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On Positive Value of Information in Risk Sharing*

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Abstract

We develop a novel argument why better public information can help countries to insure against idiosyncratic risk. Representative agents of developing and industrial countries receive public and private signals on their future income realization and engage in risk-sharing contracts with limited enforceability. Better public information has two opposite effects. First, it has a detrimental effect on risk sharing by limiting risk-sharing possibilities as emphasized by Hirshleifer (1971). Second, it mitigates the adverse selection problem resulting from private information which improves risk sharing. We find that better public information in developing countries ameliorates risk sharing in both developing and industrial countries.

JEL classification: E21, D31, D52
Keywords: Social value of information; Sovereign risk; Limited enforcement

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1 Introduction

International risk sharing has important consequences for social welfare. When countries' engagement in international risk sharing is absent or not efficient, households in those countries suffer from undesirable fluctuations in their consumption levels (Baxter, 2011, Cole and Obstfeld, 1991). Kose, Prasad, and Terrones (2009) and Bai and Zhang (2012) find that compared to industrial countries, developing countries exhibit low degrees of international risk sharing. For this reason, especially developing countries would benefit from better insurance possibilities. In this paper, we investigate the role of public and private information on idiosyncratic country risk in international risk sharing.

In particular, we ask whether there is positive social value of public information in international risk sharing. The question is motivated by the fact that developing countries are not only characterized by low degrees of risk sharing but typically also rank low in terms of quality of publicly available data and timely access to information. As illustrated in Table 1 based on data from the Penn–World Tables, developing countries are mainly found in the low-data quality categories while industrial countries fall rather in the better categories.[1] Thereby, a low data quality grade can result for example because information on real GDP is not reported and thus incomplete or when real GDP figures are sensitive with respect to the particular method used in aggregation.

The motivation for the release of the data quality rankings is to encourage countries to improve the quality of their data and increase transparency in the future. While the general view is that more transparency is beneficial, the view is at odds with the conventional wisdom on information and risk sharing as represented by the Hirshleifer (1971) effect. Releasing better information on idiosyncratic country risk realizations before trading harms social welfare by limiting risk-sharing opportunities in international financial markets. This result does not depend on whether the information is public (shared by everyone) or private (known to the individual only).

In this paper, we aim to make two contributions – one is theoretical and one is quantitative. We consider a dynamic version of the Hirshleifer model with both public and private information. As our main theoretical result, we provide a novel argument for the case of transparency and show that better public information on idiosyncratic country risk before trading can result in better risk sharing. Our key quantitative finding is that better public information in developing countries does not only benefit households in these countries but also bears positive spillover effects for households in industrial countries.

In a world economy, representative agents of small developing and industrial countries insure against country-specific shocks to real GDP by engaging in risk sharing contracts on international financial markets. Contract enforceability is limited because in any period following the realization of idiosyncratic risk countries have the option to walk away from the arrangement with the consequence of exclusion from international risk sharing as in Grossman and Huyck (1988) and Bulow and Rogoff (1989). Limited contract enforcement is motivated by the lack of a supra-national authority to internationally plan in force insurance contracts.

[1] See also Dawson, DeJuan, Seater, and Stephenson (2001) who provide a similar table.

Table 1: Real GDP data quality and stage of development

	Data quality				
Stage of development	A	B	C	D	Total
Industrial countries	18	10	11	0	39
Developing countries	0	3	76	30	109

Notes: Own computations based on data-quality rankings taken from Penn-World-Tables (2008); "A" denotes the highest data quality, "D" the lowest data quality; In 2008, the World Bank classifies developing countries are countries with an annual gross national per-capita income of US$ 11,905 and less.

Each period, countries receive private and public signals on their future idiosyncratic income realization. The public signals capture information on country's future GDP that is publicly provided, for example in the Penn World Tables. Information is the key element in agents' trade-off between future insurance and current incentives resulting from limited enforcement of contracts. While both sources of information lead to an evaporation of insurable risk, private information further harms risk sharing and induces additional welfare costs by giving rise to an adverse selection problem: countries have an incentive to understate their true willingness to share the income risk, resulting in a tracking challenge in the design of optimal insurance contracts.

As our main novel theoretical result, we formally show that better public information can be beneficial for risk sharing when private information is sufficiently precise. The key difference to Hirshleifer is that we consider both sources of information jointly. Better public information has then two opposite effects. First, more informative public signals result in a detrimental welfare effect by evaporating risk-sharing opportunities via the conventional Hirshleifer channel. Second – and this is the new effect here – more informative public signals facilitate a better tracking of agents' true willingness to share the income risk which mitigates the adverse section problem and increases social welfare. When private information is sufficiently precise, the positive *Tracking effect* dominates the negative *Hirshsleifer effect* and *ex-ante* countries prefer informative public signals.

More precise public information does not result in higher welfare by facilitating better individual decisions which is known as the Blackwell (1953) effect.[2] On the contrary and as in Hirshleifer (1971), the direct effect of better public signals alone is detrimental to risk sharing and welfare. Only when private information is sufficiently precise, better public signals can improve risk sharing by easing the adverse selection problem.

We calibrate the world economy to match cross-country income and cross-country consumption characteristics for developing and industrial countries. To account for the differences between developing and industrial countries with respect to risk sharing and quality of public information, information precision differs across countries. While public information is better in the industrial countries, the private information friction plays a more important role in the developing countries to capture the lower degrees of risk sharing observed for these countries.

[2] Papers that consider environments with a Blackwell effect are e.g., Gottardi and Rahi (2014) and Eckwert and Zilcha (2003).

Thus, the model interprets the difference in data quality and risk sharing across countries as an asymmetric information friction that is more severe for the developing than for the industrial countries.

As our main quantitative result, we find that better public information in developing countries not only improves risk sharing in these countries but also yield positive spillover effects for risk sharing in industrial countries. Thereby, the developing countries benefit relatively more than the industrial countries: their annual consumption equivalent increases permanently by up to 0.83% while the increase in industrial countries amounts up to 0.26%, respectively.

Related Literature This paper contributes to the literature on the social value of information (Hirshleifer, 1971, Lepetyuk and Stoltenberg, 2013, Morris and Shin, 2002, Schlee, 2001). With our application to international risk sharing, we connect to the quantitative literature on sovereign risk (Arellano, 2008, Bai and Zhang, 2012). Methodologically, our study relates to the literature on efficient distribution with private information and efficient risk sharing with limited commitment (Atkeson and Lucas, 1992, 1995, Krueger and Perri, 2011).

Hirshleifer (1971) is among the first to point out that perfect information makes risk-averse agents ex-ante worse off if such information leads to evaporation of risks that otherwise could have been shared in a competitive equilibrium. Schlee (2001) provides general conditions under which better public information about idiosyncratic risk is undesirable. We provide a dynamic version of the environments considered by Hirshleifer (1971) and Schlee (2001) and allow for two sources of information, private and public. If private information was uninformative, better public information would also harm risk sharing and decrease social welfare in our environment.

In a global games framework, Morris and Shin (2002) show that when the coordination of agents is driven by strategic complementarities in their actions, better public information on aggregate risks may be undesirable in the presence of private information on these risks. Lepetyuk and Stoltenberg (2013) show that better information on aggregate risk can harm the insurance of idiosyncratic risk. The main difference to Morris and Shin (2002) and Lepetyuk and Stoltenberg (2013) is that we analyze the social value of information on idiosyncratic risk in international risk sharing.

Arellano (2008) and Bai and Zhang (2012) consider decentralized economies in which small countries engage in international risk sharing using non-contingent bonds. Arellano (2008) focusses on the interaction of sovereign default with output, consumption and interest rates over the business cycle. Bai and Zhang (2012) ask whether financial liberalization helped to improve on the insurance of idiosyncratic country risk. In this paper, we analyze optimal insurance of country risk using a complete set of securities but with limited enforcement of contracts. In this environment, we study the role of private and public information for international risk sharing in developing and industrial countries.

Atkeson and Lucas (1992, 1995) are the first to analyze efficient distribution of a continuum of agents in economies with private information in general equilibrium. Krueger and Perri (2011) analyze how public insurance provided by a progressive tax scheme affects social welfare when agents engage in private insurance arrangements consistent with rational participation incentives. Our contribution is to analyze the social value of information with two sources of information in insurance arrangements that are consistent with rational participation constraints.

The remainder of the paper is organized as follows. In the next section, we present our economic environment. In Section 3, we analyze the social value of information in a model with memoryless contracts. Section 4 is devoted to the analysis of optimal allocations. In Section 5, we evaluate the importance of better public information for developing and industrial countries in international risk sharing. The last section concludes.

2 Environment

Consider a world economy that is populated by a continuum of representative households of small countries indexed i and limited enforcement of contracts. The time is discrete and indexed by t from zero onward. Households have identical preferences over consumption streams

$$U(\{c_t\}_{t=0}^{\infty}) = (1-\beta)\,\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{1}$$

where the instantaneous utility function $u : \mathbb{R}_+ \to \mathbb{R}$ is strictly increasing, strictly concave and satisfies the Inada conditions.

The income stream of household i, $\{y_t^i\}_{t=0}^{\infty}$, is governed by a stochastic process where the set of possible realizations in each period is time invariant, ordered and finite: $y_t^i \in Y \equiv \{y_1, ..., y_N\} \subseteq \mathbb{R}_{++}$, and y^t denotes the history $(y_0, ...y_t)$. Income realizations are independent across households and evolve across time according to a first-order Markov chain with constant transition probabilities $\pi(y'|y)$. The Markov chain induces a unique invariant distribution of income $\pi(y)$ such that average income is $\bar{y} = \sum_y y\pi(y)$. Current-period income is publicly observable and the transition probabilities of the Markov chain are public knowledge.

Each period $t \geq 0$, household i receives a public signal $k_t^i \in Y$ and a private signal $n_t^i \in Y$ that inform about her income realization in the next period. The private signal will give rise to an adverse selection problem. Both signals have as many realizations as income states and are assumed to follow i.i.d. processes, i.e., $\mathrm{p}(k_{t+1} = y_j | k_t = y_j) = \mathrm{p}(k_{t+1} = y_j) = \pi(y_j)$. The precision of the public signal is denoted by $\kappa = \mathrm{p}(y_{t+1} = y_j | k_t = y_j)$, while the precision of the private signal is given by $\nu = \mathrm{p}(y_{t+1} = y_j | n_t = y_j)$. Uninformative signals are characterized by precision $1/N$. The publicly observable part of the state vector is $s_t = (y_t, k_t)$, $s_t \in S$, where $S = Y \times Y$. Let $s^t = (y^t, k^t)$ denote the public history of the state $(s_0, ...s_t)$. The history of the public and private state is denoted by $\theta^t = \left(s^t, n^t\right)$, with $\theta_t \in \Theta = S \times Y$. The conditional distribution of signals and income is a time-invariant Markov chain described by transition matrices P_θ, P_s with the conditional probabilities $\pi(\theta'|\theta)$ and $\pi(s'|s)$.[3]

Households differ with respect to their initial utility entitlements w_0 and the initial state θ_0. For all θ^t, the utility allocation is $h = \{h_t(w_0, s^t)\}_{t=0}^{\infty}$ and the consumption allocation c can be obtained as $c = \{C(h_t(w_0, s^t))\}_{t=0}^{\infty}$, where $C : \mathbb{R} \to \mathbb{R}_+$ is the inverse of the instantaneous utility function u. Thus, the allocation is assumed to be contingent solely on the public state, private information is not contractible. Nevertheless, the allocation depends on private information because contract enforcement is limited.

[3] The computation of the conditional probabilities can be found in Appendix A.7.

As illustrated in Figure 1, each period after receiving the current income and the two signals, households can decide to participate in international risk-sharing contracts implementing the allocation c or to deviate into autarky forever consuming only their income. Households have no incentive to default if the allocation satisfies enforcement or participation constraints for each history θ^t and all periods t

$$(1-\beta)u(C(h_t(w_0,s^t))) + \beta \sum_{\theta_{t+1}} \pi(\theta_{t+1}|\theta^t) U_{t+1}(\{C(h_\tau(w_0,s^\tau))\}_{\tau=t+1}^\infty) \geq$$
$$(1-\beta)u(y_t) + \beta \sum_{y_{t+1}} \pi(y_{t+1}|\theta_t) U_{t+1}(\{y_\tau\}_{\tau=t+1}^\infty) = U^{Aut}(\theta_t), \quad (2)$$

with $U_\tau = (1-\beta)\,\mathrm{E}_\tau \sum_{t=\tau}^{\infty} \beta^{t-\tau} u\left(c_t\right)$ and $U^{Aut}(\theta_t)$ as the value of the outside option (autarky).

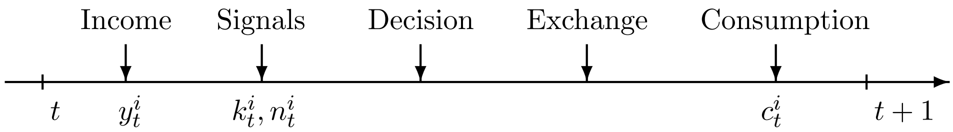

Figure 1: Timing of events with public and private information

Let Φ_0 be a distribution over initial utility promises w_0 and the initial shocks s_0. In the following definitions, we summarize the notions of constrained feasible and optimal allocations.

Definition 1 *An allocation* $\{h_t(w_0,s^t)\}_{t=0}^\infty$ *is constrained feasible if*

(i) the allocation delivers the promised utility w_0

$$w_0 = \mathrm{U}(\{C(h_t(w_0,s^t))\}_{t=0}^\infty); \quad (3)$$

(ii) the allocation satisfies enforcement constraints (2) *for each history* θ^t

(iii) and the allocation is resource feasible

$$\sum_{\theta^t} \int (C(h_t(w_0,s^t)) - y_t)\pi(s^t|s_0)\,\mathrm{d}\,\Phi_0 \leq 0. \quad (4)$$

Atkeson and Lucas (1992, 1995) show by applying a duality argument that optimal allocations can be computed either by directly maximizing households' utility over the distribution Φ_0 or, alternatively, by minimizing resource costs to deliver the promises made in Φ_0.

