Loss Modification Incentives for Insurers under Expected Utility and Loss Aversion

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Abstract

Given the possibility to modify the probability of a loss, will a profit-maximizing insurer engage in loss prevention or is it in his interest to increase the loss probability? This paper investigates this question. First, we calculate the expected profit maximizing loss probability within an expected utility framework. We then use Kőszegi and Rabin's (2006, 2007) loss aversion model to answer the same question for the case where consumers have reference-dependent preferences. Largely independent of the adopted framework, we find that the optimal loss probability is sizable and for many commonly used parameterizations much closer to $1/2$ than to 0. Previous studies have argued that granting insurers market power may incentivize them to engage in loss prevention activities, this to the benefit of consumers. Our results show that one should be cautious in doing so because there are conceivable instances where the insurer's interests in modifying the loss probability to against those of consumers.

1 Introduction

An insurer's profits depend on the amount consumers are willing to pay for protection against a potential loss in excess of the expected value of the policy, the risk premium. This risk premium in turn is a function of both the severity of the loss and the probability that a loss happens. It seems only natural for profit-maximizing insurers to influence either or both of these risk management parameters whenever possible. Despite this connection, and in sharp contrast to the extensive literature that deals with the insuree's incentives to engage in self-protection and self-insurance[1], attention for the loss-modification incentives by insurers has however been very limited.

[1]Starting with Ehrlich and Becker (1972), see Gollier *et al.* (2013) and the references therein.

Two notable exceptions are the contributions by Schlesinger and Venezian (1986, 1990) who point out that insurers often lobby Congress to implement policies aimed at loss prevention (e.g. keep drunk drivers off the road) or loss reduction (e.g. mandatory airbags and better bumpers on new automobiles (Schlesinger and Venezian, 1990, p. 84). Within an expected-utility framework with risk-averse consumers, they formalize the decision problem of a risk-neutral monopolistic insurer who has the possibility to modify the status-quo loss probability p_0. When any loss modification efforts are costless, the insurer has incentives to invest in loss prevention services prior to any insurance sales[2] when the status-quo probability p_0 exceeds the profit-maximizing probability p^*. Because the insurer always sets the risk premium such that the consumer's utility when buying insurance is marginally higher than the expected utility of being uninsured, and because the latter is decreasing in the loss probability, any *reduction* in loss probability will unambiguously increase consumer welfare.

On the other hand, in case $p_0 < p^*$, the interest of the insurer to increase the loss probability unambiguously goes against those of consumers. It is remarkable that this possibility is rather easily dismissed by Schlesinger and Venezian as largely irrelevant with the argument that insurers' initiatives to purposely increase the loss probability are "likely to meet with public resistance and possible regulatory restraint" (Schlesinger and Venezian, 1990).[3]

In our opinion, this view that society provides sufficient checks and balances to prevent insurers from taking actions against the interest of consumers may prove too optimistic. Whereas insurers' loss reduction activities are easy to monitor because they companies are happy to advertise them[4], any efforts made to increase the loss probability may well go unobserved. This holds especially for feasible but omitted loss-prevention activities. Who for example can tell whether insurers do everything within their means to increase car safety or to fight obesity? Even when the insurer has no means to raise the actual loss probability, it may be in his interest to try to increase the subjective loss probability as perceived by consumers since a successful attempt will have the same effect on his profits.

We therefore believe that the question how likely situations with $p^* > p^0$ are to occur deserves further study and exactly this is the aim of this paper. For if these situations are rare, there is not much reason to worry. If, on the other hand, it is likely that $p^* > p^0$, one should be careful in giving

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[2] These efforts are non client-specific.

[3] Schlesinger and Venezian (1986, p. 232) use a similar argument to limit the subsequent analysis ("for the sake of concreteness") to the case $p^* < p_0$.

[4] For example, insurers provide a variety of loss preventions services to reduce the probability of cars theft (http://www.aig.co.uk/motor-fleet-loss-control_2538_367524.html) or the number of hospital visits by offering free gym memberships to increase citizen's enthusiasm for physical exercises (http://articles.washingtonpost.com/2012-01-12/politics/35439261_1_gym-membership-medicare-advantage-health-insurance) or by offering free medical check-ups.

insurers incentives to modify the loss probability. As Schlesinger and Venezian (1990) point out, these incentives to engage in loss reduction are absent in a competitive market where any (increases in the) risk premium due to the lower expected loss will be competed away immediately. Any analysis that ignores the possibility that insurers may desire to increase the loss probability (or the magnitude of the loss) will therefore too easily reach the conclusion that consumers are better off when insurers are granted market power.

Whether consumers are better off in an imperfectly or perfectly competitive market thus depends on: a) the sign of the difference between the optimal and status-quo loss-probability (p^* and p_0), and b) the magnitude of the risk premium an insurer is able to charge when he has market power. In a numerical illustration for the case in which consumers' utility functions are characterized by identical, constant relative risk aversion (CARA), Schlesinger and Venezian (1990) calculate critical loss probabilities p^c: if $p^0 > p^c$ ($p^0 < p^c$), consumers are – in terms of expected utility – better (worse) off in a market with a loss probability p^* and a monopolistically priced policy than in a competitive market where insurance is sold at the actuarial value of the policy (that is, at the expected loss $p_0 L$, with a zero risk premium). Their results show that as long at the initial loss probability $p_0 \leq 1/2$, consumers in this economy are never better off in a monopolistic insurance market.

The current paper extends the numerical analysis in Schlesinger and Venezian (1990) by considering a richer variety of consumer risk preferences to identify the value of the optimal loss probability p^* in these alternative economies and keeping in mind that the higher p^*, the less likely it is that the (unobserved) initial loss probability p_0 will exceed p^*. Within the expected utility framework, we distinguish between the situation case where consumers face absolute risks and the case where the risks are proportional to their wealth. Health risks are mostly independent of one's wealth and therefore an example of the former, home insurance an example of the latter since more wealthy people tend to live in more valuable houses.

Second, the consumer's decision whether or not to buy insurance can be viewed as a choice between a certain amount and a lottery. Since the publication of Schlesinger and Venezian's original work and following the seminal contribution on prospect theory by Kaheman and Tversky (1979), evidence has accumulated showing that expected utility theory may not adequately describe people's attitudes towards risky choices (Rabin, 2000). Prospect theory assumes that people have reference-dependent preferences: when faced with a risky decision, their decision is not solely based on the implications for their absolute wealth level but on the change in wealth compared to a reference level. A second key tenet of prospect theory is loss aversion: in evaluating risks, people attach greater weight to

potential losses than to equivalent gains. A natural next step is to study the implications of prospect theory for firm behavior.[5] We apply the reference-dependent utility model introduced by Kőszegi and Rabin (2006, 2007) to extend our analysis of the insurer's loss prevention activities to situations where consumers have reference-dependent preferences. This approach is novel and complements other contributions that study the implications of the Kőszegi-Rabin framework on firm strategy and competition in non-insurance markets (Heidhues and Kőszegi, 2008, 2010; Carbajal and Ely, 2013). Models of loss aversion have also been applied in the field of insurance, but most of these contributions focus on the household's decision-making problem rather than on the implications for the optimal strategy for insurance companies (Hu and Scott ,2007; Sydnor, 2010, and Barseghyan et al.,2013).[6]

Our main result is that for level of risk aversion commonly found in the literature, both the expected utility specifications and the prospect theory models yield profit-maximizing loss probabilities of around one half. This value is higher many of loss probabilities consumers face for everyday risks.[7] The implication of this is that it is likely that an insurer with market power and unconstrained by regulation and public opinion would find in its interest to raise the loss probability to the detriment of consumers.

Our paper not only is an extension to the original work by Schlesinger and Venezian but can also be viewed as a useful counterweight to other papers that conclude that consumers may benefit from insurer market power. McKnight *et al.* (2012) for example find in a recent empirical study that insurers pay less than the uninsured for certain health services and conclude from this that "market power for insurers can offset provider market power. (p.10)" Our analysis shows that this conclusion may be context-specific.

2   Expected utility framework

In this section, we deal with the optimal loss-size problem in the expected utility framework. We assume that consumers are risk-averse with a twice differentiable utility function of final wealth W with $U'(\cdot) > 0$ and $U''(\cdot) < 0$. The monopolistic insurer is risk-neutral. We follow Schlesinger and Venezian (1986, 1990) and consider only full coverage insurance and assume complete information for both parties. This allows us to abstract away from issues of deductibles, moral hazard and adverse

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[5]As Barberis (2013, p. 188) notes: "When consumers have prospect theory preferences, firms may adopt a corresponding strategy for price setting."

[6]Barberis (2013) contains a summary of this literature.

[7]See for example `http://en.wikipedia.org/wiki/List_of_countries_by_traffic-related_death_rate` for a list of traffic-related death rates for various countries.

selection. Whereas they consider both the case where loss prevention activities can be bundled with an insurance policy and the case where the insurers can alter the loss probability only before selling insurance, we focus on the latter case.

