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# Market Power in Bilateral Oligopoly Markets with Nonexpandable Infrastructures

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## Abstract

We consider price-fee competition in bilateral oligopolies with perfectly-divisible goods, non-expandable infrastructures, concentrated agents on both sides, and constant marginal costs. We define and characterize stable market outcomes. Buyers exclusively trade with the supplier with whom they achieve maximal bilateral joint welfare. Prices equal marginal costs. Threats to switch suppliers set maximal fees. These also arise from a negotiation model that extends price competition. Competition in both prices and fees necessarily emerges. It improves welfare compared to price competition, but consumer surpluses do not increase. The minimal infrastructure achieving maximal aggregate welfare differs from the one that protects buyers most.

## JEL Classification:

C78, L10, L14, D43, R10.

## Keywords:

Assignment Games, Infrastructure, Negotiations, Non-linear pricing, Market Power

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# 1 Introduction

Oligopolistic competition often requires some infrastructure with costly transportation. This can vary from costs of postal services when purchasing books or computers from online shops to physical infrastructure required to transport perfectly-divisible goods such as water, chemicals, electricity and natural gas. Often, in such contexts, payments for goods consist of two parts: a price per unit and a fee, i.e. a fixed monetary amount. Transportation costs may differ per supplier and per customer, for example tariffs for postal services typically distinguish between domestic and foreign destinations. Also, product modifications in the intermediate goods market to meet specifications set by heterogeneous customers can be treated as relation-specific transport cost. The importance of heterogeneous transportation costs in trade is not at par with standard microeconomics, with the notable exceptions of Hotelling (1929), Salop (1979), or Economides (1986) for markets with infinitely-many small customers and indivisible goods. Also, oligopolistic competition in both prices and fees receives limited attention. Oi (1971), Schmalensee (1981) and Schmalensee (1982) analyze two-part pricing for monopoly markets where the supplier lacks information about the consumers' willingness to pay for a perfectly-divisible good. Two-part pricing in oligopoly settings with perfect information is studied in e.g. Calem and Spulber (1984), Harrison and Kline (2001), or Kanemoto (2000). In our study, we analyze competition in both prices and fees in an oligopolistic market with a finite number of concentrated buyers and suppliers, all heterogeneous, on a given non-expandable infrastructure that is needed to transport some perfectly-divisible good. Marginal costs of both production and transportation are constant and relation-specific. One aim of this study is to analyze oligopolistic competition in prices and fees, the quantities traded and how these are affected by the non-expandable infrastructure.

Markets with a few concentrated suppliers and a few concentrated buyers, who all can exercise market power, are referred to as bilateral oligopolies. It is well known that concentration on the supply side increases market power and that it has negative consequences

for consumer welfare and aggregate social welfare, see e.g. Tirole (1988) or Motta (2004). Conditions under which these results extend to bilateral oligopolies are derived in e.g. Bloch and Ghosal (1997), Bloch and Ferrer (2001), or Amir and Bloch (2009). Galbraith (1952) is probably the first author who has argued that concentrated buyers can also have countervailing power that can restrain the market power of suppliers. Hence, especially in bilateral oligopolies with high concentration on both sides, the relationship between concentration, market power and efficiency is much more complex, and only a few studies have investigated this relationship both theoretically and empirically. Of those studies, several have tested the countervailing power hypothesis, and there appears to be evidence that buyer concentration negatively affects the market power of suppliers, see e.g. Scherer and Ross (1990) for a review of this literature that was initiated by Lustgarten (1975). Also, Schumacher (1991) supports the countervailing power hypothesis in a study based on US manufacturing industries. All studies emphasize that threats to switch orders from one supplier to another strengthen a buyer's bargaining position.<sup>1</sup> A second aim of this study is to investigate the relationship between market concentration on both sides, market power and efficiency in bilateral oligopolies. We also characterize the minimal infrastructure that achieves maximal aggregate social welfare and the one that protects buyers most from the supply side's market power.

