TI 2013-202/VI
Tinbergen Institute Discussion Paper

The Interest Rate and Capital Durability, and the Importance of Methodological Pluralism

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July 15, 2013

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*We are grateful to Cees Withagen for his comments and encouragement.

Abstract

Champions of sustainable growth often call for more durable production technologies with less capital depreciation. As investment in more durable capital is encouraged by lower interest rates, we investigate whether policy makers can steer the economy towards a path with low interest rates in order to stimulate more durable capital formation. We study this question from the viewpoint of two different macroeconomic paradigms, with three different modeling strategies, and get three fundamentally different and even contradicting answers. As none of these paradigms can claim to be superior to the other one, we argue that all modeling strategies may yield valuable insights, which leads to nuanced and careful policy advice. The paper is therefore an illustration of the importance of methodological pluralism in addressing macro-environmental questions where the interest rate takes center stage.

Keywords: interest rate, capital durability, depreciation rate, sustainability, methodological pluralism

JEL Codes: B4, E22, E43, O44, Q5

1 Introduction

Champions of sustainable growth often call for more durable production technologies with less capital depreciation: less capital depreciation causes less waste, and therefore less pollution; furthermore, it decreases replacement investment, which frees up resources that can be spent on mitigation and abatement expenditures.

Investment in more durable capital is encouraged by lower interest rates. Suppose, for instance, that firms can invest in two types of capital: cheap but not very durable capital, with a high depreciation rate; and expensive but durable capital, with a low depreciation rate. Lower interest rates increase the present discounted value of future capital services, and therefore make investment in the more durable type of capital relatively more attractive compared with investment in the less durable type of capital. This raises the question whether the economy can be steered towards a path with low interest rates in order to stimulate investment in more durable capital.

The objective of this paper is not to give a definite answer to this question: that would be far too ambitious given the state of the art in macroeconomics. Our objective is rather to show how different macroeconomic paradigms suggest different answers, and to illustrate the importance of methodological pluralism in addressing such macro-environmental questions.

Methodological pluralism, advocated by (among others) Samuels (1998), is the view that if one methodology cannot be shown to be superior to all others, analyses should not be rejected solely on the basis of methodological considerations. This principle is well established in ecological economics (see, for instance, Norgaard, 1989). But its importance does not seem to be widely acknowledged by macroeconomists (at least not in the academic world).

We will focus on the two most prominent macroeconomic paradigms: *old-style* neo-classical macroeconomics, which *assumes* some stylized relations between macroeconomic variables and then studies their implications; and *new* neo-classical macroeconomics or *modern* macroeconomics, which makes assumptions about the microeconomic foundations of the macroeconomy and then *derives* relations between macroeconomic variables. We acknowledge upfront that both paradigms are inappropriate for many issues related to sustainability.[1] But they suffice for the purposes of this paper: we will argue that

[1] Many references explain why, but two standard references are Georgescu-Roegen (1971) and Daly (1977); see also Daly and Farley's textbook (Daly and Farley, 2003); note that most of the criticism focuses on old-style neo-classical macroeconomics, but can equally well be raised against modern macroeconomics. Some key building blocks for an alternative macroeconomic framework are suggested by Rezai et al. (2013).

nor the old-style neo-classical methodology, nor the modern macroeconomic methodology is unequivocally superior to the other one; and we will show that they shed a very different light on the question what policy makers can do to decrease interest rates and stimulate investment in more durable capital.

The paper proceeds as follows. In the next section, we present the supply side and the equilibrium condition in the market for loanable funds in an economy with two types of capital, differentiated by their depreciation rate. We then explain in section 3 the different approaches which old-style neo-classical and modern macroeconomists would take to complete the model with a description of aggregate saving. We work out these different modeling strategies in sections 4 and 5. Section 6 summarizes what we have learnt, and concludes.

2 The set-up

In this section, we present the supply side and the equilibrium condition in the market for loanable funds, which both the old-style neo-classical macroeconomist in section 4 and the modern macroeconomist in section 5 will take as given.

Consider a perfectly competitive economy where all firms have access to a Cobb-Douglas production technology with capital and labor input. There are two types of capital. Both types are perfect substitutes for each other in the aggregate production function, but they have different depreciation rates: K_1 wears out quickly and has a high depreciation rate δ_1; K_2, at the other hand, is more durable and has a lower depreciation rate δ_2. Labor input is denoted by L and labor effectiveness by A. Aggregate output Y is then given by

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha} = (K_{1,t} + K_{2,t})^{\alpha}(A_t L_t)^{1-\alpha} \quad \text{with } 0 < \alpha < 1 \tag{1}$$

... where K_t denotes the aggregate capital stock in period t.

Labor effectiveness A grows at an exogenous, non-negative growth rate g. The laws of motion of the two types of capital depend on their depreciation rate and investment:

$$K_{1,t+1} = (1-\delta_1)K_{1,t} + I_{1,t} \tag{2}$$
$$K_{2,t+1} = (1-\delta_2)K_{2,t} + I_{2,t} \quad \text{with } 0 \leq \delta_2 < \delta_1 \leq 1 \tag{3}$$

...where $I_{1,t}$ and $I_{2,t}$ are the additions to the capital stock because of investment in, respectively, the less durable and the more durable type of capital.

The more durable capital good is more expensive than the less durable capital good: whereas the price of one unit of K_1 is normalized to 1, the price of one unit of K_2 is equal to p, with $p > 1$. Perfect competition implies then that the representative firm's first-order condition for investment is given by the following complentary slackness (c.s.) conditions:[2]

$$\alpha \frac{Y_t}{K_t} \leq r_t + \delta_1 \quad \text{and} \quad K_{1,t} \geq 0 \quad \text{with c.s.} \tag{4}$$

$$\alpha \frac{Y_t}{K_t} \leq p(r_t + \delta_2) \quad \text{and} \quad K_{2,t} \geq 0 \quad \text{with c.s.} \tag{5}$$

...where r_t is the interest rate in period t.

The representative firm's first-order condition for labor input is

$$(1 - \alpha) \frac{Y_t}{L_t} = w_t \tag{6}$$

...where w_t is the wage in period t. Note that we will assume that labor input is always equal to labor supply, and that labor supply remains constant over time and is normalized to 1.

We end this (incomplete) set-up of the model with the equilibrium condition in the market for loanable funds. We assume that aggregate saving, S, is always completely channeled towards investment:

$$S_t = I_t \quad \text{where} \quad I_t = I_{1,t} + pI_{2,t} \tag{7}$$

How aggregate saving is determined, is different in different paradigms - we will come back to this in the next section.

