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Market structure and the pricing of new products: a nested logit approach with asymmetric firms ☆

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Abstract

This article investigates competition in a market with an emerging technology using a discrete choice model to analyze demand and welfare. We focus on industry structure and investigate the impact of different market structures on demand for the new technology and on welfare. The car market serves as a prime example of such a market, where electric vehicles (EV's) represent the new technology competing with standard cars with internal combustion engines (ICV's). To analyze such a market, we use a nested logit model. In contrast to earlier literature, we allow firms to be asymmetric and active in multiple nests, with different numbers of variants in each nest, which can add up to any market share. Additionally, we add to existing literature by considering the case where substitutability between firms is stronger than between technologies, by nesting products by technology instead of by firm. We find implicit analytical solutions for the equilibrium mark-ups which can be used when there are two nests in the market; within that restriction firms can be asymmetric. Numerically, we find that EV sales are higher if offered by a new entrant only selling EV's as opposed to when it is supplied by a firm selling variants of both types. We present an index based on mark-up differences between variants in the market, which can be used to a priori determine whether a change in market structure would increase or decrease welfare. These results are general to the nested logit model, and the index can thus be used in any market, as long as the market is sufficiently accurately described by the nested logit model.

Keywords: Nested logit model, asymmetry, market structure, welfare indices, emerging technology
JEL Codes: D430, D600, L110, L910

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1. Introduction

We investigate competition in a market with an emerging technology using a discrete choice model. We focus on industry structure and investigate the impact of different market structures on demand for the new technology and on welfare. We use the car market as an illustration, where electric vehicles (EV's) represent the new technology competing with standard cars with internal combustion engines (ICV's). To analyze this market, we use a nested logit model.

Many interesting papers have been written on the nested logit model, however most of these focus on finding symmetric equilibria. This assumption is not realistic in most markets, especially markets with an emerging technology. Therefore, our model explicitly allows firms to be asymmetric.

The nested logit was first introduced by Ben-Akiva (1973) as a sequential choice model. McFadden (1987) showed that the nested logit can also be derived non-sequentially using a random utility model where the random component of utility has a generalized extreme value distribution. This is the derivation mainly used today. The nested logit builds upon the multinomial logit model, which was first introduced by Luce (1959), and subsequently developed by McFadden (1974).

When the nested logit model is analyzed analytically in the context of competition, it is usually assumed that each firm 'owns' a nest of products, so that consumers perceive the degree of differentiation between products of the same firm to be smaller than between products of different firms. Industry structure in the nested logit with symmetric firms all owning one nest has been investigated by Anderson et al. (1992). They find that more firms enter the market when the degree of heterogeneity between products in different nests increases; and these firms offer fewer variants each. The increase is akin to higher customer loyalty, increasing prices and profit. This causes new firms to enter, so that the existing firms reduce the number of variants offered. Conversely, when the degree of heterogeneity between products in the same nest increases, fewer firms will enter the market; however, they will offer more variants each. The intuition is similar.

Anderson et al. (1992) also compare this market equilibrium to the social optimum. The market solution has too many firms in the market, offering too few products. The total number of products is too low compared to the social optimum. Varela-Irimia (2012) analyzes a model in which firms are allowed to be active in multiple nests. However, here the additional assumption is made that the market share of the firm in each nest is the same. This assumption allows for simpler expressions, however it reduces the model in essence to a multinomial logit model in multiple markets, which only differ in their market size. Not surprisingly, the paper finds that firm profit in this model is equal to firm profit in a pure multinomial setting.[1]

[1] Many other papers have looked at nested logit model from an analytical standpoint. They focus on aspects of the model other than industry structure. Anderson & de Palma (1992),

Our analysis differs from the previous papers in that we relax the assumption of symmetric firms. We allow firms to be active in multiple nests and to own different amounts of variants in each nest, which can add up to any market share.

Introducing asymmetry comes at a cost, namely that explicit analytical solutions for prices and profit do no exist in this model. However, we will present implicit solutions for equilibrium mark-ups. Additionally, we find explicit equilibria numerically.

As mentioned before, we use the car market as an illustration in this paper, but the model can be applied to any market which is sufficiently accurately described by the nested logit model. Specifically, our model can be used to investigate what happens when a new technology is competing with an established technology in the market.

Additionally, we add to existing literature by considering the case where substitutability between firms is stronger than between technologies. Since we focus on the effect of an emerging technology, we nest the products in a different way than usually done in the literature; we group products into nests by technology instead of by firm. One nest consists of ICV's, the second nest comprises of EV's.

Each consumer has an idiosyncratic preference for a technology and variant. For example, some consumers might be very environmentally-conscious and therefore prefer EV's. Other consumers might value being ahead of the curve and have a preference for new technologies, while others might be afraid to try new technologies and prefer established ones. Beggs et al. (1981) study the factors influencing willingness to pay for vehicles and find that the dispersion in parameters is highest for an EV dummy, meaning that the value consumers attach to a car, simply because it is an EV or an ICV, varies substantially within the population.[2]

We find an implicit solution for equilibrium mark-ups which can be used when there are two nests in the market; within that restriction firms can be asymmetric. There can be any number of variants, any number of firms, these firms can be active in one or two nests and can have any market share in these nests.

Then, we numerically investigate the effect of market structure on equilibrium outcomes; specifically demand for the new technology and welfare. To isolate the effect of market structure, the number of varieties in the market is fixed. To simulate the fact that EV's are relatively new to the market, it is assumed that there are more ICV varieties than there are EV varieties. We then compare equilibrium outcomes for different possible market structures, ranging

Mizuno (2003), Liu (2006), Li & Huh (2011) and Gallego & Wang (2013) amongst others focus on the existence and uniqueness of equilibria. Anderson et al. (1987), Anderson et al. (1988) and Verboven (1996) have looked at similarities between the logit model and representative consumer models.

[2]Many more paper have looked into demand for EV's. For example Dagsvik et al. (2002); Crist (2012).; Dimitropoulis et al. (2013)

from monopoly to a situation where each firm only produces one variant.

We find that EV sales are higher if offered by a new entrant only selling EV's as opposed to a firm selling variants of both types; as a firm selling both types can recoup some of the lost EV demand with its ICV demand. Additionally, EV sales are higher if the ICV's in the market are offered by fewer (and thus more concentrated) firms. Not surprisingly, firm profit is highest when the market is concentrated and only a small number of firms sell the available variants. On the contrary, consumer surplus is higher when there are more firms in the market.

We also investigate which industry structure would be best for total welfare. We present an index which can be used to a priori determine whether a change in market structure would increase or decrease welfare.

As stated before, these results are general to the nested logit model, and the index can thus be used in any market, as long as the market is sufficiently accurately described by the nested logit model.

The next section introduces the model. In section 3 we discuss analytical results. Section 4 discusses the numerical equilibrium. Finally, the paper concludes and discusses directions for further research.

