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Financial Fragility, Sovereign Default Risk and the Limits to Commercial Bank Bail-outs

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Abstract

We analyse the poisonous interaction between bank rescues, financial fragility and sovereign debt discounts. In our model balance sheet constrained financial intermediaries finance both capital expenditure of intermediate goods producers and government deficits. The financial intermediaries face the risk of a (partial) default of the government on its debt obligations. We analyse the impact of a financial crisis, first under full government credibility and then with an endogenous sovereign debt discount. We introduce long term government debt, which gives rise to the possibility of capital losses on bank balance sheets. The negative feedback effects from falling bond prices on the economy are shown to increase with the average duration of the government bonds, as higher interest rates on new debt lead to capital losses on banks' holding of existing long term (government) debt. The associated increase in credit tightness leads to a negative amplification effect, significantly increasing output losses and declines in investment after a financial crisis. We introduce sovereign default risk through the existence of a maximum sustainable level of debt, derived from the maximum level of taxation that is politically feasible. When close to this limit, sovereign discounts emerge reflecting potential defaults on debt, creating a strong link between sovereign default risk and financial fragility emerges. A debt-financed recapitalisation of the financial intermediaries causes bond prices to drop triggering capital losses at the bank under intervention. This mechanism shows the limits to conventional bank bail-outs in countries with fragile public creditworthiness, limits that became very visible during the Great Recession in Southern Europe.

Keywords: 'Financial Intermediation; Macrofinancial Fragility; Fiscal Policy; Sovereign Default Risk'

JEL classification: E44; E62; H30

"The decision to downgrade the Kingdom of Spain's rating reflects the following key factors:

1. The Spanish government intends to borrow up to EUR 100 billion from the European Financial Stability Facility (EFSF) or from its successor, the European Stability Mechanism (ESM), to recapitalise its banking system. This will further increase the country's debt burden, which has risen dramatically since the onset of the financial crisis....."; Moody's downgrades Spanish Sovereign bonds, June 13th 2012

"Today's actions reflect, to various degrees across these banks, two main drivers:

(i) Moody's assessment of the reduced creditworthiness of the Spanish sovereign, which not only affects the government's ability to support the banks, but also weighs on banks' standalone credit profiles....."; Moody's downgrades 28 Spanish banks by one to four notches 6 days later.

1 Introduction

The same day Moody's downgraded 28 Spanish banks, the political leaders of the G20 declared that: *"Against the backdrop of renewed market tensions, Euro area members of the G20 will take all necessary measures to [..........] break the feedbackloop between sovereigns and banks".* And this concern is more than political hype, as Table 1 shows: sovereign debt exposure is in the order of total bank equity. In all periphery countries except Cyprus, sovereign debt exposure exceeds the Tier-1 capital of the banks holding the debt, sometimes by a very substantial margin; in Spain banks' sovereign debt holdings equal 150% of Tier-1 capital, in Italy almost 200% and in Greece almost 250%. These data should make clear that, with sovereign debt exposure so high among especially the Southern European banks, stress in the sovereign debt market will have a very destabilizing impact on the financial system.
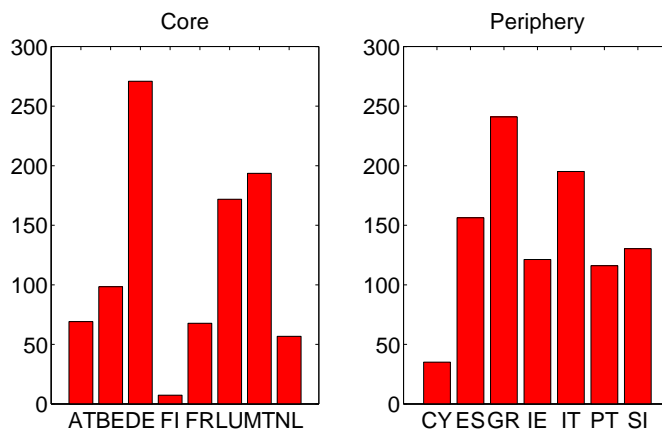
Figure 1: Figure displaying the sovereign debt exposure of banks in the eurozone as a percentage of their total tier 1 capital. Core: AT: Austria, BE: Belgium, DE: Germany, FI: Finland, FR: France, LU: Luxemburg, MT: Malta, NL: Netherlands. Periphery: CY: Cyprus, ES: Spain, GR: Greece, IE: Ireland, IT: Italy, PT: Portugal, SI: Slovenia. *Source:* European Banking Authority (2011) and Keith Kuester, personal communication.

The home bias in those sovereign debt holdings differs accross the eurozone. It averages a

Country	Total billions euros	as a percentage of 2011 GDP
Britain	873	50.0%
Germany	646	25.1
Denmark	613	256.1
Spain	575	53.6
Ireland	571	365.2
France	371	18.6
Belgium	359	97.4
Netherlands	313	52.0
Sweden	162	41.8
Italy	130	8.2
Greece	129	59.9
Austria	94	31.3
Portugal	77	45.0
Poland	68	18.3
Finland	54	28.5
Slovenia	13	35.4
Hungary	10	10.3
Latvia	9	46.2
Luxembourg	9	20.9
Cyprus	5	27.0
Total E.U.	5,086	40.3

Table 1: Total bank bailouts approved (2008 to September 2012). *Source: European Commission, as reported in IHT (2013).*

high 60% in the periphery countries, with Greece as an outlier at almost 80% (EBA (2011)). The homebias is less in the Northern countries, where the ratio averages about 20%, although again with a possibly surprising outlier: in Germany almost 60% of sovereign debt holdings is domestic sovereign debt.

Moreover, bank interventions led to very substantial increases in public debt, thereby completing the circle of dependence between sovereigns and commercial banks. When the financial crisis hit in October 2008, governments across advanced economies had to recapitalise their financial system. The U.S. adopted the T.A.R.P. program of $700 billion that in the end was mostly used to recapitalise various financial institutions. And interventions in Europe were even larger as a proportion of the intervening countries' GDP. Table 1 shows that the size of European interventions in financial institutions ranges from a relatively low 8.2% of GDP for Italy, to the mind boggling number for Ireland, 365.2%. The average for the E.U. is more than 40% of GDP, so the bank interventions have had a major impact on the aggregate stock of outstanding sovereign debt. It should be clear that interventions this large will have an impact on bond prices, and from there potentially feed back on bank's balance sheets through increased risk premia, lower bond prices and further capital losses.

Of course capital losses will only occur if the debt is of a significant maturity. Figure 2 shows

that the average maturity of the sovereign debt portfolios is between 4 and 6 years for the banks in the periphery of the eurozone, and somewhat longer in the core countries of the eurozone (6-8 years).
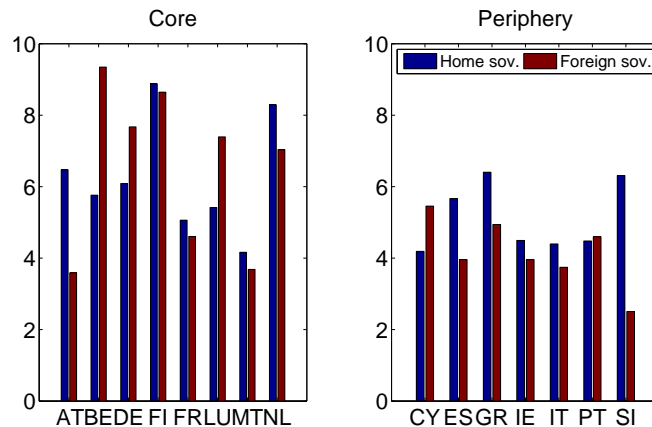

Figure 2: Figure displaying the average maturity of the sovereign debt exposure of the banking sector, where a distinction is made between domestic sovereign debt and foreign sovereign debt. *Source*: European Banking Authority (2011) and Keith Kuester, personal communication.

This implies that sovereign debt problems that cause yields to rise and prices to fall, will inflict substantial capital losses on the financial intermediaries. These capital losses will reduce net worth of the banks, which may well start off a vicious circle as banks increase credit spreads and interest rates, thereby crowding out credit to the private sector, with potentially harmful consequences for investment, tax revenues and long term growth. Lower tax revenues and higher interest rates increase deficits further, leading to further rounds of crowding out and a larger stock of debt, again increasing sovereign discounts. This amplification mechanism and the restrictions it implies on the ability of governments to intervene in and rescue their national commercial banks form the topic of this paper. The key point of this paper is that the negative amplification cycle triggered by the feedback loops back and forth between weak banks and weak governments severely limits the ability of governments to support their financial sector in situations of distress. When sovereign risk premia rise and bond prices go down, governments might not even be *able* to intervene and support their financial sector economy, contrary to what is commonly assumed in contemporaneous macroeconomic models used to analyse financial crises, that the government always has pockets deep enough to finance any possible intervention.

That this concern is not just a theoretical artefact was clearly shown in the case of Spain. Before the financial crisis started, the Spanish economy experienced a housing boom. When the bubble burst, Spanish banks were left with big losses on their real estate portfolios, effectively wiping out their net worth. This, in turn, depressed the flow of credit to the private sector, and contributed to the ensuing recession. At the same time government deficits soared, and within a couple of years Spanish debt rose from 35% of GDP to more than 80% of GDP (Spanish Ministry of Economic Affairs, 2012). The Spanish government decided to restructure the Spanish financial system in May 2012, and committed to debt-financed public recapitalisations in case banks would not be able to raise new capital privately. It was expected that the new flow of credit by a recapitalised

banking system would restart the economy, and thereby improve the long term budget position of the Spanish government, which should be reflected in lower yields on current Spanish government bonds. Instead, yields soared, undermining any effect of the intended bank recapitalisation, and the Spanish government had to apply for external funds from the ESM to be transmitted directly to the banks on June 25th, 2012. Similar problems have emerged accross Southern Europe, from Greece to Ireland.

In order to capture the above described dynamics, we build a dynamic stochastic general equilibrium model that incorporates balance sheet constrained financial intermediaries supplying loans both to firms and to the government (i.e. they hold sovereign debt on their balance sheet). We also explicitly introduce sovereign risk. The methodological innovation is the fact that we combine financial intermediaries in our macromodel that are balance sheet constrained while holding both corporate loans and government bonds subject to sovereign default risk. Through this channel we capture the interconnectedness between the financial system and the fiscal problems of the government.

We introduce long term government bonds in a way similar to Woodford (1998, 2001), through a variable maturity structure of government debt captured by the parameter ρ, through which we can obtain any duration between 1 period bonds ($\rho = 0$) and perpetuals, or 'consols' ($\rho = 1$)[2]. Introducing maturity structure allows us to capture the stylised facts from figure 2. Introducing maturities longer than the one period bonds commonly used in macroeconomic models is important because of the link with capital losses for the already balance sheet constrained commercial banks in the model. The longer the maturity of the government bonds, the higher the capital losses for the financial intermediaries, and the more pronounced the adverse effects on the economy in case of a financial crisis.

Long term government debt is commonly thought of as stabilizing because of lower roll over risk; while that is doubtlessly true, we show there is another side to this whereby long term debt may in fact exacerbate a given financial crisis. We do not try to derive an optimal maturity structure balancing these two conflicting effects on financial fragility; instead, more modestly, we take the maturity structure as given, and show how lengthening the maturity structure strengthens a poisonous link between financial fragility and sovereign weakness in the debt market.

Sovereign default risk is captured by postulating a so called maximum level of taxation that is politically feasible. We then map this maximum level of taxation into a maximum level of debt. We assume that the government follows a core tax policy that guarantees intertemporal solvency in the no default setup and compute the amount of new debt that needs to be issued in order to finance all government obligations, and compare this with the maximum level of debt that is still politically feasible. If the so-called level of no default debt is smaller than the maximum level of debt, the government honors its obligations and does not default; when the no-default level exceeds the maximum level, a (partial) default occurs bringing back the number of government bonds to the maximum number possible.

We first use the model to assess the effect of varying the maturity of the government bonds on the impact of a financial crisis. We then proceed to investigate the effect of a recapitalisation of the financial sector that is announced at the onset of a financial crisis, but implemented 4 quarters later, reflecting realistic delays in implementing rescue programs. This will introduce anticipation effects coming in before the recapitalisation itself due to the forward looking nature of the model. We finally introduce sovereign default risk, and compare the same recapitalisation exercise but now in the presence of endogenous sovereign default risk. In particular, we want to investigate whether and

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[2]We are indebted to a referee for suggesting this approach.

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how financial sector bailout programs affect sovereign default risk, and whether sovereign default risk can feed back to the financial sector, thereby undermining the rescue action and creating an amplification mechanism exacerbating the initial impact of a financial shock.

Relation to the literature

Since the start of the credit crisis, the theoretical literature with general equilibrium models containing financial frictions is growing, although Bernanke et al. (1999) preceded the crisis. Gertler and Karadi (2011) introduce financial intermediaries that are balance sheet constrained by an agency problem between the deposit holders and the bank owners. This gives rise to an endogenous leverage constraint, which becomes more binding when net worth is reduced by for example a negative shock to the quality of the loans. Several others have a similar mechanism, for example Kiyotaki and Moore (1997), Gertler and Kiyotaki (2010), and Kirchner and Van Wijnbergen (2012), who include financial intermediaries holding short term government debt besides loans to the private sector. The current paper extends that model by introducing long term government bonds and sovereign default risk. Woodford (1998, 2001) introduces long term government debt by assuming that the government is financed through a bond with infinite maturity. The stream of payments that the holder receives, though, decreases each period by a factor $\rho \leq 1$, thereby creating a bond with an effective duration that depends on the factor ρ. We follow this approach to modeling maturity. Gertler and Karadi (2012) also extend the number of assets held by financial intermediaries by letting them hold a long term government bond in the form of a perpetuity, a case that is encompassed as a special case in the setup used in this paper (for $\rho = 1$). The introduction of government bonds financed by financial intermediaries creates a second amplification mechanism, whereby increased government bond issuance, in order to stimulate the economy, can crowd out financing of the private sector. These papers, however, do not take into account the possibility of a government default. Acharya et al. (2011) have a setup containing both financial sector bailouts and sovereign default risk, but their analysis occurs within a partial equilibrium setup. Acharya and Steffen (2012), in their empirical research on systemic risk of the Euopean banking sector, find that European banks have been at the center of the two major systemic crises that have faced the financial system since 2007, and specifically that markets have demanded more capital from banks with high sovereign debt exposures to peripheral countries, thereby inidicating that sovereign debt holdings from those countries are a major contributor to systemic risk.

Questions concerning the costs and benefits of long term government debt and the optimal maturity structure are discussed in Cole and Kehoe (2000), Chatterjee and Eyigungor (2012), and Arellano and Ramanarayanan (2012). Obviously, the optimal maturity structure depends on more than simply the risk and size of capital losses that financial intermediaries incur on their sovereign debt portfolio. Designing the optimal maturity structure is not the ambition of this paper however. Staggered price setting and price stickiness go back to Calvo (1983) and Yun (1996). Sovereign default risk is captured in Arellano (2008), which contains an endogenous default mechanism somewhat similar in outcome to our approach. She finds a maximum level of debt conditional on the income shock. Uribe (2006) contains a government that defaults on part of its government debt each period in such a way that the expected future tax receipts and liabilities match afterwards. Schabert and Van Wijnbergen (2011) introduce sovereign default risk by assuming that there exists a maximum level of taxation that is politically feasible and stochastic. Investors do not know this maximum level ex-ante, but they do know the distribution, and can deduce that the risk of a default is increasing with government debt. Davig et alii (2011) do not model an explicit default, but they do assume a maximum level of taxation, or 'fiscal limit', exists.