Note that with this set-up, there are (in principle) three possibilities. The first possibility is that $r_t + \delta_1 = p(r_t + \delta_2)$, such that firms are indifferent between both types of capital; this requires that $r_t = \bar{r}$, where

$$\bar{r} = \frac{\delta_1 - p\delta_2}{p - 1} \tag{8}$$

The second possibility is that $r_t + \delta_1 < p(r_t + \delta_2)$, such that firms only use the less durable type of capital; this is the case if $r_t > \bar{r}$. The third possibility is the opposite: if $r_t + \delta_1 > p(r_t + \delta_2)$, firms only use the more durable type of capital; this happens if $r_t < \bar{r}$.

These three possibilities imply three possible types of steady states (which may or may not exist, as will become clear in sections 4 and 5):[3]

[2] For the sake of simplicity, we assume that investment can be negative. If investment *cannot* be negative, the right-hand sides of the first inequalities in (4) and (5) do not simply depend on r_t, but on all future interest rates as well. The main results of the paper would not change, however.

[3] See the appendix for the derivations.

1. An *intermediate* steady state, where firms use both types of capital. In an intermediate steady state, denoted with a superscript $I*$, the capital stock and aggregate saving are given by

$$K_t^{I*} = K_{1,t}^{I*} + K_{2,t}^{I*} = \left(\frac{p-1}{p}\frac{\alpha}{\delta_1 - \delta_2}\right)^{\frac{1}{1-\alpha}} A_t \tag{9}$$

$$S_t^{I*} = (\delta_1 + g)K_{1,t}^{I*} + p(\delta_2 + g)K_{2,t}^{I*} \tag{10}$$

An intermediate steady state requires that $K_{1,t}^{I*} > 0$, $K_{2,t}^{I*} > 0$ and $r^{I*} = \bar{r}$.

2. A *low durability* steady state, where firms only use the less durable type of capital. In a low durability steady state, denoted with a superscript $L*$, the capital stock and aggregate saving are given by

$$K_t^{L*} = K_{1,t}^{L*} = \left(\frac{\alpha}{r^{L*} + \delta_1}\right)^{\frac{1}{1-\alpha}} A_t \quad \text{and} \quad K_{2,t}^{L*} = 0 \tag{11}$$

$$S_t^{L*} = (\delta_1 + g)K_t^{L*} \tag{12}$$

A low durability steady state requires that $r^{L*} \geq \bar{r}$.

3. A *high durability* steady state, where firms only use the more durable type of capital. In a high durability steady state, denoted with a superscript $H*$, the capital stock and aggregate saving are given by

$$K_t^{H*} = K_{2,t}^{H*} = \left(\frac{\alpha}{p(r^{H*} + \delta_2)}\right)^{\frac{1}{1-\alpha}} A_t \quad \text{and} \quad K_{1,t}^{L*} = 0 \tag{13}$$

$$S_t^{H*} = p(\delta_2 + g)K_t^{H*} \tag{14}$$

A high durability steady state requires that $r^{H*} \leq \bar{r}$.

We will assume that the economy is initially in a *low durability* steady state. The question which we will explore in the next sections is then whether policy makers can push the interest rate below $\bar{r}$, to steer the economy towards a *high durability* steady state. In sections 4 and 5, we will analyze the answers of an old-style neo-classical and a modern macroeconomist. But we first spend a section to compare their different strategies to complete the model with a description of aggregate saving.

3 Two paradigms, and three ways to model aggregate saving

The previous section described the economy's supply side and the equilibrium condition in the market for loanable funds. What is still lacking, is a description of the determinants of aggregate saving: if we know how aggregate saving is determined, we can solve the model and see how the interest rate is determined.

Ideally, we would like to model aggregate saving by collecting microeconomic evidence about the economic agents' consumption and saving behavior, and by then carefully aggregating this evidence, making abstractions and simplifications where appropriate (as we do not want a map of scale 1 to 1). Unfortunately, this strategy, with microfoundations built from the *bottom up* (Hoover, 2006; and De Grauwe, 2009), is not feasible: the available microeconomic evidence is widely insufficient (Hansen and Heckman, 1996; and Browning, Hansen, and Heckman, 1999); and even if we had sufficient microeconomic evidence, any attempt to aggregate this evidence is likely to succumb to the curse of dimensionality, given the amount of heterogeneity and behavioral complexity in the real world. Macroeconomists are therefore forced to make some drastic simplifications. The way how these simplifications are carried out, is different in different macroeconomic paradigms.

Old-style neo-classical macroeconomists argue that the microeconomic foundations of macroeconomic variables are way to complex to model in a reasonable way. They therefore sidestep careful microfoundations and simply postulate some key relations between macroeconomic variables, loosely based on a mix of macroeconomic evidence and common sense. A typical neo-classical modeling strategy of aggregate saving, for instance, is to assume that aggregate saving is a constant fraction of aggregate income - sometimes defended by arguing that the share of investment in GDP looks fairly stable over long time spans in most industrialized countries. This is what Solow did in his growth model (Solow, 1956), and is the strategy which we will follow in the next section.

Modern macroeconomists, however, find this approach *ad hoc*: relations between aggregate variables may not be stable, and may change if economic agents behave strategically in anticipation of or in response to policy changes (Lucas, 1976); furthermore, they argue, as old-style neo-classical economists remain silent about the economic agents' objectives, they cannot carry out a proper welfare analysis. Some microeconomic foundations are therefore indispensible, according to modern macroeconomists. But as it is not possible to build these microfoundations from the *bottom up*, with a tight link to microeconomic evidence, modern macroeconomists build their microfoundations from the *top down*: they think about how much behavioral complexity, heterogeneity and institutional detail a model can swallow without becoming mathematically intractable, and then try to strike a balance between mathematical transparency, intuitive appeal and realism at the micro level. A typical modern macroeconomic approach for questions where the focus is on the supply side (as in this paper) is then to assume that heterogeneity and behavioral details of households are irrelevant for the question at hand, and to assume that the economy is populated by households who live infinitely long and all have the same preferences and budget constraints - which implies that their consumption and saving decisions can be derived by maximizing the utility of an immortal *representative* household. As we will see in section 5, this yields strong implications for the steady state interest rate and steady state aggregate saving. Sometimes, an alternative strategy is therefore followed, where the economy is assumed to be populated by overlapping generations - almost always generations with two-period lives as longer lives quickly make the math unwieldy. We will follow both modeling strategies - with a representative agent, and with overlapping generations - in section 5.

