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Decision Making in Incomplete Markets with Ambiguity – A Case Study of a Gas Field Acquisition

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Decision Making in Incomplete Markets with Ambiguity – A case study of a gas field acquisition*

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Abstract

We apply utility indifference pricing to solve a contingent claim problem, valuing a connected pair of gas fields where the underlying process is not standard Geometric Brownian motion and the assumption of complete markets is not fulfilled. First, empirical data are often characterized by time-varying volatility and fat tails; therefore we use Gaussian GAS (Generalized AutoRegressive Score) and GARCH models, extending them to Student's t-GARCH and t-GAS. Second, an important risk (reservoir size) is not hedgeable. Thus markets are incomplete which also makes preference free pricing impossible and thus standard option pricing inapplicable. Therefore we parametrize the investor's risk preference and use utility indifference pricing techniques. We use Lease Square Monte Carlo simulations as a dimension reduction technique. Moreover, an investor often only has an approximate idea of the true probabilistic model underlying variables, making model ambiguity a relevant problem. We show empirically how model ambiguity affects project values, and importantly, how option values change as model ambiguity gets resolved in later phases of the projects considered. We show that traditional valuation approaches will consistently underestimate the value of project flexibility and in general lead to overly conservative investment decisions in the presence of time dependent stochastic structures.

Classification-JEL: C61, D81, G1, G31, G34, Q40

1 Introduction

Firms need project evaluation techniques for many purposes: capital budgeting assessment, risk management, mergers and acquisitions (M&A) and so forth. The most popular and well-adopted evaluation method over the past decades is the net present value (NPV) approach, for whose calculations only one time discount rate and a series of future cash flows are required. The NPV approach is simple and straightforward, but to achieve that needs strong assumptions and therefore suffers from rigidity and inflexibility. Problems arise when investors believe that they may benefit from the flexibilities embedded in the projects: within a NPV framework, there is no way of quantifying the benefits of such flexibilities. As a consequence, NPV structurally underestimates the value of projects with flexible investment opportunities.

Real option valuation (ROV), which quantifies the value of embedded flexibilities through option pricing techniques, is a more appropriate tool for projects with flexibilities, for instance, a not-to-exceed value for M&A activities. Before applying any option evaluation methods, additional analytical procedures need to be carefully executed. The investor first has to reformulate the development plan into a strategic one, which exploits all the inherent managerial flexibilities embedded in the investment project. Next, in order to determine an optimal investment strategy, the investor has to consider mainly three aspects: the dynamics of the underlying asset returns, the constraints on the investment strategy, and the value of the investor's strategy. Each aspect affects the final decisions significantly and has to be carefully taken into consideration.

Volatile Volatility The volatility of underlying processes obviously matters for option pricing problems. Van Wijnbergen and Zhao [2013] show in an earlier application of ROV to an energy elated project that a Gaussian GARCH specification outperforms one which assumes constant variance in modeling the dynamics of the underlying asset returns, in this case gas prices. They also show that switching from constant variance to Gaussian GARCH has a dramatic impact on option values, so modeling the structure of volatility matters. In this paper, we again consider GARCH models but also a more general volatility modeling approach, Generalized Autoregressive Score (GAS) models which are capable of capturing some unique characteristics of the latent volatilities of the time series. This GAS model family, first proposed and developed by Harvey and Chakravarty [2008] and Creal et al. [2013], is a more general set-up compared to GARCH models. By adding the first derivatives of its likelihood function as the latent factor of dynamics, this model takes full advantage of the changing directions of likelihood. As shown in Creal et al. [2013], the GAS framework demonstrates superior features and better empirical fit over Gaussian GARCH models, which is also consistent with our findings in this paper in case of Dutch gas prices. Note that GARCH(1,1) processes can be seen as a special case of a more general GAS(1,1) structure, which allows a direct Likelihood ratio test of one specification against the other.

Furthermore, the diagnostic test on residuals from both Gaussian GARCH and Gaussian GAS models rejects the normality assumption. This feature is commonly found in financial data, often referred to as "black swans": extreme outcomes happen more often than people expect, which results in the failure of normal distribution assumptions. Therefore, we proceed on with Student's t-GARCH and t-GAS models, aiming to capture the fat-tail characteristics of the data. The estimated degrees of freedom for both models are smaller than 4 and statistically significant, which again confirms our fat-tail hypothesis. Hence a simple Gaussian assumption would undermine the high occurrence of extreme events and lead to incorrect descriptions of the data.

To our knowledge, this is the first study to derive option pricing results under a process of t-GAS and compare the results with option valuations based on the constant variance and results based on Gaussian GARCH assumptions. Even though we solve the problem within a real-option set-up, the results are doubtlessly relevant to the valuation of finance options traded in the market.

Incomplete Markets One difficulty of applying standard option techniques to real life problems is that often, the decision maker is facing an asset pricing problem in an incomplete market, where not all underlying risks are hedgeable through the market. For instance, in our gas field valuation problem, reservoir uncertainty cannot be hedged away in any existing market. In fact even price risk cannot be fully hedged since the derivative market built on the Dutch gas contracts is still young and immature[1]. Yet another cause of market incompleteness is the stochastic volatility characterizing the underlying process driving asset returns (gas prices), because the dynamics of the second moment of the process cannot be hedged through the market either. Therefore, classical option pricing models such as the Black-Scholes formula (Black and Scholes [1973]), which lay their foundations on the assumption of complete markets, are not applicable. In an incomplete markets environment; in fact preference free (risk neutral) pricing becomes impossible: an individual's risk preference has to be parametrized and be taken into account. Accordingly, utility indifference pricing is an appropriate valuation method to be applied for our real option problem.

Model Ambiguity Another important issue in decision-making problem involves model ambiguity. This occurs when the decision maker is uncertain about the true probabilistic model, which is often referred to as Knightian uncertainty (Knight [1921]) or model ambiguity. Note that a decision problem with ambiguity is different from one under risk: the latter refers to a decision problem with the true probability distribution known and the former is one without the true probability known. Model ambiguity is a realistic and robust assumption because an investor often does not have access to the true probabilistic model underlying relevant variables and may only have an approximation for the true one at the best.

In applications like the one analyzed in this paper concerning gas field evaluation, model

[1] The Dutch gas spot market is called Title Transfer Facility (TTF). The first TTF Natural Gas Options were launches by ICE in December 2011.

ambiguity occurs often due to the relatively unsophisticated existing technology for reservoir size estimation. Geophysicists estimate parameters for reservoir distribution based both on imperfect exploration data, often supplemented by insights derived from their own past experiences, which makes model ambiguity a particularly important issue for valuation problems.

Choice of Discount Rate In an incomplete market setting, a proper discount rate should not only reflects the decision maker's risk aversion and his/her time discount value, but also the structure of the uncertainty embedded in the project itself. The appropriate choice of discount rates is not always clear to the decision maker. In fact, managers are struggling in determining the time discount factor, especially for individual projects. Often, as Borison [2005] point out, the WACC (weighted average cost-of-capital) is used without clearly identifying its risk coverage, i.e., whether it reflects private risk only or the overall investment risk. In this paper, we decompose the discount rate and discuss how each aspect inherent in discount rate determination affects the decision making process.

Structure of this paper This paper is arranged as follows. Section 2 reviews related literature on real options, model ambiguity, GARCH and GAS models, etc. Our representative gas field case study problem is described in Section 3. The econometric models and option pricing models are explained in the following section. Section 5 demonstrates the results and Section 6 concludes.

2 Literature Review

Real Option Valuation McDonald and Siegel [1986] initiated the application of option pricing technology to decisions involving irreversible "real" projects. They solve for optimal investment rules using a contingent claim setting and find significantly positive value of waiting. Following their work, Pindyck [1991] provides methodology for practitioners. He emphasizes two major characteristics of investment opportunities: irreversible expenditure and postponement of execution. These features both have a profound effect on investment decision, and share similarities with financial options.

Borison [2005] criticizes existing applications of real option theory for requiring assumptions that are not satisfied in practice, thereby invalidating the pricing methodologies chosen. He surveys the applicability and assumptions of all existing approaches, including the classic approach (Brennan and Schwartz [1985], Amram and Kulatilaka [1999]), the subjective approach (Howell [2001]), the MAD approach (Copeland et al. [1994]) , the revised classic approach (Dixit and Pindyck [1994]), and the integrated approach (Smith and Nau [1995], Smith and McCardle [1998]). All of them except the last one assume market completeness (hedgeable risks), which is however rarely the case in real-life problems where real options are under consideration. Furthermore, the first two explicitly assume the underlying asset follows a constant variance Geometric Brownian Motion process(GBM), which is also not always a good approximation of real life situations.

Discount rate According to a survey conducted by Mukerji and Tallon [2001], the most popular valuation method chosen by CFOs is discounted cash flow, i.e. the Net Present Value method (NPV). However, when applying this approach, the CFOs are (understandably...) not clear on the choice of discount rate. Apparently, they typically use one of the following four discount rates: the acquiring firm's weighted average cost of capital, the acquiring firm's cost of equity, the target's weighted average cost of capital, or other rates such as the target's cost of equity. Each discount rate has its pros and cons, and the choice may also depend on the (size of the) M&A project itself. This may confuse the CFOs and lead to biased (too optimistic or pessimistic) results. And in fact project structures may be such that the use of any constant discount rate is wrong because the risk structure changes over time .

Dynamic processes of underlying assets As is explained in Van Wijnbergen and Zhao [2013], the dynamic of gas prices follows a complicated structure with time varying volatilities, which cannot be captured by a GBM process. Therefore the classic approach, the subjective approach, and the revised classic approaches mentioned before become inapplicable, since they rely on the GBM assumption for their pricing formulas.

In this paper, we consider GAS/GARCH models accounting for volatility, where both are able to reproduce the volatile volatility. Creal et al. [2012] explain that GAS models can be specialized into GARCH models by selecting appropriate factors. They also compare different dynamic copula models and conclude that the likelihood information is extensively exploited under a GAS framework. As shown in Andres [2014], the model with dynamic scores outperforms autoregressive conditional duration (ACD) models in terms of the rate of convergence and reliability. Note that an ACD model, as proposed in Engle and Russell [1998], is analogous to a Gaussian GARCH model.

Furthermore, the financial data often contain fat-tails: extreme outcomes happen too often that a normal distribution is not capable of accounting for the outliers. By applying GARCH/DCC and GAS models to global equity returns, Creal et al. [2011] find that t-GAS produces highly persistent estimated factors and improves loglikelihood substantially.

Real options and Incomplete market As mentioned above and in Borison [2005], most real option approaches assume market completeness, which results in problematic applications. For example, the subjective approach uses subjective probability; therefore, it is incapable of shading lights on the market trading price. The MAD approach argues that traditional NPV serves as an unbiased replicated portfolio; however, still, the no-arbitrage assumption cannot be satisfied with this argument only, because the arbitrage opportunity may exist due to the use of subjective data. Several attempts have been made for resolving the incomplete market problem.

For example, Smith and Nau [1995], Smith and McCardle [1998] remedy the issue by assuming a partial complete market and solve the partial market incompleteness by utility indifference pricing. Carmona [2009] states the effectiveness of utility indifference pricing mechanism for option pricing problem in a incomplete market, where risk preferences are built into the model to acknowledge risks.