2. Model setup

The market for personal transport vehicles is analyzed using a nested multinomial logit model. As opposed to the standard multinomial logit model, the nested logit allows for different degrees of substitutability between products, by grouping similar products into 'nests'. Consumers perceive products within a nest as closer substitutes than products in different nests.

In the basic model, consumer m has the following utility function

$$U_{j,i} = a_{ev} - ap_{j,i} + \varepsilon_{j,i,m} \tag{2.0.1}$$

Here a represents the marginal utility of income, which is assumed to be equal and constant for all consumers. $p_{j,i}$ represents the price of product j in nest i. a_{ev} is a dummy parameter that represents the relative attractiveness of EV cars. It will always be zero for ICV cars. A negative value for a_{ev} means that utility of the EV is relatively lower, for example because of lacking infrastructure around EV's. The random component, $\varepsilon_{j,i,m}$, represents consumer m's intrinsic preference for each car. This value is fully known to the consumer. Consumers make rational decisions and buy the product that gives them the highest utility. Firms, however, do not know the value of $\varepsilon_{j,i,m}$ for each individual consumer; they only know the distribution over all consumers. We assume a generalized extreme value distribution for $\varepsilon_{j,i,m}$, leading to the nested multinomial logit model.

As stated before we are specifically interested in how industry structure affects welfare. This means that we are interested in the relative values for demand and welfare over the different industry structures; and not in the absolute values in any one structure. This allows us to keep the utility function very simple. We only deterministically include price and an EV parameter, while other product characteristics are captured by the random component. It allows us to investigate the effect of market structure on equilibrium outcomes, while keeping the model simple and tractable.

Since we focus on competition when an emerging technology is present, we group products into nests by technology. One nest comprises of ICV cars, and another of EV cars.

There is also a third nest where the firms in the market are not active. It captures an (exogenously and socially optimally priced) outside option produced under constant returns to scale, which can be interpreted as a generalized measure of all other transport modes.[3]

The market is assumed to be of size one, so that demand for each product is simply given by the probability of buying that product. Demand for each product j in nest i is given by

$$Pr(j,i) = Pr(j|i)Pr(i) \tag{2.0.2}$$

Here $Pr(j,i)$ represents the probability of buying product j from nest i. $Pr(j|i)$ represents the demand of product j, conditional on nest i being selected, and $Pr(i)$ is the probability that a product from nest i is selected. The conditional demand is given by

$$Pr(j|i) = \frac{\exp(\frac{a_{ev} - ap_{j,i}}{\mu})}{\sum_{j \in S(i)} \exp(\frac{a_{ev} - ap_{j,i}}{\mu})} \tag{2.0.3}$$

$S(i)$ represents the set of products in nest i, μ represents the heterogeneity in unobserved product characteristics of preferences between products within one nest.

Demand for a nest is given by

$$Pr(i) = \frac{\exp(\frac{v_i}{\theta})}{\sum_i \exp(\frac{v_i}{\theta})} \tag{2.0.4}$$

Here v_i represents the maximum expected utility from nest i. It is given by

$$v_i = \mu \ln \sum_{j \in S(i)} \exp\left(\frac{a_{ev} - ap_{j,i}}{\mu}\right) \tag{2.0.5}$$

θ is the heterogeneity of unobserved preferences between nests. Let $\theta \geq \mu$, heterogeneity of preferences between nests must be at least as large as between products within one nest. Note that as θ goes to infinity $Pr(i)$ goes to $\frac{1}{n}$, where n is the number of nests in the model. When θ goes to infinity, differentiation

[3] The outside good is assumed to be a homogeneous good that is assumed to be supplied by many infinitesimal perfectly competitive firms, that all sell the product at the same price, which is equal to their (equal) marginal costs.

between products in different nests goes to infinity, in the sense that deterministic utility, consisting of price and the EV dummy, plays a negligible part in the decision of which nest to buy from. The probability of a consumer buying from each nest then becomes $1/n$. Furthermore, when μ goes to zero, utility is determined solely by the deterministic utility, since differentiation between products in the same nest is negligible. Thus the probabilities become deterministic, where $Pr(j|i)$ is 1 for the alternative with the highest deterministic utility, and zero for the others.
Firms are assumed to maximize profits, which are given by

$$\pi_k = \sum_i \sum_{j \epsilon S(i)} (p_{j,i} - c_{j,i}) Pr(j,i) \delta^k_{j,i} \tag{2.0.6}$$

Here, $\delta^k_{j,i}$ is an indicator value that equals 1 if firm k sells product j in nest i, and 0 otherwise. c_{ji} is the marginal cost of product j in nest i.

We will investigate welfare measures in the different industry structures. Consumer surplus is given by the log-sum of the expected utility of each nest:

$$CS = \frac{1}{a} \theta \ln \left[\sum_i \exp\left(\frac{v_i}{\theta}\right) \right] \tag{2.0.7}$$

Adding consumer surplus and the profits of all firms gives aggregate welfare.

$$W = CS + \sum_k \pi_k \tag{2.0.8}$$

From this equation we can find the maximum possible welfare, W_{max}, by optimizing it with respect to all prices as a social planner would set them. As an indicator for efficiency, we use the aggregate welfare divided by the maximum possible welfare.

$$E = \frac{W}{W_{max}} \tag{2.0.9}$$

3. Analytical results

3.1. Mark-up

Firms maximize profit with respect to prices. We cannot derive explicit results for equilibrium prices; however we can find implicit results, where equilibrium mark-up is expressed in terms of choice probabilities. These expressions give us the equilibrium relation between quantities and prices, which would also be very useful for empirical analysis. After maximizing and some rearranging we find the following first order condition (for derivation, we refer to Appendix 1), for the mark-up m^k_i:

$$m^k_i = \frac{\mu\theta^2 \left[1 - s(i',k)\right] + \mu^2 \theta s(i',k)}{a \left[\prod_i \left\{\theta \left[1 - s(i,k)\right] + \mu \left[1 - Pr(i)\right] s(i,k)\right\} - \prod_i \left\{\mu Pr(i) s(i,k)\right\}\right]} \tag{3.1.1}$$

In appendix 2, we show that this equilibrium is unique. This relation holds if there are two nests in which firms are active, with any number of variants and firms active in those nests. $s(i, k)$ represents the market share firm k has in nest i:

$$s(i, k) = \sum_{j \in S(i)} \left\{ Pr(j|i)\delta^k_{j,i} \right\} \tag{3.1.2}$$

Our result for mark-up (3.1.1) is not easy to interpret; however we can instantly see that the mark-up of products in the same nest and produced by the same firm will be the same. In particular, note that the mark-up only depends on market share within a nest, $s(i, k)$, and on the demand for a nest, $Pr(i)$; it does not depend on any variables that are specific to products, only to nests. Note that the denominator is equal for all products of the same firm, irrespective of the nest. Therefore, the differences in mark-ups between products in different nests owned by the same firm are caused by differences in the numerator. As a result, the differences between firms are caused by the relative market shares in each nest. We can see this more clearly if we derive the relative mark-up for products of firm k directly, by finding the ratio of its mark-ups in the different nests.

$$\frac{m^k_i}{m^k_{i'}} = \frac{[\theta + (\mu - \theta)s(i', k)]}{[\theta + (\mu - \theta)s(i, k)]} \tag{3.1.3}$$

One has to keep in mind that this formula does not give a causal relationship between market share and mark-ups, but rather shows us how these two compare in equilibrium. The formula shows that in equilibrium, firms set prices such that higher mark-ups in a nest correspond with higher market share in that nest.

