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River Sharing and Water Trade

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River sharing and water trade*

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Abstract

We analyse river sharing games in which a set of agents located along a river shares the available water. Using coalition theory, we find that the potential benefits of water trade may not be sufficient to make all agents in the river cooperate and acknowledge property rights as a prerequisite for trade. Specifically, a complete market for river water may not emerge if there are four or more agents along the river. Instead, a partial market may emerge where a subset of agents trades river water while other agents can take some of the river water that passes their territory.

Keywords: river sharing; water trade; market emergence; property rights; coalition stability

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1 Introduction

With complete markets and price-taking behaviour, trade leads to an efficient allocation of resources. For some goods or resources the complete markets assumption is not satisfied because property rights are not defined or because some agents cannot participate in the market. In such cases, where complete markets will not emerge, the allocation of resources is not efficient. For instance, when property rights are not enforced, some agents may opt for coercion and grab resources from others. Since Haavelmo (1954), such settings have been analysed using a variety of approaches (e.g. Bush and Mayer, 1974; Buchanan, 1975; Skaperdas, 1992; Muthoo, 2004; Piccione and Rubinstein, 2007).

Here, we focus on river sharing, a setting where inefficient allocation is commonplace, precisely because of the lack of well-defined property rights. In transboundary rivers, water rights are usually contested and grabbing of river water occurs as a result (Ansink and Weikard, 2009). River sharing games consist of a unidirectional water body and a set of agents who are located along the river and abstract water for beneficial use, such as irrigation. Béal et al. (2013) and Ansink and Houba (2015) provide recent surveys of the river sharing literature that emerged after an early contribution by Ambec and Sprumont (2002). Some recent contributions to this literature are by Van den Brink et al. (2012), Ambec et al. (2013), Houba et al. (2015) and Ansink and Weikard (2012, 2015). We argue that the structure of river sharing obstructs the formation of property rights and thereby the emergence of markets for river water (Wang, 2011). The key reason is that the unidirectionality of water flow dictates that upstream agents have the opportunity to take any available water from the river. In absence of well-defined property rights, they also have the power to do so.

We will see that the potential benefits of water trade may not be sufficient to make all agents in the river cooperate and acknowledge property rights as a prerequisite for trade. Instead, partial markets may emerge where a subset of agents trades river water, with the possibility that other agents take some of the river water that passes their territory. We interpret such a subset as a 'coalition', whose members mutually acknowledge the property rights to river water. Therefore we can use the tools of coalition theory to analyse equilibria of the river sharing game. Specifically, we model the emergence of a market for river water using a two-stage open-membership cartel game, commonly used in the literature on International Environmental Agreements (IEAs) (cf. Carraro and Siniscalco, 1993; Barrett, 1994; Benchekroun and Long, 2012). At stage 1 of this game, agents announce whether or not they join the coalition and, given these decisions, at stage 2, the coalition members and singletons choose their water use levels. To assess (internal and external) stability of the resulting coalitions (cf. D'Aspremont et al., 1983), we have to adapt the standard IEA model for two distinct features of the river sharing setting. First, our model has a spatial structure due to the agents being ordered along the river. Water cannot be transported upstream and any water to be delivered to a downstream coalition member via the territory of an intermediate singleton can be seized. We refer to this as leakage. Second, different from the cases discussed in most of the IEA literature, water is not a public good. Its use is rivalrous. While water is excludable within an agent's territory, its trade is hampered as long as property rights are not acknowledged.

Two recent papers are closely related to ours. Ambec and Ehlers (2008) analyse river sharing as a cooperative game with externalities, focusing on distributions of river water that are in the core. They specifically assess distributions that are derived from legal doctrines on international river sharing. In this paper, we do not constrain ourselves to specific river sharing doctrines but rather assess whether trade in river water emerges at all. This is also why we are not interested in the core, but rather assess all possible coalition structures for their stability. A second related study is by Gengenbach et al. (2010), who assess the formation and stability of coalitions for water pollution abatement along a river. The abatement game they study is similar to a standard public goods game, but has a spatial structure. Remarkably, coalition stability is not impacted by the spatial (linear) structure when compared with standard IEA games, except for cases with corner solutions. In this paper, we use a similar approach to analyse river sharing, focusing on water quantity rather than quality. Differences in results arise, however, because water use is rivalrous whereas abatement of water pollution is not.

We have three main results. In Theorem 1 we characterise the equilibria of the river sharing game and, hence, provide insights into the formation and stability of coalitions for river sharing and the emergence of trade in river water. In Theorem 2 we show that there is no leakage in equilibrium. In Theorem 3 we show that large coalitions are not necessarily stable, which implies partial rather than complete markets for river water. Specifically, our results show that a coalition with more than three agents may not be stable. As a result, an inefficient allocation of water may result. Note that these results are derived for the particular setting of river sharing, but apply more broadly to settings where both trade and coercion take place (Houba and Weikard, 2009). Our paper contributes to explaining the small number of participants in most river sharing agreements (Dinar, 2007; Giordano et al., 2014). In the setting of our paper, such agreements are the contracts that are signed by coalitions to define water rights. We will discuss this issue briefly in the closing section.

The paper proceeds as follows. In Section 2 we introduce the river sharing game. In Section 3 we present our main results. In Section 4 we provide an example of an unstable grand coalition which proves our main result and illustrates that markets for river water may fail to emerge. In Section 5 we conclude.