Model ambiguity The concern for modeling ambiguity can be traced back to Knight [1921], where ambiguity is also described as uncertainty. The essential difference between risk and (Knightian) uncertainty (or ambiguity, as we refer to it here) is whether the true probability is known or unknown. The breakthrough made by Gilboa and Schmeidler [1989] solves the ambiguity problem numerically through a maximin utility with multiple priors, by assuming the agent is ambiguity averse and therefore considers the worse case scenario.

Camerer and Weber [1992] give an extensive survey on ambiguity aversion, including both theoretical and empirical analysis. In earlier experimental studies, e.g. Heath and Tversky [1991], subjects were shown to exhibit strong ambiguity aversion in many circumstances. However, the results for the effect of ambiguity on asset prices are not always coherent. For example, Camerer and Kunreuther [1989] show that even though ambiguity has changed the market structure, it did not affect the asset prices systematically; whereas, Sarin and Weber [1993] draw a different conclusion, and claim that ambiguity drives prices down slightly but significantly. Furthermore, market incompleteness and model ambiguity may mutually reinforce each other. Mukerji and Tallon [2001] argue that the markets are less complete due to the effect of ambiguity aversion.

Chen and Epstein [2002] investigate the effect of ambiguity by considering multiple-priors utility. Their model is able to decompose excess returns into a risk premium and an ambiguity premium. Maccheroni et al. [2006] consider models of decision-making under ambiguity of variational preferences, which focus both on multiple preferences and on multiple priors as in Chen and Epstein [2002].

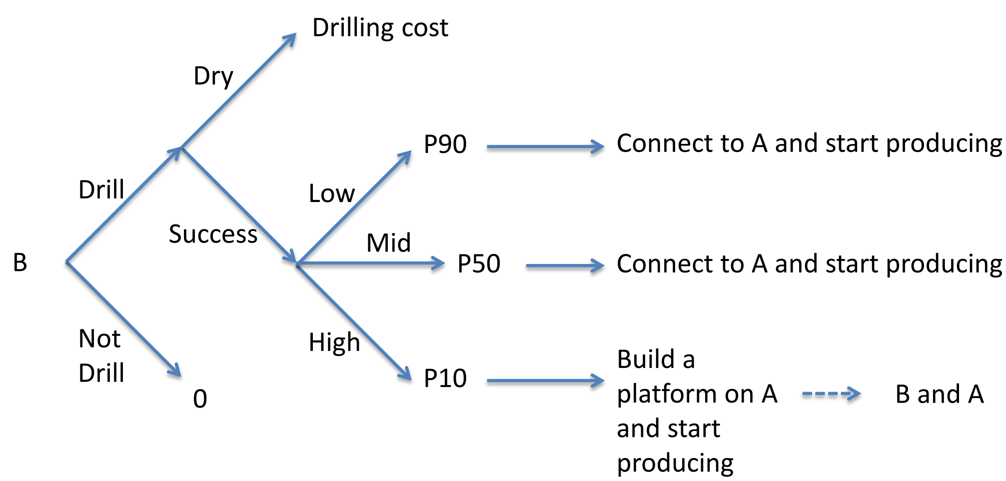
Real options and LSMC: Non-European options There is no closed form solution to the option valuation problems we analyze since we are adopting the GARCH and GAS frameworks. And there is an additional complication in that our problem has endogenous exercise moments, so it has an American option style character, but for American options no analytical solution exists either (in fact it is more appropriate to characterize the options as Bermuda type because of the discrete exercise dates). We solve the valuation problem using Stochastic Dynamic Programming, and reduce the dimensionality problem common to this approach by using the Least Squares Monte Carlo approach proposed by Longstaff and Schwartz [2001]. The continuation value of the claim is then approximated as a function of the state variables by repeated application of regression techniques. A flowchart (Figure 24) in the Appendix explains how this method works.

3 Problem Description

A and B are two gas fields geographically close to each other. The recoverable size of Field A is currently estimated to be low and not economically attractive by itself. However, if the development of Field B turns out to be successful, the reservoir estimate of Field A can be revisited and a more precise estimate might then be expected. So given the possible information updates, the investor designs a strategic developing plan as displayed in Figure 1. As is shown, if the drilling on B is successful and the reservoir of Field B turns out to be high, the producer may decide to build a new platform on Field A, which allows the production of A and B at the same time due to a platform's larger capacity compared to a pipeline. Thus Field A can be considered as an extension option on Field B, which should be exercised only when the reservoir of B reveals a good state (P10[2]).

[2]The definition of P10 is explained in Section 3.1.

Figure 1: Developing Plan



Our aim is to value Field B appropriately, so that the valuation can serve as a reserve price for the acquisition of the area containing both A and B. Van Wijnbergen and Zhao [2013] provide a suitable option pricing model for this acquisition valuation problem. In this paper, we still use the Least Square Monte Carlo method for simulating and obtaining option prices.

3.1 Reservoir Distribution

Possibility of success (POS) stands for the probability of a successful drilling. So the probability of a dry well is $1 - POS$. Based on a drilling success, the investor expects a recovery size R. Three commonly used assumptions for the distribution of R are a triangular distribution, the lognormal distribution, and a variant on the latter, the truncated lognormal distribution.

The triangular distribution is the industry standard for capital budgeting problems due to its simplicity. It considers only three outcomes of R, namely cases $P10$, $P50$, and $P90$. Naturally in line with the concept of a CDF (cumulative density function), reservoir amount R_{P10} of $P10$

case means the probability that the realization of the gas reserve is lower than R_{P10} is 90%. This simple representation is popular among decision makers, because it provides reasonable and easy proxies for low/medium/high reserve size cases, based on which a scenario analysis can be set up for capital budgeting procedures.

Despite the simplicity of triangular distribution, geophysicists prefer a lognormal distribution which provides more insights into the reservoir distribution. However, a standard lognormal distribution ranges from 0 to infinity, which is unrealistic in case of a gas reservoir. Therefore, we apply a truncated lognormal distribution for the simulation of the reservoir volume size. To fully approximate the estimations provided by the geophysicists, the reservoir distribution sometimes cannot solely be characterized by a single truncated lognormal one. For example, the reserve size of Field B here is well described by a sum of two weighted lognormal probability distribution functions. The two lognormal distributions are truncated at 99% quantile, one with parameters (-0.1772, 0.5336)[3] and a weight of 0.8661, the other with parameters (-26.6623, 0.0002)[4] and a weight of 0.1339.

In the case of Field A, its POS and reservoir size R are given as one point estimate. But the POS may be updated from the exploration of Field B and the ambiguity of A, if any, may disappear too. Note that even if the reservoir size of A in our case study is given as one number instead of a whole distribution, our methodology can still be easily adjusted to one with complicated distributions, where the input of the reservoir size would be replaced by a simulated number from a distribution, such as a lognormal distribution, or a weighted lognormal distribution, etc. Furthermore, the information update could be also extended to that of a more precise distribution estimation of A after B's development. We give an example of such an extension in Section 5.4.

[3](Mean, standard deviation).
[4](Mean, standard deviation).

3.2 Option characteristics of the exploration/valuation problem

The strategic plan followed by the firm is divided into two steps. First, the firm might wait and meanwhile observe the market price of gas to decide whether to start developing B or not. This decision has to be made within three years, due to the remaining life of the relevant exploration licenses. This setup means that the firm has a wait-and-see Bermuda-type[5] option on B with a maturity of three years. Once this development option is exercised, the firm may build a platform and further develop A based on a not-worse-than-P10 realized reservoir amount of B (Figure 1), which can thus be seen as the unlocking of a European option. Thus, Project B has multiple and compound option characteristics with a sequential structure, whose values are calculated in Section 5.

4 Methodology

Below, we present the econometric model used for fitting and predicting gas prices and the utility indifference pricing setting for the option pricing problem. We apply both GARCH and GAS models to analyze weekly returns obtained from Title Transfer Facility (TTF), the Dutch gas market.

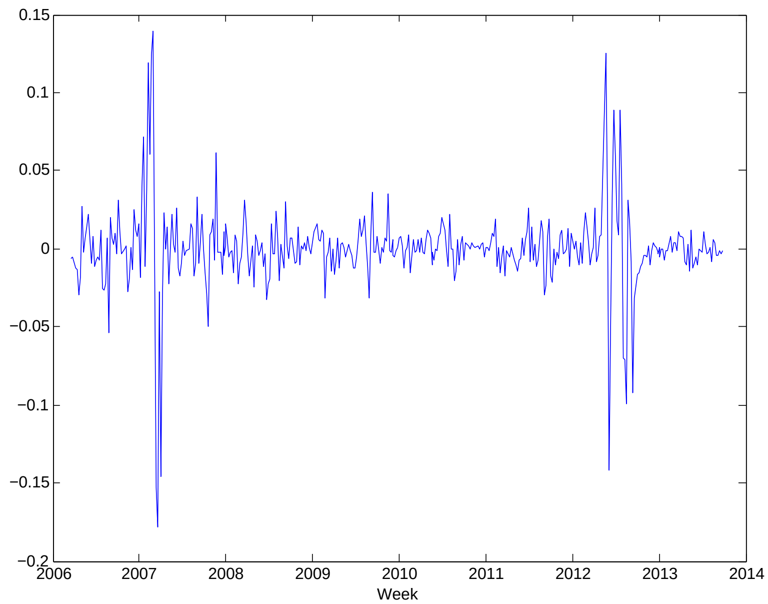
4.1 Generalized Autoregressive Score (or GAS) Models

The gas weekly return series ranges from Jan 4, 2005 to Oct 2, 2013, shown in Figure 2. The time series is stationary by both Dickey-Fuller test and Phillips–Perron test.

The Generalized Autoregressive Score model follows Creal et al. [2012]. y_t is the demeaned weekly log return of gas on the TTF market and has a probability distribution function $p\left(y_t|f_t;\theta_t\right)$, where f_t stands for unobserved time-varying factors and θ_t contains unknown parameters.

[5]A Bermuda option is an American option with a set of predetermined exercise timing possibilities.

Figure 2: TTF Weekly Logarithmic Return Data



$$
\begin{aligned}
y_t &= \sigma_t \varepsilon_t \\
f_{t+1} &= \omega + A s_t + B f_t \\
s_t &= S_t \nabla_t \\
S_t &= -\mathbb{E}_{t-1}\left[\nabla_t \nabla_t'\right]^{-1} \\
\nabla_t &= \frac{\partial \log p\left(y_t | f_t; \theta_t\right)}{\partial f_t}
\end{aligned}
$$

The scaling matrix S_t equals the Fisher information matrix and ∇_t stands for the "score" as in "Generalized Autoregressive Score". Hence, s_t is also called the scaled score function. We assume that ε_t follows a standard normal (Gaussian) distribution but also investigate the possibility of fatter tails by basing the GAS model on a student's t distribution with estimated degrees of freedom, thereby testing for normality. The model collapses into a Gaussian GARCH or a t-GARCH one with the appropriate assumptions on the factor f_t as we show below in the Appendix 7.1.1; so GARCH is embedded in GAS, which allows for a simple loglikelihood test.

