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Penalizing Cartels: The Case for Basing Penalties on Price Overcharge

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Abstract

In this paper we set out the welfare economics based case for imposing cartel penalties on the cartel overcharge rather than on the more conventional bases of revenue or profits (illegal gains). To do this we undertake a systematic comparison of a penalty based on the cartel overcharge with three other penalty regimes: fixed penalties; penalties based on revenue, and penalties based on profits. Our analysis is the first to compare these regimes in terms of their impact on *both* (i) the prices charged by those cartels that do form; and (ii) the number of stable cartels that form (deterrence). We show that the class of penalties based on profits is identical to the class of fixed penalties in all welfare-relevant respects. For the other three types of penalty we show that, for those cartels that do form, penalties based on the overcharge produce lower prices than those based on profit) while penalties based on revenue produce the highest prices. Further, in conjunction with the above result, our analysis of cartel stability (and thus deterrence), shows that penalties based on the overcharge out-perform those based on profits, which in turn out-perform those based on revenue in terms of their impact on *each* of the following welfare criteria: (a) average overcharge; (b) average consumer surplus; (c) average total welfare.

JEL Classification: L4 Antitrust Policy, K21 Antitrust Law, D43 Oligopoly and Other Forms of Market Imperfection, C73 Stochastic and Dynamic Games; Repeated Games

Keywords: Antitrust Enforcement, Antitrust Law, Cartel, Oligopoly, Repeated Games.

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1. Introduction

Cartels are still very active throughout the world and pervasive in a wide variety of markets - despite increased enforcement in the form of much higher fines and other sanctions and the implementation of leniency policies. As Levenstein and Suslow (2012) report for the US, the country with probably the toughest sanctions[5] regime in the world "from 1992 – 2010 there were approximately 700 DoJ cartel convictions or over 36 per year". Indeed, "fifty new criminal cartel cases were filed in 2013".[6] Empirical evidence suggests that "while antitrust is the most important force leading to cartel break-up ……there are limitations to the effectiveness of these policies as currently designed".[7]

In this paper, we argue that the widely employed current designs of one of the most important enforcement tools in the fight against cartels – namely monetary penalties are flawed and that this does indeed limit the effectiveness of this tool. We propose an alternative design that could significantly improve the effectiveness of monetary penalties.. Specifically, our objective is to set out the welfare economics framework that supports the case for Competition Authorities to switch the base on which penalties for cartels are set away from the conventional bases of revenue or illegal gains and instead to base the penalty on the cartel overcharge.

To make this case we analyze the impact of various penalty regimes that have been widely considered and analyzed in the literature on: (i) the price charged by any given cartel; (ii) cartel stability and hence the number of cartels that form; and finally (iii) the overall level of welfare induced by the different regimes.[8]

We use a repeated Bertrand oligopoly model that allows us to compare both the price and the deterrence effects of the four major types of fining structures investigated in the literature. These four types are: fines based on revenue (see e.g. Bageri et al. (2013) and Katsoulacos and Ulph (2013)); fines based on illegal gains (see e.g. Harrington (2004, 2005) or Houba et al. (2010, 2012)); fines based on cartel overcharge (see e.g. Buccirossi and Spagnolo (2007) and Katsoulacos and Ulph (2013)); and fixed fines (see e.g. Rey (2003) or Motta and Polo (2003)).

[5] In US sanctions take the form of monetary penalties as in all other countries plus treble damages and criminal convictions.

[6] See Levenstein and Suslow (2014).

[7] See in particular the recent series of papers by Levenstein and Suslow (2011, 2012, 2014) that contain reviews and extensive references to the relevant literature.

[8] The impact of the toughness of the penalty regime on the cartel pricing behavior has been addressed in Katsoulacos and Ulph (2013). However, they have relied on a static game, have not analyzed the impact of the penalty structure on cartel stability and have not examined the deterrence implication of the various penalty structure employed in practice, as we do here. In his seminal article, Harrington (2005) has also shown that price dependent penalties (that are based on damages) imply that the steady state cartel price will be below the simple monopoly price and that the toughness of the penalty regime (the size of the damage multiplier) is one of the factors that reduce the equilibrium cartel price. However, he does not provide comparisons of all the alternative penalty regimes examined here. Also, and most importantly, the possible deterrence effects of various penalty structures in conjunction with their direct price effects have not been systematically analyzed in literature on antitrust so far.

While other papers have considered the properties of each of the four penalty regimes and made some limited comparisons between them, a major contribution of this paper is to undertake a systematic comparison of all four regimes in terms of *both* the prices set by those cartels that form *and* on the deterrence of potential stable cartels (which we sometimes refer to as cartel stability in short).

In setting out our arguments we also make two important methodological contributions. First we extend the repeated Bertrand model proposed in Houba et al. (2010, 2012) to capture the effect of the cartel stability condition on cartel pricing behavior. This allows us to bridge the standard critical discount factor approach to the analysis of collusion (see e.g. Tirole (1988) or Motta and Polo (2003)) to profit maximizing decisions by the cartel members (with continuum of prices, which can be chosen by the cartel). This latter approach has been proposed in e.g. Block et al. (1981) or Harrington (2004, 2005). Second, we provide a framework within which we integrate the impact of penalty regimes on the price setting behaviour of cartels that do form with their deterrent effects and this provides an evaluation of the overall impact of different penalty regimes.

Our first result is that the class of profits-based penalties is identical to the class of fixed penalties in terms of all the welfare-relevant outcomes they produce – price, deterrence. Anything that can be achieved by one type of penalty can be achieved by the other using an equivalent level of penalty.

Consequently we confine attention to three penalty regimes – those based on profits, those based on revenue, and those based on the cartel overcharge.

In terms of the price set by those cartels that do form, we show that proportional fines based on overcharges are more successful in terms of their effect on price when compared to proportional fines based on revenues or illegal gains. Specifically, we show that:

- penalties based on illegal gains lead cartels to set the monopoly price.
- if the penalty is imposed on revenue then the cartel price will be *above* the monopoly price, and, moreover, the need to maintain cartel stability can require that the cartel sets a *minimum price* which increases towards the choke price (at which demand drops to zero) as it becomes increasingly difficult to maintain stability;
- if the penalty is imposed on overcharge then the cartel price will be *below* the monopoly price, and, moreover, the need to maintain cartel stability can require that the cartel sets a *maximum price* which decreases towards the competitive price as it becomes increasingly difficult to maintain stability.

- moreover, these conclusions do not depend on the toughness of the individual penalty regimes[9] – where toughness reflects both the penalty rate and the probability of detection.

Turning to the *deterrence impact of different penalty structures,* an important contribution of this paper is to provide for the first time a full analysis of how this is influenced by penalty rates and by other enforcement and market related parameters, such as the detection rate and the elasticity of market demand.

To start with we show that, as expected, deterrence depends on the toughness of the penalty regime, and that if each regime is made sufficiently tough all cartels can be deterred. This implies that in order to meaningfully compare the effects of using different penalty bases on deterrence and hence overall welfare we need to ensure that each penalty regime is in some sense equally tough.[10] We consider two concepts of equal toughness.

The first is *deterrence equivalence*: the same fraction of all stable cartels that could potentially form do in fact form. Given the above results on the prices set by cartels that do form, it is clear that under deterrence equivalence penalties on the overcharge out-perform those on profits which in turn out-perform penalties based on revenue.

However competition authorities – and courts – are not concerned solely with deterrence, they also want penalties that are reasonable and proportionate. So the second criterion of equal toughness that we consider is that of *penalty revenue equivalence*: on average[11] the size of the penalty actually paid by any cartel that forms and is subsequently detected and penalized should be the same. We again demonstrate that in terms of each of the following criteria: average overcharge, average consumer surplus, average total welfare (consumer plus producer surplus) penalties on the overcharge out-perform those on profits which in turn out-perform penalties based on revenue.

While, as we show, there can be some tension between these two different notions of "equal toughness" we also show that this has no effect on one of our central conclusions

[9]Apart from the trivial case where either the probability of detection or the penalty rate is zero, in which case there is effectively no penalty regime and it does not matter on which base the non-existent penalty might have been based.

[10]As we will show, achieving a given level of toughness under a revenue based penalty regime requires the penalty rate to vary according to the elasticity of demand in the industry. An estimate of this can be obtained by the use of what Farrell and Shapiro (2008) have proposed in the case of mergers through the application of Critical Loss Analysis: "revealed preference information (to) make inferences about preferences based directly on observed choices". Here, and to paraphrase their argument in relation to mergers, "one can make inferences about demand sensitivity as gauged by …real firm(s) based on (their collusive) choice of price…. The idea is captured by the Lerner equation".

[11]Since, as indicated, the price set by any cartel that does form under both a revenue-based penalty regime and an overcharge-based penalty regime will potentially vary depending on the intrinsic difficulty of holding the cartel together, so too will the actual penalty paid. So all we can require is that on average the penalty paid should be the same.

that, however one resolves this tension, penalties based on the overcharge welfare dominate those based on profits.[12]

Our clear policy recommendation is therefore that Competition Authorities should switch to a penalty structure that uses the price overcharge as the base on which the penalty is imposed.[13] In essence the reason is that overcharge based fines are preferable, since they target the price, which is causing the damage to consumers. Profit based fines are a weaker instrument since they do not target the price directly, but target firms' earnings, while revenue based fines have strongly counterproductive effects as originally also shown in Bageri et al. (2013) and Katsoulacos and Ulph (2013).

Overcharge-based fines are not just superior at a theoretical welfare-economics level. It is likely that implementation of overcharge-based fines in practice is no more difficult than the next best alternative (in terms of welfare induced) – a profits-based penalty[14]. Although establishing the counterfactual can be tricky, competition authorities have to obtain estimates anyway during the investigation in order to establish whether a group of firms really has driven up the price. And certainly such information is needed in order to obtain estimates of illegal gains. Further, developments in economics and econometrics make it possible to estimate cartel overcharges with reasonable precision or confidence, as regularly done in order to assess damages.

The rest of this paper is organized as follows. Section 2 discusses the current sentencing guidelines. Section 3 outlines the model. In Section 4 we derive all the main formulae for pricing, deterrence and various welfare indicators under each of the four penalty regimes. In section 5 we undertake a systematic comparison of the various regimes in terms of prices, deterrence and various measures of overall welfare. Section 6 concludes.

[12] More precisely, for any degree of toughness of the overcharge-based regime and for any degree of toughness of the profit-based regime which lies above the level required to achieve equivalent deterrence to the overcharge regime but below that which is required to achieve an equivalent level of penalty revenue as the overcharge regime, the overcharge-based regime is welfare superior to the profits-based regime in terms of average overcharge, average consumer surplus, average total welfare.

[13] It is important to note that in this paper we are concerned solely with the question of which of the various alternative penalty bases is superior in terms of its welfare implications and not with the different issue of whether current cartel penalty rates are or are not too high. There is an extensive theoretical and empirical literature on this latter question which is reviewed, for example, in Katsoulacos and Ulph (2013). Their results support the recent evidence by Allain et.al (2011) and Boyer et.al (2011) that current rates are not too low and indicate that higher rates (on a revenue base) will not necessarily lead to lower cartel prices.

[14] Bageri et. al. (2013) provide additional arguments to those presented below for preferring a profit-based penalty regime to a revenue-based regime (they do not consider an overcharge-based regime).

2. Brief Review of the Current Sentencing Guidelines[15]

This section demonstrates through a brief review that revenue-based penalties is the norm in all major jurisdictions with caps that are based on either revenue (EU) or on illegal gains (US).

To start with, in the EU, a violation of the cartel prohibition constitutes an administrative offence. In order to ensure transparency of this enforcement procedure, the EC published new penalty guidelines in 2006 refining the methodology that has been applied so far (since 1998). Under these guidelines, fines are calculated in the following way: First, the Commission determines a basic amount which may be adjusted afterwards due to aggravating and mitigating elements. The basic amount is calculated by taking into account the undertaking's *relevant turnover* (of the last year of the cartel), the gravity and the duration of the infringement, as well as an additional amount of about 15% - 25% of the value of sales in order to achieve deterrence. For cartels, the proportion of the relevant turnover is set "at the higher end of the scale"[16] which is 30%. Additional uplifts or reductions are then made when certain aggravating or attenuating circumstances exist. However, the maximum amount of the fine imposed shall not exceed the cap of 10% of annual *worldwide turnover* of the undertaking in the preceding business year.

In the US, cartels are prosecuted as criminal offences, and sentences are imposed by a non-specialized court. The courts use the US Sentencing Guidelines (USSG) as a consulting tool regarding the appropriate form and severity of punishment for offenders. According to these guidelines, both pecuniary and non-pecuniary penalties may be imposed: fines on firms and individuals, as well as imprisonment of individuals involved in the cartel. With regards to fines on firms, the process of their assessment begins with the calculation of a base fine. To determine the base fine, a percentage of the *volume of affected commerce*, that is, of total sales from the relevant market, is taken into account. The USSG suggests that 20% of the volume of affected commerce can be used as a good proxy. This volume of affected commerce covers the entire duration of the infringement. Once the amount of the base fine has been calculated, aggravating and mitigating elements are taken into consideration. However, the final fine for undertakings must not exceed a maximum statutory limit which is the greatest of 100 million USD *or* ***twice*** *the gross pecuniary gains the violators derived from the cartel or twice the gross pecuniary loss caused to the victims*.

Most other OECD countries follow the lead of the US and EU on one or both dimensions. For example, in the UK the starting point for calculating antitrust fines is a fraction of the relevant turnover, i.e. affected commerce; the cap on fines is set at 10% of the undertaking's global turnover, exactly as is the case in the EU.

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[15] See also Bageri et.al. (2013).

[16] 2006 EU Guidelines.

As Bageri et.al. (2013) note, "(E)nforcement costs often justify the use of simple rules of thumb that are easier to implement, although they are not optimal. However, …..basing fines on a firm's affected commerce rather than on collusive profits …..is likely to create large distortions……(and) empirically-based simulations suggest that the deadweight losses produced by these distortions can be very large…..It is worth noting that, in the US case, this rule of thumb does not produce any saving in enforcement costs, because the cap on fines prescribed by the USSG requires courts to calculate firms' collusive profits anyway……". As we noted above, this implies that estimates of overcharges can also be made, indeed, even more easily.

3. The Model

We consider an infinitely-repeated Bertrand oligopoly model in the presence of antitrust enforcement.[17] Antitrust enforcement consists of the probability to detect a cartel and a fine schedule. If the firms collude, they will be detected probabilistically and fined according to the fine structure. Given the detection probability and the fine schedule, the firms will collude at a price that maximizes their future profit, supported by a simple trigger strategy profile.

In each of infinitely many periods, $n \geq 2$ firms compete in prices in a homogeneous oligopoly model with linear demand function of the form[18]

$$Q = 1 + \varepsilon - p, \quad 0 < \varepsilon < 1,$$

Where p denotes price and Q is the quantity supplied to the market. Symmetric marginal costs are denoted by c and, consistent with the structure of the demand function, are normalized to 1.