Definition 2 *An allocation* $\{h_t(w_0,s^t)\}_{t=0}^\infty$ *is optimal if it is constrained feasible and either*

(i) maximizes households' ex-ante utility over the distribution Φ_0

$$\mathrm{E}\,U(c) = \int U(\{c_t(w_0,s^t)\}_{t=0}^\infty)\,\mathrm{d}\,\Phi_0$$

(ii) or, alternatively there does not exist another constrained feasible allocation $\{\hat{h}_t(w_0, s^t)\}_{t=0}^{\infty}$ *with respect to* Φ_0 *that requires fewer resources in at least one period* t

$$\exists t: \quad \sum_{\theta^t} \int (C(\hat{h}_t(w_0, s^t) - C(h_t(w_0, s^t))\pi(s^t|s_0)\,\mathrm{d}\,\Phi_0 < 0.$$

The second part of the definition is relevant with a continuum of agents and when allocations depend on the state history. Using the first approach, there would be a continuum of promise-keeping constraints (3) the planner would have to respect which makes the approach to optimize directly over ex-ante utility impossible. The first approach can be applied however in the absence of promises as a state variable such that allocations just depend on the current state but not on the history. We refer to such allocations as memoryless allocations which are defined as follows.

Definition 3 *An allocation* $\{h_t(w_0, s^t)\}_{t=0}^{\infty}$ *is a memoryless allocation (denoted* h_{ML}*) if:*

$$\forall s^t \quad \{h_t(w_0, s^t)\}_{t=0}^{\infty} = \{h_t(s_t)\}_{t=0}^{\infty} \equiv h_{ML}.$$

Due to the absence of an efficient revelation mechanism, private information is not contractible and optimal allocations summarized in Definition 2 are in general not (constrained) efficient. Obstfeld and Rogoff (1996) argue in Chapter 6.3 that the absence of an efficient revelation mechanism is reasonable in the context of international risk sharing. Risk-sharing contracts with revelation captured by truth-telling constraints would require that the contract is the only financial commitment countries can make or that countries' other trades are perfectly observable which is difficult in the context of sovereign risk.

Optimal allocations that depend only on the history of the public state can further be interpreted as allocations stemming from a competitive equilibrium as shown in Kehoe and Levine (1993) and Krueger and Perri (2011). In Appendix A.10, we provide a decentralization with period-zero trading of a complete set of state-contingent consumption claims. Atkeson and Lucas (1992) show that such a full decentralization is in general not possible for efficient allocations with truth telling.[4]

The direct dependency on public information further captures the idea that trade in international financial markets takes place with respect to the observable information on country risk but private information is important for understanding the incentives of countries to honor or to default on international risk sharing contracts. These incentives further play a key role in shaping equilibrium allocations and interest rates.

In the next section, we employ the simplified structure resulting from memoryless allocations to derive the analytical results on the positive value of public information in risk sharing.

[4] In a robustness exercise, we find a quantitative similar positive effect of public information when private information is contractible (see Appendix A.6).

3 Positive value of public information

In this section, we provide our main analytical results on the positive welfare effect of public information that emerges when risk sharing is partial. Further, we analyze how the precision of public and private information affect the conditions for full risk sharing and autarky.

3.1 An environment with memoryless allocations

To analytically characterize the welfare effect of public information, we abstract in this section from a number of features. First, we assume that the set of possible income realizations consists of two states, a high income $y_h = \bar{y} + \delta_y$ and a low income $y_l = \bar{y} - \delta_y$, where $\delta_y > 0$ is the standard deviation of income. Second, the states are equally likely and the income realizations are independent across time and agents. Correspondingly, signals can indicate either a high income ("good" or "high" signals) or a low income ("bad" or "low" signals) in the future. By definition, a memoryless allocation does not depend on past events but just on current realizations of the state.[5]

A utility allocation in this simplified setting is $h_{ML} = \{u\left(c_i^j\right)\}$, with $c_i^j = C[h(y = y_j, k = y_i)]$ and social welfare is

$$\mathrm{E}\, U(c) = (1-\beta)\frac{1}{4}\sum_{t=0}^{\infty}\sum_{j\in\{l,h\}}\sum_{i\in\{l,h\}}\beta^t u\left(c_i^j\right). \tag{5}$$

Resource feasibility requires in each period t

$$\frac{1}{4}\sum_{j\in\{l,h\}}\sum_{i\in\{l,h\}} c_i^j = \frac{1}{2}\sum_{j\in\{l,h\}} y_j. \tag{6}$$

Further, constrained feasibility requires that allocations are consistent with rational incentives to participate. For example, the enforcement constraints of high-income households with high public and high private signals are

$$\begin{aligned}(1-\beta)u(c_h^h) + \frac{\beta(1-\beta)\left[\kappa\nu V_{rs}^h + (1-\kappa)(1-\nu)V_{rs}^l\right]}{\kappa\nu + (1-\kappa)(1-\nu)} + \beta^2 V_{rs} \geq \\ (1-\beta)u(y_h) + \frac{\beta(1-\beta)\left[\kappa\nu u(y_h) + (1-\kappa)(1-\nu)u(y_l)\right]}{\kappa\nu + (1-\kappa)(1-\nu)} + \beta^2 V_{out},\end{aligned} \tag{7}$$

while the constraints for high-income agents with low public and high private signals read

$$\begin{aligned}(1-\beta)u(c_l^h) + \frac{\beta(1-\beta)\left[(1-\kappa)\nu V_{rs}^h + \kappa(1-\nu)V_{rs}^l\right]}{(1-\kappa)\nu + \kappa(1-\nu)} + \beta^2 V_{rs} \geq \\ (1-\beta)u(y_h) + \frac{\beta(1-\beta)\left[(1-\kappa)\nu u(y_h) + \kappa(1-\nu)u(y_l)\right]}{(1-\kappa)\nu + \kappa(1-\nu)} + \beta^2 V_{out},\end{aligned} \tag{8}$$

[5] The absence of history dependence implies that optimal memoryless allocations with efficient revelation can only depend on public information. Thus, all results in this section apply if we were to consider contractible private information.

with

$$V_{rs}^h = \frac{1}{2}\left[u(c_h^h) + u(c_l^h)\right], \quad V_{rs}^l = \frac{1}{2}\left[u(c_h^l) + u(c_l^l]\right)$$

and

$$V_{rs} = \frac{1}{2}\left[V_{rs}^h + V_{rs}^l\right], \quad V_{out} = \frac{1}{2}\left(u(y_h) + u(y_l)\right).$$

As a next step, we provide the definition of an optimal memoryless allocation.

Definition 4 *An optimal memoryless arrangement (ML arrangement) is a consumption allocation $\{c_i^j\}$ that maximizes households' utility (5) subject to resource feasibility (6) and enforcement constraints in all periods $t \geq 0$.*

In an influential paper, Grossman and Huyck (1988) entertain an equivalent formulation to compute optimal memoryless arrangements. In their formulation, representative agents in small countries engage in risk-sharing arrangements provided by risk neutral foreign insurers that act under perfect competition. Representative households then maximize expected utility (5) subject to participation incentives and a zero-profit condition for the insurers (corresponding to the resource constraint in our formulation)

$$\frac{1}{4}\sum_{j\in\{l,h\}}\sum_{i\in\{l,h\}} \tau_i^j = 0,$$

where $\tau_i^j = y_j - c_i^j$ are insurance transfers. A negative value denotes a payment to the country and a positive value an insurance premium paid by the country.

The optimal memoryless arrangement exists and is unique. The arrangement and social welfare are continuous functions in the precision of the public and private signal. The proof follows from the maximum theorem under convexity and is omitted here. Further, one can show that in memoryless arrangements only participation constraints of high-income agents can be binding.[6] Allocations depend only on public information but not on private information. Thus, for each public state allocations must be consistent with the participation incentives of households with high private signals as the highest outside option value. For this reason, only constraints (7) and (8) can be binding in the optimal memoryless arrangement.

3.2 Information, perfect risk sharing and autarky

Optimal memoryless arrangements may feature either perfect risk sharing (all agents consume $\bar{y}$ in all states), no insurance against income risk (autarky, all agents consume their income in all states) or partial risk sharing. The first case is analyzed in the following proposition.

Proposition 1 (Perfect Risk Sharing) *Consider the optimal memoryless arrangement.*

1. *There exists a unique cutoff point, $0 < \bar{\beta}(\kappa, \nu) < 1$, such that for any discount factor $1 > \beta \geq \bar{\beta}(\kappa, \nu)$ the optimal allocation for any signal precision is perfect risk sharing.*

2. *The cutoff point $\bar{\beta}(\kappa, \nu)$ is increasing in the precision of the public and private signal.*

[6] A proof for this result can be found for example in Lepetyuk and Stoltenberg (2013). In optimal history-dependent arrangements also participation constraints of low-income agents are occasionally binding.

The proof is provided in Appendix A.1. The cutoff point for perfect risk sharing is determined by the tightest participation constraints which are the ones of high income agents with good public and private signals. The long-term gains from risk sharing can only outweigh the desire to leave the arrangement if agents are sufficiently patient. Furthermore, the value of the outside option at the tightest participation constraint is increasing in the precision of both signals.

On the other extreme, autarky may be the only constrained feasible memoryless allocation. In the next proposition, we provide conditions for this case.

Proposition 2 (Autarky) *Consider the case when the participation constraints of high-income agents (7) and (8) are binding. Let* $z_1 = \kappa\nu/[\kappa\nu + (1-\kappa)(1-\nu)]$ *and* $z_2 = (1-\kappa)\nu/[(1-\kappa)\nu + \kappa(1-\nu)]$*. If and only if*

$$u'(y_h) + \beta\frac{z_1 + z_2}{2}[u'(y_h) + u'(y_l)] - \beta u'(y_l) - \frac{\beta^2}{1-\beta}\frac{1}{2}[u'(y_l) - u'(y_h)] \geq 0 \tag{9}$$

autarky is the optimal memoryless arrangement.

The proof is provided in Appendix A.2.

The more patient agents are the more restrictive **is** the condition stated in the proposition. The effect of information is source dependent. Thereby, the terms $1-z_1$ and $1-z_2$ as the weights to the low-income state in the next period capture the relevance of insurance for high-income agents. An increase in precision of the private signal decreases the importance of insurance by increasing z_1 and z_2, making it more likely that autarky is the only constrained feasible arrangement. The effect of an increase in public information precision leads to an increase in the importance of insurance as the sum of z_1 and z_2 decreases whenever private information is informative. The increase in importance of insurance makes it less likely that autarky is the optimal memoryless arrangement.

Up to here, we analyzed the effects of information on the conditions that make perfect risk sharing or the complete absence of risk sharing optimal memoryless allocations. In the following, we consider the empirically most common case of partial risk sharing in which information precision directly affects the allocation.

3.3 Information and partial risk sharing

As a first step, we consider both sources of information separately. In this case, the traditional Hirshleifer result applies and increases in signal precision are welfare reducing. This is illustrated in Figure 2. Increases in private information precision have a more detrimental effect on welfare than increases in public signal precision. While the release of both types of information limit risk-sharing possibilities, private information induces additional welfare costs. The additional welfare costs arise because households true willingness to share the income risk is not directly observable.

The picture changes when we consider both sources of information jointly. While private information continues to have negative welfare effect (see Proposition 4 in Appendix A.5), public information yields positive welfare effects when private signals are sufficiently informative. In

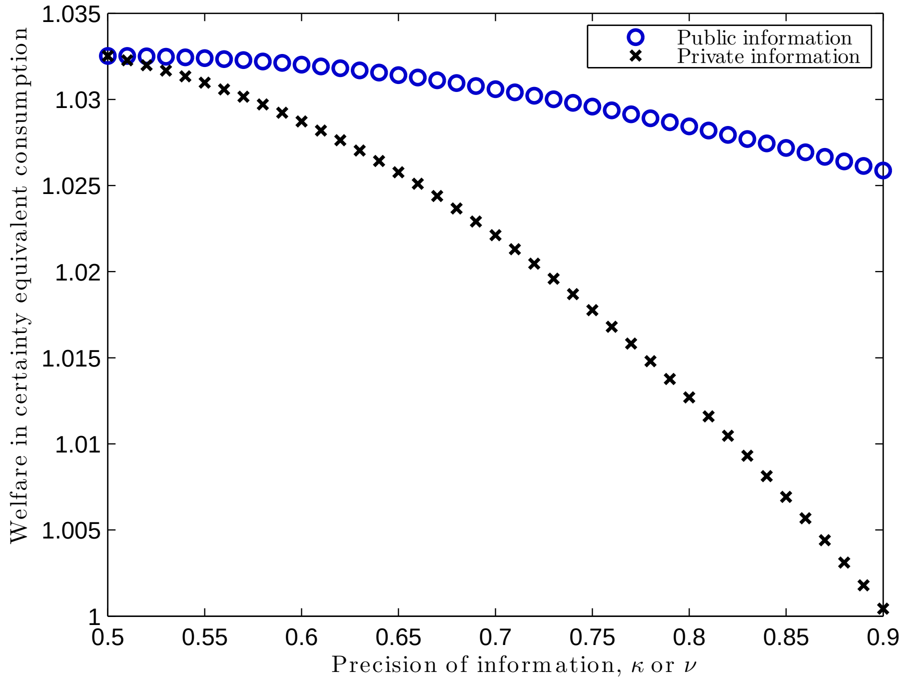

Figure 2: Separate welfare effects of public and private information.

the following theorem, we summarize our main theoretical result on the positive value of public information in risk sharing.

Theorem 1 (Positive Value of Public Information) *Consider the case when the participation constraints (7) and (8) of high-income agents are binding. Assume that autarky is not the only constrained feasible memoryless arrangement. Then there exists a precision of the private signal $\bar{\nu}$, such that for $\nu \geq \bar{\nu}$ and $\nu \in [0.5, 1)$, social welfare is increasing in the precision of the public signal κ over $\kappa \in [0.5, 1)$.*

The proof is provided in Appendix A.3. The logic of the proof is as follows. First, we show that for an uninformative private signal the social value of information is negative, while for a perfectly informative signal the effect is positive. Continuity then implies that there exist a level of private information for which the welfare effect of better public information is positive.

There is a negative and a positive effect of releasing better public information when private signals are informative. First – and more conventionally – more precise public information in advance of trading limits risk-sharing possibilities. Second – and this is the new effect here – more informative public signals facilitate a better tracking of households' true willingness to share the income risk which increases social welfare.