4.2 Estimation Results and Diagnostic Tests

Our econometric analysis shows that the Gaussian GAS model yields a higher log-likelihood value of 672.36, comparing to 614.76 for the Gaussian GARCH model. Thus a Gaussian GARCH model outperforms a Gaussian GAS model by 9% in terms of log-likelihood. And the kernel density plots and the QQ-plots in Figure 3 imply that the residuals from both Gaussian GARCH and Gaussian GAS models present fat tail leading to a rejection of the normality hypothesis. Therefore we proceed with a Student's t-based GAS model, to capture the impact of the fat tails embedded in the data.

Figure 3: Residual Tests

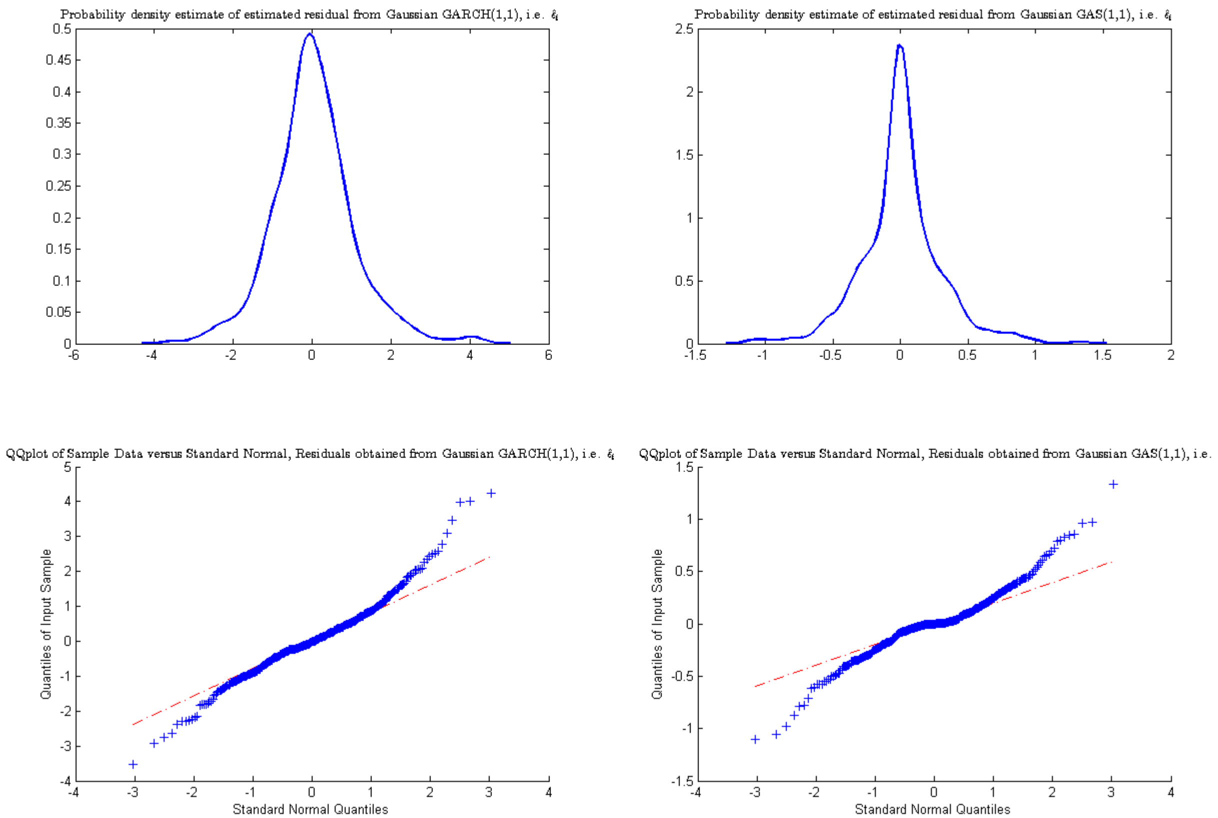


Table 4 lists the estimation results from all four models considered. The estimated degree of freedom for the Student's t distribution is 3.95 and 3.91 for Student's t-GARCH and Student's t-GAS model respectively, and is significant for both models[6]. In addition, the loglikelihood from the models with Student's t distribution is significantly larger than the one from those with Gaussian distribution, which also confirms our findings from the residual plots. The result of a

[6]Arguably more important, the degrees of freedom parameter is very significantly lower than the number where the difference between t-distribution and the normal becomes negligible (higher than 30).

Figure 4: Estimation Results

	Gaussian		Student's t	
	GARCH	GAS	GARCH	GAS
omega	4.6086 (1.4039)	0.1823 (0.6241)	3.7312 (1.2269)	0.2447 (0.6235)
A	0.3822 (9.4001)	0.2245 (8.7826)	0.2860 (7.3775)	0.2434 (7.6351)
B	0.9803 (57.7331)	0.9329 (56.8993)	0.9704 (32.7113)	0.9205 (35.0938)
nu (Degree of Freedom)			3.9511 (5.0168)	3.9096 (4.6893)
LogLikelihood	-614.759	-672.361	-595.759	**-593.894**

(Note: t statistics are reported in parentheses.)

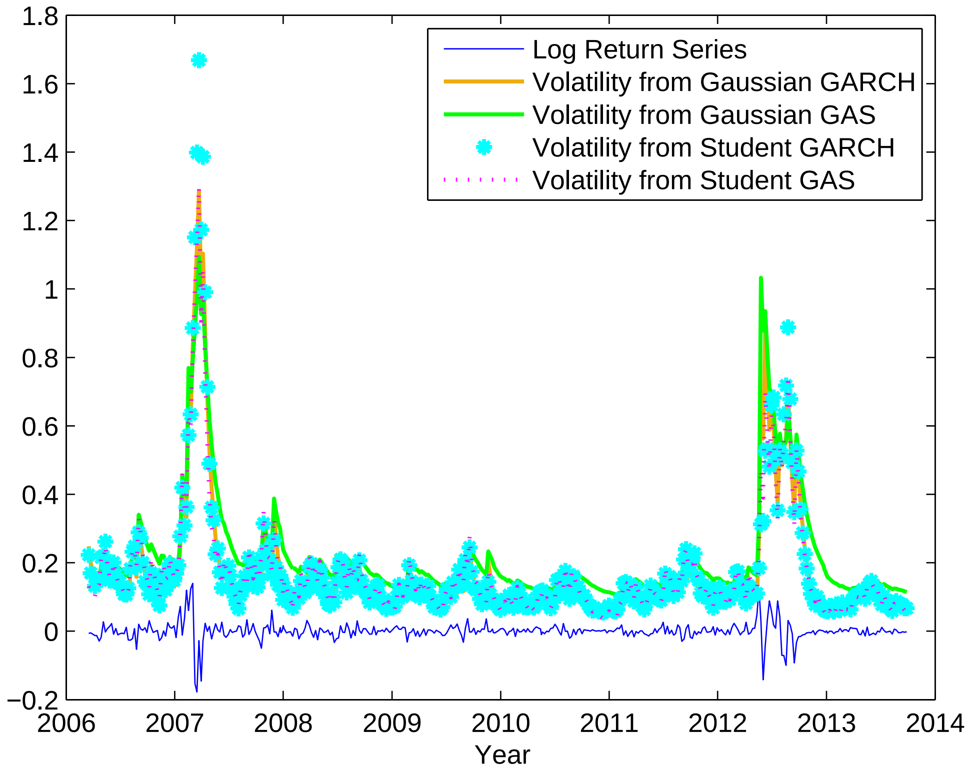
loglikelihood-ratio test[7] between Gaussian-GAS and Student's t-GAS is significant, implying the existence of fat-tails in the data series.

Figure 5 demonstrates the variances σ_t^2 estimated from the two models. As can be seen, a model with constant variance assumption is not able to capture all the relevant features of this time series adequately. All Gaussian/ Student's-t GARCH/ GAS models are able to characterize the high volatility periods, compared to a constant volatility model. As illustrated, two highly volatile periods in early 2007 and mid 2012 can be easily observed correspondingly to Figure 2.

The Student's t-GAS model produces a slightly more smooth volatility series than the Student's t-GARCH model: In the high volatility periods, e.g. early 2007, the estimated volatility given by a Student's t-GAS model is smaller than one by a Student's t-GARCH model. This observation is attributed to the feature of Student's t-GAS model, which adjusts quickly to new observations. Also it implies that extreme outcomes of returns does not necessarily stem from high volatility, the occurrence of tail events can result in the rare outcomes as well.

[7] $LR = 2\left(-593.894 + 672.361\right) = 156.934 > \chi^2\left(1\right).$

Figure 5: Comparison of Estimated Volatility with Gaussian/ Student's-t GARCH/ GAS Models



4.3 Utility Indifference Pricing

Van Wijnbergen and Zhao [2013] applied an integrated approach adjusted from Smith and Nau [1995], Smith and McCardle [1998] based on the assumption that (A) gas price risk can be hedged so risk neutral valuation can be used in that dimension,but (B) reservoir risk is not hedgeable so we used a preference based valuation method in that dimension. Although we used a Gaussian GARCH(1,1) model for gas prices in our earlier paper, which violates the constant variance property necessary for the applicability of risk neutral pricing methods, we used a local variant on risk neutral pricing as proposed by Duan (1995). But that mixed approach cannot be used here because the general GAS models do not satisfy Duan's conditions necessary for the applicability of his local variant on risk neutral pricing. Since now the gas price volatility risks are unhedgeable too, we thus simply assume that neither risk can be hedged and adopt multidimensional utility indifference pricing for both the gas price and reservoir size risks. Also, for comparison with methods more used in practice, we present results adopting the so called Cost-of-capital method where a range of discount rates is used instead of explicit preference parameters.

5 Results

The number of Monte Carlo simulations is 100,000. Here POS for drilling at A is 90% and POS for drilling at B is 30%.

5.1 Cost-of-capital Method

5.1.1 NPVs v.s. Option Values

The results over a range of values for the cost-of-capital (3% - 15%) rates are shown in Figure 6. Figure 6a gives the results based on assuming a t-GARCH(1,1) process for gas prices, and Figure 6b the same set of results but now based on assuming a t-GAS(1,1) structure for the volatility process of gas prices. In both sub-figures, option values and NPVs of both fields are declining as the assumed cost of capital increases, as one should expect given the time structure of cash flows. The horizontal solid (red) line stands for a break-even project, which sets a standard for accepting and rejecting investment projects. The gray circled line illustrates the net present value of B at time zero without any option values counted. The dashed gray line stands for the net present value of the strategic plan without the wait-and-see Bermuda option on B, i.e. a fixed starting time at t=0.

The first interesting result stems from comparing the two graphs: the option values assuming t-GAS(1,1) are about 0.5-1 million euros higher than the project values based on assuming a t-GARCH(1,1) specification. Note that the t-GARCH and t-GAS models explains the data with similar power in terms of loglikelihood, therefore this difference of option values result from the different volatility structure predicted by two models.