In the absence of a cartel, the competitive equilibrium would be the unique Bertrand-Nash equilibrium in which all firms set price at marginal cost, so $p^N = c = 1$. p^N is therefore what is sometimes referred to as the "but-for" price – the counterfactual price that would have arisen had there not been a cartel. Notice that, given the specification of the demand function, in this competitive equilibrium output will be $Q^N = \varepsilon$. Given the normalization of price, ε also measures the industry revenue earned in the competitive equilibrium, that is $\varepsilon = R^N = p^N Q^N$. In addition $\varepsilon = -\dfrac{\partial p}{\partial Q}\dfrac{Q^N}{p^N}$ is the inverse price elasticity of demand evaluated at the competitive equilibrium. So, in this

[17]Several elements of this model are borrowed from the analysis in Houba, Motchenkova and Wen (2010, 2011, 2012).
[18]Similar demand structure has been analyzed in Katsoulacos and Ulph (2013).

model, ε is a parameter that reflects the underlying competitiveness of any given industry in which a cartel might form. It has three different, though related, interpretations.[19]

The Nash/competitive equilibrium is the best possible outcome in terms of static social welfare and generates zero profits for all firms. This implies that the actual profits earned by the cartel are also the illegal gains – profits in excess of those that would have been earned had the cartel not been in place.[20] Notice that, given our assumptions, if a cartel formed that did not include every firm in the industry, then Bertrand competition between the cartel and the fringe would drive price down to marginal cost, and reproduce the competitive equilibrium. So if any cartel is to form and drive up price it must include all firms in the industry. The socially worst outcome is when all firms collude at the monopoly price $p^M = 1 + \frac{\varepsilon}{2}$.

We now assume that in every period the n firms decide whether to collude and if so, at what price. If the firms collude at price $p > c = 1$, total cartel illegal profits will be $\pi(p) = (p-1)(1+\varepsilon - p)$, while total cartel revenue is $R(p) = p(1+\varepsilon - p)$. Both functions are continuous and concave in p.

The price-overcharge is the extent to which price is raised above its "but-for" level (or Nash level). This can be expressed either as the absolute increase in price $\theta = p - p^N = p - 1 > 0$ or as the percentage increase $\theta = \frac{p - p^N}{p^N} > 0$. Given our normalization assumption that $p^N = 1$, these two interpretations take the same numerical value.

We assume that:

- due to limited resources of the CA, cartels will not be detected for sure and we denote the probability of detecting a cartel by β, $0 \le \beta < 1$; β is constant across all cartels – in particular it is independent of the price chosen by a cartel and of whether or not a given cartel in a given industry has previously been detected;
- if a cartel is detected and investigated, the CA will decide for sure that the cartel has acted in anti-competitive fashion and impose a penalty;
- all this is common knowledge.

Given these assumptions, there is no reason why, having been detected, a cartel shouldn't just re-form - and we assume that this is indeed what happens. A similar

[19] The particular interpretation will be reflected in the dimensions of the units in which the particular interpretation is being made. Thus as a measure of competitive output, Q^N, the units that apply will be of the quantities in which the output of that industry are produced; as a measure of industry revenue in the competitive equilibrium, R^N, the units will be units of currency that apply in that particular industry; as a measure of the inverse elasticity it will be a pure dimensionless number.

[20] Throughout the paper we use the terms "illegal gains" and "profits" interchangeably.

assumption that the cartel reestablishes after each conviction has been adopted in Motta and Polo (2003).

Let F denote the penalty imposed on a cartel that has been detected and successfully prosecuted. In the subsequent sections we will consider a number of alternative fine schedules/ structures, in all of which the fine actually paid by a cartel potentially depends on the price set by the cartel. So we consider:

- fines on illegal gains, $F_{\pi}(p) = \psi\pi(p) = \psi\left(p-1\right)\left(1+\varepsilon-p\right)$;
- fines on revenue, $F_{R}(p) = \varphi R(p) = \varphi p\left(1+\varepsilon-p\right)$;
- fixed fines, $F(p) \equiv F$.

Here $\psi \geq 0$ and $\varphi \geq 0$ are the *penalty rates* that apply, respectively, in the profit and revenue-based regimes.[21] Notice that for each of these three penalty regimes the penalty base – and the penalty itself – are denominated in units of currency.

We will contrast all of these penalty bases with fines based on the overcharge. However, as noted above, the overcharge, θ, is not denominated in units of currency, and, given our normalization of the competitive price, can be given two interpretations. If we think of the overcharge as measuring the absolute increase in the price, then, in order to have a penalty base that is denominated in units of currency we have to multiply the overcharge by some measure of output, and the natural one to use is the competitive output, Q^N . So the penalty base will be $\left(p-p^N\right).Q^N = \left(p-1\right).Q^N = \theta\varepsilon$. Alternatively if we think of the overcharge as the percentage increase in price – and so a pure number – in order to have a penalty base denominated in currency we need to multiply this by something which is also measured in units of currency, and the natural one to use is revenue earned in the competitive equilibrium, $R^N = p^N Q^N$. In this case the penalty base for an overcharge-based penalty regimes will be $\dfrac{p-p^N}{p^N}.R^N = \theta\varepsilon$. So, in either interpretation the penalty base for the overcharge-based regime is $\theta\varepsilon$, and is denominated in units of currency. If we let $\eta \geq 0$ denote the *penalty rate* in an overcharge-based penalty regime, then the fine under an overcharge-based regime is $F_O(p) = \eta\theta\varepsilon = \eta\varepsilon\left(p-1\right)$.

In the analysis that follows we will take the penalty rates η, ψ, φ to be constants. So when we talk about different penalty regimes we are talking about the use of different *penalty bases*.[22]

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[21] These are pure numbers that convert a base denominated in currency into a penalty that is also denominated in currency

[22] One could also think of penalty regimes in which the *penalty rates* depend on the overcharge even though the *penalty base* is something other than the overcharge. Houba et al. (2010), Jansen and Sorgard (2012) and Katsoulacos and Ulph (2013) have shown that the price-reducing effect of the profit based fines can be improved, when the penalty rate depends on the overcharge. In this paper we rule out this possibility and

If the firms collude at price p in any period, the expected per period profit to every firm is $\frac{\pi(p)-\beta F(p)}{n}$. This process then repeats every period, and each firm takes account of the discounted continuation payoff of remaining in the cartel with discount factor δ.[23] We focus on the class of equilibria that are supported by simple grim-trigger strategies: firms collude at price p in every period- recognizing that their collusion will be detected by the CA in any given period with probability β, $0<\beta\leq 1$[24] and that, if detected, the cartel will certainly be prosecuted and have the penalty $F(p)$ imposed. However, if any firm deviates in any period by undercutting prices, the firms will revert to the static Nash/competitive equilibrium in all future periods. With such strategies, the present value of a firm's expected profit from being in the cartel is given by

$$V\left[p,F(.)\right]=\frac{\pi(p)-\beta F(p)}{n(1-\delta)}.$$

In order to support such an equilibrium, no firm should have incentive to deviate, which is the case if and only if

$$V\left[p,F(.)\right]=\frac{(p-1)(1+\varepsilon-p)-\beta F(p)}{n(1-\delta)}\geq(p-1)(1+\varepsilon-p) \qquad (1)$$

where the right hand side of the condition is the profit a firm would receive for just one period from deviating by undercutting the cartel price.[25]

Let $\Delta=n(1-\delta)$ measure what we call the *intrinsic difficulty* of keeping a cartel together. This is increasing in *n* since the more firms there are in the cartel the smaller the share of cartel profits accruing to any one firm, whereas by deviating all the cartel profits accrue to just one firm. It is also a decreasing function of δ since the more weight firms put on the future the greater the value of staying in the cartel and not just grabbing the one-period profits from deviating.

Notice that as long as $\beta F\geq 0$ then the cartel stability condition (1) can only hold if $\Delta\leq 1$, so in all that follows we will confine attention to values of $\Delta\in\left[0,1\right]$.

We define the *maximum critical difficulty*, $\overline{\Delta}$, as the value of Δ at which the cartel stability condition just holds.[26] Notice that if there is no antitrust enforcement – i.e.

analyze pure effects of the different penalty bases. However it is easy to show in our model as well that if the penalty rate is proportional to the overcharge, the resulting cartel prices will be lower under each penalty structure that we consider. In addition, penalty rates which vary with price overcharge are not in line with the current sentencing guidelines as discussed in section 2. So we concentrate on the effects of various *penalty bases* in this paper.

[23] As noted we adopt Motta and Polo (2003) assumption that cartel reestablishes after each conviction.

[24] We rule out the possibility of β = 0 since then there would be no enforcement regime and so the question of what base to use for penalties would be irrelevant.

[25] As in Motta and Polo (2003), we assume that price-deviating firms will not be prosecuted. Alternative assumptions are examined in Spagnolo (2004), Buccirossi and Spagnolo (2007), Chen and Rey (2012), Jansen and Sorgard (2014).

$\beta = 0$ - then the cartel stability condition just reduces to $\Delta \le 1$, and so we have the standard result that $\overline{\Delta} = 1$. It is obvious that when there is enforcement (i.e. $\beta > 0$) - the assumption made throughout this paper - it will be more difficult to keep a given cartel together and the maximum critical level of difficulty will fall, so we will have $\overline{\Delta} \le 1$. In the next section we will explore precisely how $\overline{\Delta}$ varies both within and across penalty regimes.

For any given stable cartel that forms we define the cartel price induced by a given penalty regime as the price, p^C, $1 \le p^C \le 1 + \varepsilon$ that yields the highest expected profit from being in the cartel subject to the sustainability condition (1). Formally:

$$p^C\left[F(.)\right] = \arg\max_p \quad \frac{\pi(p) - \beta F(p)}{\Delta} \quad \text{subject to} \quad \frac{\pi(p) - \beta F(p)}{\Delta} \ge (p-1)(1+\varepsilon - p)$$

The consumer surplus and total welfare induced by a cartel in a particular industry charging a price p are given by

$$CS(p) = \frac{(1+\varepsilon - p)^2}{2}, \; TW(p) = \frac{1}{2}\left[\varepsilon^2 - (p-1)^2\right].^{27} \qquad (2)$$

Our objective is to understand how the particular structure of fines – i.e. the choice of penalty base - affects:

(i) the price, p^C, $1 \le p^C \le 1 + \varepsilon$ charged by any given stable cartel that does form;

(ii) the cartel stability condition as reflected in the *maximum critical difficulty*, $\overline{\Delta} \le 1$, for which stable cartels will form, which determines the number of cartels that form and so the *deterrence effect* of a given penalty regime;

(iii) the **overall price** that emerges under a given penalty regime, which is defined as the price, p^C, charged by the cartel over those values of Δ, $0 \le \Delta \le \overline{\Delta}$ for which stable cartels exist, and the competitive price $p = 1$ for those values of Δ, $\overline{\Delta} \le \Delta \le 1$ for which no stable cartel exists;

(iv) the average[28] level of welfare induced by that penalty structure, as reflected in

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[26]Note that any maximum critical difficulty translates into minimum critical discount factor $\underline{\delta} = 1 - \frac{\overline{\Delta}}{n}$, which in the absence of antitrust enforcement can be rewritten as $\underline{\delta} = 1 - \frac{1}{n}$, which is the standard critical discount factor in infinitely repeated Bertrand setting with n symmetric firms (see e.g. Tirole 1988).

[27]Notice that as long as the price set by the cartel lies strictly between the competitive price and the choke price, that is as along as $1 < p < 1 + \varepsilon$, these are both strictly decreasing functions of p.

[28]The average here, and throughout the paper is over those values of Δ, $0 \le \Delta \le \overline{\Delta}$ for which stable cartels exist, and those values of Δ, $\overline{\Delta} \le \Delta \le 1$ for which no stable cartel exists and so the price is just the

(a) the average price/overcharge;
(b) the average Consumer Surplus;
(c) the average total welfare. .

4. Cartel Pricing, Cartel Stability and Welfare

In this section we derive the impact of each of the four alternative types of fine structures identified above on cartel pricing and on cartel stability (by which we mean the maximum critical difficulty of holding a cartel together).[29] We combine these into a number of measures of overall/average welfare.

4.1. Penalty on Profits

A penalty on illegal profits is given by $F_\pi(p) = \psi(p-1)(1+\varepsilon-p)$, where ψ is the constant penalty rate. The stability condition for each individual cartel member is given by

$$\frac{(1-\beta\psi)(p-1)(1+\varepsilon-p)}{\Delta} \geq (p-1)(1+\varepsilon-p). \qquad (3)$$

In what follows we let $\tau_\pi = \beta\psi$ denote the *toughness of the profits-based penalty regime*. This is a natural measure of toughness, reflecting as it does both the probability of detection and the penalty rate. Notice that in order for there to be any p, $1 \leq p \leq 1+\varepsilon$, for which the cartel stability condition (3) holds it is necessary that $0 \leq \tau_\pi \leq 1$.

It also clearly follows from (3) that the *maximum critical difficulty* under a profits-based penalty regime that is implemented with toughness τ_π, $0 \leq \tau_\pi \leq 1$ is

$$\bar{\Delta}_\pi\left(\tau_\pi\right) = 1 - \tau_\pi \leq 1. \qquad (4)$$

For those stable cartels that do form, i.e. for those for whom $0 \leq \Delta \leq \bar{\Delta}_\pi\left(\tau_\pi\right)$, the cartel price is:

$$p_\pi^C\left(\Delta, \tau_\pi\right) = \arg\max_p V\left[p, F_\pi(\cdot)\right] = p^M = 1 + \frac{\varepsilon}{2}. \qquad (5)$$

competitive price. In the absence of any theoretical or empirical evidence on the distribution of Δ, in calculating these averages we assume it is uniformly distributed on [0,1].

[29] The existing literature has not examined the implications of antitrust penalties for cartel stability in a systematic manner.

Thus fines based on illegal profits induce the cartel to set the monopoly price $p^M = 1 + \frac{\varepsilon}{2}$, independently of the toughness of the penalty regime.[30] This is because the penalty just produces an equi-proportionate reduction in net expected profits. So maximizing net profits is equivalent to maximizing gross profits and the price that is set is the same as that which would have arisen had no penalty regime been in place – the monopoly price. However, a tougher regime lowers the maximum critical difficulty and so makes it less likely that stable cartels will form.

In order to combine these two elements to provide an overall picture of how a particular penalty regime affects both the price set by those stable cartels that do form, and the extent to which certain cartels that might possibly have formed are deterred from doing so we proceed as follows. For all values of $\Delta \in [0,1]$ we define the **overall price** that would emerge under a profits-based regime, $p_\pi(\Delta, \tau_\pi)$, to be the price that would be set by the cartel for those ranges of Δ, $0 \le \Delta \le \bar{\Delta}_\pi(\tau_\pi)$ for which stable cartels exist, and the competitive price for those values of Δ, $\bar{\Delta}_\pi(\tau_\pi) \le \Delta \le 1$ for which no stable cartel forms. Formally:

$$p_\pi(\Delta, \tau_\pi) = \begin{cases} 1 + \frac{\varepsilon}{2}, & 0 \le \Delta \le 1 - \tau_\pi \\ 1, & 1 - \tau_\pi \le \Delta \le 1 \end{cases}. \tag{6}$$

This is shown in Figure 1 below.