2 Model description

Financial frictions are introduced in a manner similar to the approach pioneered by Gertler and Karadi (2011), but in our set up banks extend credit to firms but also hold public sector debt on their balance sheet, like in Kirchner and van Wijnbergen (2012). Furthermore we introduce long term government debt and the possibility of a (partial) sovereign default. The government issues debt to financial intermediaries and raises taxes in a lump sum fashion from households to finance its expenditures and meet debt service obligations of its existing debt. The default probability is increasing in the real debt burden in a manner specified more fully below. The other part of the public sector is a central bank that is in charge of monetary policy. It sets the nominal interest rate on the deposits that the households bring to the financial intermediaries. The private sector consists of financial intermediaries and a non-financial sector that includes households and firms. The non financial sector consists of capital producing firms that buy investment goods and used capital, and convert these into capital that is sold to the intermediate goods producers. The intermediate goods producers use the capital as an input, together with labor, to produce intermediate goods for the retail firms. Future gross profits are pledged to the financial intermediaries in order to obtain funding, hence the profits of the intermediate goods producers are zero in equilibrium. Each intermediate goods producer produces a differentiated product. The retail firms repackage and sell the retail products to the final goods producer. Every retail firm is a monopolist and charges a markup for his product. The final goods producers buy these goods and combine them into a single output good. The final good is purchased by the households for consumption, by the capital producers to convert it into capital, and by the government. The household maximizes life-time utility subject to a budget constraint, which contains income from deposits, profits from the firms, both financial and non-financial, and from labor. The income is used for consumption, lump sum taxes and investments in deposits.

2.1 Household

The household sector consists of a continuum of infinitely lived households that exhibit identical preferences and asset endowments. A typical household consists of bankers and workers. Every period, a fraction f of the household members is a banker running a financial intermediary. A fraction $1 - f$ of the household members is a worker. At the end of every period, all members of the household pool their resources, and every member of the household has the same consumption pattern. Hence there is perfect insurance within the household, and the representative agent representation is preserved. Every period, the household earns income from the labor of the working members and the profits of the firms, which are owned by the household. And deposits are paid back with interest. The household uses these funds to buy goods for consumption or deposits them in financial intermediaries (but not the ones owned by the family, in order to prevent self-financing). The household members derive utility from consumption and leisure, with habit formation in consumption, in order to more realistically capture consumption dynamics, as in Christiano et al. (2005). Households maximize expected discounted utility

$$\max_{\{c_{t+s}, h_{t+s}, d_{t+s}\}_{s=0}^{\infty}} E_t \Big[\sum_{s=0}^{\infty} \beta^s \Big(\log \big(c_{t+s} - \upsilon c_{t-1+s} \big) - \Psi \frac{h_{t+s}^{1+\varphi}}{1+\varphi} \Big) \Big], \quad \beta \in (0,1), \quad \upsilon \in [0,1), \quad \varphi \geq 0$$

where c_t is consumption per household, and h_t are hours worked, subject to the following budget constraint:

$$c_t + d_t + \tau_t = w_t h_t + (1 + r_t^d)d_{t-1} + \Pi_t$$

The household optimizes with respect to the budget constraint. Intermediary deposits d_{t-1} are deposited at $t-1$; they receive interest r_t^d and repayment of principal at time t. w_t is the real wage rate, τ_t are the lump sum tax payments the household has to pay to the government, and Π_t are the profits from the firms that are owned by the households. The profits of the financial intermediary are net of the startup capital for new bankers, as will be explained below. The first order conditions are now given by:

$$c_t \quad : \quad \lambda_t = (c_t - vc_{t-1})^{-1} - v\beta E_t\big[(c_{t+1} - vc_t)^{-1}\big] \tag{1}$$

$$h_t \quad : \quad \Psi h_t^\varphi = \lambda_t w_t \tag{2}$$

$$d_t \quad : \quad 1 = \beta E_t\Big[\Lambda_{t,t+1}(1 + r_{t+1}^d)\Big] \tag{3}$$

λ_t is the Lagrange multiplier of the budget constraint. The stochastic discount factor $\Lambda_{t,t+i} = \lambda_{t+i}/\lambda_t$ for $i \geq 0$.

2.2 Financial intermediaries

Financial intermediaries lend funds obtained from households to intermediate goods producers and the government. The banker's balance sheet is given by:

$$p_{j,t} = n_{j,t} + d_{j,t}$$

where $p_{j,t}$ are the assets of bank j in period t, $n_{j,t}$ and $d_{j,t}$ denote the net worth and deposits of bank j. The financial intermediary invests its funds in claims issued by the intermediate goods producer, and in government bonds. Hence the asset side of the bank's balance sheet has the following structure:

$$p_{j,t} \quad = \quad q_t^k s_{j,t}^k + q_t^b s_{j,t}^b$$

where $s_{j,t}^k$ are the number of claims on the intermediate goods producers with price q_t^k, and $s_{j,t}^b$ the number of government bonds acquired by intermediary j, at a price q_t^b. The claims on the producers pay a net real return r_{t+1}^k at the beginning of period $t + 1$. Government bonds pay a net real return r_{t+1}^b at the beginning of period $t + 1$. Financial intermediaries earn those returns on their assets, and pay a return on the deposits. The difference between the two is equal to the increase in the net worth from one period to the next. The balance sheet of intermediary j then evolves as follows:

$$
\begin{aligned}
n_{j,t+1} \quad &= \quad (1 + r_{t+1}^k)q_t^k s_{j,t}^k + (1 + r_{t+1}^b)q_t^b s_{j,t}^b - (1 + r_{t+1}^d)d_{j,t} + n_{j,t+1}^g - \tilde{n}_{j,t+1}^g \\
&= \quad (r_{t+1}^k - r_{t+1}^d)q_t^k s_{j,t}^k + (r_{t+1}^b - r_{t+1}^d)q_t^b s_{j,t}^b + (1 + r_{t+1}^d)n_{j,t} + \tau_{t+1}^n n_{j,t} - \tilde{\tau}_{t+1}^n n_{j,t}
\end{aligned}
$$

where $n_{j,t+1}^g = \tau_{t+1}^n n_{j,t}$ denotes net worth provided by the government to the financial intermediary j (for example a capital injection). $\tilde{n}_{j,t+1}^g = \tilde{\tau}_{t+1}^n n_{j,t}$ denotes the repayment of government support received in previous periods.

The financial intermediary maximizes expected profits. The probability that the banker has to exit the industry next period equals $1 - \theta$, in which case he will bring his net worth $n_{j,t+1}$ to the household. So θ is the probability that he will be allowed to continue operating. The banker discounts these outcomes by the stochastic discount factor $\beta\Lambda_{t,t+1}$, since financial intermediaries are owned by households. The banker's objective is then given by the following recursively defined maximand:

$$V_{j,t} = \max E_t \Big[\beta\Lambda_{t,t+1} \big\{ (1-\theta) n_{j,t+1} + \theta V_{j,t+1} \big\} \Big]$$

where $\Lambda_{t,t+1} = \lambda_{t+1}/\lambda_t$. We conjecture the solution to be of the following form, and later check whether this is the case:

$$V_{j,t} = \nu_t^k q_t^k s_{j,t}^k + \nu_t^b q_t^b s_{j,t}^b + \eta_t n_{j,t} \tag{4}$$

Like in Gertler and Karadi (2011), bankers can divert a fraction λ of the assets at the beginning of the period, and transfer these assets costlessly back to the household. If that happens, the depositors will force the intermediary into bankruptcy, but will only be able to recover the remaining fraction $1 - \lambda$ of the assets of the financial intermediary. Hence lenders will only supply funds if the gains from stealing are lower than the continuation value of the financial intermediary. This gives rise to the following constraint:

$$V_{j,t} \geq \lambda(q_t^k s_{j,t}^k + q_t^b s_{j,t}^b) \quad \Rightarrow \quad \nu_t^k q_t^k s_{j,t}^k + \nu_t^b q_t^b s_{j,t}^b + \eta_t n_{j,t} \geq \lambda(q_t^k s_{j,t}^k + q_t^b s_{j,t}^b) \tag{5}$$

The optimization problem can now be formulated in the following way:

$$\max_{\{q_t^k s_{j,t}^k, q_t^b s_{j,t}^b\}} V_{j,t}, \quad \text{s.t.} \quad V_{j,t} \geq \lambda(q_t^k s_{j,t}^k + q_t^b s_{j,t}^b)$$

From the first order conditions we find that $\nu_t^b = \nu_t^k$. Hence the leverage constraint (5) can be rewritten in the following way:

$$\nu_t^k(q_t^k s_{j,t}^k + q_t^b s_{j,t}^b) + \eta_t n_{j,t} \geq \lambda(q_t^k s_{j,t}^k + q_t^b s_{j,t}^b) \Rightarrow q_t^k s_{j,t}^k + q_t^b s_{j,t}^b \leq \phi_t n_{j,t}, \quad \phi_t = \frac{\eta_t}{\lambda - \nu_t^k} \tag{6}$$

where ϕ_t denotes the ratio of assets to net worth, which can be seen as the leverage constraint of the financial intermediary. The intuition for the leverage constraint is straightforward: a higher shadow value of assets ν_t^k implies a higher value from an additional unit of assets, which raises the continuation value of the financial intermediary, thereby making it less likely that the banker will steal. A higher shadow value of net worth η_t implies a higher expected profit from an additional unit of net worth, while a higher fraction λ implies that the banker can steal a larger fraction of assets, which induces the household to provide less funds to the banker, resulting in a lower leverage ratio everything else equal. Substitution of the conjectured solution into the right hand side of the Bellman equation gives the following expression for the continuation value of the financial intermediary:

$$\begin{aligned} V_{j,t} &= E_t \Big[\Omega_{t+1} n_{j,t+1} \Big], \\ \Omega_{t+1} &= \beta\Lambda_{t,t+1} \big\{ (1-\theta) + \theta[\eta_{t+1} + \nu_{t+1}^k \phi_{t+1}] \big\} \end{aligned}$$

Ω_{t+1} can be thought of as a stochastic discount factor that incorporates the financial friction. Now

substitute the expression for next period's net worth into the expression above:

$$
\begin{aligned}
V_{j,t} &= E_t\Big[\Omega_{t+1}n_{j,t+1}\Big] = E_t\Big[\Omega_{t+1}\big\{(1+r^k_{t+1})q^k_t s^k_{j,t} + (1+r^b_{t+1})q^b_t s^b_{j,t} - (1+r^d_{t+1})d_{j,t} + n^g_{j,t+1} - \tilde{n}^g_{j,t+1}\big\}\Big] \\
&= E_t\Big[\Omega_{t+1}\big\{(r^k_{t+1} - r^d_{t+1})q^k_t s^k_{j,t} + (r^b_{t+1} - r^d_{t+1})q^b_t s^b_{j,t} + (1 + r^d_{t+1} + \tau^n_{t+1} - \tilde{\tau}^n_{t+1})n_{j,t}\big\}\Big]
\end{aligned}
$$
(7)

After combining the conjectured solution with (4), we find the following first order conditions:

$$
\eta_t = E_t\Big[\Omega_{t+1}\big(1 + r^d_{t+1} + \tau^n_{t+1} - \tilde{\tau}^n_{t+1}\big)\Big]
$$
(8)

$$
\nu^k_t = E_t\Big[\Omega_{t+1}\big(r^k_{t+1} - r^d_{t+1}\big)\Big]
$$
(9)

$$
\nu^b_t = \nu^k_t = E_t\Big[\Omega_{t+1}\big(r^b_{t+1} - r^d_{t+1}\big)\Big]
$$
(10)

$$
\Omega_{t+1} = \beta\Lambda_{t,t+1}\big\{(1-\theta) + \theta[\eta_{t+1} + \nu^k_{t+1}\phi_{t+1}]\big\}
$$

2.2.1 Financial sector support

We assume that support provided to an individual intermediary, if provided, will be proportional to the intermediary's net worth in the previous period. Hence individual financial support is given by:

$$
\begin{aligned}
n^g_{j,t} &= \tau^n_t n_{j,t-1}, \quad \zeta \le 0, \quad l \ge 0 \\
\tau^n_t &= \zeta(\xi_{t-l} - \xi)
\end{aligned}
$$

Repayment of the support is parametrized proportionally to the sector's net worth in the period preceding the pay back period:

$$
\tilde{n}^g_{j,t} = \tilde{\tau}^n_t n_{j,t-1}
$$

where $\tilde{\tau}^n_t$ is a scaling factor that is obviously time dependent and incorporates the return paid by the sector to the government over the support funds.

2.2.2 Aggregation of financial variables

Integrating the individual balance sheets of the financial intermediaries yields the aggregate balance sheet of the financial sector:

$$
p_t = n_t + d_t
$$
(11)

Aggregation over the asset side of the balance sheet gives the composition of the aggregated financial system:

$$
p_t = q^k_t s^k_t + q^b_t s^b_t
$$
(12)

ϕ_t does not depend on firm specific factors, so we can aggregate the leverage constraint (6) across financial intermediaries to link sector wide assets and net worth:

$$
p_t = q^k_t s^k_t + q^b_t s^b_t = \phi_t n_t
$$
(13)

The share of assets invested in private loans is given by:

$$\omega_t = q_t^k s_t^k / p_t \tag{14}$$

At the end of the period, only a fraction θ of the current bankers will remain a banker, while the remaining fraction $1 - \theta$ will become a worker. Bankers only pay out dividends at the moment they quit the banking business. If they do not quit, they retain their net worth and carry it into the next period. So the aggregate net worth of the continuing bankers at the end of the period equals:

$$n_{e,t} = \theta \big[(r_t^k - r_t^d) q_{t-1}^k s_{t-1}^k + (r_t^b - r_t^d) q_{t-1}^b s_{t-1}^b + (1 + r_t^d) n_{t-1} \big]$$

Exiting bankers bring their net worth into the household's income. A fraction $1 - \theta$ of the f bankers leaves the financial industry each period, equal to a fraction $(1-\theta)f$ of the household. The same fraction of the household will enter the financial industry next period. We assume that the household will provide a starting net worth to the new bankers proportional to the assets of the old bankers, equal to a fraction $\chi/(1-\theta)$ of the assets of the old bankers, as in Gertler Karadi (2011). Hence the aggregate net worth of the new bankers will be equal to:

$$n_{n,t} = \chi p_{t-1}$$

Then the total net worth at the end of the period, after the lottery has decided which bankers will leave the industry, is:

$$
\begin{aligned}
n_t &= n_{e,t} + n_{n,t} + n_t^g - \tilde{n}_t^g \\
&= \theta \big[(r_t^k - r_t^d) q_{t-1}^k s_{t-1}^k + (r_t^b - r_t^d) q_{t-1}^b s_{t-1}^b + (1 + r_t^d) n_{t-1} \big] \\
&+ \chi p_{t-1} + n_t^g - \tilde{n}_t^g
\end{aligned}
\tag{15}
$$

where n_t^g and $\tilde{n}_t^g$ are aggregate financial sector support, respectively payback of (earlier) financial support. Since individual support is proportional to the individual intermediary's net worth, it is straightforward to get aggregate financial sector support:

$$n_t^g = \zeta(\xi_{t-l} - \xi) n_{t-1} \tag{16}$$

Similarly, we can aggregate financial sector payback:

$$\tilde{n}_t^g = \tilde{\tau}_t^n n_{t-1} \Rightarrow \tilde{\tau}_t^n = \tilde{n}_t^g / n_{t-1} \tag{17}$$

We derive the expression for $\tilde{n}_t^g$ below, in section 2.4.

2.3   Production side

The production side of the economy is modeled in by now standard NeoKeynesian fashion. We have a continuum of intermediate goods producers indexed by $i \in [0,1]$ borrowing from the financial intermediary to purchase the capital necessary for production. With the proceeds from the sale of the output and the sale of the capital after it has been used, the firms pay workers and pay back the loans to the financial intermediary. The capital producers buy the capital that has been used, and transform the used capital, together with the goods purchased from the final goods producers, into new capital. This new capital is sold to the intermediate goods producers, who will use it for production next period. A continuum of retail firms, indexed by $f \in [0,1]$, repackage the products

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bought from the intermediate goods producers to produce a unique differentiated retail product. The retail firms sell their products to a continuum of final goods producers. The products are differentiated, so each individual retail firm has "local" monopoly power, and charges a markup. A randomly selected fraction ψ of all retail firms can not change prices in a given period. The final goods producers convert the inputs from the retail firms into final goods. Due to perfect competition, profits are zero in equilibrium, and the final goods are sold to the households, the government, and the capital producers.