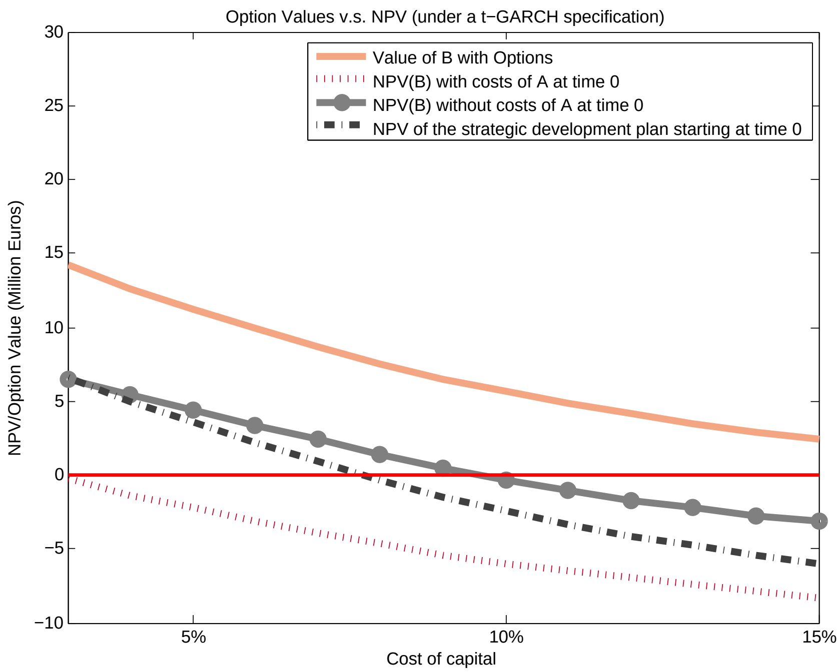
As is shown in Figure 6b (and Table 1 in the Appendix), with a t-GAS specification, the gray circled line intersects the break-even line at a cost-of-capital of 10%, so on a *NPV should be positive* criterium, the firm would reject the entire project B for any cost-of-capital higher than (or equal to) 10%. But taking into account the various waiting option values changes that outcome: The discounted net project value with all options incorporated values more than doubles for a WACC of 3%, declines with higher values for the WACC but the overall project value with options included stays significantly positive for all values of the WACC considered. It does decline with higher discount rates, obviously, because the high CAPEX come upfront but the revenues come later in time. Note also that a platform may have further uses that we do not incorporate: for example, it can be used for gas storage at a later stage. Nevertheless, it is evident that the strategic development plan including waiting option is worthwhile because the project value including option values stays positive given cost-of-capitals varying from 3% to 15%. A second conclusion one can draw is that incorporating the option values is a meaningful exercise: otherwise the wrong investment decision would be taken under a wide range of cost of capital estimates.

Similar patterns can be found under the t-GARCH specification with slightly lower option values, as demonstrated in Figure 6a (and Table 1 in the Appendix). For example, the break-even point of NPV of B is at a cost-of-capital of 9.5%, compared to 10% in case of a t-GAS model.

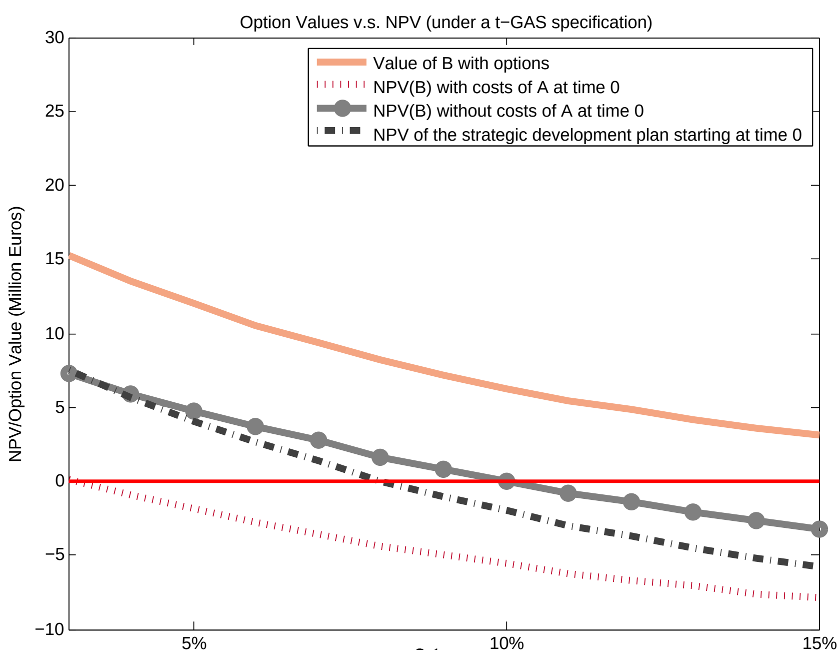
The strategic plan is valued less than the NPV of sole project B. Later we show that the strategic plan becomes valuable if more information will be brought in in the future.

Figure 6: Option pricing results

(a) Student's t-GARCH



(b) Student's t-GAS



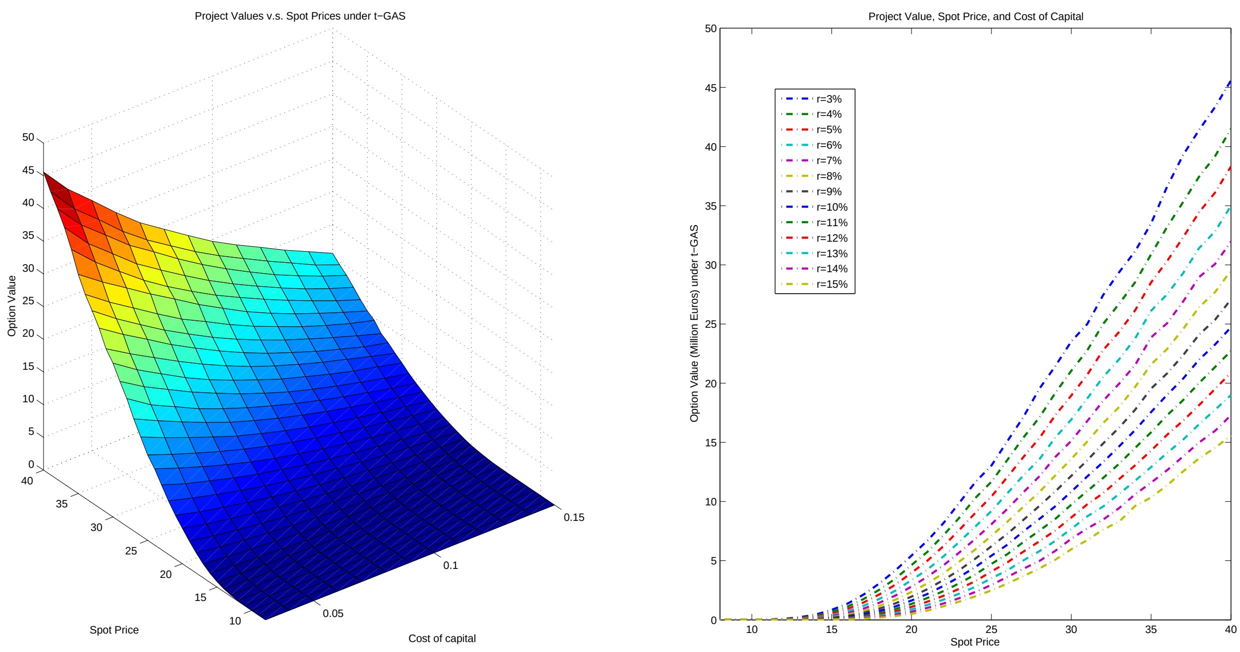
From here on we will not report the GARCH results anymore[8]. They are obviously qualitatively similar to the GAS based results, but the GAS specification has a stronger basis in the econometric results of our data analysis.

5.1.2 Good State v.s. Bad State

As in regular option pricing theory, the option value in our analysis depends on the current market state, in this case the gas price, since the econometric analysis suggests that the best prediction for the future return is mainly influenced by the current state. Figure 7 shows that the value of the project increases with the spot market price. For example, when the spot price is lower than 15 euros per megawatt hour, the option value is close to zero, so for spot prices that low the project has not only negative NPV but also almost worthless options, resulting in a definite rejection at all discount rates.

[8] Corresponding GARCH based results can be found in the Appendix.

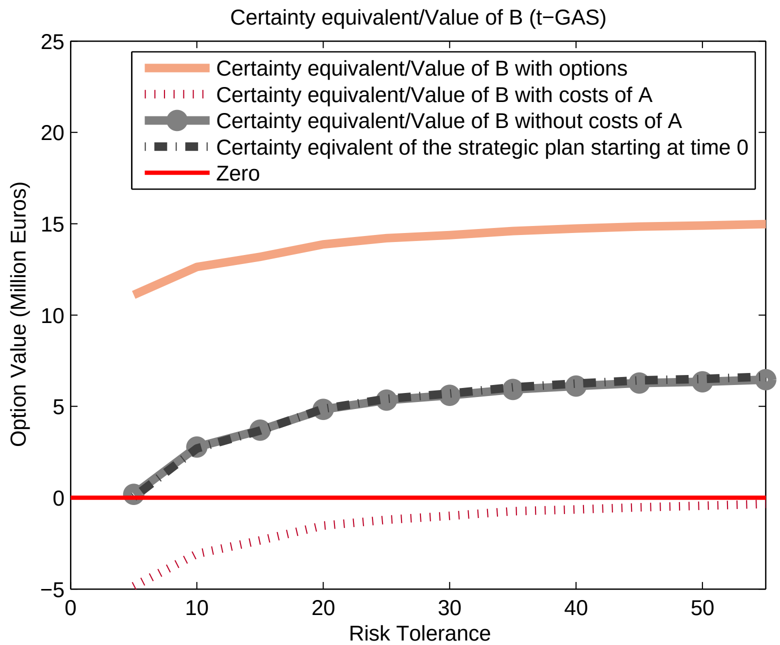
Figure 7: Spot prices v.s. option value under a Student's t-GAS specification



5.2 Utility Indifference Pricing (UIP)

We assume the investor has an exponential utility function: $u_t(x_t) = -exp(-x_t/\rho_t)$, where ρ_t represents the decision maker's risk tolerance. A high ρ_t implies a high tolerance for risk (low risk aversion). Our basic criterion then relies on the discounted value of certainty equivalence cash flows where the certainty equivalence is calculated using a specific value for ρ. Given the degree of risk tolerance, the certainty equivalent therefore represents the project value, in comparison to the NPVs used above.

Figure 8: Utility Indifference Pricing (Student's t-GAS)



The option values calculated based on utility indifference pricing for a range of values for the risk tolerance parameter ρ are given in Figure 8 and Table 2 in the Appendix. For both models, the option values ranges between 10 to 15 million euros. As one should expect, option values are increasing in the investor's risk tolerance[9]. Or, to put it differently, the more risk averse an investor is, the less value she/he attaches to a risky project. It is evident that taking into account the option values once again leads to higher project values, and to a different outcome in terms of the decision to proceed or not. Note that the valuation increases initially steadily as the risk tolerance of the decision maker goes up from 5 to 40; however, from a risk tolerance of 40 onwards, the valuation flattens out. These findings are similar to the results obtained by

[9] This may sound plausible, but it is not trivial: note that such a result is typical for real options (unhedgeable risk) set ups only. For standard risk neutral option pricing methodology to be applicable all risks need to be hedgeable and option values do not depend on risk aversion in such a hedgeable risk environment.