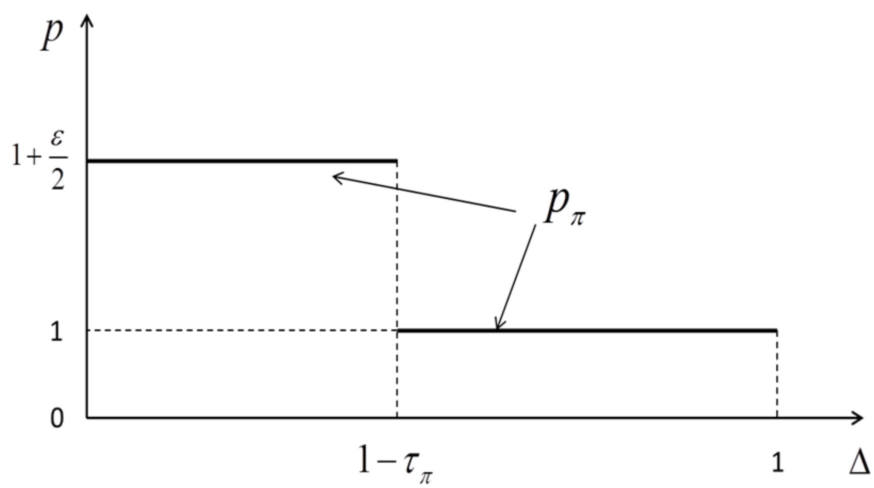

Figure 1: Overall Price Under Profits-Based Penalty

[30] Houba et al. (2010) show that if the probability of detection and the penalty rate are constant then the same result holds for more general demands.

Next, based on the assumption that Δ is uniformly distributed on [0,1],we can define the average overcharge, consumer surplus and total welfare under a profits-based regime that is implemented with toughness $\tau_\pi, \quad 0 \le \tau_\pi \le 1$ as:

$$\overline{O}_\pi\left(\tau_\pi\right) = \int_0^1 p_\pi\left(\Delta, \tau_\pi\right) d\Delta - 1 = \frac{\varepsilon}{2}\left(1 - \tau_\pi\right) \tag{7}$$

$$\overline{CS}_\pi\left(\tau_\pi\right) = \int_0^1 CS\left[p_\pi\left(\Delta, \tau_\pi\right)\right] d\Delta = \frac{\varepsilon^2}{8}\left(1 - \tau_\pi\right) + \tau_\pi \frac{\varepsilon^2}{2} = \frac{\varepsilon^2}{8}\left(1 + 3\tau_\pi\right) \tag{8}$$

$$\overline{TW}_\pi\left(\tau_\pi\right) = \int_0^1 TW\left[p_\pi\left(\Delta, \tau_\pi\right)\right] d\Delta = \frac{3\varepsilon^2}{8}\left(1 - \tau_\pi\right) + \tau_\pi \frac{\varepsilon^2}{2} = \frac{\varepsilon^2}{8}\left(3 + \tau_\pi\right) \tag{9}$$

The formulae that appear on the RHS of (7), (8) and (9) follow from straightforward integration using (6) and (2).

Finally, notice that the profits made by a stable cartel that has formed and has set the monopoly price are $\frac{\varepsilon^2}{4}$. Consequently, under a profits-based penalty regime that is implemented with toughness $\tau_\pi, \quad 0 \le \tau_\pi \le 1$, the average fine that is paid by any cartel that is detected and penalized is

$$\overline{F}_\pi\left(\tau_\pi\right) = \psi \frac{\varepsilon^2}{4} = \frac{1}{\beta}\left(\tau_\pi \frac{\varepsilon^2}{4}\right) \tag{10}$$

and so is directly proportional to the toughness of the profits-based penalty regime.

4.2. Fixed Penalty

A fixed penalty is given by $F(p) \equiv F$, where F is the constant absolute penalty. Under this regime the cartel stability condition (1) becomes

$$\begin{aligned} &\frac{(p-1)(1+\varepsilon-p) - \beta F}{\Delta} \ge (p-1)(1+\varepsilon-p) \\ \Rightarrow \quad &\beta F \le (1-\Delta)(p-1)(1+\varepsilon-p) \end{aligned} \tag{11}$$

In order for there to be any price that a cartel can set and still satisfy the stability condition it must be the case that

$$\beta F \le (1-\Delta).\underset{p}{MAX}(p-1)(1+\varepsilon-p) = \frac{(1-\Delta)\varepsilon^2}{4} \Rightarrow \beta\left(\frac{4F}{\varepsilon^2}\right) \le 1-\Delta \tag{12}$$

This implies that the *maximum critical difficulty* of a fixed penalty regime is:

$$\overline{\Delta}_F = 1 - \beta\frac{4F}{\varepsilon^2} \le 1. \tag{13}$$

For those stable cartels that do form, i.e. for those for whom $0 \le \Delta \le \overline{\Delta}_F$, the cartel price is:

$$p_F^C = \arg\max_p V\left[p, F(\cdot)\right] = p^M = 1 + \frac{\varepsilon}{2}. \tag{14}$$

This implies that fixed fines induce the cartel to set the monopoly price $p^M = 1 + \frac{\varepsilon}{2}$, independently of the toughness of the penalty regime.[31] The reason is clear: a fixed penalty acts just like an expected fixed cost, and, as we know, fixed costs have no effect on pricing decisions. However, a tougher regime lowers the *maximum critical difficulty* and so makes it less likely that stable cartels will form.

Consequently the **overall price** that would emerge under a fixed penalty regime is

$$p_F\left(\Delta, F\right) = \begin{cases} 1 + \frac{\varepsilon}{2}, & 0 \le \Delta \le 1 - \beta \frac{4F}{\varepsilon^2} \\ 1, & 1 - \beta \frac{4F}{\varepsilon^2} \le \Delta \le 1 \end{cases}. \tag{15}$$

Recalling that $\tau_\pi = \beta\psi$, then from (6) and (15) we have the following:

Proposition 1 *The class of fixed penalty regimes is identical to the class of profit-based penalty regimes in the sense that:*

(i) *for every value of $\psi \ge 0$ arising in some profits-based penalty regime, there is a fixed penalty $F = \psi \frac{\varepsilon^2}{4} \ge 0$ such that the associated fixed penalty regime will induce exactly the same welfare-relevant outcomes: cartel price, deterrent effects (maximum critical difficulty), overall price etc.*

(ii) *Conversely for every fixed penalty $F \ge 0$ associated with some fixed penalty regime, there is a penalty rate $\psi = \frac{4F}{\varepsilon^2} \ge 0$ on profits such that the associated profits-based penalty regime will induce exactly the same welfare-relevant outcomes: cartel price, deterrent effects (maximum critical difficulty), overall price.*

In what follows we will therefore ignore fixed penalty regimes and confine our attention solely to a comparison of profits-based penalty regimes, revenue-based regimes and overcharge-based regimes.

[31] Harrington (2005) shows that if the probability of detection and the penalty rate are constant then the same result holds for more general demands.

4.3. Penalty on Revenue

A penalty on revenue is given by $F_R(p) = \varphi p(1+\varepsilon - p)$, where φ is the constant penalty rate.[32] The cartel stability condition (1) now becomes

$$\frac{(p-1)(1+\varepsilon-p)-\beta\varphi p(1+\varepsilon-p)}{\Delta} \geq (p-1)(1+\varepsilon-p). \tag{16}$$

Provided output $Q = 1+\varepsilon - p$ is positive, we can divide both sides of (16) by output to obtain

$$\begin{aligned} &\frac{(p-1)-\beta\varphi p}{\Delta} \geq (p-1) \\ \Rightarrow \quad & p(1-\beta\varphi-\Delta) \geq 1-\Delta \end{aligned} \tag{17}$$

So we have the following novel result:

Proposition 2: *When penalties are imposed on revenue then, in order to maintain stability, the cartel has to set a* ***minimum*** *price*

$$p_R^{MIN} = \frac{1-\Delta}{1-\beta\varphi-\Delta} = \frac{1}{1-\dfrac{\beta\varphi}{1-\Delta}}. \tag{18}$$

Corollary: *The minimum price is strictly increasing in*

(i) *the toughness of the penalty regime as reflected in* β, φ*;*

(ii) *the difficulty of holding the cartel together,* Δ.

Proof: Straightforward implication of partial differentiating the expression in (18) with respect to Δ, β, φ. ■

Notice that in order to have any chance of making positive profits the cartel must have positive output, so the minimum price as defined in (18) must be less than or equal to the choke price $1+\varepsilon$. After re-arranging this implies that the *maximum critical difficulty* under a revenue-based penalty regime is:

$$\bar{\Delta}_R = 1 - \frac{\beta\varphi(1+\varepsilon)}{\varepsilon}.$$

If we now let $\tau_R = \dfrac{\beta\varphi(1+\varepsilon)}{\varepsilon}$ denote the "industry-adjusted" measure of toughness of the revenue-based penalty regime[33], we have

[32] By constant we mean that, within a given industry, it doesn't vary depending on the amount of revenue a cartel makes. As discussed below it may however differ across industries depending on the inverse elasticity of demand, ε.

$$\overline{\Delta}_R\left(\tau_R\right)=1-\tau_R\,, \tag{19}$$

and since, as noted above, for the cartel stability condition to hold we must have $0\leq \overline{\Delta}_R \leq 1$ it follows that the toughness parameter must satisfy $0\leq\tau_R\leq 1$. Notice that the measure of toughness captures the probability of successful detection and prosecution, β, and the penalty rate, φ, but, for constant values of these, toughness will vary across industries depending on the elasticity of demand, ε.[34]

Using this definition of toughness the minimum price as defined in (18) becomes

$$p_R^{MIN}=\frac{1}{1-\dfrac{\varepsilon\tau_R}{\left(1+\varepsilon\right)\left(1-\Delta\right)}}, \tag{20}$$

which is a strictly increasing function of the toughness parameter, τ_R.

If the cartel stability condition (16) is not binding, the unconstrained profit maximizing cartel price is

$$p_R{}^C=\arg\max_p V\left[p,F_R(\cdot)\right]=\frac{1+\varepsilon}{2}+\frac{1}{2\left(1-\beta\varphi\right)}=1+\frac{\varepsilon}{2}\left[1+\frac{\tau_R}{1+\varepsilon\left(1-\tau_R\right)}\right]\geq p^M .$$

Notice that this unconstrained cartel price is a strictly increasing in τ_R and that as $\tau_R\to 1,\quad p_R^C\to 1+\varepsilon$ - that is the unconstrained cartel price tends towards the choke price, $1+\varepsilon$.

Taking into account the minimum cartel price required to achieve stability, it follows that the price set by those stable cartels that do exist under a revenue-based penalty regime– i.e. those cartels for which $\Delta\leq\overline{\Delta}_R\left(\tau_R\right)=1-\tau_R$ - is:

$$p_R{}^C\left(\Delta,\tau_R\right)=MAX\left\{\frac{1}{1-\dfrac{\varepsilon\tau_R}{\left(1+\varepsilon\right)\left(1-\Delta\right)}},\quad 1+\frac{\varepsilon}{2}\left[1+\frac{\tau_R}{1+\varepsilon\left(1-\tau_R\right)}\right]\right\}\geq p^M \tag{21}$$

This confirms that fines based on revenues not only do not reduce the cartel price below the monopoly price, but actually push the price above the monopoly price. This distortionary effect of antitrust fines based on revenues was first identified in Bageri et al. (2013).[35] Moreover, it follows from (21) that the tougher the penalty regime, i.e. the

[33] For a given value of φ the degree of toughness will vary across industries depending on the inverse elasticity. Alternatively one can think of holding the degree of toughness constant across all industries by letting $\varphi=\varphi'\dfrac{\varepsilon}{1+\varepsilon}$.

[34]Alternatively, in order to have a constant level of toughness across all industries it would be necessary for the penalty rate to vary with elasticity of demand.

[35] However this was done in the context of a static model that did not permit the examination of stability conditions and how these are affected by penalties. In particular they did not establish the existence of a minimum price under a revenue-based penalty regime.

higher is τ_R, the higher will be each of the two terms in the MAX expression and so the cartel price and the greater the distortion. This was also previously shown in Katsoulacos and Ulph (2013), albeit in a more static context.

The intuition is straightforward – a penalty on revenue lowers expected average and marginal revenue but not marginal costs, so leading cartels to reduce output and drive up price. The tougher the penalty the bigger the reduction in marginal revenue and so output.

It is straightforward to show that the value of Δ at which the stability condition just bites, i.e. the two terms in (21) are equal, is:

$$\tilde{\Delta}_R(\tau_R) = (1-\tau_R)\frac{\left[1+\varepsilon(1-\tau_R)\right]}{\left[1+\varepsilon(1-\tau_R)+\tau_R\right]} \leq 1-\tau_R = \overline{\Delta}_R(\tau_R). \tag{22}$$

Once again, for all $\Delta \in [0,1]$ we define the **overall price** that would emerge under a revenue-based regime as the price that would emerge taking account of both the price set by those cartels that do form and the (competitive) price that would prevail were no cartel to form. It follows from (21) and (22) that this takes the form:

$$p_R(\Delta,\tau_R) = \begin{cases} 1+\dfrac{\varepsilon}{2}\left[1+\dfrac{\tau_R}{1+\varepsilon(1-\tau_R)}\right], & 0 \leq \Delta \leq \tilde{\Delta}_R \\ \dfrac{1}{1-\dfrac{\varepsilon\tau_R}{(1+\varepsilon)(1-\Delta)}}, & \tilde{\Delta}_R \leq \Delta \leq \overline{\Delta}_R = 1-\tau_R \\ 1, \quad \overline{\Delta}_R = 1-\tau_R \leq \Delta \leq 1 \end{cases} \tag{23}$$

Given the definition of the *maximum critical level of difficulty,* $\overline{\Delta}_R(\tau_R)$, it is easy to see that as $\Delta \to \overline{\Delta}_R$ from below, $p_R \to 1+\varepsilon$ - the choke price. So the overall price under a revenue-based regime is highly discontinuous at $\overline{\Delta}_R$. It is illustrated in Figure 2.

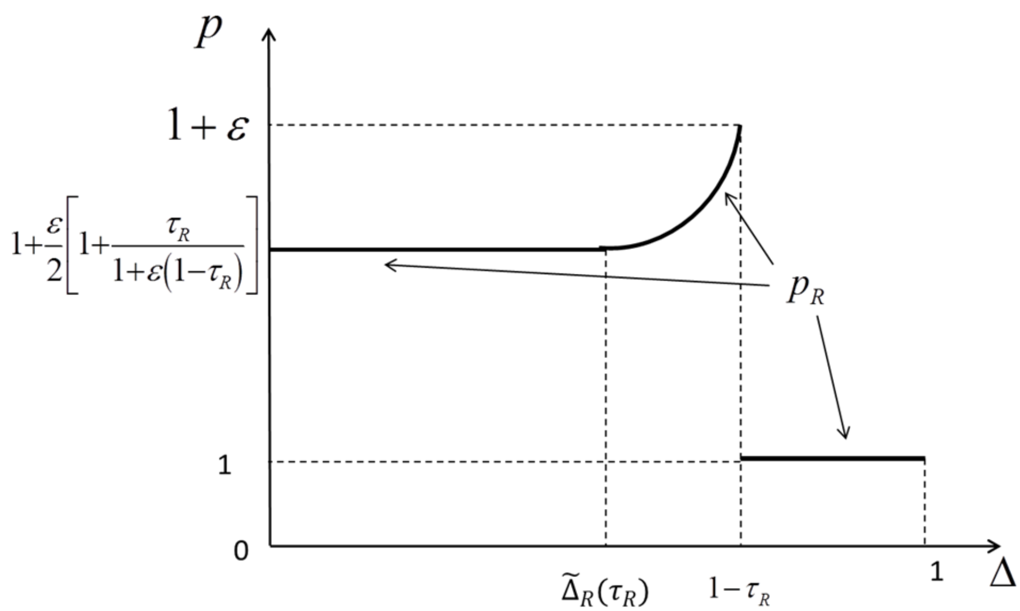

Figure 2: Overall Price Under Revenue-Based Penalty

Based on the assumption that Δ is uniformly distributed on [0,1],we can define the average overcharge, average consumer surplus and average total welfare under a revenue-based regime that is implemented with toughness τ_R as:

$$\overline{O}_R\left(\tau_R\right)=\int_0^1 p_R\left(\Delta,\tau_R\right)d\Delta-1 \tag{24}$$

$$\overline{CS}_R\left(\tau_R\right)=\int_0^1 CS\left[p_R\left(\Delta,\tau_R\right)\right]d\Delta \tag{25}$$

$$\overline{TW}_R\left(\tau_R\right)=\int_0^1 TW\left[p_R\left(\Delta,\tau_R\right)\right]d\Delta\,. \tag{26}$$

However, given the complexity of the price formula (23) we have been unable to get closed-form analytical expressions for these terms.