The mark-up result 3.1.1 is a more general version of the result found by Anderson et al. (1992, pp. 251). They show that in the situation where each firm owns all products in one nest (and only in that nest), the mark-up is equal to:

$$m^k_i = \frac{\theta}{1 - Pr(i)} \tag{3.1.4}$$

We can analyze the same situation with our expression for mark-up above by setting $s(i, k) = 1$ and $s(i', k) = 0$, $a = 1$. Then we find:

$$m^k_i = \frac{\mu\theta^2}{\mu\theta\,[1 - Pr(i)]} = \frac{\theta}{1 - Pr(i)} \tag{3.1.5}$$

Thus, our model generalizes the model of Anderson et al. (1992), and when firms are completely symmetric, our result reduces to the result found there. However, we emphasize that our result also applies to situations where multiple firms are present in one nest, and when firms supply products in multiple nests.

3.2. Lerner-index

To further understand how firms set mark-up, we may rewrite 3.1.1 in a way it references the well-known Lerner index. First, we find aggregate elasticities and then substitute them in the mark-up expression 3.1.1. The derivations of the elasticities can be found in Appendix 3. The final result takes the form of an adjusted Lerner index.

$$\frac{p_{j,i} - c_{j,i}}{p_{j,i}} = \frac{1}{-\epsilon_{own,i,k}} + \frac{p_{j,i'} - c_{j,i'}}{p_{j,i'}} \frac{\epsilon_{cross,i,k}}{-\epsilon_{own,i,k}} \tag{3.2.1}$$

This is an extended version of the standard Lerner index for monopoly. It is also an adaptation of the Lerner index for multi-product monopoly in a Cournot setting, which is given by (Belleflamme & Peitz, 2010, pp. 28):

$$\frac{p_j - c_j}{p_j} = \frac{1}{\epsilon} + \frac{p_{j'} - c_{j'}}{p_{j'}} \frac{\partial Q_j / \partial p_j}{-\partial Q_{j'} / \partial p_j} \tag{3.2.2}$$

Compare this to our expression which can be rewritten as:

$$\frac{p_{j,i} - c_{j,i}}{p_{j,i}} = \frac{1}{-\epsilon_{own,i,k}} + \frac{p_{j,i'} - c_{j,i'}}{p_{j,i'}} \frac{\partial Pr(j,i)/\partial p_{j,i'}}{-\partial Pr(j,i)/\partial p_{j,i}} \frac{p_{j,i'}}{p_{j,i}} \tag{3.2.3}$$

The expressions 3.2.2 and 3.2.3 are very similar. However, in our formula the additional term comes from multiple nests, not from multiple products. Additionally, the derivatives in the additional term vary in which quantity is used in the Cournot expression, and in which price is used in ours. This stems from the fact that in our model firms maximize profit over price, not quantities.

Equation 3.2.1 is adjusted to reflect the fact that there are multiple products per firm, and that the products are in different nests. The interpretation of the equilibrium mark-up is now much simpler. Mark-up is given by the standard inverse elasticity of demand,[4] plus an additional term to reflect that the firm offers products in multiple nests. If a firm would produce products in one nest only, this additional term equals zero. Otherwise, since the products are strategic substitutes, the extra term is positive. Thus, when a firm produces products in two nests, its mark-up increases. The intuition is that lost market share from an increase in the price in one nest is partly compensated by extra revenue from sales in the other nest. The ratio of elasticities in the additional term shows that the mark-up increases when the positive effect of a price change in another nest is larger than the negative effect of a price change in the current nest.

We can use this expression to explore what happens in two particular cases, monopoly and a situation with many infinitesimal firms. In monopoly both $s(i,k)$ and $s(i',k)$ are equal to one. If we use these values in equation 3.2.1, and substitute the expression for the Lerner index of the other nest and simplify, we find

$$p_{j,i} - c_{j,i} = \frac{\theta}{a\left[1 - Pr(i) - Pr(i')\right]} = \frac{\theta}{aPr(0)} \tag{3.2.4}$$

Here $Pr(0)$ represents demand for the outside option. The same expression holds for products in the other nest. The formula shows that the mark-up of all

[4] Note that this is the aggregate elasticity, where all prices of the same firm in the same nest change by the same amount.

products will be equal, irrespective of the nest. When perceived differentiation between nests (θ) increases, the mark-up will also increase; since this reflects the decrease in substitutability with the outside good. Intuitively, mark-up depends negatively on a; as utility decreases faster with price, firms set lower prices. Additionally, there is a negative relation between the demand for the outside option and mark-up.

As before, recall that 3.2.4 is an implicit expression, thus we cannot say that as demand for the outside product decreases, the mark-up will increase. Rather, these are two effects that may be caused by a third effect, for example decreased attractiveness of the outside option. Whenever the outside option is less attractive, the monopolist has more market power and will therefore set a higher price. Additionally, note that the mark-up in 3.2.4 does not depend on any within-nest variables, such as μ and $Pr(j|i)$.

The opposite is true for the case of very small firms, when $s(i,k)$ and $s(i',k)$ approach zero. When we set both to zero in 3.2.1, the expression becomes a constant:

$$p_{j,i} - c_{j,i} = \frac{\mu}{a} \tag{3.2.5}$$

In this case, the expression does not depend on any between-nest parameters (note, however, that μ is equal to θ in nests with only one variant). Firms do not take competitors in other nests into account, as competition within the nest fully dominates for infinitesimal firms. Each firm sets the same mark-up for its products, which increases in the perceived differentiation of products within a nest, and decreases in the marginal utility of money. Note that the mark-up is not equal to zero, since products are differentiated. μ gives the degree of differentiation in non-price variables. When μ increases, the mark-up increases; since products are more differentiated. Conversely, when μ is equal to zero, differentiation disappears and all products are priced at marginal cost.[5]

3.3. Welfare rankings

We now investigate the effect of industry structure on welfare. In particular, in this section we discuss various indices that can be used to asses two industry structures in terms of expected welfare. For policy purposes, it would be very important to have a predictor of the efficiency of different market structures in terms of the pricing and uptake of new technologies. This is even more important when welfare cannot be computed directly, for example because (marginal) cost cannot be observed. One index competition agencies use is the Herfindahl-index (HHI), which was developed independently by Hirschman (1945) and Herfindahl (1950). It measures the degree of concentration in a market:

$$HHI = \sum_k \left(\sum_i \sum_{j \in S(i)} Pr(j,i)\delta_{ji}^k \right)^2 = \sum_k (s_k)^2 \tag{3.3.1}$$

[5]For a discussion of what happens when μ and θ take extreme values, see section 2.