Van Wijnbergen and Zhao [2013].

One interesting observation can be made by comparing Figure 8 and Figure 6b. In Figure 8, when the risk tolerance of the investor increases and she/he becomes risk neutral, the valuations are similar to the ones from Cost-of-capital method with a cost-of-capital of 3%. Since we assume a risk free rate of 3%, these two methods coincide provided complete market assumption holds.

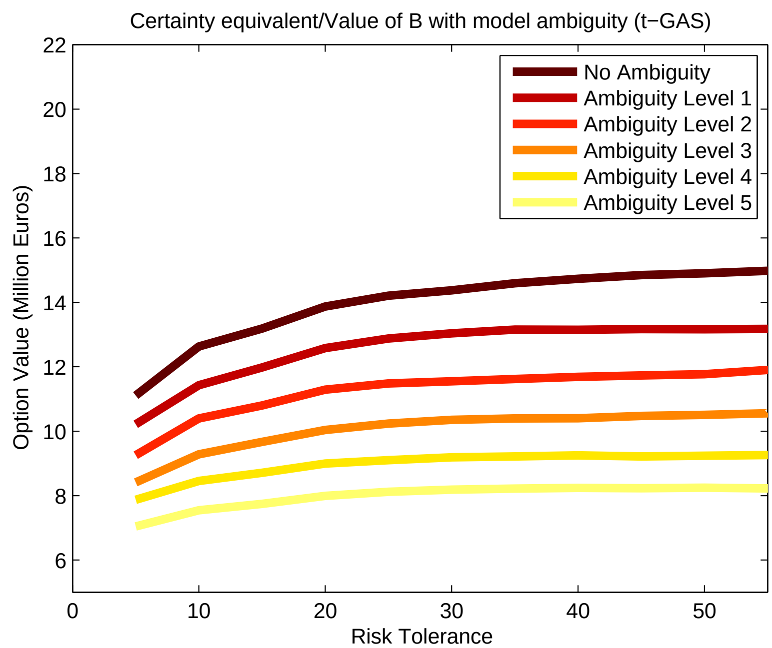
As one should expect, the UIP approach leads to a comparison between the outcomes based on the different stochastic specifications of the volatility processes that is similar to what we saw comparing the outcomes under different cost of capital values. For all levels of risk tolerance, the t-GAS based analysis leads to slightly higher valuations than the t-GARCH based approach due to higher estimated volatilities. We do not show the plots based on the GARCH results any further.

5.3 Model Ambiguity

In the discussion sofar we have proceeded on the assumption that specific values for the reserve levels were unknown, but that their probability distribution was known with full certainty. Of course that is overly optimistic: there is ambiguity about the distribution itself, model ambiguity in short. Therefore, in this subsection, we take this ambiguity into consideration and show how it affects the project valuation. We also assume that the investor is ambiguity averse, which means she/he considers the worst-case scenario when facing ambiguity. In technical terms, the investor follows a Minimax strategy: take the minimum value of the maximized outcomes/valuations over the different distributional possibilities (see Gilboa and Schmeidler [1989]). We assume in this example ambiguity on the mean of the reservoir distribution only, and we take the variance of the reservoir distribution as known from the geological structure of the locations. Initially we assume the same ambiguity on the reservoir sizes of both A and B. Of course we can apply similar methods based on the assumption of different ambiguity levels for A and B. Of special interest is the case where ambiguity levels get reduced when information becomes available half way the project. We consider that possibility explicitly in the next section, 5.3.1.

Figure 9 shows that the project value is declining with higher ambiguity levels. The levels chosen here are only for demonstration purpose. A higher ambiguity level means the decision maker is less certain about the mean of the reservoir distribution, which leads to a lower level of valuation due to the Minimax strategy followed. As can be seen, given a high level of ambiguity, the valuation differences between decision makers with different risk tolerance shrink accordingly.

Figure 9: Option Values with Persistent Model Ambiguity



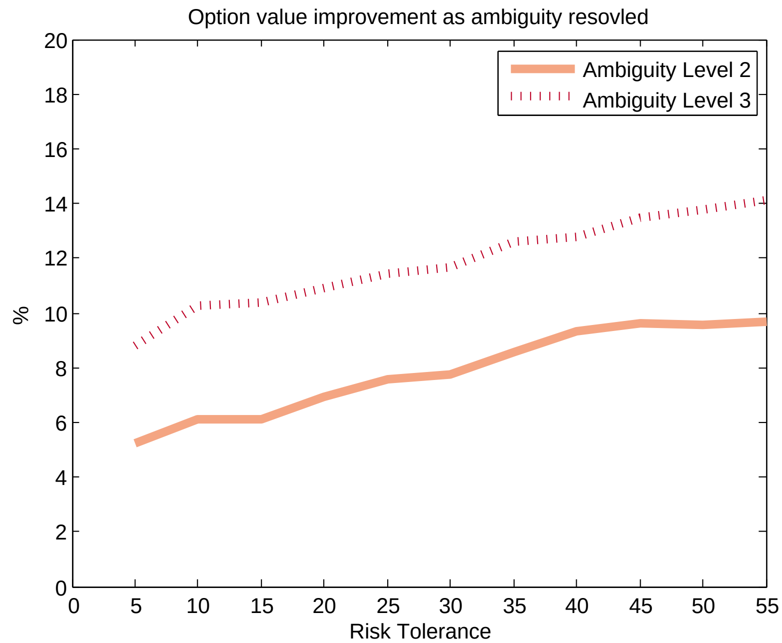
These results have interesting implications for insurance. The mirror image (up and down) of these graphs can be interpreted as how much the agent would be willing to pay for insurance against a certain risk the agent faces. It implies that for a given ambiguity level, risk averse agents are likely to buy an insurance product because they attach a high value to the insurance. On the contrary, risk tolerant agents would be less likely to buy such an insurance. On the other hand, for those people with the same risk tolerance, the decision of purchasing the insurance contract depends on her/his ambiguity level on the underlying processes. For example, people with high ambiguity levels tend to buy an insurance comparing to those with low ambiguity levels.

5.3.1 Project values when ambiguity is resolved halfway the process

In the preceding section, we introduced persistent ambiguity, i.e. uncertainty about the probability structure that remains constant over time. However it is more reasonable to assume that once production in B has started, more information about A, and more specifically, about the probability distribution of possible outcomes of A, will become available, since the geological structures of B and A are related. And declining ambiguity again brings in rewards for waiting, in a sense once again real option value. We explore the additional value project B gets if its exploration reduces ambiguity over well A once B is brought in production. In particular, we focus on reservoir A ambiguity only, and assume it gets resolved after starting on reservoir B. In other words, starting on B leads not only to more specific information but also narrows down the range of distributional possibilities.

For simplicity and focus we demonstrate the effect for the case where there is just ambiguity about A, which gets resolved once B is brought into operation. The no-ambiguity case is obviously the same as shown in Figure 9. But the interesting results come once we assume that starting on B leads to reduced ambiguity on A because the fields are contiguous. If the ambiguity level of A is at a particular level at the beginning and we know that ambiguity disappears after the development of B, then the difference between no-ambiguity and the project value at that particular Ambiguity-level should be added to the project value of B. 10 makes the point for the two moderate ambiguity level (Level 2 and Level 3): it shows the option values that resolution of ambiguity leads to as a percentage of the original project value of B with ambiguity persistent, and for different levels of risk tolerance.

Figure 10: Ambiguity in Field A Only (Student's t-GAS)



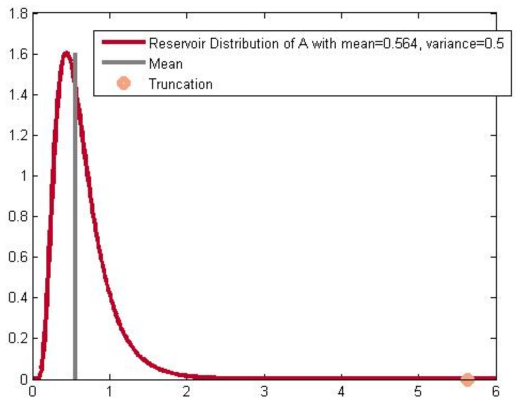
It is clear from Figure 10 that option values go up with risk tolerance and also increase as the initial ambiguity level that gets resolved is higher. And the option value numbers are substantial: in this example the increase in project value due to the reduction in ambiguity ranges between approximately 5 and 15% of the original project value depending on risk tolerance and level of pre-existing ambiguity.

5.4 Reservoir Correlation

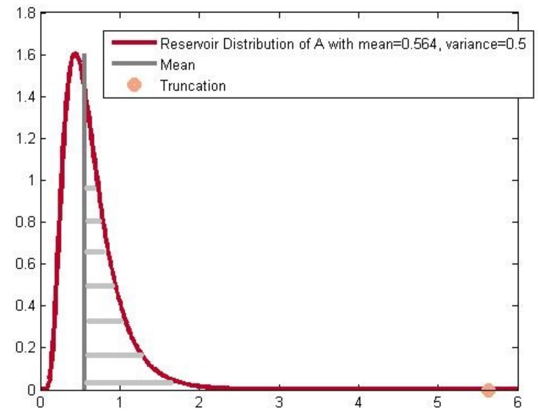
Finally we consider another plausible example of correlated information, for when the reservoir size of A follows a truncated lognormal distribution, with mean equal to the one mass point before, and with variance equal to 0.5, shown in Figure 11a. Assume now that if B turns out to be a successful development, the information about the reservoir size distribution of A will be updated correspondingly. This can be interpreted as one example of ambiguity reduction. Figure 11 shows some possible distribution updates for the distribution of A, with the original distribution given in Figure 11a. The following diagrams 11b-d show three different ways the distributional information could change: In Figure 11b we show how the distribution changes when the truncation point shifts inwards, the range of possible outcomes narrows down, as in the shadow area displayed in Figure 11b. Alternatively, the mean could shift, Figure 11c shows an example where the mean shifts up. Finally, mean and truncation points could be left unchanged but the variance could be reduced as information from B becomes available (Figure 11d). In what follows we focus on the case where the truncation point shifts inwards once B has started up, the case shown in Figure 11b, to demonstrate how our option technique works. We again present the results both for the Cost-of-capital approach and for UIP.