2 The river sharing game

2.1 Preliminaries

Consider a set N of $n \geq 2$ agents ordered along a river. Agent 1 is the most upstream and n the most downstream; agent i is upstream of j whenever $i < j$. $U_i \equiv \{1, 2, \ldots, i-1\}$ denotes the set of i's upstream predecessors, while $D_i \equiv \{i+1, i+2, \ldots, n\}$ denotes the set of i's downstream followers. On the territory of each agent, the total amount of water in the river increases by inflow $e_i \geq 0$, which originates from e.g. rainfall and tributaries. Each agent i abstracts $x_i \geq 0$ units of water from the river. We assume that all abstracted water is used. The amount of water available to an agent depends on water use by upstream agents. Let the total available water on the territory of agent i be denoted by

$$E_i \equiv e_i + \sum_{j \in U_i} (e_j - x_j). \tag{1}$$

Water use cannot exceed availability:

$$x_i \leq E_i, \ \forall i \in N, \tag{2}$$

but some amount $u_i \geq 0$ of available water may be left in the river and so is unused:

$$u_i \equiv E_i - x_i. \tag{3}$$

Benefits $b_i(x_i)$ of water use—benefits net of abstraction costs—are strictly concave. We assume that $b_i(x_i)$ is differentiable for all $x_i \geq 0$. Denote by $\hat{x}_i$ the satiation point of water use for agent i such that $b_i'(\hat{x}_i) = 0$. Without loss of generality (Ambec and Ehlers, 2008, Remark 2) we assume water scarcity by taking satiation points as weakly larger than inflow for all agents:

Assumption 1. $\hat{x}_i \geq e_i \ \forall i \in N$.

Agents maximize their benefits of water use and may increase these benefits by trading water with others. Water trade implies that water is shared such that the total benefits of water use of the trading agents is maximized (Wang, 2011), given that other agents may take some of the river water that passes their territory. This type of trade is often institutionalized in a river sharing agreement that defines the property rights to water as well as compensation for water sharing.

We model the emergence of such agreements as a two-stage open-membership cartel game as is common in the literature on IEAs. At stage 1, each agent decides whether or not to sign the agreement. We denote this choice by $\sigma_i \in \{0,1\}$, where $\sigma_i = 0$ and $\sigma_i = 1$ mean that i is a non-signatory or a signatory, respectively. The choices of all agents result in a coalition structure $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_n)$ which corresponds to a set of signatories $S = \{i|\sigma_i = 1\}$. We refer to this set as the 'coalition' and to signatories as 'members'. At stage 2 of the game, both members and singletons choose their water use levels x_i. Coalition members choose their water use level in order to maximise coalition payoff and do so by distributing their available water to the downstream members with the highest marginal benefits, provided this is possible and profitable. Singletons choose their water use levels in order to maximise individual payoffs by using all available water up to their satiation point.

We will say that a coalition S with coalition structure σ is 'connected' if for all $i,k \in S$ (that is, $\sigma_i = \sigma_k = 1$) there does not exist $j \notin S$ (that is, $\sigma_j = 0$) with $i < j < k$. Furthermore, it is useful to refer to the partition of the coalition S that consists of its largest connected subsets as the 'components' of S. We denote the set of these components by $C(S)$, whose elements $T \in C(S)$ are the coalition components. For example, a river sharing game with 6 agents and a coalition $S = \{1,2,4,5\}$ has two coalition components $\{1,2\}$ and $\{4,5\}$. If a coalition passes water from one component to another, it may be affected by 'leakage'. Leakage occurs when intermediate singletons seize some or all of this passing water, if they are not satiated.

Formally, leakage $l_i(u_i)$ consists of member i's unused water u_i that does not reach the next downstream member $k = \min(S \cap D_i)$:

$$l_i(u_i) \equiv \min\left(u_i, \sum_{i<j<k} \left(\hat{x}_j - e_j\right)\right), \; i,k \in S, \; k = \min(S \cap D_i). \tag{4}$$

Obviously, leakage cannot occur between neighbouring members, i.e. within a component of a coalition. The amount of water received by coalition member k from his nearest upstream member i, is $r_k(u_i) \equiv u_i - l_i(u_i)$. That is, the amount of water passed from i to k equals the amount of water unused by i minus leakage (the water used by intermediate singletons). We have

$$r_k(u_i) = \begin{cases} 0 & \text{for} \quad 0 \le u_i \le \sum_{i<j<k}(\hat{x}_j - e_j) \\ u_i - \sum_{i<j<k}\left(\hat{x}_j - e_j\right) & \text{for} \quad u_i > \sum_{i<j<k}(\hat{x}_j - e_j). \end{cases} \tag{5}$$

Equation (5) shows that received water $r_k(u_i)$ is a piecewise function of unused upstream water, with positive amounts only if all intermediate singletons are satiated. Note that for $i, k \in S$ with $k = \min(S \cap D_i)$, we have $E_k = e_k + r_k(u_i) = x_k + u_k$.

Water sharing is severely constrained by leakage. In Section 3 we will see that leakage may obstruct the emergence of water trade when gains from trade are insufficient to satisfy intermediate players' demands.