Consider a monopolistic insurance market where consumers have an wealth W and face a wealth prospect $W - x$ where W is the present value of lifetime income and x a binary random variable that takes the value L with probability p and 0 otherwise. A key element in our model is that the insurer has the ability to costlessly change p. Consumer i will buy insurance if and only if:

$$U_i(W - R) \geq (1 - p)U_i(W) + pU_i(W - L), \tag{1}$$

with R denoting the premium.[8] The insurer's decision problem is to set the premium R and the loss probability p at values that maximize the insurer's expected profits:

$$\pi(p) = (R - pL) \sum_{i=1}^{N} I[U_i(W - R) \geq (1 - p)U_i(W) + pU_i(W - L)], \tag{2}$$

where N denotes population size and $I[\cdot]$ an indicator function. The first term denotes the expected profit per insuree and the summation gives the aggregate demand for insurance. Schlesinger and Venezian (1986, 1990) focus on the case where consumers have identical risk preferences, that is, $U_i(\cdot) = U(\cdot)$. In this case, demand for insurance is either N or 0 for any (R, p)-combination. For any given p, a profit-maximizing insurer will set the price of the policy $R(p)$ such that $U(W - R) = (1 - p)U(W) + pU(W - L)$. That is

$$R(p) = W - U^{-1}[U(W) - p(U(W) - U(W - L))] = W - CE(p), \tag{3}$$

with $CE(p)$ denoting the certainty equivalent to the wealth prospect $W - x$. This price equals the actuarial value of the policy, pL (i.e. the expected loss), plus a fixed fee equal to the consumer's risk premium.[9]

For this general setup, Schlesinger and Venezian show that for any loss size $L < W$, there exists a unique loss probability p^* that maximizes the insurer's expected profit. This situation is illustrated in Figure 1. $p = p^*$ maximizes the horizontal distance between the certainty equivalent $(CE(p))$ and the wealth prospect $W - x$. If this optimal probability p^* is smaller than the status-quo probability p_0 in the market, the monopolistic insurer has incentives to invest in loss prevention activities. In a

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[8]We assume that when consumers are indifferent between taking insurance or not, they choose to insure.

[9]For concave utility functions it follows from Jensen's inequality that $U(W - pL) \geq pU(W - L) + (1 - p)U(W)$ which is equivalent to $W - pL \geq U^{-1}[pU(W - L) + (1 - p)U(W)]$ because of $U' > 0$. Thus $R(p) = W - U^{-1}[pU(W - L) + (1 - p)U(W)] \geq pL$. That is, for any p, $R(p)$ is such that the insurer's expected profits $R(p) - pL$ are non-negative.

perfectly competitive market, insurers do not have an incentive to engage in loss prevention, because any increase in margin due to these activities will be competed away. Whether consumers are better off in a monopolistic or a competitive market depends on whether any reduction in loss probability compensates for the policy being priced above its actuarial value in the monopoly market.
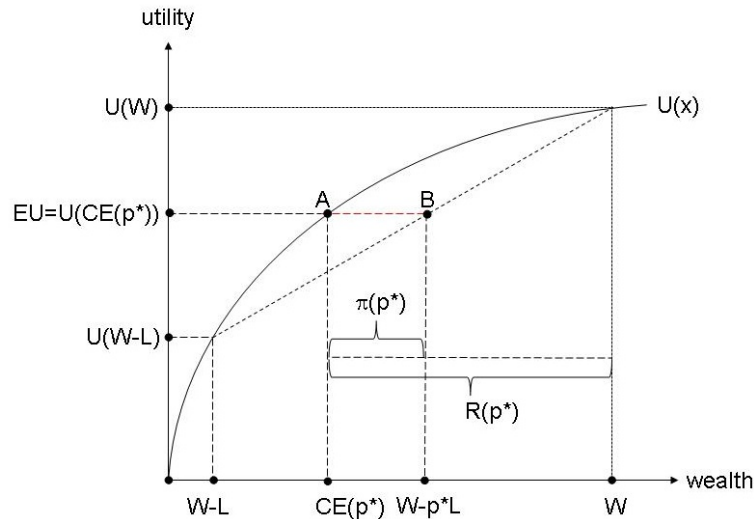
Figure 1: The expected profit maximizing loss probability p^*.

2.1 Absolute risks

Schlesinger and Venezian (1990) present a quantitative analysis of their model. Their setting can be thought of as one where consumers have to choose between a lottery of the form $l = p \circ -L \oplus (1-p) \circ 0$ or avoiding the lottery by paying $R(p)$. That is, consumers go uninsured against the risk to lose of an absolute sum L with probability p or they buy insurance. They assume a representative consumer with preferences that exhibit constant absolute risk aversion (CARA):

$$U(W) = 1 - e^{-\theta W} \tag{4}$$

with $\theta > 0$ the level of risk aversion. CARA preferences makes the decision to insure independent of a consumer's initial wealth level W.

For convenience, we repeat the main results. They show that (p. 88), for a given loss size L, the loss probability that maximizes the insurer's profits equals

$$p^*(\theta) = \frac{1}{\theta L} - \frac{1}{e^{\theta L} - 1} \tag{5}$$

The critical probability p^c which makes consumers as well off in a monopolistic market as in a com-

petitive market equals

$$p^c \equiv p^* + (R(p^*) - p^*L)/L. \tag{6}$$

Note that $p^c L = R(p^*)$. The term on the left hand side is the actuarially fair price consumers pay for coverage in a competitive market with loss probability p^c, the right hand side the monopolistically priced policy with loss probability p^*. Figure 2 depicts the optimal and critical loss probabilities for different loss sizes L. The left panel shows that the optimal probability is decreasing in the potential loss L consumers face. One can easily check the following result for the limiting cases of zero and infinite potential loss.

Result 1.

$$\lim_{L \to 0} p^*(\theta) = 1/2, \qquad \lim_{L \to \infty} p^*(\theta) = 0 \qquad (\theta > 0).$$

Proof: All proofs are in the Appendix.
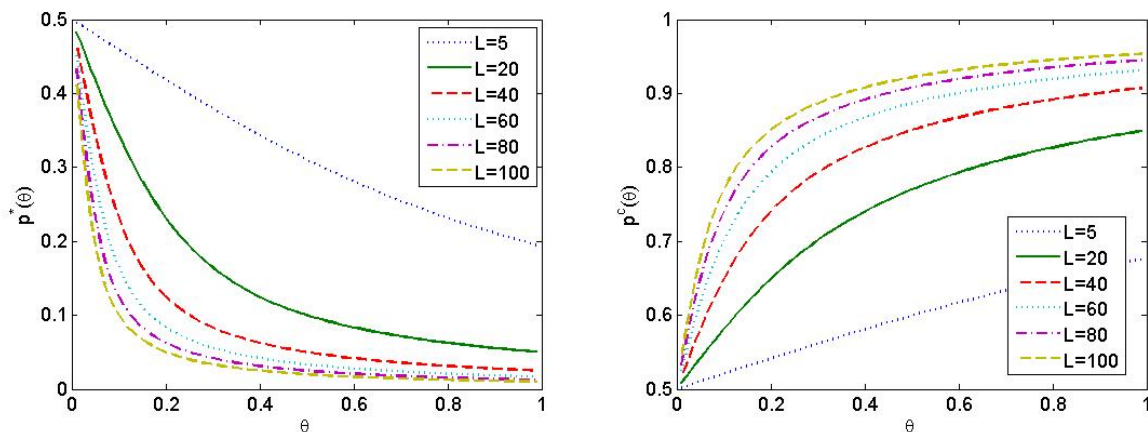
Figure 2: Plots of $p^*(\theta)$ (left panel) and $p^c(\theta)$ (right panel) under different calibrations of $L = 20, 40, 60, 80$ and 100 for $\theta \in [0.01, 0.99]$.

This means that, independent of the consumers level of risk aversion, the insurer has an interest in pushing down the status-quo loss probability as long as the loss L is sufficiently large, as for, say, hospital expenses; for small losses, the insurer has an incentive to inflate the status-quo loss probability to the detriment of consumers, unless one believes that the status-quo loss probability exceeds 0.5. Although hard evidence is absent, we do observe that insurance against small losses is often offered at a high price compared to the coverage. This implies that anyone who buys such policies is either extremely risk averse or perceives the loss as highly likely to happen to him or her.[10]

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[10]For example, a two-year insurance that covers breakage of prescription glasses with a value up to £100 costs £9 (http://www.visionexpress.com/glasses/buyers-guide/breakage-protection/).

The left panel of Figure 2 shows that for given L, the optimal loss probability is decreasing in θ. This is because in selecting the loss probability, the insurer has to trade-off the negative effect of decreasing p on consumers' willingness to pay (insuring against a loss is more valuable the higher the expected loss) against the positive impact a lower loss probability has on the fraction of clients suffering an actual loss (which reduces the insurer's cost). For CARA utility and a given loss L, when society becomes more risk-averse the second effect dominates, such that the insurer lowers p when people become more risk-averse.