The main theorem is illustrated in Figure 3 that depicts social welfare as a function of public information precision for different precisions of private information. When private information is uninformative or not precise (see the upper two functions), the negative Hirshleifer effect dominates and social welfare is decreasing in public signal precision. For $\nu = 0.7$, the negative and the positive effect neutralize each other. However, when private signals are sufficiently precise the latter positive effect dominates the negative effect, and social welfare increases in public signal precision (see the lower two functions).

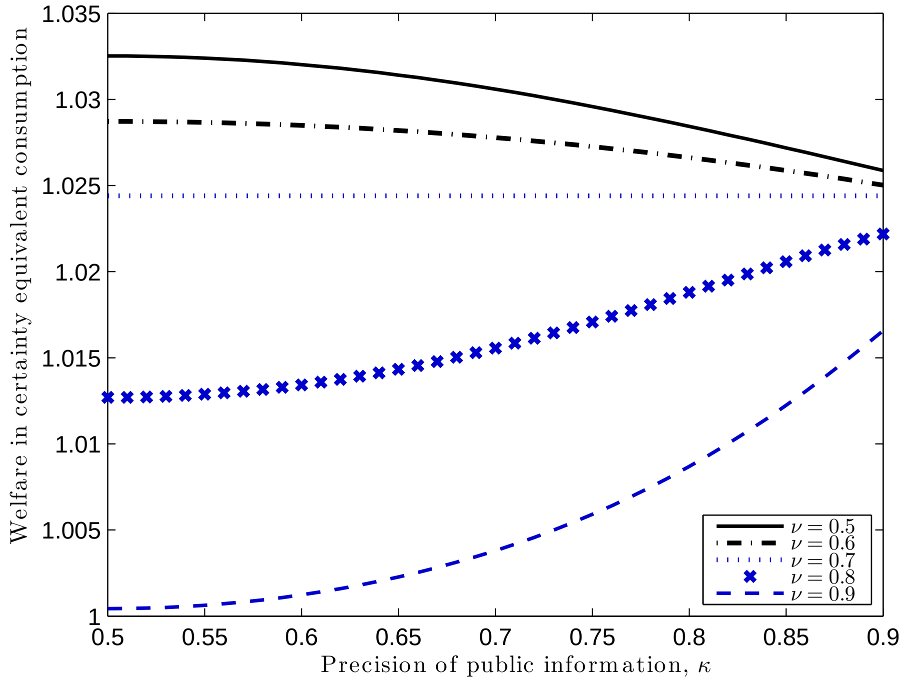

Figure 3: Welfare effects of public information for different precisions of private information.

To gain intuition, consider an increase in the precision of the public signal. By (7) and (8), this results in an increase in the value of the outside option for high-income agents with a good public signal and a decrease for agents with a bad public signal as illustrated in Figure 4.

As captured by the changes in the outside option values, agents with a bad signal are more willing while the agents with a good signal are less willing to share their current high income. When the private signal is uninformative (see the lower part of Figure 4 with $\nu = 0.5$), the changes in the value of the outside option of high-income agents with a good signal ($V^h_{h,out}$) and with a bad signal ($V^h_{l,out}$) are symmetric.

The high-income agents with a good public signal have a lower marginal utility of consumption and thus require more additional resources than the high-income agents with a bad public signal are willing to give up. In sum, average consumption of high-income agents increases which by resource feasibility reduces the risk-sharing possibilities for low-income agents. As a consequence, the allocation becomes riskier ex ante and social welfare decreases.

When the private signal is sufficiently informative and public-signal precision increases (see the upper part of Figure 4 for $\nu = 0.9$), the value of the outside option of high-income agents with a good public signal increases less than the outside option of high-income agents with a bad public signal decreases (in absolute terms). The asymmetric change in the outside option creates room for redistribution from high to low-income agents stemming from high-income agents with low public signals. For a sufficiently informative private signal, better public information facilitates additional risk-sharing transfers between high-and low-income agents, and social welfare increases.

As our main result, we have shown that better public information can be beneficial for risk sharing if private information is sufficiently precise. To ease the exposition, we focused

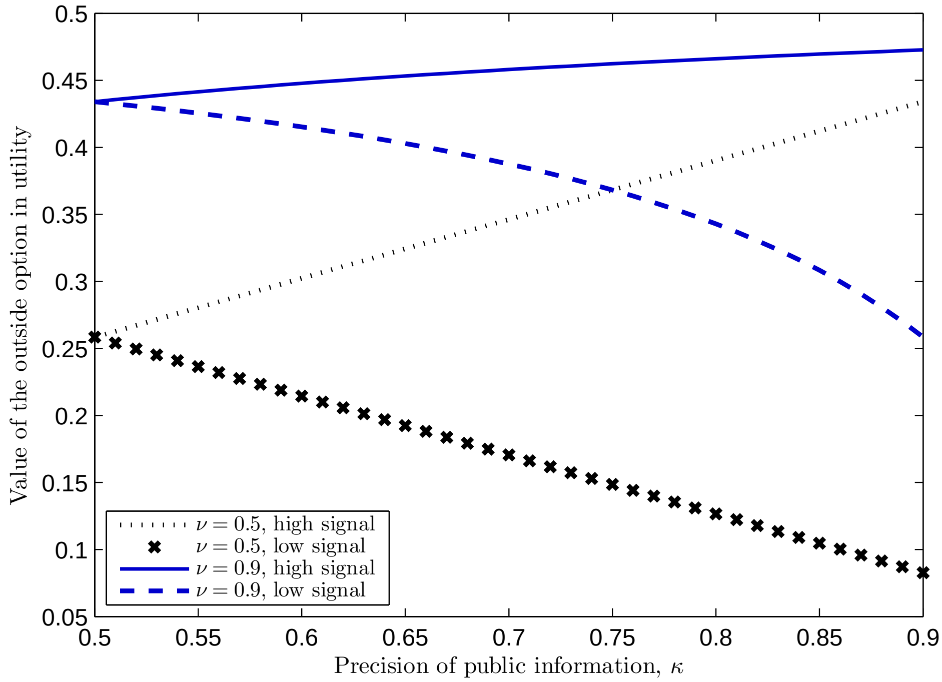

Figure 4: Outside option values of high-income agents as a function of public information precision for different precisions of private information.

on memoryless allocations that are in general not optimal allocations. In the next section, we proceed with optimal allocations that can depend on the history of the state.

4 Optimal allocations

To compute optimal stationary history-dependent allocations, we apply the methodology provided by Atkeson and Lucas (1992, 1995) and Krueger and Perri (2011). As in the previous section, income is assumed to follow an i.i.d. process with two equally likely states.

4.1 History-dependent arrangements

Optimal stationary history-dependent allocations can be computed by solving recursive planner problems. The planner (or financial intermediary) is responsible for allocating resources to a particular household. There are many planners and they can inter-temporally trade resources with each other at the given shadow price $1/R$ with $R \in (1, 1/\beta]$. The equilibrium interest rate is the interest rate that guarantees resource feasibility.

Given a utility promise w, a public state $s = (y, k)$, a constant R, the planner chooses a portfolio of current utility h and future promises $w'(s')$ for each future income realization y' and signal k'. The portfolio $(h, \{w'(s')\})$ is required to minimize the discounted resources costs

$$V(w(s), s) = \min_{h, \{w'\}} \left[\left(1 - \frac{1}{R}\right) C(h) + \frac{1}{R} \sum_{s'} \pi(s'|s) V(w'(s'), s') \right] \tag{10}$$

to deliver the promised value $w(s)$ and to satisfy the participation constraints

$$w(s) = (1-\beta)h + \beta \sum_{s'} \pi(s'|s) w'(s') \tag{11}$$

$$w'(s') \geq U^{Aut}(s', n'), \forall s', n'. \tag{12}$$

A stationary allocation $\{h_t(w_0, s^t)\}_{t=0}^{\infty}$ is a candidate for an optimal allocation if it is induced by an optimal policy from the functional equation above with $R > 1$ and satisfies the resource constraint (4) with equality. The difference to a memoryless allocation is that there can be more than one current utility $h(s)$ for each s, depending on the history s^t captured by promises as a state variable which allows for better risk sharing.

The solution is characterized by the first order conditions:

$$u'[c(w(s), s)] \geq \beta R u' \left[c\left(w'(w(s), s); s'\right)\right] \tag{13}$$

$$u'[c(w(s), s)] = \beta R u' \left[c\left(w'(w(s), s); s'\right)\right], \ \textit{if } w'(\cdot; s') > U^{Aut}(s', n'), \tag{14}$$

where we have applied that $C'(h) = 1/u'(c)$ and the envelope condition

$$\lambda = \frac{\partial V(w, s)}{\partial w},$$

with λ as the Lagrange multiplier associated with the promise keeping constraint (11). Following analogous arguments as in Krueger and Perri (2011), $w' \in W = [\underline{w}, \overline{w}]$, $\underline{w} = \min_{s' \in S} U^{Aut}(s', n' = y_N)$ and $\overline{w} = \max_{\theta' \in \Theta} U^{Aut}(\theta')$. The state space $Z = W \times S$ induces the Borel σ-algebra $\mathcal{B}(Z)$. The Markov process for income and public signals together with the policy functions w' generate a law of motion for the probability measure $\Phi(w, s)$ with marginal distribution of income given by $\pi(y)$. Let $Q_R[(w, s), (\mathcal{W}, \mathcal{S})]$ be the probability conditional on R that a household with utility promise w and income-signal state s transits to the set $\mathcal{W} \times \mathcal{S}$:

$$Q_R[(w, s), (\mathcal{W}, \mathcal{S})] = \sum_{s' \in \mathcal{S}} \begin{cases} \pi(s'|s) & \text{if } w'(w, s) \in \mathcal{W} \\ 0 & \text{else} \end{cases}$$

The law of motion for the probability measure Φ is then given by

$$\Phi' = H(\Phi) = \int Q_R[(w, s), (\mathcal{W}, \mathcal{S})] \Phi(\mathrm{d}\, w \times \mathrm{d}\, y \times \mathrm{d}\, k).$$

An allocation is stationary if $\Phi = H(\Phi)$.[7]

Definition 5 *An HD arrangement is a stationary utility allocation $\{h(s, w)\}$, an invariant probability measure Φ induced by the problem (10)-(12) and R such that*

$$\int V[w(s), s] - y \, \mathrm{d}\, \Phi = 0.$$

[7] The probability measure Φ can be shown to exist and to be unique. The proof relies on a compact state space for utility promises which follows from analogous arguments as in Krueger and Perri (2011). In the absence of public signals, Broer, Klein, and Kapicka (2015) show that solving for stationary allocations with contractible private information in the infinite horizon model can be prohibitively hard, especially because the state space of promises is not necessarily compact.

4.2 Memoryless arrangements

In the analytical section, we directly compute optimal memoryless allocations by maximizing social welfare constrained by participation incentives and resource feasibility. For comparison with history-dependent arrangements, we provide here a recursive representation with $W = \mathrm{E}\, U(c)$ as social welfare.

$$W = \max_{\{h(s)\}} \sum_s \pi(s) W(s) \tag{15}$$

$$\text{s.t. } \tilde{w}\,(s,n) \geq U^{Aut}\,(s,n)\, \forall s,n \tag{16}$$

$$\text{and the resource feasibility constraint,} \tag{17}$$

with

$$W(s) = (1-\beta)h(s) + \beta \sum_{s'|s} \pi(s'|s) W'(s'),$$

and $\tilde{w}\,(s,n)$ as the lifetime utility of a household in state (s,n). Unlike in the history-dependent case, $\tilde{w}$ are not a state variable but a solution to the following functional equation

$$\tilde{w}\,(s,n) \equiv (1-\beta)\, h\,(s) + \beta \sum_{s'|s,n} \pi(s'|s,n) \tilde{w}'(s',n'). \tag{18}$$

From the definition of a memoryless allocation, we get $\forall s\ \exists! h(s)$. This implies that $\tilde{w}(s,n) = \tilde{w}'(s',n') = \tilde{w}(s',n')$ for $s = s'$ and $n = n'$ and the invariant probability measure Φ is given by the exogenous joint distribution of income and public signals. Equivalent to Definition 4 but using the recursive formulation, an optimal memoryless arrangement is defined as follows.

Definition 6 *An ML arrangement is a stationary utility allocation $\{h(s)\}$ as a solution to (15)-(17) with Φ is given by the exogenous distribution over income and public signals.*

4.3 Existence of risk sharing arrangements

In the standard case without signals, whenever there is risk sharing constrained feasible in memoryless arrangements, it is also constrained feasible in history-dependent arrangements (Krueger and Perri, 2011). In the following proposition, we generalize the standard case by accounting for public and non-contractible private information.

Proposition 3 (Existence of history dependent arrangement) *A history dependent arrangement with risk sharing exists if*

$$\beta \geq \left[\frac{u'(y_h)}{u'(y_l)} \frac{1}{2 - z_1 - z_2} \right].$$

The proof is provided in Appendix A.4.

The main messages from the proposition are first that more precise private information increases the degree of patience needed to make history-dependent arrangements constrained feasible. Second, better public information decreases the degree of patience needed to support risk sharing in history-dependent arrangements when private signals are informative. As

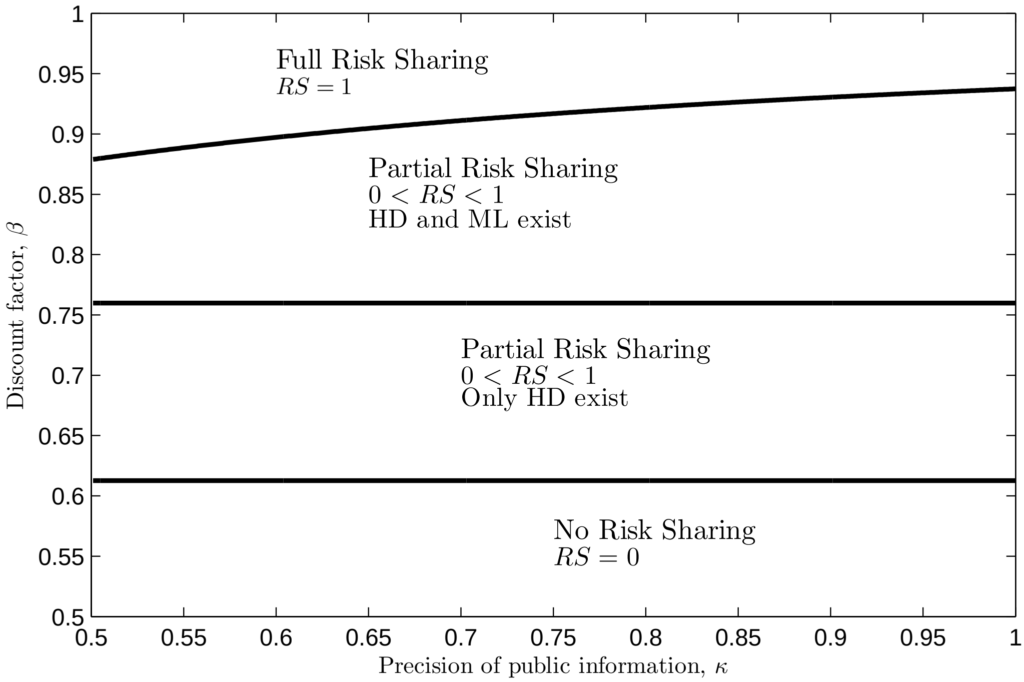

Figure 5: Risk-sharing regimes and allocations as a function of the discount factor β and precision of public information κ with uninformative private information.

summarized in the following corollary, when private signals are not informative, the condition in Proposition 3 with public information resembles the standard condition in the absence of signals.