2.2 Equilibrium river sharing

At stage 2 of the game, given a coalition S, water use decisions are taken. The solution to this sub-game is a Partial Agreement Nash Equilibrium (Chander and Tulkens, 1995, 1997). In the river setting, this equilibrium can be found using backward induction, analogous to Ambec and Ehlers (2008). We do so by interpreting the river sharing game as an extensive form game in which the players are the coalition and the $|N| - |S|$ singletons. The nodes of play in this extensive game are given by the coalition components and the singletons. Formally, let $\{C(S) \cup N \setminus S\}$ be the ordered set of nodes in the extensive game, where each element T_i of this set is either a coalition component or a singleton and T_i is upstream of T_{i+1}. Each sub-game starts at an initial node and consists of any available water and subsequent downstream nodes.

For singletons, optimal water use is given by maximisation of their individual payoff. For coalition components, optimal water use is given by maximisation of the coalition payoff, which implies that optimal water use of each coalition component takes into account its effect on the benefits of all coalition members in downstream coalition components. The solution to this game is found by calculating the optimal water use in the (sub)game starting with the first node of the extensive game, and subsequent optimal decisions in downstream nodes. Uniqueness of the resulting equilibrium follows from the strict concavity of $b_i(x_i)$.

Because we consider river sharing games with one coalition only, it is straightforward to verify that optimal water use $x_i^*(S, E_i)$ of singletons $i \notin S$ solves

$$\max_{x_i(S,E_i)} b_i\big(x_i(S, E_i)\big). \tag{6}$$

That is, each singleton maximises his individual benefits of water use, subject to water availability. Note that, by Assumption 1, programme 6 is equivalent to $x_i^*(S, E_i) = \min\{\hat{x}_i, E_i\}$. Optimal water use by coalition members is slightly more involved because decisions by the coalition are made by components, not by individual members. This is just a technical difference (i.e. consecutive nodes in an extensive game require decisions by different players in the game), and can be resolved by re-interpreting each coalition member as a separate component. Doing so, given that coalitions may consist of more than one component, each with one or more members, optimal water use $x_i^*(S, E_i)$ of coalition members $i \in S$ solves

$$\max_{x_i(S,E_i)} b_i\big(x_i(S,E_i)\big) + \sum_{j \in D_i \cap S} b_j\Big[x_j^*\Big(S, E_j\big(x_i(S,E_i)\big)\Big)\Big]. \tag{7}$$

Solving programme 6 and programme 7 for respectively all singletons and coalition members yields the equilibrium levels of water use for all agents, denoted $x^* = (x_1^*, .., x_n^*)$. Using $(e_1, .., e_n)$ we obtain the equilibrium available water, $E_i^* \equiv e_i + \sum_{j \in U_i}(e_j - x_j^*)$, and the equilibrium unused water, $u_i^* = E_i^* - x_i^*$, for each agent.

2.3 Stability

In Section 2.2 we established for every coalition structure a unique vector of water use levels x^*. Each x^* induces a partition function $v(S)$ that establishes the coalition payoff and individual payoffs to singletons. Singletons' payoffs equal the benefits of water use and are denoted by $v_i(S) = b_i(x_i(S, E_i)), i \notin S$. Coalition payoffs are $v_S(S) = \sum_{i \in S} b_i(x_i(S, E_i))$ and may be redistributed among coalition members. The partition function in combination with a sharing rule induces a valuation function that gives a payoff vector $(v_i(S))_{i \in N}$ for all $S \subseteq N$. We assume that the coalition uses a sharing rule for distributing the coalition payoff that satisfies the *Claim Rights Condition* (Weikard, 2009), if this is possible:

Condition 1 (Claim Rights Condition). *We have $v_i(S) \geq v_i(S_{-i})$ if and only if $v_S(S) \geq \sum_{i \in S} v_i(S_{-i})$ for all $i \in S$ and all $S \subseteq N$.*

Condition 1 says that a coalition member receives at least his outside option payoff (his 'claim') if and only if the coalition payoff is large enough to satisfy all claims.

At stage 1 of the coalition formation game, we are interested in analysing Nash equilibria. A Nash equilibrium at the coalition formation stage is found by applying the concepts of internal and external stability (D'Aspremont et al., 1983). A coalition is internally stable when no coalition member $i \in S$ wants to leave the coalition; it is externally stable when no singleton $j \notin S$ wants to join the coalition:

Condition 2 (Internal stability). $v_i(S) \geq v_i(S_{-i}) \ \forall i \in S$.

Condition 3 (External stability). $v_i(S) > v_i(S_{+i}) \ \forall i \notin S$.[1]

[1]Note that we use a strict inequality sign, which implies that an agent will always join the coalition if he is indifferent between joining or not.

It is clear that no coalition member has an incentive to leave the coalition under any sharing rule that meets Condition 1, provided the coalition payoff exceeds the sum of claims. Internal stability is therefore guaranteed whenever it can possibly be obtained. Note that the two stability concepts are linked (Weikard, 2009, Lemma 1):

Lemma 1. *Under Condition 1, a coalition S is externally unstable if and only if there exists $j \in N \setminus \{S\}$ such that coalition S_{+j} is internally stable.*

We will invoke Lemma 1 in Section 3 as it allows to focus on internal stability only. For the stability analysis that follows below it will also be useful to notice that river sharing agreements are superadditive. Any enlargement of an agreement generates payoffs that are at least as large as the payoffs of the initial coalition plus the payoff of the joining player. More formally, superadditivity is defined as follows:

Definition 1 (Superadditivity). *A partition function v is superadditive if for all $S \subset N$ and all $i \in N \setminus \{S\}$ it holds that $v_{S_{+i}}(S_{+i}) \geq v_S(S) + v_i(S)$.*

2.4 Two benchmark situations

Using programmes 6 and 7 and the stability conditions identified above, we first present two extreme benchmark situations, All Singletons and the Grand Coalition.