Finally under a revenue-based penalty regime that is implemented with toughness τ_R, $0\le\tau_R\le1$ we can define the average fine that is collected from stable cartels that form and are subsequently detected and penalized as

$$\overline{F}_R\left(\tau_R\right)=\frac{\varphi}{\overline{\Delta}_R\left(\tau_R\right)}\int_0^{\overline{\Delta}_R(\tau_R)} R\left[p_R\left(\Delta,\tau_R\right)\right]d\Delta$$

i.e.

$$\overline{F}_R\left(\tau_R\right)=\frac{1}{\beta}\left[\frac{\varepsilon\tau_R}{(1+\varepsilon)\left(1-\tau_R\right)}\int_0^{1-\tau_R} p_R\left(\Delta,\tau_R\right)\left[1+\varepsilon-p_R\left(\Delta,\tau_R\right)\right]d\Delta\right] \tag{27}$$

Once again, we have been unable to obtain a closed-form analytical expression for this term. An important point to notice is that the average fine raised on detected cartels is zero both when $\tau_R = 0$ - because then there is no penalty – and when $\tau_R = 1$ - because then $p_R^C = 1 + \varepsilon \Rightarrow R\left(p_R^C\right) = p_R^C\left(1 + \varepsilon - p_R^C\right) = 0$ and so the penalty base is totally eroded.[36] Formally:

$$\overline{F}_R(0) = 0 = \overline{F}_R(1). \tag{28}$$

4.4. Penalty on Overcharge

A penalty on the cartel overcharge is given by $F_O(p) = \eta\varepsilon\theta = \eta\varepsilon(p-1)$, where η is the constant penalty rate. The cartel stability condition now becomes

$$\frac{(p-1)(1+\varepsilon-p) - \beta\eta\varepsilon(p-1)}{\Delta} \geq (p-1)(1+\varepsilon-p). \tag{29}$$

Provided $p > 1$ and so the cartel sets a price above the but-for price, we can divide both sides of (29) by $(p-1)$ to obtain:

$$Q = 1+\varepsilon - p \geq \frac{\beta\eta\varepsilon}{1-\Delta} \quad \Leftrightarrow \quad p \leq 1+\varepsilon - \frac{\beta\eta\varepsilon}{1-\Delta}$$

So, depending on the magnitude of the term $\frac{\beta\eta\varepsilon}{1-\Delta}$, in order to maintain stability, the cartel may be forced to set a minimum level of output (equivalently a maximum price). Therefore we have the following result:

Proposition 3: *When penalties are imposed on the overcharge then, in order to maintain stability, the cartel has to set a* ***maximum*** *price*

$$p_O^{MAX} = 1+\varepsilon - \frac{\beta\eta\varepsilon}{1-\Delta}. \tag{30}$$

Corollary*: The maximum price is a strictly decreasing function of:*

(i) *the toughness of the penalty regime, as reflected in the parameters* β, η;

(ii) *the difficulty of holding cartel together,* Δ.

Proof: Straightforward implication of partially differentiating the expression in (30) with respect to Δ, β, η.

In order for the cartel to have any chance of making profit the maximum price must be greater than or equal to the marginal cost (= 1). Hence, from (30), the maximum critical difficulty is:

$$\overline{\Delta}_O = 1 - \beta\eta .$$

[36]This is the exact analogy of the Laffer curve.

If we now let $\tau_O = \beta\eta$ denote the measure of toughness of the overcharge-based penalty regime, we have

$$\bar{\Delta}_O\left(\tau_O\right) = 1 - \tau_O, \tag{31}$$

and since, as noted above, for the cartel stability condition to hold we must have $\bar{\Delta}_O \le 1$ it follows that the toughness parameter must satisfy $0 \le \tau_O \le 1$. Notice that the measure of toughness captures the probability of successful detection and prosecution, β, and the penalty rate, η. Using this notation the maximum price as defined in (30) becomes

$$p_O^{MAX} = 1 + \varepsilon\left(1 - \frac{\tau_O}{1-\Delta}\right), \tag{32}$$

Provided that the cartel stability constraint is not binding, the unconstrained profit maximizing cartel price is

$$p_O{}^C = \arg\max_p V\left[p, F_O(\cdot)\right] = 1 + \frac{\varepsilon}{2} - \frac{\beta\eta\varepsilon}{2} = 1 + \frac{\varepsilon}{2}\left(1-\tau_O\right) \le p^M .$$

Notice that this unconstrained cartel price is strictly decreasing in τ_O and that as $\tau_O \to 1, \quad p_O^C \to 1$ - that is the unconstrained cartel price tends towards the competitive price (=1).

Hence, taking into account the maximum sustainable cartel price, we see that the price set by those stable cartels that do exist is:

$$p_O{}^C\left(\Delta,\tau_O\right) = MIN\left[1 + \varepsilon\left(1 - \frac{\tau_O}{1-\Delta}\right), 1 + \frac{\varepsilon}{2}\left(1-\tau_O\right)\right] \le p^M . \tag{33}$$

Notice that the critical value of Δ at which the maximum price constraint bites, i.e. the terms in (33) are equal, is

$$\tilde{\Delta}_O\left(\tau_O\right) = \frac{1-\tau_O}{1+\tau_O} \le 1 - \tau_O = \bar{\Delta}_O . \tag{34}$$

As before for all $\Delta \in [0,1]$ we define the **overall price** that would emerge under an overcharge-based regime to be the price that would emerge taking account of both the price set by those cartels that do form and the price that would prevail were no cartel to form. From (33) and (34) this is given by :

$$p_O(\Delta,\tau_O) = \begin{cases} 1 + \frac{\varepsilon}{2}\left(1-\tau_O\right), & 0 \le \Delta \le \frac{1-\tau_O}{1+\tau_O} \\ 1 + \varepsilon\left(1 - \frac{\tau_O}{1-\Delta}\right), & \frac{1-\tau_O}{1+\tau_O} \le \Delta \le 1 - \tau_0 \\ 1, \quad 1 - \tau_0 \le \Delta \le 1 & \end{cases} \tag{35}$$

Notice that, for $\tilde{\Delta}_O(\tau_O) \le \Delta \le \overline{\Delta}_O(\tau_O)$, the overall price is a strictly decreasing function of Δ, with $p_O^C \to 1$ as $\Delta \to \overline{\Delta}_O$. It is illustrated in Figure 3.

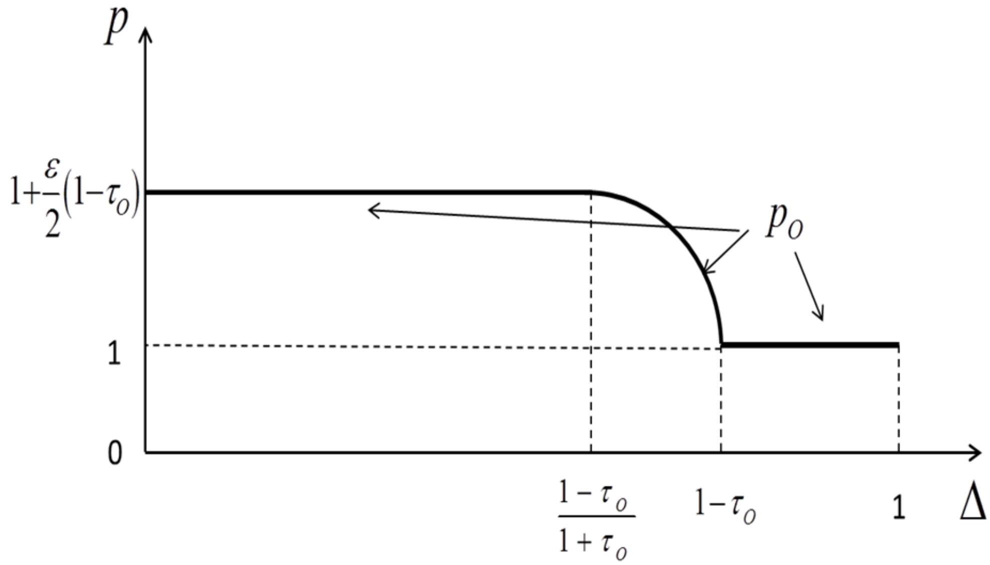

Figure 3: Overall Price Under Overcharge-Based Penalty

Further, based on the assumption that Δ is uniformly distributed on [0,1], we can define the average overcharge, average consumer surplus and average total welfare under an overcharge-based penalty regime that is implemented with toughness τ_O, $0 \le \tau_O \le 1$ as:

$$\overline{O}_O(\tau_O) = \int_0^1 p_0(\Delta, \tau_O)\, d\Delta - 1 = \frac{\varepsilon}{2}\left\{(1-\tau_O) - 2\tau_O\left[\ln(2) - \ln(1+\tau_O)\right]\right\} \tag{36}$$

$$\overline{CS}_O(\tau_O) = \int_0^1 CS\left[p_O(\Delta, \tau_O)\right] d\Delta = \frac{\varepsilon^2}{8}\left[1 + 6\tau_O - 3\tau_O^2\right] \tag{37}$$

$$\overline{TW}_O(\tau_O) = \int_0^1 TW\left[p_O(\Delta, \tau_O)\right] d\Delta = \frac{\varepsilon^2}{8}\left\{\left(3 + 2\tau_O - \tau_O^2\right) + 8\tau_O\left[\ln(2) - \ln(1+\tau_O)\right]\right\} \tag{38}$$

Finally under an overcharge-based penalty regime that is implemented with toughness τ_O, $0 \le \tau_O \le 1$ we can define the average fine that is collected from stable cartels that form and are subsequently detected and penalized as

$$\begin{aligned}\overline{F}_O(\tau_O) &= \frac{\eta\varepsilon}{1-\tau_O}\int_0^{1-\tau_O} O_O(\Delta, \tau_0)\, d\Delta \\ &= \frac{1}{\beta}\left\{\frac{\varepsilon^2}{2}\frac{\tau_O}{1-\tau_O}\left\{(1-\tau_O) - 2\tau_O\left[\ln(2) - \ln(1+\tau_O)\right]\right\}\right\}\end{aligned} \tag{39}$$

Notice that

$$\overline{F}_O(0) = 0 = \overline{F}_O(1)\ ^{37} \qquad (40)$$

The first result arises because, although the penalty base is positive, the penalty rate is zero, while the second result arises because, although the penalty rate is positive, the penalty base is zero since, as the degree of toughness tends to 1, the cartel price is driven down to the competitive price and there is no overcharge.

The derivation of the formulae that appear on the RHS of (36)-(39) is given in Appendix 1.

5. Comparisons within and across Penalty Regimes

In this section we draw on the results of the previous section to undertake an analysis of how the various welfare indicators in which we are interested – price, deterrence, average surplus etc. – are affected by both within-regime factors such as toughness and the nature of the industry (as captured by the inverse elasticity of demand, ε) but also by across regime changes that arise from switching to different penalty bases. We begin with a number of background remarks.

- Recall that from Proposition 1 a profits-based penalty regime is equivalent to a fixed penalty regime when implemented with the same toughness. So in this section we focus on a comparison between: profit-based regimes; revenue-based regimes and overcharge-based regimes.
- If effectively there are no penalties – i.e. if $\tau_\pi = \tau_R = \tau_O = 0$ - then, under all three regimes, cartels exist for all Δ, $0 \le \Delta \le 1$, and just charge the monopoly price $p^M = 1 + \frac{\varepsilon}{2}$[38] - so all regimes are identical. In other words, if no penalties are imposed it doesn't matter on which base you don't impose them.
- Similarly if all regimes are implemented with maximum toughness - i.e. if $\tau_\pi = \tau_R = \tau_O = 1$- then, under all three regimes the *maximum critical difficulty* is zero and the overall price is just the competitive price,[39] and so, once again, all three regimes are identical. In other words, if no stable cartels ever form it doesn't matter on which base you would have penalized a non-existent cartel
- So in what follows we undertake comparisons on the assumption that all regimes are implemented with a degree of toughness lying between 0 and 1.[40]

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[37] This follows from the formula in (39) by using l'Hopital's rule.

[38]This can be easily seen by setting $\tau_\pi = \tau_R = \tau_O = 0$ into equations (4), (6), (19), (23), (31), and (35)

[39]This can be easily seen by setting $\tau_\pi = \tau_R = \tau_O = 1$ into equations (4), (6), (19), (23), (31), and (35)

[40]Formally $0 < \tau_\pi < 1; 0 < \tau_R < 1; 0 < \tau_O < 1$.

5.1 Comparison of prices set by stable cartels that form

We start with a comparison across the three regimes of the prices set by those cartels that do form.[41] This largely summarises results established in Section 4. We start with:

***Proposition 4**: Penalties based on either profits (illegal gains) or on revenue are **ineffective** in reducing the cartel price below the monopoly price. In particular:*

(i) With penalties on illegal gains (profits) the cartel price remains equal to the monopoly price, $p_\pi^C = p^M$ and, moreover, is independent of the toughness of the profits-based regime τ_π, $0 < \tau_\pi < 1$.

(ii) Penalties on revenue are distortionary and produce a cartel price that

a. lies between the monopoly price and the choke price – i.e. $p^M < p_R^C < 1 + \varepsilon$;

b. is strictly increasing in the toughness of the revenue-based penalty regime, τ_R, $0 < \tau_R < 1$.

c. tends towards the choke price $1 + \varepsilon$ as $\tau_R \to 1$

These results can all be readily established formally by an inspection of equations (5) and (21). Their intuition is clear. Whatever the degree of toughness, fines based on illegal gains just produce a proportional reduction in expected profits. Consequently maximizing net profits is equivalent to maximising gross profits and so leads the cartel to set the same price that would have prevailed had there been no enforcement. However penalties based on revenue lower expected marginal revenue but do not affect marginal cost, thus inducing cartels to cut output and drive up price. Moreover, the tougher the penalty regime the greater the reduction in expected marginal revenue and consequently output and so the higher the price.

We next have:

***Proposition 5**: Penalties on overcharges are **effective** in producing a cartel price that*

(i) lies between the monopoly price and the competitive – i.e. $1 < p_O^C < p^M$;

(ii) is a strictly decreasing function of the toughness of the overcharge regime τ_O, $0 < \tau_O < 1$;

(iii) tends towards the competitive price as $\tau_O \to 1$.

These results can readily be established formally by an inspection of equation (33) Again, the intuition is clear. Penalties based on the overcharge directly target the pricing decision of the cartel and so reduce the incentive to raise price significantly above its "but-for" level. The tougher the regime, the greater is the incentive to keep price down.