2.3.1 Capital Producers

At the end of period t, when the intermediate goods firms have produced, the capital producers buy the remaining stock of capital $(1-\delta)\xi_t k_{t-1}$ from the intermediate goods producers at a price q_t^k. They combine this capital with goods bought from the final goods producers (investment i_t) to produce next period's beginning of period capital stock k_t. This capital is being sold to the intermediate goods producers at a price q_t^k. We assume that the capital producers face convex adjustment costs when transforming the final goods bought into capital goods, set up such that changing the level of gross investment is costly. Hence we get:

$$k_t = (1-\delta)\xi_t k_{t-1} + (1-\Psi(\iota_t))i_t, \quad \Psi(x) = \frac{\gamma}{2}(x-1)^2, \quad \iota_t = i_t/i_{t-1} \tag{18}$$

ξ_t represents a capital quality shock which will be discussed later. Profits are passed on to the households, who own the capital producers. The profit at the end of period t equals:

$$\Pi_t^c = q_t^k k_t - q_t^k(1-\delta)\xi_t k_{t-1} - i_t$$

The capital producers maximize expected current and (discounted) future profits (where we substitute in (18)):

$$\max_{\{i_{t+i}\}_{i=0}^\infty} E_t\Big[\sum_{i=0}^\infty \beta^i \Lambda_{t,t+i}\Big(q_{t+i}^k\big(1-\Psi(\iota_{t+i})\big)i_{t+i} - i_{t+i}\Big)\Big]$$

Differentiation with respect to investment gives the first order condition for the capital producers:

$$q_t^k\big(1-\Psi(\iota_t)\big) - 1 - q_t^k \iota_t \Psi'(\iota_t) + \beta E_t \Lambda_{t,t+1} q_{t+1}^k \iota_{t+1}^2 \Psi'(\iota_{t+1}) = 0$$

which gives the following expression for the price of capital:

$$\frac{1}{q_t^k} = 1 - \frac{\gamma}{2}\Big(\frac{i_t}{i_{t-1}}-1\Big)^2 - \frac{\gamma i_t}{i_{t-1}}\Big(\frac{i_t}{i_{t-1}}-1\Big) + \beta E_t\Big[\Lambda_{t,t+1}\frac{q_{t+1}^k}{q_t^k}\Big(\frac{i_{t+1}}{i_t}\Big)^2\gamma\Big(\frac{i_{t+1}}{i_t}-1\Big)\Big] \tag{19}$$

2.3.2 Intermediate Goods Producers

There exists a continuum of intermediate goods producers indexed by $i \in [0,1]$. Each of these firms produce a differentiated good. The intermediate goods producers obtain funds from the financial intermediaries by pledging next period's profits, so banks are exposed to downside risk. We assume that there are no financial frictions between the financial intermediaries and the intermediate goods

producers. The securities issued by the intermediate goods producers are best considered as state-contingent debt, like in Gertler and Kiyotaki (2010)[3]. The price of the claims is equal to q_{t-1}^k, and pay a state-contingent net real return r_t^k in period t. The production technology of the intermediate goods producers is given by:

$$y_{i,t} = a_t(\xi_t k_{i,t-1})^\alpha h_{i,t}^{1-\alpha}, \quad \log(a_t) = \rho_a \log(a_{t-1}) + \varepsilon_{a,t}, \quad \log(\xi_t) = \rho_\xi \log(\xi_{t-1}) + \varepsilon_{\xi,t}$$

Both (log of) total factor productivity a_t and capital quality ξ_t are AR(1) processes driven by random shocks $\varepsilon_{a,t} \sim N(0, \sigma_a^2)$ and $\varepsilon_{\xi,t} \sim N(0, \sigma_\xi^2)$. The intermediate goods producer acquires the capital at the end of period $t-1$ and uses it for production in period t. The capital quality shock ξ_t occurs at the beginning of period t, so $\xi_t k_{i,t-1}$ is the effective stock of capital used for production in period t. A negative realization of $\varepsilon_{\xi,t}$ lowers the quality of the capital stock, hence the return on the claims of the financial intermediary will be lower. The intermediate goods producer hires labor $h_{i,t}$ for a wage rate w_t after the shock ξ_t has been realized. When the firm has produced in period t, the output is sold for price m_t to the retail firms. m_t is the relative price of the intermediate goods with respect to the price level of the final goods, i.e. $m_t = P_t^m/P_t$. A fraction δ of the capital stock $\xi_t k_{i,t-1}$ is used up in the production process. The intermediate goods producing firms sell back what is left of the effective capital stock to the capital producers for the end-of-period price of q_t^k and thus receives $q_t^k(1-\delta)\xi_t k_{i,t-1}$. Hence period t profits are:

$$\Pi_{i,t} = m_t a_t(\xi_t k_{i,t-1})^\alpha h_{i,t}^{1-\alpha} + q_t^k(1-\delta)\xi_t k_{i,t-1} - (1+r_t^k)q_{t-1}^k k_{i,t-1} - w_t h_{i,t}$$

The intermediate goods producing firms maximize expected current and future profits using the household's stochastic discount factor $\beta^s \Lambda_{t,t+s}$ (since they are owned by the households), taking all prices as given:

$$\max_{\{k_{t+s}, h_{t+s}\}_{s=0}^\infty} E_t\Big[\sum_{s=0}^\infty \beta^s \Lambda_{t,t+s} \Pi_{i,t+s}\Big]$$

The first order conditions belonging to this problem are given by:

$$k_{i,t} \quad : \quad E_t\Big[\beta\Lambda_{t,t+1}q_t^k(1+r_{t+1}^k)\Big] = E_t\Big[\beta\Lambda_{t,t+1}\big(\alpha m_{t+1}y_{i,t+1}/k_{i,t} + q_{t+1}^k(1-\delta)\xi_{t+1}\big)\Big]$$

$$h_{i,t} \quad : \quad w_t = (1-\alpha)m_t y_{i,t}/h_{i,t}$$

In equilibrium profits will be zero. By substituting the first order condition for the wage rate into the zero-profit condition $\Pi_{i,t} = 0$, we can find an expression for the ex-post return on capital:

$$r_t^k = (q_{t-1}^k)^{-1}\big(\alpha m_t y_{i,t}/k_{i,t-1} + q_t^k(1-\delta)\xi_t\big) - 1$$

Now we rewrite the first order condition for labor and the expression for the ex-post return on capital to find the factor demands:

$$k_{i,t-1} = \alpha m_t y_{i,t}/[q_{t-1}^k(1+r_t^k) - q_t^k(1-\delta)\xi_t]$$

$$h_{i,t} = (1-\alpha)m_t y_{i,t}/w_t$$

---

[3]It is therefore better to think of the claims of financial intermediaries as equity. Occhino and Pescatori (2010) explicitly model loans to producers with a fixed face value, where the goods producers have the possibility of defaulting on the loans. We refrain from explicitly modelling this default possibility, and note the equity characteristics of debt in the real world when firms are short of funds to pay off the loans.

By substituting the factor demands into the production technology function, we get for the relative intermediate output price m_t :

$$m_t \;\; = \;\; \alpha^{-\alpha}(1-\alpha)^{\alpha-1}a_t^{-1}\left(w_t^{1-\alpha}\left[q_{t-1}^k(1+r_t^k)\xi_t^{-1}-q_t^k(1-\delta)\right]^{\alpha}\right) \tag{20}$$

2.3.3 Retail firms

Retail firms purchase goods $(y_{i,t})$ from the intermediate goods producing firms for a nominal price P_t^m, and convert these into retail goods $(y_{f,t})$. These goods are sold for a nominal price $P_{f,t}$ to the final goods producer. It takes one intermediate goods unit to produce one retail good $(y_{i,t}=y_{f,t})$. All the retail firms produce a differentiated retail good by assumption, therefore operate in a monopolistically competitive market, and charge a markup over the input price earning them profits $(P_{f,t}-P_t^m)y_{f,t}$.

Each period, only a fraction $1-\psi$ of retail firms is allowed to reset their price, while the ψ remaining firms are not allowed to do so, like in Calvo (1983) and Yun (1996). The firms allowed to adjust prices are randomly selected each period. Once selected, they set prices so as to maximize expected current and future profits, using the stochastic discount factor $\beta^s\Lambda_{t,t+s}(P_t/P_{t+s})$:

$$\max_{P_{f,t}} E_t\left[\sum_{s=0}^{\infty}(\beta\psi)^s\Lambda_{t,t+s}(P_t/P_{t+s})[P_{f,t}-P_{t+s}^m]\right]y_{f,t+s}$$

where $y_{f,t}=(P_{f,t}/P_t)^{-\epsilon}y_t$ is the demand function. y_t is the output of the final goods producing firms, and P_t the general price level. Symmetry implies that all firms allowed to reset their prices choose the same new price (P_t^*). Differentiation with respect to $P_{f,t}$ and using symmetry then yields:

$$\frac{P_t^*}{P_t} \;\; = \;\; \frac{\epsilon}{\epsilon-1}\frac{E_t\sum_{s=0}^{\infty}(\beta\psi)^s\lambda_{t+s}P_{t+s}^{\epsilon}P_t^{-\epsilon}m_{t+s}y_{t+s}}{E_t\sum_{s=0}^{\infty}(\beta\psi)^s\lambda_{t+s}P_{t+s}^{\epsilon-1}P_t^{1-\epsilon}y_{t+s}}$$

Defining the relative price of the firms that are allowed to reset their prices as $\pi_t^*=P_t^*/P_t$ and gross inflation as $\pi_t=P_t/P_{t-1}$, we can rewrite this as:

$$\pi_t^* \;\; = \;\; \frac{\epsilon}{\epsilon-1}\frac{\Xi_{1,t}}{\Xi_{2,t}} \tag{21}$$

$$\Xi_{1,t} \;\; = \;\; \lambda_t m_t y_t + \beta\psi E_t\pi_{t+1}^{\epsilon}\Xi_{1,t+1} \tag{22}$$

$$\Xi_{2,t} \;\; = \;\; \lambda_t y_t + \beta\psi E_t\pi_{t+1}^{\epsilon-1}\Xi_{2,t+1} \tag{23}$$

The aggregate price level equals:

$$P_t^{1-\epsilon} = (1-\psi)(P_t^*)^{1-\epsilon}+\psi P_{t-1}^{1-\epsilon}$$

Dividing by $P_t^{1-\epsilon}$ yields the following law of motion:

$$(1-\psi)(\pi_t^*)^{1-\epsilon}+\psi\pi_t^{\epsilon-1}=1 \tag{24}$$

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2.3.4 Final Goods Producers

Final goods firms purchase intermediate goods which have been repackaged by the retail firms in order to produce the final good. The technology that is applied in producing the final good is given by $y_t^{(\epsilon-1)/\epsilon} = \int_0^1 y_{f,t}^{(\epsilon-1)/\epsilon} df$, where $y_{f,t}$ is the output of the retail firm indexed by f. ϵ is the elasticity of substitution between the intermediate goods purchased from the different retail firms. The final goods firms face perfect competition, and therefore take prices as given. Thus they maximize profits by choosing $y_{f,t}$ such that $P_t y_t - \int_0^1 P_{f,t} y_{f,t} df$ is maximized. Taking the first order conditions with respect to $y_{f,t}$, gives the demand function of the final goods producers for the retail goods. Substitution of the demand function into the technology constraint gives the relation between the price level of the final goods and the price level of the individual retail firms:

$$
\begin{aligned}
y_{f,t} &= (P_{f,t}/P_t)^{-\epsilon} y_t \\
P_t^{1-\epsilon} &= \int_0^1 P_{f,t}^{1-\epsilon} df
\end{aligned}
$$

2.3.5 Aggregation

Substituting $y_{f,t} = y_{i,t} = y_t (P_{f,t}/P_t)^{-\epsilon}$ into the factor demands derived earlier yields:

$$
h_{i,t} = (1-\alpha)m_t y_{f,t}/w_t, \quad k_{i,t-1} = \alpha m_t y_{f,t}/[q_{t-1}^k(1+r_t^k) - q_t^k(1-\delta)\xi_t]
$$

Aggregation over all firms i gives us aggregate labor and capital:

$$
h_t = (1-\alpha)m_t y_t \mathcal{D}_t/w_t, \quad k_{t-1} = \alpha m_t y_t \mathcal{D}_t/[q_{t-1}^k(1+r_t^k) - q_t^k(1-\delta)\xi_t]
$$

where $\mathcal{D}_t = \int_0^1 (P_{f,t}/P_t)^{-\epsilon} df$ denotes the price dispersion. It is given by the following recursive form:

$$
\mathcal{D}_t = (1-\psi)(\pi_t^*)^{-\epsilon} + \psi \pi_t^\epsilon \mathcal{D}_{t-1} \tag{25}
$$

The aggregate capital-labor ratio is equal to the individual capital-labor ratio:

$$
k_{t-1}/h_t \;=\; \alpha(1-\alpha)^{-1}w_t/[q_{t-1}^k(1+r_t^k) - q_t^k(1-\delta)\xi_t] = k_{i,t-1}/h_{i,t} \tag{26}
$$

Now calculate aggregate supply by aggregating $y_{i,t} = a_t(\xi_t k_{i,t-1})^\alpha h_{i,t}^{1-\alpha}$:

$$
\int_0^1 a_t(\xi_t k_{i,t-1})^\alpha h_{i,t}^{1-\alpha} di \;=\; a_t \xi_t^\alpha \left(\frac{k_{t-1}}{h_t}\right)^\alpha \int_0^1 h_{i,t} di = a_t(\xi_t k_{t-1})^\alpha h_t^{1-\alpha}
$$

while aggregation over $y_{i,t}$ gives:

$$
\int_0^1 y_{i,t} df = y_t \int_0^1 (P_{f,t}/P_t)^{-\epsilon} df = y_t \mathcal{D}_t
$$

So we get the following relation for aggregate supply y_t:

$$
y_t \mathcal{D}_t = a_t(\xi_t k_{t-1})^\alpha h_t^{1-\alpha} \tag{27}
$$

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2.4 Government

The government issues b_t bonds in period t, and raises $q_t^b b_t$ with q_t^b the market price of bonds. We parametrize the maturity structure of government debt like Woodford (1998, 2001): maturity is introduced by assuming that one government bond issued in period t pays out r_c units (in real terms) in period $t+1$, ρr_c real units in period $t+2$, $\rho^2 r_c$ real units in period $t+3$ etc. This is equivalent to a payout of r_c plus ρ times one newly issued bond in period $t+1$, with a value of $r_c + \rho q_{t+1}^b$. So ρ pins down the maturity of government debt, and government debt service in period t is $(r_c + \rho q_t^b) b_{t-1}$. The duration[4] of public debt is $1/(1-\beta\rho)$. The government also raises revenue by levying lump sum taxes on the households. Government purchases are constant: $g_t = G$. Furthermore the government may provide assistance to the financial intermediaries by injecting capital n_t^g, and it receives repayment of support administered previously ($\tilde{n}_t^g$). So the budget constraint becomes:

$$
\begin{aligned}
q_t^b b_t + \tau_t + \tilde{n}_t^g &= g_t + n_t^g + (r_c + \rho q_t^b) b_{t-1} = g_t + n_t^g + \left(\frac{r_c + \rho q_t^b}{q_{t-1}^b}\right) q_{t-1}^b b_{t-1} \implies \\
q_t^b b_t + \tau_t + \tilde{n}_t^g &= g_t + n_t^g + \left(1 + r_t^b\right) q_{t-1}^b b_{t-1}
\end{aligned} \tag{28}
$$

r_t^b is the real return on government bonds:

$$
1 + r_t^b = \frac{r_c + \rho q_t^b}{q_{t-1}^b} \tag{29}
$$

The tax rule of the government is given by a rule which Bohn (1998) has shown secures sustainability:

$$
\tau_t = \bar{\tau} + \kappa_b(b_{t-1} - \bar{b}) + \kappa_n n_t^g, \quad \kappa_b \in (0,1], \quad \kappa_n \in [0,1] \tag{30}
$$

$\bar{b}$ is the steady state level of debt. κ_n controls the way government transfers to the financial sector are financed. If $\kappa_n = 0$, support is financed by new debt. $\kappa_n = 1$ implies that the additional spending is completely financed by increasing lump sum taxes. We parametrize government support as follows:

$$
\begin{aligned}
n_t^g &= \tau_t^n n_{t-1}, \quad \zeta \le 0, \quad l \ge 0 \\
\tau_t^n &= \zeta(\xi_{t-l} - \xi)
\end{aligned} \tag{31}
$$

Thus the government provides funds to the financial sector if $\zeta < 0$ (a negative shock to the quality of capital). Depending on the value of l, the government can provide support instantaneously ($l = 0$), or with a lag ($l > 0$). Furthermore, ϑ indicates the extent to which the government needs to be repaid:

$$
\tilde{n}_t^g = \vartheta n_{t-e}^g, \quad \vartheta \ge 0, \quad e \ge 1 \tag{32}
$$

$\vartheta = 0$ means the support is a gift from the government. In case $\vartheta = 1$, the government aid is a zero interest loan, while a $\vartheta > 1$ implies that the financial intermediaries have to pay interest over the support received earlier[5]. The parameter e denotes the amount of time after which the government aid has to be paid back.