In the All Singletons benchmark, the coalition structure is $\sigma = (0, \ldots, 0)$ so that $S = \emptyset$ and programme (6) applies to all agents. Each agent only takes into account his individual benefits in choosing his water use, constrained only by water availability. The equilibrium of this game is found by applying programme (6) recursively to each agent, starting upstream with agent 1 who solves the programme subject to $x_1 \leq E_1 = e_1$. His water use $x_1(\emptyset, E_1)$ determines E_2 according to (1) and the allocation to agent 2 is found by solving the programme subject to $x_2 \leq E_2 = e_1 + e_2 - x_1(\emptyset, E_1)$, and so on. We call this allocation the All Singletons allocation.

Remark 1. *Because by Assumption 1 we have $\hat{x}_i \geq e_i \ \forall i \in N$, application of programme* (6) *implies that $\hat{x}_i \geq x_i(\emptyset, E_i) = e_i \ \forall i \in N$ and therefore $E_i = e_i \ \forall i \in N$. Hence, by Assumption 1, in the All Singletons allocation there exists no water passing, and agents are only satiated in case Assumption 1 holds with equality.*

In the Grand Coalition benchmark, the coalition structure is $\sigma = (1, \ldots, 1)$ so that $S = N$ and programme (7) applies to all agents. Each agent takes into account his individual benefits as well as benefits to all downstream agents in choosing his water use. Unlike in the All Singletons benchmark, here the water use of agent i depends on downstream marginal benefits. The solution to this game is found by solving the sub-game starting with agent 1, who takes into account subsequent decisions by downstream agents. We call this allocation the Grand Coalition allocation.

Remark 2. *Marginal benefits in the Grand Coalition allocation decrease (weakly) when moving downstream (Ambec and Sprumont, 2002). (Weakly) decreasing marginal benefits are caused by the water availability constraint* (2). *The Grand Coalition is efficient and corresponds to the efficient allocation due to downstream bilateral trading as described by Wang (2011).*

Clearly, given the difference between the two maximisation programmes (6) and (7), water use levels in the grand coalition are different from those in the All Singletons benchmark. In general, water is efficiently allocated in the Grand Coalition because water is passed from agents with low marginal benefits to agents with higher marginal benefits subject only to the water availability constraint (2). Only for agents 1 and n we can generally establish differences in water use between the All Singletons and Grand Coalition equilibria. Irrespective of marginal benefits we have $x_1(N, E_1) \leq x_1(\emptyset, E_1)$ and $x_n(N, E_n) \geq x_n(\emptyset, E_n)$. That is, water use by agent 1 in the Grand Coalition is weakly lower than his water use in the All Singletons benchmark, while the reverse holds for agent n.

3 River sharing agreements

A priori there is no reason why either the All Singletons or the Grand Coalition would result as an equilibrium of the river sharing game. Some agents in the Grand Coalition may prefer to free-ride (internal instability) while some singletons may prefer cooperation to the All Singletons benchmark (external instability). Hence, we now turn to the analysis of partial markets for river water using the tools of coalition theory. We analyse coalition stability by applying the stability concepts introduced in Section 2 and using the water use levels determined by programmes (6) and (7). This allows us to derive our three main results. Theorem 1 provides the equilibrium properties of river sharing games. Theorem 2 establishes that stable coalitions do not suffer from leakage. Finally, Theorem 3 shows that every coalition with two or three agents is internally stable, but larger coalitions may not be internally stable. This implies that partial rather than complete markets for river water may emerge as equilibria in the river sharing game.

3.1 Equilibrium properties

The following lemma presents two causal relations between water passing and the amount of water received, which helps to prove Theorem 1, introduced below.

Lemma 2. *For $i,k \in S$ with $k = \min(S \cap D_i)$, in equilibrium we have*

$$u_i = 0 \iff r_k(u_i) = 0, \text{ or, equivalently,}$$
$$u_i > 0 \iff r_k(u_i) > 0.$$

Proof. By (7) and given that $\hat{x}_i \geq e_i \ \forall i \in N$ (see Assumption 1), it is never optimal for coalition member i to leave a positive amount of unused water $u_i : 0 < u_i \leq \sum_{i<j<k} \left(\hat{x}_j - e_j\right)$ that does not meet the leakage threshold, see (5). By (6), singletons $j : i < j < k$ will use all this water so that $r_k(u_i) = 0$. No water reaches the next downstream coalition member k. It follows immediately that $r_k(u_i) = 0$ implies $u_i = 0$ and that $u_i > 0$ implies $r_k(u_i) > 0$. By (5) we have that $u_i = 0$ implies $r_k(u_i) = 0$ and that $r_k(u_i) > 0$ implies $u_i > 0$. □