None of these two paradigms can claim to be superior to the other one. The old-style neo-classical analysis hinges upon the assumption that the aggregate saving rate remains constant over time. But it is not certain that this will be the case: if firms switch to a more durable production technique with less replacement investment, the demand for loanable funds may go down, which may lead to a lower aggregate saving rate and more consumption. So the Lucas critique may well be relevant - which calls for a modern macroeconomic model with microfoundations. Unfortunately, such a model has its shortcomings too, as its microfoundations almost inevitably lack a sound empirical foundation. Advocates of the modern macroeconomic paradigm argue that this does not matter: modern macroeconomic models are tested by checking to what extent their microfoundations yield macroeconomic implications that are observed in the real world; and the more dimensions in which this is the case, the more trustworthy are their predictions (Lucas, 1980; and Kydland and Prescott, 1996). But this point of view is severely contested (see, for instance, Hansen and Heckman, 1996; Solow, 2007 and 2008; Colander et al., 2009; and Caballero, 2010).[4] Furthermore, virtually all modern macroeconomic models fail to mimic some basic facts in the financial markets, such as the equity premium, the yield curve, the forward premium, and even the risk-free interest rate[5] - which happens to be the key variable in this paper. Finally, note that as the microfoundations in modern macroeconomic models lack a sound empirical foundation, the lack of microfoundations in the old-style neo-classical analysis is not a sufficient reason to dismiss its assumption that the aggregate

[4]Doubts about the modern macroeconomic research agenda have even lead to hearings in U.S. Congress in September 10, 2009, and July 20, 2010, and the foundation of the Institute for New Economic Thinking (INET).

[5]Gabaix (2012) provides an overview of several puzzles in the macro-finance literature.

saving rate remains constant over time: it may well be that we will be able to derive this assumption once we have more microeconomic evidence of the consumption and saving behavior of economic agents and the mathematical techniques to aggregate it.[6]

Given this state of affairs, the best thing we can do is to embrace methodological pluralism (in the spirit of Samuels, 1998), and to consider both paradigms in turn. We do this in sections 4 and 5, and compare the policy implications of both paradigms in section 6.

4 The old-style neo-classical approach

In this section, we complete the set-up of section 2 in an old-style neo-classical way. Recall that old-style neo-classical macroeconomists simply *postulate* relations between aggregate variables. Let us therefore follow Solow (1956), and assume that aggregate saving is a constant fraction s of aggregate output:

$$S_t = sY_t \quad \text{with } 0 < s < 1 \tag{15}$$

To fix ideas, we also assume that $s < \alpha$, such that the economy is dynamically efficient (which implies that a higher aggregate saving rate leads to more steady state output net of investment).

We first derive the implications of these assumptions for the model's steady states and transitional dynamics. We then use the model to find the answer of an old-style neo-classical economist to the question whether policy makers can steer the economy towards a path with low interest rates in order to stimulate investment in the more durable capital type.

4.1 Steady states

Substituting the old-style neo-classical saving equation (15) in equations (12) and (14) shows how much capital there would be in a low and high durability steady state, if these steady states exist; from equations (11) and (13) follows then the interest rate that would prevail in these steady states:

$$r^{L*} = \left(\frac{\alpha}{s} - 1\right)\delta_1 + \frac{\alpha}{s}g \tag{16}$$

$$r^{H*} = \left(\frac{\alpha}{s} - 1\right)\delta_2 + \frac{\alpha}{s}g \tag{17}$$

[6]Wren-Lewis (2007) raises a similar argument.

We therefore find that for given values of s, α and g, the interest rate that would prevail in a high durability steady state is always lower than in a low durability steady state: $r^{H*} < r^{L*}$. Recall now that an intermediate steady state, a low durability steady state, and a high durability steady state require that, respectively, $r^{I*} = \bar{r}$, $r^{L*} \geq \bar{r}$, and $r^{H*} \leq \bar{r}$. This leads then to the following results:

1. if $r^{L*} > r^{H*} \geq \bar{r}$, only a low durability steady state exists;
2. if $\bar{r} \geq r^{L*} > r^{H*}$, only a high durability steady state exists;
3. if $r^{L*} \geq \bar{r} \geq r^{H*}$, both a low durability and a high durability steady state exist, as well as an intermediate steady state.[7]

Essential for these results is that $r^{H*} < r^{L*}$, so it is worthwhile to investigate why this is the case. According to equations (4) and (5), the interest rate is the extra product of a marginal increase in the value of the capital stock, *minus* the depreciation rate. In a high durability steady state, both terms are lower than in a low durability steady state: the depreciation rate is lower; but the extra product of a marginal increase in the value of the capital stock is also lower, as the constant saving rate supports a higher steady state capital stock if the depreciation rate is lower. *A priori*, it is therefore not clear whether a lower depreciation rate implies a lower or a higher interest rate: in principle, it can go both ways. However, it turns out that the Cobb-Douglas specification for the production function combined with the assumption that the economy is dynamically efficient is sufficient to make sure that the extra product of a marginal increase in the value of the capital stock decreases more than the depreciation rate if the economy moves from a low to a high durability steady state - which then implies that the interest rate also decreases if the economy switches from a low to a high durability steady state.

Clearly, this is not a universal law, and very specific to the model. However, if $r^{H*} < r^{L*}$, multiple equilibria may emerge: a high durability steady state with a low interest rate, which encourages all firms to invest in the more durable capital type; and a low durability steady state with a high interest rate, which encourages all firms to invest in the less durable capital type; which then also implies the existence of an intermediate steady state with an interest rate where firms are indifferent between both capital types. Below, we will explore the policy implications of the possibility of multiple equilibria. But we first describe the model's transitional dynamics.

[7] The appendix presents a proof of the existence of an intermediate steady state in this case.

4.2 Transitional dynamics

The transitional dynamics are presented in figure 1.[8]

If only one steady state exists, the model is very similar to the Solow (1956) model: if the capital stock is below (above) its steady state value, the capital-output ratio grows (declines), gradually converging to its steady state value. The only complication is that at some point firms may switch to a different capital type. Suppose, for instance, that initially the interest rate is above $\bar{r}$ such that firms only use the less durable capital type, but that the economy is on a convergence path towards a high durability steady state. The capital-output ratio will then gradually increase and the interest rate will gradually decline, until the interest rate reaches $\bar{r}$. At this point, aggregate saving will not be used anymore to increase the capital-output ratio, but to change the composition of the aggregate capital stock: as the less durable capital goods is replaced by more durable capital goods (which are also more expensive), the capital-output ratio, and therefore also the interest rate, remains constant. When all the less durable capital goods are replaced by more durable capital goods, aggregate saving is used again to increase the capital-output ratio, and the economy continues its convergence path to the high durability steady state.

If the low and high durability steady states both exist, as well as an intermediate steady state, the initial conditions determine to which steady state the economy converges. If the interest rate is above (below) $\bar{r}$, the economy converges to the low (high) durability steady state. If the interest rate is equal to $\bar{r}$, and the share of the less durable capital type in the aggregate capital stock is higher than in the intermediate steady state, the more durable capital goods are replaced by less durable capital goods, until the whole capital stock is of the less durable type, after which the capital-output ratio will decline until the economy reaches the low durability steady state; and vice versa if the interest rate is $\bar{r}$ and the share of the less durable capital type in the aggregate capital stock is less than in the intermediate steady state.

4.3 Policy implications

Suppose now that the economy is initially in a low durability steady state, with the interest rate above $\bar{r}$. Can policy makers then push the interest rate below $\bar{r}$ and steer the economy towards a high durability steady state?