The right panel of Figure 2 shows the critical loss probabilities for different loss sizes L. Note that for all values of L and θ, the status-quo probability has to exceed 0.5 for consumers to be better off in a monopoly market. In most cases it has to be higher than 0.7. For example, for $\theta = 0.3$ and $L = 40$, $p^c \approx 0.79$ and $p^* \approx 0.08$. Why are consumers not better off in a monopoly market despite the impressive reduction in loss probability? The reason is that the monopolistic insurer sets the price of the policy equal to the price that would be obtained under competition with the higher loss probability: $R(0.08) = p^c L \approx 31.7$. Figure 3 illustrates this point by showing the ratio between the actual price of the policy $R(p^*)$ and its actuarial value $p^* L$. For $L = 5$ the risk premium seems reasonable, but as L increases, consumers are willing to pay a premium dozens of times the actuarial value, which implies absurdly high degree of risk aversion. This result is a direct consequence of the observation first made by Rabin (2000) and Rabin and Thaler (2001) that under CARA utility, the refusal of small bet implies absurd levels of risk aversion for large bets. In sum, when consumers are endowed with CARA preferences, the instances where they are better off in a monopolistic than a competitive insurance market seem to be fairly few.
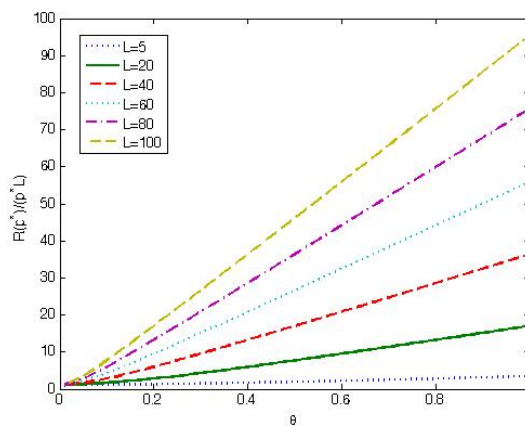

Figure 3: Plot of the $R(p^*)/(p^* L)$ ratio for the loss sizes $L = 20, 40, 60, 80, 95$ and 100 and initial wealth $W = 100$.

2.2 Proportional risks

We next extend the analysis to the case where consumer preferences are characterized by constant relative risk aversion (CRRA). CRRA models are more common than CARA in the recent literature of insurance markets.[11] CRRA utility is given by

$$U(W) = \begin{cases} W^{1-\theta}/(1-\theta) & \text{for} \quad \theta \neq 1, \\ \ln W & \text{for} \quad \theta = 1. \end{cases} \tag{7}$$

Since offering insurance is only profitable if there are risk-averse individuals, we limit attention to the case $\theta > 0$, ruling out situations where $\theta = 0$ (risk-neutrality) or $\theta < 0$ (risk-seeking).

By inserting (7) into the profit function (2) and taking the derivative with respect to p, we obtain the following general expression for the profit-maximizing loss probability as a function of the risk aversion parameter θ[12]:

$$p^*(\theta) = \frac{W^{1-\theta} - [\frac{L(1-\theta)}{W^{1-\theta}-(W-L)^{1-\theta}}]^{\frac{1-\theta}{\theta}}}{W^{1-\theta} - (W-L)^{1-\theta}}. \tag{8}$$

In the remainder of this section, we focus on the situation in which consumers face a loss proportional to their initial or discounted lifetime wealth, $L = \delta W$. In other words, they face a lottery of the form $l = p \circ -\delta W \oplus (1-p) \circ 0$. This seems an appropriate description for decisions concerning e.g. home insurance. With potential losses proportional to wealth, the optimal probability becomes wealth independent and equation (8) reduces to:

$$p^*(\theta) = \left[1 - \left(\frac{\delta(1-\theta)}{B} \right)^{\frac{1-\theta}{\theta}} \right] B^{-1} \text{ with } B = 1 - (1-\delta)^{1-\theta}. \tag{9}$$

We have the following results:

Result 2.

1. $\lim_{\theta \to 0} p^*(\theta)\big|_{\delta=1} = 1 - e^{-1}$,

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[11]Just to mention some recent examples, Barseghyan et al. (2013), Sydnor (2010), Kaplan and Violante (2010) and Brown and Finkelstein (2008) have all applied CRRA utility to describe risk aversion and insurance choice.

[12]Insert (7) and (3) into profit function (2), taking first-order condition and we arrive at

$$\begin{aligned} \pi(p) &= R(p) - pL = W - CE(p) - pL = W - U^{-1}[U(W) - p(U(W) - U(W-L))] - pL \\ &= W - (W^{1-\theta} - p(W^{1-\theta} - (W-L)^{1-\theta}))^{\frac{1}{1-\theta}} - pL; \\ \pi'(p) &= -\frac{1}{1-\theta}(W^{1-\theta} - p(W^{1-\theta} - (W-L)^{1-\theta}))^{\frac{1}{1-\theta}-1}(-W^{1-\theta} + (W-L)^{1-\theta}) - L = 0 \\ &\Rightarrow (W^{1-\theta} - p(W^{1-\theta} - (W-L)^{1-\theta}))^{\frac{1}{1-\theta}-1} = \frac{L(1-\theta)}{W^{1-\theta} - (W-L)^{1-\theta}} \\ &\Rightarrow p^* = \frac{W^{1-\theta} - [\frac{L(1-\theta)}{W^{1-\theta}-(W-L)^{1-\theta}}]^{\frac{1-\theta}{\theta}}}{W^{1-\theta} - (W-L)^{1-\theta}}. \end{aligned}$$

2. $p^*(1/2) = 1/2$,

3. $\lim_{\delta \to 1} p^*(\theta) = 1 - (1-\theta)^{\frac{1-\theta}{\theta}}$,

4. $\lim_{\delta \to 0} p^*(\theta) = 1/2$.

It is most insightful to discuss the implications of these properties together with the graphs in Figure 4 that show the development of the optimal and critical loss probabilities for different values of θ and δ.[13] Again, as for CARA utility, we observe that the optimal p is decreasing with the level of risk-aversion among the population. The right panel of Figure 4 shows that for all sizes of the potential loss and all levels of risk aversion, the status-quo probability has to exceed 0.5 for consumers to be better off in a monopoly market. Again, the instances where consumers are better off in a monopolistic insurance market seem to be few.

The left panel of Figure 4 and Result 2 show that for values of the risk aversion parameter $\theta \leq 1/2$, $p^*(\theta) \geq 1/2$ $\forall \delta$. That is, a monopolistic insurer will not have any incentive whatsoever to push loss probabilities below 0.5 if consumers are only mildly risk averse. Moreover, according to property 4, the optimal loss probability is 0.5 for any level of risk aversion in the limiting case $\delta \downarrow 0$. The figure shows that only in case of $\delta \geq 0.95$ and high levels of risk aversion, the optimal loss probability drops to values importantly lower than 0.5. The reason is that in this case, a lowering of the loss probability only has a very limited impact on the price the insurer can charge while significantly reducing the expected cost. Wakker (2008) mentions that when large amounts of money are at stake, utility functions with $\theta > 1$ tend to best fit empirical data, such that the combination of high-δ/high-θ may not be that rare in practice, see also Hartley *et al.* (2013).

The right panel of Figure 4 shows that, as in the CARA case, for any level of risk aversion and loss size, the status-quo probability has to exceed 0.5 for consumers to be better off in a monopoly market. The instances that give the insurer the strongest incentives to reduce the loss probability are exactly those for which the status-quo probability has to be very high in order for consumers to benefit from being in a monopolistic instead of a competitive market. So also for CRRA utility, we conclude that consumers are better off in a monopolistic insurance market only when the potential loss is close to one's initial wealth and consumers have a high index of relative risk aversion.

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[13]We would like to point out that, other than ease of exposition, there no reason to neglect values of $\theta > 1$ (see Wakker (2008, p. 1330-1332)).
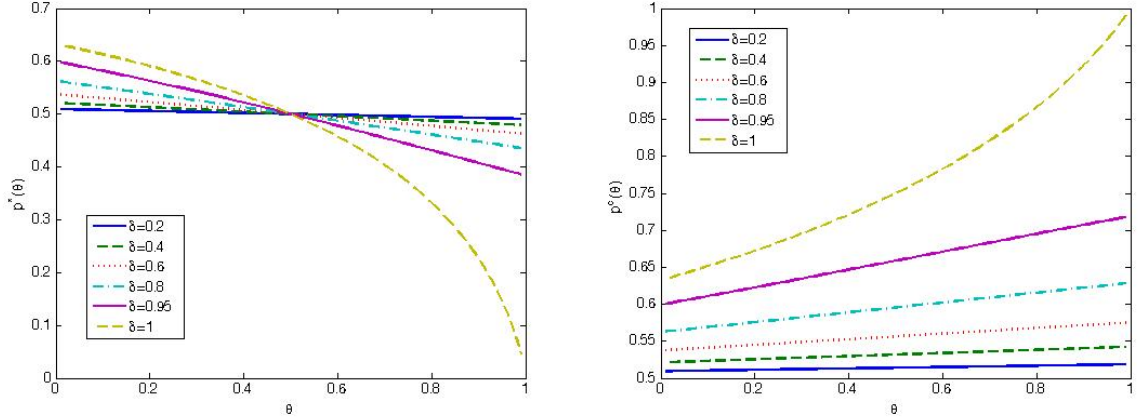
Figure 4: Plots of $p^*(\theta)$ (left panel) and $p^c(\theta)$ (right panel) for $\delta = 0.2, 0.4, 0.6, 0.8, 0.95$ and 1.