Lemma 2 states that the coalition passes water if this is beneficial to the coalition and otherwise not. One consequence of the second part of Lemma 2 is that whenever $u_i > 0$, we know that $u_i > l_i(u_i)$ and that any singleton j with $i < j < k$ must be satiated so that $x_j = \hat{x}_j$. Note that leakage does not necessarily prevent formation of coalitions or destabilise cooperation within the coalition. As long as leakage is sufficiently small there are gains from water trade by forming a coalition. Water passing has implications for the payoffs to both coalition members and singletons. Suppose that, given a coalition S, there exists a singleton $j \notin S$ and members $i,k \in S$ with $k = \min(S \cap D_i)$ such that $i < j < k$. In case $u_i = 0$, member k does not receive any water from i even though this may have increased the coalition payoff. Hence, the components that members i and k belong to do not increase their payoff beyond the sum of what these components can achieve individually. In case $u_i > 0$ and $r_k(u_i) > 0$, the components that members i and k belong to increase the coalition payoff through the delivery of water from i to k. In addition, the singleton j benefits from this water passing, if he was not satiated before.

Lemma 2 suffices to establish the following theorem which is inspired by Kilgour and Dinar (2001). The theorem describes equilibrium properties of the river sharing game, in terms of the marginal benefits of water use and water passing.

Theorem 1. *Consider any coalition $S \subseteq N$ with $i,k \in S$ such that $k = \min(S \cap D_i)$. Exactly one of the following statements is true.*

(a) $b_i'(x_i) = b_k'(x_k)$ *and* $r_k(u_i) \geq 0$,

(b) $b_i'(x_i) < b_k'(x_k)$ *and* $r_k(u_i) > 0$ *with* $x_i = 0$,

(c) $b_i'(x_i) < b_k'(x_k)$ *and* $r_k(u_i) = 0$ *with* $x_i = E_i$,

(d) $b_i'(x_i) > b_k'(x_k)$ *and* $r_k(u_i) = 0$ *with* $x_i = E_i$,

Proof. Consider any coalition $S \subseteq N$ with $i, k \in S$ such that $k = \min\{l \in S \cap D_i\}$. We establish statements (a), (b), (c) and (d) consecutively.

Consider a water allocation vector x of the river sharing game equilibrium such that $b_i'(x_i) = b_k'(x_k)$ and $r_k(u_i) \geq 0$. By programmes (6) and (7), no water is wasted upstream so that $E_i \leq \hat{x}_i$. By programme (7) and given that $b_i'(x_i) = b_k'(x_k)$, agent i has no incentive to change his water use level x_i, independent of possible leakage. This establishes statement (a).

Suppose that (a) does not hold. There are two possibilities, either $b_i'(x_i) < b_k'(x_k)$ or $b_i'(x_i) > b_k'(x_k)$. We analyse both cases, starting with the first.

Consider a water allocation vector x of the river sharing game equilibrium such that $b_i'(x_i) < b_k'(x_k)$ and $x_i = E_i$. By programmes (6) and (7), no water is wasted upstream so that $E_i \leq \hat{x}_i$. Pick $\epsilon > 0$ with $\epsilon < x_i$ and consider a related vector x' such that $x_i' = x_i - \epsilon$. This implies $u_i' = E_i - x_i' = \epsilon$. By Lemma 2, $u_i' = \epsilon$ implies $r_k'(u_i') > 0$ and any intermediate singletons are satiated so that by (7), agent i should pass more water. There are two options. Either he continues passing water until $u_i = E_i$ but still we have $b_i'(x_i) < b_k'(x_k)$. This establishes statement (b). Or he continues passing water until $b_i'(x_i) = b_k'(x_k)$. But this contradicts that $b_i'(x_i) < b_k'(x_k)$. Hence, if $b_i'(x_i) < b_k'(x_k)$ then $r_k(u_i) = 0$ so that, by Lemma 2, $x_i = E_i$. This establishes statement (c).

Next, consider a water allocation vector x of the river sharing game equilibrium such that $b_i'(x_i) > b_k'(x_k)$ and $x_i = E_i$. By programmes (6) and (7), no water is wasted upstream so that $E_i \leq \hat{x}_i$. Again, pick $\epsilon > 0$ with $\epsilon < x_i$ and consider a related vector x' such that $x_i' = x_i - \epsilon$. This implies $u_i' = E_i - x_i' = \epsilon$. By Lemma 2, $u_i' = \epsilon$ implies $r_k'(u_i') > 0$. Even in the best possible case with $r_k'(u_i') = \epsilon$, because $b_i'(x_i) > b_k'(x_k)$, coalition payoff under x_i' is lower than under x_i, so that by (7) x_i' cannot be optimal. Hence, if $b_i'(x_i) > b_k'(x_k)$ then $r_k(u_i) = 0$ so that, by Lemma 2, $x_i = E_i$. This establishes statement (d). □

Theorem 1 outlines the importance of water availability, water passing, and leakage for the marginal benefits of neighbouring coalition members. Case (a) holds for situations with or without water passing, in which water availability or leakage do not constrain the optimal allocation of water over the two coalition members. The next two cases hold whenever the downstream member has higher marginal benefits that cannot be realised. In case (b) because of water shortage and in case (c) because leakage prevents agent i from passing water to j. Case (c) extends the theorem by Kilgour and Dinar (2001) by taking into account the impact of leakage in addition to water shortage as in statement (b). Note that it is not possible to have $b_i'(x_i) < b_k'(x_k)$ with $0 < x_i < E_i$ and $r_k(u_i) > 0$ as the latter implies that intermediate singletons are satiated so that an additional unit of passed water would increase the coalition payoff v_S. Case (d) holds whenever the upstream partner has higher marginal benefits but cannot increase his water use due to the water availability constraint (2).