The Herfindahl-index sums the squared market shares of all firms in the market. In monopoly, the index should be one. When the market consists of many small firms, the index will approach zero. A higher index thus indicates a more concentrated market.

Alternatively, we could calculate the Herfindahl only for the two product nests. In our model this would mean considering the car firms only and excluding outside option. We have assumed the latter is produced by many infinitesimal firms, which means that their individual market shares approach zero and therefore drop out of the standard formula for the Herfindahl index. The adjusted Herfindahl first weighs the market share of each product by the total market share of the car firms only. Then, the formula becomes

$$HHI^{adj} = \sum_k \left(\frac{s_k}{Pr(icv) + Pr(ev)} \right)^2 \tag{3.3.2}$$

Third, we introduce here a new index, which is based on mark-up differences between products and thus requires that (marginal) costs can be observed:

$$\sum_i \sum_{j \in S(i)} \sum_{j' \in S(i) \forall j \neq j'} \left\{ \frac{1}{2} \cdot |\, (p_{j,i} - c_{j,i}) - (p_{j',i} - c_{j',i}) \,| \cdot [Pr(j,i) \cdot Pr(j',i)] \right\} \tag{3.3.3}$$

The most efficient outcome in the market, by perfect competition, is when the mark-up of all products is zero; when $p_{j,i} = c_{j,i}$. Then, total welfare is maximized. The index we introduce here compares all pairs of products in the market; for each pair it takes the difference in mark-up and multiplies it by the market shares of the products. Thus, we get an index which takes the 'deviations' from perfect competition, and multiplies them by their weight (market share) in the market. When the statistic is low, welfare is high. Thus, according to this index, welfare is expected to be highest when the mark-up is equal for each product, as with perfect competition. Welfare in the market is maximized when the mark-up of all products is zero; when $p_{j,i} = c_{j,i}$. The statistic could also be zero when mark-up is positive, but equal, for all products. We have seen in the previous section that mark-up in monopoly is equal for all products; however, the mark-up of the outside option could be different. In our model the outside option is represented by perfectly competitive firms, with zero mark-up. Since the mark-up in monopoly is high, the mark-up difference between the outside option and monopoly products will be high. Thus, the index will not be zero in monopoly in our model. However, our statistic shows that monopoly could be very beneficial to welfare, depending on the assumptions made on the outside option.

All the indices discussed here can be used in the same way. A higher value for the index corresponds to a lower welfare. Thus, competition agencies could use our new index in the same way as the Herfindahl-index to investigate the welfare effect of a merger, for example, as long as mark-up or marginal costs can be observed.

The next section discusses how the different indices perform numerically.

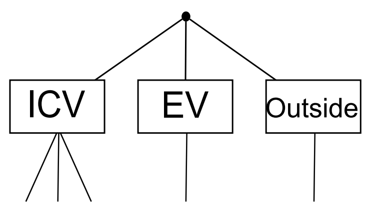
4. Numerical results

Due to the lack of analytical (closed-form) results, we now process with numerical analysis of our model. For the basic model, we assume $a_{ev} = 0$, $\mu = 0.5$, $\theta = 1$, $p_0 = 1$, and all production costs equal zero. We will discuss later if and how results change when these parameters are changed.

For the numerical analysis, the variants offered in each nest are given, but the firms producing them vary. Since the EV technology is relatively new, we assume there are less EV variants in the market than ICV variants. A similar assumption could be expected for new technologies in markets other than vehicles. To keep the analysis tractable, the total number of variants in the market is small; we have three ICV variants and one EV variant.

The nested structure is illustrated in Figure 1.

Figure 1: Nested structure



We will consider different configurations of firms supplying the three ICV variants and the single EV variant, by assigning the identity of a firm to the four lower level alternatives. The four numbers represent the industry structures and show which firm produces each product. The first three numbers represent the ICV's and the fourth the EV. For example, 1111 represents monopoly and 1234 means each product is produced by a different firm, where firm 4 produces the EV.

Table 1 shows the results for prices, demands and profits as defined in section 1. Additionally, the table shows consumer surplus, total welfare, efficiency and the Herfindahl-indices. The different industry structures are ordered increasingly by efficiency (from now on, we call this the efficiency ranking).

4.1. EV demand

To understand the welfare patterns shown in table 1, we first discuss the impact of industry structure on EV demand. We are specifically interested in what happens to demand for the new technology (EV demand) for different market structures. Table 2 shows the ranking in terms of equilibrium EV sales. The top row shows the ranking in the basic model. First of all, note that EV demand is highest in the industry structures where the EV is produced by a separate firm; a firm that does not also produce an ICV. A firm that produces

Table 1: Results in basic model

	1111	1112	1121	1122	1233	1123	1234
Price							
1st ICV	firm 1: 2.00	firm 1: 1.57	firm 1: 1.07	firm 1: 0.96	firm 1: 0.69	firm 1: 0.91	firm 1: 0.68
2nd ICV	firm 1: 2.00	firm 1: 1.57	firm 1: 1.07	firm 1: 0.96	firm 2: 0.69	firm 1: 0.91	firm 2: 0.68
3rd ICV	firm 1: 2.00	firm 1: 1.57	firm 2: 0.78	firm 2: 0.88	firm 3: 0.80	firm 2: 0.73	firm 3: 0.68
EV	firm 1: 2.00	firm 2: 1.35	firm 1: 1.57	firm 2: 1.44	firm 3: 1.37	firm 3: 1.26	firm 4: 1.23
Demand							
1st ICV	firm 1: 0.11	firm 1: 0.12	firm 1: 0.14	firm 1: 0.17	firm 1: 0.20	firm 1: 0.16	firm 1: 0.19
2nd ICV	firm 1: 0.11	firm 1: 0.12	firm 1: 0.14	firm 1: 0.17	firm 2: 0.20	firm 1: 0.16	firm 2: 0.19
3rd ICV	firm 1: 0.11	firm 1: 0.12	firm 2: 0.25	firm 2: 0.20	firm 3: 0.17	firm 2: 0.22	firm 3: 0.19
EV	firm 1: 0.18	firm 2: 0.26	firm 1: 0.17	firm 2: 0.18	firm 3: 0.17	firm 3: 0.20	firm 4: 0.19
Outside option	0.50	0.37	0.30	0.29	0.25	0.26	0.24
Profit							
firm 1	1.00	0.57	0.57	0.32	0.14	0.28	0.13
firm 2		0.35	0.20	0.44	0.14	0.16	0.13
firm 3					0.37	0.26	0.13
firm 4							0.23
Total profit	1.00	0.92	0.77	0.76	0.65	0.70	0.62
Consumer surplus	-0.306	-0.015	0.219	0.251	0.377	0.336	0.431
Total welfare	0.697	0.913	0.982	1.010	1.032	1.037	1.052
Efficiency	0.616	0.807	0.868	0.893	0.912	0.916	0.930
HHI	0.25	0.20	0.27	0.26	0.20	0.19	0.14
HHI^{adj}	1.00	0.50	0.54	0.52	0.36	0.35	0.25
Mark-up Index	0.50	0.37	0.303	0.272	0.237	0.242	0.21