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[41] Strictly speaking, for this comparison to be valid we have to assume
$0 \le \Delta \le MIN\left[\overline{\Delta}_\pi(\tau_\pi), \overline{\Delta}_R(\tau_R), \overline{\Delta}_O(\tau_O)\right]$

The conclusions of Propositions 4 and 5 can be combined in the following

Proposition 6: *For all* τ_π, $0 < \tau_\pi < 1$, τ_R, $0 < \tau_R < 1$ *and* τ_O, $0 < \tau_O < 1$ $p_O^C < p^M = p_\pi^C < p_R^C$.

This shows that, in relation to the prices[42] set by stable cartels that form, we can get a clear ranking across the three penalty regimes that is independent of the precise degree of toughness of each regime. This remarkably strong conclusion will be important in the analysis in Section 5.3

5.2 Deterrence Effects: Comparative Static Properties

The desirability of alternative penalty regimes depends not just on their effect on the price charged by any given cartel, but also on their effects on the number of cartels that are formed and remain stable - their effect on deterrence. In this sub-section we begin to examine these latter effects by undertaking a comparative static analysis of how the *maximum critical difficulty* of holding a cartel together is affected by the penalty regime and market conditions. We have:

Proposition 7:

(i) For all three penalty regimes, the maximum critical difficulty of holding a cartel together is decreasing in the toughness of the penalty regime, and goes to zero as the degree of toughness goes to 1;

(ii) For revenue-based penalty regime the maximum critical difficulty is also increasing in the inverse elasticity parameter.

Proof: Follows immediately from equations (4), (19) and (31) defining the maximum critical difficulty under all three regimes.

5.3. Welfare Effects of Using Different Penalty Bases

Precisely because, as we have just seen, the deterrent effects of any given penalty regime is so sensitive to the toughness with which it is implemented, it follows that the overall price and hence the various measures of average consumer surplus etc. are also going to be very sensitive to the toughness with which any given regime is implemented. So using, say, revenue rather than the overcharge as a penalty base may produce better welfare outcomes if the revenue-based penalty is implemented very toughly while the overcharge-based penalty is implemented very weakly. Consequently if we want to undertake a meaningful analysis of the consequences for various indicators of welfare – price, deterrence etc. - of using different bases on which to impose penalties, we have to do so holding the degree of toughness constant in some sense. There are a number of

[42] And hence the associated levels of consumer surplus and total welfare

possible interpretations of what it might mean for regimes to be equally tough. In what follows we consider two: deterrence equivalence and penalty revenue equivalence.

5.3.1 Deterrence Equivalence

One fairly natural interpretation of what it might mean for each regime to be equally tough is that the fraction of cartels deterred is exactly the same across all three regimes – i.e. the maximum critical difficulty is the same across all three regimes. Formally we require that the toughness parameters τ_π, τ_R, τ_O are such that, for some $\overline{\Delta}^*, \quad 0 < \overline{\Delta}^* < 1$

$$\overline{\Delta}_\pi\left(\tau_\pi\right) = \overline{\Delta}_R\left(\tau_R\right) = \overline{\Delta}_O\left(\tau_O\right) = \overline{\Delta}^*. \tag{41}$$

If we denote the toughness parameters for which this is true by $\tau_\pi^*, \tau_R^*, \tau_O^*$, then clearly $\tau_\pi^* = \tau_R^* = \tau_O^* = 1 - \overline{\Delta}^*$ But, from Proposition 6 this immediately implies:

$$\begin{gathered}\forall\Delta, \quad 0 \le \Delta \le \overline{\Delta}^*: \\ p_O\left(\Delta, \tau_O^*\right) = p_O^C\left(\Delta, \tau_O^*\right) < p_\pi\left(\Delta, \tau_\pi^*\right) = p_\pi^C\left(\Delta, \tau_\pi^*\right) < p_R\left(\Delta, \tau_R^*\right) = p_R^C\left(\Delta, \tau_R^*\right)\end{gathered} \tag{42}$$

and

$$\forall\Delta, \quad \overline{\Delta}^* \le \Delta \le 1: \quad p_O\left(\Delta, \tau_O^*\right) = p_\pi\left(\Delta, \tau_\pi^*\right) = p_R\left(\Delta, \tau_R^*\right) = 1 \tag{43}$$

This gives us:

Proposition 8: *If we impose <u>deterrent equivalence</u> for any* $\overline{\Delta}^*, \quad 0 < \overline{\Delta}^* < 1$ *then*

(i) $\overline{O}_O\left(\tau_O^*\right) < \overline{O}_\pi\left(\tau_\pi^*\right) < \overline{O}_R\left(\tau_R^*\right)$

(ii) $\overline{CS}_O\left(\tau_O^*\right) > \overline{CS}_\pi\left(\tau_\pi^*\right) > \overline{CS}_R\left(\tau_R^*\right)$

(iii) $\overline{TW}_O\left(\tau_O^*\right) > \overline{TW}_\pi\left(\tau_\pi^*\right) > \overline{TW}_R\left(\tau_R^*\right)$.

Proof: (i) follows by using (42) and (43) and integrating over all $\Delta \in [0,1]$. (ii) and (iii) follow by noting from (2) that consumer surplus and total welfare are strictly decreasing functions of price.

Figure 4 shows the superiority of the overcharge based regime for overall prices under deterrence equivalence.

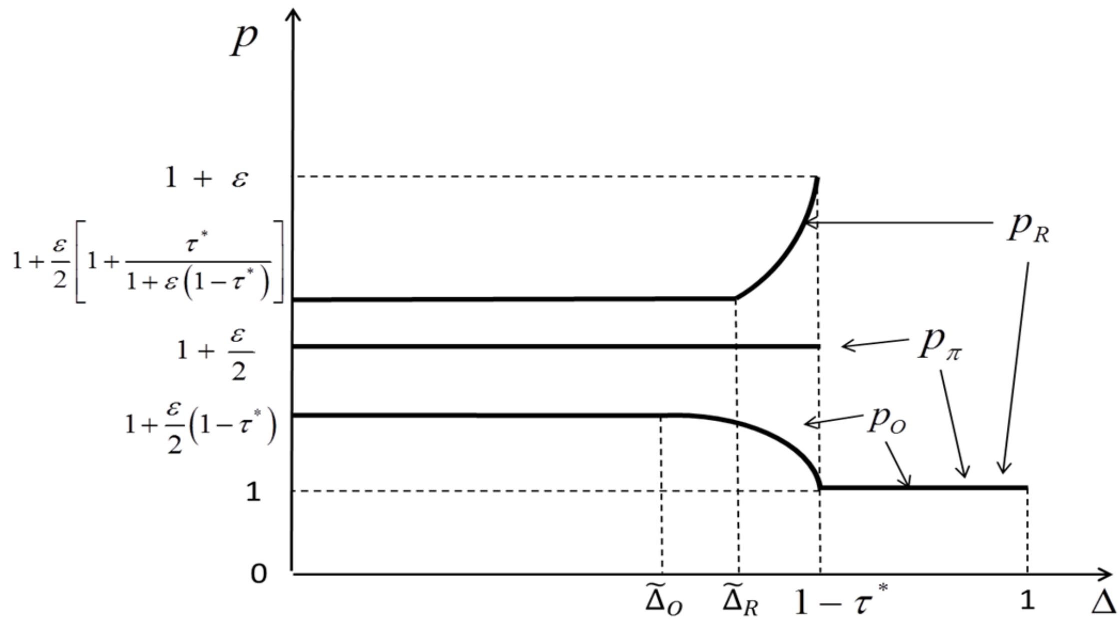

Figure 4: Overall Prices Under Deterrence Equivalence

The intuition is simple. Profit or revenue based regimes could only perform better in terms of overall prices if the toughness parameters in these regimes led to cartel deterrence and thus competitive (Bertrand) pricing at even lower values of Δ (the critical difficulty of forming cartels) than the values of Δ that lead to deterrence and competitive pricing with the toughness parameter of the overcharge-based regime. Under deterrence equivalence the toughness parameters and the values of Δ above which we get deterrence and competitive pricing are equalized across the three regimes, so the overcharge regime leads unambiguously to lower overall prices. As Proposition 8 shows, whatever welfare indicator we use, an overcharge-based penalty regime welfare dominates a profit-based regime which in turn welfare dominate a revenue-based penalty regime.

We also have the following simple corollary relating to the comparison of an overcharge-based penalty regime with a profits-based regime.

Corollary 8 *If an overcharge-based regime is at least as tough as a profit-based regime then it welfare dominates the profit-based regime. Formally, for all* (τ_π, τ_O) $0 < \tau_\pi \le \tau_O < 1$

(i) $\overline{O}_O(\tau_O) < \overline{O}_\pi(\tau_\pi)$;

(ii) $\overline{CS}_O(\tau_O) > \overline{CS}_\pi(\tau_\pi)$;

(iii) $\overline{TW}_O(\tau_O) > \overline{TW}_\pi(\tau_\pi)$.

Proof: From (35) and (6) it follows that $\forall\Delta,\ 0 \le \Delta < 1-\tau_\pi$, $p_O(\Delta,\tau_O) < p_\pi(\Delta,\tau_\pi) = 1+\frac{\varepsilon}{2}$, while $\forall\Delta,\ 1-\tau_\pi \le \Delta \le 1$, $p_O(\Delta,\tau_O) = p_\pi(\Delta,\tau_\pi) = 0$. So (i) follows by integration over $\Delta \in [0,1]$; (ii) and (iii) follow by noting from (2) that consumer surplus and total welfare are strictly decreasing functions of price.

5.3.2 Penalty Revenue Equivalence

An alternative notion of equal toughness is that of penalty revenue equivalence, namely that, on average[43] the size of the penalty actually paid by any cartel that forms and is subsequently detected and penalized should be the same whatever penalty base is in operation. There are two related reasons for considering such a criterion.

The first is the obvious one that, since we are using different bases, which, in any given case could be of a different size – e.g. profit will be lower than revenue – then imposing a penalty at, say, 50% on one base will produce a very different revenue than imposing a penalty at 50% on the other. So rather than just comparing the rates at which various penalties are imposed, it makes sense to correct for the difference in the size of the bases by requiring that the absolute amount of penalty revenue raised is the same.

The second reason is that competition authorities and, especially, courts are not just interested in deterrence. They care about factors such as proportionality and so may not want convicted cartels to end up paying massively different fines just because a different base is used on which to impose penalties. We return to these issues in Section 5.3.3.

We now set out more formally what we mean by revenue equivalence and its implications for the welfare rankings of different penalty regimes. For reasons that will become clear we will break the comparison into two parts: we first compare an overcharge-based regime with a profit-based regime; we then compare a profit-based regime with a revenue-based regime.

5.3.2.1 Overcharge vs Profits

Suppose that an overcharge-based penalty regime is implemented with toughness $\tau_O,\ 0 < \tau_O < 1$, and so, for any cartel that is detected and penalized, generates on average[44] the penalty revenue $\overline{F}_O(\tau_O)$ as defined in equation (39). If we let $\tau_\pi^e(\tau_O)$

[43]Since, as indicated, the price set by any cartel that does form, under both a revenue-based penalty regime and an overcharge-based penalty regime will potentially vary depending on the intrinsic difficulty of holding the cartel together, so will the actual penalty paid. So all we can require is that on average the penalty paid should be the same.

[44]Because the overcharge varies with the critical level of difficulty $\Delta \in \left[0, \overline{\Delta}(\tau_R)\right]$ so too will the precise penalty revenue obtained, so all we can determine is the average penalty revenue.

denote the toughness of the profits-based penalty regime that is penalty revenue equivalent to τ_O then this is defined by the condition:

$$\overline{F}_\pi\left[\tau_\pi^e(\tau_0)\right]=\overline{F}_O(\tau_O)$$

which, from (10) and (39) implies:

$$\tau_\pi^e(\tau_O)=\frac{2\tau_O}{1-\tau_O}\left\{(1-\tau_O)-2\tau_O\left[\ln(2)-\ln(1+\tau_O)\right]\right\}. \qquad (44)$$

However, because $\overline{F}_O(\tau_O)$ is inverse U-shaped with $\overline{F}_O(\tau_O)\to 0$ as $\tau_O\to 1$ so too will be $\tau_\pi^e(\tau_O)$. In other words because a very tough overcharge based regime will erode the penalty base and drive penalty revenue to zero as $\tau_O\to 1$, if the only condition we imposed for revenue equivalence was (44) we would end up comparing extremely tough overcharge-based regimes with extremely weak profit-based regimes, which would violate our requirement that these regimes are equally tough. So the second condition we impose to ensure equal toughness is that we confine attention to values of $\tau_O\in(0,1)$ for which $\tau_\pi^e(\tau_O)$ is monotonic increasing.[45]

Having defined $\tau_\pi^e(\tau_O)$ we can use (7) – (9) to define $\overline{O}_\pi\left[\tau_\pi^e(\tau_0)\right], \overline{CS}_\pi\left[\tau_\pi^e(\tau_0)\right], \overline{TW}_\pi\left[\tau_\pi^e(\tau_0)\right]$ which are, respectively the average overcharge, consumer surplus and total welfare that would accrue under a profits-based regime that was implemented with a degree of toughness that is penalty-revenue equivalent to the degree of toughness τ_O with which the overcharge-based penalty regime is implemented. We then have the following powerful proposition .

Proposition 9: *If we impose penalty-revenue equivalence then:*

(i) $\overline{O}_O(\tau_O)<\overline{O}_\pi\left[\tau_\pi^e(\tau_O)\right]$

(ii) $\overline{CS}_O(\tau_O)>\overline{CS}_\pi\left[\tau_\pi^e(\tau_O)\right]$

(iii) $\overline{TW}_O(\tau_O)>\overline{TW}_\pi\left[\tau_\pi^e(\tau_O)\right]$

Proof: See Appendix 2.

[45]It is straightforward to show that this restriction implies $\tau_O\le 0.465,\quad \tau_\pi\le 0.427$. Following the work of Bryant and Eckard (1991), it is widely accepted that in practice only about 15% of cartels are detected, and so $\beta=0.15$, while the discussion in Section 2 suggests that the maximum penalty rate that is ever likely to be imposed on profits is 200%. Taken together this implies that in practice $\tau_\pi=\beta\psi\le 0.3$ and so the restricting attention to the range of values for which $\tau_\pi^e(\tau_O)$ is monotonic increasing is fully consistent with the range of values for the toughness of profits-based regimes that we see in practice.

So in terms of all our welfare criteria an overcharge-based penalty regime welfare dominates a profits-based regime that is implemented with a penalty-revenue-equivalent degree of toughness.