Contrary to the Grand Coalition allocation—see Remark 2—marginal benefits do not necessarily decrease (weakly) when moving downstream. In case (b) of Theorem 1, marginal benefits may increase due to water shortage, and in case (c) due to a sufficiently large leakage threshold. Such leakage blocks the passing of water that would have increased the coalition payoff v_S. This case (c) is the only case where leakage due to water seizing by singletons affects the efficiency of water allocation, as case (a) describes the efficient equilibrium and cases (b) and (d) describe inefficiency due to water shortage or the water availability constraint.

All agents (weakly) prefer the river sharing game equilibrium over the All Singletons benchmark. To see why, note that coalition members maximise the coalition payoff. By (7), the coalition payoff is at least as large as the sum of payoffs if all coalition members behave as singletons. By Condition 1, this assures that each coalition member prefers the river sharing game equilibrium over the All Singletons benchmark. A related observation can be made for singletons. In the All Singletons benchmark, each agent uses $x_i = e_i$. In the river sharing game, the only difference for singletons is that there may be water passing between coalition members so that leakage may increase their water use to $x_i(S) > e_i$. Hence, singletons too prefer the equilibrium of the river sharing game over the All Singletons benchmark.

3.2 Stable coalitions

We now turn to the stability analysis of coalitions and show how leakage is obstructing the emergence of water trade since it limits the size of stable coalitions.

First, we show how the river sharing game differs from standard IEA games. A common assumption in the IEA literature is that agents are homogeneous, which allows analytical solutions (Barrett, 1994). In this paper, this assumption would imply that agents have the same benefit functions and water inflow, but they obviously differ in their location along the river.

Remark 3. *In a river sharing game with homogeneous agents such that* $e_i = e \ \forall i \in N$ *and*

$b_i(x_i) = b(x_i)$ $\forall i \in N$, there are no possible gains from water trade. Therefore, each agent is indifferent between joining and not joining the coalition. By Condition (3) *the Grand Coalition forms, but this coalition is not welfare-improving.*

Our second main result is established in the following theorem.

Theorem 2. *In the river sharing game, there does not exist a stable coalition with leakage.*

Proof. The proof is by contradiction. Consider the stable coalition $S \subseteq N$ and coalition members $i, k \in S$ such that $k = \min(S \cap D_i)$, and consider leakage. By (4) and Lemma 2, leakage implies that there is a singleton $j : i < j < k$ such that $u_i > 0$ and $r_k(u_i)$. This implies that agent j is satiated with payoff $v_j(S) = b_j(\hat{x}_j)$. Now consider coalition S_{+j}. By super-additivity of the game, $v_{S_{+j}}(S_{+j}) \geq v_S(S) + v_j(S)$. By Condition 1, coalition S_{+j} is internally stable and by Lemma 1, this implies that coalition S is externally unstable, a contradiction. □

Theorem 2 indicates that leakage will not occur in equilibrium. One option is that water-seizing singletons will always join the coalition since the forgone benefits due to leakage are sufficient to compensate him. This option is proven for two-agent coalitions in part (ii) of Theorem 3 below. Alternatively, leakage may prevent a coalition from being stable. Therefore, absence of leakage in equilibrium does not imply that leakage is an unimportant aspect of the river sharing game.

For the remaining results it is useful to introduce the following definition of effectiveness.

Definition 2 (Effectiveness)**.** *Coalition S is effective for agent $k \in S$ if and only if, for $i = \max\{l \in S \cap U_k\}$ we have $r_k(u_i) > 0$.*

Effectiveness refers to member k receiving water from the nearest upstream coalition member. It allows us to distinguish between two types of claims c_j, see Condition 1, as established in the following lemma.

Lemma 3. *Claim c_j by agent j in coalition $S \subseteq N$ is either low, c_j^L, or high, c_j^H. We have $c_j^L = b_j(e_j)$ and $c_j^H = b_j(\hat{x}_j)$.*

Proof. A claim by agent j in coalition $S \subseteq N$ is based on his outside option payoff which he would receive under coalition S_{-j}. Clearly, if agent j is either the most upstream or most downstream member of S, then by programmes (6) and (7), no water is wasted upstream, so that agent j has a low claim $c_j^L = b_j(e_j)$.

Next, consider the remaining case where agent j is not the most upstream or downstream member of S. Consider coalition members i and k with $i < j < k$ and such that $i = \max\{l \in$

$S \cap U_j\}$ while $k = \min\{l \in S \cap D_j\}$. In words, there are no other coalition members located between i and j nor between j and k. Consider coalition S_{-j}. There are two options. If $u_i = 0$, because no water is wasted upstream, $E_j = e_j$ so that agent j has a low claim $c_j^L = b_j(e_j)$. If $u_i > 0$, then by Lemma 2 we have $r_k(u_i) > 0$ and therefore $E_i > \hat{x}_i$, so that agent j has a high claim $c_j^H = b_j(\hat{x}_j)$. □

Lemma 3 implies that agent j has a high claim only when coalition S_{-j} is effective for agent k, who is the nearest downstream coalition member. Otherwise, agent j has a low claim. The following lemma shows that coalitions of various types are all internally stable.