[8] All transitional dynamics in this paper can be derived in a similar way as for the Solow model, the Ramsey model and the Diamond-Samuelson overlapping generations model (even though the derivations are somewhat more laborious as they have to deal with three regimes, depending on whether the interest rate is above, equal to or below $\bar{r}$).

The old-style neo-classical analysis in this section suggests that this is possible, as long as policy makers can sufficiently increase the aggregate saving rate s: if s sufficiently increases such that r^{L*} falls below $\bar{r}$, the low durability steady state ceases to exist, and the economy converges to a high durability steady state (see equations (16) and (17), and the three results that follow from them). How policy makers can increase the aggregate saving rate s is not explained in the model; but cuts in government consumption or incentives for private saving come to mind as possibilities that policy makers may want to contemplate.

Note that it may not be necessary to increase s permanently. Suppose that $r^{L*} \geq \bar{r} \geq r^{H*}$, such that for the initial level of s a low and a high durability steady state co-exist, together with an intermediate steady state. It is then sufficient to increase s temporarily, until the capital stock is equal to K^{I*} and the share of the more durable type of capital is higher than in the intermediate steady state. From that point onwards, the economy will converge to a high durability steady state, even if s is set back to its initial level. The intuition for this is that with multiple equilibria, a high durability equilibrium is self-enforcing: if all firms use the more durable type of capital, the steady state capital stock that is supported by the aggregate saving rate is so large that the interest rate is below $\bar{r}$ - which makes it efficient for firms to invest in the more durable type of capital.

5 The modern macroeconomic approach

Let us now follow the modern macroeconomic paradigm, and derive aggregate saving from microfoundations. As we discussed in section 3, there are two modeling strategies which we can follow: we can derive aggregate saving by maximizing the utility of an immortal representative household, or we can start from an overlapping generations structure. We consider both strategies in turn.

5.1 A model with an immortal representative household

Let us first assume that the world is populated by a representative household whose utility is a discounted stream of felicity derived from aggregate consumption C. To keep the math tractable, it is common to assume that the felicity function is of the CRRA-class:

$$U_t = \sum_{s=t}^{\infty} \left(\frac{1}{1+\rho} \right)^{s-t} \frac{C_s^{1-\gamma} - 1}{1-\gamma} \quad \text{where } \rho > 0 \text{ and } \gamma > 0 \tag{18}$$

The parameter ρ is the representative household's subjective discount rate and $1/\gamma$ is the elasticity of intertemporal substitution.

The representative household owns all capital and labor in the economy, such that his budget constraint is given by:

$$\begin{aligned}\sum_{s=t}^{\infty}\left(\prod_{s'=t+1}^{s}\frac{1}{1+r_{s'}}\right)C_s &= (K_{1,t}+pK_{2,t})(1+r_t)\\ &\quad+\sum_{s=t}^{\infty}\left(\prod_{s'=t+1}^{s}\frac{1}{1+r_{s'}}\right)(w_s-T_s)\end{aligned} \tag{19}$$

...where w_t is his labor income (which is equal to his wage as his labor supply is equal to one), and T_t denotes the lump sum taxes which he has to pay to the government. Assume that the representative household maximizes his utility (18) subject to his budget constraint (19), while taking as given the initial capital stock, current and future factor prices w_s and r_s, and current and future lump sum taxes T_s. This yields the following Euler equation:

$$C_t^{-\gamma} = \frac{1+r_{t+1}}{1+\rho}C_{t+1}^{-\gamma} \tag{20}$$

Substituting in the budget constraint (19) to eliminate future consumption levels, yields then C_t as a function of the current capital stock and current and future factor prices and tax payments.

Government consumption G and taxes T are exogenous. For the sake of simplicity, we assume that government consumption is a constant fraction σ of aggregate output, and that the government always has a balanced budget:

$$G_t = T_t = \sigma Y_t \quad \text{with } 0 \leq \sigma < 1 \tag{21}$$

The model is then completed with the equilibrium condition in the goods market: $Y_t = C_t + I_t + G_t$.

We now derive the model's steady states and transitional dynamics, and discuss its policy implications.

5.1.1 Steady states

In a steady state, aggregate consumption grows at the rate of technological progress, g. From the Euler equation (20) follows then the steady state interest rate r^*. As r^* does not depend on the type of capital which firms use, we find that

$$r^* = r^{H*} = r^{L*} = r^{I*} = (1+\rho)(1+g)^{\gamma} - 1 \tag{22}$$

...where the parameter values are restricted such that $r^* > g$ (otherwise the infinite sums in the budget constraint (19) are infinitely large).

Recall again that an intermediate steady state, a low durability steady state, and a high durability steady state require that, respectively, $r^{I*} = \bar{r}$, $r^{L*} \geq \bar{r}$, and $r^{H*} \leq \bar{r}$. This leads then to the following results:

1. if $r^* > \bar{r}$, only a low durability steady state exists;
2. if $r^* < \bar{r}$, only a high durability steady state exists;
3. if $r^* = \bar{r}$, the initial conditions determine whether the economy converges to a low durability, a high durability or an intermediate steady state.

Note the difference with the old-style neo-classical analysis. In the old-style neo-classical analysis, the aggregate saving rate is exogenous, and the steady state interest rate is endogenous. In the modern macroeconomic analysis with an immortal representative agent, however, the steady state interest rate is *de facto* exogenous (as it is fully determined by the rate of technological progress and the representative household's preferences), while the aggregate saving rate is endogenous (as the representative household always adjusts his consumption to make sure that the Euler equation is satisfied). This has strong policy implications, which we will discuss when we have described the transitional dynamics of the model.

5.1.2 Transitional dynamics

The transitional dynamics are presented in figure 2.

If $r^* \neq \bar{r}$, the transitional dynamics are similar to the transitional dynamics in the old-style neo-classical model: the capital stock will gradually converge to its steady state value, possibly passing through a period where the capital stock of one type is converted to capital of the other type.

If $r^* = \bar{r}$, the initial conditions determine which steady state the economy will converge to: if the interest rate is initially above $\bar{r}$ such that firms only use the less durable capital type, the capital-output ratio will gradually increase and the interest rate will decline, until the interest rate reaches $\bar{r}$ - at which point the economy will be in its steady state; and vice versa if the interest rate is initially below $\bar{r}$; if the interest rate is initially equal to $\bar{r}$, the economy starts off in a steady state and remains there. So if $r^* = \bar{r}$, the initial conditions determine the composition of the steady state capital stock, and therefore also the steady state level of aggregate saving.

5.1.3 Policy implications

Suppose now that the economy is initially in a low durability steady state. From equation (22) follows then that $r^* = (1+\rho)(1+g)^\gamma > \bar{r}$. As long as policy makers cannot change the rate of technological progress or the preferences of the representative household, it is not possible for them to push the interest rate below $\bar{r}$ and to steer the economy towards a high durability steady state.