2.2.1 Heterogeneous risk attitudes

So far, we have assumed representative consumers. Insurers however operate in markets where consumers differ in their risk attitudes and for this reason, we now lift the assumption to see whether how this will affect our results.[14] Since there is no closed form solution for $p^*(\theta)$ in this case, we revert to simulation and present numerical results.

In line with Holt and Laury (2002), who estimate the coefficient of risk aversion for most subjects in a laboratory experiment to be in the $0.3 - 0.5$ range, we draw individual risk preferences θ_i from the distribution $N(0.4, 0.1)$. To find the distribution of profit maximizing $(R(p^*), p^*)$-combinations for a given proportional loss δ, we follow a three-step procedure: First we generate a total of $N = 1000$ consumers $(\theta_1, \theta_2, \ldots, \theta_{1000})$, with θ_j independent draws from $N(0.4, 0.1)$. Each consumer has initial wealth fixed at $W = 100$. Second we determine for each given loss probability p the optimal premium by calculating the quantity sold and profits obtained for each possible value of the premium $R \in [pL : 0.01 : W]$; we then repeat this step for each probability $p \in \{0, 0.01, \ldots 1.00\}$ and select the probability p^* for which $\pi(p^*, R(p^*)) \geq \pi(p', R(p')), \forall p' \in [0 : 0.01 : 1]$. We repeat these three steps $T = 1000$ times in order to arrive at distributions of the optimal p^* and other market characteristics such that the percentage of consumers that takes out insurance and consumer welfare.

Table 1 gives the simulation results for different values of δ. The table shows that, similar to the homogeneous CRRA case with $\theta < 0.5$, the optimal loss probability is increasing in δ but close to one half for all values of δ considered. The equilibrium fraction of consumers insured is very similar for different values of δ. Figure 5 shows for $\delta = 0.2$ the simulated distributions of the optimal loss

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[14]We assume that the insurer only knows the distribution $f(\theta)$ of θ such that he cannot engage in first-degree price discrimination.

probability p^*, the insurer's profits, the premium $R(p^*)$ set and the number of consumers that decides to buy insurance.

Loss size	Probability		Premium		Profit		Percentage	
(δ)	$\overline{p}^*$	s.e.	$\overline{R}$	s.e.	$\overline{\pi(p^*)}$	s.e.	insured	s.e.
0.01	0.490	(0.00)	0.49	(0.01)	0.30	(0.00)	79.13	(1.30)
0.05	0.490	(0.01)	2.46	(0.04)	8.10	(0.12)	81.02	(1.24)
0.10	0.491	(0.01)	4.95	(0.09)	33.30	(0.48)	83.16	(1.19)
0.20	0.493	(0.01)	10.03	(0.15)	141.10	(2.06)	81.86	(2.61)
0.40	0.500	(0.01)	20.78	(0.32)	644.30	(9.35)	82.08	(2.45)
0.60	0.504	(0.01)	32.32	(0.47)	1704.40	(24.63)	81.75	(2.53)
0.80	0.518	(0.01)	46.14	(0.44)	3842.50	(56.31)	81.79	(2.54)
0.90	0.528	(0.00)	54.66	(0.43)	5820.40	(86.24)	81.47	(2.53)
0.99	0.550	(0.01)	66.48	(0.54)	9668.00	(150.09)	80.26	(2.63)

Table 1: The simulation results for CRRA utility with $\theta \sim N(0.4, 0.1)$. Standard errors in parentheses.
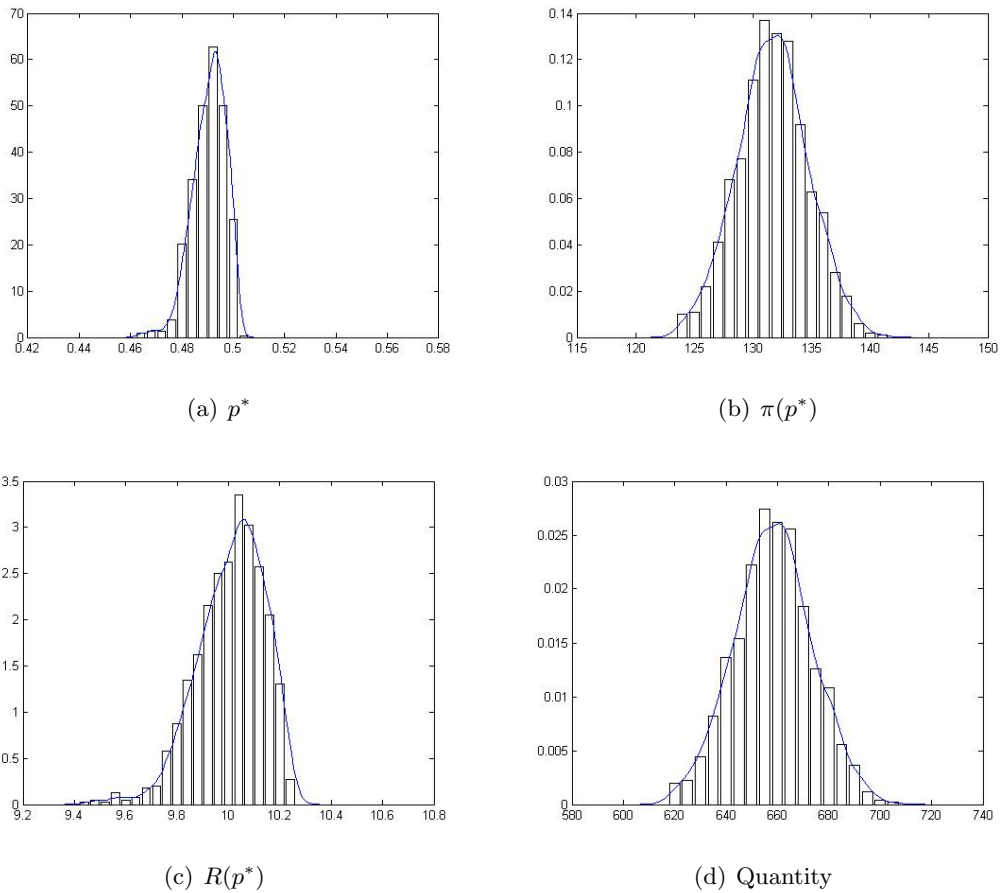

(a) p^*

(b) $\pi(p^*)$

(c) $R(p^*)$

(d) Quantity

Figure 5: Normal Kernel density estimations and scatter plots for the simulation results ($\delta = 0.2$)

3 Reference-dependent utility

In the expected-utility model, recent changes in wealth do not affect the utility one derives from one's current wealth. That is, a wealth level of $2 million gives you the same utility independent of whether you gained $1 million or lost $3 million compared to yesterday. Rabin (2000) has shown that this limited framework is unable to explain risk aversion over relatively small stakes because anything but virtually risk neutrality over small stakes will implies absurd risk aversion over larger stakes. Based on this, Rabin and Thaler (2001) conclude that economists should abandon the expected-utility hypothesis.

Samuelson (2005, p. 90) notes that although this is the common way expected utility appears in theoretical models, there are no fundamental objections to defining utility over initial wealth and changes in wealth. Kőszegi and Rabin (2006, 2007) develop such a model of reference-dependent utility in which the utility derived from a riskless wealth outcome consists of two components: a - traditional - intrinsic "consumption utility" that is a function of the wealth outcome only, plus a reference-dependent gain-loss utility. Subsequent studies have applied this model to topics as disparate as cross-country differences in trust levels (Bohnet et al., 2010), a monopolistic firm's pricing strategies when consumers have reference-dependent preferences (Heidhues and Kőszegi, 2010; Carbajal and Ely, 2013), price variation and competition intensity (Heidhues and Kőszegi, 2008) and dynamic models of consumption plans (Kőszegi and Rabin, 2009).

This section analyzes the behavior of a profit-maximizing insurer who can influence loss probabilities in the reference-dependent utility framework. Our objective is to see whether the main finding of the previous section – the profit-maximizing loss probability is around 0.5 for commonly observed levels of risk aversion – is upheld in this context. To this end, we first present the Kőszegi and Rabin (2007) model.[15]

The key element of Kőszegi and Rabin (2007) is that a person's utility not only depends on her riskless wealth outcome $w \in \mathbb{R}$, but also on a riskless reference level of wealth $r \in \mathbb{R}$.[16] A representative consumer's total utility is given by

$$u(w|r) \equiv m(w) + \mu(m(w) - m(r)), \tag{10}$$

---

[15]Sydnor (2010, Section F) contains a nice discussion how standard prospect theory cannot fully explain insurance purchases, but newer models such as Köszegi and Rabin (2006, 2007), can.

