defined in the left hand sides of equations (43) and (44), and equation (47) respectively).

Method *W1* encapsulates methods *A1*, *F1*, *D1*, *I1*, *N1*, *W2* and *W3*: whenever a change in an arbitrary endogenous variable e in $\wp$ occurs, *W1* calls consecutively those of the Methods *A1*, *F1*, *D1* and

---

[19] Earlier contributions in the field (e.g. RELU-TRAN by Anas and Liu, 2007) have proposed a detachment of the economic part of the equilibrium from the stochastic user part. This detachment is possible under certain mathematical manipulations of the model presented here. See Anas and Tikoudis (2015) for a detailed discussion over the compatibility and the relative computational efficiency of the two types of models.

[20] To be consistent with the Walras' law, one equation, together with an endogenous price has to be excluded from the system. Here we chose arbitrarily to exclude the zero profit condition of the assembly industry in (49) and to normalize the price of the composite, by setting $p = 1$. Therefore, Method 2c excludes this equation, which is used upon convergence to check if the equilibrium is correct.

I1 that can access e in order to use it as an input (see above). Call of *A1* implies a subsequent call of *W2* because systematic utilities (i.e. the input of *W2*) are altered by *A1*. In contrast, changes in produced quantities Q_j^S, H_i^S or Y imply that *A1* and *W2* are skipped because household behavior remains intact in the current iteration.

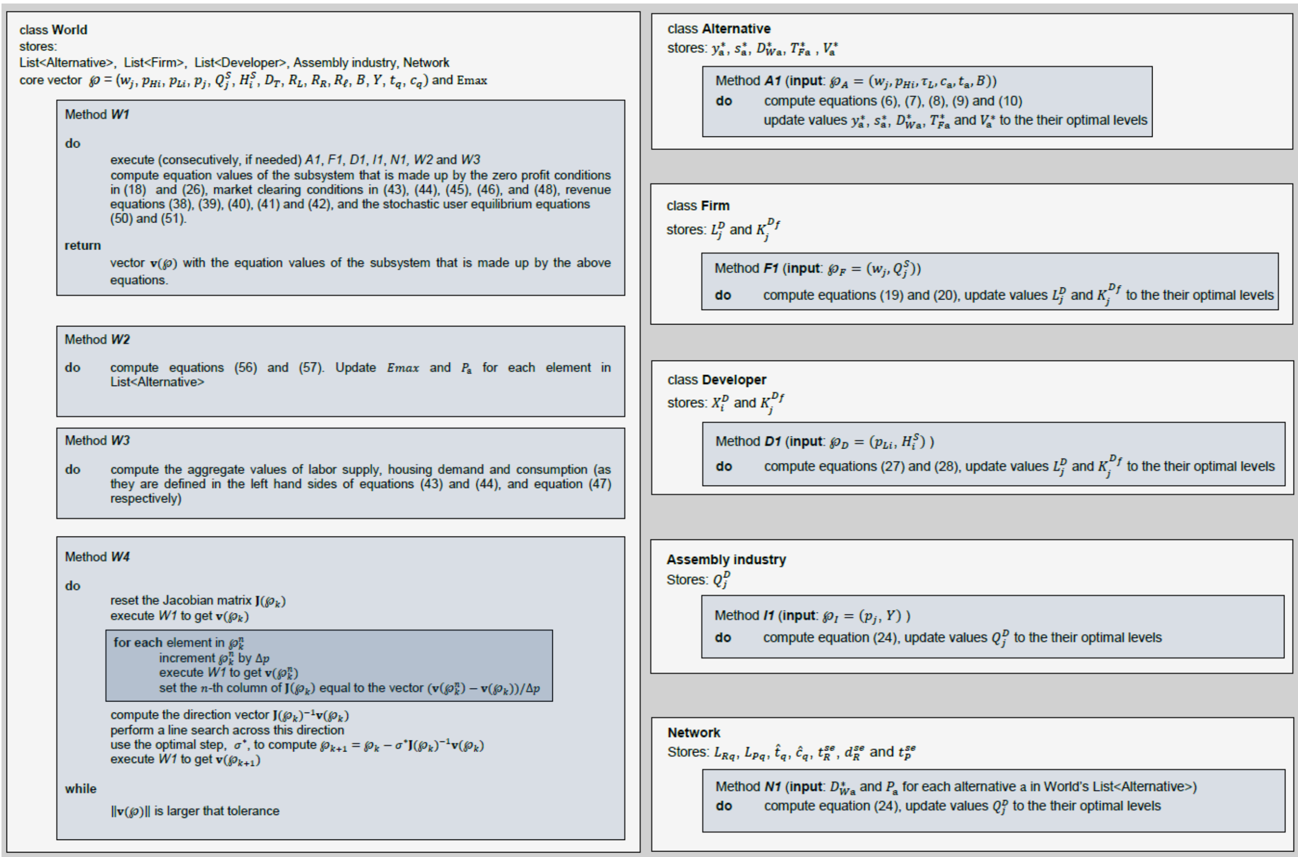

Fig. 1. Schematic depiction of the object-oriented solution algorithm.

Because *W1 encapsulates* all the optimal economic behavior of the agents that class *World* contemplates, equilibrium is reached when the norm of the subsystem returned by this method is sufficiently close to zero. Method *W4* performs the *numerical tâtonnement process* that computes the equilibrium. This is a variant of the Newton method with a line search. At each iteration k, *W4* calls *W1* multiple times. Initially, this is to compute the distance of $\wp_k$ from the equilibrium, i.e. the vector $\mathbf{v}(\wp_k)$. Then, N_1 times to numerically approximate the Jacobian matrix at $\wp_k$, $\mathbf{J}(\wp_k)$, using finite differences. At each of these calls, n, *W1* returns the vector $\mathbf{v}(\wp_k^n)$, where $\wp_k^n$ differs from $\wp_k$ in its n-th element by Δp. The n-th column of $\mathbf{J}$ is therefore the vector $(\mathbf{v}(\wp_k^n) - \mathbf{v}(\wp_k))/\Delta p$. When the direction vector, $\mathbf{J}^{-1}\mathbf{v}$, is computed, *W4* uses a line search method which calls *W1* multiple times, in order to evaluate different step sizes, σ. When the optimal step is found, *W4* updates the core vector:

$$\wp_{k+1} = \wp_k - \sigma^* \mathbf{J}^{-1} \mathbf{v}(\wp_k), \tag{52}$$

it calls *W1* to return $\mathbf{v}(\wp_{k+1})$ and checks if the distance from equilibrium, i.e. $\|\mathbf{v}(\wp_{k+1})\|$ has fallen below a prespecified tolerance level. If not, *W4* repeats itself. Figure 1 summarizes the architecture of the program.

4. Application to the area of Randstad: key data and calibration.

The model is calibrated in order for the relative values of the endogenous variables to exhibit the maximum degree of resemblance to those of Randstad (see introduction) in the base equilibrium. In order to ensure that the research question can be addressed without disturbing the computational tractability of the model, a resolution which comprises 18 zones has been chosen. As shown in Figure 2, each zone represents a group of municipalities, which share either similar commuting patterns or a common strategic position in the road network.[21] The four largest employment and residential centers (Amsterdam & Amstelveen, Utrecht, Rotterdam and the Hague) constitute separate zones.[22] The data used for the calibration of the model regard primarily commuting flows between the 18 zones of the model. The 18×18 *origin-destination* (hereafter, *OD*) matrix has been computed using CBS microdata for fully employed workers in year 2012. The variation of labor supply across different *OD*-pairs is not observed.