Table 2: EV demand rankings for outside prices of 1, 3 and 5

	low						high
$p_0 = 1$	1121	1233	**1111**	1122	1234	1123	1112
$p_0 = 3$	1121	1233	1122	1234	**1111**	1123	1112
$p_0 = 5$	1121	1233	1122	1234	1123	**1111**	1112

products in multiple nests, can capture some of the lost demand due to high price with products in other nests; and will thus ask higher prices, decreasing equilibrium demand. For example, in Table 1 compare market structures 1233 and 1234. Price and mark-ups for the EV are lower and demand is higher in market structure 1234. The same happens to the third ICV, while the prices and demand of the first two ICV's are hardly changed. To understand the welfare patterns shown in table 1, we first discuss the impact of industry structure on EV demand. We are specifically interested in what happens to demand for the new technology (EV demand) for different market structures. Table 2 shows the ranking in terms of equilibrium EV sales. The top row shows the ranking in the basic model. First of all, note that EV demand is highest in the industry structures where the EV is produced by a separate firm; a firm that does not also produce an ICV. A firm that produces products in multiple nests, can capture some of the lost demand due to high price with products in other nests; and will thus ask higher prices, decreasing equilibrium demand. For example, in Table 1 compare market structures 1233 and 1234. Price and mark-ups for the EV are lower and demand is higher in market structure 1234. The same happens to the third ICV, while the prices and demand of the first two ICV's are hardly changed.

Moreover, given that a specialized firm produces the EV, EV equilibrium demand is higher when the ICV nest is more concentrated. The intuition is straightforward. When the ICV nest is more concentrated, prices are higher, so that the EV nest as a whole becomes more attractive. These results are robust to changes in parameters. When we change μ, θ or marginal cost, the ranking does not change. When we change the outside price, an interesting pattern arises. Ignoring monopoly (structure 1111), the ranking remains constant when the outside price increases. But as is shown in table 2, when the outside price increases monopoly does change places in the ranking. Namely, EV sales in monopoly increase relative to the other market structures. To understand why, we need to take a closer look at the pricing behavior of the firms in the different market structures in equilibrium.

For most industry structures, EV equilibrium price is higher than ICV equilibrium price (see Table 1). This reflects the stronger differentiation of the EV variants, which allows firms to set a higher price. However, in monopoly, EV price is exactly equal to ICV price; while with the 1112 structure, for example, EV price is actually lower. Based solely on these results we would expect EV demand to be highest in the market structure 1112, next highest in monopoly and then the other structures. We actually see this ranking when the outside price is very high ($p_0 = 5$).

However, there is a second force at work. When the outside price is lower, monopoly falls in the ranking. Since, for monopoly, prices are always high compared to the other industry structures, EV sales are relatively low. However, when the outside price increases, the sensitivity of the total demand for cars with respect to price decreases, since the outside option becomes a less attractive option. Then, prices of all firms in all industry structures rise. However, demand elasticities are higher when price is higher. Thus, if prices in all industry structures increase due to a decrease in the price sensitivity (outside price increase), firms that start from a lower price will increase their price faster. The result is that prices converge, which means prices in other structures approach the monopoly price, so that monopoly increases in the EV demand ranking: the effect that it sets a relatively low price for its EV compared to its ICV's, dominates the fact that in monopoly prices are higher than in other structures.

Another pattern emerges when θ is increased. Just as an increase in the outside price, an increase in θ decreases the price sensitivity of demand, since products are perceived to be more differentiated. When θ is increased, EV demand in monopoly increases relative to the other market structures, due to the same (counteracting) effects as with an outside price increase. This result shows us that monopoly is not necessarily bad for EV demand. In fact, if for some reason prices in the car market are high, monopoly does relatively well in terms of EV demand.

The results presented so far are based on the model where there is only one EV in the market. Thus, the EV seller does not have any competitors in the same nest. As explained in the introduction, we assume consumers view products between nests as more differentiated than products within a nest. Consequently, variants face stronger competition from products in the same nest than from products in different nests. Thus, the EV variant faces less competitive pressure in the market than the other variants. Market structure 1234 illustrates the stronger EV position. Table 1 shows that firm 4 (the EV firm) asks almost double the price of the other variants and still faces the same demand as the others. Moreover, also notice that in general, the EV product is more profitable than the other products.

As stated before, there are many reasons why, in real markets, EV's are less attractive to consumers than ICV's, making our base model unrealistic in that respect. We can adapt the values of our parameters to reflect real markets. Our sensitivity analysis revealed that the results discussed here do not change. As discussed in the introduction, one of the reasons why EV sales are lagging is high production costs (and consequently, high price). We have therefore also repeated the analysis assuming that EV products have higher marginal costs of production. As expected, the price of EV's is now relatively higher compared to the price of ICV's; therefore, absolute EV demand decreases. However, the change does not affect the market structure ranking in terms of EV sales, thus the previous conclusions remain valid. Another reasons for lagging EV sales is range anxiety, reducing the utility and willingness to pay for EV's. We have introduced this into the model by the variable a_{ev}, which was set to zero for the base model. If we set increasingly negative values for a_{ev}, the absolute EV demand in each industry structure decreases. However, the relative demand (the ranking in terms of EV demand), remains unchanged.

Thus, when we lower the attractiveness of the EV nest relative to the other nests, whether it is through the supply or the demand side, the results regarding the effect of market structures hold.

In conclusion, we do find that industry structure has an important effect on the market and demand for a new technology such as EV's. We find evidence that traditional ICV firms will be less successful in bringing EV's to the market, because they recapture some of the lost EV sales with their ICV sales. Specialized firms will sell more EV cars to the market in total. Thus, the results suggest that, for policy makers to stimulate EV's, they may have to subsidize specialized firms over traditional ICV firms. Of course, disadvantages of subsidizing should be taken into account as well; however we have shown that there might be a reason to expect that specialized firms set prices that would stimulate a more rapid take-off of the new product variant. Moreover, we have found that a complete monopoly is not necessarily bad for EV sales. When, for example, non-car travel modes (the outside option) are very expensive, monopoly can perform well in terms of EV sales.

4.2. Welfare

Table 1 orders industry structures by efficiency. Since no scale externalities are involved, monopoly is least efficient as expected, while the structure with the highest number of firms is most efficient. In fact, the table clearly shows that when there are more firms in the market, the market efficiency is higher. We can also rank the different structures in terms of consumer surplus and profit. Not surprisingly, more firms in the market leads to higher consumer surplus and lower total profit. In fact, the profit and CS rankings are completely opposite of each other. Hence, the most profitable structure for firms is worst for consumer surplus. The second-best in terms of profit, is second-worst in terms of consumer surplus, etc.