Figure 11: Reservoir Correlation Examples

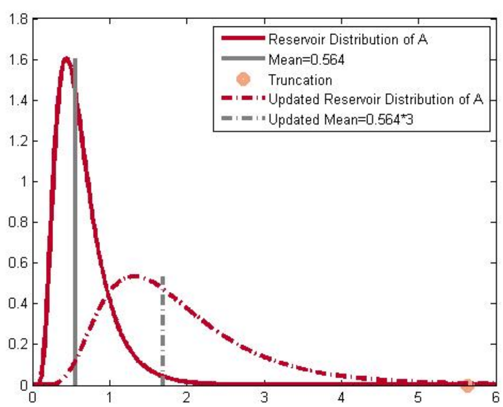
(a) A has a truncated lognormal distribution



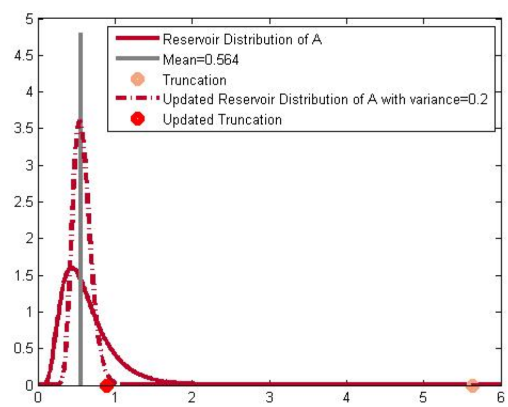
(b) Truncation Update



(c) Mean Update



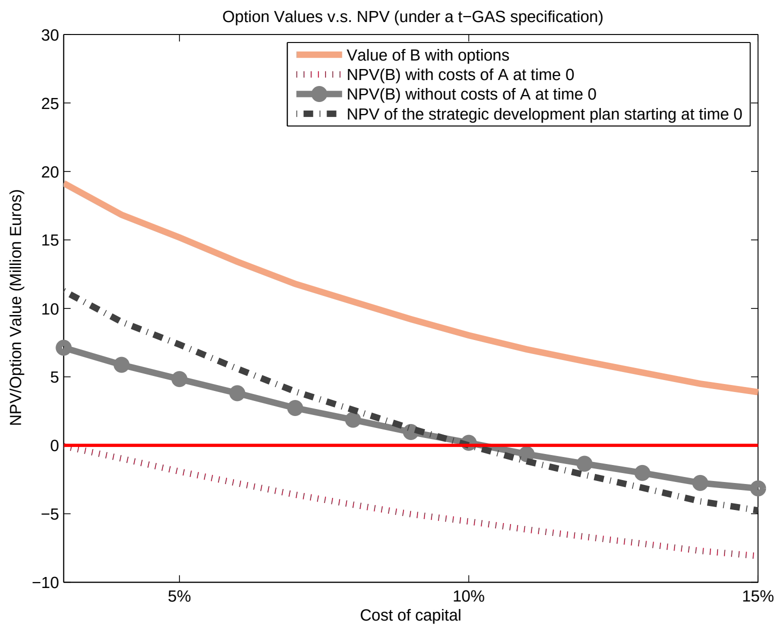
(d) Variance Update



5.4.1 Cost-of-capital method

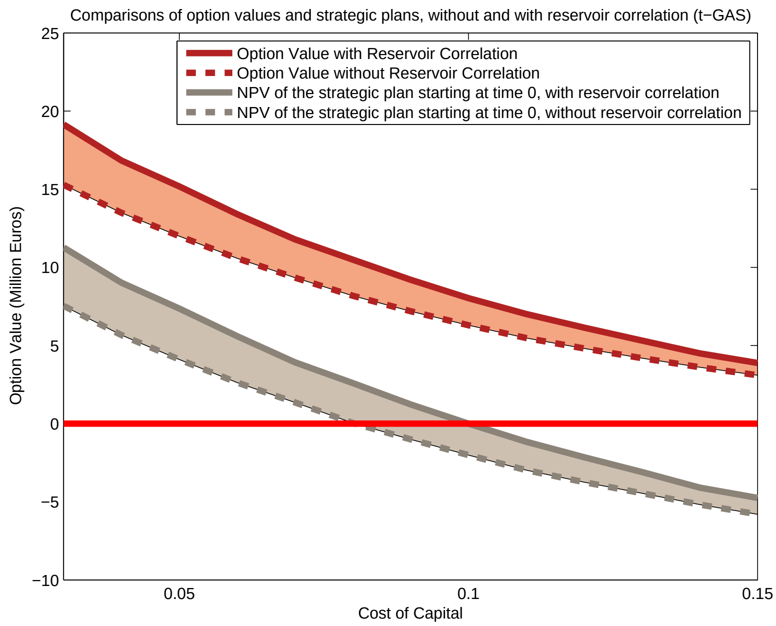
By comparing Figure 12 with Figure 6, one can easily find out that this reservoir correlation has increased the option value by about 1 to 5 million over the range of cost-of-capital rates considered (as also shown in Figure 13). The reservoir correlation of course does not change the NPV of B, therefore the red dashed line and the gray circled line stay the same as in Figure 6. Moreover, the strategic plan now outperforms the stand-alone project B for low cost-of-capital estimates; this happens for rates below 10% under a t-GAS specification.

Figure 12: Option Pricing Results for the General Case with Reservoir Correlation (Student's t-GAS)



Comparison with the case without reservoir correlation Furthermore, the shadow areas in Figure 13 represent the differences of values between the projects with and without reservoir correlation. It is evident that both the option and strategic plan are valued higher when reservoir correlation exists. In other words, the halfway resolution of reservoir distribution ambiguity/correlation increases the project value significantly by adding option value.

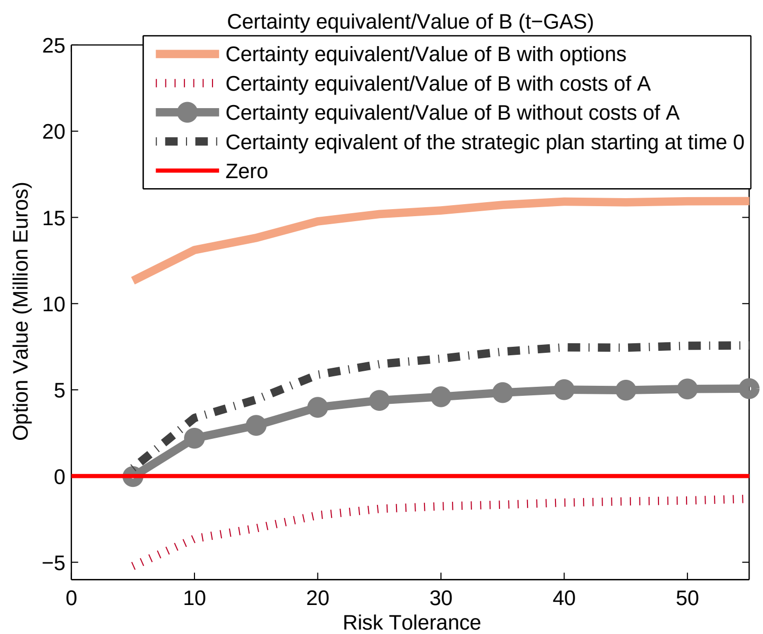
Figure 13: Comparison: Option Values With and Without Reservoir Correlation (Student's t-GAS)



5.4.2 Utility Indifference Pricing

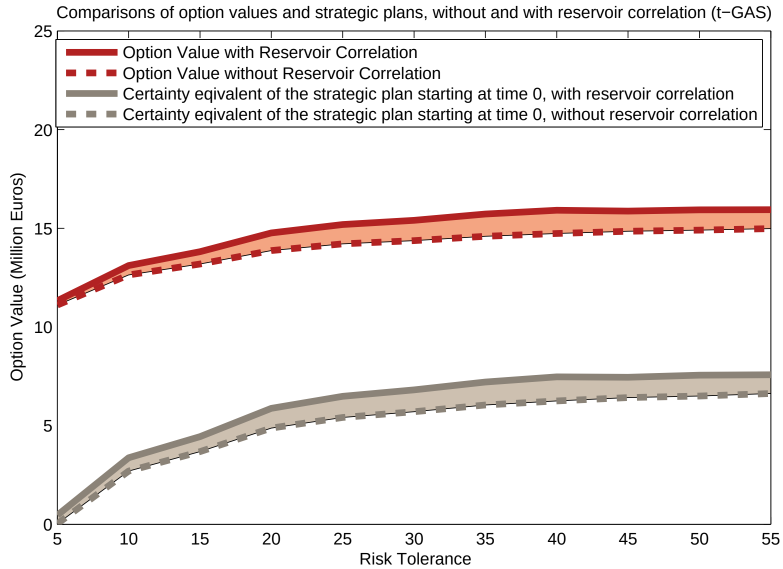
When we base the evaluation on UIP instead of on fixed cost-of-capital estimates, Similar to the comparison in Section 5.4.1, the strategic plan presented brings in more revenues (in NPV terms) than project B on its own, shown in Figure 14 and Figure 15. But the more important point is that reduction of uncertainty, this time a narrowing down of the range of possible outcomes, once again leads to substantial option values and correspondingly higher project value. Again, ignoring option values and information acquisition would lead to overly conservative project valuation and excessively conservative project decisions.

Figure 14: Utility Indifference Pricing Results for the General Case with Reservoir Correlation (Student's t-GAS)



Comparison with the case without reservoir correlation Similar to Figure 13, the shadow areas in Figure 15 represent the differences of values when comparing the projects with and without reservoir correlation.

Figure 15: Comparison: Option Values With and Without Reservoir Correlation (Student's t-GAS)



Once again, the fact that ambiguity on A is reduced once B has been brought into operation causes the option values to increase: the more future information gets updated as the project moves ahead, the higher the initial project value.

6 Conclusion

This paper has focused on the real option approach to solving a contingent claim problem as an alternative method for decision making under uncertainty. We incorporate many aspects that complicate asset pricing problems, such as incomplete markets and unhedgeable risks, dynamic release of distributional information and non-normal volatility assumptions, ll of which invalidate traditional risk neutral approaches to asset pricing. Utility indifference pricing is applied in face of market incompleteness and t-GARCH/t-GAS models are used for volatility modeling of gas prices. We show in a real world example that the Student's t-GARCH/ -GAS model, with its fatter tails, fits the observed data better than the Gaussian GARCH/GAS model in terms of loglikelihood ratio.

We also take the analysis one step further by introducing deep uncertainty, of the type that cannot be summarized by formulating a probability density function, because it concerns uncertainty about that very density function. In the literature this sort of uncertainty is referred to as Knightian uncertainty or, the word we prefer, model ambiguity. In our case study we show that the existence of model ambiguity reduces asset values in a risk averse world and will ceteris paribus lead to more conservative project continuation decisions. But we also introduce a new angle to this debate by pointing out that for time structured projects with correlated distributions, a new source of option value can emerge. If executing one part of the project leads to reduced model ambiguity concerning the later components of the project, the initial blocks acquire additional option values, which in our case study are shown to be substantial. As the ambiguity level decreases with project progress, the initial project becomes more valuable due to the information that will be brought in along with development. The value of projects that allow for that sort of flexibility will be underestimated consistently by more traditional NPV-based valuation approaches. In our real world case study, the biases are shown to be substantial.