The calibration is treated as a series of optimization problems, one at each stage. At any arbitrary iteration k of *stage A*, a *genetic* algorithm draws a population of random preference vectors $\boldsymbol{v_A} = (\pi_0, \pi_1, \alpha, \beta, \gamma)$ from a ball $B\left(\boldsymbol{v}_A^{k-1}, r^k\right) = \{\boldsymbol{v_A} : d(\boldsymbol{v_A}, \boldsymbol{v}_A^{k-1}) \leq r^k\}$ of radius r^k centered at the survivor preference vector from iteration $k-1$, i.e. $\boldsymbol{v}_A^{k-1}$. At each draw, the model is solved using the algorithm discussed in section 3. Upon solution, the following information is recorded: i) average expenditure share on consumption, $E_{sh}(\boldsymbol{v_A^k}, C)$, housing, $E_{sh}(\boldsymbol{v_A^k}, H)$, and transport, $E_{sh}(\boldsymbol{v_A^k}, T)$, and ii) average time shares on labor, $T_{sh}(\boldsymbol{v_A^k}, L)$, leisure, $T_{sh}(\boldsymbol{v_A^k}, \ell)$, and commuting, $T_{sh}(\boldsymbol{v_A^k}, c)$.

The *objective functions* (to be minimized) are:

$$\hbar_{A0}(\boldsymbol{v_A^k}) = \left[E_{sh}(\boldsymbol{v_A^k}, C) - 0.65\right] + \left[E_{sh}(\boldsymbol{v_A^k}, H) - 0.30\right], \tag{53}$$

and

[21] An initial selection excluded municipalities with population below 20000 inhabitants. A first grouping of municipalities into clusters (zones) was made in order to merge neighboring municipalities with populations between 20000 and 180000 inhabitants that share similar labor supply patterns (towards municipalities with a population over 180000 inhabitants). Further refinements rived some of these groupings into smaller parts, to account for the fact that some municipalities had access to more than one major highway link and would therefore hold larger monopoly power (ceteris paribus) had they been granted fiscal autonomy to perform road pricing on their own.

[22] The included zones are: 1) Amsterdam and Amstelveen, 2) municipalities between Amsterdam and Utrecht across highway A2, 3) eastern suburbs of Amsterdam along highway A1, including Diemen, Muiden, Weesp and Naarden, 4) cluster of municipalities from Bussum, all the way on A1 to the crossing with A27, and across A27 all the way to Utrecht, 5) The cluster of Amersfoort, Soest and Zeist, municipalities located across A1 and A28 to the northeast of Utrecht, 6) Utrecht, 7) west suburbs of Utrecht (Montfoord, Woerden) across A12, 8) South suburbs of Utrecht (IJsselstein, Houten, Nieuwegein, Vianen) across A2 and A27, 9) Almere and Lelystad on A6, 10) northeast suburbs of Rotterdam, built around A12 and A20, 11) Rotterdam, 12) southeast suburbs of Rotterdam around A15, 13) municipalities located between Rotterdam and the Hague (e.g. Delft, Zoetermeer), 14) the Hague, 15) municipalities located north of the Hague, around Leiden, which are accessed through A4 and A44, 16) municipalities located southwest of Amsterdam (Haarlem, Haarlemermeer), with the area including Schiphol airport, 17) cluster of municipalities located across A9 from Haarlem up to Alkmaar, and 18) northwest suburbs of Amsterdam (Zaanstad, Purmerend) accessed through parts of A10, A8 and A7.

$$\mathscr{k}_{A1}(\boldsymbol{v}_{\boldsymbol{A}}^{k}) = [T_{sh}(\boldsymbol{v}_{\boldsymbol{A}}^{k}, L) - 0.60] + [T_{sh}(\boldsymbol{v}_{\boldsymbol{A}}^{k}, c) - 0.10]. \tag{54}$$

The benchmark consumption share (0.65) and housing expenditure share (0.30), which imply a transport expenditure share of 0.05, have been chosen to be in rough accordance with the expenditure profile of the average household in Western Europe. Similarly, assuming a day of 13½ hours (excluding the essential time needed for physical rest and necessary activities of the household from the full 24-hour day), the benchmark labor time share (0.60) corresponds to a working day of approximately eight hours; therefore, the one-way benchmark commuting time (0.10) corresponds to approximately 40 minutes. The rest of the time endowment, i.e. approximately four hours, is spent on leisure activities.[23]

The vector that survives the iteration is the one with the lowest objective values $\mathscr{k}_{A0}(\boldsymbol{v}_{\boldsymbol{A}}^{k})$ and $\mathscr{k}_{A1}(\boldsymbol{v}_{\boldsymbol{A}}^{k})$. As these objective values get closer to zero the radius r^k that defines the search area around the survivor vector $\boldsymbol{v}_{A}^{k-1}$ becomes smaller. The algorithm terminates when there the *Pareto frontier* is reached, i.e. no superior vector is drawn for a large number of trials.

On stage B, the same algorithm draws vectors of $\boldsymbol{v}_{\boldsymbol{B}} = (\beta, \gamma, \xi_{1R})$ i.e. of the parameters that are mainly responsible for the level of traffic and the average equilibrium speed. The objective function here is:

$$\mathscr{k}_{B}(\boldsymbol{v}_{\boldsymbol{B}}) = [t_{rel}(\boldsymbol{v}_{\boldsymbol{B}}) - 2.3], \tag{55}$$

where $t_{rel}(\boldsymbol{v}_{\boldsymbol{B}})$ is the average (across links) *equilibrium-to-free-flow* travel time ratio.[24] Assuming a free flow speed of 120 km per hour, the benchmark value (2.3) implies an average *equilibrium speed* of 52 kilometers per hour. This speed is roughly consistent with the average commuting speed reported for large US cities in the national household travel survey (Federal Highway Administration, 2004), and plausible if someone takes into account bottlenecks, traffic lights, parking search time and other events not modeled explicitly in this application.[25] For the public transport mode, we assume a constant speed of 60 km per hour, i.e. half of the average private mode speed.

Table 1. Residential and employment percentages in benchmark equilibrium and data**.**

Zone	*Residential share (model)*	*Residential share (data)*	*Employment share (model)*	*Employment share (data)*
Amsterdam & Amstelveen	0.145	0.148	0.179	0.182
southeast Amsterdam suburbs	0.026	0.021	0.016	0.012
east Amsterdam suburbs	0.015	0.011	0.017	0.015
northeast Utrecht suburbs	0.039	0.037	0.038	0.037
east Utrecht suburbs	0.035	0.036	0.037	0.035
Utrecht	0.050	0.049	0.054	0.061

[23] The calibrated model shares are: $E_{sh}(\boldsymbol{v}_{\boldsymbol{p}}^{k}, C) = 0.643$, $E_{sh}(\boldsymbol{v}_{\boldsymbol{p}}^{k}, H) = 0.307$, $E_{sh}(\boldsymbol{v}_{\boldsymbol{p}}^{k}, T) = 0.050$, $T_{sh}(\boldsymbol{v}_{\boldsymbol{p}}^{k}, L) = 0.591$, $T_{sh}(\boldsymbol{v}_{\boldsymbol{p}}^{k}, \ell) = 0.299$ and $T_{sh}(\boldsymbol{v}_{\boldsymbol{p}}^{k}, c) = 109$.

[24] The calibrated ratio $t_{rel}(\boldsymbol{v}_{\boldsymbol{B}})$ is 2.274.