Theorem 3. *The following is true in the river sharing game:*

(i) Every two-agent coalition is internally stable.

(ii) For every two-agent coalition $S = \{i, k\}$ with $i < k$ and S is effective for k, it holds that $S' = \{i, i+1, ..., k-1, k\}$ is internally stable.

(iii) Every three-agent coalition is internally stable.

(iv) A coalition S of at least four agents is not necessarily internally stable.

Proof. The proof is for each part separately.
Part (i): This follows immediately from super-additivity of the game.
Part (ii): By effectiveness of S for k we have $r_k(u_i) > 0$ and all singletons j such that $i < j < k$ are satiated. By Part (i) we know that S is internally stable such that claims of agents i and k are satisfied. We can now add any agent j such that $i < j < k$ and by super-additivity $v_{S_{+j}}(S_{+j}) \geq v_S(S) + v_j(S)$. Because claims in S do not change when j joins, super-additivity is sufficient to guarantee internal stability of S_{+j}. The argument can be repeated for all $j \in \{i+1, ..., k-1\}$.
Part (iii): Consider coalitions $S = \{i, j, k\}$ and $S' = \{i, k\}$ with $i < j < k$. There are two cases. If S' is not effective for k, then $r_k(u_i') = 0$, which implies that claims are c_i^L, c_j^L, c_k^L. By super-additivity these claims can be met by S such that the internal stability Condition 2 holds. If S' is effective for k, then $r_k(u_i') > 0$, which implies that claims are c_i^L, c_j^H, c_k^L. Since $v_j(S') = c_j^H$ and by super-additivity these claims can be met by S such that the internal stability Condition 2 holds.
Part (iv): We prove this part by an example of a four-agent coalition that is internally unstable even if the sharing rule satisfies the Claim Rights Condition 1. The example is provided in Section 4. □

Combining parts (i) and (iii) of Theorem 3 with Lemma 1, we have that every one-agent (i.e. All Singletons) and every two-agent coalition is externally unstable.

The arguments used in the proof of parts (i) to (iii) of Theorem 3 do not generalise to larger coalitions. The reason is that the sum of claims of the intermediate coalition members (i.e. those coalition members that are not the most upstream or downstream member of S) may be too high for the coalition to compensate. By Condition 1, this implies that internal stability is not satisfied. Specifically, such a situation may arise if (i) the coalition S consists of more than three agents, (ii) the claims of the intermediate members are high (Lemma 3), and (iii) the coalition $S' = \{\min\{S\}, \max\{S\}\}$, is not effective for $\max\{S\}$ (part (ii) of Theorem 3). An example, provided in the next section, illustrates how the determinants of stability interact and proves our result that internally stability for river sharing games with more than three players cannot be guaranteed.

4 Example of an unstable Grand Coalition

In this section we develop an example that proves part (iv) of Theorem 3. We will show that a coalition of more than three agents may not be internally stable and, hence, that there is no guarantee that a river sharing agreement would include all agents in the river. In other words, water trade may fail to emerge or may not reach efficient levels such that some gains from trade remain unexploited. We construct our example using the three conditions identified at the end of Section 3.2. That is, we consider a river sharing game with four agents and the Grand Coalition $S = N = \{1, 2, 3, 4\}$. We choose our parameter values such that both the claims of the intermediate members are high and the coalition $\{1, 4\}$ is not effective. Clearly, if $\{1, 4\}$ were effective, then by part (ii) of Theorem 3, N is stable and the river sharing agreement would cover all four agents. Given that $\{1, 4\}$ is ineffective, the condition of high claims for intermediate members assures that the overall gains from water trade are insufficient to satisfy all claims.

We construct our example using the following quadratic benefit function:

$$b_i(x_i) = \frac{1}{s_i^2} \cdot \left(2s_i x_i - x_i^2\right). \tag{8}$$

The s_i parameter in Equation 8 is equal to the satiation level of agent i so that $s_i = \hat{x}_i$. In addition, the satiation benefits are normalised to one so that $b_i(s_i) = 1$.

Parameter values for e_i and s_i in our example are provided in Table 1. Column 4 of Table 1 indicates that there are potential gains from water trade. Water used by agent 1 can be profitably passed to agents 2–4 in order to increase the total benefits of water use. In

Table 1: An example river sharing game with four agents: levels of inflow, satiation points, and marginal benefits for the All Singletons benchmark.

i	e_i	s_i	$b_i'(e_i)$
1	2.00	3	0.22
2	0.75	1	0.50
3	0.75	1	0.50
4	0.55	1	0.90

the Grand Coalition, the marginal benefits of water use can be equalised among all agents.