Corollary 1 *Consider uninformative private signals ($\nu = 0$). A history dependent arrangement with risk sharing exists if $\beta \geq u'(y_h)/u'(y_l)$.*

For memoryless arrangements with $\nu = 0.5$, the corresponding condition for risk sharing in optimal memoryless allocation follows from Proposition 2 and requires $\beta u'(y_l) > (2 - \beta)u'(y_h)$.

Comparing the two conditions, history-dependent arrangements with risk sharing require a lower degree of patience to be constrained feasible. This is illustrated in Figure 5 in $\kappa - \beta$ space. When the discount factor β is sufficiently high, the gains of future insurance are high and risk sharing is perfect ($RS = 1$) which is pictured in the upper region of the figure. However, as public signals becomes more informative, constrained feasibility of perfect risk sharing requires a higher discount factor to be consistent with voluntary participation. As the discount factor decreases, there is first a region that supports risk sharing in memoryless and history-dependent arrangements. Further decreasing the discount factor leads to a region in which partial risk sharing can be only supported in history-dependent arrangements and eventually autarky is the only constrained feasible allocation.

When private signals are informative the conditions summarized in Propositions 2 and 3, imply that memoryless arrangements can provide risk sharing for some precision of private information when history-dependent arrangements cannot. This result is illustrated in Figure 6 for uninformative public signals. For low levels of private information, there is a region (in the left bottom of the figure) in which risk sharing can be supported in history-dependent arrangements

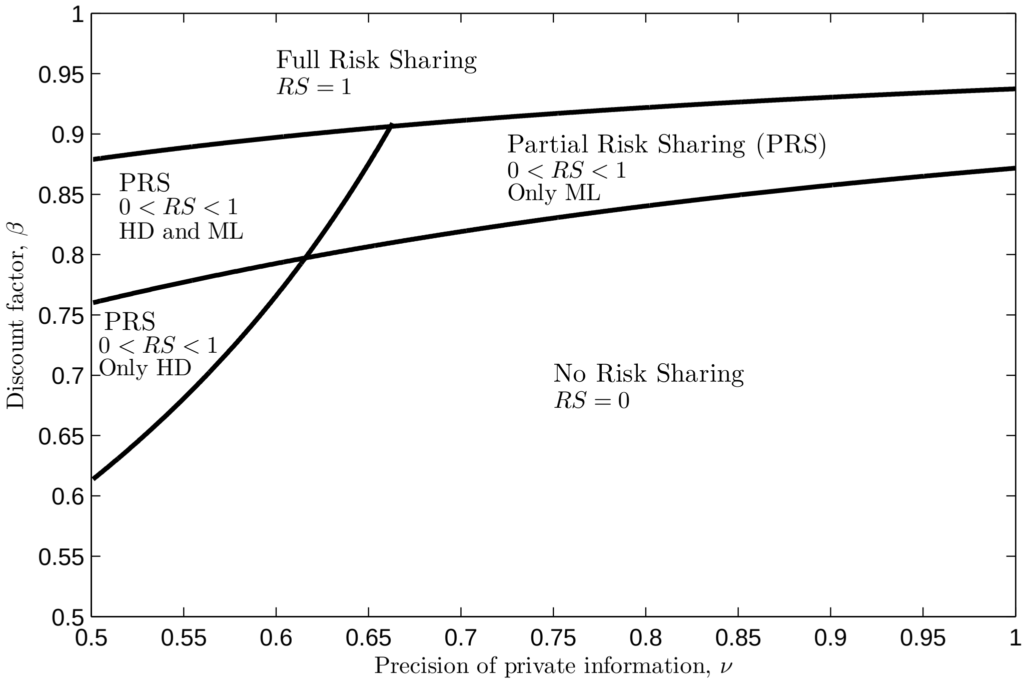

Figure 6: Risk-sharing regimes and allocations as a function of the discount factor β and precision of private information ν with uninformative public information.

but not in memoryless arrangements, and the optimal arrangement is history dependent. When precision of private information increases a region with both types of arrangements emerges. While – in principle – optimal allocations here could be either memoryless or history dependent, we have found in all our numerical experiments that history-dependent arrangements yield higher welfare. The joint region is followed by a region in which only memoryless arrangements can provide risk sharing. Thus, when private information is very precise only memoryless arrangements can provide insurance and constitute then the optimal arrangement.

5 Quantitative evaluation

In this section, we quantitatively analyze how better public information affects welfare of developing and industrial countries in international risk sharing. In the preceding sections, we posit that all countries face signals of the same precision. In this section, motivated by the data quality differentials between developing and industrial countries reported in Table 1, we extend the model to allow for heterogeneous quality of information. As our key result in this section, we find that better public information in developing countries has not only positive welfare effects for these countries but also positive spillover effects for risk sharing in industrial countries.

5.1 Information and risk-sharing environments

Agents in our model correspond to different representative households across countries. The world economy comprises two groups of countries of equal measure, a continuum of developing countries indexed by D with measure 1/2 and a continuum of industrial countries indexed by I. While we assume that the groups of countries face the same income process, the two type of countries differ with respect to the importance of private and public information. Thus, developing and industrial countries are heterogeneous in information. We consider two different risk-sharing environments.

First and similar to the environment outlined in Section 2, we consider only trade that occurs *Within* each group of countries, i.e., between agents with the same precision of private and public signals. To distinguish developing and industrial countries, each group of countries is characterized by a different precision of public and private signals.

The *Within-and-between* or *World risk sharing* environment considers risk sharing between agents with the same but also with agents with different precision of signals. The solution concept and equilibrium features extend naturally from the within-group environment. For given stationary distributions $\Phi(w_{0,D}, s)_D, \Phi(w_{0,I}, s)_I$, an optimal stationary allocation delivers the promises $w_{0,D}, w_{0,I}$, satisfies participation constraints for developing and industrial countries and is resource feasible:

$$\frac{1}{2}\sum_{\theta^t}\int \left[C(h_t(w_{0,D}, s^t)) - y_t\right] \pi^D(s^t|s_0)\,\mathrm{d}\,\Phi_{0,D} \\ + \frac{1}{2}\sum_{\theta^t}\int \left[(C(h_t(w_{0,I}, s^t)) - y_t\right] \pi^I(s^t|s_0)\,\mathrm{d}\,\Phi_{0,I} \leq 0. \tag{19}$$

Further, there is no other stationary constrained feasible allocation that requires less resources or delivers higher world ex-ante utility. Optimal allocations are computed by jointly solving the recursive problems (10)-(12) and (15)-(16) for developing and industrial countries. The recursive problems induce invariant probability measures $\Phi(w_D, s)_D$ and $\Phi(w_I, s)_I$ conditional on the inter-temporal price R_W.

5.2 Calibration

We start with standard values for the specification of preferences. The instantaneous utility function features log utility and we a choose a discount factor of $\beta = 0.85$. We evaluate the welfare effects of information using annual cross-country data. In 2004, income generated by developing countries added up to 25 percent of world income. Using this fraction, we set the unconditional standard deviation of the idiosyncratic logged income risk to $\sigma_y = 0.25$ as the weighted average of the income volatilities of the two groups of countries estimated in Bai and Zhang (2012) using data from 1987–2004 . The mean of logged income is normalized to zero and we approximate the income distribution as an i.i.d. process with two states such that the state θ_t comprises eight states.[8]

Given the vector of all model-relevant parameters Ψ, we will measure the degree of risk sharing, $RS\,(\Psi)$ by the ratio of the unconditional standard deviations of logged consumption σ_c and logged income σ_y:

$$RS\,(\Psi) = 1 - \frac{\sigma_c}{\sigma_y}.$$

[8] The cardinality of the set of all θ_t is N^3 which makes computing optimal allocations a challenging task. A description of the numerical solution algorithm can be found in Appendix A.9 .

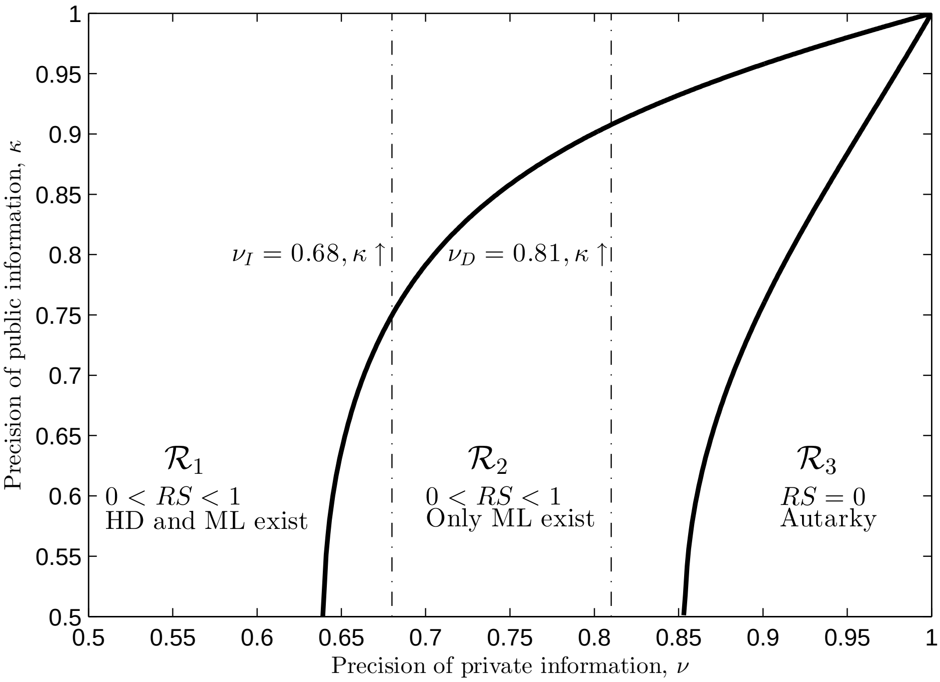

Figure 7: Risk-sharing regimes and allocations as a function of the precision of public information κ and private information ν. Within-group risk sharing.

Lower $RS(\Psi)$ means that only a small portion of idiosyncratic income risk is insured because then σ_c is relatively close to σ_y. On the opposite, values of $RS(\Psi)$ close to unity describe an economy in which risk sharing comes close to the first best.

To capture the stylized fact that developing countries rank low in terms of data quality and access to information, we assume that private signals are informative $\nu_D > 0.5$ while public signals are uninformative for these countries ($\kappa_D = 0.5$). Industrial countries are characterized by the same degree of private and public signal precision ($\kappa_I = \nu_I$).

Further, industrial countries are more successful in insuring against idiosyncratic income fluctuations than developing countries. Bai and Zhang (2012) and Kose et al. (2007, 2009) find risk-sharing coefficients RS of 0.1 for developing and 0.4 for industrial countries.[9] These degrees of risk sharing are our calibration targets. Risk sharing in the model decreases monotonically in the precision of private information (see also Proposition 4 in Appendix A.5). This allows us to uniquely identify the unobserved precision of private information by varying it until equilibrium consumption in our model mimics the degrees of risk sharing for the two country groups found in the data.

Throughout the following analysis, we assess the welfare effects of better public information by comparing welfare measured in annual certainty equivalent consumption with $\kappa_i = 0.5$ to welfare when public information is as precise as the calibrated precision of private information, $\kappa_i = \nu_i$. The percentage welfare change of country group k induced by the change of public signals precision of country group l is denoted $\Delta\kappa_{k,l}$.

[9] Backus, Kehoe, and Kydland (1992) find a similar degree of risk sharing for industrial countries.

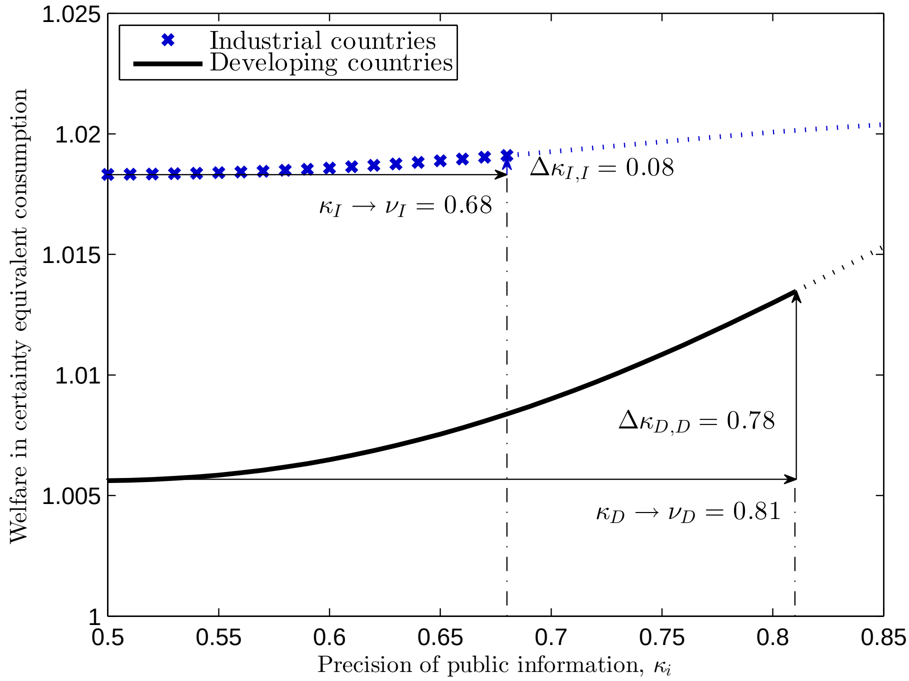

Figure 8: Social welfare as a function of public information precision, κ. Within-group risk sharing

5.3 Within-country group risk sharing

To match the calibration targets, we find that the private information friction plays an important role in the developing countries with $\nu_D = 0.81$ and results in a low degree of risk sharing in these countries. Industrial countries on the other hand, are less prone to the information friction as captured by a lower degree of private information precision ($\nu_I = \kappa_I = 0.68$).