[25] To facilitate calibration, we have assumed that ξ_{0R}^{se} and ξ_{1R}^{se} are constant across road links. We have fixed the free-flow parameter ξ_{0R} to 0.25. Multiplying this with 13½ hours (the assumed time endowment) yields approximately 3.375 hours per unit of distance or, equivalently, a speed of 0.296 units of distance per hour. Setting the unit of distance equal to 405.4 km, ξ_{0R} = 0.25 implies a free-flow speed of 120 km per hour.

southwest Utrecht suburbs	0.016	0.011	0.014	0.009
southeast Utrecht suburbs	0.031	0.028	0.029	0.025
Almere & Lelystad	0.035	0.038	0.031	0.027
northeast Rotterdam suburbs	0.041	0.039	0.038	0.034
Rotterdam	0.098	0.102	0.117	0.124
southeast Rotterdam suburbs	0.072	0.073	0.053	0.056
cluster between Rotterdam and the Hague	0.127	0.124	0.109	0.107
the Hague	0.077	0.081	0.091	0.094
Leiden and suburbs	0.050	0.048	0.038	0.038
cluster around Schiphol airport	0.049	0.053	0.066	0.071
northwest Amsterdam suburbs	0.046	0.051	0.040	0.039
northeast Amsterdam suburbs	0.049	0.050	0.033	0.034

On stage C, we calibrate the *employment* and *population shares* of each zone to fit those observed in data, by adjusting the parameter vector $\boldsymbol{v_C} = (\mathrm{z}_i, \mathrm{z}_j, \zeta_j)$ while keeping the rest of the parameters fixed.[26] This part of the algorithm is rather heuristic: it increases (decreases) z_i for residential zones that attract fewer (more) households than those observed in data, and adjusts z_j in the same manner to bring employment density in alignment with data. However there is an asymmetry between the two adjustments: while increasing the average utility of local amenities, z_i, attracts more residents and presses housing and land prices upwards, increasing the employment-specific constants, z_j, increases labor supply and therefore (ceteris paribus) pushes wages in these zones downwards.[27] This downward pressure is partially due to the fact that we have accounted for a unique type of skill in the model. We offset it by simultaneously adjusting the cost share of the local intermediate on the cost of the composite good, i.e. by adjusting ζ_j; this ensures that the main labor attractor zones offer a higher wage in equilibrium, as data suggest.[28] Table 1 juxtaposes the employment and residential shares in the benchmark equilibrium against the shares observed in data.

The fourth stage of the algorithm adjusts the parameter vector $\boldsymbol{v_D} = (\mathrm{z}_R, \mathrm{z}_P, \mathrm{z}_t)$, i.e. the mode-specific constants and the transit specific constant, in order for the probability of commuting by car to be sufficiently close to 65%. This is in general alignment with the observed rates in the Netherlands. Furthermore, the algorithm renders commutes with modal shift unlikely (below 5%).[29]

[26] Throughout the entire calibration process, parameter λ is fixed to 3.0. We have chosen to abstract from agglomeration effects, which are going to be studied thoroughly in a separate contribution. We have therefore fixed total factor productivity, A_j, to 1.0 in each zone.

[27] Relative housing prices (base zone 18) are in the range of 0.59 to 1.67. Relative land prices vary between 0.46 and 2.60. Unfortunately, the model produces a poor spatial variation in terms of floor-to-area ratios, which varies in the limited range between 1.16 and 1.95. This is mainly due to the equilibrium elasticity of substitution between residential space and consumption that gives rise to large residential consumption in zones with very low population density.

[28] This holds because an increased ζ_j implies (in general) higher demand for the intermediate goods produced in those zones (see equation (46)); and this, in turn, implies higher wages.

[29] See Schwanen et al. (2001) for an analysis of the modal split and urban form based on the Dutch National Travel Survey of 1998.

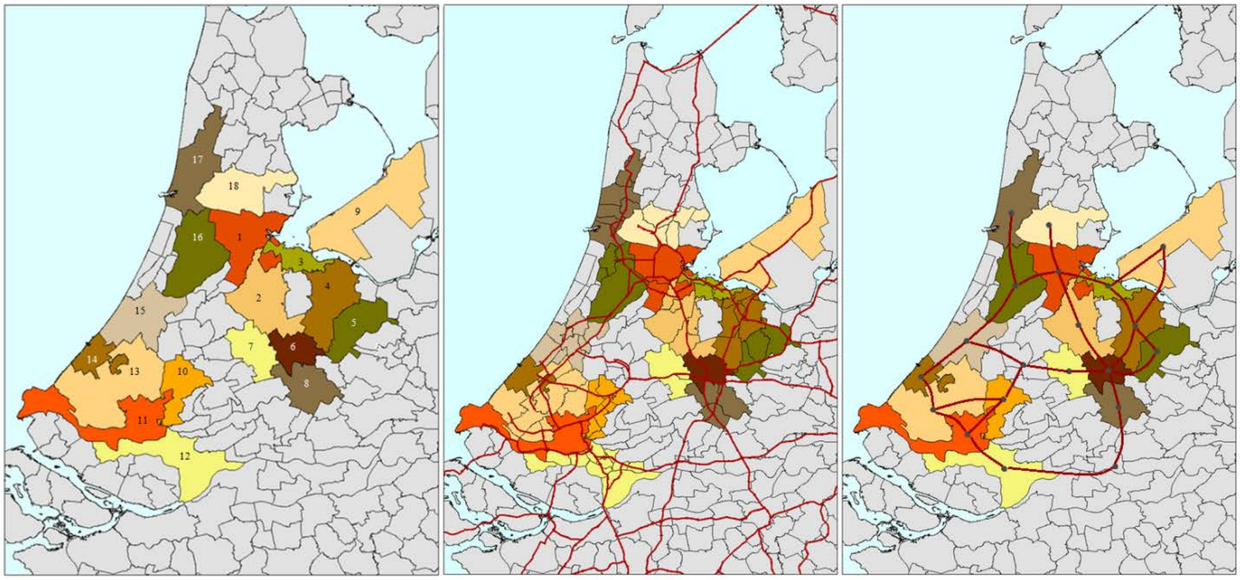

Fig. 2. Spatial configuration of the model: zonal aggregation (left), main highways of the road network (middle) and network representation (right).

For the rest, we assume a uniform labor income tax rate of 35% which is in accordance with the generic tax rates in Western Europe and North America (25-45%). In equilibrium, the annual pecuniary cost of car use per kilometer is 0.177% of total disposable (after tax) income. Setting this income to the plausible level of €30000, this annual cost turns out to be slightly above €53. The corresponding annual kilometer cost with public transport is approximately €16.[30] The fixed cost of the public transport operator is set to zero in benchmark equilibrium. There are no price subsidies in public or private transport thus pricing is efficient. From (5) it is straightforward that the value of time has an upper bound which is the after-tax wage, $w_j(1-\tau_L)$. The expected value of time is 0.737 of the wage, with a standard deviation of 0.151. These values are in line with those proposed at several previous studies.

Contrary to stylized models, in which the *elasticity of labor supply* may attain a uniform value (e.g. Parry and Bento, 2001), or monocentric city models, in which it may vary across residence locations, here we consider an equilibrium elasticity of labor supply (with respect to the labor tax) by origin-destination pair.[31] This elasticity is in accordance with values considered in previous studies and it varies considerably in the plausible range between -0.028 and -0.296. Appendix A provides a complete summary of the endogenous, exogenous variables and parameters (including their values) of the model.

5. Policy analysis

[30] Exogenous price of car use per unit of distance is set to 0.05. Dividing by the number of kilometers per unit of distance (i.e. 405.4), and then by 0.0697 (the equilibrium after-tax income) yields an annual kilometer cost equal to 0.00177 of after tax income.