---

[4]duration is defined as: $\frac{\sum_{j=1}^{\infty} j \beta^j (\rho^{j-1} r_c)}{\sum_{j=1}^{\infty} \beta^j (\rho^{j-1} r_c)}$

[5]The case where $\vartheta > 1$ happened in the Netherlands, where financial intermediaries received government aid with a penalty rate of 50 percent.

2.5 Central Bank

The Central Bank sets the nominal interest rate on deposits r_t^n according to a standard Taylor rule, in order to minimize output and inflation deviations:

$$r_t^n = (1 - \rho_r)\big(r^n + \kappa_\pi(\pi_t - \bar\pi) + \kappa_y \log(y_t/y_{t-1})\big) + \rho_r r_{t-1}^n + \varepsilon_{r,t} \tag{33}$$

where $\varepsilon_{r,t} \sim N(0, \sigma_r^2)$, and $\kappa_\pi > 0$ and $\kappa_y > 0$. The parameter $\bar\pi$ is the target inflation rate. We choose $\kappa_\pi > 1, \kappa_y > 0$, (leaning against the wind). The real interest rate on deposits then equals:

$$1 + r_t^d = (1 + r_{t-1}^n)/\pi_t \tag{34}$$

2.6 Market clearing

Equilibrium requires that the number of claims owned by the financial intermediaries (s_t^k) must be equal to aggregate capital (k_t), while the number of government bonds owned by the financial sector (s_t^b) must be equal to the number of bonds issued by the government (b_t):

$$s_t^k = k_t \tag{35}$$
$$s_t^b = b_t \tag{36}$$

Goods market clearing requires that the aggregate demand equals aggregate supply:

$$c_t + i_t + g_t = y_t \tag{37}$$

3 Extension with government default

3.1 The default process

The government follows a simple tax rule consistent with the long term sustainability requirements outlined in Bohn (1998):

$$\tau_{t+1} = \bar\tau + \kappa_b(b_t - \bar b) + \kappa_n n_{t+1}^g$$

But we furthermore assume that there is a maximum level beyond which taxes are not politically sustainable anymore, like in Schabert and Van Wijnbergen (2011), or like the 'fiscal limit' in Davig et alii (2011). This translates in a maximum level of debt that can be sustained, and introduces the possibility of (partial) default if shocks trigger higher levels of debt than the maximum level of debt implied by the 'fiscal limit'. Such a maximum level of taxes should probably be stochastic, as in Schabert and Van Wijnbergen (2011), and possibly time-varying; but for simplicity and without much loss of generality we follow Davig et alii (2011) in assuming it to be fixed and known to be equal to τ^{max}. The maximum level of debt b_t^{max} implied by this fiscal limit equals:

$$b_t^{max} = \bar b + \frac{\tau^{max} - \bar\tau}{\kappa_b} \tag{38}$$

As shown in section 2.4, the Woodford (1998, 2001) maturity structure leads to a government liability before financing of the primary deficit equal to $L_t^q = (r_c + \rho q_t^b)b_{t-1}$. Thus in the absence of government default, the end of period debt would become:

$$q_t^b \tilde b_t = L_t^q + g_t + n_t^g - \tau_t - \tilde n_t^g$$

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where $\tilde{b}_t$ denotes the level of government debt if the government would not default on its obligations. The constraint can be rewritten in the following way:

$$q_t^b \tilde{b}_t + \tau_t + \tilde{n}_t^g = g_t + n_t^g + \left(r_c + \rho q_t^b\right) b_{t-1} \tag{39}$$

As long as the level of debt that the government needs to issue in order not to default $(\tilde{b}_t)$ is smaller than the maximum level of debt b_t^{max}, the actual government debt b_t will be equal to the no default level of government debt $\tilde{b}_t$. But when $\tilde{b}_t > b_t^{max}$, we assume that the government defaults on a large enough fraction of its outstanding debt and debtservice obligations to bring the actual end-of-period debt down to b_t^{max}:

$$b_t = \begin{cases} \tilde{b}_t & \text{if } \tilde{b}_t \leq b_t^{max}; \\ b_t^{max} & \text{if } \tilde{b}_t > b_t^{max}. \end{cases} \tag{40}$$

We assume that the government achieves this outcome through orderly renegotiation with its creditors. Since creditors have rational expectations, they know that they will not be able to get more from the government than what the government can raise through the maximum level of taxes τ^{max}. The debt/tax limit may nevertheless become binding because of random shocks to the system affecting debt both directly and through the government's tax and expenditure rules. We abstain from free-rider problems among creditors and assume all creditors participate in the renegotiation. The government partially reneges on its debtservice obligations, applying the same discount Δ_t as used in the debt restructuring.[6] We can see the debt structure b_t as the blue solid line in figure 7 (see appendix). It is informative to write the debt level structure (40) in the following way:

$$b_t = \min\left(\tilde{b}_t, b_t^{max}\right) = b_t^{max} - \max\left(b_t^{max} - \tilde{b}_t, 0\right) \tag{41}$$

The second term is like the payout of a put option at maturity with underlying process $\tilde{b}_t$ and strike price b_t^{max}; see Claessens and Van Wijnbergen (1993) who apply such a model in their evaluation of the Mexican Brady plan debt restructuring using option pricing methodology for ex ante valuation. As an ex post default function, however, (41) is not differentiable at $b_t = b_t^{max}$ which creates severe problems in solving the model. We therefore approximate the ex post default rule by its ex ante option pricing based valuation formula. Since option prices close to maturity are a good approximation to option-payouts at maturity, our option based formula approximates (41) closely but without differentiability problems. This is described in detail in the appendix.

3.2 Default and the government budget constraint

Of course this default process has implications for the government budget constraint. When $\tilde{b}_t \leq b_t^{max}$, $\Delta_t = 0$ but when $\tilde{b}_t > b_t^{max}$, the old government debt b_{t-1} is restructured by converting the old bonds into new bonds against a pro-rata discount high enough to avoid overshooting the maximum debt level. Both coupon payments and all existing bonds are reduced by a factor $(1 - \Delta_t)$. This implies that the government saves an amount equal to $\Delta_t(r_c + \rho q_t^b)b_{t-1}$ on new debt issuance. The flow budget constraint of the government in period t thus becomes:

$$q_t^b b_t = L_t^q + g_t + n_t^g - \tau_t - \tilde{n}_t^g - \Delta_t(r_c + \rho q_t^b)b_{t-1}$$

---

[6]For analytical simplicity and without loss of generality we choose an equal discount percentage for both current debtservice and for the existing stock of debt.

with $\Delta_t = 0$ when there is no default. This can be rearranged to get:

$$q_t^b b_t + \tau_t + \tilde{n}_t^g \;\; = \;\; g_t + n_t^g + (1 - \Delta_t)(r_c + \rho q_t^b) b_{t-1} \tag{42}$$

3.3 Financial intermediaries and default

The returns to the financial intermediaries holding the sovereign debt are of course affected. Call the default inclusive return r_t^{b*} , which is given by:

$$1 + r_t^{b*} = (1 - \Delta_t)\left(1 + r_t^b\right) = (1 - \Delta_t)\left(\frac{r_c + \rho q_t^b}{q_{t-1}^b}\right) \tag{43}$$

This definition of the return on sovereign debt captures the complete direct impact of the possible default on financial intermediaries holding the debt, so we do not need to change anything after the introduction of posible sovereign defaults other than replacing r_t^b by r_t^{b*}. This in turn implies that the expression for the leverage constraint remains unchanged, as well as the expressions for the shadow value of private loans and net worth, and that the equations for the shadow value of government bonds and the law of motion of net worth of financial intermediaries only need minor adjustment (replacement of r_t^b by r_t^{b*}):

$$\nu_t^b \;\; = \;\; E_t\left[\Omega_{t+1}\left(r_{t+1}^{b*} - r_{t+1}^d\right)\right] \tag{44}$$

$$n_t \;\; = \;\; \theta\left[\left(r_t^k - r_t^d\right)q_{t-1}^k s_{t-1}^k + \left(r_t^{b*} - r_t^d\right)q_{t-1}^b s_{t-1}^b + (1 + r_t^d)n_{t-1}\right] + \chi p_{t-1} + n_t^g - \tilde{n}_t^g \tag{45}$$

4 Calibration

4.1 No default version

We calibrate the model on a quarterly frequency. The parameter values can be found in table 2. Most of the parameters are common in the literature on DSGE models, or frequently used in models containing financial frictions. We mostly follow the calibration of Gertler and Karadi (2011). This is the case for the subjective discount factor β, the degree of habit formation υ, the Frisch elasticity of labor supply φ^{-1}, the elasticity of substitution among intermediate goods ϵ, the price rigidity parameter ψ, the effective capital share α, and the investment adjustment cost parameter γ. The calibration of the financial variables is also taken from Gertler and Karadi (2011). The steady state leverage ratio is set to 4, while the credit spread Γ is set to 100 basis points annually (which amounts to $\Gamma = 0.0025$), which coincides with the pre-2007 spreads in US financial data between BAA corporate and government bonds. The parameter θ is calibrated by taking the average survival period ($\Theta = 1/(1 - \theta)$) to be equal to 36 quarters, or $\theta = 0.9722$. The parameters in the Taylor rule are set to conventional values.

The feedback from government debt on taxes is set to a value such that both the model with and without default are stable. We calibrate the steady state ratios of investment and government spending over GDP, $\bar{i}/\bar{y}$ and $\bar{g}/\bar{y}$ to 20 percent (a reasonable value for OECD countries), by calibrating the depreciation parameter δ. The fixed payment in real terms that the holder of government bonds receives each period is set to 0.04. Different values have been tried but do not significantly affect the results. In our base case we set the maturity parameter at $\rho = 0.96$, equivalent to an

Parameter	Value	Definition
Households		
β	0.990	Discount rate
υ	0.815	Degree of habit formation
Ψ	3.409	Relative utility weight of labor
φ	0.276	Inverse Frisch elasticity of labor supply
Financial Intermediaries		
λ	0.3863	Fraction of assets that can be diverted
χ	0.0021	Proportional transfer to entering bankers
θ	0.9722	Survival rate of the bankers
Γ	0.0025	Steady state credit spread $E[r^k - r^d]$
Intermediate good firms		
ϵ	4.176	Elasticity of substitution
ψ	0.779	Calvo probability of keeping prices fixed
α	0.330	Effective capital share
Capital good firms		
γ	1.728	Investment adjustment cost parameter
δ	0.0494	Depreciation rate
Autoregressive components		
ρ_z	0.95	Autoregressive component of productivity
ρ_ξ	0.66	Autoregressive component of capital quality
ρ_r	0	Interest rate smoothing parameter
Policy		
r_c	0.04	Real payment to government bondholder
ρ	0.96	Parameter government debt duration (5 yrs)
κ_b	0.050	Tax feedback parameter from government debt
κ_π	1.500	Inflation feedback on nominal interest rate
κ_y	0.125	Output feedback on nominal interest rate
Default parameters		
$\bar{\Delta}$	0.005	Steady state share of default indicator
Shocks		
σ_z	0.010	Standard deviation productivity shock
σ_ξ	0.050	Standard deviation capital quality shock
σ_r	0.0025	Standard deviation interest rate surprise shock
Option parameters		
r	-0.0273	Compounded risk-free interest rate
σ	0.5031	Standard deviation underlying process
T	0.1107	Time to maturity

Table 2: Table with model parameters.

average duration of 5 years. The steady state bond price, though, changes as we vary ρ in the first experiment, the parameter governing the average duration of government debt. In order for different maturities to be comparable, we must make sure that the fraction of government debt on the balance sheet of the financial intermediaries does not change. Hence we calibrate on the outstanding government liabilities as a fraction of GDP $\bar{q}_b \bar{b} / \bar{y}$ instead of $\bar{b} / \bar{y}$, and set it equal to 2.4, implying an annual debt-to-GDP ratio of 60 percent. Even though government financing by financial intermediaries accounts for only a small part in the US, most financial friction models have been calibrated on US data. We follow the conventional calibration. The purpose of this paper is not to perform quantitative exercises specifically focused on the debt-distressed European periphery economies; our aim is more generally to highlight the relevant mechanisms, leaving calibration on European data for the future. We perform robustness checks to make sure that the mechanism does not depend on a specific set of parameters. We assume more aggressive monetary policy in the face of a credit crisis, and hence set $\rho_r = 0$ in times of crisis. We think this captures the way central banks reacted when the financial crisis erupted. A credit crisis is represented by a negative shock to capital quality ξ_t of 5 percent on impact, with an autocorrelation coefficient $\rho_\xi = 0.66$, as in Gertler and Karadi (2011).

4.2 Default calibration

In this section we describe the calibration when sovereign default risk is introduced. The calibration of the real economy is not affected by the introduction of sovereign default risk. For the financial sector, the steady state bond price $\bar{q}_b$ changes, and hence $\bar{b}$. We calibrate the maximum level of government liabilities $\bar{q}_b \bar{b}_{max} / \bar{y}$ to be at 90% of annual steady state GDP. Different values could have been chosen, but the main point of the paper is to show the mechanisms that interplay when debt levels get close to the maximum level of debt. The steady state fraction of government liabilities $\bar{q}_b \bar{b} / \bar{y}$ on the balance sheet of financial intermediaries does not change, since it is still calibrated to hit the 60% annual steady state output target. The reason for this freedom is the fact that we have a new variable, the level of debt in case of no government debt $\tilde{b}_t$, and the steady state tax rate that we can adjust in order to still be able to hit our original targets. The steady state default probability is set at a rather conservative estimate of $\bar{\Delta} = 0.005$, which implies an annual default probability of 2%, which is small given the observed bond spreads in the European periphery.

There are 2 ways in which we calibrate the model, which are in detail described in the appendix. We apply calibration strategy 1 when far away from the debt limit. In this case we always apply $\bar{\Delta} = 0.005$. When the steady state level of government debt comes close to the maximum level of debt, though, this strategy can not be followed anymore, due to numerical problems. This is the case for the last exercise, in which we investigate a delayed recapitalisation when the steady state government liabilities are at 80% of annual steady state GDP. First we find the parameters of the (option) approximation when $\bar{q}_b \bar{b} / \bar{y}$ is at 75% of annual steady state GDP. Calibration strategy 1 cannot go further than this when the maximum level of government liabilities is at 90% of annual steady state GDP. We therefore change to calibration strategy 2 which can still be applied, and calibrate the model at 80% of annual steady state GDP. This changes the steady state default probability to $\bar{\Delta} = 0.0068$.