We proceed by comparing coalition $\{1,3,4\}$ and the Grand Coalition which suffices to prove part (iv) of Theorem 3. Consider coalition $\{1,3,4\}$. For this coalition we obtain the equilibrium water allocation $x = (1.40, 1.00, 0.82, 0.82)$ with coalition payoff $v_{\{1,3,4\}} = 0.72 + 0.97 + 0.97 = 2.65$. Singleton agent 2 benefits from leakage and is completely satiated at $x_2 = s_2 = 1$. This coalition is effective for agent 3 (but note that part (ii) of Theorem 3 does not apply here) since $v_{\{1\}} + v_{\{3,4\}} = 2.64 < 2.65 = v_{\{1,3,4\}}$. That is, if the coalition would not deliver water to agent 3, the coalition payoff would be lower. Therefore,

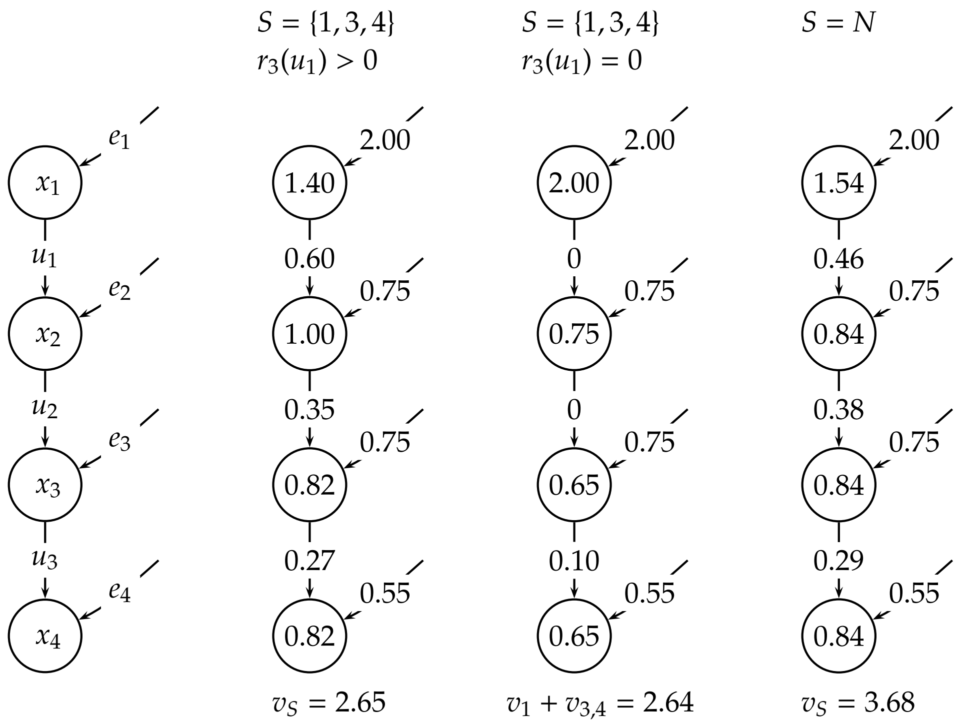

Figure 1: Inflow, passed water, and water use for $S = N$ and $S = \{1,3,4\}$ (with and without water passing and leakage); nodes are agents and arrows indicate water flows. Note that the numbers for water use provided may not sum to total available water $\sum_{i \in N} e_i = 4.05$ due to rounding.

agent 2 has a high claim in the Grand Coalition, which equals $c_2^H = b_2(\hat{x}_2) = 1$. A similar argument can be made for coalition $\{1,2,4\}$, which establishes that agent 3 has a high claim in the Grand Coalition, which equals $c_3^H = b_3(\hat{x}_3) = 1$. Agents 1 and 4 have low claims in the Grand Coalition (see proof of Lemma 3). Their claims are $c_1^L = b_1(e_1) = 0.89$ and $c_4^L = b_4(e_4) = 0.80$. Hence, the sum of claims is $0.89 + 1 + 1 + 0.80 = 3.69$.

Next consider the Grand Coalition. The optimal water allocation is achieved when the cheap water from upstream agent 1 is delivered to agent 4. The resulting water allocation is $x = (1.54, 0.84, 0.84, 0.84)$ and the coalition payoff is $v_N = 0.76+0.97+0.97+0.97 = 3.68$. The Grand Coalition payoff (3.68) is insufficient to meet the sum of claims (3.69) and, hence, the Grand Coalition is not stable. Inflow, passed water, and water use for coalitions $\{1,3,4\}$ (with and without water passing and leakage) and the Grand Coalition are illustrated in Figure 1.

5 Conclusion

In this paper we have established that the potential benefits of water trade in transboundary rivers may not be sufficient to make all agents in the river cooperate and acknowledge property rights as a prerequisite for trade. One of our main results shows that such leakage does not occur in equilibrium. In addition, we identify the equilibrium properties of river sharing games and we find that large coalitions are not necessarily stable. This implies that partial markets are more likely to emerge, in which a subset of agents trades river water, while others choose not to join such an agreement.

Given that river water is not a public good, one would expect transboundary trade in river water to emerge spontaneously. In this paper we show that this is not necessarily true, mainly due to the constraints imposed by unidirectional river flow. Our results help to explain the predominance of 'small' river sharing agreements in transboundary rivers and the difficulties often seen in negotiations on river sharing agreements (cf. Just and Netanyahu, 1998). For example, Giordano et al. (2014) find that only a quarter of all transboundary river treaties include all countries in the river basin to which they apply. While the failures of river sharing negotiations have often been explained using arguments based on political feasibility (LeMarquand, 1977; Dinar, 2000; Zawahri et al., 2015), in this paper we show that 'large' agreements may be economically unattractive from (some) individual countries' perspective. Our results put recent research on river sharing in a broader perspective and demonstrate that the benefits of cooperative water use may not be reaped easily.

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