[31] This is approximated numerically using finite differences. That is, first compute the labor supply matrix in the benchmark equilibrium, $\boldsymbol{L_0^S}$, whose element in the i-th row and j-collumn is $\boldsymbol{L_{0}^{S}}_{ij} = \sum_{\mathrm{a}\in C}\{I(i,j|\mathrm{a})\ P_{\mathrm{a}} D_{W\mathrm{a}}^{*}\}$. Then let the tax rate τ_L increase by $\Delta\tau_L$. Compute the tax rate change in monetary units for each employment zone: this is $d\tau_j = \Delta\tau_L w_j$. Solve for the general equilibrium and compute the new labor supply matrix $\boldsymbol{L_1^S}$. The approximation for the elasticity of labor supply for the OD-pair a_{ij} is $\left[\left(\boldsymbol{L_{1}^{S}}_{ij} - \boldsymbol{L_{0}^{S}}_{ij}\right)/d\tau_j\right]\left[\left(\tau_L w_j\right)/\boldsymbol{L_{0}^{S}}_{ij}\right]$.

We now consider a series of interventions that are highly relevant for policy analysis: the Pigouvian toll, partial taxation of the network, uniformly-priced and differentiated cordon tolls around the three largest employment zones of Randstad (Amsterdam, Rotterdam and the Hague). In section 5.1 we assume that revenue recycling takes its simplest form, i.e. a lump-sum transfer to the representative household. In section 5.2 we search for realistic revenue recycling strategies that may reverse the findings of section 5.1.

5.1 Lump-sum revenue recycling

While the Pigouvian toll is the most efficient intervention in a setting where road traffic externalities pose the only failure in the economy, a series of contributions have shown that large welfare losses may occur from a such policy in the presence of a pre-existing distortionary tax that remains intact (Parry and Bento 2001; Tikoudis et al., 2015a).[32]

Since road congestion is generated by identical vehicles, the *marginal external congestion cost* (*mecc*) of an additional unit of traffic is independent of both the origin and destination of the vehicle that enters the link, as well as of the characteristics of its driver. To compute this cost, we need to decompose the traffic demand of the link by *user type*, i.e. by alternative $\mathrm{a} = \{\mathrm{a}_{ij}, q\}$. This is because every additional trip that uses link $l_C^{(se)}$ delays all drivers in the same link by $\ell_R^{se}\xi_{1R}^{se}$, as (29) suggests.[33] However, this delay is valued differently by each user type, as the value of time varies not only across *OD*-pairs, but also across routes.[34] Therefore, to compute the link-specific marginal external congestion cost, $mecc_R^{se}$, we have to: go through *each* alternative $\mathrm{a} = \{\mathrm{a}_{ij}, q\}$ in the choice set $\mathcal{C}$, check if the associated route q implies the use of $l_R^{(se)}$ (using again the indicator function $I\left(l_R^{(se)}|q\right)$ as in (31)) and, if yes, compute how many users of this type are using the link (i.e.$NP_\mathrm{a}D_{W\mathrm{a}}^*$); then, this demand has to be multiplied with their value of time (v_a). Summing across all alternatives yields the *aggregate value of time* in $l_R^{(se)}$. Multiplying this with the delay per user, $\ell_R^{se}\xi_{1R}^{se}$, yields:

$$mecc_R^{se} = \ell_R^{se}\xi_{1R}^{se} \underbrace{\sum_{\mathrm{a}\in\mathcal{C}} \left\{ \underbrace{I\left(l_R^{(se)}|q\right) NP_\mathrm{a}D_{W\mathrm{a}}^*}_{\substack{demand\ from\ users \\ of\ type\ \mathrm{a}}} v_\mathrm{a} \right\}}_{aggregate\ value\ of\ time\ in\ l_R^{(se)}}. \tag{56}$$

Using the above formula, we calculate the *quasi first-best* Pigouvian toll scheme and apply it in our model. The scheme causes considerable welfare losses that account for approximately 1.83% of the after-tax, disposable labor income, although it is efficient from an environmental point of view (i.e. in terms of

[32] To the knowledge of the author, this is the first study to approach the issue using real data from a polycentric network.
[33] This is simply the derivative of link travel time in (29) with respect to the demand/load in the link.
[34] There is, therefore, a unique value of time for each alternative, since by definition $\mathrm{a} = \{\mathrm{a}_{ij}, q\}$.

reducing traffic).[35] On the other hand, the same toll rule applied in a setting where the labor tax is set at 0% has been confirmed to be *first-best* and welfare increasing.[36]

This finding corroborates earlier computations of the welfare effects from Pigouvian pricing under pre-existing distortionary labor taxation in the setting of Parry and Bento (2001). In this contribution, the *environmental*, or *Pigouvian effect* (i.e. the first dividend) from a road pricing policy, despite positive in itself, falls short of the negative *tax interaction effect* at the margin of the no-toll equilibrium.[37] As a result, the optimal congestion taxes are not only below their Pigouvian levels (as expressed in (56)); they are essentially negative.

However, Tikoudis et al. (2015a) illustrate a case in which *partial network taxation* that takes the form of a cordon toll in a monocentric serial network may not only be less harmful than the *quasi first-best* Pigouvian toll, but it can also reverse the sign of the welfare effect, even in the case in which a pre-existing labor tax exceeds 40%. The key driver behind this result was that the cordon toll can be imposed in a certain distance from CBD, affecting only the subgroup of the population that provides labor relatively inelastically, provided that the elasticity of labor supply falls with distance from the CBD.

To test if the above result still holds in our polycentric setting, we perform additional computations regarding systems of *uniformly*- and *differentially-priced cordon tolls* around zones 1, 11 and 14 (i.e. Amsterdam-Amstelveen, Rotterdam and the Hague respectively). For the *uniformly-priced* cordon toll system, it holds that:

$$\tau_R^{se} = \begin{cases} \bar{\tau}_R & if\ e \in \{1, 11, 14\} \\ 0 & otherwise. \end{cases} \tag{57}$$

For the *differentially-priced* cordon toll system, the toll is:

$$\tau_R^{se} = \begin{cases} \bar{\tau}_{Re} & if\ e \in \{1, 11, 14\} \\ 0 & otherwise. \end{cases} \tag{58}$$

In the benchmark equilibrium considered (where τ_L = 0.40) the above measures prove to be less harmful than the Pigouvian toll. This is because a large part of the network (i.e. the majority of road links) is left untaxed. These untaxed links generally decrease the size of the environmental/transport benefits that can be realized with a given pricing scheme, but this decrease is small relative to the labor market effect. That is, setting the road tax to zero in a large part of the network reduces significantly the private cost of labor supply (and therefore the marginal excess burden of the labor tax), especially for alternatives that involve long commutes. But despite cordon toll systems provide an improvement vis-à-vis the Pigouvian toll, they still generate welfare losses. Therefore, the finding by Tikoudis et al. (2015a) is probably network specific, as it fails to be confirmed in a less stylized, mixed network. Consequently, optimal cordon charges in the level of the labor tax considered are negative. The threshold labor tax values at which the three pricing schemes (*quasi first-best* Pigouvian, uniform cordon system, differentiated cordon system) generate positive welfare effects are 18%, 24% and 25.5% respectively.

---

[35] Considering an average, annual after tax income of € 30000, this welfare loss is € 549 per capita.

[36] Setting the labor tax at 0% leaves the road traffic externalities as the only remaining source of inefficiency in the model. The researcher can use the Pigouvian equilibrium as a starting point to check whether deviations from the toll rule in (56) can increase utility. Such an increase is a sign of model misspecification.

[37] The *environmental* or *Pigouvian effect* is the welfare gain from a marginal adjustment of the externality tax. When the latter is set at its Pigouvian level, the above effect is zero. The *tax interaction effect* is the welfare loss from the erosion of the base that corresponds to a distortionary tax, caused by a marginal increase of the environmental tax.