Real option approaches have been known for a long time, but have by and large been dismissed in practice because real world problems quickly lead to what is called the curse of dimensionality. Their solution requires solving quintessentially non-linear stochastic dynamic optimization problems, and the numerical problems solving those become rapidly insurmountable as the problem's dimensionality increases. We demonstrate however that a dimension reduction approach long used in the solution of problems posed by the valuation of American options (i.e. options with endogenous exercise timing) can also be applied successfully to the stochastic dynamic programming (SDP) problems arising in complex high dimensionality real option problems. We analyze a real world case study, the valuation of two connected off shore gas fields in the presence of price uncertainty with variable volatility (which we analyze using the Generalized Autoregressive Score or GAS models), intertemporally correlated uncertainty and even model ambiguity concerning the reservoir size of the two connected gas fields. Our analysis shows very substantial payoffs to explicitly introducing asymmetric stochastic variance modeling, substantial option values in the presence of unhedgeable risks (although in that case we show them to depend on preferences) and the importance for decision making of taking into account (declining) model ambiguity. We show that traditional valuation approaches will consistently underestimate the value of project flexibility and in general lead to overly conservative investment decisions in the presence of time dependent stochastic structures.

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7 Appendix

7.1 GAS Models

7.1.1 Gaussian GARCH model

The model above can be reduced to a Gaussian GARCH model if $f_t = \sigma_t^2$ and $\varepsilon_t \sim N(0,1)$, i.e.

$$\begin{aligned} y_t &= \sigma_t \varepsilon_t \\ \sigma_{t+1}^2 &= \omega + A\left(y_t^2 - \sigma_t^2\right) + B\sigma_t^2 \end{aligned}$$

where ω, A, and $B - A$ are parameters in a classical Gaussian GARCH model.

7.1.2 Gaussian GAS model

Alternatively, if take $f_t = \log \sigma_t^2$, we obtain a Gaussian GAS(1,1) model, i.e.

$$\begin{aligned} y_t &= \sigma_t \varepsilon_t \\ \log \sigma_{t+1}^2 &= \omega + A\left(\frac{y_t^2}{\sigma_t^2} - 1\right) + B \log \sigma_t^2 \end{aligned}$$

In this model, next period's variance depends in a linear manner on a constant, the current period's variance and the square of the standardized observations, $\frac{y_t^2}{\sigma_t^2}$.

7.1.3 Student's t GARCH model

If the error term ε_t follows a Student's t distribution with degree of freedom ν, then it again becomes a t-GARCH model. Similarly, if we still fit it into a GAS framework, the model can be written as follows:

$$\begin{aligned} y_t &= \sigma_t \varepsilon_t \\ \sigma_{t+1}^2 &= \omega + A\frac{\nu+3}{\nu}\left(\left(1 + \frac{y_t^2}{\nu-2}\right)^{-1} \frac{\nu+1}{\nu-2} y_t^2 - \sigma_t^2\right) + B\sigma_t^2 \end{aligned}$$

7.1.4 Student's t GAS model

A Student's t GAS(1,1) model is obtained by choosing $f_t = \log \sigma_t^2$, and $\varepsilon_t \sim t(\nu)$.

$$\begin{aligned} y_t &= \sigma_t \varepsilon_t \\ \log \sigma_{t+1}^2 &= \omega + A\frac{\nu+3}{\nu}\left(\left(1 + \frac{y_t^2}{\nu-2}\right)^{-1} \frac{(\nu+1)\, y_t^2}{(\nu-2)\, \sigma_t^2} - 1\right) + B \log \sigma_t^2 \end{aligned}$$

7.2 Results under a specification of the gas price volatility process as a Student's t-GARCH model

Figure 16: Spot prices v.s. option value under a Student's t-GARCH specification

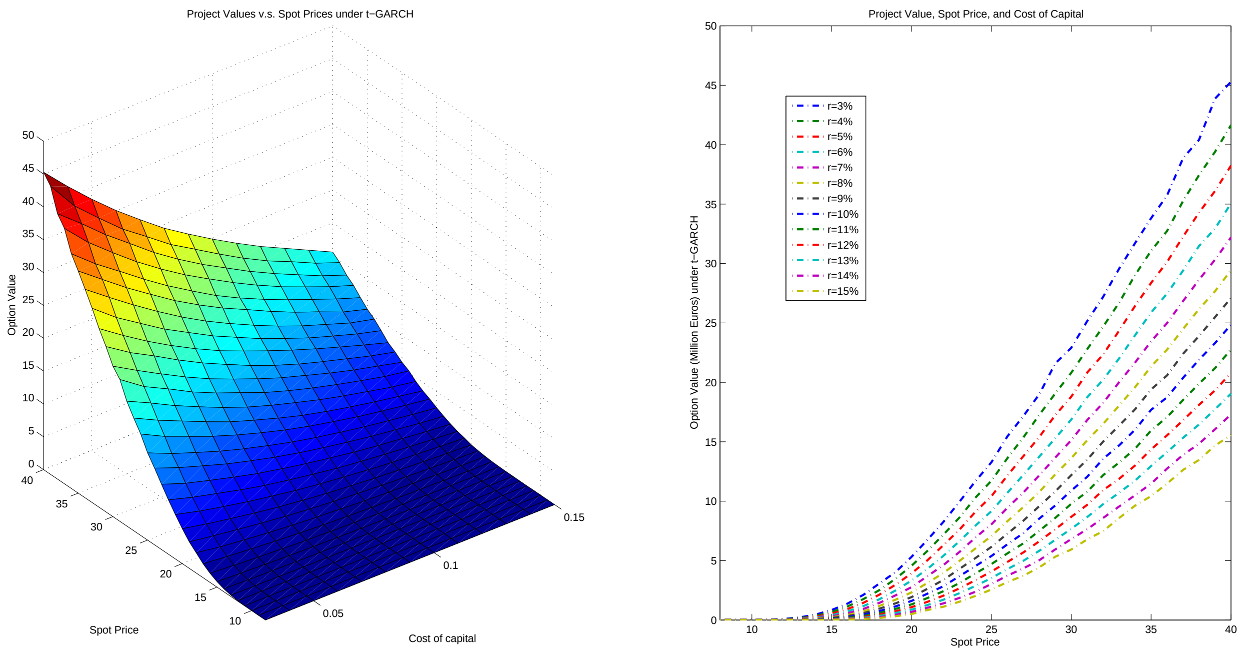


Figure 17: Utility Indifference Pricing (Student's t-GARCH)

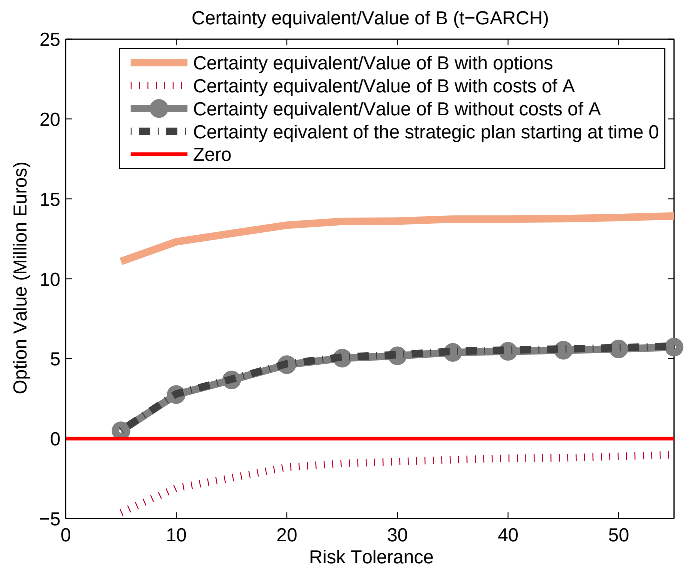


Figure 18: Option Values with Persistent Model Ambiguity (t-GARCH)

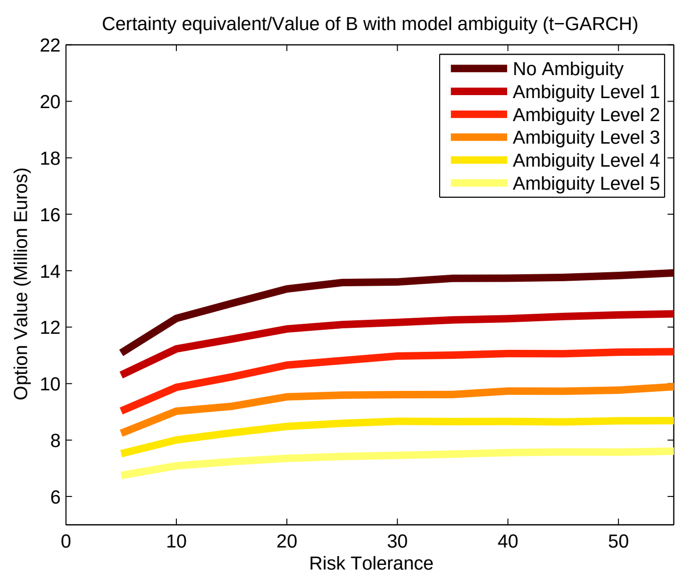


Figure 19: Ambiguity in Field A Only (t-GARCH)

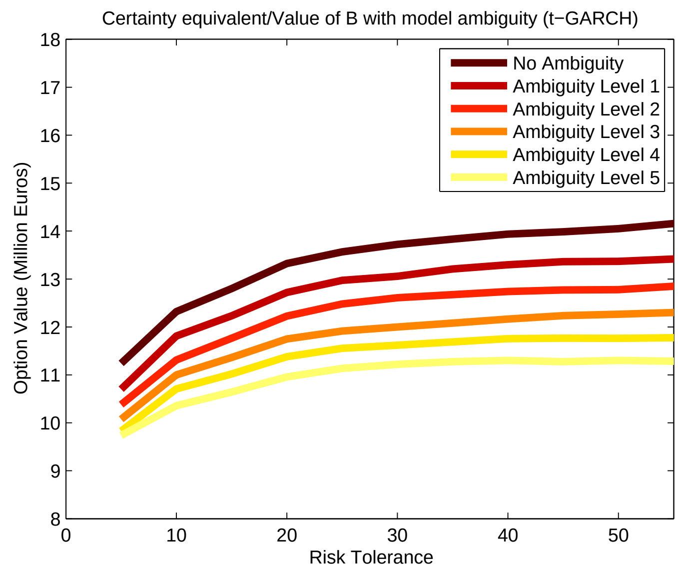


Figure 20: Option Pricing Results for the General Case with Reservoir Correlation (t-GARCH)

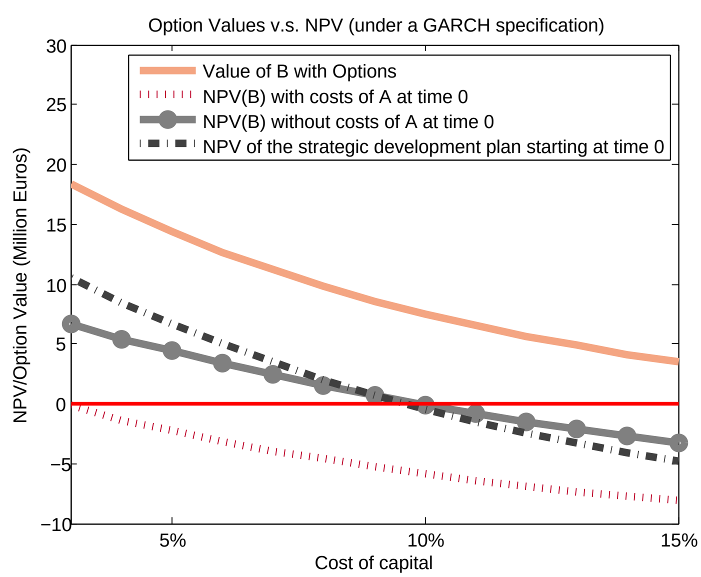


Figure 21: Comparison: Option Values With and Without Reservoir Correlation (Gaussian GARCH)