As an appropriate equilibrium concept for bilateral oligopolies we propose a stability concept that balances, on the one hand, generality without too much specific institutional details to capture a variety of bargaining processes.<sup>2</sup> On the other hand, this equilibrium concept takes into account the essence of market power or bargaining power of suppliers and buyers, including threats to switch orders from one supplier to another. As the equilibrium concept, we impose stability against deviating coalitions of traders: prices, fees and traded

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<sup>1</sup>Another threat by which buyers can strengthen their bargaining position is to start upstream production themselves. Inderst and Wey (2007) and Dana (2012) investigate the threat of forming buyer groups that will act on behalf of their members, which requires some form of either legal or credible commitment. In our study we only look at the threat of switching orders.

<sup>2</sup>For that reason, we do not impose explicit price setting mechanisms such as in e.g. the market game proposed in Shapley and Shubik (1977).

quantities are in equilibrium if there are no alternative prices, fees and quantities such that either no buyer wants to join another supplier's clientele, or no supplier wants to reduce or expand his clientele. Stability is defined on the non-expandable infrastructure.

We characterize all stable market outcomes and show that these have a lattice structure and are bilaterally efficient. Bilateral efficiency means that positive quantities in each pairwise trade maximize the bilateral joint welfare of this pair, which consists of the standard consumer and producer surplus, taking all other trades as given. Therefore, bilateral efficiency implies relation-specific marginal-cost pricing in order to realize the maximal bilateral joint welfare. This result implies that all stable market outcomes are also Pareto efficient: the equilibrium quantities are the same as if all firms were price-takers. Thus, market power under oligopolistic competition in prices and fees does not necessarily cause distortions as opposed to oligopolistic competition in prices, where deadweight losses are unavoidable. This result is similar to those reported in e.g. Oi (1971), but different from the results in e.g. Calem and Spulber (1984), or Harrison and Kline (2001).

The relation-specific fee distributes the joint welfare between the buyer and supplier, where higher fees can be seen as a reflection of higher supply side market power. From the perspective of buyers, zero fees yield maximal consumer surplus. In any stable market outcome, suppliers may trade with several buyers. Each buyer, however, exclusively trades with his most-efficient supplier, i.e., the one with the lowest cost, on the infrastructure in order to achieve the maximal bilateral joint welfare, but this makes each buyer vulnerable to market power exercised by his most-efficient supplier. Such market power, however, is limited by each buyer's threat to trade with his second-efficient supplier on the infrastructure. Therefore, a buyer's maximal fee is bounded from above by the difference of the maximal bilateral joint welfare levels that can be achieved by trading with his most-efficient supplier and second-efficient supplier. The set of stable fees has a lattice structure. All these results can be linked to similar results for assignment games in matching markets with indivisible goods, see Shapley and Shubik (1972), Roth and Sotomayor (1990), Camina (2006), and

Sozanski (2006).

The stability concept induces unique relation-specific prices and unique relation-specific positive quantities traded, but the non-degenerate ranges of relation-specific fees reveal indeterminacy. We do not regard indeterminacy as a critique, because it allows for flexibility in filling in the precise institutional details of the market and the distribution of bargaining or market power. In the bargaining literature, there are many bargaining protocols that each induce different market outcomes. In our study, we modify the one-sided proposal-making model from the literature on matching markets, see e.g. Roth and Sotomayor (1990), to model extreme market or bargaining power by one side of the market. The modified negotiation model features simultaneous price-fee proposals by agents from one side of the market to all connected agents on the opposite side of the market and, if accepted, the agents who accept choose quantities. We are especially interested in the supply side making such proposals and compare the outcomes to those arising from oligopolistic (differentiated Bertrand) price competition. Suppliers propose marginal-cost pricing and the maximal stable fees, all relation-specific. Buyers accept the offer from their most-efficient supplier and demand the joint-welfare maximizing quantities. Compared to oligopolistic price competition, competition in both prices and fees improves aggregate joint welfare but buyers are not better off. Finally, when buyers propose, then we have the relation-specific marginal-cost pricing and no (or zero) fees result. The latter can be seen as the outcome under perfect competition.