In a setting where traffic externalities would be the only failure, i.e. when τ_L is set to zero, the above schemes would be second-best to Pigouvian pricing.[38] Since (in this case) tax interaction effects are absent, the latter produces significant welfare gains, accounting for approximately 0.33% of the after-tax labor income.[39] Because a large portion of the total external costs is generated on the links ending on the above three large employment centers, the relative efficiency of the above second-best measures is high: they are found to capture 48.2% and 48.8% of the Pigouvian gains respectively. This result is roughly in accordance with Mun et al. (2005).

5.2 Intelligent revenue recycling (Labor tax cuts)

The general failure of pricing policies with lump-sum revenue recycling leads naturally to the investigation of policies that return the road tax revenue in the form a labor tax cut.[40] Because the labor tax is distortionary to begin with, this cut generates a positive welfare effect that is widely known in the theory of double dividend as the *revenue-recycling effect*. When the latter effect is strong enough to offset the *tax interaction effect*, the second dividend emerges.

The welfare gains, as well as the link charges of the *optimal road tax scheme* are of special interest to the policy maker, because they provide the theoretical benchmark against which other second-best interventions (e.g. cordon tolls, fuel taxes and area fees) can be compared. In the context of this paper, the above policy leaves the charge in each road link free to acquire any value, even a negative one.[41] Computing the optimal tax scheme, we find that link charges exhibit a large variation around their Pigouvian levels.[42] This deviation (roughly) ranges from -400% to +50%, with 27 of the 52 links considered in the study receiving subsidies (negative charges).[43] The second dividend emerges in six links; the remaining links receive a positive charge below the marginal external cost of congestion (Table *B1* in Appendix B provides the values for each link).

The size of these deviations underlies a surprising finding, namely that the *quasi first-best Pigouvian* pricing scheme is welfare decreasing even in the case in which road tax revenue finances labor tax cuts. The result may at first appear counter-intuitive, since a set of Pigouvian tolls replaces part of a distortionary tax in the form of a *revenue-neutral tax swap*. However, nothing prevents the tax interaction

---

[38] Here, we keep all parameters to their calibrated values and set τ_L equal to zero.

[39] Despite being in the range proposed by prior literature (e.g. Anas and Hiramatsu, 2013), the reader may find this value relatively low. A key contributing factor to this result is that the model bounds the value of time to a ceiling that equals the nominal wage associated with each alternative.

[40] To compute the equilibrium for a policy with the labor tax cut, one has to fix B to the value it obtains in the benchmark equilibrium, and replace B with τ_L in the vector of endogenous variables, $\wp = (w_j, p_{Hi}, p_{Li}, p_j, Q_j^S, H_i^S, D_T, R_L, R_R, R_\ell, B, Y, t_q, c_q)$, as defined in section 3.

[41] This necessitates the exclusion of cyclical paths, which has been discussed in section 2.1.

[42] The optimal tax has been computed using variants of *Newton-Raphson (NR)* and *BFGS* methods with *line search*. The *NR* algorithms we used proceed with sequential updates of the road tax, in each iteration for a set of links that share the same start node *s* or the same end node *e*. The gradient vector and the Hessian matrix are computed exclusively for this set of links using finite differences. The *BFGS* algorithms we used produce updates for the entire network at each iteration, using approximations of the Hessian matrix. In both cases, a line search method is employed to maximize the value of the objective function across the proposed direction. To test for local extrema points we repeat the computations with different starting values.

[43] This result is in line with Tikoudis et al. (2015a). With a parameterization similar to the one used here, it is shown that the *location-based* optimal road tax scheme in a monocentric city can be non-monotonic when the benchmark labor tax is high. Considering the monocentric city as a serial network with a unique destination implies that the above result can be expressed as a link-based road tax scheme in which the most distant links receive negative charges.

effect in many of the network links from being strong enough to offset the *sum* of the environmental (Pigouvian) and revenue recycling effects at the margin of the *no-toll* equilibrium. Since the *optimal road tax scheme* lies far away from the *quasi first-best Pigouvian*, the latter may, as it turns out to be the case here, be welfare decreasing.[44]

Apart from the above schemes, we compute the optimal charges and welfare levels for: i) a *uniformly-priced* and ii) a *differentially-priced* cordon toll system around zones 1, 11, 14 (i.e. around Amsterdam, Rotterdam and the Hague). Because these schemes leave a large set of links untaxed, especially most of those receiving a negative charge in the optimal scheme, they generate considerable welfare gains: their efficiency relative to the optimal tax scheme is 28.5% and 32% respectively. In the case of the differentiated cordon toll, the road tax revenue can finance a labor tax cut of approximately 0.55%. The results suggest that the entire tax reform might be feasible from a political point of view as well, since the above tax cut can take the form of a regional labor subsidy, and the optimal cordon toll prices (which vary between 1.15% and 1.50% of the average wage) are found in the range of urban tolls already imposed with considerable levels of public acceptability elsewhere in Europe (e.g. Oslo and Stockholm).

6. Concluding remarks

This study presented an application of a polycentric general equilibrium model of transport and land-use designed to capture the exact *interaction* between the set of the externality-correcting road taxes in a network and the spatially uniform, distortionary labor tax. Because the latter underlies labor supply, it indirectly affects the level of traffic during commuting hours. Earlier contributions provided strong insights, but were referring to more abstract, spaceless or monocentric settings.

In this paper we derived new insights by incorporating the two core mechanisms that generate the double dividend (i.e. the tax interactions and revenue-recycling effects) in a spatial setting with multiple externalities (i.e. a road network where each link is modeled as a different congestible facility), some of which may be left untaxed (partial taxation). Among others, this setting facilitates the identification of circumstances, under which partial taxation of these externalities may be Pareto preferred vis-à-vis the textbook *quasi first-best Pigouvian toll* (marginal external cost pricing) not only with revenue returned lump-sum, but also in the form of a distortionary tax cut.

The model has a clear geographical reference, i.e. the polycentric urban conglomeration in the area of Randstad, which comprises the country's four largest cities (Amsterdam, Rotterdam, the Hague and Utrecht). In line with more stylized models, the introduction of a quasi first-best Pigouvian toll in the base calibration is shown to generate considerable welfare losses. Partial taxation that involves a system of cordon tolls around the largest employment zones of the conglomeration is found to mitigate these losses. *Surprisingly*, marginal external cost pricing is found to be welfare decreasing even with revenue used to finance a labor tax cut. The computation of the optimal tax scheme reveals that this is, among others, due to the large deviation of optimal link charges from their Pigouvian levels; in a significant part of the network links are charged negatively. With labor tax cuts, a system of differentially-priced cordons around Amsterdam, Rotterdam and the Hague leaves most of these links unpriced and is found to capture approximately 27% of the gains generated by the optimal tax scheme. The optimal charges and the labor tax reduction indicate that the entire reform may be politically feasible. That is, charges vary between

[44] The welfare loss from the quasi first-best Pigouvian toll in this case accounts (in absolute value) for approximately % of the gains of the optimal tax scheme.

1.15% and 1.50% of the average wage and the resulting labor tax cut (which can be implemented as a regional labor subsidy) is approximately 0.55%.