[16]The difference between the models introduced in Kőszegi and Rabin (2007) and Kőszegi and Rabin (2006) is that utility in the latter depends on a multi-dimensional consumption bundle and reference bundle. We follow Kőszegi and Rabin (2007), which uses a version with a one-dimensional utility function.

with the term $m(w)$ the intrinsic consumption utility and the term $\mu(m(w) - m(r))$ the reference-dependent gain-loss utility. The model assumes that the reference point r relative to which a consumer evaluates an outcome is stochastic because a consumer may be uncertain about outcomes. When w is drawn according to the probability measure $F(\cdot)$, utility is given by

$$U(F|G) = \int \int u(w|r)dG(r)dF(w). \tag{11}$$

The model makes the simplifying assumption that preferences are linear in probabilities: For a given reference point, the stochastic wealth outcome is evaluated according to its expected reference-dependent utility. This in contrast to prospect theory (Kahneman and Tversky, 1979; Barberis, 2013) that allows decision weights to be a non-linear function of the objective probabilities in order to accommodate the commonly observed phenomenon that people tend to overweigh small probabilities and underweigh large probabilities.[17] Kőszegi and Rabin (2007) make five assumptions on the properties of the gain-loss utility $\mu(\cdot)$ of which we repeat for convenience assumption A2 (capturing loss aversion for large stakes) and A3 (diminishing sensitivity):

A2 If $y > x > 0$, then $\mu(y) + \mu(-y) < \mu(x) + \mu(-x)$.

A3 $\mu''(x) \leq 0$ for $x > 0$ and $\mu''(x) \geq 0$ for $x < 0$.

In our analysis, we will use the same parametrization as Kőszegi and Rabin (2007): $\mu(x) = \eta x$ for $x > 0$, and $\mu(x) = \eta\lambda x$ for $x \leq 0$. In this parametrization, $\eta > 0$ is the weight that consumers attach to gain-loss utility, and $\lambda > 1$ is their coefficient of loss aversion. As in the previous section, consumers have to decide whether they wish to face the risk of losing L of their initial wealth W with probability p or to buy insurance against this risk by paying a premium R. Again, we assume that people choose to buy insurance as long as the expected utility of being insured is at least equal to the expected utility of staying uninsured.

To close the model, one needs to determine the appropriate reference point. Although there is little empirical evidence on the determinants of reference points, Kőszegi and Rabin (2006, 2007) make the case for a rational expectations assumption: A person's reference point has to be consistent with the beliefs about the outcome this person held in the recent past. For example, an employee who had been expecting a salary of $100,000 and should assess a salary of $90,000 not as a gain but as a loss.[18]

Kőszegi and Rabin (2007) consider three attitudes towards risk and give an example for $L = 100$,

---

[17]This simplification may lead us to underestimate the demand for insurance for low-probability losses.

[18]Their main reasons for assuming rational expectations are that it maintains modeling discipline and that there is empirical evidence indicating that reference points are influenced by expectations (Post et al., 2008).

$p = 0.5$ and $R = 55$. First they look at unanticipated risks, where the agent's reference point is fixed. An agent for example expects to retain the status quo of 0. In this case, buying insurance will inflict a sure loss of 55 whereas the no-insurance option gives a 50% chance to lose 100. Due to the diminishing sensitivity assumption, the agent will not buy insurance. For the context we consider however, the instances where agents do anticipate the exposure to risk seem more appropriate. In these situations, the agent correctly predicts the choice set she faces. Within this class, Kőszegi and Rabin (2007) distinguish between UPE/PPE risk attitudes and CPE risk attitudes.

In the unacclimating personal equilibrium (UPE), the time between the decision (take insurance or not) and the outcome (a loss occurs or not) is sufficiently short that the agent does not adapt her expectations. That is, she will evaluate the gain-loss utility of the outcome relative to the expected outcome without coverage, and the agent knows she will evaluate outcomes this way (the rational expectations assumption). Kőszegi and Rabin (2007) mention as examples, insurance choice for short-term rentals such as cars and skis. In terms of the earlier example, in deciding whether or not to take insurance, she will infer that

    a taking insurance by paying 55 will induce a either feeling of losing 55 with probability $1 - p = 0.5$ (in case no loss occurs) or a feeling of gaining 45 (in case a loss does occur);

    b not taking insurance will either lead to a mixed feeling of status quo and gaining 100 (in case no loss occurs) or a mixed feeling of status quo and loosing 100 (in case a loss does occur).

In the choice-acclimating personal equilibrium (CPE), it is assumed that the time between the moment of deciding and the moment of the outcome is sufficiently long to adapt expectations. That is, if the agent decides not to take insurance, this choice will determine her reference point at the time the relevant wealth outcome occurs and the possibility that she could have taken insurance does not enter the gain-loss calculation.[19] If she decides to take insurance, this will determine her reference point and the possibility that she could have chosen not to insure does not enter the gain-loss calculation. This situation adequately describe choice for travel and flight insurance. To return to the Kőszegi and Rabin example, the agent will rightly infer that

    a taking insurance by paying 55 will not lead to any gain-loss utility because at the moment of the outcome, the risk that was once there will be forgotten;

---

[19]Phrased a bit differently, the CPE is defined as the decision that maximizes expected utility given that it determines both the reference lottery and the outcome lottery. (Kőszegi and Rabin, 2007).

b not taking insurance will, just as in the UPE situation either lead to a mixed feeling of status quo and gaining 100 (in case no loss occurs) or a mixed feeling of status quo and loosing 100 (in case a loss does happen).

So, compared to UPE, taking insurance will be more attractive in a CPE context because it is never felt as a loss. The implication of the insurance being relatively more attractive is that agents are more risk averse when they anticipate a risk and the possibility buy insurance coverage. We now continue with calculating the optimal loss probabilities under UPE and CPE.

3.1 Optimal loss probability under UPE risk attitudes

In the remainder of this section, we assume that the consumption utility is linear, $m(w) = w$. This is a reasonable assumption for modest scale risks. If being insured is the reference point, the expected utility of a consumer with initial endowment W who decides to buy insurance by paying a premium R equals

$$
\begin{aligned}
U(F|F) &= \sum\sum u(w|r)f(w)f(r)drdw \\
&= f(W-R)f(W-R)[m(W-R) + \mu(m(W-R) - m(W-R))] \qquad (12) \\
&= m(W-R),
\end{aligned}
$$

where the last equality follows because i) in case of being covered, there is no uncertainty in the final wealth received, $f(W-R) = 1$; ii) if being insured is the reference point, the probability measure of the reference point has mass 1 at $W - R$ as well. There is no feeling of loss or gaining in this case.

If being insured is the reference point but the consumer decides not to buy insurance, her the expected utility is:

$$
\begin{aligned}
U(F'|F) &= \sum\sum u(w|r)f'(w)f(r)drdw \\
&= f'(W-0)f(W-R)[m(W) + \mu(m(W) - m(W-R))] \\
&+ f'(W-L)f(W-R)[m(W-L) + \mu(m(W-L) - m(W-R))] \qquad (13) \\
&= (1-p)[m(W) + \mu(m(W) - m(W-R))] + p[m(W-L) + \mu(m(W-L) - m(W-R))],
\end{aligned}
$$

where the last equality follows from $f'(W-L) = 1 - f'(W) = p$ and $f(W-R) = 1$: without insurance, the wealth outcome is $W - L$ with probability p and $(W-L)$ otherwise; the reference point is $(W-R)$ with probability 1. Applying Kőszegi and Rabin's (2007) definition, the decision to buy insurance is an UPE if $U(F|F) \geq U(F'|F)$.

Assuming that consumers will buy insurance whenever the expected utility of being insured is at least as large as the expected utility of not being insured, a risk-neutral monopolistic insurer who aims to maximize expected profits will set the loss probability p such that $R - pL$ is maximal, conditional on $U(F|F) \geq U(F'|F)$. In order to find an explicit solution for p^*, we use the same parametrization of the reference-dependent gain-loss utility $\mu(\cdot)$ as Kőszegi and Rabin (2006, 2007): $\mu(x) = \eta x$ for $x > 0$, and $\mu(x) = \eta\lambda x$ for $x \leq 0$, with $\eta > 0$ the relative weight that consumers attach to gain-loss utility, and $\lambda > 1$ the coefficient of loss aversion. Given this specification:

$$U(F|F) \geq U(F'|F) \Leftrightarrow W - R \geq (1-p)[W + \eta R] + p[W - L - \lambda\eta(L - R)]$$

$$\Leftrightarrow W - R \geq W + (1-p)\eta R - pL - p\lambda\eta(L - R).$$

We arrive at the following result (a detailed derivation is provided in Appendix A.3):

Result 3. *In an economy where consumers' attitude towards risk is characterized by UPE, the loss probability p^* that maximizes the expected-profits of a monopolistic insurer equals*

$$p^* = \frac{\sqrt{(1 + \lambda\eta)(1 + \eta)} - \eta - 1}{\eta(\lambda - 1)}, \tag{14}$$

and the corresponding price of the insurance is

$$R(p^*) = \frac{L(1 + \lambda\eta)(\sqrt{(1 + \lambda\eta)(1 + \eta)} - \eta - 1)}{\eta(\lambda - 1)\sqrt{(1 + \lambda\eta)(1 + \eta)}} = p^* L \sqrt{\frac{1 + \lambda\eta}{1 + \eta}} \tag{15}$$

One easily sees $\lambda > 1$ guarantees positive expected profits per insuree, $R(p^*) - p^* L$. Note that, different from the expected-utility framework, the loss size L does not appear as an argument. A number of other properties of p^* are stated in the following corollary.