5 Results

As a prelude to the main results about the interaction between financial fragility, sovereign debt and commercial bank rescues, we first investigate the effect of a financial crisis, initiated through a credit (capital quality) shock in the model, to set the stage for the interventions that form the main topic of this paper. A special point of interest highlighted in this subsection is the crucial importance of the maturity of sovereign debt. We then analyse the consequences of a classic commercial bank recapitalization by the government, where we realistically assume that the recaps are implemented 4 quarters after the announcement. Then we show that introducing sovereign default risk exacerbates the poisonously negative interactions between sovereign debt holdings of commercial banks and debt financed rescue attempts.

5.1 The macroeconomic impact of a financial crisis

As a prelude to the analysis of the interaction between sovereign debt and bank rescues, we first set the benchmark, a financial crisis without government intervention to support banks. Like Gertler and Karadi (2011), we model a financial crisis as a decrease in the capital quality ξ_t. A deterioration of capital quality induces losses at the financial intermediaries on the loans provided to the intermediate goods producers. As a consequence, the net worth of the financial intermediaries decreases, and hence the intermediaries become more balance sheet constrained, and the credit spread increases by almost 120 basis points in the short maturity case (which corresponds to an average maturity of 2 quarters). The lower quality of capital also decreases the expected productivity from the capital that is purchased with the loans, and because of the lower net worth, the financial intermediaries have to cut back on lending, which further reduces the price of capital. Because of classic Dornbusch style overshooting, the price collapse leads to a higher expected return on the loans once the shock has hit. In response, financial intermediaries sell government bonds, thereby pushing down the current bond price, which in turn inflicts further capital losses on the financial intermediaries. This process continues until the forward looking expected return on government bonds has increased sufficiently to make the intermediaries willingly hold the outstanding stock of bonds. In the process, financial net worth falls further; the intermediaries' balance sheet further deteriorates, raising borrowing costs and so on. What we see (in the plots collected in figure 3) is a pro-cyclical amplification cycle whereby investment and eventually capital drop by more than 10 percent.

A third balance sheet effect that plays a role was highlighted by Kirchner and Van Wijnbergen (2012) and is due to crowding out by government debt. Government spending g_t is fixed; then government borrowing is primarily affected by the bond price q_t^b. Since the increase in the (expected) interest rates on private loans pushes up the (expected) interest rates on government debt, the bond price drops. Besides inflicting capital losses on the financial intermediaries, it also increases the number of bonds the government needs to issue for a given amount of expenditures. Issuing more bonds implies that there are more creditors to which the government has to pay the fixed payment r_c, implying higher borrowing needs in future periods. Hence a smaller proportion of the intermediaries' balance sheets is available for financing the capital purchases of the intermediate goods producers. This effect is amplified because the size of the balance sheet is reduced as well, due to a tightening of the balance sheet constraint.

The lower capital quality reduces the productivity of the capital. Wages fall as a consequence, as do profits from the production sector and the financial intermediaries, and (except in the period of
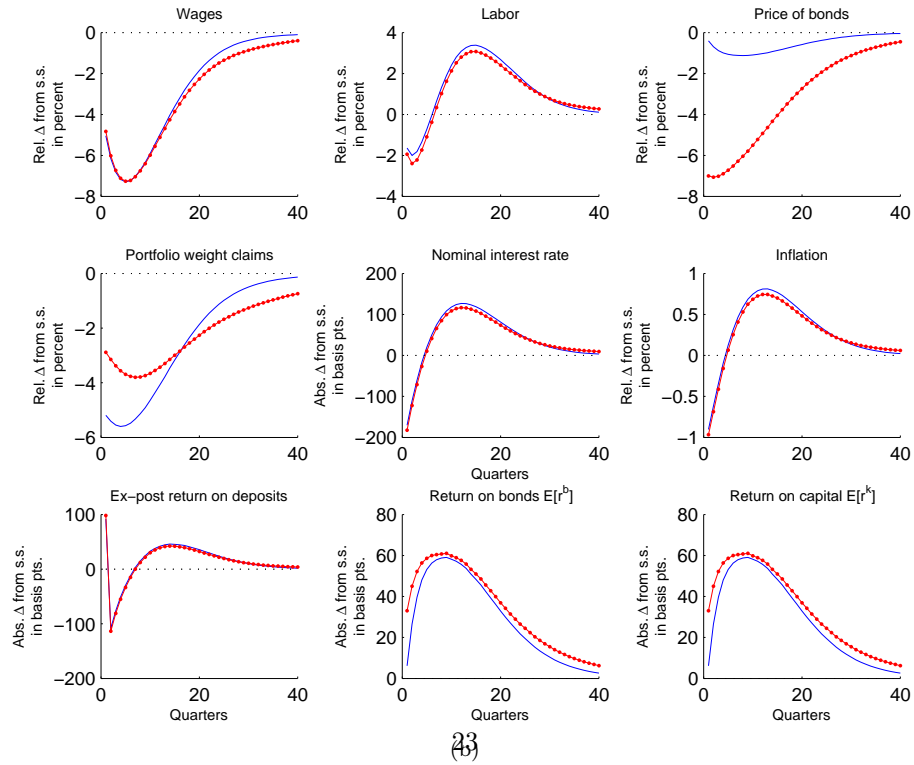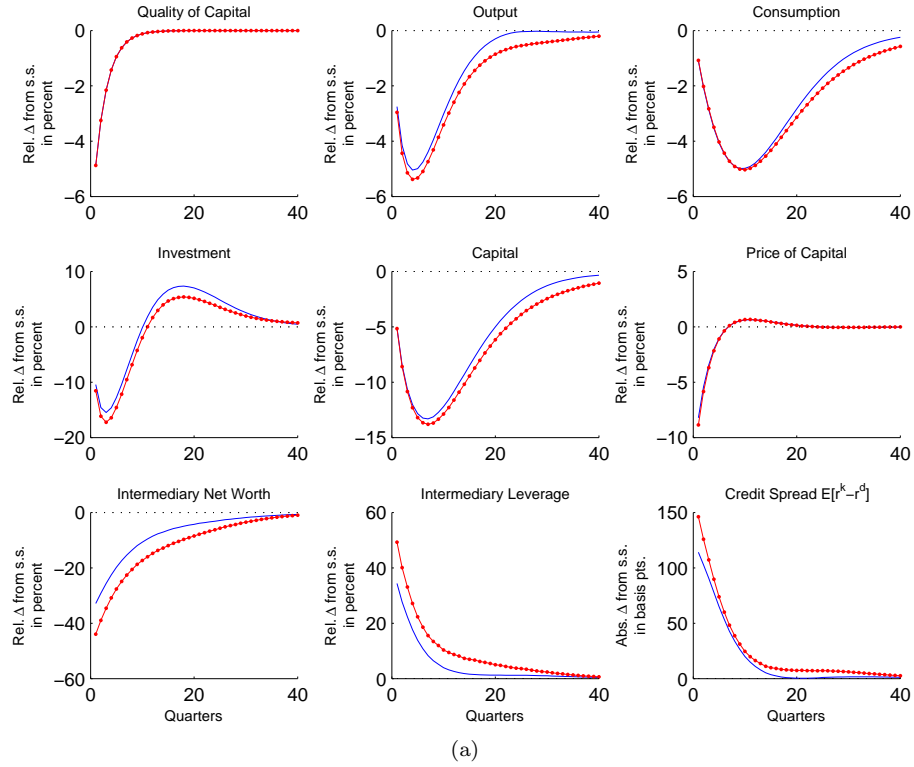
Figure 3: Plot of the impulse response functions for the model without default for $\rho = 0.5$ (blue) and $\rho = 0.96$ (red,-.-.). The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state, and no additional government policy is implemented.

the shock) the real return on deposits. Since we assume a very aggressive monetary policy response (the smoothing parameter is set to zero after the crisis hits), nominal rates initially fall to such an extent that the Zero Lower Bound is actually violated for one period. For more realistic values of the smoothing parameter the ZLB is not violated. Obbviously the household's budget constraint is tightened, and consumption falls: We see that output and consumption are reduced by more than 4 percent. The model also reproduces the Reinhart Rogoff (2009) finding of long recessions after financial crises: after 40 quarters the economy has still not recovered completely from the initial shock.

Figures 3a and 3b compare the case where $\rho = 0.5$, which coincides with an average duration of 2 quarters (solid blue line), with the case $\rho = 0.96$, which corresponds to an average duration of 5 years (dashed red line). The impact of a longer maturity structure is very clear: the longer the maturity of the government bonds, the larger the drop in the bond price to even 7%, thereby increasing the capital losses faced by financial intermediaries, and a further deterioration of the net worth of the financial intermediaries. Hence they become more balance sheet constrained, as can be seen from an increase in the credit spread to almost 150 basis points. The tightening of the balance sheet induces the financial intermediaries to charge higher expected returns on both bonds and private loans, thereby reducing demand for new private loans, which in turn decreases the price of capital and investment. The drop in asset prices further erodes net worth, further raising interest rates etc. The effects on the real economy are clear: output drops further, and a decrease in investment pushes down the capital stock. The reduction in capital further reduces the demand for labor, and lowers wages and profits from the firms in the economy, which tightens the household budget constraint, further pushing down consumption.

Figure 4 elaborates on the importance of the maturity of sovereign debt. The figure shows the average deviation from the steady state in percentages (except the credit spread, which is in absolute deviation in basis points) for selected variables as a function of the average duration of government bonds. The average deviation is taken over the first 40 periods after the capital shock hits the economy in period 1. By assumption, the government does not engage in additional policy. 1 quarter duration corresponds to $\rho = 0$. $\rho = 1$ corresponds to perpetual bonds (consols), which we list in the figure as an average duration of 100 quarters. The maturity parameter ρ is transformed into an average duration in quarters q through the formula $q = 1/(1 - \beta\rho)$.

Figure 4 shows that a longer maturity of sovereign bonds substantially increases the impact of a crisis. The mechanism should be clear: longer maturities introduce larger capital losses on the stock of bonds on bank balance sheets, giving the negative feedback between bank holdings of sovereign debt and financial fragility another perverse twist. The relation is strikingly nonlinear. The average deviation of all variables drops substantially when increasing the average duration from 1 quarter to approximately 30 quarters after which the pace of the decline is lower. The average output decline is higher by about a half over the range considered; the capital stock decline increases by about a quarter. This is triggered by an almost doubling of the decline in net worth and a substantially higher increase in credit spread as maturities lengthen. Clearly, the maturity structure of government bonds is an important channel for further capital losses on bank balance sheets during financial crises, with substantial adverse macroeconomic consequences..
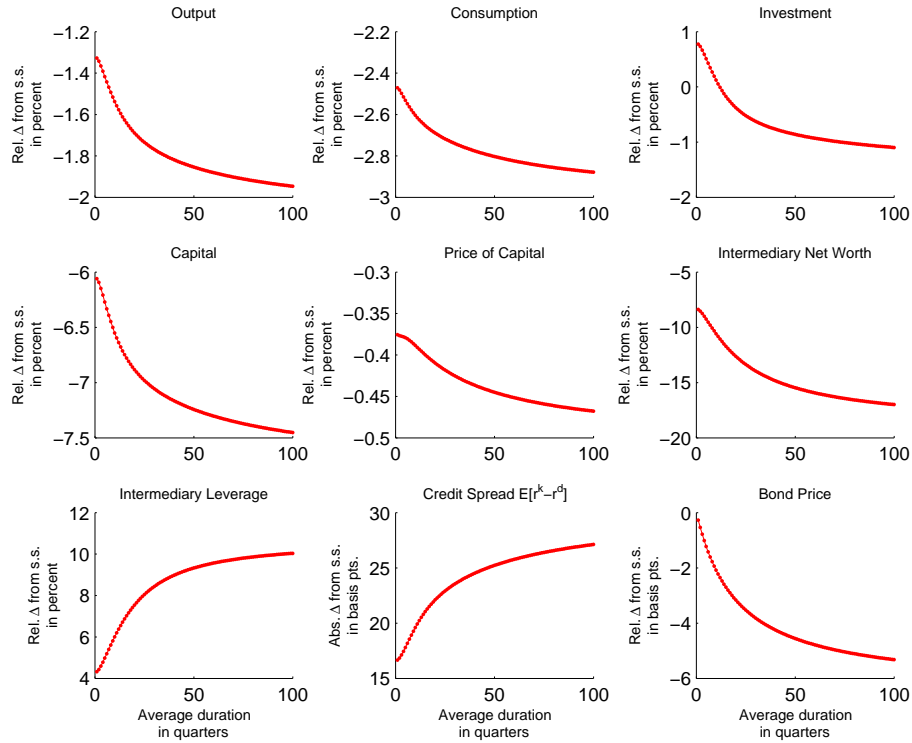
Figure 4: Figure showing the average deviation from the steady state in percentages (except the credit spread, which is in absolute deviation in basis points) for selected variables vs. average duration of government bonds. The average is taken over the first 40 periods, where the financial crisis is initiated when a negative capital quality shock of 5 percent relative to the steady state hits the economy in period 1. The government does not engage in additional policy. 1 quarter duration corresponds to $\rho = 0$, while an average duration of 100 quarters corresponds practically speaking to the case of perpetual bonds, or 'consols', $\rho = 1$. The maturity parameter ρ is transformed into an average duration in quarters q through the formula $q = 1/(1 - \beta\rho)$.

5.2 Financial crisis and government response: the effect of a (delayed) recapitalisation

Since low capitalisation is at the root of the credit tightening and macroeconomic fall out after a financial crisis, a recapitalisation is a logical response and has been the mainstay of government intervention on both sides of the Atlantic. We evaluate the impact of a recapitalisation of the financial system of 1.25% of annual steady state GDP by an issuance of new government bonds. We assume that the recap is announced immediately after the crisis hits, but that implementing it takes time: in our policy experiment 4 quarters. The measure is designed to improve financing conditions: the recap alleviates the balance sheet constraint. On the other hand, the recap requires the government to issue more bonds, which will cause a drop in the bondprice due to an increased supply, which in turn affects the balance sheet of the banks which hold bonds in their asset portfolio. Figure 5 compares the case of no additional policy with the delayed recapitalisation. The maturity parameter is set at $\rho = 0.96$, which corresponds to an average maturity of sovereign debt of about 5 years, about the average for the Eurozone area.

We see that the *announcement* of the recapitalisation already has an impact effect prior to the actual policy implementation: the financial intermediaries anticipate the recapitalisation, which raises the continuation value of the financial intermediaries and so immediately relaxes the balance sheet constraint. Due to the relaxation of the balance sheet constraint, the supply of loans (demand for private sector liabilities) and demand for government bonds increases, driving up the price of these assets, thereby reducing the losses on the private loans triggered by the initial capital shock, as well as leading to lower losses on government bonds. Net worth increases with respect to the no policy case, further reducing the losses of the financial intermediaries caused by the initial deterioration of the quality of capital, as well as reducing the credit spread by 30 basis points. When the actual recapitalisation is implemented, a further drop in the credit spread of approximately 70 basis points occurs. Investment increases by almost 5 percentage points, due to a higher demand for capital.