Acknowledgements

I would like to express my gratitude to Professors Alex Anas, Erik Verhoef, Jos van Ommeren, Kay Axhausen, John Rust and Kenneth Judd for their constructive and influential comments in earlier versions of the paper. Furthermore, the useful comments and feedback from participants in the North American Regional Science Council conference (Atlanta, November 2013), the 2nd European Association for Research in Transportation (Stockholm, September 2013), the poster session during the Zurich Initiative for Computational Economics (Zurich, February 2014), the meetings of i-PriSM project and the internal seminars in Tinbergen Institute and Swiss Federal Institute of Technology (ETH) are highly appreciated. The author is indebted to the Netherlands Organization for Scientific Research (NWO Grant 434-09-021) and to the European Research Council (AdG Grant #246969 OPTION) for financial support.

Appendix A: notation

Table A1. *Variables, prices and policy instruments*

y_{a}	*consumption of a composite good*	Y^S	*supply of composite good (numéraire)*
s_{a}	*housing consumption*	Q_j^D	*intermediate demand by assembly industry*
$T_{F\mathrm{a}}$	*leisure*	s_i^S	*supply of floor space by local developer*
$D_{W\mathrm{a}}$	*labor supply*	K_i^{Da}	*demand for capital by local developer*
M_{a}	*full income*	X_i^D	*demand for land by local developer*
v_{a}	*value of time*	d_R^{se}	*demand for road link s→e*
τ_R^{se}	*toll on road link s→e*	t_R^{se}	*travel time for the road link s→e*
t_{a}	*assumed commuting time*	t_P^{se}	*travel time for the public transport link s→e*
c_{a}	*assumed commuting cost*	$\hat{t}_q$	*resulting commuting time*
P_{a}	*alternative's choice probability*	$\hat{c}_q$	*resulting commuting cost*
V_{a}^*	*maximum (indirect) utility obtained*	p_j	*price of intermediate produced in zone j*
L_j^D	*labor demand by local firm*	Q_j^S	*intermediate supply by local firm*
K_j^{Df}	*capital demand by local firm*	D_T	*transport provision deficit*
R_L	*total labor tax revenue*	R_R	*total road tax revenue*
R_ℓ	*aggregate land rents*	Y^D	*demand for the composite good (numéraire)*
w_j	*wage at zone j*	p_{Hi}	*housing price at zone i*
p	*price of composite good (numéraire)*	p_{Li}	*price of land at zone i*
$L_{P\mathrm{a}}$	*kilometers generated with public transport under alternative* $\mathrm{a} = \{i, j, q\}$	$L_{R\mathrm{a}}$	*kilometers generated with car under alternative* $\mathrm{a} = \{i, j, q\}$
τ_L	*labor income tax rate*	B	*lump-sum transfer (exogenous income)*

p_g	*cost of car use inputs (gasoline, vehicle depreciation, etc.) per unit of distance (as faced by households)*	$\hat{p}_g$	*import price of car use inputs (gasoline, vehicle depreciation, etc.) per unit of distance (as faced by the importer)*
p_P	*per passenger price for a unit of distance commute with public transport (as faced by households)*	$\hat{p}_P$	*per passenger cost for a unit of distance commute with public transport (as faced by government)*

Notes: the subscript **a** denotes that the variable is conditional on the choice of a given alternative. Because every route q corresponds to a unique alternative a, subscripts that refer to a specific route q can be replaced by a (e.g. $L_{R\mathrm{a}} = L_{Rq}$).

Table A2. *Network and choice notation*

$\mathcal{J}$	*an order set of J zones*	i	*an index pointing at the i-th zone of* $\mathcal{J}$
j	*an index pointing at the j-th zone of* $\mathcal{J}$	$\mathcal{C}_{OD}$	*a set of all possible origin-destinations*
a_{ij}	*an arbitrary origin-destination pair i→i*	$l_m^{(se)}$	*an arbitrary link from node s to node e with transport mode m*
q	*route: a sequence of neighboring links* $l_m^{(se)}$	ℓ_m^{se}	*the length of link* $l_m^{(se)}$
$\mathcal{Q}(\mathrm{a}_{ij})$	*the set of all routes q that are compatible to* a_{ij}*, i.e. they depart from the i-th zone and terminate to the j-th zone*	a	*alternative: an arbitrary origin destination pair,* a_{ij}*, coupled with a route* $q \in \mathcal{Q}(\mathrm{a}_{ij})$*. Denoted as* $\mathrm{a} = \{\mathrm{a}_{ij}, q\} = \{i, j, q\}$
$\mathcal{C}$	*Choice set containing all possible alternatives*	$I\left(l_m^{(se)} \mid q\right)$	*indicator function that equals one if route q contains link* $l_m^{(se)}$ *and zero otherwise*
$I(j \mid \mathrm{a})$	*indicator function that equals one if alternative* a *implies the j-th zone of* $\mathcal{J}$ *as destination*	$I(i \mid \mathrm{a})$	*indicator function that equals one if alternative* a *implies the i-th zone of* $\mathcal{J}$ *as origin*

Appendix B: optimal tolls against the marginal external cost of congestion

Table B1. *Optimal toll and marginal external congestion cost (by link) in the case of labor tax cut revenue recycling.*

start node → end node	toll	mecc	start node → end node	toll	mecc
1→2	0.00261	0.00237	2→1	-0.00143	0.00307
1→3	0.00225	0.00256	3→1	-0.00086	0.00287
1→16	0.00381	0.00414	16→1	0.00064	0.00484
1→18	0.00018	0.00137	18→1	-0.00448	0.00205
2→6	-0.00002	0.00265	6→2	0.00285	0.00284
4→6	-0.00048	0.00304	6→4	0.00000	0.00276
5→6	-0.00215	0.00159	6→5	-0.00427	0.00104
8→6	-0.00013	0.00190	6→8	0.00201	0.00181
7→6	-0.00180	0.00251	6→7	0.00150	0.00235
3→9	-0.00035	0.00134	9→3	-0.00490	0.00126
3→4	0.00022	0.00315	4→3	0.00216	0.00367
4→9	0.00011	0.00184	9→4	-0.00573	0.00158
4→5	-0.00449	0.00096	5→4	-0.00190	0.00099
8→100	-0.00127	0.00133	100→8	0.00033	0.00172
12→100	0.00033	0.00327	100→12	-0.00127	0.00253

11→12	*-0.00182*	*0.00199*	*12→11*	*-0.00097*	*0.00310*
11→10	*0.00285*	*0.00321*	*10→11*	*0.00436*	*0.00356*
11→13	*-0.00057*	*0.00213*	*13→11*	*0.00479*	*0.00296*
13→14	*0.00191*	*0.00246*	*14→13*	*0.00013*	*0.00176*
14→15	*0.00432*	*0.00338*	*15→14*	*-0.00036*	*0.00368*
15→16	*-0.00077*	*0.00790*	*16→15*	*-0.00154*	*0.00642*
16→17	*-0.00384*	*0.00231*	*17→16*	*-0.00423*	*0.00296*
15→101	*0.00096*	*0.00248*	*101→15*	*0.00153*	*0.00320*
10→101	*0.00021*	*0.00302*	*101→10*	*-0.00091*	*0.00222*
7→101	*-0.00052*	*0.00241*	*101→7*	*0.00036*	*0.00288*
10→13	*-0.00299*	*0.00342*	*13→10*	*0.00200*	*0.00498*

Notes: Values have been computed using a BFGS algorithm. Different starting values generated by the no-toll and the Pigouvian quasi first-best equilibrium were used, providing a convergence to (roughly) the same optimum.

Appendix C: Replicability of simulation experiments

The following tables provide the calibrated values of parameters used in the simulation experiments, as well as the values of the exogenous variables in the model. A vector of satisfactory initial values of the endogenous variables is available upon request.