The assumption of an immortal representative household has therefore a very strong policy implication: it rules out *per construction* any role for the interest rate as a long run policy variable. Note especially that the share of government consumption and taxes in aggregate output does not affect the steady state which the economy converges to: in the long run, any change in government consumption will be offset by a change in private consumption, and will not affect aggregate saving or the interest rate.

5.2 A model with overlapping generations

Let us now turn to the alternative modeling strategy in the modern macroeconomic paradigm, and assume that the world is populated by overlapping generations. Every generation lives for two periods: in the beginning of every period, a new generation is born, and at the end of every period, the oldest generation dies. The generation born in period t consists of a representative household whose utility depends on the felicity which he derives from his consumption in period t, $C_{t,t}$, *plus* the felicity which he derives from his consumption in period $t+1$, $C_{t,t+1}$, discounted with a subjective discount rate ρ. To allow for closed-form solutions, the felicity function is often assumed to be logarithmic:

$$U_t \;=\; \ln C_{t,t} + \frac{1}{1+\rho}\ln C_{t,t+1} \quad \text{with } \rho > 0 \tag{23}$$

In his first period of life, he supplies one unit of labor, earns labor income w, pays a lump sum tax T to the government, consumes part of his disposable income, and saves the rest to finance his second-period retirement consumption. In his second period of life, he does not earn any labor income, and consumes his savings. His intertemporal budget constraint is therefore given by:

$$C_{t,t} + \frac{1}{1+r_{t+1}}C_{t,t+1} \;=\; w_t - T_t \tag{24}$$

Maximizing his utility (23) subject to his budget constraint (24) gives:

$$C_{t,t} \;=\; (1-s_Y)(w_t - T_t) \tag{25}$$

$$C_{t,t+1} \;=\; s_Y(w_t - T_t)(1+r_{t+1}) \tag{26}$$

...where the saving rate of the young generation, s_Y, is given by:

$$s_Y = \frac{1}{2+\rho} \tag{27}$$

For government consumption G and taxes T, we make a similar assumption as assumption (21) in the model with an immortal representative household, but we now restrict the share of government consumption to be less than the share of labor income (in order to ensure that consumption levels are positive):

$$G_t = T_t = \sigma Y_t \quad \text{with } 0 \leq \sigma < 1-\alpha \tag{28}$$

The equilibrium condition in the goods market is then given by $Y_t = C_t + I_t + G_t$, where $C_t = C_{t,t} + C_{t-1,t}$.

Finally, we derive an expression for aggregate saving. The young generation saves by buying from the old generation the capital stock that has not depreciated by the end of period t and by investing in new capital; equation (25) shows that this saving is equal to $s_Y(w_t - T_t)$. The old generation consumes its capital income as well as the revenues from the sale of the not-depreciated part of its capital stock, and therefore *dissaves* $(1-\delta_1)K_{1,t} + p(1-\delta_2)K_{2,t}$. Aggregate saving in period t is then given by

$$S_t = s_Y(w_t - T_t) - (1-\delta_1)K_{1,t} - p(1-\delta_2)K_{2,t} \tag{29}$$

5.2.1 Steady states

Use the first-order condition (6) and assumption (28) to rewrite the first term at the right-hand-side of equation (29) as a function of Y_t; eliminate Y_t with the production function (1); and substitute in equations (12) and (14). This yields expressions for the capital stock in a low and high durability steady state, if these steady states exist; substituting in equations (11) and (13) yields then the interest rate that would prevail in these steady states:

$$r^{L*} = \frac{\alpha}{1-\alpha-\sigma}\frac{1+g}{s_Y} - \delta_1 \tag{30}$$

$$r^{H*} = \frac{\alpha}{1-\alpha-\sigma}\frac{1+g}{s_Y} - \delta_2 \tag{31}$$

From these two equations follows that $r^{H*} > r^{L*}$, for given values of σ, α and g. It is then straightforward to show that

1. if $r^{H*} > r^{L*} > \bar{r}$, only a low durability steady state exists;
2. if $\bar{r} > r^{H*} > r^{L*}$, only a high durability steady state exists;
3. if $r^{H*} \geq \bar{r} \geq r^{L*}$, only an intermediate steady state exists.[9]

[9]The appendix presents a proof.

This is very different from what we have found before: it is the opposite of what we found in the old-style neo-classical analysis, where $r^{H*} < r^{L*}$; but it is also different from what we found in the modern macroeconomic model with an immortal representative agent, where $r^{H*} = r^{L*}$.

The reason why we now find that $r^{H*} > r^{L*}$ is as follows. The young generation saves a constant fraction of aggregate output, and uses this saving to acquire the aggregate capital stock in the next period. As a result, the steady state ratio of aggregate output to the value of the aggregate capital stock does not depend on the depreciation rate. From the first-order conditions (4) and (5) follows then that the steady state interest rate is higher if the depreciation rate is lower, and vice versa.

And just as in the model with an immortal representative household, the aggregate saving rate endogenously adjusts: recall that the young generation saves a constant fraction of aggregate output, while the old generation *dissaves* its revenues from the sale of the not-depreciated part of the aggregate capital stock; as the steady state ratio of the value of the aggregate capital stock to aggregate output is the same in all steady states, we find that a *lower* depreciation rate causes more *dissaving* by the old generation rate, and therefore a lower aggregate saving rate.

Finally note that, in contrast with the old-style neo-classical analysis, this overlapping generations model does not generate multiple equilibria. We will come back to this when we discuss this model's policy implications. But we first turn to its transitional dynamics.

5.2.2 Transitional dynamics

The transitional dynamics are presented in figure 3.

If the steady state is a low or high durability steady state, the transitional dynamics are similar to the transitional dynamics in the old-style neo-classical model and in the modern macroeconomic model with an immortal representative household: the capital stock will gradually converge to its steady state value, possibly passing through a period where capital of one type is converted to capital of the other type.

If $r^{H*} \geq \bar{r} \geq r^{L*}$, such that the steady state is of the intermediate type, the transitional dynamics are essentially the same as well. Suppose, for instance, that the interest rate is initially above $\bar{r}$, and that all capital is of the less durable type. The capital-output ratio is then less than what it would be in a low durability steady state. As a result, aggregate saving is more than what is necessary to keep the capital-output ratio constant, such that the capital-output ratio gradually increases and the interest rate gradually declines. This goes on until the interest rate reaches $\bar{r}$. At this point, aggregate saving will be used to replace (cheap) capital of the less durable type by (more expensive) capital of the more durable type, until the economy has converged to its intermediate steady state, where aggregate saving is just sufficient to keep the capital-output ratio constant.

5.2.3 Policy implications

Suppose now that the economy is initially in a low durability steady state, with the interest rate above $\bar{r}$. Is it then possible for policy makers to push the interest rate below $\bar{r}$, and to steer the economy towards a high durability steady state?