The CS and profit rankings appear to be robust to all changes in variables. However, the efficiency ranking, which depends on both CS and profit, changes constantly when parameters change. While CS and profit rankings stay constant, their absolute values do change when parameters change. For example, when the outside price increases, all firms can set higher prices, thus total profit in all structures increases. Then, total welfare will be more determined by profit and less by CS, thus the total welfare ranking will change; it will go towards the total profit ranking.

Interestingly, our results show that lower concentration, as measured by the HHI, is not necessarily better for efficiency, or even consumer surplus. The adjusted Herfindahl-index is more intuitive that the standard HHI, since it gives 1 in the monopoly situation, as we would expect. A higher value for the adjusted Herfindahl-Index, means higher concentration and leads to lower total welfare.

The correlation between the adjusted Herfindahl-index and total welfare (in the basic model) is as high as -0.95, while for the standard Herfindahl it is only -0.43. However, as parameters change from the base model, the correlations decrease. Nevertheless, the correlation of the HHI^{adj} does stay below -0.9, making it a relatively good prediction.

The new index 3.3.3 works very well for our model; it has at least -0.99 correlation with total welfare, and is the best performing statistic we have. However, it does rely on cost information, which is not always available. Our results show that when cost information is not available, the adjusted Herfindahl is a good alternative.

5. Conclusion

The model we have presented here gives us new insights into the applicability and implications for the nested logit model in the analysis of the market up-take of a new technology under oligopolistic competition with asymmetric firms. We have seen how industry structure affects demand for an emerging technology. We have also shown what determines total welfare when markets can be described by the nested logit model and have presented and compared statistics that can be used to a priori predict which market structure will be best in terms of total welfare. This information is very useful for competition agencies investigating a possible merger.

Our main analytical result, relevant for the general literature looking into the nested logit model is our implicit solution for mark-up. It is a generalization of an earlier result that can be used when there are two nests present in the model, for any number of variants and firms active in the market. In the future, it might be helpful to further extent this formula, so that it can be used for any number of nests as well.

Furthermore, we have analyzed the effect of industry structure on emerging technologies. We have found that emerging technologies will experience higher sales if produced by a specialized firm. Surprisingly, we have also found that complete monopoly is not necessarily bad for the demand for the new technology. This is due to the fact that the monopoly combines higher average prices (across all alternatives) with lower price differences between alternatives.

This research can easily be extended in various directions. For example, we have not discussed externalities in this paper; using our illustration of the car market, these could be social environmental benefits from using an EV instead of an ICV. Additionally, we have not studied cost synergies for multiple products produced by the same firm. Finally, we have used a very simple demand function in this paper. This also could be extended in many ways, for example, by including network benefits. These are topics we intend to address in follow-up research.

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Appendices

Appendix 1: Finding the FOC

The profit function of any firm k is given by

$$\Pi_k = \sum_i \sum_{j\in S(i)} \{(p_{j,i} - c_{j,i}) Pr(j,i)\delta^k_{j,i}\}$$

Where $\delta^k_{j,i}$ is a dummy, which is equal to 1 if firm k sells product j in nest i

Pick a product $j*$ in nest i, such that $\delta^k_{j,i}$=1. Suppose there are two nests. i and i' Then the FOC is given by:

$$\begin{aligned}\partial\Pi_k/\partial p_{j*,i} =& Pr(j*,i) + (p_{j*,i} - c_{j*,i})\frac{\partial Pr(j*,i)}{\partial p_{j*,i}} \\ &+ \sum_{j\in S(i), j\neq j} \left\{(p_{j,i} - c_{j,i})\frac{\partial Pr(j,i)}{\partial p_{j*,i}}\delta^k_{j,i}\right\} \\ &+ \sum_{j\in S(i')} \left\{(p_{j,i'} - c_{j,i'})\frac{\partial Pr(j,i')}{\partial p_{j*,i}}\delta^k_{j,i}\right\} = 0\end{aligned}$$

Denote $p_{j,i} - c_{j,i}$ by $m_{j,i}$

$$\begin{aligned}&\Rightarrow Pr(j*,i) \\ &+ m_{j*,i}\left[\frac{a}{\mu}Pr(j*,i)[Pr(j*|i) - 1] + \frac{a}{\theta}Pr(j*,i)Pr(j*|i)[Pr(i) - 1]\right] \\ &+ \sum_{\substack{j\in S(i)\\ j\neq j*}} \left\{m_{j,i}\left[\frac{a}{\mu}Pr(j*,i)Pr(j|i) + \frac{a}{\theta}Pr(j*,i)Pr(j|i)[Pr(i) - 1]\right]\delta^k_{j,i}\right\} \\ &+ \sum_{j\in S(i')} \left\{m_{j,i'}\left[\frac{a}{\theta}Pr(j*,i)Pr(j,i')\right]\delta^k_{j,i}\right\} = 0\end{aligned}$$

Now $Pr(j*,i)$ cancels

$$\begin{aligned}\Rightarrow& 1 + m_{j*,i}\left[\frac{a}{\mu}[Pr(j*|i) - 1] + \frac{a}{\theta}Pr(j*|i)[Pr(i) - 1]\right] \\ &+ \sum_{j\in S(i), j\neq j*} \left\{m_{j,i}\left[\frac{a}{\mu}Pr(j|i) + \frac{a}{\theta}Pr(j|i)[Pr(i) - 1]\right]\delta^k_{j,i}\right\} \\ &+ \sum_{j\in S(i')} \left\{m_{j,i'}\left[\frac{a}{\theta}Pr(j,i')\right]\delta^k_{j,i}\right\} = 0\end{aligned}$$

Take $\frac{a}{\mu} m_{j*,i}$ to the left

$$\begin{aligned}\frac{a}{\mu} m_{j*,i} = 1 + m_{j*,i} &\left[\frac{a}{\mu} Pr(j*|i) + \frac{a}{\theta} Pr(j*|i)[Pr(i)-1]\right] \\ &+ \sum_{j \in S(i), j \neq j*} \left\{ m_{j,i} \left[\frac{a}{\mu} Pr(j|i) + \frac{a}{\theta} Pr(j|i)[Pr(i)-1]\right] \delta^k_{j,i} \right\} \\ &+ \sum_{j \in S(i')} \left\{ m_{j,i'} \left[\frac{a}{\theta} Pr(j,i')\right] \delta^k_{j,i} \right\}\end{aligned}$$

Then merge the second and third term on the right into one sum:

$$\begin{aligned}\frac{a}{\mu} m_{j*,i} =& 1 + \sum_{j \in S(i)} \left\{ m_{j,i} \left[\frac{a}{\mu} Pr(j|i) + \frac{a}{\theta} Pr(j|i)[Pr(i)-1]\right] \delta^k_{j,i} \right\} \\ &+ \sum_{j \in S(i')} \left\{ m_{j,i'} \left[\frac{a}{\theta} Pr(j,i')\right] \delta^k_{j,i} \right\}\end{aligned}$$