Figure 7 pictures the conditions for allocations different from autarky from Propositions 2 and 3 in a $\nu - \kappa$ space. For the calibrated values of private signal precision, optimal allocations for developing and industrial countries are memoryless but with increases in public signal precision change to history dependent. The change occurs for industrial countries for precisions $\kappa_I \geq 0.75$, for developing countries for precisions $\kappa \geq 0.91$.

Quantitatively, we find that risk sharing and social welfare are improving with better public information for both type of countries. Thus, private information is sufficiently precise in the sense of Theorem 1. The insurance improvement for the developing countries is sizeable, risk sharing increases from 0.10 to 0.25. If we were to decrease the quality of public information for the industrial countries to $\kappa_I = 0.5$, we would find a drop in risk sharing ratio of two percent. Figure 8 depicts the welfare effects of better public information. For developing countries, the positive welfare effect of information amounts permanently to 0.78 percent in annual certainty equivalent consumption when moving from uninformative signals to public signals with the same precision as private signals in the developing countries. The welfare gains of industrial countries are smaller and amount to 0.08 percent.[10]

[10] In our calibration, we assume $\kappa_I = \nu_I$, thus, an alternative way of thinking about this number is to take it as the magnitude of welfare losses that the industrial countries would incur if the data quality on their idiosyncratic income risk was to deteriorate.

Table 2: Welfare effects of public information with world risk sharing

	Within Ψ		*World risk sharing* Ψ	
	Developing	Industrial	Developing	Industrial
Risk sharing, $RS(\Psi)$	0.21	0.33	0.40	0.10
Signal precision, ν_i	0.81	0.68	0.98	0.63
Welfare effect, $\Delta\kappa_{i,D}$	0.34	0.17	0.88	0.10

Notes: World risk sharing. $\Delta\kappa_{i,D}$ is percentage change in welfare of country type $i \in D, I$ for an increase from $\kappa_D = 0.5$ to $\kappa_D = \nu_D$.

5.4 World risk sharing

In reality, industrial and developing countries engage in risk sharing contracts with each other. This case is analyzed in the *within-and-between* environment or *World risk sharing.* In this section, we deliver our key quantitative result that better public information in developing countries leads not only to better risk sharing in developing but also gives rise to positive spillover effects for risk sharing in industrial countries.

Within-versus world risk sharing To gain intuition on the difference between within-and world risk sharing, we start with the calibrated values of signal precision from the within-group risk sharing model, i.e., $\nu_D = 0.81$ and $\nu_I = 0.68$. As displayed in the first column of Table 2, after opening up trade between the country groups risk sharing increases roughly by half for the developing countries and decreases by seven percent for the industrial countries compared to the within-group risk sharing in the previous section.

The changes are a consequence of the possibility to smooth consumption not only across income states within each country group, but also to smooth it across income states *across* country groups. To compensate for the low degree of risk sharing in developing countries, the optimal allocation features additional transfers from industrial to developing countries. As a consequence, matching the target risk sharing ratios, requires now more precise private signals in developing countries with $\nu_D = 0.98$ and less precise signals in industrial countries with $\nu_I = 0.63$ (see the second column of Table 2).

Quantitatively, we find that better public information in developing countries affects risk sharing and welfare in developing and industrial countries. As displayed in the second column of Table 2, risk sharing in developing countries increases almost by a factor of three from the calibrated value of 0.10 to 0.29. The change in insurance for the industrial countries is less pronounced, from 0.40 to 0.42. Both countries – developing and industrial countries – can realize welfare gains from better public information in developing countries of 0.88 and 0.10 percent, respectively. Thus, better information in developing countries has positive spillover effects for industrial countries.

The logic behind the positive spillover effect of better public information in developing countries can be explained as follows. As a first direct effect, better public information results in better risk sharing within the groups of developing countries because private information is sufficiently precise in these countries in the sense of Theorem 1. As a second and indirect

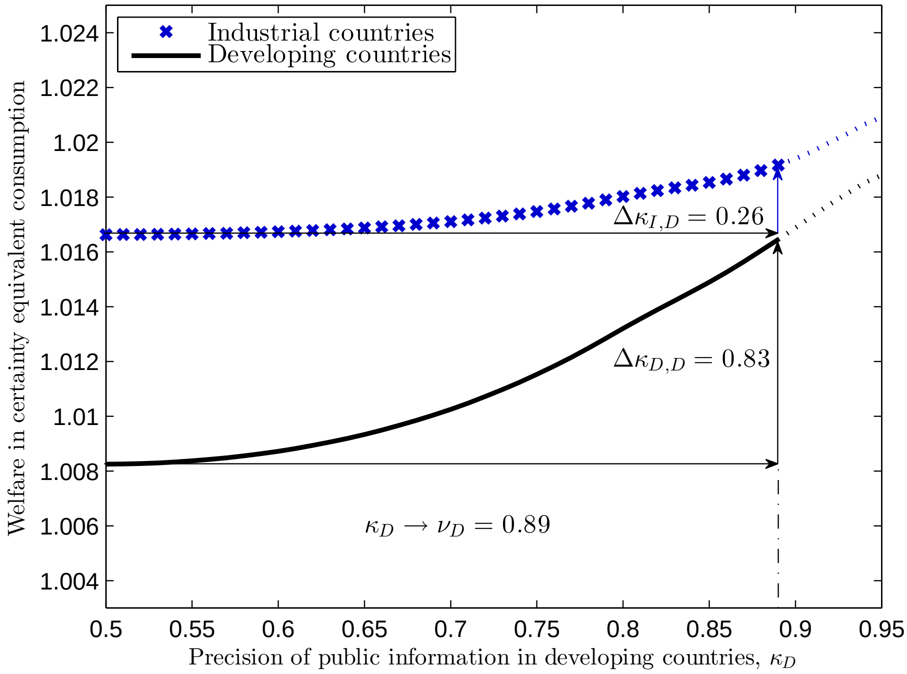

Figure 9: Social welfare as a function of public information precision in the developing countries, κ_D. World risk sharing with savings in autarky.

effect, the better risk sharing in developing countries allows for lower transfers from industrial countries in the optimal allocation. The additional resources can then be used to improve risk sharing in industrial countries as well.

Savings in the outside option Up to here, we assumed that in autarky agents can only consume their endowments but cannot save. Bai and Zhang (2012) argue in favor of introducing consumption smoothing in autarky by self insurance as a realistic feature available to countries in the state of default. To integrate this feature, we introduce the possibility of savings in autarky. While households loose all their consumption claims, they can store a quantity of the good with a return R_A such that $R_W \geq R_A > 0$. Alternatively, it can be thought of as saving in a non-state contingent bond with borrowing excluded. Thus, the value of the outside option is a solution to an optimal savings problem that can be written in recursive form as follows

$$v_{R_A}(\theta, a) = \max_{0 \leq a' \leq y + aR_A} \left[(1-\beta)u(aR_A + y - a') + \beta \sum_{\theta'} \pi(\theta'|\theta) v'_{R_A}(\theta', a') \right],$$

where a denote savings. The outside option is off-equilibrium and households loose all their claims on current and future consumption. Hence, the value of the outside option depends on R_A and is given by

$$U^{Aut}(\theta_t) = v_{R_A}(\theta_t, 0).$$

We set $R_A = \beta + 0.05$, and find that savings in the outside option further restrict the possibilities for risk sharing by increasing the outside option values of high-income agents. Correspondingly, the calibrated private signal precisions are with $\nu_I = 0.89$ and $\nu_I = \kappa_I = 0.58$

lower than in case agents are only allowed to consume their income but not to save (see the second column of Table 2).

Compared to the case without savings, we compute similar improvements in risk sharing in developing countries from better public data quality in these countries; risk sharing increases from 0.10 to 0.29. In industrial countries, risk sharing improves from 0.40 to 0.46 indicating a stronger spillover effect than in the case without savings. The response of welfare of the two groups of countries as a function of κ_D displayed in Figure 9 confirms the risk sharing findings. Welfare improvements amount to 0.83 percent for developing and 0.26 percent for industrial countries, respectively.

6 Conclusions

We have developed a novel argument why more precise public information can be beneficial for risk sharing. We have shown that if private information is sufficiently precise, the positive tracking effect dominates the negative conventional Hirshleifer (1971) effect of better public signals, and as a result, risk sharing and social welfare increase. Quantitatively, we have found that better public information in developing countries not only improves risk sharing in these countries but also yields positive spill-over effects for industrial countries.

The positive effect of public information is not only relevant for international risk sharing. The effect also applies to optimal risk sharing in other environments where private information is an important factor. One example includes commercial banks facing idiosyncratic liquidity shocks against which they seek insurance. Banks have private information about their future liquidity position, and there is also a public source of information on liquidity risk of individual banks provided by the Federal Reserve System (Fed). Recently, the Fed moved towards more transparency in providing more timely information on banks' liquidity risk than ever before in its history. While the transparency move is puzzling and detrimental according to the standard view, our model provides a rationale for the case of transparency.

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A Appendix

A.1 Proof of Proposition 1

Let $\bar{V}_{rs} = u(\bar{y})$ be social welfare under perfect risk sharing. First, perfect risk sharing provides the highest ex-ante utility among the consumption-feasible allocations. The existence of $\bar{\beta}(\kappa, \nu)$ follows from monotonicity of participation constraints in β and $\bar{V}_{rs} > V_{out}$. A higher β increases the future value of perfect risk sharing relative to the allocation in the equilibrium without transfers, leaving the current incentives to deviate unaffected. Therefore, if the participation constraints are not binding for $\bar{\beta}(\kappa, \nu)$, they are not binding for any $\beta \geq \bar{\beta}(\kappa, \nu)$. The cutoff point is characterized by the tightest participation constraint (7), i.e., by the participation constraint with the highest value of the outside option. Solving this constraint yields a unique solution for $\bar{\beta}(\kappa, \nu)$ in $(0, 1)$ because $u(\bar{y}) < u(y_h)$. Second, the tightest constraint at the first best allocation is

$$u(\bar{y}) \geq (1-\beta)u(y_h) + \beta(1-\beta)z_1 u(y_h) + \beta(1-\beta)(1-z_1)u(y_l) + \beta^2 V_{out},$$

$z_1 = \kappa\nu/[\kappa\nu + (1-\kappa)(1-\nu)]$. Differentiating the constraint fulfilled with equality with respect to κ using the implicit function theorem gives

$$\frac{\partial \bar{\beta}(\kappa, \nu)}{\partial \kappa} = \frac{\beta(1-\beta)[u(y_h) - u(y_l)]\frac{\partial z_1}{\partial \kappa}}{D} > 0,$$

results in a positive sign for $y_h > y_l$,

$$\frac{\partial z_1}{\partial \kappa} = \frac{\nu(1-\nu)}{[\kappa\nu + (1-\kappa)(1-\nu)]^2} > 0$$

and

$$D = (1-\beta)[u(y_h) - z_1 u(y_h) - (1-z_1)u(y_l)] + \beta[z_1 u(y_h) + (1-z_1)u(y_l) - u(y_l)] > 0.$$

For $\nu \in [0.5, 1)$, the cutoff point increases strictly in precision of the public signal. Similarly, taking the derivative with respect to the ν results in a positive sign as well (a strictly positive sign for $\kappa \in [0.5, 1)$).

A.2 Proof of Proposition 2

The optimal memoryless arrangement can be analyzed as a fixed-point problem expressed in terms of the period value of the arrangement.

The fixed-point problem is constructed as follows. Let $W = \mathrm{E}\,U(c)$ be the unconditional expected value of an arrangement before any signal has realized. We restrict attention to $W \in [V_{out}, \bar{V}_{rs})$ because per assumption participation constraints for high-income households are binding. The binding participation constraints are given by the following

$$u(c_h^h) + \frac{\beta\left\{\kappa\nu[u(c_h^h)+u(c_l^h)]/2 + (1-\kappa)(1-\nu)u(2\bar{y}-(c_h^h+c_l^h)/2)\right\}}{\kappa\nu+(1-\kappa)(1-\nu)}$$

$$= u(y_h) + \frac{\beta\left[\kappa\nu u(y_h)+(1-\kappa)(1-\nu)u(y_l)\right]}{\kappa\nu+(1-\kappa)(1-\nu)} + \frac{\beta^2}{1-\beta}(V_{out}-W), \tag{20}$$

$$u(c_l^h) + \frac{\beta\left\{(1-\kappa)\nu[u(c_h^h)+u(c_l^h)]/2 + \kappa(1-\nu)u(2\bar{y}-(c_h^h+c_l^h)/2)\right\}}{(1-\kappa)\nu+\kappa(1-\nu)}$$

$$= u(y_h) + \frac{\beta\left[(1-\kappa)\nu u(y_h)+\kappa(1-\nu)u(y_l)\right]}{(1-\kappa)\nu+\kappa(1-\nu)} + \frac{\beta^2}{1-\beta}(V_{out}-W), \tag{21}$$

and resource feasibility is used. The objective function of the problem to compute the optimal memoryless arrangement is given by the following expression

$$V_{rs}(W) \equiv \frac{1}{4}\left[u(c_h^h(W)) + u(c_l^h(W)) + 2u(2\bar{y}-(c_h^h(W)+c_l^h(W))/2)\right].$$

The optimal memoryless arrangement should necessary solve the fixed-point problem $W = V_{rs}(W)$. We will show that $V_{rs}(W)$ is strictly increasing. $V(W)$ is also strictly concave, therefore there exist at most two solutions to the fixed-point problem.