5.3.2.2 Profits vs Revenue

Suppose that a revenue-based penalty regime is implemented with toughness τ_R, $0<\tau_R<1$, and so, for any cartel that is detected and penalized, generates on average[46] the penalty revenue $\overline{F}_R\left(\tau_R\right)$ as defined in equation (27). If we let $\tau_\pi^e\left(\tau_R\right)$ denote the toughness of the profits-based penalty regime that is <u>penalty revenue equivalent</u> to τ_R then this is defined by the condition:

$$\overline{F}_\pi\left[\tau_\pi^e\left(\tau_0\right)\right]=\overline{F}_R\left(\tau_R\right)$$

which, from (10) and (27) implies:

$$\tau_\pi^e\left(\tau_R\right)=\frac{4}{\varepsilon(1+\varepsilon)}\left[\frac{\tau_R}{1-\tau_R}\int_0^{1-\tau_R} p_R\left(\Delta,\tau_R\right)\left[1+\varepsilon-p_R\left(\Delta,\tau_R\right)\right]d\Delta\right] \tag{45}$$

Now because, as noted above in Section 4.3, $\overline{F}_R\left(\tau_R\right)$ is inverse U-shaped with $\overline{F}_R\left(0\right)=0; \overline{F}_R\left(\tau_R\right)\to 0$ as $\tau_R\to 1$ so too will be $\tau_\pi^e\left(\tau_R\right)$. So, just as in the previous sub-section, the second condition we impose to ensure equal toughness is that we confine attention to values of $\tau_R\in\left(0,1\right)$ for which $\tau_\pi^e\left(\tau_R\right)$ is monotonic increasing. Notice however, that, unlike the expression for $\tau_\pi^e\left(\tau_O\right)$ that appears in equation (44), the parameter ε appears explicitly in the expression for $\tau_\pi^e\left(\tau_R\right)$, so we have to allow for the possibility that this function takes a somewhat different shape for different values of ε.[47]

Having defined $\tau_\pi^e\left(\tau_R\right)$ we can use (7) – (9) to define $\overline{O}_\pi\left[\tau_\pi^e\left(\tau_R\right)\right], \overline{CS}_\pi\left[\tau_\pi^e\left(\tau_R\right)\right], \overline{TW}_\pi\left[\tau_\pi^e\left(\tau_R\right)\right]$ which are, respectively the average overcharge, consumer surplus and total welfare that would accrue under a profits-based regime that was implemented with a degree of toughness that is <u>penalty-revenue</u>

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[46]Because the overcharge varies with the critical level of difficulty $\Delta\in\left[0,\overline{\Delta}\left(\tau_R\right)\right]$ so too will the precise penalty revenue obtained, so all we can determine is the average penalty revenue.

[47]Although we cannot get close-form analytical solutions for $\tau_\pi^e\left(\tau_R\right)$, calculations performed using *Mathematica* – which are set out in Appendix 3 - suggest that (a) for all values of $\varepsilon\in\left[0,1\right]$ the function $\tau_\pi^e\left(\tau_R\right)$ takes its maximum value when $\tau_R=0.585$ independently of ε; (b) the associated values of $\tau_\pi^e\left(0.585\right)$ range from 0.8 to 0.83 depending on the value of ε, so we are effectively imposing the restriction $\tau_\pi\le 0.8$. As discussed above in the previous sub-section this is fully consistent with the values for the toughness of profit-based regimes that we observe in practice.

equivalent to the degree of toughness τ_R with which the revenue-based penalty regime is implemented. Once again, these could vary with the parameter ε. Nevertheless we have the following powerful proposition:.

Proposition 10: *If we impose penalty-revenue equivalence then, for all values of* $\varepsilon \in (0,1]$*:*

(i) $\overline{O}_R(\tau_R) > \overline{O}_\pi\left[\tau_\pi^e(\tau_R)\right]$

(ii) $\overline{CS}_R(\tau_R) < \overline{CS}_\pi\left[\tau_\pi^e(\tau_R)\right]$

(iii) $\overline{TW}_R(\tau_R) < \overline{TW}_\pi\left[\tau_\pi^e(\tau_R)\right]$

Proof: Although we don't have close-form analytical expressions for average overcharge etc. under a revenue-based penalty regime, these results are readily obtained using *Mathematica*[48] – see Appendix 3.

So in terms of all our welfare criteria a revenue-based penalty regime is unambiguously dominated by a profits-based regime that is implemented with a penalty-revenue-equivalent degree of toughness.

5.3.3 Deterrence Equivalence vs Revenue Equivalence

In the previous two subsections we have shown that whether one uses deterrence equivalence or penalty-revenue equivalence as a way of equating the degree of toughness of the various regimes, an overcharge-based regime welfare dominates a profits-based regime which in turn welfare dominates a revenue-based regime.

In this subsection we focus on the comparison between a profits-based regime and an overcharge-based regime, but we generalize the results of the previous two sub-sections, by showing that, in a sense to be made precise, an overcharge-based penalty regime welfare dominates a profits-based regime however one resolves any potential tensions between these two criteria.

To see what these tensions might be, consider Figure 5 below. The red-dashed curve shows the function $\tau_\pi^e(\tau_O)$ in (44) which, for any $\tau_O \in [0,1]$ shows the value of $\tau_\pi \in [0,1]$ that is necessary to achieve penalty revenue equivalence. As shown in Appendix 2, this function takes its maximum at $\tau_O = 0.465$ with the associated value $\tau_\pi = 0.427$. It is straightforward to show that the derivative of this function at the point $\tau_O = 0$ is 2.

In the same diagram we have also plotted in blue-solid the line $\tau_\pi = \tau_O$ - the i.e 45^o line – which is the locus of points that achieve deterrence equivalence. This has to lie

[48]We are grateful to Sean Slack, a PhD student at the University of St Andrews for his able research assistance in producing all the *Mathematica* results reported in Appendix 3.

below $\tau_\pi^e(\tau_O)$ for $\tau_O \approx 0$ and will cut the curve $\tau_\pi^e(\tau_O)$ somewhat to the left of its turning point. It is straightforward to show that this occurs where $\tau_O = \tau_\pi = 0.424$.

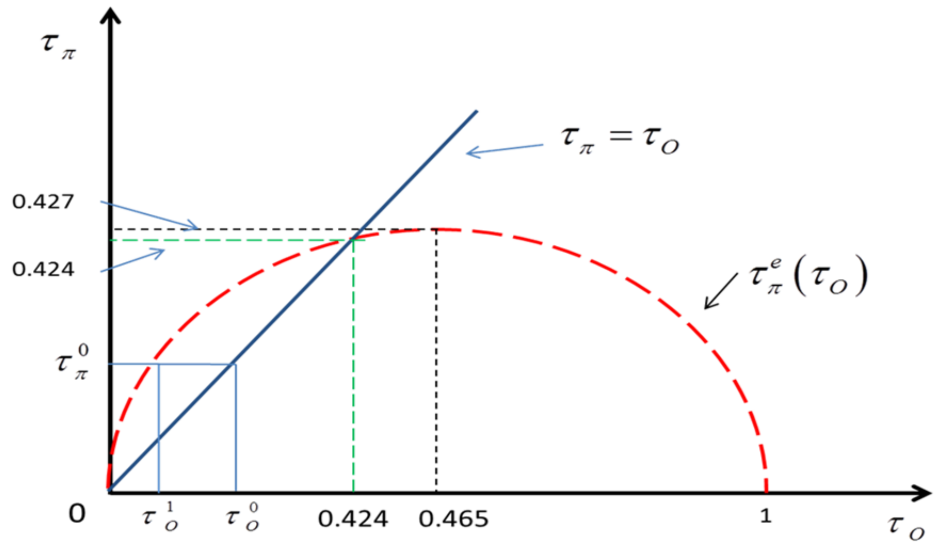

Figure 5: Deterrence Equivalence vs Revenue Equivalence

Suppose a CA was currently using a profits-based penalty where, as noted in Section 2, the penalty rate would be at most 200%. Since, as noted in footnote 40, a widely accepted value of for the probability of detection is $\beta = 0.15$, it follows that we could take the value of toughness of the profits based penalty regime to be $\tau_\pi^0 \le 0.3 < 0.424$. To be concrete, suppose that the CA used a 100% penalty on profits, so $\tau_\pi^0 = 0.15$, and the penalty policy would deter 15% of all cartels that might have formed.

Now suppose the CA considered switching to an overcharge-based penalty regime, but initially with the intention of having the same degree of deterrence, which it would achieve by imposing a 100% penalty on the overcharge base as defined in Section 3 - and so have toughness $\tau_O^0 = 0.15$. From Proposition 8 we know that such a switch would be unambiguously welfare increasing, since the number of cartels that formed would be the same but the overcharge arising for those cartels that did form would be lower by a factor of between 15% and 100% depending on the intrinsic difficulty of holding the cartel together. However, because, from Figure 5, $\tau_\pi^0 < \tau_\pi^e(\tau_O^0)$, it follows that this switch of penalty base would increase the amount of penalty revenue raised. Indeed, by using (44) to calculate $\tau_\pi^e(0.15) \approx 0.241$ we see that penalty revenue would increase by 61% simply due to this switch of base. This may be deemed a good thing if it was felt that the penalty revenue raised under the profit-based regime did not adequately reflect the severity of the damage caused by cartels. But if it was felt that the penalty revenue raised under the profit regime was proportionate then, if penalty revenues went up purely because of the switch to the new base, prosecuted cartels might appeal and courts might strike the penalties down on the grounds of being disproportionate. So there may be pressure to impose a lower level of penalty under the overcharge regime.

From Proposition 9 we know that if the Competition Authority switched to an overcharge-based regime but now set a penalty rate that was penalty revenue equivalent to a 100% penalty on profits then this switch of penalty base would once again be unambiguously welfare enhancing. Since $\tau_\pi^e(0.085) \approx 0.15$ this would require setting a penalty on the overcharge base of 57% so now only 8.5% of all potential cartels would be deterred. However the fact that the switch of bases is welfare enhancing implies that the beneficial effects of the overcharge-based regime on the prices of those cartels that do form[49] would dominate the weaker deterrence effects. Nevertheless the lower level of deterrence could create pressure to adopt a tougher policy.

From this discussion we see that switching from a profits-based regime to an overcharge-based could involve some balancing of the desire to achieve revenue equivalence and to achieve deterrence equivalence. The following result – a significant generalization of Proposition 9 - shows that **however a Competition Authority chooses to strike this balance**, the switch from a profits-based regime to an overcharge-based regime is unambiguously welfare enhancing.

Proposition 11 *For all* $\tau_O \in [0,1]$ *such that* $\tau_O \le \tau_\pi^e(\tau_O)$[50] *and for all* $\tau_\pi, \tau_O \le \tau_\pi \le \tau_\pi^e(\tau_O)$ *an overcharge-based penalty regime welfare dominates a profits-based penalty regime.*
Proof: See Appendix 4

Note that this implies that, for the range of pairs of toughness parameters (τ_π, τ_O) defined in the statement of the Proposition, the overcharge-based regime can welfare dominate the profits-based regime even when, in deterrence terms, the profits-based regime is the tougher of the two regimes.

[49] Compared to the outcome under the profit-based regime, the overcharge-based penalty regime would lower the overcharge by between 8.5% and 100% depending on the the intrinsic difficulty of holding the cartel together.
[50] This condition ensures that there is a potential tension between deterrence equivalence and revenue equivalence. As noted, it is satisfied as long as $\tau_O \le 0.424$.

6. Concluding Remarks

We analyze the impact of various antitrust penalty regimes on: (i) the price charged by any given cartel; (ii) cartel stability; and finally, (iii) the overall level of consumer and total welfare induced by the different regimes.

For this analysis we use a repeated Bertrand oligopoly model that allows us to compare both the price and the deterrence effects of the four major types of fining structures. These four types include fines based on revenues, fines based on illegal gains, fines based on the cartel overcharge, and fixed fines. Further, we make an important methodological contribution by extending the standard repeated Bertrand models that are employed for the analysis of collusion to show that when penalties are introduced on either revenue or on the cartel overcharge then the cartel stability condition can influence the price set by the cartel. This allows us to bridge the standard critical discount factor approach to the analysis of collusion (see e.g. Tirole (1988) or Motta and Polo (2003)) to profit maximizing decisions by the cartel members (with continuum of prices, which can be chosen by the cartel). This latter approach has been proposed in e.g. Block et al. (1981) or Harrington (2004, 2005).

While other papers have considered the properties of some of the four penalty regimes and made some limited comparisons between them, the contribution of this paper is to undertake a systematic comparison of all four regimes in terms of *both* the prices set by those cartels that form *and* on cartel stability (deterrence) and hence the number of cartels that form. We examine deterrence under both deterrence equivalence and penalty-revenue equivalence.

Our analysis leads to the conclusion that there is absolutely no support from welfare economics for the currently widely utilized fining structures (mainly based on revenues). Penalties based on overcharges are welfare superior to all the other penalty structures. While it has not being our purpose in this paper to go into issues of implementation of an alternative penalty regime based on overcharges there are two points that should be made. First, developments in economics and econometrics make it possible to estimate overcharges from a cartel infringement with reasonable precision or confidence, as regularly done to assess damages. Second, the question of which penalty regime to implement must be answered taking into account both relative welfare levels induced under each regime and the relative costs of implementation. To paraphrase the position already expressed recently in another paper it is probably time to change the distortive policies currently utilized - that make revenue so central for calculating fines, if the only thing the welfare costs of the distortions buy for us is saving the costs of data collection and overcharges estimation.[51]

[51] Bageri et.al. (2013) also make this point in a static setting having compared revenue and profit-based regimes and arguing in favor of the latter. In our repeated game setting we also find that a profit-based regime is welfare superior to a revenue-based regime. In addition, we show that an overcharge-based regime is superior to a profit-based regime.

Appendix 1: Derivation of Formulae (36) – (39).

From equation (35) in paper we have the following formula for overcharge:

$$O_O(\Delta,\tau_O)=\frac{\varepsilon}{2}\begin{cases}(1-\tau_O), & 0\le\Delta\le\dfrac{1-\tau_O}{1+\tau_O}\\ 2\left(1-\dfrac{\tau_O}{1-\Delta}\right), & \dfrac{1-\tau_O}{1+\tau_O}\le\Delta\le 1-\tau_0\\ 0, & 1-\tau_0\le\Delta\le 1\end{cases}\qquad\text{(A1.1)}$$

Easy to check that this is a continuous function of Δ. Notice also that in the limiting cases $O_O(\Delta,0)=\frac{\varepsilon}{2}\quad 0\le\Delta\le 1;\quad O_O(\Delta,1)=0\quad 0\le\Delta\le 1$. This implies, in particular, that if $\tau_O=0$ - and so effectively no sanctions exist on cartels - then stable cartels exist for all $\Delta\in[0,1]$ and just set the monopoly price. The average overcharge is

$$\overline{O}_O(\tau_O)=\int_0^1 O_O(\Delta,\tau_0)\,d\Delta=\frac{\varepsilon}{2}X\,,\qquad\text{(A1.2)}$$

where, by using (A1.1) and performing the integration,

$$\begin{aligned}X&=(1-\tau_O)\frac{1-\tau_O}{1+\tau_O}+2\int_{\frac{1-\tau_O}{1+\tau_O}}^{1-\tau_O}\left(1-\frac{\tau_O}{1-\Delta}\right)d\Delta\\&=(1-\tau_O)\frac{1-\tau_O}{1+\tau_O}+2(1-\tau_O)\frac{\tau_O}{1+\tau_O}+2\tau_O\left[\ln(1-\Delta)\right]_{\frac{1-\tau_O}{1+\tau_O}}^{1-\tau_O}\\&=(1-\tau_O)+2\tau_O\left[\ln(\tau_O)-\ln\left(\frac{2\tau_O}{1+\tau_O}\right)\right]\\&=(1-\tau_O)-2\tau_O\left[\ln(2)-\ln(1+\tau_O)\right]\end{aligned}\qquad\text{(A1.3)}$$

Combining (A1.2) and (A1.3) gives

$$\overline{O}_O(\tau_O)=\frac{\varepsilon}{2}\left\{(1-\tau_O)-2\tau_O\left[\ln(2)-\ln(1+\tau_O)\right]\right\},\qquad\text{(A1.4)}$$

which is equation (36) in the paper.
The average penalty revenue raised on those stable cartels that do form and are subsequently caught and penalised is:

$$\overline{F}_O(\tau_O)=\frac{\eta\varepsilon}{1-\tau_O}\int_0^{1-\tau_O}O_O(\Delta,\tau_0)\,d\Delta\,,\qquad\text{(A1.5)}$$