Corollary 1. *The loss probability p^* as given in equation (14) has the following properties*

1. $\frac{\partial p^*}{\partial \eta} < 0$.

2. $\lim_{\eta\downarrow 0} p^* = 1/2$.

3. $\lim_{\eta\to\infty} p^* = \frac{\sqrt{\lambda}-1}{\lambda-1}$.

The first property says that the optimal loss probability is decreasing with the relative importance of the gain-loss utility. Taken together, the properties inform us that for a given λ, $p^* \in \left[\frac{\sqrt{\lambda}-1}{\lambda-1}, \frac{1}{2}\right]$. Empirical studies typically find estimates of the loss aversion parameter λ of around 2.25 (Kahneman, Knetsch and Thaler, 1990; Tversky and Kahneman, 1992; Gill and Prowse, 2012). Such an estimate

implies an lower bound for the optimal loss probability of 0.4. So, again, we find values of p^* much closer to $1/2$ than to 0.

Another possible UPE is the situation where no insurance is the reference point and the decision not to buy insurance gives the consumer a higher expected utility than buying insurance, that is: $U(F'|F') \geq U(F|F')$. Kőszegi and Rabin (2006) propose that in cases with multiple equilibria, an individual will choose her "favorite" equilibrium, the one that gives the highest *ex ante* expected utility if followed through. This leads to the concept of 'preferred personal equilibrium' (PPE) as an equilibrium selection mechanism: the PPE is the most preferred UPE. In our case, deciding to buy insurance is a PPE if $U(F|F) \geq U(F'|F')$. The assumption of profit-maximization by the insurer rules out that $U(F|F) < U(F'|F')$ because in that case, his profits would be zero and because – as we will show in the next section – there is always a feasible loss probability p such that his expected profits are non-negative and $U(F|F) \geq U(F'|F')$ holds.

3.2   Optimal loss probability under CPE risk attitudes

One of the implications of Kőszegi and Rabin's model is that buying insurance is more attractive when consumers have CPE instead of UPE risk attitudes. This implies that insurers are better off when consumers can buy insurance well ahead of time. We explore this possibility in this section. The expected utility of taking insurance $U(F|F)$ does not change and equals (12). The expected utility of the decision not to buy insurance, given that the reference point is also "no insurance", equals

$$
\begin{aligned}
U(F'|F') =& \sum\sum u(w|r)f'(w)f'(r)drdw \\
=& f'(W-0)f'(W-0)[m(W-0)+\mu(m(W-0)-m(W-0))] \\
& + f'(W-0)f'(W-L)[m(W-0)+\mu(m(W-0)-m(W-L))] \\
& + f'(W-L)f'(W-0)[m(W-L)+\mu(m(W-L)-m(W-0))] \\
& + f'(W-L)f'(W-L)[m(W-L)+\mu(m(W-L)-m(W-L))] \\
=& (1-p)^2 m(W) + p^2 m(W-L) \\
& + p(1-p)[m(W)+m(W-L)+\mu(m(W)-m(W-L))+\mu(m(W-L)-m(W))].
\end{aligned}
$$
(16)

Without insurance, the wealth outcome is W with probability $f'(W) = 1-p$ and $(W-L)$ with probability $f'(W-L) = p$. The reference point is 'no insurance' in which case likewise the outcome is W with probability $(1-p)$ and $(W-L)$ otherwise.

Buying insurance is a choice-acclimating personal equilibrium if $U(F|F) \geq U(F'|F')$ for all F'. The difference between UPE and CPE is that in the latter case, the reference point adjusts to the

decision. The monopolistic insurer sets p such that the expected profits are maximized under the condition that $U(F|F) \geq U(F'|F')$. Equating $U(F'|F')$ in equation (16) to $U(F|F)$ in equation (12) shows that in equilibrium, the expected profit margin of the insurer equals

$$R - pL = -p(1 - p)[\mu(L) + \mu(-L)]$$

Since we know from assumption $A3$ that $\mu(+L) + \mu(-L) < 0$, expected profits are maximized when $p^* = 1/2$. We state this result formally:

Result 4. *In an economy where consumers' attitude towards risk is characterized by CPE, the loss probability p^* that maximizes the expected-profits of a monopolistic insurer equals $1/2$.*

Note that this result is reached without assuming any specific parametrization for the gain-loss utility function. Figure 6 provides some intuition for this result. In the figure, the loss-averse utility function $U(F'|F')$ of equation (16) is convex with respect to p.[20] When $p = p^*$, an individual's utility equals $U(p^*)$ if she is loss-averse and $(W - p^*L)$ if risk-neutral. Since we assume linear consumption utility, the certainty equivalent equals $CE(p) = U(CE(p))$. Thus the expected profit equals the distance marked by the vertical dotted line. The optimal loss probability p^* maximizes the distance between $U(F'|F')$ and the expected wealth line $W - pL$, which is the point p where $U'(p)$ equals the slope of the expected wealth line, which is $-L$. This maximal distance is attained when $p^* = 1/2$ because $U'(F'|F') = -L + (1 - 2p)[\mu(L) + \mu(-L)]$.

---

[20]Because we assume linear consumption utility, plotting the wealth level at the horizontal axis, as in Schlesinger and Venezian (1986, Figure 1) leads to linear utility curves. For this reason, we use the decision variable p as the variable at the horizontal axis.

To show that $U'(F'|F')$ is convex, take the first order and second order derivatives w.r.t. p:

$$U'(F'|F') = -2(1 - p)W + 2p(W - L) + (1 - 2p)[2W - L + \mu(L) + \mu(-L)]$$
$$= -L + (1 - 2p)[\mu(L) + \mu(-L)];$$
$$U''(F'|F') = -2[\mu(L) + \mu(-L)].$$

Because $\mu(L) > 0$, $\mu(-L) < 0$ and $|\mu(L)| < |\mu(-L)|$, $U''(F'|F') > 0$ and thus $U(F'|F')$ is convex. When $0 \leq p \leq \frac{1}{2}$, $U'(F'|F') < 0$ for sure; when $\frac{1}{2} < p \leq 1$, we have

$$U'(F'|F') = 0 \Rightarrow \hat{p} = \frac{1}{2} - \frac{L}{2[\mu(L) + \mu(-L)]}.$$

For $\frac{1}{2} < p < \hat{p}$, $U'(F'|F') < 0$; and for $\hat{p} < p \leq 1$, $U'(F'|F') > 0$. The utility function first decreases in p and then increases after some point $\hat{p} > \frac{1}{2}$.
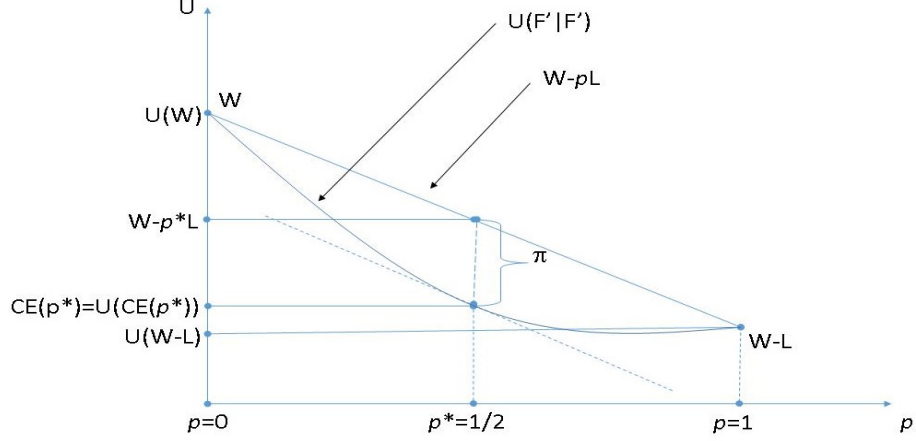
Figure 6: CPE risk attitudes and the risk premium with linear consumption utility.

For the specific parametrization $\mu(x) = \eta x$ for $x > 0$ and $\mu(x) = \lambda \eta x$ for $x \leq 0$, the profit maximizing premium and profits are equal to

$$R(p^*) = p^* L - p^* \eta L + p^* \lambda \eta L + p^{*2} \eta L - p^{*2} \lambda \eta L = \frac{1}{2} L - \frac{1}{4} \eta L + \frac{1}{4} \lambda \eta L, \tag{17}$$

and

$$\pi(p^*) = \frac{1}{4} \eta L (\lambda - 1). \tag{18}$$

The premium R^* is increasing in the weight of the gain-loss utility in the utility function and in λ. This means that, in line with intuition, an insurer can attain higher profits, the more an individual weighs losses relative to gains.

Our result that $p^* = 1/2$ when consumers have CPE risk attitudes is qualitatively similar to the results for UPE risk attitudes and for the expected utility model. Compared to the UPE case, individuals are more inclined to take out insurance because they are more risk-averse when they can commit to the choice ahead of time. The model we discuss in this section only considers a representative agent economy. Note however that for CPE risk attitudes, heterogeneity in either η or λ does not change our result because p^* does not depend on these values.