From the participation constraints (20) and (21), the derivative of $V(W)$ is given by

$$V_{rs}'(W) = \frac{1}{4}\left[(u'(c_h^h)-u'(c_l))\frac{\partial c_h^h}{\partial W} + (u'(c_l^h)-u'(c_l))\frac{\partial c_l^h}{\partial W}\right]$$

which is strictly increasing in W because perfect risk sharing is not constrained feasible which implies that $\partial c_h^h/\partial W$ and $\partial c_l^h/\partial W$ are negative and $c_h^h, c_l^h \neq \bar{y}$.

By construction, one solution to the fixed-point problem is V_{out}. The concavity of $V_{rs}(W)$ implies that the derivative of $V_{rs}(W)$ at V_{out} is higher than at any partial risk-sharing allocation. Therefore, autarky is the optimal memoryless arrangement if the derivative of $V_{rs}'(w)$ at V_{out} must be smaller than or equal to 1 which implies

$$V_{rs}'(W) = \frac{1}{4}\left[(u'(y_h)-u'(y_l)\right]\left(\frac{\partial c_h^h}{\partial W} + \frac{\partial c_l^h}{\partial W}\right)\Bigg|_{\{c_i^j\}=\{y_j\}} \leq 1$$

The two derivatives are

$$\frac{\partial c_h^h}{\partial W} = -\frac{\begin{vmatrix} \beta^2 & P_{c_l^h} \\ \beta^2 & Q_{c_l^h} \end{vmatrix}}{\begin{vmatrix} P_{c_h^h} & P_{c_l^h} \\ Q_{c_h^h} & Q_{c_l^h} \end{vmatrix}}, \quad \frac{\partial c_l^h}{\partial W} = -\frac{\begin{vmatrix} P_{c_h^h} & \beta^2 \\ Q_{c_h^h} & \beta^2 \end{vmatrix}}{\begin{vmatrix} P_{c_h^h} & P_{c_l^h} \\ Q_{c_h^h} & Q_{c_l^h} \end{vmatrix}},$$

with the auxiliary derivatives P, Q as the partial derivatives of the binding participation constraints (20) and (21) evaluated at the autarky allocation given by

$$
\begin{aligned}
P_{c_h^h} &= (1-\beta)[u'(y_h) + \frac{\beta}{2} z_1 u'(y_h) - \frac{\beta}{2}(1-z_1)u'(y_l)] \\
P_{c_l^h} &= P_{c_h^h} - (1-\beta)u'(y_h) \\
Q_{c_l^h} &= (1-\beta)[u'(y_h) + \frac{\beta}{2} z_2 u'(y_h) - \frac{\beta}{2}(1-z_2)u'(y_l)] \\
Q_{c_h^h} &= Q_{c_l^h} - (1-\beta)u'(y_h).
\end{aligned}
$$

Using these expression, the sum of the partial derivatives with respect to W evaluated at $\{c_i^j\} = \{y_j\}$ is given by

$$
\begin{aligned}
\left(\frac{\partial c_h^h}{\partial W} + \frac{\partial c_l^h}{\partial W}\right) &= -\frac{2\beta^2(1-\beta)u'(y_h)}{(1-\beta)u'(y_h)[P_{c_h^h} + Q_{c_l^h} - (1-\beta)u'(y_h)]} \\
&= \frac{-4\beta}{(1-\beta)\left[u'(y_h)\left(z_1 + \frac{2}{\beta} + z_2\right) - (2 - z_1 - z_2)u'(y_l)\right]}.
\end{aligned}
$$

Using this expression in $V_{rs}'(W)$ and collecting terms eventually results in

$$
u'(y_h) + \beta\frac{z_1+z_2}{2}[u'(y_h) + u'(y_l)] - \beta u'(y_l) - \frac{\beta^2}{1-\beta}\frac{1}{2}[u'(y_l) - u'(y_h)] \geq 0.
$$

Under this condition, the optimal memoryless arrangement is the outside option. If the condition is strictly negative then there exists an alternative constrained feasible allocation that is preferable to the outside option. This result is used in the main theorem.

This condition can be related to the corresponding condition in case of uninformative signals ($\kappa = \nu = 0.5$). In this case, $z_1 = z_2 = 0.5$, and the condition reads

$$
\begin{aligned}
& u'(y_h)\left[(1-\beta) + \beta\frac{1-\beta}{2} + \frac{\beta^2}{2}\right] - u'(y_l)\beta\left[1 - \beta - \frac{1-\beta}{2} + \frac{\beta}{2}\right] \geq 0 \\
& \Leftrightarrow u'(y_h)\left(1 - \frac{\beta}{2}\right) - u'(y_l)\frac{\beta}{2} \geq 0 \\
& u'(y_h) \geq \frac{\beta}{2-\beta}u'(y_l),
\end{aligned}
$$

which corresponds to the condition derived in Krueger and Perri (2011) and in Lepetyuk and Stoltenberg (2013). From the other end, suppose that autarky is the optimal memoryless arrangement and that participation constraints (7) and (8) are binding. Then, the value of this arrangement $W_{out} = V_{out}$ must be a solution to the fixed-point problem. This requires that the slope of $V_{rs}(W)$ at $\{c_i^j\} = \{y_j\}$ must be necessarily smaller than or equal to unity. Otherwise, due to the concavity of $V_{rs}(W)$, there exists another solution to the fixed-point problem with an allocation that Pareto dominates the autarky allocation.

A.3 Proof of Theorem 1

Take the derivative of social welfare with respect to κ

$$\frac{\partial V_{rs}}{\partial \kappa} = \frac{1}{4}\left[u'(c_h^h)\frac{\partial c_h^h}{\partial \kappa} + u'(c_l^h)\frac{\partial c_l^h}{\partial \kappa} - u'(c^l)\left(\frac{\partial c_h^h}{\partial \kappa} + \frac{\partial c_l^h}{\partial \kappa}\right)\right] \equiv I(\nu,\kappa), \quad (22)$$

where we used that in the optimal memoryless arrangement consumption of low-income agents is the same for a good and a bad public signal, $c_h^l = c_l^l = c^l$ and that u is Inada. The derivatives of consumption with respect to the signal precision follow from the implicit function theorem. For the latter, re-write the two binding participation constraints

$$\begin{aligned} F(c_h^h, c_l^h) \equiv{} & (1-\beta)u(c_h^h) + \beta(1-\beta)z_1 V_{rs}^h + \beta(1-\beta)(1-z_1)V_{rs}^l + \beta^2 V_{rs} \\ & - (1-\beta)u(y_h) - \beta(1-\beta)\left(z_1 u(y_h) + (1-z_1)u(y_l)\right) - \beta^2 V_{out} = 0, \end{aligned}$$

$$\begin{aligned} G(c_h^h, c_l^h) \equiv{} & (1-\beta)u(c_l^h) + \beta(1-\beta)z_2 V_{rs}^h + \beta(1-\beta)(1-z_2)V_{rs}^l + \beta^2 V_{rs} \\ & - u(y_h) - \beta(1-\beta)\left(z_2 u(y_h) + (1-z_2)u(y_l)\right) - \beta^2 V_{out} = 0, \end{aligned}$$

with $z_1 \geq z_2$ defined as before as

$$z_1 = \frac{\kappa\nu}{\kappa\nu + (1-\kappa)(1-\nu)}$$

and

$$z_2 = \frac{(1-\kappa)\nu}{(1-\kappa)\nu + \kappa(1-\nu)}$$

In the following, it is useful to employ the sum of derivatives of high-income agents' consumption with respect to κ which is given by

$$\frac{\partial c_h^h}{\partial \kappa} + \frac{\partial c_l^h}{\partial \kappa} = \frac{x_2 F_{c_l^h} + x_1 G_{c_l^h}}{F_{c_h^h} G_{c_l^h} - F_{c_l^h} G_{c_h^h}} - \frac{x_2 F_{c_h^h} + x_1 G_{c_h^h}}{F_{c_h^h} G_{c_l^h} - F_{c_l^h} G_{c_h^h}}$$

where

$$x_1 = -\frac{\partial F}{\partial \kappa} = \beta(1-\beta)(u(y_h) - V_{rs}^h - u(y_l) + V_{rs}^l)\frac{\nu(1-\nu)}{(\kappa\nu + (1-\kappa)(1-\nu))^2} \geq 0$$

$$x_2 = \frac{\partial G}{\partial \kappa} = \beta(1-\beta)(u(y_h) - V_{rs}^h - u(y_l) + V_{rs}^l)\frac{\nu(1-\nu)}{((1-\kappa)\nu + \kappa(1-\nu))^2} \geq 0.$$

The partial derivatives are

$$F_{c_h^h} = (1-\beta)\left[u'(c_h^h) + \beta\frac{z_1}{2}u'(c_h^h) - \beta\frac{1-z_1}{2}u'(c^l)\right] + \frac{\beta^2}{4}\left[u'(c_h^h) - u'(c^l)\right]$$
$$F_{c_l^h} = (1-\beta)\left[\beta\frac{z_1}{2}u'(c_l^h) - \beta\frac{1-z_1}{2}u'(c^l)\right] + \frac{\beta^2}{4}\left[u'(c_l^h) - u'(c^l)\right]$$
$$G_{c_l^h} = (1-\beta)\left[u'(c_l^h) + \beta\frac{z_2}{2}u'(c_l^h) - \beta\frac{1-z_2}{2}u'(c^l)\right] + \frac{\beta^2}{4}\left[u'(c_l^h) - u'(c^l)\right]$$
$$G_{c_h^h} = (1-\beta)\left[\beta\frac{z_2}{2}u'(c_h^h) - \beta\frac{1-z_2}{2}u'(c^l)\right] + \frac{\beta^2}{4}\left[u'(c_h^h) - u'(c^l)\right].$$

Autarky is not the only constrained feasible allocation. This implies that at the optimal memoryless allocation the following holds

$$F_{c_h^h}G_{c_l^h} - F_{c_l^h}G_{c_h^h} > 0. \tag{23}$$

Together with the properties of the utility function $u(\cdot)$, this establishes the unique existence of the continuous differentiable functions $c_h^h(\kappa), c_l^h(\kappa)$, while at the autarky allocation, (23) is negative (see Proposition 2), i.e.,

$$F_{c_h^h}G_{c_l^h} - F_{c_l^h}G_{c_h^h}\Big|_{\{c_i^j\}=\{y_j\}} = u'(y_h) + \beta\frac{z_1+z_2}{2}[u'(y_h) + u'(y_l)]$$
$$- \beta u'(y_l) - \frac{\beta^2}{1-\beta}\frac{1}{2}[u'(y_l) - u'(y_h)] < 0.$$

For $\nu = 0.5$, $x_1 = x_2 = x > 0$, $c_h^h \geq c_l^h$ and $\partial c_h^h/\partial\kappa \geq 0$[11], the derivative with respect to κ satisfies

$$I(0.5,\kappa) \leq \frac{1}{4}[u'(c_l^h) - u'(c^l)]\left(\frac{\partial c_h^h}{\partial\kappa} + \frac{\partial c_l^h}{\partial\kappa}\right).$$

which is negative whenever the sum of derivatives with respect to κ on the right-hand side is positive. Evaluated at $\nu = 0.5$, the sum of derivatives reads (with a strict inequality for $\kappa > 0.5$)

$$\frac{\partial c_h^h}{\partial\kappa} + \frac{\partial c_l^h}{\partial\kappa} = x\frac{F_{c_l^h} + G_{c_l^h} - F_{c_h^h} - G_{c_h^h}}{F_{c_h^h}G_{c_l^h} - F_{c_l^h}G_{c_h^h}}$$
$$= x\frac{(1-\beta)(1+\beta\frac{z_1+z_2}{2} + \frac{\beta^2}{(1-\beta)}\frac{1}{2})(u'(c_l^h) - u'(c_h^h))}{F_{c_h^h}G_{c_l^h} - F_{c_l^h}G_{c_h^h}} \geq 0.$$

For $\nu \to 1$, we get $c_h^h \to c_l^h = c^h$, the derivative of social welfare with respect to κ is

$$I(1,\kappa) = \frac{1}{4}[u'(c^h) - u'(c^l)]\left(\frac{\partial c_h^h}{\partial\kappa} + \frac{\partial c_l^h}{\partial\kappa}\right),$$

[11] At the optimal memoryless arrangement with binding participation constraints, $F_{c_l^h} + G_{c_l^h} > 0$ follows from the first order conditions. For $\nu = 0.5$, $x_1 = x_2 \geq 0$ which implies the positive sign of the derivative when autarky is not the only constrained feasible memoryless arrangement.

and the sum of derivatives is negative

$$\frac{\partial c_h^h}{\partial \kappa} + \frac{\partial c_l^h}{\partial \kappa} \leq 0$$

$$\Leftrightarrow \frac{-\frac{x_2}{x_1}(F_{c_h^h} - F_{c_l^h}) + G_{c_l^h} - G_{c_h^h}}{F_{c_h^h}G_{c_l^h} - F_{c_l^h}G_{c_h^h}} = (1-\beta)\frac{\left(1 - \frac{\kappa^2}{(1-\kappa)^2}\right)u'(c^h)}{F_{c_h^h}G_{c_l^h} - F_{c_l^h}G_{c_h^h}} \leq 0,$$

where we used that for $\nu \to 1$, $F_{c_l^h} \to G_{c_h^h}$, $F_{c_h^h} \to G_{c_l^h}$ and $x_1/x_2 = \kappa^2/(1-\kappa)^2$; for $\kappa > 0.5$, the strict inequality applies. Existence of $\bar{\nu}$ follows from continuity of $I(\nu, \kappa)$ for $\nu \in [0.5, 1)$.