Equations (30) and (31) suggest that this is possible if policy makers sufficiently decrease σ, the share of government consumption and taxes in aggregate output. Note, however, that it is now not enough to decrease σ such that r^{L*} falls below $\bar{r}$, as in the old-style neo-classical case: policy makers have to decrease σ further, until $r^{H*} < \bar{r}$; otherwise the economy gets stuck at an intermediate steady state.

Note also that, in contrast to the old-style neo-classical analysis with multiple equilibria, government consumption and taxes have to decrease not just temporarily, but permanently.

6 Conclusions

We raised the question whether policy makers can steer the economy towards a path with low interest rates in order to stimulate durable capital formation. We analyzed this question with three different models, belonging to two different paradigms, and got fundamentally different and even contradicting answers.

The reason why we got different answers is that each research strategy models aggregate saving in a different way: a typical old-style neo-classical analysis simply assumes a constant aggregate saving rate; in modern macroeconomic models, at the other hand, aggregate saving endogenously adjusts such that the economy converges to an exogenously determined long run interest rate (in the case of an immortal representative household) or an exogenous long run capital-output ratio (in the case of overlapping generations).

None of these modeling strategies is unequivocally based on empirical evidence, however. They are simply designed to satisfy the demands of a paradigm. And as the old-style neo-classical and the modern macroeconomic paradigm are incommensurable, in the sense of Thomas Kuhn (1961), there is no rational reason (in Kuhn's sense) to reject one paradigm in favor of the other one. So the best we can do, is to embrace methodological pluralism (Samuels, 1998), and to see what we can learn from both of them.

The message which we take away from this exercise is then as follows:

1. Policy makers may be able to steer the economy towards a path with low interest rates if they can permanently increase the aggregate saving rate, for instance by a permanent cut in government consumption. But it may not be straightforward to achieve this: a permanent cut in government consumption, for instance, may be offset by a permanent increase in private consumption.

2. If the aggregate saving rate remains constant over time, switching to the more durable type of capital will in the long run increase the aggregate capital stock (because of less capital depreciation) and drive down the interest rate. As lower interest rates encourage firms to switch to the more durable type of capital, an equilibrium where all firms use the more durable type of capital may be self-enforcing, not requiring any government intervention at all. However, it cannot be ruled out that switching to the more durable type of capital affects the interest rate in the opposite way: the aggregate saving rate may not remain constant, and may actually decrease so much that the interest rate does not decrease, but increases.

Note how much less nuanced our conclusions would have been if we had focused on only one paradigm. If we had limited our analysis to the old-style neo-classical paradigm, we would not have realized that a permanent cut in government consumption may be offset by a permanent increase in private consumption, and may therefore not lead to higher aggregate saving and lower interest rates; and we would not have realized that switching to the more durable type of capital may not decrease, but increase the interest rate. At the other hand, if we had only considered the modern macroeconomic paradigm, we would not have realized that an equilibrium where all firms use the more durable type of capital may be self-enforcing. Both paradigms therefore provide a different perspective and yield different intuitions. So both paradigms should be considered in order to reach policy advice that is as nuanced and unbiased as possible.

At the same time, however, it should be noted that the assumption of an immortal representative household in the modern macroeconomic paradigm is extremely limiting for issues related with sustainability. In many sustainability questions, the interest rate plays a crucial role, as it determines how much private firms care for the future. But in an economy with an immortal representative household, the long run interest rate is fully determined by the subjective discount rate of the representative household and the long run consumption growth rate, both of which are largely beyond the sphere of influence of policy makers. So it is important to realize that the immortal representative household in the modern macroeconomic paradigm rules out *per construction* any role for the interest rate as a policy variable to enhance sustainability.

Appendix: Mathematical derivations

In this appendix, we derive some of the results used in sections 2, 4 and 5.

A.1 Derivations for section 2

In section 2, we presented expressions (9) and (10) for intermediate steady states, expressions (11) and (12) for low durability steady states, and expressions (13) and (14) for high durability steady states. These expressions can be derived as follows.

Start from the first-order conditions (4) and (5). In intermediate and low durability steady states, the first inequality in (4) holds with equality; in high durability steady states, the first inequality in (5) holds with equality. Eliminate Y_t with the production function (1) and reshuffle to find K_t as a function of r_t - this yields expressions (11) and (13). Expression (9) is found by replacing r_t with expression (8) for $\bar{r}$.

Consider now the production function (1). As L_t remains constant over time and A_t grows at rate g, the aggregate capital stock K_t must also grow at rate g in a steady state. From the laws of motion (2) and (3) follows then that in a steady state, $I_{1,t}$ must be equal to $(\delta_1 + g)K_{1,t}$, while $I_{2,t}$ must be equal to $(\delta_2 + g)K_{2,t}$. Substituting in equation (7) yields then expressions (10), (12) and (14).

A.2 Derivations for section 4

We now prove the claim in section 4 that in the old-style neo-classical analysis an intermediate steady state exists if $r^{L*} > \bar{r} > r^{H*}$.

First recall that we concluded in section 2 that an intermediate steady state requires that $K_{1,t}^{I*}$ and $K_{2,t}^{I*}$ are positive, where $K_{1,t}^{I*}$ and $K_{2,t}^{I*}$ follow from equation (10) and the fact that $K_t = K_{1,t} + K_{2,t}$. So an intermediate steady state requires that

$$K_{1,t}^{I*} = \frac{S_t^{I*} - p(\delta_2 + g)K_t^{I*}}{(\delta_1 + g) - p(\delta_2 + g)} > 0 \tag{A.1}$$

$$K_{2,t}^{I*} = \frac{(\delta_1 + g)K_t^{I*} - S_t^{I*}}{(\delta_1 + g) - p(\delta_2 + g)} > 0 \tag{A.2}$$

Replace S_t^{I*} with assumption (15), and then use first-order conditions (4) and (5) and expressions (16) and (17) to rewrite $K_{1,t}^{I*}$ and $K_{2,t}^{I*}$ as functions of $\bar{r}$, r^{L*} and r^{H*}:

$$K_{1,t}^{I*} = \frac{\bar{r} - r^{H*}}{(\delta_1 + g) - p(\delta_2 + g)} p \frac{s}{\alpha} K_t^{I*} > 0 \tag{A.3}$$

$$K_{2,t}^{I*} = \frac{r^{L*} - \bar{r}}{(\delta_1 + g) - p(\delta_2 + g)} \frac{s}{\alpha} K_t^{I*} > 0 \tag{A.4}$$

Now recall that the economy is dynamically efficient, such that $\alpha > s$. From equation (17) follows then that $r^{H*} > g$. If $r^{L*} > \bar{r} > r^{H*}$, we then find that $\bar{r} > g$ as well. Replacing $\bar{r}$ by expression (8) and reshuffling shows then that $(\delta_1 + g) - p(\delta_2 + g) > 0$. We then find that if $r^{L*} > \bar{r} > r^{H*}$, the inequalities (A.3) and (A.5) are satisfied, such that an intermediate steady state exists.