But at the moment the recap is implemented, debt issue goes up and the bond price drops again, although marginally so. In fact it is more accurate to say that the gradual increase in bond prices is reversed, an actual discrete price drop is prevented by intertemporal arbitrage, since everything has been priced in at time zero, when the intervention was announced. We see that the positive effects from a capital injection dominate the effects of an increased debt issuance, as the asset prices remain above the no policy case. The positive effects on the real economy are clear as well: higher investment pushes up the capital stock. A higher capital stock increases the marginal product of labor, inducing firms to offer higher wages, which causes households to increase their labor supply. This leads to higher consumption with respect to the no policy case, also because of increased profits/reduced losses from the financial intermediaries. The recap has unambiguously positive macroeconomic effects.
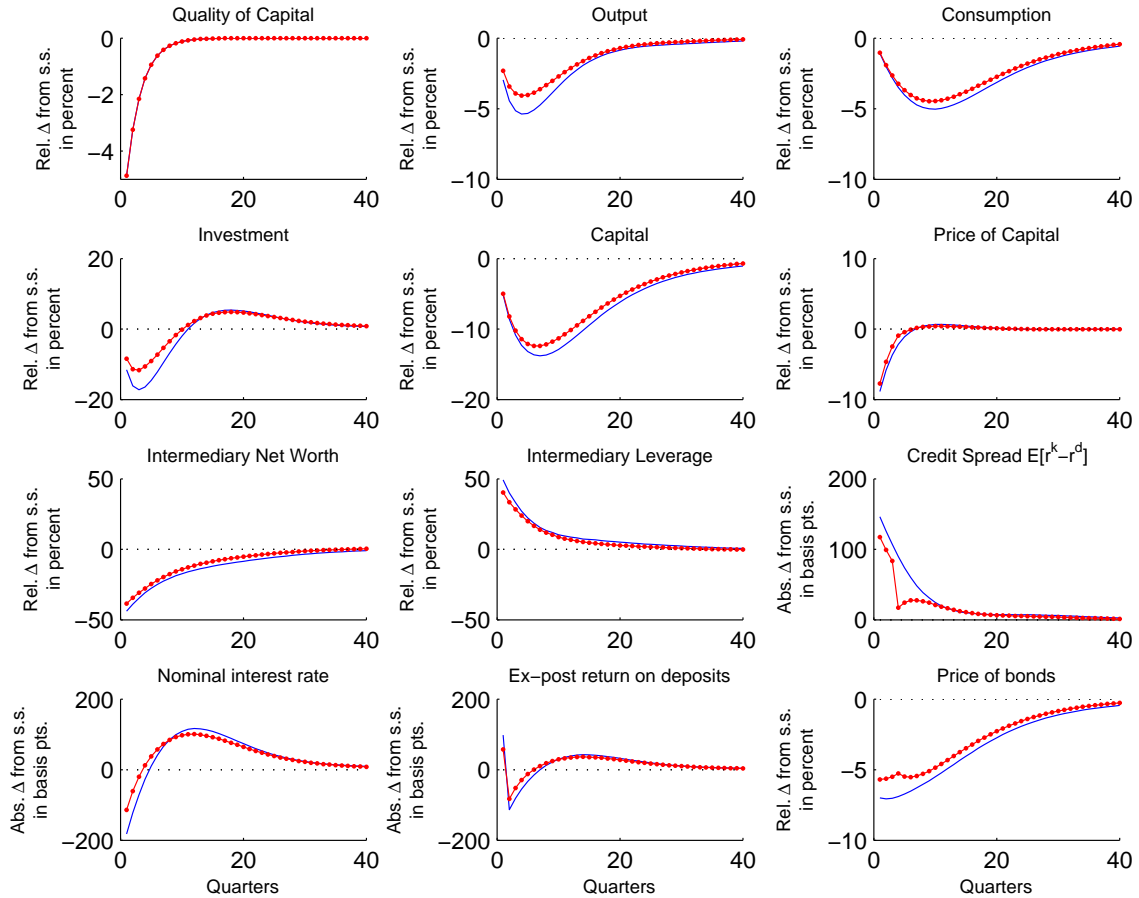
Figure 5: Plot of the impulse response functions for the model without default with no additional government policy (blue) and a delayed recapitalisation of the financial sector (red,-.-.) occurring 4 quarters after the financial crisis hits and equal to 1.25% of annual steady state GDP. The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.

5.3   Sovereign risk and the limits of government intervention

The analysis of the commercial bank recap has sofar ignored the issue of sovereign debt discounts even though a standard government induced recap involves substantial issues of new public debt. The Spanish and Irish experience with large scale commercial bank rescues through the issue of new public debt and/or public guarantees of private claims has indicated, however, that such debt financed bank rescues do undermine capital market confidence in the public sector and its debt. This, in turn, may jeopardise the impact of the initial bank rescue action when the same banks hold sovereign debt on their balance sheet. To analyse this conflict, we introduce sovereign risk into the model. The specifics of the calibration strategies can be found in the appendix. We once again analyse the impact of a recap equal to 1.25% of annual steady state output, and announced at the onset of the financial crisis but implemented 4 quarters later. Figure 6 compares the macroeconomic responses set against the same experiment but without sovereign risk.

The graphs clearly show that the government default possibility *does* have a significant effect on the economy. The announcement of a recapitalisation immediately causes the bond price to drop by an additional 5% with respect to the no default case. Investors subsequently anticipate the extra bond issuance necessary for financing the recapitalisation, and the accompanying increase in the sovereign risk discount. This causes additional losses at the financial intermediaries and a further tightening of the balance sheet constraints of those intermediaries. As a consequence, the cost of capital (required return) shoots up as the price drops on impact. The effects on the real economy are clear: investment goes down further, pushing down the capital stock, wages go down, and thereby the supply of labor. Consumption eventually decreases more than in the no default case. Clearly sovereign default risk affects the economy substantially as increased sovereign risk premia are translated into lower bond prices that further inflict capital losses on the financial intermediaries.

The drop in the bond price clearly shows the limits of government intervention: the bonds are issued to increase bank capitalisation, but their very issuance causes prices to fall, triggering subsequent losses on commercial banks' sovereign asset portfolios that substantially offset the impact of the recap they are financing. The sovereign risk impact of the recap makes it more difficult to implement effective support measures. This is reminiscent of the failed recapitalisation of the Spanish banking sector that the Spanish government (Spanish Ministry of Economic Affairs 2012) tried to implement in May 2012, by committing to provide new capital to banks if they could not raise it privately. It was expected that the recapitalisation of the Spanish financial sector would relax the debt overhang problems, and create additional room for financing the Spanish private sector. Contrary to these expectations, though, bond yields on Spanish government debt shot up and bond prices went down commensurately when the recap was announced, and the Spanish government had to apply for a direct financial sector bailout by the EFSF/ESM, whereby the bank risk was taken over by those financial institutions. Our model highlights the importance of the poisonous nexus between banks and sovereigns in times of financial fragility.
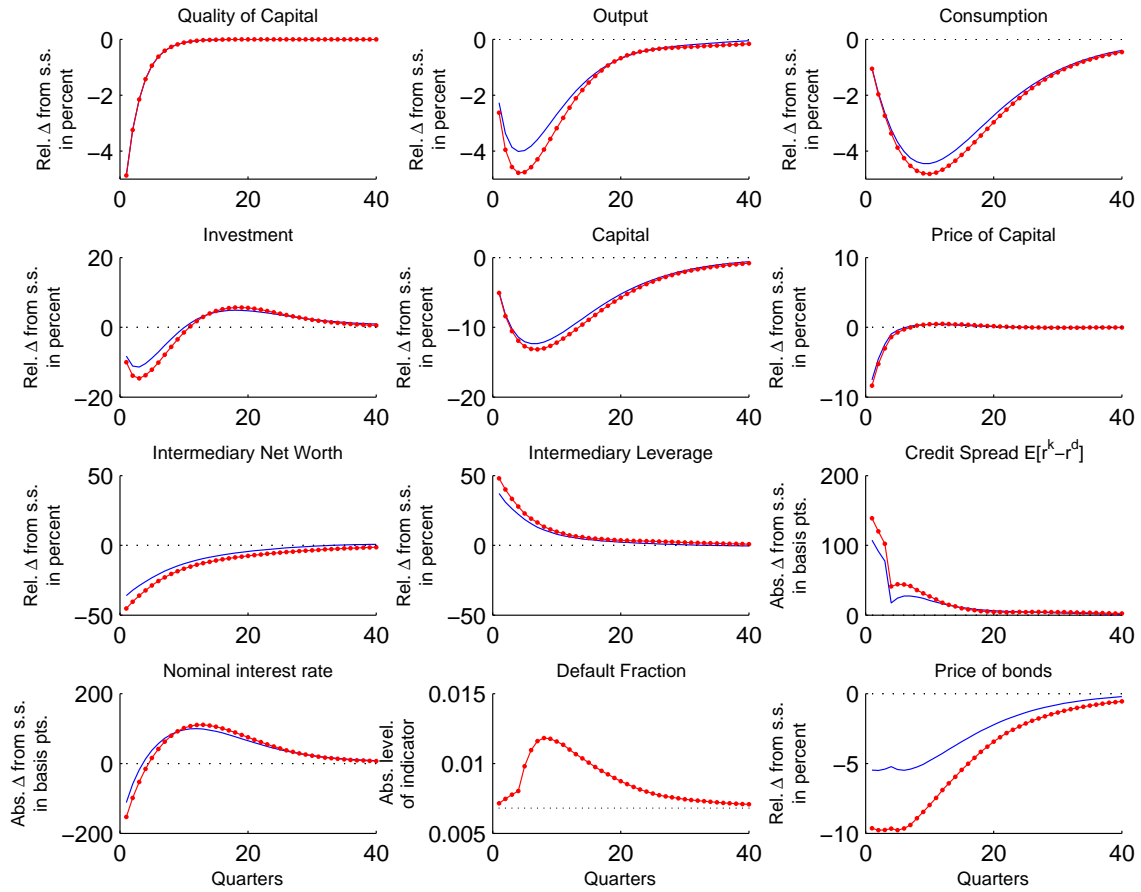
Figure 6: Plot of the impulse response functions comparing the case with no default (blue,-) and default (red,-.-.) in case of a delayed recapitalisation of the financial sector of 1.25 percent of annual steady state GDP that is announced at the start of the financial crisis but implemented 4 quarters later. The financial crisis is initiated through an initial negative capital quality shock of 5 percent relative to the steady state.

6   Robustness checks

In the appendix we extensively perform several robustness checks to make sure that the results that we report are due to the mechanisms described, and not due to other frictions in the model. We summarize the results below.

6.1   RBC version

First we investigate the effect of price-stickiness by performing the same experiments as in the main text, but now the model does not contain price-stickiness and monetary policy anymore, i.e. an RBC (real business cycle) model. The results are reported in figure 8, 9, and 10. It is clear that the main mechanisms work in the same way as the model that contains price-stickiness, although the effects of increasing the maturity parameter ρ are small. A delayed recapitalisation still has positive effects in the no default case. Introducing sovereign default risk has an effect on the real economy when close to the debt limit, although the effects are again smaller than in the model with price-stickiness and monetary policy. So price-stickiness and monetary policy amplify the negative consequences for the economy. In case of a financial crisis, financial intermediaries need the real rate of return on deposits to be as low as possible, because the lower the return, the more net worth is left in the financial intermediaries, and the less balance sheet constrained they are. The central bank, though, is primarily concerned with price stability and to a lesser extent (under the current calibration) with the output gap. This causes deflation in the period of the shock, and since the nominal interest rate was set in the period before the shock, the real return on deposits increases by 100 basispoints. This, however, harms financial intermediaries, because their funding costs increase significantly, thereby causing more balance sheet tightening in a financial crisis, with all the adverse consequences for the real economy that result.

6.2   Errors due to approximation default function

But to what extent influences the approximation method for the default function the results? On the one hand, the effects of the sovereign default risk will be stronger under the approximation method, due to the fact that the government already starts to default on a small part of the debt before the maximum debt level is reached, as can be seen from figure 7 (see appendix). This implies that capital losses will arise before the maximum debt level is reached, which tightens the leverage constraint. At the same time, we see that the possibility of a sovereign default gives the same results as in the case of no default when far away from the debt limit, while the effects only show up when we get close to the maximum level of debt, which is to be expected. The approximation is worst at the kink, but in our simulations we never reach this point. Besides that, it is reasonable to expect a sovereign risk premium to arise before the maximum debt level is reached, because it is always possible that a big shock arrives that pushes the no default level of debt $\tilde{b}_t$ over the maximum level of debt b_t^{max}. The increased interest rate on sovereign bonds induces an additional capital loss through a lower market price for bonds, while the fraction of debt over which the government defaults is small. Hence the current approximation method will probably exacerbate the effects from sovereign default risk slightly, but we do not think that the error in doing so is significant, since the fraction over which the government defaults remains below 0.012 of outstanding government debt.

6.3 Inverse labor supply elasticity

As a last check we assume a low Frisch elasticity in figure 11(see appendix). The results do not change qualitatively, except for more persistence. The effects of the shock are less severe, because labor does not fall as much, depsite lower wages. This improves the ex-post return on capital with respect to the case with a high Frisch elasticity, and hence investment and output do not fall by as much. The same persistence, though, prevents the workers from increasing the labor supply when the capital quality is improving, and hence the negative effects of the capital quality shock are more persistent.

7 Conclusion

We have investigated the poisonous interaction between financial fragility, commercial bank holdings of public debt and sovereign risk. We do so by introducing sovereign default risk into a model with financial intermediaries that hold public debt in their asset portfolio and are subject to a leverage constraint. We show that the maturity structure of government debt matters a great deal. Issuing more debt as part of a bank recapitalisation exercise causes interest rates to rise. But at longer maturities this leads to larger capital losses for the very banks the debt issue is supposed to recapitalise, and the more so the longer is the average maturity of the sovereign debt outstanding.

We subsequently introduce sovereign default risk and show that the real effects from sovereign default risk are substantial, leading to further difficulties when trying to recapitalise banks after a financial crisis. Higher debt issuance undermines sovereign credibility and thus leads to larger sovereign debt discounts, further increasing the capital losses on sovereign debt holdings of the banks being intervened. The additional capital losses cause the leverage constraint to tighten more, thereby exacerbating the link between sovereign debt problems and financial fragility, through which sovereign debt problems have a negative impact on the corporate sector: (required) returns increase, making finance more expensive, asset prices go down, balance sheet constraints are tightened, with substantial subsequent negative impact on investment, consumption and output. Obviously, the bond price is most significantly affected: the price drop is doubled compared with the case of no sovereign default. This also triggers dynamic problems: the government has to issue more bonds to finance the same intervention, which increases the payments that the government has to make in the future, thereby increasing future debt issuance and default probabilities, causing a further amplification through which the drop in the bond price increases.

The mechanisms we highlighted in this paper bring out the limits to traditional bank intervention when commercial banks have sovereign bonds on their balance sheets. Capital losses on longer maturity bonds in commercial bank portfolios, through higher interest rates and/or through increased sovereign risk discounts triggered by increased fears of default, undermine the recapitalisation for which the bonds are issued to begin with. We do not draw any conclusions on whether longer maturity debt should be avoided; although shorter maturities would limit interest rate related capital losses, it would also expose the government to higher roll-over risk, an issue we do not address in this paper.

The link between the sovereign debt-financial fragility nexus with the European debt crisis, and especially the failed Spanish recapitalisation attempt in 2012 is clear: our model predicts the increase in sovereign risk discounts and drop in bond prices when announcing a debt financed bank recapitalisation that caused the Spanish government's recapitalisation program announced in May 2012 to fail before even being implemented.