Table C1. *Values of parameters and exogenous variables*

α	0.65	z_P	-5.0	ζ_{11}	0.140	z_{I5}	-1.47	z_{I17}	0.00	z_{J11}	2.60	p_g	0.05
β	0.35	z_t	-3.0	ζ_{12}	0.040	z_{I6}	0.20	z_{I18}	0.20	z_{J12}	0.80	$\hat{p}_g$	0.05
γ	0.40	ζ_1	0.210	ζ_{13}	0.055	z_{I7}	-4.0	z_{J1}	4.10	z_{J13}	2.90	p_P	0.015
π_0	23.6	ζ_2	0.015	ζ_{14}	0.110	z_{I8}	-1.55	z_{J2}	-2.80	z_{J14}	2.00	$\hat{p}_P$	0.015
π_1	3.1	ζ_3	0.015	ζ_{15}	0.040	z_{I9}	-1.82	z_{J3}	-2.80	z_{J15}	-0.40	R	0.001
λ	3.0	ζ_4	0.025	ζ_{16}	0.060	z_{I10}	-0.70	z_{J4}	-0.40	z_{J16}	1.50		
δ	0.3	ζ_5	0.025	ζ_{17}	0.035	z_{I11}	2.20	z_{J5}	-0.50	z_{J17}	0.10		
θ	0.3	ζ_6	0.070	ζ_{18}	0.035	z_{I12}	1.30	z_{J6}	0.15	z_{J18}	-0.70		
ξ_{0R}^{se}	0.25	ζ_7	0.020	z_{I1}	3.70	z_{I13}	2.70	z_{J7}	-4.10	A_j	1.0		
ξ_{1R}^{se}	17.0	ζ_8	0.035	z_{I2}	-2.50	z_{I14}	2.00	z_{J8}	-1.40	T	1.0		
ξ_{0P}^{se}	0.50	ζ_9	0.035	z_{I3}	-3.90	z_{I15}	0.00	z_{J9}	-1.40	N	1.0		
z_R	-2.5	ζ_{10}	0.035	z_{I4}	-1.25	z_{I16}	0.00	z_{J10}	-0.60	F	0.0		

Notes: Capital shares are assumed to be constant across zones for both firms ($\delta = 0.3$) and developers ($\theta = 0.3$). Volume delay function parameters are constant across all (road and rail) links; this is a weak assumption because all links represent large parts of a highway system which, at this level of aggregation, is relatively homogenous. Total factor productivity is uniform over space: differences in factor employment (including job concentration) and output level are generated through non-uniform cost shares (ζ) of the assembly industry. This prevents wages from displaying a spatial variation that would be incompatible with data.

Table C2. *Land endowment ($\bar{X}$) of each zone*

1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*
.208	*.222*	*.068*	*.223*	*.158*	*.095*	*.127*	*.140*	*.363*	*.126*	*.206*	*.233*	*.378*	*.082*	*.204*	*.236*	*.208*	*.187*

Notes: Surface endowments are in accordance with the municipal aggregations displayed in Figure 2.

Appendix D: Numerical approximation of the key *double-dividend* effects

We finally approximate numerically the double dividend effects discussed in the text. The reader should bear in mind that, given the size, detail and complexity of the model, an analytic decomposition of a total effect (i.e. the total welfare change caused by a marginal increase in the toll of an arbitrary link) in its Pigouvian, Tax Interaction and Revenue Recycling components is not possible.

Instead, what we attempt here is an improvised adoption of the general idea behind decomposition formulas that have appeared with significant variations in the literature of double-dividend (Bovenberg and de Mooij, 1994; Goulder et al. 1999; Parry and Bento, 2000; Bento and Jacobsen, 2007; Carbone and Smith, 2008; Bento et al., 2011). Define the *Pigouvian effect* in link $l_R^{(se)}$ as:

$$P^{se} = (\tau_R^{se} - mecc_R^{se})\frac{\mathrm{d}(d_R^{se})}{\mathrm{d}\tau_R^{se}}, \tag{D1}$$

where the total derivative of the link load (i.e. the total demand for the road link, d_R^{se}, as defined in (31)) with respect to the link toll is approximated by using finite differences (i.e. by computing d_R^{se} in two general equilibria between which τ_R^{se} has been increased) and $mecc_R^{se}$ denotes the Pigouvian level of the toll as given by (56).

Next, define the total (general equilibrium) derivative of labor tax revenue (R_L as it is expressed in equation (39)) as:

$$\frac{\mathrm{d}R_L}{\mathrm{d}\tau_L} = \frac{\mathrm{d}[\tau_L LS(\tau_L)]}{\mathrm{d}\tau_L} = \underbrace{LS(\tau_L)}_{\substack{first-order\\ effect}} + \underbrace{\tau_L \frac{\mathrm{d}LS(\tau_L)}{\mathrm{d}\tau_L}}_{\substack{effect\ from\ the\ erosion\\ of\ tax\ base}}, \tag{D2}$$

where $\frac{\mathrm{d}LS(\tau_L)}{\mathrm{d}\tau_L}$ is approximated again using finite differences. We define the *marginal excess burden of the labor tax* as:

$$M = -\frac{\left\{\tau_L \frac{\mathrm{d}LS(\tau_L)}{\mathrm{d}\tau_L}\right\}}{\left\{LS(\tau_L) + \tau_L \frac{\mathrm{d}LS(\tau_L)}{\mathrm{d}\tau_L}\right\}}. \tag{D3}$$

Finally, we compute the *tax interaction effect* in the link $l_R^{(se)}$ as:

$$TI^{se} = (1 + M)\frac{\mathrm{d}R_L}{\mathrm{d}\tau_R^{se}}, \tag{D4}$$

and the revenue recycling effect in the same link as:

$$RR^{se} = M\frac{\mathrm{d}R_R}{\mathrm{d}\tau_R^{se}}. \tag{D5}$$

Table D1 presents the sum of P^{se}, TI^{se} and RR^{se} computed for each link in the base, no-toll equilibrium. In general, the sign of the total effect is in accordance with the sign and the magnitude of the optimal toll (toll characterization), although the alignment is imperfect (the sign is opposite in a few cases). The imperfection can be attributed to three non-mutually-exclusive factors: i) local (instead of global) optimum reached by the optimization algorithm (in this case most probably the global optimum lies in the close neighborhood of the local optimum, since the difference in this case is hard to detect using heuristic tests with different starting values), ii) the above approximation is incomplete, in the sense that residual effects are missing or analytic formulas differ from the above, and iii) numerical errors in the above approximations.

Table D1. *Double dividend* (DD) *approximation of the total effect in the untolled (base) equilibrium versus optimal link toll characterizations: double dividend emerges* (++), *positive without double dividend* (+), *approximately zero* (≈0), *negative* (-).