A.3 Derivations for section 5

We now prove the claim in section 5 that in the modern macroeconomic model with overlapping generations an intermediate steady state exists if $r^{H*} \geq \bar{r} \geq r^{L*}$.

Recall from section 2 that an intermediate steady state requires that $K_{1,t}^{I*}$ and $K_{2,t}^{I*}$ are both positive. $K_{1,t}^{I*}$ and $K_{2,t}^{I*}$ can be derived by substituting the saving equation (29) in (10), taking into account that $K_t = K_{1,t} + K_{2,t}$. We then find that an intermediate steady state requires that

$$K_{1,t}^{I*} = \frac{pK_t^{I*} - \frac{s_Y}{1+g}(1-\alpha-\sigma)Y_t^{I*}}{p-1} \tag{A.5}$$

$$K_{2,t}^{I*} = \frac{\frac{s_Y}{1+g}(1-\alpha-\sigma)Y_t^{I*} - K_t^{I*}}{p-1} \tag{A.6}$$

...where we replaced w_t and T_t by expressions (6) and (28) (taking into account that $L_t = 1$). Now use first-order conditions (4) and (5) and expressions (30) and (31) to rewrite $K_{1,t}^{I*}$ and $K_{2,t}^{I*}$ as functions of $\bar{r}$, r^{L*} and r^{H*}:

$$K_{1,t}^{I*} = \frac{p(r^{H*} - \bar{r})}{(p-1)(r^{H*} + \delta_2)} K_t^{I*} > 0 \tag{A.7}$$

$$K_{2,t}^{I*} = \frac{\bar{r} - r^{L*}}{(p-1)(r^{L*} + \delta_1)} K_t^{I*} > 0 \tag{A.8}$$

We then find that if $r^{H*} \geq \bar{r} \geq r^{L*}$, the inequalities (A.7) and (A.8) are satisfied, such that an intermediate steady state exists.

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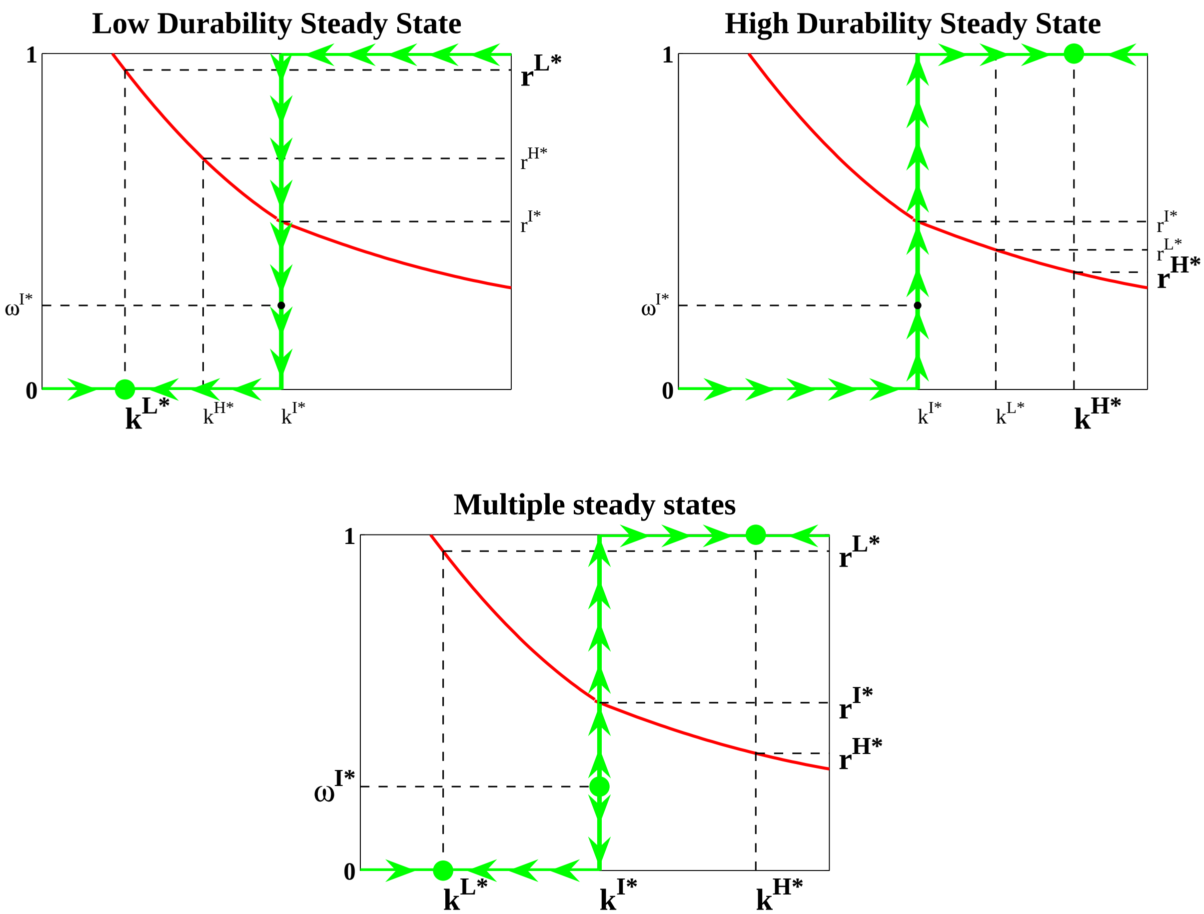
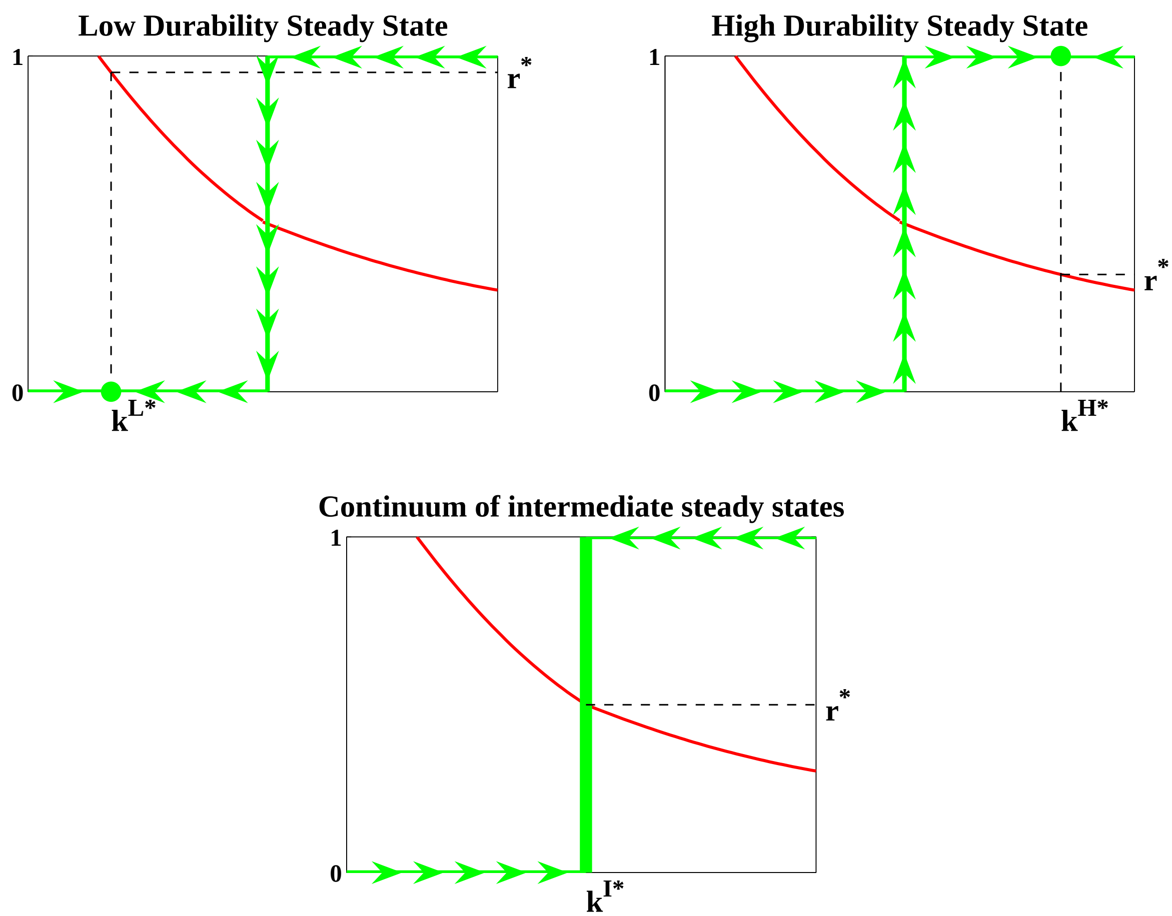
Figure 1:
The old-style neo-classical analysis