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9 Appendix

9.1 Derivation of structural equations financial intermediaries in presence of sovereign default risk

The introduction of sovereign default risk changes the equations for the financial intermediaries. In this section we show that the net worth of an individual intermediary is given by the same expression as the case with no default, except for the fact that we replace r_t^b by r_t^{b*}. Hence we only have to replace r_t^b in the equations governing the financial intermediaries by r_t^{b*}, and include the expression for r_t^{b*} in the first order conditions. We start by observing that the funds obtained from selling the bonds in period $t+1$, that were purchased in period t, are reduced by $1 - \Delta_{t+1}$, just as the fixed real payment r_c per bond is reduced to $(1 - \Delta_{t+1})r_c$. Hence the law of motion for the net worth of an individual intermediary changes into the following equation:

$$
\begin{aligned}
n_{j,t+1} &= \left(1 + r_{t+1}^k\right) q_t^k s_{j,t}^k + (1 - \Delta_{t+1}) r_c s_{j,t}^b + (1 - \Delta_{t+1}) \rho q_{t+1}^b s_{j,t}^b - (1 + r_{t+1}^d) d_{j,t} + n_{j,t+1}^g - \tilde{n}_{j,t+1}^g \\
&= \left(1 + r_{t+1}^k\right) q_t^k s_{j,t}^k + (1 - \Delta_{t+1}) \left(r_c + \rho q_{t+1}^b\right) s_{j,t}^b - (1 + r_{t+1}^d) d_{j,t} + n_{j,t+1}^g - \tilde{n}_{j,t+1}^g \\
&= \left(1 + r_{t+1}^k\right) q_t^k s_{j,t}^k + (1 - \Delta_{t+1}) \frac{\left(r_c + \rho q_{t+1}^b\right)}{q_t^b} q_t^b s_{j,t}^b - (1 + r_{t+1}^d) d_{j,t} + n_{j,t+1}^g - \tilde{n}_{j,t+1}^g \\
&= \left(1 + r_{t+1}^k\right) q_t^k s_{j,t}^k + (1 - \Delta_{t+1}) \left(1 + r_{t+1}^b\right) q_t^b s_{j,t}^b - (1 + r_{t+1}^d) d_{j,t} + n_{j,t+1}^g - \tilde{n}_{j,t+1}^g \\
&= \left(r_{t+1}^k - r_{t+1}^d\right) q_t^k s_{j,t}^k + \left(r_{t+1}^{b*} - r_{t+1}^d\right) q_t^b s_{j,t}^b + (1 + r_{t+1}^d) n_{j,t} + \tau_{t+1}^n n_{j,t} - \tilde{\tau}_{t+1}^n n_{j,t}
\end{aligned}
$$

where r_t^{b*} is given by:

$$
1 + r_t^{b*} = (1 - \Delta_t) \left(1 + r_t^b\right) = (1 - \Delta_t) \left(\frac{r_c + \rho q_t^b}{q_{t-1}^b}\right)
$$

We replace r_t^b by r_t^{b*} in the equation for the shadow value of government bonds, and the law of motion of net worth:

$$
\begin{aligned}
\nu_t^b &= E_t \left[\Omega_{t+1} \left(r_{t+1}^{b*} - r_{t+1}^d\right) \right] \\
n_t &= \theta \left[\left(r_t^k - r_t^d\right) q_{t-1}^k s_{t-1}^k + \left(r_t^{b*} - r_t^d\right) q_{t-1}^b s_{t-1}^b + (1 + r_t^d) n_{t-1} \right] + \chi p_{t-1} + n_t^g - \tilde{n}_t^g
\end{aligned}
$$

The other equations for the financial intermediaries remain the same.

9.2 Approximation of the default function

We can also write the debt level structure (40) in the following way:

$$
b_t = \min \left(\tilde{b}_t, b_t^{max} \right) = b_t^{max} - \max \left(b_t^{max} - \tilde{b}_t, 0 \right) \tag{46}
$$

We can interpret the second term of the new debt level as the payoff of a put option at maturity with underlying process $\tilde{b}_t$ and strike price b_t^{max}. The formula for b_t, however, does not have a defined derivative at $\tilde{b}_t = b_t^{max}$. Therefore we apply an approximation for the payoff structure of the put option, and use the option pricing formula, which gives the price of the put option when time to maturity is equal to T, compounded risk-free interest rate r, and volatility of the underlying

process σ. This is an *approximation* to the actual mapping from $\tilde{b}_t$ to b_t, and has in this sense no economic interpretation in our model. The dashed line in figure 7 is the approximation to the *actual* mapping of $\tilde{b}_t$ to b_t. We then get the following approximation for b_t, with $\Phi(\cdot)$ denoting the standard normal CDF, which is indeed continuous:

$$b_t = b_t^{max} - put_t \tag{47}$$

$$put_t = Xe^{-rT}\Phi(-d_{2,t}) - S_t\Phi(-d_{1,t}) \tag{48}$$

$$d_{1,t} = \frac{\log(S/X) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \tag{49}$$

$$d_{2,t} = \frac{\log(S/X) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \tag{50}$$

$$X = b_t^{max} \tag{51}$$
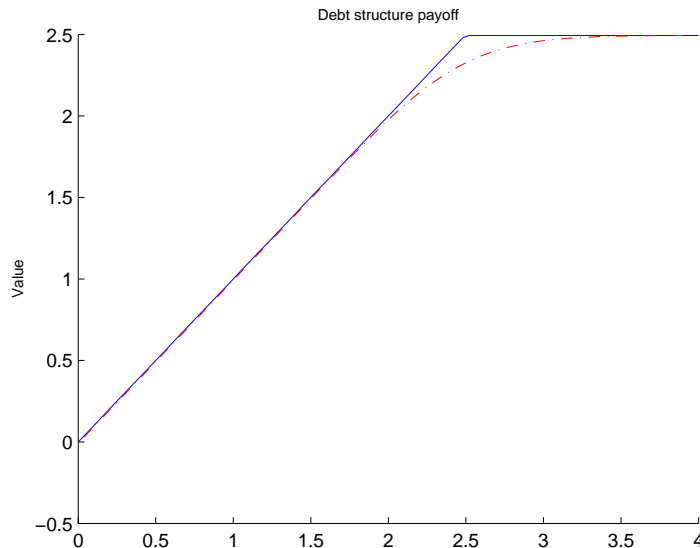
$$S_t = \tilde{b}_t \tag{52}$$

Figure 7: Plot showing the mapping from the no default level of debt $\tilde{b}_t$ to the actual debt level b_t. The solid blue line is the actual mapping, while the dashed red line is an approximation to it, where option pricing formulas have been used.

9.3 Calibration strategies for the default function

In this section we will write down the 2 calibration strategies regarding the sovereign debt in the current paper, since other parts of the model are straightforward to calibrate. The steady state

value of a variable x_t is denoted by $\bar{x}$. We have the following 2 equations from the financial interemediaries' problem, from which we can derive the steady state return on bonds ex-post a possible default.

$$\nu_t^k = E_t\left[\Omega_{t+1}\left(r_{t+1}^k - r_{t+1}^d\right)\right]$$

$$\nu_t^b = E_t\left[\Omega_{t+1}\left(r_{t+1}^{b*} - r_{t+1}^d\right)\right]$$

$$\Rightarrow E_t\left[\Omega_{t+1}\left(r_{t+1}^k - r_{t+1}^{b*}\right)\right] = 0$$

From these equations it is clear that $\bar{r}_k = \bar{r}_{b*}$. Now we have the following equation for the maximum level of debt:

$$b_t^{max} = \bar{b} + \frac{E_t\left[\tau_{t+1}^{max}\right] - \bar{\tau}}{\kappa_b} \tag{53}$$

The government budget constraint in case of no default by the government is equal to:

$$q_t^b \tilde{b}_t + \tau_t + \tilde{n}_t^g = g_t + n_t^g + \left(r_c + \rho q_t^b\right)b_{t-1}$$

$$= g_t + n_t^g + \frac{\left(r_c + \rho q_t^b\right)}{q_{t-1}^b}q_{t-1}^b b_{t-1} \Longrightarrow$$

$$q_t^b \tilde{b}_t + \tau_t + \tilde{n}_t^g = g_t + n_t^g + \left(1 + r_t^b\right)q_{t-1}^b b_{t-1} \tag{54}$$

The mapping from the number of no default bonds to the actual number of bonds is given by:

$$b_t = b_t^{max} - \max\left(b_t^{max} - \tilde{b}_t, 0\right) \approx b_t^{max} - put\left(b_t^{max}, \tilde{b}_t\right) \tag{55}$$

The actual number of government bonds is given by:

$$q_t^b b_t + \tau_t + \tilde{n}_t^g = g_t + n_t^g + \left(1 - \Delta_t\right)\left(r_c + \rho q_t^b\right)b_{t-1}$$

$$= g_t + n_t^g + \left(1 - \Delta_t\right)\frac{\left(r_c + \rho q_t^b\right)}{q_{t-1}^b}q_{t-1}^b b_{t-1}$$

$$= g_t + n_t^g + \left(1 - \Delta_t\right)\left(1 + r_t^b\right)q_{t-1}^b b_{t-1} \Longrightarrow$$

$$q_t^b b_t + \tau_t + \tilde{n}_t^g = g_t + n_t^g + \left(1 + r_t^{b*}\right)q_{t-1}^b b_{t-1} \tag{56}$$

together with the ex-post default return on bonds:

$$1 + r_t^{b*} = \left(1 - \Delta_t\right)\left(1 + r_t^b\right) \tag{57}$$

and the return on bonds before default:

$$1 + r_t^b = \frac{\left(r_c + \rho q_t^b\right)}{q_{t-1}^b} \tag{58}$$

Throughout the paper we assume that the financial intermediaries do not receive any support from the government in the steady state, nor are they paying back support in the steady state, i.e.

$\bar{n}_g = \bar{\tilde{n}}_g = 0$. Finally, we have the option pricing formulas:

$$put_t = b_t^{max} e^{-rT} \Phi(-d_{2,t}) - \tilde{b}_t \Phi(-d_{1,t}) \tag{59}$$

$$d_{1,t} = \frac{\log\left(\tilde{b}_t/b_t^{max}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \tag{60}$$

$$d_{2,t} = \frac{\log\left(\tilde{b}_t/b_t^{max}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \tag{61}$$

Calibration strategy 1

The first strategy targets $\bar{q}_b\bar{b}$, $\bar{q}_b\bar{b}_{max}$ and $\bar{\Delta}$, for which we take 60%, respectively 90% of annual GDP and $\bar{\Delta} = 0.005$. Since we know $\bar{q}_b\bar{b}$, the steady state return on bonds after default $\bar{r}_{b*}$ and $\bar{g}$ (which is calibrated to be 20% of steady state output), we can find the steady state level of taxes from (56):

$$\bar{q}_b\bar{b} + \bar{\tau} = \bar{g} + (1 + \bar{r}_{b*})\bar{q}_b\bar{b} \Longrightarrow$$
$$\bar{\tau} = \bar{g} + \bar{r}_{b*}\bar{q}_b\bar{b}$$

Since we know the steady state default fraction $\bar{\Delta}$ and the ex-post return on bonds, we can calculate $\bar{r}_b$ through (57):

$$1 + \bar{r}_{b*} = \left(1 - \bar{\Delta}\right)(1 + \bar{r}_b) \Longrightarrow$$
$$\bar{r}_b = \frac{1 + \bar{r}_{b*}}{1 - \bar{\Delta}} - 1$$

Since we know r_c, we can find the steady state bond price through (58):

$$1 + \bar{r}_b = \frac{r_c + \rho\bar{q}_b}{\bar{q}_b} \Longrightarrow \bar{q}_b(1 + \bar{r}_b) = r_c + \rho\bar{q}_b \Longrightarrow$$
$$\bar{q}_b = \frac{r_c}{1 + \bar{r}_b - \rho}$$

Now that the steady state bond price is known, we can find the steady state number of bonds and the maximum number of bonds $\bar{b}$ and $\bar{b}_{max}$. Since we know the return on bonds $\bar{r}_b$, we can find the number of bonds if the government does not default:

$$\bar{q}_b\bar{\tilde{b}} + \bar{\tau} = \bar{g} + (1 + \bar{r}_b)\bar{q}_b\bar{b}$$

Now that we have found steady state number of bonds $\bar{b}$, the maximum number of bonds in the steady state $\bar{b}_{max}$, and the steady state number of bonds in case the government does not default $\bar{\tilde{b}}$. With these 3 numbers, and the requirement that the derivative of the put option $-\Phi(-d_{1,t})$ is equal to -0.99 (otherwise the default probability goes down when debt increases, which is counterintuitive), we can find the variables r, σ and T from the option pricing formulas.

Calibration strategy 2

The second strategy targets $\bar{q}_b \bar{b}$, and $\bar{q}_b \bar{b}_{max}$, and takes the option pricing parameters r, σ and T as given. We calibrate $\bar{q}_b \bar{b}$ to be equal to 80% of annual GDP, while $\bar{q}_b \bar{b}_{max}$ is equal to 90% of annual GDP. Since $\bar{g}$ is also known, we can find the steady state level of taxes from (56):

$$\bar{q}_b \bar{b} + \bar{\tau} = \bar{g} + (1 + \bar{r}_{b*}) \bar{q}_b \bar{b} \Longrightarrow$$
$$\bar{\tau} = \bar{g} + \bar{r}_{b*} \bar{q}_b \bar{b}$$

Since we know $\bar{q}_b \bar{b}$ and $\bar{q}_b \bar{b}_{max}$, we can divide the two to find the ratio $\bar{b}_{max}/\bar{b}$. Now we look at the option pricing formulas, and remember that the parameters r, σ and T are given. We rewrite the put option in the following way:

$$
\begin{aligned}
b_t &= b_t^{max} - put_t = b_t^{max} - \left\{ b_t^{max} e^{-rT} \Phi(-d_{2,t}) - \tilde{b}_t \Phi(-d_{1,t}) \right\} \\
&= b_t^{max} - b_t^{max} e^{-rT} \Phi(-d_{2,t}) + \tilde{b}_t \Phi(-d_{1,t})
\end{aligned}
$$

Division by b_t gives the following expression:

$$
1 = \frac{b_t^{max}}{b_t} - \frac{b_t^{max}}{b_t} e^{-rT} \Phi(-d_{2,t}) + \frac{\tilde{b}_t}{b_t} \Phi(-d_{1,t}) \tag{62}
$$

Now we look at the formula for $d_{1,t}$:

$$
\begin{aligned}
d_{1,t} &= \frac{\log\left(\tilde{b}_t / b_t^{max}\right) + \left(r + \frac{\sigma^2}{2}\right) T}{\sigma \sqrt{T}} \\
&= \frac{\log\left(\frac{\tilde{b}_t}{b_t} \frac{b_t}{b_t^{max}}\right) + \left(r + \frac{\sigma^2}{2}\right) T}{\sigma \sqrt{T}} = f\left(\frac{\tilde{b}_t}{b_t}, \frac{b_t^{max}}{b_t}\right) \tag{63}
\end{aligned}
$$

similarly we find for $d_{2,t}$:

$$
\begin{aligned}
d_{2,t} &= \frac{\log\left(\tilde{b}_t / b_t^{max}\right) + \left(r - \frac{\sigma^2}{2}\right) T}{\sigma \sqrt{T}} \\
&= \frac{\log\left(\frac{\tilde{b}_t}{b_t} \frac{b_t}{b_t^{max}}\right) + \left(r - \frac{\sigma^2}{2}\right) T}{\sigma \sqrt{T}} = g\left(\frac{\tilde{b}_t}{b_t}, \frac{b_t^{max}}{b_t}\right) \tag{64}
\end{aligned}
$$

Hence we see that equations (62), (63) and (64) only depend on the ratios b_t^{max}/b_t and $\tilde{b}_t/b_t$. We know the first ratio in steady state, so we can solve for the steady state ratio $\tilde{\bar{b}}/\bar{b}$. We also see that regarding the default function, it does not matter whether we calibrate on the number of bonds b_t or the value of government liabilities $q_t^b b_t$, since the bond *ratios* are the variables that matter. Now we can find $\tilde{\bar{b}}/\bar{b}$, and hence we find $\bar{q}_b \tilde{\bar{b}} = \bar{q}_b \bar{b} \left(\tilde{\bar{b}}/\bar{b}\right)$. Since we know $\bar{q}_b \tilde{\bar{b}}$, we can find $\bar{r}_b$ from (54):

$$\bar{q}_b \tilde{\bar{b}} + \bar{\tau} = \bar{g} + (1 + \bar{r}_b) \bar{q}_b \bar{b} \Longrightarrow$$
$$\bar{r}_b = \frac{\bar{q}_b \tilde{\bar{b}} + \bar{\tau} - \bar{g}}{\bar{q}_b \bar{b}} - 1$$