By using A1.1 and performing the integration it is easy to establish that

$$\overline{F}_O(\tau_O)=\frac{\varepsilon^2}{2\beta}\frac{\tau_O}{1-\tau_O}\left\{(1-\tau_O)-2\tau_O\left[\ln(2)-\ln(1+\tau_O)\right]\right\}\qquad\text{(A1.6)}$$

Which is equation (39) in the paper. Notice that

- $\overline{F}_O(0)=0$ - because, while the penalty base is positive the penalty rate is zero

- $\overline{F}_O(1)=0$ [52] - because, while the penalty rate is positive the penalty base is zero

Taken together these give equation (40) in the text.
By substituting equation (35) in the paper into the formula given in (2), consumer surplus under an overcharge-based penalty regime is:

$$CS_O(\Delta,\tau_O)=\frac{\varepsilon^2}{8}\begin{cases}(1+\tau_O)^2, & 0\le\Delta\le\dfrac{1-\tau_O}{1+\tau_O}\\ 4\left(\dfrac{\tau_O}{1-\Delta}\right)^2, & \dfrac{1-\tau_O}{1+\tau_O}\le\Delta\le1-\tau_0\\ 4, & 1-\tau_0\le\Delta\le1\end{cases}\tag{A1.7}$$

So, performing the integration, average consumer surplus is

$$\overline{CS}_O(\tau_O)=\int_0^1 CS_O(\Delta,\tau_0)\,d\Delta=\frac{\varepsilon^2}{8}Y\tag{A1.8}$$

where

$$\begin{aligned}Y&=(1+\tau_O)^2\frac{1-\tau_O}{1+\tau_O}+4\tau_O^2\int_{\frac{1-\tau_O}{1+\tau_O}}^{1-\tau_O}\frac{1}{(1-\Delta)^2}\,d\Delta+4\tau_O\\&=(1-\tau_O^2)+4\tau_O^2\left[\frac{1}{1-\Delta}\right]_{\frac{1-\tau_O}{1+\tau_O}}^{1-\tau_O}+4\tau_O\\&=(1-\tau_O^2)+4\tau_O^2\left[\frac{1}{\tau_O}-\frac{1+\tau_O}{2\tau_O}\right]+4\tau_O\\&=(1-\tau_O^2)+2\tau_O(1-\tau_O)+4\tau_O\\&=1+6\tau_O-3\tau_O^2\end{aligned}\tag{A1.9}$$

Combining (A1.8) and (A1.8) gives equation (37) in the main text.
By substituting equation (35) in the paper into the formula given in (2) for total welfare, we see that total welfare under an overcharge-based penalty regime is:

$$TW_O(\Delta,\tau_O)=\frac{\varepsilon^2}{8}\begin{cases}(3-\tau_O)(1+\tau_O), & 0\le\Delta\le\dfrac{1-\tau_O}{1+\tau_O}\\ 4\left[\dfrac{2\tau_O}{1-\Delta}+\left(\dfrac{\tau_O}{1-\Delta}\right)^2\right], & \dfrac{1-\tau_O}{1+\tau_O}\le\Delta\le1-\tau_0\\ 4, & 1-\tau_0\le\Delta\le1\end{cases}\tag{A1.10}$$

So average total welfare under an overcharge-based penalty regime is:

$$\overline{TW}_O(\tau_O)=\int_0^1 TW_O(\Delta,\tau_0)\,d\Delta=\frac{\varepsilon^2}{8}Z\ ,\tag{A1.11}$$

[52] This follows straightforwardly from (A1.6) by using l'Hopital's rule

where

$$Z = (3-\tau_O)(1-\tau_O) + 4\tau_O^2\left[\frac{1}{1-\Delta}\right]_{\frac{1-\tau_O}{1+\tau_O}}^{1-\tau_O} - 8\tau_O\left[\ln(1-\Delta)\right]_{\frac{1-\tau_O}{1+\tau_O}}^{1-\tau_O} + 4\tau_O$$

$$= 3+\tau_O^2+4\tau_O^2\left[\frac{1}{\tau_O}-\frac{1+\tau_O}{2\tau_O}\right]-8\tau_O\left[\ln(1+\tau_O)-\ln(2)\right] \qquad \text{(A1.12)}$$

$$= (3+2\tau_O-\tau_O^2)+8\tau_O\left[\ln(2)-\ln(1+\tau_O)\right]$$

Combining (A1.11) and (A1.12) produces:

$$\overline{TW}_O(\tau_O) = \frac{\varepsilon^2}{8}\left\{(3+2\tau_O-\tau_O^2)+8\tau_O\left[\ln(2)-\ln(1+\tau_O)\right]\right\} \qquad \text{(A1.13)}$$

which is equation (38) in the paper.

Appendix 2:
Comparing Overcharge-based and Profit- based Penalty Regimes under Penalty Revenue Equivalence

(i) Revenue Equivalence

If we let $\tau_\pi^e(\tau_O)$ denote the toughness of the profits-based penalty regime that is penalty revenue equivalent to τ_O then, as noted in equation (44) in the text

$$\tau_\pi^e(\tau_O) = \frac{2\tau_O}{1-\tau_O}\left\{(1-\tau_O)-2\tau_O\left[\ln(2)-\ln(1+\tau_O)\right]\right\} \qquad \text{(A2.1)}$$

As noted in the text:

- the RHS of (A2.1) is zero both when $\tau_O = 0$ and when $\tau_O = 1$[53], so this is an inverse U-shaped relationship;
- as noted in footnote 36 this function takes its maximum value when $\tau_O = 0.465$ for which the associated maximum value of τ_π is $\tau_\pi = 0.427$;
- if we are using revenue-equivalence as a way of defining the sense in which the penalty regimes might be said to be equally tough, it makes no sense to compare an overcharge based penalty regime with $\tau_O \approx 1$ - and so very high deterrence effects; very low overcharges set by those stable cartels that do form ; and consequently very low penalty revenue when they are detected - with a profits-based regime which, in order to have the same very low level of penalty revenue would have to have $\tau_\pi \approx 0$ - and so virtually no deterrence effects;
- so we want to confine attention to the range of values of $\tau_O > 0$ for which the function $\tau_\pi^e(\tau_O)$ is monotonically increasing – namely $\tau_O \leq 0.465$.

[53] Easily seen by using l'Hopital's Rule

However the results established below hold for all $\tau_O \in (0,1)$ and so hold *a fortiori* for $\tau_O \in (0, 0.465]$.

(ii) Comparing the two penalty regimes

To undertake the comparison of the welfare outcomes produced by an overcharge-based regime that is implemented with toughness τ_O, $0 < \tau_O < 1$ with those produced by a penalty-based regime that is implemented with a revenue-equivalent level of toughness $\tau_\pi^e(\tau_O)$, we proceed as follows. Equations (36) – (39) in the text give the expressions for the three welfare indicators under an overcharge regime implemented with toughness τ_O, $0 < \tau_O < 1$. Equations (7) to (9) in the text give the analogous expressions for average overcharge etc. under a profits-based penalty regime implemented with toughness τ_π, $0 < \tau_\pi < 1$. So in each of these expressions we set τ_π equal to $\tau_\pi^e(\tau_O)$ and compare the resulting expression with the formula for the corresponding welfare indicator from equations (36) – (39).

Taking these in turn:

(a) Average overcharge

From (7) and (36) in the text we have:

$$\frac{\overline{O}_O(\tau_O)}{\overline{O}_\pi\left[\tau_\pi^e(\tau_O)\right]} = \frac{(1-\tau_O) - 2\tau_O\left[\ln(2) - \ln(1+\tau_O)\right]}{1 - \left\langle \frac{2\tau_O}{1-\tau_O}\left\{(1-\tau_O) - 2\tau_O\left[\ln(2) - \ln(1+\tau_O)\right]\right\}\right\rangle} \tag{A2.2}$$

If $\tau_O > 0$ then, after a bit of re-arranging, we see that

$$\frac{\overline{O}_O(\tau_O)}{\overline{O}_\pi\left[\tau_\pi^e(\tau_O)\right]} < 1 \quad \Leftrightarrow \quad \frac{1-\tau_0}{1+\tau_0} < 2\left[\ln(2) - \ln(1+\tau_O)\right] \tag{A2.3}$$

Result 1: $$\frac{\overline{O}_O(\tau_O)}{\overline{O}_\pi\left[\tau_\pi^e(\tau_O)\right]} < 1 \quad \forall \tau_0, \quad 0 < \tau_O < 1$$

Proof: Let $\Phi(\tau_O) \equiv 2\left[\ln(2) - \ln(1+\tau_O)\right] - \frac{1-\tau_0}{1+\tau_0}$. We have $\Phi(0) = 2\ln(2) - 1 > 0$; $\Phi(1) = 0$. Moreover $\Phi'(\tau_O) = \frac{1-\tau_0}{(1+\tau_0)^2} - \frac{1}{1+\tau_O} = \frac{-2\tau_0}{(1+\tau_0)^2} < 0 \; \forall \tau_O \in (0,1)$. So RHS of (A2.3) holds $\forall \tau_O \in (0,1)$.

(b) Average Consumer Surplus

From equations (8) and (37) in the text we have:

$$\frac{\overline{CS}_O(\tau_O)}{\overline{CS}_\pi\left[\tau_\pi^e(\tau_O)\right]}=\frac{\left[1+6\tau_O-3\tau_O^2\right]}{1+3\left\langle\frac{2\tau_O}{1-\tau_O}\left\{(1-\tau_O)-2\tau_O\left[\ln(2)-\ln(1+\tau_O)\right]\right\}\right\rangle} \tag{A2.4}$$

If $\tau_O>0$ then, after a bit of re-arranging, it is easy to see that:

$$\frac{\overline{CS}_O(\tau_O)}{\overline{CS}_\pi\left[\tau^e(\tau_O)\right]}>1 \Leftrightarrow \quad 4\left[\ln(2)-\ln(1+\tau_O)\right]>1-\tau_0 \tag{A2.5}$$

Result 2: $\frac{\overline{CS}_O(\tau_O)}{\overline{CS}_\pi\left[\tau_\pi^e(\tau_O)\right]}>1 \quad \forall\tau_0, \quad 0<\tau_O<1$.

Proof. Let $\Phi(\tau_O)\equiv 4\left[\ln(2)-\ln(1+\tau_O)\right]+\tau_0-1$. We have $\Phi(0)=4\ln(2)-1>0$; $\Phi(1)=0$. Moreover $\Phi'(\tau_O)=1-\frac{4}{1+\tau_O}=\frac{\tau_O-3}{1+\tau_O}<0\ \forall\tau_O\in(0,1)$. So RHS of (A2.5) holds $\forall\tau_O\in(0,1)$.

(c) Average Total Welfare

From equations (9) and (38) in the main text we have:

$$\frac{\overline{TW}_O(\tau_O)}{\overline{TW}_\pi\left[\tau_\pi^e(\tau_O)\right]}=\frac{\left(3+2\tau_O-\tau_O^2\right)+8\tau_O\left[\ln(2)-\ln(1+\tau_O)\right]}{3+\left\langle\frac{2\tau_O}{1-\tau_O}\left\{(1-\tau_O)-2\tau_O\left[\ln(2)-\ln(1+\tau_O)\right]\right\}\right\rangle} \tag{A2.6}$$

If $\tau_O>0$ then, after a bit of re-arranging, it is easy to see that:

$$\frac{\overline{TW}_O(\tau_O)}{\overline{TW}_\pi\left[\tau_\pi^e(\tau_O)\right]}>1 \Leftrightarrow \quad 4\left[\ln(2)-\ln(1+\tau_O)\right]>\frac{\tau_0(1-\tau_0)}{2-\tau_0} \tag{A2.7}$$

Result 3: $\frac{\overline{TW}_O(\tau_O)}{\overline{TW}_\pi\left[\tau_\pi^e(\tau_O)\right]}>1 \quad \forall\tau_0, \quad 0<\tau_O<1$.

Proof: Let $\Phi(\tau_O)=4\left[\ln(2)-\ln(1+\tau_O)\right]-\frac{\tau_0(1-\tau_0)}{2-\tau_0}$ Note that $\Phi(0)=4\ln(2)>0$, while $\Phi(1)=0$. Also $\Phi(\tau_O)$ is monotonically decreasing on the interval (0,1), since $\Phi'(\tau_O)=-\left[\frac{18(1-\tau_O)+\tau_O^2(1+\tau_O)}{(1+\tau_O)(2-\tau_O)^2}\right]<0\ \forall\tau_O\in(0,1)$. Hence, we can conclude that $\Phi(\tau_O)>0 \quad \forall\tau_O\in(0,1)$ and so RHS of (A2.7) holds $\forall\tau_O\in(0,1)$.

Appendix 3:

Comparing Profit- based and Revenue-based Penalty Regimes under Penalty Revenue Equivalence

(i) Revenue Equivalence

If we let $\tau_\pi^e(\tau_R)$ denote the toughness of the profits-based penalty regime that is penalty revenue equivalent to a revenue-based penalty regime that is implemented with toughness τ_R, $0 < \tau_R < 1$ then, as noted in equation (45) in the text,

$$\tau_\pi^e(\tau_R) = \frac{4}{\varepsilon(1+\varepsilon)} \cdot \left[\frac{\tau_R}{1-\tau_R} \int_0^{1-\tau_R} p_R(\Delta, \tau_R)\left[1+\varepsilon - p_R(\Delta,\tau_R)\right] d\Delta \right] \qquad \text{(A3.1)}$$

While we cannot get close-form analytical solutions for $\tau_\pi^e(\tau_R)$, it is easy to see that $\tau_\pi^e(0) = \tau_\pi^e(1) = 0$ and so $\tau_\pi^e(\tau_R)$ is inverse U-shaped. Unlike formula (44) for $\tau_\pi^e(\tau_O)$, the formula for $\tau_\pi^e(\tau_R)$ contains ε, so we have to allow for the possibility that the precise shape of $\tau_\pi^e(\tau_R)$ varies with ε. This is confirmed through using *Mathematica* to calculate $\hat{\tau}_R = \underset{\tau_R \in (0,1)}{\arg\max}\, \tau_\pi^e(\tau_R)$ and $\hat{\tau}_\pi^e = \underset{\tau_R \in (0,1)}{MAX}\, \tau_\pi^e(\tau_R)$ for a range of values of ε in (0,1] with an interval 0.05. The results are reported in Table 1.

Table 1: Turning Points for $\tau_\pi^e(\tau_R)$

ε	$\hat{\tau}_R$	$\hat{\tau}_\pi^e$
0.05	0.585	0.832
0.10	0.585	0.829
0.15	0.585	0.826
0.20	0.585	0.823
0.25	0.585	0.821
0.30	0.585	0.818
0.35	0.585	0.816
0.40	0.585	0.815
0.45	0.585	0.813
0.50	0.585	0.811
0.55	0.585	0.810
0.60	0.585	0.808
0.65	0.585	0.807
0.70	0.585	0.806
0.75	0.585	0.805
0.80	0.585	0.803
0.85	0.585	0.802
0.90	0.585	0.802
0.95	0.585	0.801
1.0	0.585	0.800

Table 1, suggests that:

(a) for all values of $\varepsilon \in (0,1]$ the function $\tau_\pi^e(\tau_R)$ takes its maximum value when $\tau_R = 0.585$ independently of ε;

(b) the associated values of $\tau_\pi^e(0.585)$ range from 0.8 to 0.83 depending on the value of ε.