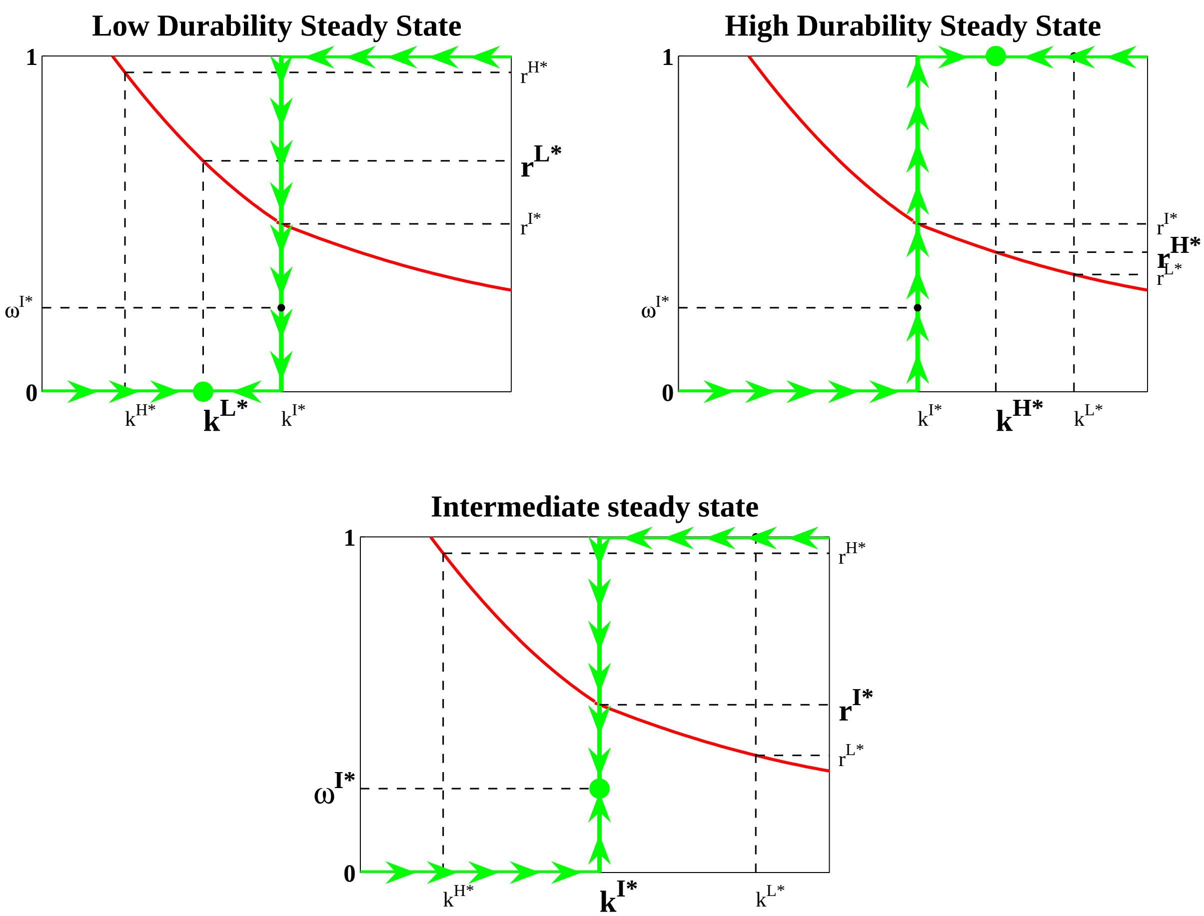
<u>*Note*</u>: The horizontal axis measures the capital-output ratio, $k = K/Y$. The left vertical axis measures the share of the more durable capital type in the aggregate capital stock, $\omega = k_2/k$ The right vertical axis measures the interest rate. The arrows show the transitional dynamics as a function of ω. Steady states are indicated in bold. Recall that $r^{I*} = \bar{r}$.

Figure 2:
The modern macroeconomic analysis
with an immortal representative household



<u>*Note*</u>: The horizontal axis measures the capital-output ratio, $k = K/Y$. The left vertical axis measures the share of the more durable capital type in the aggregate capital stock, $\omega = k_2/k$ The right vertical axis measures the interest rate. The arrows show the transitional dynamics. as a function of ω. Steady states are indicated in bold. Recall that $r^{I*} = \bar{r}$.

Figure 3:
The modern macroeconomic analysis with overlapping generations



Note: The horizontal axis measures the capital-output ratio, $k = K/Y$. The left vertical axis measures the share of the more durable capital type in the aggregate capital stock, $\omega = k_2/k$ The right vertical axis measures the interest rate. The arrows show the transitional dynamics as a function of ω. Steady states are indicated in bold. Recall that $r^{I*} = \bar{r}$.