Now we can find the steady state bond price from (58):

$$
\begin{aligned}
1 + \bar{r}_b &= \frac{r_c + \rho \bar{q}_b}{\bar{q}_b} \implies \bar{q}_b \left(1 + \bar{r}_b\right) = r_c + \rho \bar{q}_b \implies \\
\bar{q}_b &= \frac{r_c}{1 + \bar{r}_b - \rho}
\end{aligned}
$$

after which we know $\bar{b}$, $\bar{b}_{max}$ and $\bar{\bar{b}}$. From (57) we can find the steady state default probability:

$$
\begin{aligned}
1 + \bar{r}_{b*} &= \left(1 - \bar{\Delta}\right)\left(1 + \bar{r}_b\right) \implies \\
\bar{\Delta} &= 1 - \frac{1 + \bar{r}_{b*}}{1 + \bar{r}_b}
\end{aligned}
$$

9.4   Robustness checks

In this section we sho the various IRFs for several robustness checks discussed in Section 6 in the main body of the paper.
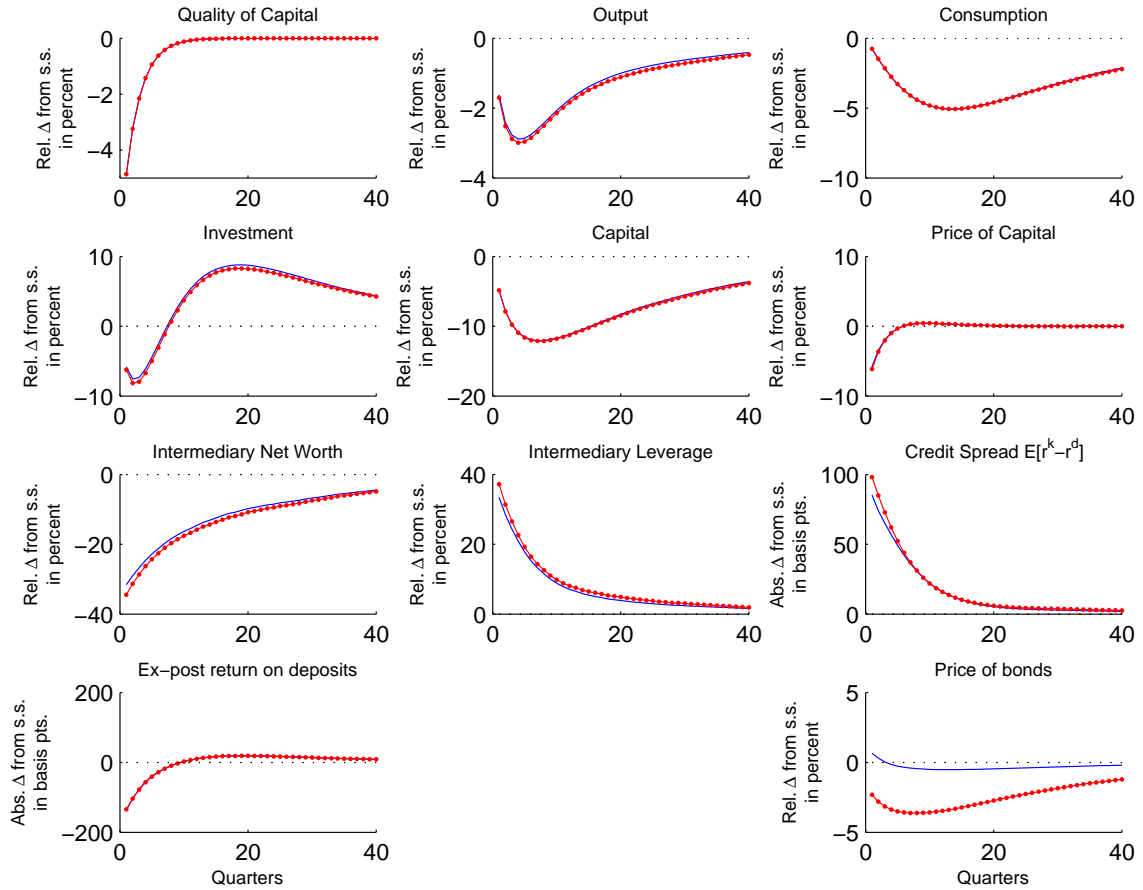
Figure 8: Plot of the impulse response functions for the model (RBC version) without default for $\rho = 0.5$ (blue) and $\rho = 0.96$ (red,-.-.). The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state, and no additional government policy is implemented.
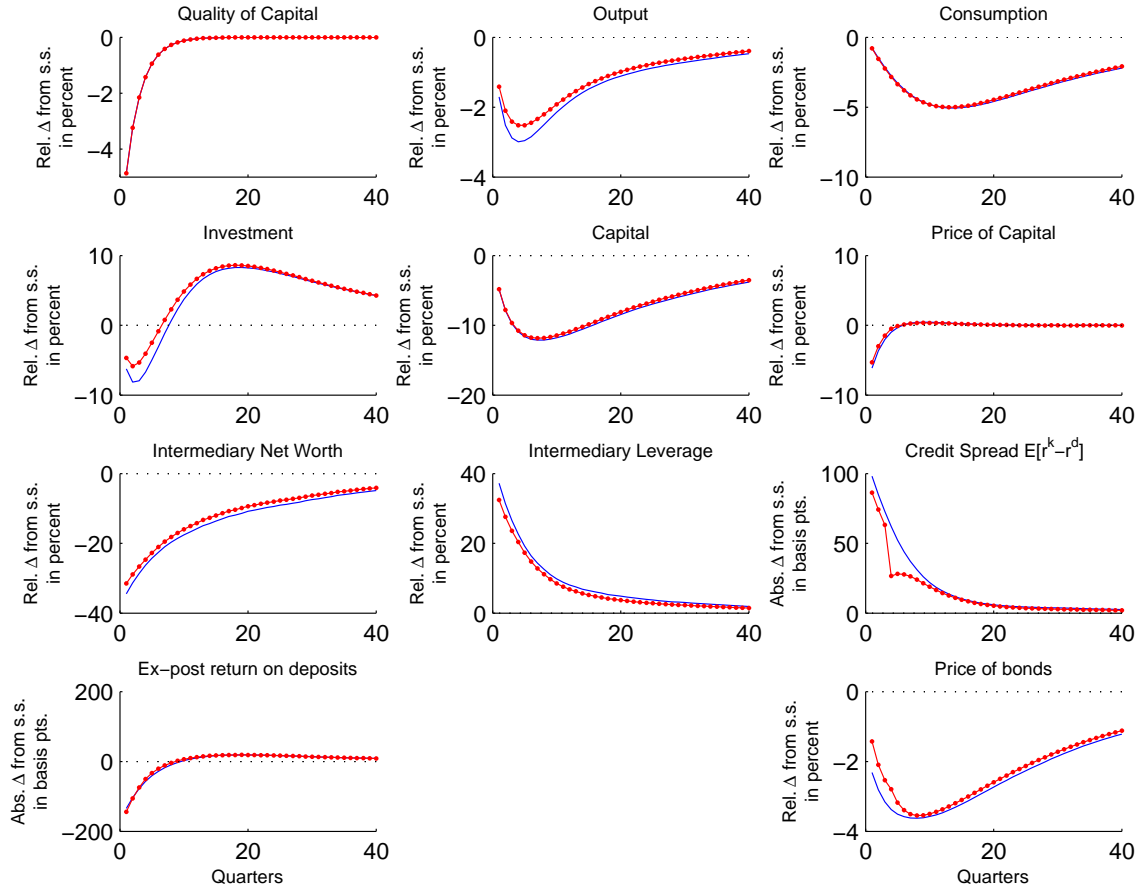
Figure 9: Plot of the impulse response functions for the model (RBC version) without default with no additional government policy (blue) and a delayed recapitalization of the financial sector (red,-.-.) occurring 4 quarters after the financial crisis hits and equal to 1.25% of annual steady state GDP. The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.
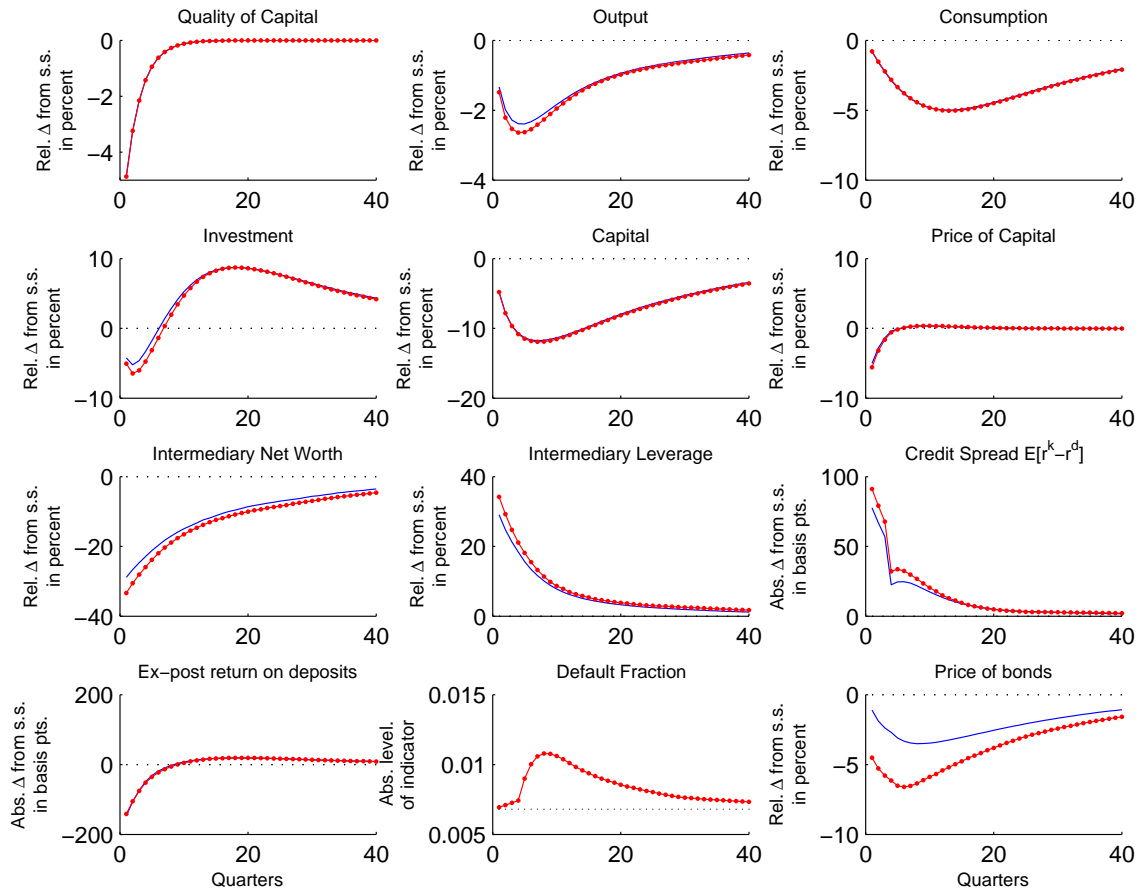
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Figure 10: Plot of the impulse response functions (RBC version) comparing the case with no default (blue,-) and default (red,-.-.) in case of a delayed recapitalization of the financial sector of 1.25 percent of annual steady state GDP that is announced at the start of the financial crisis but implemented 4 quarters later. The financial crisis is initiated through an initial negative capital quality shock of 5 percent relative to the steady state. Contrary to the previous cases, the value of the government debt is now 80% of annual GDP.
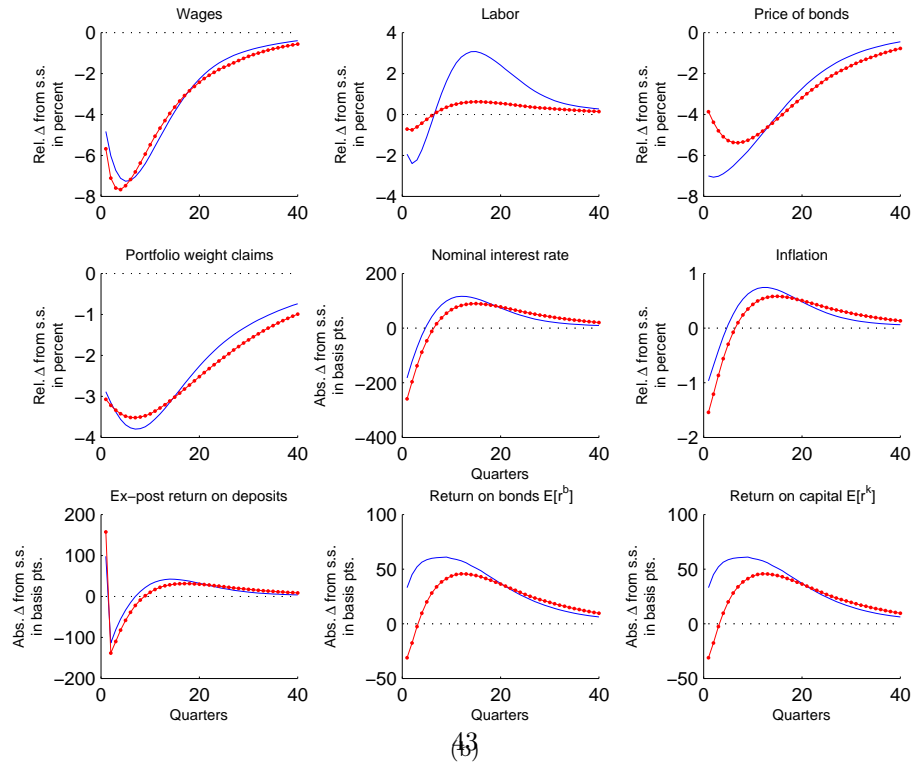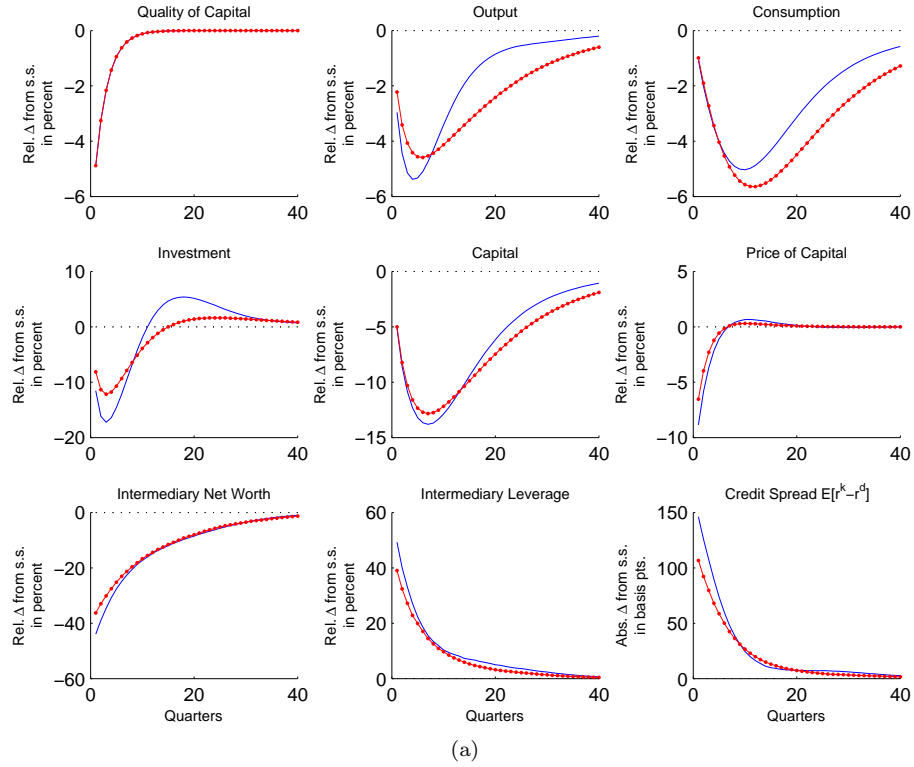
Figure 11: Plot of the impulse response functions for the model without default ($\rho = 0.96$) with the Frisch elasticity at $\varphi = 0.276$ (blue), and for $\varphi = 4$ (red,-.-.). There is no additional government policy, and the crisis is initiated through a negative capital quality shock of 5% of steady state.