3.3 Numerical example

We conclude this section with a small numerical example. Assume that the consumer with a gain-loss coefficient $\lambda = 2.25$ has to decide whether or not to insure against a risk that leads to a loss $L = 10$ with probability p^*. Table 2 compares for different values of η the expected-profit maximizing loss probability p^* and premium for the case where consumers have UPE risk attitudes with the case where

	UPE				CPE			
η	p^*	$R(p^*)$	π	$\frac{R(p^*)}{p^*L}$	p^*	$R(p^*)$	π	$\frac{R(p^*)}{p^*L}$
0.1	0.486549	5.13	0.27	1.06	0.5	5.31	0.31	1.06
0.5	0.456571	5.43	0.87	1.19	0.5	6.56	1.56	1.31
1	0.439608	5.60	1.21	1.27	0.5	8.13	3.13	1.63
5	0.411714	5.88	1.77	1.43	0.5	20.63	15.63	4.13
10	0.406235	5.94	1.88	1.46	0.5	36.25	31.25	7.25
50	0.401315	5.99	1.97	1.49	0.5	161.25	156.25	32.25
100	0.400662	5.99	1.99	1.50	0.5	317.50	312.50	63.50

Table 2: Numerical example of the optimal loss probability and premium when $L = 10$, $\lambda = 2.25$, and consumers have either UPE or CPE risk attitudes.

they have CPE risk attitudes. The table also gives the expected profits per insuree and the ratio of the premium charged ($R(p^*)$) and the actuarial value of the policy (p^*L).

In line with the analytical results, the numerical results show that as the gain-loss utility receives higher weight, the optimal loss probability decreases in the UPE case. The premium and expected profits are increasing in η, both for UPE as for CPE. Table 2 confirms that the monopolistic insurer is able to attain higher expected profits when consumers have CPE preferences. This difference is very sizable: whereas in the UPE case, the premium rises to about 1.5 times the actuarial value, it rises to 63 times the actuarial value in the CPE case. This is reminiscent of our earlier findings for the expected utility model were consumers were endowed with CARA preferences (see Figure 3).

4 Conclusions

This paper follows up on the original contributions by Schlesinger and Venezian (1986, 1990) who first investigated the incentives for loss-modification by profit-maximizing insurers. They concluded that granting insurers market power might benefit consumers because this might trigger them exert efforts to bring down the ex ante loss or the probability with which such a loss occurs. In this original work, the possibility of increases in the loss probability that would harm consumers receives relatively little attention, because it is "likely to meet with public resistance and possible regulatory restraint." (Schlesinger and Venezian, 1990)

In this theory paper, we calculate for a number of settings the value of the profit-maximizing loss probability with the idea that the higher this value, the less likely it is that the initial loss probability is even higher and the less likely that consumers would be better off in an insurance market with less competition. First we consider the expected-utility framework. We repeat the

analysis in Schlesinger and Venezian (1990) for an economy in which consumers are endowed with CARA preferences, which describes the case where consumers face absolute losses. Next we describe the situation where consumers have CRRA preference, which describes situations where they have to choose whether or not to insure against a potential loss proportional to their wealth. In both cases, the optimal loss probabilities only come close to zero if consumers are highly risk averse (CARA) or are highly risk averse and face the risk of losing a large fraction of their initial wealth (CRRA).

In the second part of the paper, we use the more recent loss aversion theory to analyze the insurer's problem of finding the optimal loss probability in case the consumers have reference-dependent preferences. We use the reference-dependent utility model developed by Kőszegi and Rabin's (2006, 2007) to show that under the assumption of linear consumption utility, the optimal loss probability is 0.5 when consumers have CPE risk attitudes and between 0.4 and 0.5 when consumers have UPE risk attitudes and a gain-loss coefficient of 2.25, a value often found in empirical studies.

Our main conclusion therefore is that in most commonly used specifications, the loss probability that maximizes a monopolistic insurer's profits is closer to 1/2 than to 0, independent of whether we adopt an expected-utility framework or take the perspective of loss-averse consumers. As a consequence, the instances where consumers are better off in a monopolistic than in a competitive insurance market seem to be fairly few. Our results culminate in the advice that one needs to be cautious to bestow market power on insurers with the argument that this will incentivize them to engage in loss reduction activities that will benefit consumers.

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A   Appendix with proofs

A.1   Proof of Result 1

$$\lim_{L\to 0} p^*(\theta) = \lim_{L\to 0} \frac{1}{\theta L} - \frac{1}{e^{\theta L}-1} = \lim_{L\to 0} \frac{e^{\theta L}-1-\theta L}{\theta L(e^{\theta L}-1)}$$

$$= \lim_{L\to 0} \frac{\theta e^{\theta L}-\theta}{\theta e^{\theta L}-\theta+\theta^2 L e^{\theta L}}$$

$$= \lim_{L\to 0} \frac{\theta^2 e^{\theta L}}{\theta^2 e^{\theta L}+\theta^2 e^{\theta L}+\theta^2 L\theta e^{\theta L}} = \frac{\theta^2}{\theta^2+\theta^2} = \frac{1}{2},$$

where we apply the rule of L'Hôspital twice, respectively in step 2 and 3.

$$\lim_{L\to\infty} p^*(\theta) = \lim_{L\to\infty} \frac{1}{\theta L} - \frac{1}{e^{\theta L}-1} = 0-0 = 0.$$

A.2   Proof of Result 2

1. $\lim_{\theta\to 0} p^*(\theta)\big|_{\delta=1} = 1-e^{-1}$.

Inserting $L = W$ into equation (8) gives

$$
\begin{aligned}
\lim_{\theta\to 0} p^*(\theta)\big|_{\delta=1} &= \lim_{\theta\to 0}\left[1-\frac{\left(\frac{W(1-\theta)}{W^{1-\theta}}\right)^{\frac{1-\theta}{\theta}}}{W^{1-\theta}}\right] \\
&= \lim_{\theta\to 0}\left[1-\frac{W^{1-\theta}(1-\theta)^{\frac{1-\theta}{\theta}}}{W^{1-\theta}}\right] = 1-\lim_{\theta\to 0}(1-\theta)^{-1+1/\theta} \\
&= 1-\lim_{\theta\to 0}\frac{(1-\theta)^{1/\theta}}{1-\theta} = 1-\frac{\lim_{\theta\to 0}(1-\theta)^{1/\theta}}{\lim_{\theta\to 0}(1-\theta)} \qquad\qquad \text{(A.1)}\\
&= 1-\lim_{\theta\to 0}(1-\theta)^{1/\theta} = 1-\lim_{\theta\to 0}e^{\ln(1-\theta)/\theta} \\
&= 1-e^{\lim_{\theta\to 0}[\ln(1-\theta)/\theta]} = 1-e^{\lim_{\theta\to 0}(-1/(1-\theta))/1} = 1-e^{-1},
\end{aligned}
$$

where the second to last equality follows from application of L'Hôpital's rule.

2. $p^*(1/2) = 1/2$.

Define $A \equiv W^{1-\theta} - (W-L)^{1-\theta}$. This allows one to rewrite equation (8) as

$$p^*(\theta) = W^{1-\theta}/A - \left([L(1-\theta)]^{(1-\theta)/\theta}\right)/A^{1/\theta} = W^{1-\theta}/A - \left([L(1-\theta)]^{1-\theta}/A\right)^{1/\theta}. \qquad \text{(A.2)}$$

This gives

$$
\begin{aligned}
p^*(1/2) &= *\sqrt{W}/A - (L/2)/A^2 = \frac{\sqrt{W}A - L/2}{A^2} \\
&= \frac{\sqrt{W}(\sqrt{W} - \sqrt{W-L}) - L/2}{2(W - \sqrt{W}\sqrt{W-L} - L/2)} = 1/2.
\end{aligned}
\tag{A.3}
$$

The second to last equality follows by noting that

$$
A^2 = W + (W - L) - 2\sqrt{W}\sqrt{W-L} = 2(W - \sqrt{W}\sqrt{W-L} - L/2).
$$

3. $\lim_{\delta \to 1} p^*(\theta) = 1 - (1-\theta)^{\frac{1-\theta}{\theta}}.$

Note that $\lim_{\delta \to 1} A = W^{1-\theta}$. The limit then follows as an immediate consequence of equation (A.2):

$$
\begin{aligned}
\lim_{\delta \to 1} p^*(\theta) &= \frac{W^{1-\theta}}{\lim_{\delta \to 1} A} - \frac{[L(1-\theta)]^{(1-\theta)/\theta}}{(\lim_{\delta \to 1} A)^{1/\theta}} \\
&= \frac{W^{1-\theta}}{W^{1-\theta}} - \left(\frac{(W(1-\theta))^{1-\theta}}{W^{1-\theta}} \right)^{1/\theta} = 1 - (1-\theta)^{(1-\theta)/\theta}.
\end{aligned}
\tag{A.4}
$$