A.4 Proof of Proposition 3

First, if $\beta > u'(y_h)/u'(y_l)$, autarky is not a solution to the recursive problem because (13) is not satisfied for autarky. With informative private information, this is not sufficient. We construct an alternative distribution $\hat{\Phi}$ that dominates the stationary distribution $\Phi^{Aut}(\{U^{Aut}(y), y\}) = q(y)$ and show that the distribution is resource-feasible if the condition of the proposition is satisfied. Let $\hat{\Phi}$ be the following distribution over utility promises, income and public signals with a one-period history

$$\begin{aligned}
&\hat{\Phi}(\{U^{Aut}(y_h, k_h, n_h), y_h\}) = \pi_h\pi(k_h), \quad \hat{\Phi}(\{U^{Aut}(y_h, k_l, n_h), y_h\}) = \pi_h\pi(k_l), \\
&\hat{\Phi}(\{U^{Aut}(y_l, k_h, n_h), y_l\}) = (1-\pi_h)(1-\pi_h)\pi(k_h), \\
&\hat{\Phi}(\{U^{Aut}(y_l, k_l, n_h), y_l\}) = (1-\pi_h)(1-\pi_h)\pi(k_l), \\
&\hat{\Phi}(\{\tilde{\omega}_{k_h}^{k_h}, y_l\}) = (1-\pi_h)\pi_h\pi(k_h)\pi(k_h), \quad \hat{\Phi}(\{\tilde{\omega}_{k_l}^{k_h}, y_l\}) = (1-\pi_h)\pi_h\pi(k_h)\pi(k_l), \\
&\hat{\Phi}(\{\tilde{\omega}_{k_h}^{k_l}, y_l\}) = (1-\pi_h)\pi_h\pi(k_l)\pi(k_h), \quad \hat{\Phi}(\{\tilde{\omega}_{k_l}^{k_l}, y_l\}) = (1-\pi_h)\pi_h\pi(k_l)\pi(k_l),
\end{aligned}$$

where $\tilde{\omega}_{k_i}^{k_j} = U^{Aut}(y_l, k_i, n_h) + \varepsilon_{k_i}^{k_j}$ for small $\varepsilon_{k_i}^{k_j}$ and and the upper (lower) index indicates the previous (current) period signal. In the following, we consider two equally likely income state such that $\pi_h = \pi(k) = 0.5$. Let $\{\delta_{ij}^m\}$, $i, j, m \in \{l, h\}$ be transfers in terms of utility, with the first lower index as current income, the second index as the current public signal and the upper index as previous period public signal. The transfers are implicitly defined for the low-income agents by

$$\begin{aligned}
\tilde{\omega}_{k_h}^{k_h} &= (1-\beta)(u(y_l) + \delta_{lh}^h) + \beta\left[(1-z_1)U^{Aut}(y_l) + z_1U^{Aut}(y_h)\right] \\
\tilde{\omega}_{k_h}^{k_l} &= (1-\beta)(u(y_l) + \delta_{lh}^l) + \beta\left[(1-z_1)U^{Aut}(y_l) + z_1U^{Aut}(y_h)\right] \\
\tilde{\omega}_{k_l}^{k_h} &= (1-\beta)(u(y_l) + \delta_{ll}^h) + \beta\left[(1-z_2)U^{Aut}(y_l) + z_2U^{Aut}(y_h)\right] \\
\tilde{\omega}_{k_l}^{k_l} &= (1-\beta)(u(y_l) + \delta_{ll}^l) + \beta\left[(1-z_2)U^{Aut}(y_l) + z_2U^{Aut}(y_h)\right].
\end{aligned}$$

Utility of high-income are equal to their outside options which do not depend on previous period signals which leads to $\delta_{hh}^h = \delta_{hh}^l = \delta_{hh}$ and $\delta_{hl}^h = \delta_{hl}^l = \delta_{hl}$. Transfers of high-income agents are implicitly defined by

$$U^{Aut}(y_h, k_h, n_h) = (1-\beta)(u(y_h) - \delta_{hh}) + \beta \left[(1-z_1) \frac{\tilde{\omega}_{k_l}^{k_h} + \tilde{\omega}_{k_h}^{k_h}}{2} + z_1 U^{Aut}(y_h) \right]$$
$$U^{Aut}(y_h, k_l, n_h) = (1-\beta)(u(y_h) - \delta_{hl}) + \beta \left[(1-z_2) \frac{\tilde{\omega}_{k_l}^{k_l} + \tilde{\omega}_{k_h}^{k_l}}{2} + z_2 U^{Aut}(y_h) \right],$$

where $U^{Aut}(y_h) = 0.5[U^{Aut}(y_h, k_h, n_h) + U^{Aut}(y_h, k_l, n_h)]$, and $U^{Aut}(y_l)$ defined, accordingly. The marginal utility of low-income agents before the transfer is identical for each combination of past and current public signal and low-income agents receive the same transfer, $\delta_{li}^j = \epsilon/[4(1-\beta)]$ for all i, j. Transfers of high-income agents can be then directly derived from binding participation constraints and the scheme can be summarized by

$$\delta_l \equiv \sum_{i,j} \delta_{li}^j = \frac{\epsilon}{1-\beta} \qquad \delta_{hh} = \beta\epsilon \frac{(1-z_1)}{4(1-\beta)} \qquad \delta_{hl} = \beta\epsilon \frac{(1-z_2)}{4(1-\beta)}.$$

The distribution $\hat{\Phi}$ requires the following increase in resources

$$\begin{aligned} \Upsilon &= \pi_h(1-\pi_h)c'(u(y_l))\delta_l/4 - \frac{\pi_h}{2} c'(u(y_h))(\delta_{hh} + \delta_{hl}) \\ &= \frac{\pi_h(1-\pi_h)\epsilon}{4(1-\beta)} \left[\frac{1}{u'(y_l)} - \frac{\beta(2-z_1-z_2)}{2(1-\pi_h)u'(y_h)} \right]. \end{aligned}$$

Solving for β, if

$$\beta \geq \left[\frac{(1-\pi_h)u'(y_h)}{u'(y_l)} \frac{2}{2-z_1-z_2} \right] = \left[\frac{u'(y_h)}{u'(y_l)} \frac{1}{2-z_1-z_2} \right],$$

the constructed allocation $\hat{\Phi}$ uses less resources and dominates Φ^{Aut} by making the low-income agents strictly better off.

A.5 Negative value of private information

The welfare effect of increases in private signal precision is negative both for informative and uninformative public signals. An increase in private signal precision increases the value of the outside option of all high-income agents because only the high private signal is relevant for the optimal memoryless arrangement. As a consequence, consumption of high income agents increases, and risk sharing and welfare decrease. The following proposition states this result formally.

Proposition 4 (Negative Value of Private Information) *Let participation constraints of high-income agents with a good private signal (7) and (8) be binding. Assume that autarky is not the only constrained feasible memoryless arrangement and consider* $\kappa \in [0.5, 1)$*. Then social welfare is decreasing in the precision of the private signal for* $\nu \in [0.5, 1)$*.*

Proof. Take the derivative of social welfare with respect to ν

$$\frac{\partial V_{rs}}{\partial \nu} = \frac{1}{4}\left[u'(c_h^h)\frac{\partial c_h^h}{\partial \nu} + u'(c_l^h)\frac{\partial c_l^h}{\partial \nu} - u'(c^l)\left(\frac{\partial c_h^h}{\partial \nu} + \frac{\partial c_l^h}{\partial \nu}\right)\right] \leq 0, \tag{24}$$

where we used that in the optimal memoryless arrangement consumption of low-income agents is the same for a good and a bad public signal, $c_h^l = c_l^l = c^l$ and that u is Inada. The derivatives of consumption with respect to the signal precision follow from the implicit function theorem. Further, in optimum, the following conditions holds

$$\frac{u'(c_h^h) - u'(c^l)}{F_{c_h^h} + G_{c_h^h}} = \frac{u'(c_l^h) - u'(c^l)}{F_{c_l^h} + G_{c_l^h}},$$

with both denominators being strictly positive. Using this and dividing by $u'(c_l^h) - u'(c^l) < 0$ leads to a negative derivative of social welfare with respect to ν when

$$\begin{aligned}\left(F_{c_h^h} + G_{c_h^h}\right)\frac{\alpha_1 G_{c_l^h} - \alpha_2 F_{c_l^h}}{F_{c_h^h}G_{c_l^h} - F_{c_l^h}G_{c_h^h}} + \left(F_{c_l^h} + G_{c_l^h}\right)\frac{\alpha_2 F_{c_h^h} - \alpha_1 G_{c_h^h}}{F_{c_h^h}G_{c_l^h} - F_{c_l^h}G_{c_h^h}} &\geq 0\\ \Leftrightarrow (\alpha_1 + \alpha_2)\frac{F_{c_h^h}G_{c_l^h} - F_{c_l^h}G_{c_h^h}}{F_{c_h^h}G_{c_l^h} - F_{c_l^h}G_{c_h^h}} &\geq 0\\ \Leftrightarrow \alpha_1 + \alpha_2 &\geq 0,\end{aligned}$$

with the coefficients α_1, α_2 defined as

$$\alpha_1 = -\frac{\partial F}{\partial \nu} = \beta(1-\beta)(u(y_h) - V_{rs}^h - u(y_l) + V_{rs}^l)\frac{\kappa(1-\kappa)}{(\kappa\nu + (1-\kappa)(1-\nu))^2} \geq 0$$

$$\alpha_2 = -\frac{\partial G}{\partial \nu} = \beta(1-\beta)(u(y_h) - V_{rs}^h - u(y_l) + V_{rs}^l)\frac{\kappa(1-\kappa)}{((1-\kappa)\nu + \kappa(1-\nu))^2} \geq 0,$$

where strict inequalities apply and social welfare is strictly decreasing in ν for $\kappa \in [0.5, 1)$. ∎

A.6 Contractible private information

In this section, we study the social value of public information with revelation of private information. Essentially, the model combines limited commitment with efficient distribution under private information analyzed in Atkeson and Lucas (1992). For an infinite horizon, the limited commitment in general exhibits a compact state space for utility promises while the private information is characterized by ever increasing promises. It is therefore not clear that a combination of both ingredients results in a compact state space needed to compute a unique invariant probability measure necessary for welfare comparison.

To avoid additional complications, we cast the robustness exercise in an environment with two periods and compare the effect of public information for allocations with non-contractible private information to allocations that are also contingent on the truthful reports of private signals (contractible private information). We find that the welfare effects of public information are quantitatively very similar to the case with non-contractible private information studied in the main body. For the second period, we assume that agents respect the commitments made in the first period. Otherwise, if voluntary participation were allowed in both periods, there would be no room for social insurance because agents would always choose to consume their endowments.

Non-contractible private information Let $c^j_{i,1}$ be first-period consumption of agents with public signal k_i and income y_j and $c^{jk}_{i,2}$ second-period consumption of agents with public signal k_i and income y_j in the first period and income y_k in the second period with $i, j, k \in \{l, h\}$. Normalizing discounting to one, the planner chooses $\{c^j_{i,1}\}, \{c^{jk}_{i,2}\}$ to maximize

$$\frac{1}{4}\left[u(c^h_{h,1}) + u(c^l_{h,1}) + u(c^h_{l,1}) + u(c^l_{l,1})\right] + \frac{1}{4}\left[\kappa u(c^{hh}_{h,2}) + (1-\kappa)u(c^{hl}_{h,2}) + \kappa u(c^{lh}_{h,2}) + (1-\kappa)u(c^{ll}_{h,2}) + \kappa u(c^{hl}_{l,2}) + (1-\kappa)u(c^{hh}_{l,2}) + \kappa u(c^{ll}_{l,2}) + (1-\kappa)u(c^{lh}_{l,2})\right],$$

subject to resource feasibility in the first and second period

$$\frac{1}{4}\left(c^h_{1h} + c^l_{1h} + c^h_{1l} + c^l_{1l}\right) = \frac{1}{2}\sum_{j\in\{l,h\}} y_{j,1}$$

$$\frac{1}{4}\left[\kappa\left(c^{hh}_{h,2} + c^{lh}_{h,2} + c^{hl}_{l,2} + c^{ll}_{l,2}\right) + (1-\kappa)\left(c^{hl}_{h,2} + c^{ll}_{h,2} + c^{hh}_{l,2} + c^{lh}_{l,2}\right)\right] = \frac{1}{2}\sum_{j\in\{l,h\}} y_{j,2}$$

and participation constraints. As one example, for high-income agents in the first period the participation constraints for a good public and good private signal are

$$u(c^h_{h,1}) + \left(\frac{\kappa\nu}{\kappa\nu + (1-\kappa)(1-\nu)}u(c^{hh}_{h,2}) + \frac{(1-\kappa)(1-\nu)}{\kappa\nu + (1-\kappa)(1-\nu)}u(c^{hl}_{h,2})\right) \geq u(y_{h,1}) + \left(\frac{\kappa\nu}{\kappa\nu + (1-\kappa)(1-\nu)}u(y_{h,2}) + \frac{(1-\kappa)(1-\nu)}{\kappa\nu + (1-\kappa)(1-\nu)}u(y_{l,2})\right).$$

Contractible private information As an alternative, the planner can encourage agents to truthfully report their private signals by rendering them at least as well as when lying about the private signal. This possibility of the planner gives rise to another type of incentive constraints, truth-telling constraints. In this case, the planner chooses allocations that are contingent on public information and on the reports on private information in the first period. Let $c^j_{im,1}$ be first-period consumption of agents with public signal k_i, private signal n_m and income y_j and $c^{jk}_{im,2}$ is second-period consumption defined accordingly. The planner chooses $\{c^j_{im,1}\}, \{c^{jk}_{im,2}\}$ to maximize social welfare with the corresponding weights subject to resource feasibility, participation constraints and truth-telling constraints. As one example, the truth-telling constraint of an agent with a high income, a high public and a low private signal is given

Table 3: Welfare effects of public information with private information.

	Non-contractible, $\Delta\kappa$	Contractible, $\Delta\kappa$
$\sigma_y = 0.25$	0.15	0.14
$\sigma_y = 0.2$	0.08	0.04

Note: $\Delta\kappa$ captures the relative change in welfare for an increase in public signal precision measured in certainty equivalent consumption expressed in percent.

by

$$u(c^h_{hl,1}) + \left(\frac{\kappa(1-\nu)}{\kappa(1-\nu)+(1-\kappa)\nu}u(c^{hh}_{hl,2}) + \frac{(1-\kappa)\nu}{\kappa(1-\nu)+(1-\kappa)\nu}u(c^{hl}_{hl,2})\right) \geq$$
$$u(c^h_{hh,1}) + \left(\frac{\kappa(1-\nu)}{\kappa(1-\nu)+(1-\kappa)\nu}u(c^{hh}_{hh,2}) + \frac{(1-\kappa)\nu}{\kappa(1-\nu)+(1-\kappa)\nu}u(c^{hl}_{hh,2})\right).$$

For informative private signals, the optimal allocation is characterized by higher transfers from high-income to low-income agents in the first period than with non-contractible private information. The additional transfers are stemming from agents with a low private signal. Agents with a low private signal are willing to transfer more in the first period to be insured in the low-income state in the second period because this state is likely to realize for them. To discourage these agents to lie, the corresponding consumption for a high private signal in the low-income state in the future is significantly lower. In optimum, agents with a low private signal are indifferent between lying and telling the truth. The truth-telling constraints of agents with a high private signal do not bind.