Bring $\frac{a}{\mu}$ to the right

$$\begin{aligned}m_{j*,i} =& \frac{\mu}{a} + \frac{\mu}{a} \sum_{j \in S(i)} \left\{ m_{j,i} \left[\frac{a}{\mu} Pr(j|i) + \frac{a}{\theta} Pr(j|i)[Pr(i)-1]\right] \delta^k_{j,i} \right\} \\ &+ \frac{\mu}{a} \sum_{j \in S(i')} \left\{ m_{j,i'} \left[\frac{a}{\theta} Pr(j,i')\right] \delta^k_{j,i} \right\}\end{aligned}$$

The expression on the right does not depend directly on $m_{j*,i}$. It only depends on the sum of all products for firm k in nest i, not on the specific product. Thus all products of firm k in nest i have the same expression (and thus value) for mark-up. Let this mark-up simply be m^k_i. Thus it gives the mark-up of any product produced by firm k in nest i. By the same calculations and reasoning, this also goes for products of firm k in nest i'. Let this mark-up be $m^k_{i'}$ The expression becomes

$$\begin{aligned}m^k_i =& \frac{\mu}{a} + \frac{\mu}{a} \sum_{j \in S(i)} \left\{ m^k_i \left[\frac{a}{\mu} Pr(j|i) + \frac{a}{\theta} Pr(j|i)[Pr(i)-1]\right] \delta^k_{j,i} \right\} \\ &+ \frac{\mu}{a} \sum_{j \in S(i')} \left\{ m^k_{i'} \left[\frac{a}{\theta} Pr(j,i')\right] \delta^k_{j,i} \right\}\end{aligned}$$

Take all constants out of the sum signs and simplifying (note that $Pr(j,i') = Pr(i) \cdot Pr(j|i')$)

$$\begin{aligned}m^k_i =& \frac{\mu}{a} + m^k_i (1 + \frac{\mu}{\theta}[Pr(i)-1]) \sum_{j \in S(i)} \left\{ Pr(j|i) \delta^k_{j,i} \right\} \\ &+ \frac{\mu}{\theta} m^k_{i'} Pr(i') \sum_{j \in S(i')} \left\{ Pr(j|i') \delta^k_{j,i} \right\}\end{aligned}$$

Bring m_i^k to the left

$$m_i^k \left\{ 1 - (1 + \frac{\mu}{\theta}[Pr(i) - 1]) \sum_{j \in S(i)} \left\{ Pr(j|i)\delta_{j,i}^k \right\} \right\}$$
$$= \frac{\mu}{a} + \frac{\mu}{\theta} m_{i'}^k P(i') \sum_{j \in S(i')} \left\{ Pr(j|i')\delta_{j,i}^k \right\}$$

Then solve for m_i^k and let $s(i,k)$ be $\sum_{j \epsilon m(i)} \left\{ Pr(j|i)\delta_{j,i}^k \right\}$. Rewrite to get:

$$m_i^k = \frac{\mu\theta + a\mu m_{i'}^k Pr(i') s(i',k)}{a \left\{ \theta \left[1 - s(i,k)\right] + \mu[1 - Pr(i)]s(i,k) \right\}} \tag{5.0.1}$$

Note that the mark-up of products of firm k in nest i also depends on the mark-up of products of firm k in nest i' By the same reasoning:

$$m_{i'}^k = \frac{\mu\theta + a\mu m_i^k Pr(i) s(i,k)}{a \left\{ \theta \left[1 - s(i',k)\right] + \mu[1 - Pr(i')]s(i',k) \right\}} \tag{5.0.2}$$

To find the FOC any product in nest i substitute the expression for $m_{i'}^k$ into m_i^k:

$$m_i^k = \frac{\mu\theta^2 \left[1 - s(i',k)\right] + \mu^2\theta s(i',k)}{a \left[\prod_i \left\{ \theta \left[1 - s(i,k)\right] + \mu \left[1 - Pr(i)\right] s(i,k) \right\} - \prod_i \left\{ \mu Pr(i) s(i,k) \right\} \right]}$$

Appendix 2: Uniqueness

By writing the system of equations in matrix form we can show that the implicit equilibrium is unique. Recall the two expressions for mark-up 5.0.1 and 5.0.2 and write them in matrix-form.

$$m_i^k = \frac{\mu\theta + a\mu m_{i'}^k Pr(i') s(i',k)}{a \left\{ \theta \left[1 - s(i,k)\right] + \mu[1 - Pr(i)]s(i,k) \right\}}$$

and

$$m_{i'}^k = \frac{\mu\theta + a\mu m_i^k Pr(i) \sum_{j \in m(i)} \left\{ Pr(j|i)\delta_{j,i}^k \right\}}{a \left\{ \theta \left[1 - s(i,k)\right] + \mu[1 - Pr(i')]s(i,k) \right\}}$$

The system in matrix form ($b = Ax$) becomes:

$$\begin{bmatrix} m_i^k \\ m_{i'}^k \end{bmatrix} = \begin{bmatrix} 1 & -b_i c_{i'} \\ -b_{i'} c_i & 1 \end{bmatrix} \times \begin{bmatrix} a_i \\ a_{i'} \end{bmatrix}$$

Where

$$a_i = \frac{\mu\theta}{z_i}$$
$$b_i = \frac{a\mu}{z_i}$$

$$c_i = Pr(i)s(i,k)$$
$$z_i = a\left\{\theta\left[1 - s(i,k)\right] + \mu[1 - Pr(i)]s(i,k)\right\}$$

Showing that the determinant of the matrix A is nonzero proves that the found equilibrium is unique. The determinant is given by

$$\begin{bmatrix} 1 & -b_i c_{i'} \\ -b_{i'} c_i & 1 \end{bmatrix}$$

$$= 1 - b_i b_{i'} c_i c_{i'}$$
$$= 1 - \frac{a^2 \mu^2 Pr(i) Pr(i') s(i,k) s(i',k)}{a^2 \prod_i \left\{\theta\left[1 - s(i,k)\right] + \mu\left[1 - Pr(i)\right] s(i,k)\right\}}$$

Which is equal to

$$\frac{a^2 \prod_i \left\{\theta\left[1 - s(i,k)\right] + \mu\left[1 - Pr(i)\right] s(i,k)\right\} - a^2 \mu^2 Pr(i) Pr(i') s(i,k) s(i',k)}{a^2 \prod_i \left\{\theta\left[1 - s(i,k)\right] + \mu\left[1 - Pr(i)\right] s(i,k)\right\}}$$

This expression is zero if the numerator is zero. Rewrite to get:

$$\begin{aligned} &\theta^2\left[1 - s(i,k)\right]\left[1 - s(i',k)\right] \\ &+ \sum_i \mu\theta s(i,k)\left[1 - Pr(i)\right]\left[1 - s(i',k)\right] \\ &+ \mu^2 s(i,k) s(i',k)\left[1 - Pr(i) - Pr(i')\right] = 0 \end{aligned}$$

All the elements of this equation are non-negative, so the equation is only zero if all elements are zero. Note that $\theta > 0$ and $\mu > 0$ and, because of the logit setup, $Pr(i)$ and $Pr(i')$ can never be exactly zero or 1. Additionally, since there is an outside option $1 - Pr(i) - Pr(i') = Pr(0)$ can never be zero. Only $s(i,k)$ and $s(i',k)$ could potentially be zero. From term 1 and term 3 it becomes clear that, for those to be zero, $s(i,k) = 0$ and $s(i',k) = 1$ or its reciprocal. However, with these two combinations, the second term can never be zero. Thus, it is not possible for the determinant to be zero. Clearly, we have a unique implicit solution.