4. $\lim_{\delta \to 0} p^*(\theta) = 1/2.$

$$
\begin{aligned}
\lim_{\delta \to 0} p^*(\theta) &= \lim_{\delta \to 0} \frac{W^{1-\theta} - [\frac{\delta W(1-\theta)}{W^{1-\theta} - (W - \delta W)^{1-\theta}}]^{\frac{1-\theta}{\theta}}}{W^{1-\theta} - (W - \delta W)^{1-\theta}} \\
&= \lim_{\delta \to 0} \frac{W^{1-\theta}[1 - (\frac{\delta(1-\theta)}{1 - (1-\delta)^{1-\theta}})^{\frac{1}{\theta} - 1}]}{W^{1-\theta}(1 - (1-\delta)^{1-\theta})} \\
&= \lim_{\delta \to 0} \frac{(1 - (1-\delta)^{1-\theta})^{\frac{1}{\theta} - 1} - (\delta(1-\theta))^{\frac{1}{\theta} - 1}}{(1 - (1-\delta)^{1-\theta})^{\frac{1}{\theta}}}.
\end{aligned}
$$

Let $f(\delta) = (1-\delta)^{1-\theta}$ and its Taylor series at point $\delta = 0$ is:

$$
\begin{aligned}
f(\delta) &= f(0) + \frac{f'(0)}{1!}(\delta - 0) + \frac{f''(0)}{2!}(\delta - 0)^2 + \frac{f^{(3)}(0)}{3!}(\delta - 0)^3 + \ldots \\
&= 1 - (1-\theta)\delta - \frac{(1-\theta)\theta}{2}\delta^2 - \frac{(1-\theta)(1+\theta)\theta}{6}\delta^3 - \ldots.
\end{aligned}
$$

Plug $f(\delta)$ back into $\lim_{\delta \to 0} p^*(\theta)$:

$$
\begin{aligned}
\lim_{\delta \to 0} p^*(\theta) &= \lim_{\delta \to 0} \frac{[(1-\theta)\delta + \frac{(1-\theta)\theta}{2}\delta^2 + \frac{(1-\theta)(1+\theta)\theta}{6}\delta^3 + \ldots]^{\frac{1}{\theta} - 1} - (\delta(1-\theta))^{\frac{1}{\theta} - 1}}{[(1-\theta)\delta + \frac{(1-\theta)\theta}{2}\delta^2 + \frac{(1-\theta)(1+\theta)\theta}{6}\delta^3 + \ldots]^{\frac{1}{\theta}}} \\
&= \lim_{\delta \to 0} \frac{[(1-\theta)\delta]^{\frac{1}{\theta} - 1}([1 + \frac{\theta}{2}\delta + \frac{(1+\theta)\theta}{6}\delta^2 + \ldots]^{\frac{1}{\theta} - 1} - 1)}{[(1-\theta)\delta]^{\frac{1}{\theta}}[1 + \frac{\theta}{2}\delta + \frac{(1+\theta)\theta}{6}\delta^2 + \ldots]^{\frac{1}{\theta}}} \\
&= \lim_{\delta \to 0} \frac{[1 + \frac{\theta}{2}\delta + \frac{(1+\theta)\theta}{6}\delta^2 + \ldots]^{\frac{1}{\theta} - 1} - 1}{(1-\theta)\delta[1 + \frac{\theta}{2}\delta + \frac{(1+\theta)\theta}{6}\delta^2 + \ldots]^{\frac{1}{\theta}}}
\end{aligned}
$$

$$= \lim_{\delta \to 0} \frac{(\frac{1}{\theta} - 1)X^{\frac{1}{\theta} - 2}(\frac{\theta}{2} + \frac{(1+\theta)\theta}{3}\delta + \dots)}{(1 - \theta)X^{\frac{1}{\theta}} + (1 - \theta)\delta\frac{1}{\theta}X^{\frac{1}{\theta} - 1}(\frac{\theta}{2} + \frac{(1+\theta)\theta}{3}\delta + \dots)}$$

$$= \frac{\frac{1-\theta}{\theta} \times \frac{\theta}{2}}{1 - \theta} = \frac{1}{2}, \tag{A.5}$$

whereas $X = [1 + \frac{\theta}{2}\delta + \frac{(1+\theta)\theta}{6}\delta^2 + \dots]$ and $\lim_{\delta \to 0} X = 1$; and the result in step 4 is derived from step 3 by applying l'Hôpital's rule.

A.3   Detailed solution for the insurer's optimization problem under UPE

The constrained optimization problem for the monopolistic expected-profit maximizing insurer is:

$$\max_{p,R} \ \pi = R - pL$$

$$s.t. \ (1 - p)\eta R + R - pL - p\lambda\eta(L - R) \leq 0.$$

The Lagrangian and the Karush-Kuhn-Tucker (KKT) conditions are the following:

$$\mathcal{L} = R - pL - \xi \left[(1 - p)\eta R + R - pL - p\lambda\eta(L - R)\right],$$

$$\frac{\partial \mathcal{L}}{\partial R} = 1 - \xi \left[\eta - p\eta + 1 + p\lambda\eta\right] = 0,$$

$$\frac{\partial \mathcal{L}}{\partial p} = -L - \xi \left[-\eta R - L - \lambda\eta L + \lambda\eta R\right] = 0,$$

$$\xi \geq 0,$$

$$\xi \left[(1 - p)\eta R + R - pL - p\lambda\eta(L - R)\right] = 0.$$

Case 1: The constraint is not binding and $\xi = 0$. However, this is not admissible because in that case $\frac{\partial \mathcal{L}}{\partial R} = 1 \neq 0$.

Case 2: The constraint is binding. In this case the KKT conditions can be simplified to

$$1 - \xi \left[\eta - p\eta + 1 + p\lambda\eta\right] = 0,$$

$$- L - \xi \left[-\eta R - L - \lambda\eta L + \lambda\eta R\right] = 0,$$

$$(1 - p)\eta R + R - pL - p\lambda\eta(L - R) = 0,$$

$$\xi > 0.$$

Solving these equations for p and R, one obtains:

$$p^* = \frac{\sqrt{(1 + \lambda\eta)(1 + \eta)} - \eta - 1}{\eta(\lambda - 1)},$$

$$R(p^*) = \frac{L(1 + \lambda\eta)(\sqrt{(1 + \lambda\eta)(1 + \eta)} - \eta - 1)}{\eta(\lambda - 1)\sqrt{(1 + \lambda\eta)(1 + \eta)}} = p^* L \sqrt{\frac{1 + \lambda\eta}{1 + \eta}}.$$

Since $\sqrt{(1 + \lambda\eta)(1 + \eta)} - \eta - 1 > 0$ when $\lambda > 1$, p^* and $R(p^*)$ are positive. The expected profits per

insuree equal

$$R(p^*) - p^*L = \left(\sqrt{\frac{1+\lambda\eta}{1+\eta}} - 1\right) p^*L$$

which is positive for $\lambda > 1$ and $\eta, L > 0$.

A.3.1 Proof of Corollary 1

A.4 Detailed solution for the insurer's optimization problem under CPE

The constrained optimization problem for the monopolistic expected-profit maximizing insurer is:

$$\max_{p,R} \ \pi = R - pL$$
$$s.t. \ (1-p)^2W + p^2(W-L) + p(1-p)[2W - L + \eta L - \lambda\eta L] - W + R \le 0$$

The Lagrangian and the KKT conditions for this problem are the following:

$$\mathcal{L} = R - pL - \xi((1-p)^2W + p^2(W-L) + p(1-p)[2W - L + \eta L - \lambda\eta L] - W + R),$$

$$\frac{\partial\mathcal{L}}{\partial R} = 1 - \xi = 0,$$

$$\frac{\partial\mathcal{L}}{\partial p} = -L - \xi\left[-2W + 2pW + 2pW - 2pL + (1-2p)(2W - L + \eta L - \lambda\eta L)\right] = 0,$$

$$\xi \ge 0,$$

$$\xi\left[(1-p)^2W + p^2(W-L) + p(1-p)[2W - L + \eta L - \lambda\eta L] - W + R\right] = 0.$$

Case 1: The constraint is not binding and $\xi = 0$. However, this is not admissible because in that case $\frac{\partial\mathcal{L}}{\partial R} = 1 \ne 0$.

Case 2: The constraint is binding. In this case the KKT conditions can be simplified to

$$\xi = 1,$$

$$L - 2W + 2pW + 2pW - 2pL + (1-2p)(2W - L + \eta L - \lambda\eta L) = 0,$$

$$(1-p)^2W + p^2(W-L) + p(1-p)[2W - L + \eta L - \lambda\eta L] - W + R = 0.$$

Solving these equations for p and R, one obtains:

$$p^* = \frac{1}{2},$$

$$R(p^*) = \frac{L}{2} + \frac{\eta(\lambda-1)L}{4}.$$

$R(p^*)$ is positive since $\lambda > 1$. These expected profits $R(p^*) - p^*L$ are positive because $\frac{L}{2} + \frac{\eta(\lambda-1)L}{4} - \frac{L}{2} = \frac{\eta(\lambda-1)L}{4} > 0$.