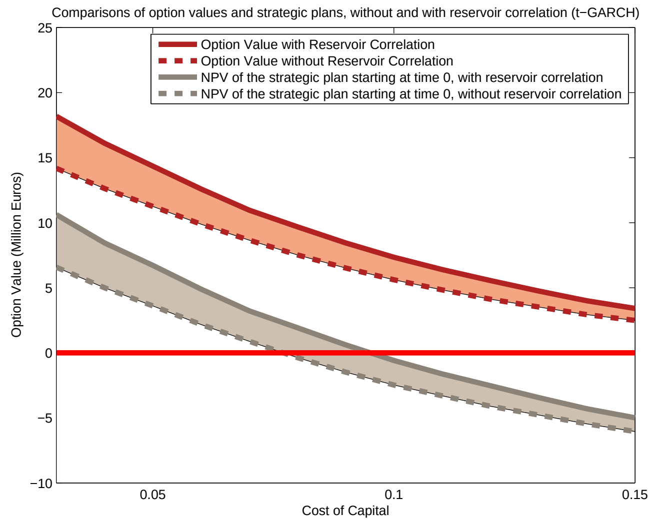


Figure 22: Utility Indifference Pricing Results for the General Case with Reservoir Correlation (t-GARCH)

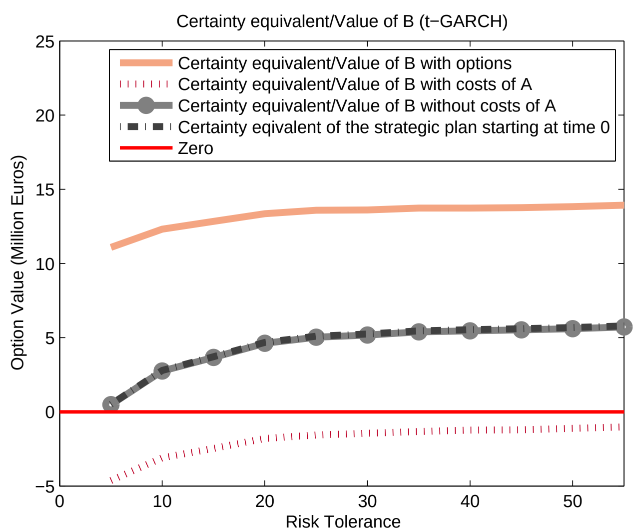
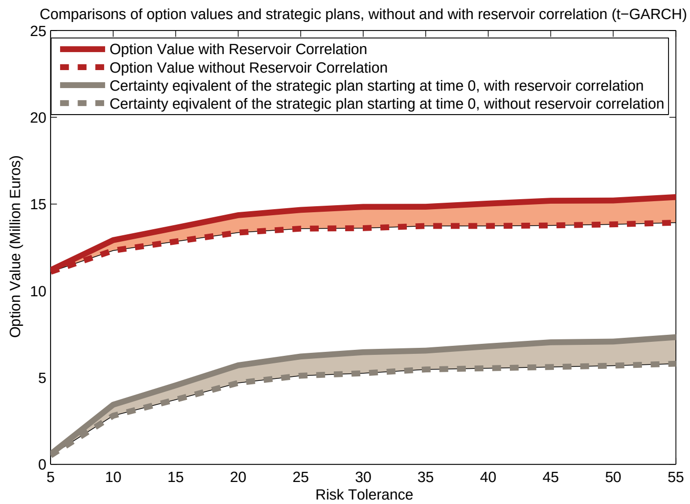


Figure 23: Comparison: Option Values With and Without Reservoir Correlation (t-GARCH)



7.3 Option Results in Details

Table 1: Option values versus NPV of B at time 0 (million euros)

Cost-of-capital	Student's t-GARCH			Student's t-GAS		
	Option Values	NPV of B at $t = 0$	Difference	Option Values	NPV of B at $t = 0$	Difference
3%	14.18	6.49	7.68	15.28	7.27	8.00
4%	12.62	5.41	7.21	13.49	5.90	7.59
5%	11.25	4.40	6.85	11.99	4.75	7.25
6%	9.89	3.35	6.54	10.57	3.67	6.91
7%	8.66	2.37	6.29	9.34	2.72	6.62
8%	7.52	1.36	6.17	8.17	1.66	6.51
9%	6.51	0.48	6.03	7.19	0.83	6.36
10%	5.61	-0.34	5.95	6.30	0.00	6.29
11%	4.83	-1.02	5.85	5.47	-0.79	6.26
12%	4.12	-1.70	5.82	4.81	-1.43	6.24
13%	3.52	-2.22	5.75	4.19	-2.04	6.23
14%	2.93	-2.81	5.74	3.61	-2.65	6.25
15%	2.48	-3.18	5.66	3.09	-3.28	6.37

Table 2: Utility Indifference Pricing Comparison (million euros)

Risk Tolerance	Option Value under a t-GARCH specification	Option Value under a t-GAS specification	Difference
5	11.09	11.11	-0.02
10	12.31	12.63	-0.32
15	12.84	13.19	-0.35
20	13.35	13.87	-0.52
25	13.58	14.21	-0.63
30	13.60	14.37	-0.77
35	13.73	14.60	-0.86
40	13.74	14.74	-1.00
45	13.76	14.85	-1.08
50	13.83	14.90	-1.08
55	13.92	14.98	-1.06

Table 3: Model Ambiguity

(a) Student's t-GARCH

Risk Tolerance	Ambiguity Level (Left to Right, Low to High)					
	No Ambiguity	Level 1	Level 2	Level 3	Level 4	Level 5
5	11.09	10.30	9.03	8.24	7.52	6.74
10	12.31	11.23	9.87	9.02	8.00	7.09
15	12.84	11.58	10.24	9.20	8.26	7.24
20	13.35	11.93	10.65	9.53	8.48	7.35
25	13.58	12.09	10.82	9.59	8.59	7.42
30	13.60	12.17	10.98	9.61	8.67	7.46
35	13.73	12.26	11.01	9.62	8.65	7.50
40	13.74	12.30	11.07	9.74	8.66	7.56
45	13.76	12.38	11.06	9.73	8.64	7.58
50	13.83	12.43	11.11	9.77	8.68	7.57
55	13.92	12.47	11.13	9.89	8.69	7.60

(b) Student's t-GAS

Risk Tolerance	Ambiguity Level (Left to Right, Low to High)					
	No Ambiguity	Level 1	Level 2	Level 3	Level 4	Level 5
5	11.11	10.21	9.25	8.41	7.87	7.05
10	12.63	11.43	10.39	9.29	8.46	7.55
15	13.19	11.98	10.80	9.67	8.71	7.74
20	13.87	12.58	11.29	10.04	9.00	7.99
25	14.21	12.88	11.48	10.24	9.10	8.12
30	14.37	13.04	11.55	10.35	9.19	8.19
35	14.60	13.16	11.62	10.40	9.22	8.22
40	14.74	13.15	11.69	10.40	9.25	8.24
45	14.85	13.17	11.73	10.47	9.22	8.23
50	14.90	13.16	11.77	10.51	9.24	8.25
55	14.98	13.18	11.90	10.56	9.26	8.22

Table 4: Model Ambiguity of A Only

(a) Student's t-GARCH

Risk Tolerance	Ambiguity Level (Left to Right, Low to High)					
	No Ambiguity	Level 1	Level 2	Level 3	Level 4	Level 5
5	11.09	10.70	10.38	10.08	9.83	9.74
10	12.31	11.81	11.31	11.00	10.71	10.36
15	12.84	12.23	11.77	11.36	11.02	10.64
20	13.35	12.72	12.23	11.75	11.38	10.96
25	13.58	12.97	12.48	11.92	11.55	11.13
30	13.60	13.06	12.61	12.00	11.62	11.22
35	13.73	13.21	12.67	12.08	11.69	11.27
40	13.74	13.30	12.74	12.16	11.75	11.30
45	13.76	13.36	12.77	12.24	11.76	11.28
50	13.83	13.37	12.78	12.26	11.76	11.30
55	13.92	13.42	12.85	12.30	11.78	11.28

(b) Student's t-GAS

Risk Tolerance	Ambiguity Level (Left to Right, Low to High)					
	No Ambiguity	Level 1	Level 2	Level 3	Level 4	Level 5
5	11.11	10.89	10.56	10.21	9.96	9.71
10	12.63	12.16	11.90	11.46	10.98	10.75
15	13.19	12.75	12.43	11.95	11.51	11.22
20	13.87	13.36	12.97	12.51	12.00	11.69
25	14.21	13.66	13.21	12.75	12.27	11.87
30	14.37	13.86	13.34	12.87	12.42	12.01
35	14.60	14.01	13.45	12.96	12.49	12.08
40	14.74	14.11	13.48	13.07	12.49	12.13
45	14.85	14.21	13.54	13.08	12.56	12.17
50	14.90	14.26	13.61	13.10	12.62	12.25
55	14.98	14.35	13.66	13.13	12.70	12.27

Table 5: Option values v.s NPV of B at time 0 (million euros) with reservoir correlation

Cost-of-capital	Student's t-GARCH			Student's t-GAS		
	Option Values	NPV of strategic plan starting at $t = 0$	Difference	Option Values	NPV of strategic plan starting at $t = 0$	Difference
3%	18.18	10.62	7.57	19.14	11.27	7.87
4%	16.11	8.44	7.66	16.83	9.01	7.82
5%	14.35	6.72	7.63	15.18	7.36	7.82
6%	12.59	4.90	7.69	13.40	5.59	7.81
7%	10.96	3.22	7.74	11.79	3.92	7.87
8%	9.68	1.92	7.76	10.49	2.59	7.90
9%	8.44	0.61	7.83	9.20	1.22	7.98
10%	7.35	-0.59	7.94	8.03	-0.01	8.04
11%	6.39	-1.62	8.01	7.01	-1.16	8.17
12%	5.55	-2.53	8.08	6.14	-2.16	8.30
13%	4.75	-3.44	8.19	5.31	-3.10	8.41
14%	3.99	-4.30	8.29	4.50	-4.10	8.60
15%	3.41	-5.00	8.41	3.88	-4.76	8.64

Table 6: Utility Indifference Pricing Comparison (million euros) with Reservoir Correlation

Risk Tolerance	Option Value under a Student's t-GARCH specification	Option Value under a Student's t-GAS specification	Difference
5	11.19	11.33	-0.14
10	12.92	13.11	-0.19
15	13.63	13.81	-0.18
20	14.35	14.76	-0.41
25	14.66	15.19	-0.53
30	14.84	15.40	-0.56
35	14.84	15.72	-0.88
40	15.02	15.92	-0.90
45	15.19	15.87	-0.68
50	15.20	15.94	-0.74
55	15.40	15.94	-0.54

Figure 24: Least Square Monte Carlo Method (Longstaff and Schwartz [2001])