startnode → endnode	*toll characterization*	*total effect-approximation*	*startnode → endnode*	*toll characterization*	*total effect-approximation*
1→2	(++)	*0.00119*	*2→1*	(-)	*0.00096*
1→3	(+)	*0.00159*	*3→1*	(-)	*-0.00019*
1→16	(+)	*0.00321*	*16→1*	(+)	*0.00624*
1→18	(+)	*0.00090*	*18→1*	(-)	*-0.00029*
2→6	(≈0)	*8E-05*	*6→2*	(++)	*0.00158*
4→6	(-)	*-0.00207*	*6→4*	(≈0)	*0.00038*
5→6	(-)	*-0.00252*	*6→5*	(-)	*-0.00110*
8→6	(≈0)	*-0.00065*	*6→8*	(++)	*-0.00231*
7→6	(-)	*0.00061*	*6→7*	(+)	*-0.00271*
3→9	(-)	*0.00024*	*9→3*	(-)	*-0.00140*
3→4	(≈0)	*-9E-05*	*4→3*	(+)	*0.00051*
4→9	(≈0)	*0.00025*	*9→4*	(-)	*-0.00192*
4→5	(-)	*-0.00197*	*5→4*	(-)	*-0.00087*
8→100	(-)	*-0.00342*	*100→8*	(+)	*-0.00029*
12→100	(+)	*-0.00029*	*100→12*	(-)	*-0.00342*
11→12	(-)	*0.00069*	*12→11*	(-)	*-0.00124*
11→10	(+)	*0.00252*	*10→11*	(++)	*0.00212*
11→13	(-)	*0.00144*	*13→11*	(++)	*0.00313*
13→14	(+)	*0.00473*	*14→13*	(≈0)	*0.00340*
14→15	(++)	*0.00643*	*15→14*	(≈0)	*0.00223*
15→16	(≈0)	*0.00786*	*16→15*	(-)	*0.00056*
16→17	(-)	*0.00027*	*17→16*	(-)	*-0.00159*
15→101	(+)	*-0.00028*	*101→15*	(+)	*0.00316*
10→101	(≈0)	*0.00378*	*101→10*	(-)	*-0.00406*
7→101	(-)	*-0.00322*	*101→7*	(+)	*0.00116*
10→13	(-)	*-0.00369*	*13→10*	(+)	*0.00312*

Notes: We classify a link toll as being approximately zero (≈0) if the absolute value of the optimal toll is below 10% of the marginal external congestion cost in this link.

References

Anas, A., 2012. The optimal pricing, finance, and supply of urban transportation in general equilibrium: a theoretical exposition. *Economics of Transportation*, 1, 64-76.

Anas, A., Hiramatsu, T., 2012. The effect of the price of gasoline on the urban economy: from route choice to general equilibrium. *Transportation Research Part A*, 46, 855-873.

Anas, A., Hiramatsu, T., 2013. The economics of cordon tolling: general equilibrium and welfare analysis. *Economics of Transportation*, 2 (1), 18-37.

Anas, A., Kim, I., 1996. General equilibrium models of polycentric urban land use with endogenous congestion and job agglomeration. *Journal of Urban Economics*, 40 (2), 232-256.

Anas, A., Liu, Y., 2007. A regional economy, land use, and transportation model (RELU-TRAN): formulation, algorithm design, and testing. *Journal of Regional Science*, 47 (3), 415-455.

Anas, A., Tikoudis, I., 2015. Numerical solution of general equilibrium models of urban transportation and land use: *decomposition* versus *all-in-one* approaches. Paper for the *Applied Urban Modeling Conference* (AUM), June 24-26, Cambridge University, UK.

Anas, A., Xu, R., 1999. Congestion, land use, and job dispersion: a general equilibrium model. *Journal of Urban Economics*, 45 (3), 451-473.

Bento, A.M., Jacobsen, M., 2007. Ricardian rents, environmental policy, and the "double-dividend" hypothesis. *Journal of Environmental Economics and Management*, 53 (1), 17-31.

Bento, A.M, Franco, S.F., Kaffine, D., 2011. Is there a double-dividend from anti-sprawl policies? *Journal of Environmental Economics and Management*, 61 (2), 135-152.

Bovenberg, A.L., 1999. Green tax reforms and the double dividend: an updated reader's guide. *International Tax and Public Finance*, 6, 421-443.

Bovenberg, A.L., de Mooij, R.A., 1994. Environmental levies and distortionary taxation. *American Economic Review*, 84 (4), 1085-1089.

Böhringer, C., Rutherford, T.F., 1997. Carbon taxes with exemptions in an open economy: a general equilibrium analysis of the German tax initiative. *Journal of Environmental Economics and Management*, 32 (2), 189-203.

Carbone, J.C., Smith, V.K., 2008. Evaluating policy interventions with general equilibrium externalities. *Journal of Public Economics*, 92 (5-6), 1254-1274.

Dagsvik, J.K., Karlström, A., 2005. Compensating variation and hicksian choice probabilities in random utility models that are nonlinear in income. *Review of Economic Studies*, 72 (1), 57-76.

Goulder, L.H., 1995. Environmental taxation and the double dividend: a reader's guide. *International Tax and Public Finance*, 2, 157-183.

Goulder, L.H., Parry, I.W.H., Williams, R.C., Burtraw, D., 1999. The cost-effectiveness of alternative instruments for environmental protection in a second-best setting. *Journal of Public Economics*, 72 (3), 329-360.

Herriges, J.A., Cling, C.L., 1999. Non-linear income effects in random utility models. *The review of Economics and Statistics*, 81 (1), 62-72.

Koster, P., Verhoef, E.T., Shepherd, S., Watling, D., 2014. Probabilistic choice and congestion pricing with heterogenous travellers and price sensitive demand. Tinbergen Institute discussion paper: TI 2014-078/VIII.

Mun, S., Konishi, K., Yoshikawa, K., 2005. Optimal cordon pricing in a non-monocentric city. *Transportation Research Part A*, 39 (7-9), 723-736.

U.S. Department of Transportation, Federal Highway Administration, 2004. *National Household Travel Survey 2001. User's guide. National sample with add-ons.* Available at: http://nhts.ornl.gov/index.shtml.

OECD, 2007. *Territorial Review: Randstad Holland, Netherlands*.

Parry, I.W.H., 2002. Comparing the efficiency of alternative policies for reducing traffic congestion. *Journal of Public Economics*, 85 (3), 333-362.

Parry, I.W.H., Bento, A.M., 2000. Tax deductions, environmental policy and the "double-dividend" hypothesis. *Journal of Environmental Economics and Management*, 39 (1), 67-96.

Parry, I.W.H., Bento, A.M., 2001. Revenue recycling and the welfare effects of road pricing. *Scandinavian Journal of Economics*, 103 (4), 645-671.

Parry, I.W.H., Bento, A.M., 2002. Estimating the welfare effects of congestion taxes: the critical importance of other distortions within the transport system. *Journal of Urban Economics*, 51 (2), 339-365.

Rhee, H.J., Yu, S., Hirte, G., 2014. Zoning in cities with traffic congestion and agglomeration externalities. *Regional Science and Urban Economics*, 44, 82-93.

Schwanen, T., Dieleman, F.M., Dijst, M., 2001. Travel behaviour in Dutch monocentric and polycentric systems. *Journal of Transport Geography*, 9 (3), 173-186.

Tikoudis, I., Verhoef, E.T., van Ommeren, J.N., 2015. On revenue recycling and the welfare effects of second-best congestion pricing in a monocentric city. Forthcoming in *Journal of Urban Economics*, doi:10.1016/j.jue.2015.06.004.

Tikoudis, I., Verhoef, E.T., van Ommeren, J.N., 2015. Second-best urban tolls in a monocentric city with housing market regulations. Tinbergen Institute discussion paper: TI 2015-006/VIII.

van Dender, K., 2004. Pricing transport networks with fixed residential location. *Regional Science and Urban Economics*, 34 (3), 289-307.

Verhoef, E.T., Nijkamp, P., Rietveld, P., 1996. Second best congestion pricing: the case of an untolled alternative. *Journal of Urban Economics*, 40 (3), 279-302.

Verhoef, E.T.., 2002a. Second-best congestion pricing in general networks. Heuristic algorithms for finding second-best optimal toll levels and toll points. *Transportation Research Part B*, 36 (8), 707-729.

Verhoef, E.T., 2002b. Second-best congestion pricing in general static transportation networks with elastic demands. *Regional Science and Urban Economics*, 32 (3), 281-310.

Verhoef, E.T., 2005. Second-best congestion pricing schemes in the monocentric city. *Journal of Urban Economics*, 58 (3), 367-388.