Appendix 3: Deriving the adjusted Lerner-Index

By calculating aggregate elasticities and substituting them in the expression for mark-up above, we can rewrite the expression into an adjusted Lerner condition. The aggregate own price elasticity is

$$\epsilon_{own,i,k} = \left.\frac{\partial Pr(j,i)}{\partial p_{j,i}}\right|_{sym,i,k} \cdot \frac{p_{j,i}}{Pr(j,i)} =$$

$$\left\{ \left.\frac{\partial Pr(i)}{\partial p_{j,i}}\right|_{sym,i,k} \cdot Pr(j|i) + \left.\frac{\partial Pr(j|i)}{\partial p_{j,i}}\right|_{sym,i,k} \cdot Pr(i) \right\} \cdot \left\{ \frac{p_{j,i}}{Pr(j,i)} \right\}$$

Thus, the change in demand when all products of firm k in nest i increase by the same amount. This will always be the case since we have see that the mark-up has to be equal for products in the same nest of the same firm. So whenever one price increases, the other prices have to increase by the same amount. Similarly, we can find what happens to the demand of products of firm k in one nest, if all prices of firm k in the other nest increase by the same amount.

$$\epsilon_{cross,i,k} = \left.\frac{\partial Pr(j,i)}{\partial p_{j,i'}}\right|_{sym,i',k} \cdot \frac{p_{j,i'}}{Pr(j,i)} =$$

$$\left\{ \left.\frac{\partial Pr(i)}{\partial p_{j,i'}}\right|_{sym,i,k} \cdot Pr(j|i) + \left.\frac{\partial Pr(j|i)}{\partial p_{j,i'}}\right|_{sym,i,k} \cdot Pr(i) \right\} \cdot \left\{ \frac{p_{j,i'}}{Pr(j,i)} \right\}$$

These expressions equal:

$$\epsilon_{own,i,k} = p_{j,i} \left(\frac{a}{\theta} s(i,k) \left[1 - Pr(i)\right] - \frac{a}{\mu} \left[1 - s(i,k)\right] \right)$$
$$\epsilon_{cross,i,k} = p_{j,i'} \frac{a}{\theta} s(i',k) Pr(i')$$

Now recall the expression for mark-up:

$$m_i^k = \frac{\mu\theta + a\mu m_{i'}^k Pr(i') s(i',k)}{a \left\{ \theta \left[1 - s(i,k)\right] + \mu [1 - Pr(i)] s(i,k) \right\}}$$

The expression includes the elasticities, so we can rewrite:

$$p_{j,i} - c_{j,i} = \frac{p_{j,i}}{-\epsilon_{own,i,k}} + (p_{j,i'} - c_{j,i'}) \frac{p_{j,i}\epsilon_{cross,i,k}}{-p_{j,i'}\epsilon_{own,i,k}}$$

Dividing both sides by $p_{j,i}$ gives the adjusted Lerner-index:

$$\frac{p_{j,i} - c_{j,i}}{p_{j,i}} = \frac{1}{-\epsilon_{own,i,k}} + \frac{p_{j,i'} - c_{j,i'}}{p_{j,i'}} \frac{\epsilon_{cross,i,k}}{-\epsilon_{own,i,k}}$$

Derivations of the aggregate derivatives

These derivatives give the change in different demand functions if all prices of firm k in nest i are increased by the same amount.

$$\begin{aligned}
\left.\frac{\partial v_i}{\partial p_{j,i}}\right|_{sym,i,k} &= \mu \frac{1}{\sum_{j\in S(i)} \exp\left[\frac{-ap_{j,i}}{\mu}\right]} \sum_{j\in S(i)} \left\{-\frac{a}{\mu}\exp\left[\frac{-ap_{j,i}}{\mu}\right]\delta^k_{ji}\right\} \\
&= -a\sum_{j\in S(i)}\left\{Pr(j|i)\delta^k_{ji}\right\} \\
&= -as(i,k) \\
\left.\frac{\partial Pr(i)}{\partial p_{j,i'}}\right|_{sym,i,k} &= 0 \\
\left.\frac{\partial v_i}{\partial p_{j,i}}\right|_{sym,i,k} &= \frac{\left\{\begin{array}{c} -\frac{a}{\theta}s(i,k)\exp\left[\frac{v_i}{\theta}\right]\sum_i \exp\left[\frac{v_i}{\theta}\right] \\ -\exp\left[\frac{v_i}{\theta}\right]\left\{-a\frac{a}{\theta}s(i,k)\exp\left[\frac{v_i}{\theta}\right]\right\}\end{array}\right\}}{\left[\exp\left[\frac{v_i}{\theta}\right]\right]^2} \\
&= -\frac{a}{\theta}s(i,k)Pr(i) + \frac{a}{\theta}s(i,k)Pr(i)^2 \\
&= -\frac{a}{\theta}s(i,k)Pr(i)\left[1-Pr(i)\right] \\
\left.\frac{\partial Pr(i)}{\partial p_{j,i'}}\right|_{sym,i',k} &= \frac{\frac{as(i',k)}{\theta}\exp\left[\frac{v_i}{\theta}\right]\exp\left[\frac{v_{i'}}{\theta}\right]}{\left[\exp\left[\frac{v_i}{\theta}\right]\right]^2} \\
&= \frac{a}{\theta}s(i',k)Pr(i)Pr(i') \\
\left.\frac{\partial Pr(j|i)}{\partial p_{j,i}}\right|_{sym,i,k} &= \frac{\left\{\begin{array}{c} -\frac{a}{\mu}\exp\left[\frac{-ap_{j,i}}{\mu}\right]\sum_{j\in S(i)}\exp\left[\frac{-ap_{j,i}}{\mu}\right] \\ -\exp\left[\frac{-ap_{j,i}}{\mu}\right]\frac{a}{\mu}\sum_{j\in S(i)}\left\{\exp\left[\frac{-ap_{j,i}}{\mu}\right]\delta^k_{ji}\right\}\end{array}\right\}}{\left[\exp\left[\frac{-ap_{j,i}}{\mu}\right]\right]^2} \\
&= -\frac{a}{\mu}Pr(j|i) + \frac{a}{\mu}Pr(j|i)\sum_{j\in S(i)}\left\{Pr(j|i)\delta^k_{ji}\right\} \\
&= -\frac{a}{\mu}Pr(j|i)\left[1-s(i,k)\right] \\
\left.\frac{\partial Pr(j|i)}{\partial p_{j,i'}}\right|_{sym,i,k} &= 0
\end{aligned}$$

Filling these derivatives into the expressions for the elasticities, gives the final aggregate demand elasticities.