Numerical comparison For the same parameter values, risk sharing is better and social welfare is higher with truth telling. Quantitatively, the model with contractible private information leads to too much risk sharing for developing countries even for completely informative private signals. For this reason, we cannot capture the low degree of risk sharing in developing countries of 10 percent. In the comparison, we therefore calibrate the precision of private information such that 40 percent of the variation in logged income is insured in the first period for $\kappa = 0.5$. We consider i.i.d. income with two income states and log preferences. The welfare effect of public information $\Delta\kappa$, is computed by comparing welfare with uninformative public signals to welfare with public signals that are as precise as the calibrated precision of private signals.

As displayed in Table 3, the social value of public information is positive in both environments and very similar. While for a higher variability of income, public information has slightly larger marginal gain with contractible private information (see first row), the reverse applies for lower variability of income (second row).

A.7 Transition laws

There are four transition laws of interest, namely:

- $\pi(s'|s) = \pi(y', k'|y, k)$,

- $\pi(s'|s,n) = \pi(y',k'|y,k,n)$,
- $\pi(y'|s,n) = \pi(y'|y,k,n)$
- $\pi(\theta'|\theta) = \pi(y',k',n'|y,k,n)$.

The conditional probability for income is

$$\pi\left(y' = y_j | k = y_m, n = y_l, y = y_i\right) = \frac{\pi\left(y' = y_j, k = y_m, n = y_l, y = y_i\right)}{\pi\left(k = y_m, n = y_l, y = y_i\right)}.$$

The denominator can be derived by using the following identity

$$\sum_{z=1}^{K} \pi\left(y' = y_z | k = y_m, n = y_l, y = y_i\right) = 1$$

which implies

$$\pi\left(k = y_m, n = y_l, y = y_i\right) = \sum_{z=1}^{K} \pi\left(y' = y_z, k = y_m, n = y_l, y = y_i\right).$$

The elements of the sum on the right hand side are products that depend on z, m, l, i and the precisions of signals (we treat the current income simply as a yet another signal). It follows

$$\pi\left(y' = y_z, k = y_m, n = y_l, y = y_i\right) = p_{iz}\kappa^{\mathbf{1}_{m=z}}\left(\frac{1-\kappa}{K-1}\right)^{1-\mathbf{1}_{m=z}} \nu^{\mathbf{1}_{l=z}}\left(\frac{1-\nu}{K-1}\right)^{1-\mathbf{1}_{l=z}},$$

where p_{iz} is the Markov transition probability for moving from income i to income z. When the signal is wrong, $m \neq Z$, we assume that all income states are equally likely. The general formula for the conditional expectations reads:

$$\pi\left(y' = y_j | k = y_m, n = y_l, y = y_i\right) = \frac{p_{ij}\kappa^{\mathbf{1}_{m=j}}\left(\frac{1-\kappa}{K-1}\right)^{1-\mathbf{1}_{m=j}} \nu^{\mathbf{1}_{l=j}}\left(\frac{1-\nu}{K-1}\right)^{1-\mathbf{1}_{l=j}}}{\sum_{z=1}^{K} p_{iz}\kappa^{\mathbf{1}_{m=z}}\left(\frac{1-\kappa}{K-1}\right)^{1-\mathbf{1}_{m=z}} \nu^{\mathbf{1}_{l=z}}\left(\frac{1-\nu}{K-1}\right)^{1-\mathbf{1}_{l=z}}}. \quad (25)$$

From here, we get the expression for $\pi(y',k'|k,n,y)$ right away upon assuming that the distribution of k' doesn't depend on y'. For all k', we get

$$\pi\left(y' = y_j, k' | k = y_m, n = y_l, y = y_i\right) = \pi(k')\frac{p_{ij}\kappa^{\mathbf{1}_{m=j}}\left(\frac{1-\kappa}{K-1}\right)^{1-\mathbf{1}_{m=j}} \nu^{\mathbf{1}_{l=j}}\left(\frac{1-\nu}{K-1}\right)^{1-\mathbf{1}_{l=j}}}{\sum_{z=1}^{K} p_{iz}\kappa^{\mathbf{1}_{m=z}}\left(\frac{1-\kappa}{K-1}\right)^{1-\mathbf{1}_{m=z}} \nu^{\mathbf{1}_{l=z}}\left(\frac{1-\nu}{K-1}\right)^{1-\mathbf{1}_{l=z}}}. \quad (26)$$

The corresponding conditional probability for the full state is for all k', n'

$$\pi\left(y' = y_j, k', n' | k = y_m, n = y_l, y = y_i\right) \\ = \pi(k')\pi(n') \frac{p_{ij}\kappa^{\mathbf{1}_{m=j}}\left(\frac{1-\kappa}{K-1}\right)^{1-\mathbf{1}_{m=j}} \nu^{\mathbf{1}_{l=j}}\left(\frac{1-\nu}{K-1}\right)^{1-\mathbf{1}_{l=j}}}{\sum_{z=1}^{K} p_{iz}\kappa^{\mathbf{1}_{m=z}}\left(\frac{1-\kappa}{K-1}\right)^{1-\mathbf{1}_{m=z}} \nu^{\mathbf{1}_{l=z}}\left(\frac{1-\nu}{K-1}\right)^{1-\mathbf{1}_{l=z}}}. \quad (27)$$

Finally, the formula for $\pi\left(y', k' | k, y\right)$ naturally follows:

$$\pi\left(y' = y_j, k' | k = y_m, y = y_i\right) = \pi(k') \frac{p_{ij}\kappa^{\mathbf{1}_{m=j}}\left(\frac{1-\kappa}{K-1}\right)^{1-\mathbf{1}_{m=j}}}{\sum_{z=1}^{K} p_{iz}\kappa^{\mathbf{1}_{m=z}}\left(\frac{1-\kappa}{K-1}\right)^{1-\mathbf{1}_{m=z}}} \forall k'. \quad (28)$$

A.8 Computing the outside options

Here we provide detailed derivations of how the values of the outside options $U^{Aut}\left(y, k, n\right)$ were computed. Let P be the transition matrix for income with elements $\pi(y'|y)$. Scrapping the time index we have:

$$U^{Aut}(y, k, n) = (1-\beta)u(y) + \beta(1-\beta)\sum_{y'} \pi(y'|k, n, y)u(y') \\ + \beta^2(1-\beta)\sum_{y''} \pi(y''|y, k, n)u(y'') + \beta^3(1-\beta)\sum_{y'''} \pi(y'''|y, k, n)u(y''') + ...$$

Now, we need to distinguish two cases depending on income autocorrelation. If income is i.i.d, we have that $\pi\left(y_n|y, k, n\right) = \pi\left(y\right), n \geq 2$. For an autocorrelated income process we have that $\pi\left(y_n|y, k, n\right) = \pi\left(y_{n-1}|y, k, n\right)^T P, n \geq 2$. Let $U^{Aut,i}$ be the outside option under iid income and $U^{Aut,p}$ be the outside option with persistent income. By the virtue of infinite sum for matrices formula we then have the following

$$U^{Aut,i}\left(y, k, n\right) = \left(1-\beta\right)u\left(y\right) + \beta\left(1-\beta\right)\pi(y'|k, n, y)^T u(y') + \beta^2 \pi\left(y''\right)^T u\left(y''\right) \\ U^{Aut,p}\left(y, k, n\right) = \left(1-\beta\right)u\left(y\right) + \beta(1-\beta)\pi(y'|k, n, y)^T\left(I - \beta P\right)^{-1} u\left(y'\right).$$

A.9 Numerical algorithm

Here we describe the design of the algorithm to solve for the HD allocations.

1. We collapse the 3-D state space to one state variable q. We order the new variable q from lowest to highest value of the outside option. Hence, we have the collapsed state space $Q = \{q_1, ..., q_L\}'$ which induces the transition law $Q_{i,j}$.

2. We construct grid for promises $W = \{w_1, ..., w_M\}$ with M reasonably larger than L.

3. We identify the set of all possible combinations of incentive-compatible promises at each state q, denoted W_{ic} where $W_{ic} = \{\left(w_{ic}^1, ..., w_{ic}^L\right)\} \in W^L : w_{ic}^j \geq U_{aut}\left(q_j\right), w_{ic}^1 \leq w_{ic}^n \leq w_{ic}^L$

where by W^L we denote the set of all L-tuples with elements from W. With this step, we endogeneize the grid on the multidimensional promises space.

4. We proceed with a Howard policy function algorithm looping over Q, W_{ic} and Q' (set of future states) until the convergence of the value function up to a pre-specified tolerance is achieved. The minimizing combinations of promised values for each q determine a Markov chain over $Q \times W_{ic}$ which we than use to find an invariant distribution of state-promised values combinations which we then use to validate the resource feasibility constraint.

5. If the demand for consumption in the optimal arrangement exceeds/is lower than the available resources we decrease/increase the interest R and do the policy iteration again until we find the market clearing R^*.

The tolerance for the Howard step is set at 10^{-5}. Because solving for an HD allocations is much more costly than finding an ML allocation, we compute the HD allocations on a loose grid on κ values.

A.10 Competitive equilibrium and decentralization

In a decentralized version of the economy, households will be heterogenous in initial asset holdings, income and public signals. Private information is not contractible and thus only relevant for participation incentives. We start with defining a competitive equilibrium as in Krueger and Perri (2011) that follows Kehoe and Levine (1993) and derive prices to decentralize the efficient allocations. Denote by $p_t(s^t)$ the period-zero price of a unit of period-t consumption faced by a household following history s^t. A household with initial wealth a_0, initial endowment y_0, and signals k_0, θ_0 chooses an allocation $\{c_t(a_0, s^t)\}_{t=0}^{\infty}$ that provides the highest utility subject to their intertemporal budget constraint

$$c_0(a_0, s_0) + \sum_{t=1}^{\infty} \sum_{\theta^t|\theta_0} p_t(\theta^t) c_t(a_0, s^t) \leq y_0 + \sum_{t=1}^{\infty} \sum_{\theta^t|\theta_0} p_t(\theta^t) y_t + a_0 \tag{29}$$

and their participation incentives for each history θ^t in each period t

$$(1-\beta)u(c_t(a_0, s^t)) + \beta \sum_{s_{t+1}} \pi(\theta_{t+1}|\theta^t) \, \mathrm{U}(\{c_\tau(a_0, s^\tau)\}_{\tau=t+1}^{\infty}) \geq$$
$$(1-\beta)u(y_t) + \beta \sum_{y_{t+1}} \pi(y_{t+1}|\theta^t) \, \mathrm{U}_{t+1}(\{y_\tau\}_{\tau=t+1}^{\infty}). \tag{30}$$

Definition 7 *A competitive equilibrium with limited commitment is a price system* $\{p_t(s^t)\}_{t=0}^{\infty}$ *and an allocation* $\{c_t(a_0, s^t)\}_{t=0}^{\infty}$ *such that*

(i) given prices, the allocation of each household (a_0, s_0) *solves the household's problem;*

(ii) all markets clear.

An efficient allocation with public information can be decentralized as a competitive equilibrium. This result is captured in the following proposition.

Proposition 5 *A stationary optimal allocation* $\{C(h_t(w_0, s^t))\}_{t=0}^{\infty}$ *can be decentralized as a competitive equilibrium allocation* $\{c_t(a_0, s^t)\}_{t=0}^{\infty}$ *with prices and initial asset holdings given by*

$$p_t(\theta^t) = \frac{\pi(s^t|s_0)}{R^t} = p_t(s^t)$$

and

$$a_0 = c(w_0, s_0) - y_0 + \sum_{t=1}^{\infty} \sum_{s^t|s_0} \frac{\pi(s^t|s_0)}{R^t}(c(w_0, s^t) - y_t).$$

Proof. The first order condition for competitive equilibrium consumption is requiring

$$\beta^t \pi(s^t|s_0) u'(c_t(a_0, s^t)) \geq \lambda p_t(\theta^t),$$

where λ is the Lagrange multiplier on the budget constraint (29). Asset prices are determined by households with the highest willingness to pay for the asset. These are unconstrained households in period $t+1$. For two consecutive periods, the first order conditions for households with slack participation constraints are

$$\beta^t \pi(s^t|s_0) u'(c_t(a_0, s^t)) = \lambda p_t(\theta^t),$$
$$\beta^{t+1} \pi(s^{t+1}|s_0) u'(c_{t+1}(a_0, s^{t+1})) = \lambda p_{t+1}(\theta^{t+1}).$$

Dividing those we obtain:

$$\beta \frac{u'(c_{t+1}(a_0, s^{t+1}))}{u'(c_t(a_0, s^t))} = \frac{\pi(s^t|s_0) p_{t+1}(\theta^{t+1})}{p_t(\theta^t) \pi(s^{t+1}|s_0)} \tag{31}$$

Consider a consumption allocation from the planner problem. In periods t and $t+1$, the optimality condition with non-binding participation constraints reads

$$\beta \frac{u'(c_{t+1})}{u'(c_t)} = \frac{1}{R}. \tag{32}$$

Combining the optimality conditions of planner and households results in

$$\frac{1}{R} = \frac{\pi(s^t|s_0) p_{t+1}(\theta^{t+1})}{p_t(\theta^t) \pi(s^{t+1}|s_0)},$$

which implies that

$$p_t(\theta^t) = \frac{\pi(s^t|s_0)}{R^t} = p_t(s^t).$$

Finally, the initial wealth that makes the allocation (w_0, s_0) affordable follows from substituting the period-zero prices in the budget constraint (29)

$$a_0 = c(w_0, s_0) - y_0 + \sum_{t=1}^{\infty} \sum_{\theta^t|\theta_0} \frac{\pi(s^t|s_0)}{R^t}(c(w_0, s^t) - y_t).$$

■