As argued in the text, to be consistent with the notion of the different penalty regimes being implemented with equal toughness, we need to confine attention to values of $\tau_R \in (0,1)$, $\tau_\pi \in (0,1)$ for which $\tau_\pi^e(\tau_R)$ is monotonic increasing. From Table 1 it follows that we are effectively imposing the restriction $\tau_\pi \leq 0.8$. As discussed in the text this restriction is fully consistent with the values for the toughness of profit-based regimes that we observe in practice.

(ii) Comparing the two penalty regimes

Equations (7) to (9) in the text give the expressions for average overcharge, average consumer surplus and average total welfare under a profits-based penalty regime. To undertake the comparison we just replace the value of τ_π in these expressions with $\tau_\pi^e(\tau_R)$ and compare with the corresponding expression for average overcharge, average consumer surplus and average total welfare under a revenue-based regime which are given in (24) – (26) in the text. Although we don't have close-form analytical expressions for average overcharge, average consumer surplus and average total welfare under a revenue-based regime, we used *Mathematica* to undertake the evaluations of these expressions and, from this, calculate the three ratios:

$$\frac{\overline{O}_\pi\left[\tau_\pi^e(\tau_R)\right]}{\overline{O}_R(\tau_R)};\ \frac{\overline{CS}_\pi\left[\tau_\pi^e(\tau_R)\right]}{\overline{CS}_R(\tau_R)}\ \frac{\overline{TW}_\pi\left[\tau_\pi^e(\tau_R)\right]}{\overline{TW}_R(\tau_R)}.$$

The calculations were undertaken for a range of values of ε in (0,1] with an interval 0.1. Rather than reporting the results for all ten values of ε, in Figures 6a-6d we present illustrative results for the four values $\varepsilon = 0.1, 0.2, 0.9, 1.0$. In each figure τ_R is on the horizontal axis, the curve $\tau_\pi^e(\tau_R)$ is shown as the blue curve taking its maximum value at $\tau_R = 0.585$, while the ratios $\frac{\overline{O}_\pi\left[\tau_\pi^e(\tau_R)\right]}{\overline{O}_R(\tau_R)}$, $\frac{\overline{CS}_\pi\left[\tau_\pi^e(\tau_R)\right]}{\overline{CS}_R(\tau_R)}$ and $\frac{\overline{TW}_\pi\left[\tau_\pi^e(\tau_R)\right]}{\overline{TW}_R(\tau_R)}$ are depicted respectively by the dashed red line, the dotted orange line and the green line. As can be seen the ratio $\frac{\overline{O}_\pi\left[\tau_\pi^e(\tau_R)\right]}{\overline{O}_R(\tau_R)}$ is below 1 for all $\tau_R \in (0,585)$, while the ratios

$\frac{\overline{CS}_{\pi}\left[\tau_{\pi}^{e}\left(\tau_{R}\right)\right]}{\overline{CS}_{R}\left(\tau_{R}\right)}$ and $\frac{\overline{TW}_{\pi}\left[\tau_{\pi}^{e}\left(\tau_{R}\right)\right]}{\overline{TW}_{R}\left(\tau_{R}\right)}$ are above 1 for all $\tau_{R}\in\left(0,0.585\right)$ thus confirming the results of Proposition 10: for all three welfare indicators a profits-based penalty regime out-performs a revenue-based regime for the range of values of $\tau_{R}\in\left(0,1\right)$ for which $\tau_{\pi}^{e}\left(\tau_{R}\right)$ is monotonic increasing. .

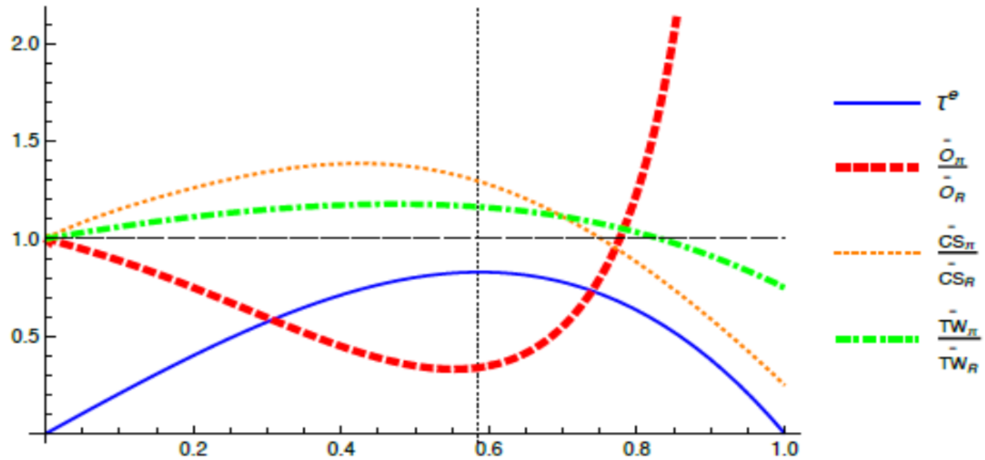

Figure 6a: Welfare Ratios for ε =0.1

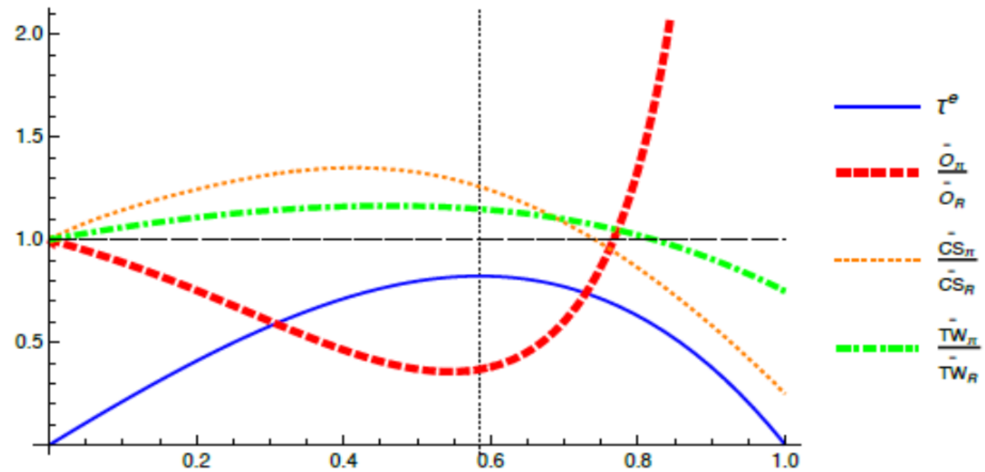

Figure 6b: Welfare Ratios for ε =0.2

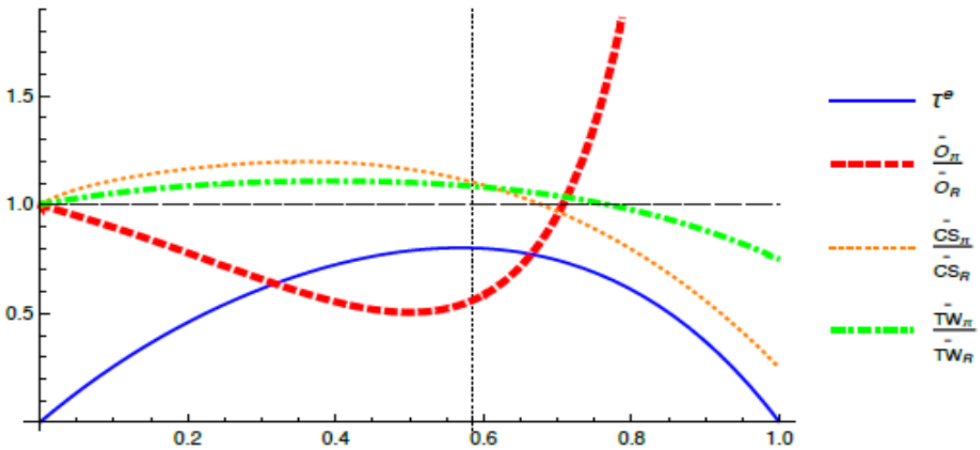

Figure 6c: Welfare Ratios for ε =0.9

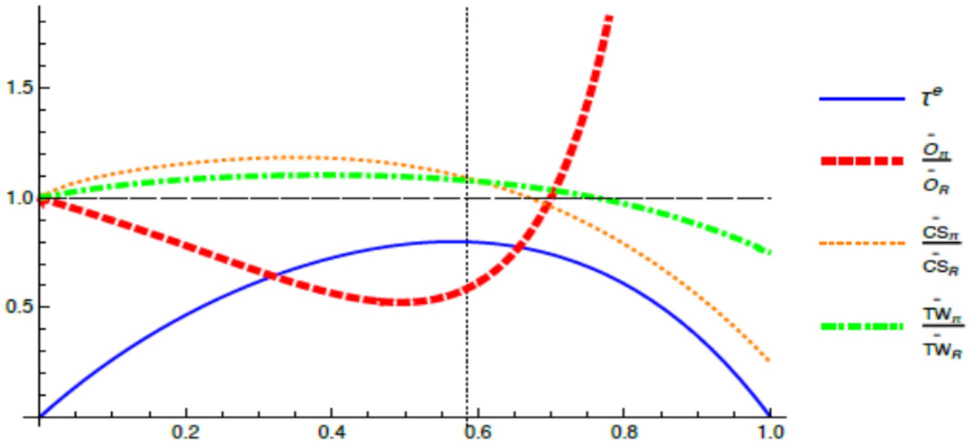

Figure 6d: Welfare Ratios for ε =1

Appendix 4: Proof of Proposition 11

Start with a $\tau_O < 0.424$ and impose deterrence equivalence so $\tau_\pi = \tau_O$. We know the overcharge-based regime welfare dominates. Now keep τ_O constant and raise τ_π systematically until $\tau_\pi = \tau_\pi^e(\tau_O)$. All the welfare indicators for the overcharge-based regime stay fixed but the profits-based regime gets better – because of its enhanced deterrence properties. But nevertheless**, despite this enhanced deterrent effect** we know that at the point $(\tau_O, \tau_\pi^e(\tau_O))$ the overcharge regime still welfare dominates the profits-based regime.

Alternatively, start with a $\tau_\pi < 0.424$ and impose revenue equivalence – so $\tau_O = \tau_O^{e-1}(\tau_\pi) < \tau_\pi$. We know the overcharge based regime welfare dominates the profits-based regime. Now keep τ_π constant and raise τ_O systematically until we have achieve deterrence equivalence $\tau_O = \tau_\pi$. This doesn't change the welfare indicators for the profits-based regime, but will improve the performance of the overcharge-based regime BOTH because of better deterrence, AND because it drives down the price for those cartels that do form.

References

Allain, M-L, Boyer, M., Kotchoni, R. & J-P. Ponssard, (2011) "The Determination of Optimal Fines in Cartel Cases: The Myth of Underdeterrence", CIRANO Discussion Paper 2011s-34

Bageri, V., Y. Katsoulacos, and G. Spagnolo (2013) "The Distortive Effects of Antitrust Fines Based on Revenue,"The Economic Journal, 123 (572), 545-557.

Becker, G.S. (1968) "Crime and Punishment: An Economic Approach," Journal of Political Economy 76, 169-217.

Block, M., F. Nold, and J. Sidak (1981) "The deterrent effect of antitrust enforcement," Journal of Political Economy89, 429-445.

Boyer M. and R. Kotchoni (2011) "The Econometrics of Cartel Overcharges", CIRANO Discussion Paper, 2011s-35

Bryant, P.G., and E.W. Eckard (1991) "Price Fixing: The Probability of Getting Caught," The Review of Economics and Statistics 73, 531-536.

Buccirossi, P. and G. Spagnolo (2007) "Optimal Fines in the Era of Whistleblowers - Should Price Fixers Still Go to Prison?" in The Political Economy of Antitrust, by V. Goshal and J. Stennek (Eds.), Elsevier: Amsterdam.

Chen, Z. and P. Rey (2007) "On the design of leniency programs," IDEI working paper number 452 (revised 2012)

Cyrenne, P. (1999) "On antitrust enforcement and the deterrence of collusive behavior," *Review of Industrial Organization* 14, 257-272.

EC (2006) "Guidelines on the Method of Setting Fines Imposed Pursuant to Article 23(2)(a) of Regulation no 1/2003," Official Journal of the European Union (2006/C 210/02), Brussels.

Fudenberg, D. and J. Tirole (1991) Game Theory, MIT Press: Cambridge.

Harrington, J. (2004) "Cartel Pricing Dynamics in the Presence of an Antitrust Authority," The Rand Journal of Economics 35, 651-673.

Harrington, J. (2005) "Optimal Cartel Pricing in the Presence of an Antitrust Authority," International Economic Review 46, 145-170.

Harrington J. (2010) "Comment on Antitrust Sanctions," Competition Policy International 6, 41-51.

Harrington J. and Chen (2007) "The impact of the corporate leniency program on cartel formation and the cartel price path. In V. Ghosal and J. Stennek (Eds.), The Political Economy of Antitrust. Elsevier.

Houba, H., E. Motchenkova, and Q. Wen (2010) "Antitrust enforcement with price-dependent fines and detection probabilities," Economics Bulletin, 30(3), 2017-2027.

Houba, H., E. Motchenkova, and Q. Wen (2011) "Antitrust Enforcement and Marginal Deterrence," (2011) Tinbergen Institute Discussion Papers 11-166/1.

Houba, H., E. Motchenkova, and Q. Wen (2012) "Competitive Prices as Optimal Cartel Prices," Economics Letters 114, 39-42.

Jensen, S. and L. Sorgard (2012) "Enforcement with heterogeneous cartels," Working paper 14/12, Institute for Research in Economics and Business Administration, Norway.

Jensen, S. and L. Sorgard (2014) "Fine schedule with heterogeneous cartels: Are the wrong cartels deterred?," mimeo, Institute for Research in Economics and Business Administration, Norway.

Katsoulacos, Y. and D. Ulph (2013) "Antitrust Penalties and the Implications of Empirical Evidence on Cartel Overcharges", The Economic Journal, 123 (572), 558-581.

Levenstein, M. C. and V. Y. Suslow (2011), "Breaking Up Is Hard to Do: Determinants of Cartel Duration", Journal of Law and Economics 54:455- 92.

Levenstein, M. C. and V. Y. Suslow (2012), "Cartels and Collusion - Empirical Evidence". Ross School of Business Paper No. 1182. Available at SSRN: http://ssrn.com/abstract=2182565

Levenstein, M. C. and V. Y. Suslow (2014) "Price fixing hits home: an empirical study of price fixing conspiracies in the US", mimeo, University of Michigan, Jan. 2014.

Motta, M. and M. Polo (2003) "Leniency Programs and Cartel Prosecution," International Journal of Industrial Organization 21, 347-379.

OECD (2002) "Fighting Hard-Core Cartels: Harm, Effective Sanctions and Leniency Programs" OECD Report 2002, OECD, Paris, France, http://www.SourceOECD.org.

Rey, P. (2003) “Towards a Theory of Competition Policy” in Advances in Economics and Econometrics: Theory and Applications by M. Dewatripont, L. Hansen, and S. Turnovsky, Eds., Cambridge University Press.

Spagnolo, G. (2004) “Optimal leniency programs," CEPR Discussion paper series, working paper number 4840, (revised 2008)

USSG (2010) "US Sentencing Guidelines (Chapter 8: Sentencing of organizations)" http://www.ussc.gov/2009guid/CHAP8.htm

Wils, W. (2007) "The European Commission's 2006 Guidelines on Antitrust Fines: A Legal and Economic Analysis" in World Competition 30.