Because each buyer only utilizes a single link among his potential trading relations on the non-expandable infrastructure, we also identify the minimal infrastructure that would generate maximal aggregate joint welfare among all infrastructures and this only requires that each buyer is linked to his most-efficient supplier among all suppliers. To reduce vulnerability of buyers to market power exercised by the supply side, we also identify the minimal infrastructure that would generate the maximal aggregate consumer surplus among all infrastructures and this requires that each buyer is linked to his most-efficient supplier and his second-most efficient supplier among all suppliers. In such setting, even though each buyer

will never utilize one of his two links, the other link needs to be present in order to have a credible threat of switching to another supplier.

The current study introduces the framework of bilateral oligopolies and explains the main ideas of competition in relation-specific prices and fees on a non-expandable infrastructure. This allows us to offer three major insights: a theory of market competition and market power in concentrated markets with a non-expandable infrastructure; identification of the non-expandable infrastructure with maximal buyer protection; and emergence of competition in prices and fees instead of oligopolistic competition in prices only. We regard a thorough understanding of non-expandable infrastructures as a first and necessary step towards an analysis of market power on expandable infrastructures under costly investment. Such analysis will be provided in a companion paper Funaki et al. (2012). Expandable infrastructures are more appropriate in the setting with less costly investment such as contractual relationships, software development for heterogeneous clients, or relation-specific investments in intermediate goods markets to meet heterogeneous buyers' specifications, as discussed in e.g. Bjornerstedt and Stenneck (2007). Nevertheless, the results for non-expandable infrastructures are relevant to analyze spot-markets on infrastructures that cannot be expanded in the short run, such as infrastructure for natural gas and oil, and relation-specific capital investments.

The paper is organized as follows. Section 2 outlines the model and in section 3 three motivating examples are provided. Oligopolistic competition in prices and fees on a non-expandable infrastructure is analyzed in Section 4. Some concluding remarks are left for Section 5.

## 2 The model

Consider a market with a finite set  $S$  of suppliers,  $|S| \geq 1$ , of some good and a finite set  $B$  of buyers,  $|B| \geq 1$ . We denote an individual supplier as  $i$  and an individual buyer as  $j$ . The set of all agents is  $N = S \cup B$ . Each agent in  $N$  is either a supplier or a buyer, that is the



sets  $S$  and  $B$  are disjoint, i.e.  $S \cap B = \emptyset$ .

Bilateral trade requires infrastructure that links supplier  $i$  and buyer  $j$ . Without such a link, a pair of buyers and suppliers cannot trade. The link between supplier  $i$  and buyer  $j$  is denoted  $ij \equiv (i, j) \in S \times B$ , and often we call  $ij$  a pair. The set of all potential links  $ij$  is denoted by  $g_N = \{ij | ij \in S \times B\}$ , which is an undirected graph. An infrastructure  $g$  on  $N$  is an arbitrary set of links  $g \subseteq g_N$ . In particular, the set of all feasible links  $g_N$  is called the complete infrastructure and  $g_0 = \emptyset$  represents the absence of any infrastructure. The collection of all networks is denoted by  $G_N = \{g | g \subseteq g_N\}$ . In what follows, we think of infrastructures as some non-expandable infrastructure inherited from the past whose building costs are sunk. For explanatory reasons, we assume that links are of unlimited capacity. Its operating and managing costs are assumed to be included in the variable transportation costs, which are defined below.

In this market, we keep track of trade flows between pairs of linked suppliers and buyers. For a given infrastructure  $g \subseteq g_N$  and a pair  $ij \in g$ , the quantity  $q_{ij} \geq 0$  denotes the flow of output from supplier  $i$  to buyer  $j$ . For convenience, we set  $q_{ij} = 0$  to represent the infeasibility of trade for any pair  $ij \notin g$ , and we denote the matrix of all trades on  $g \subseteq g_N$  as  $Q|g = (q_{ij})_{ij \in S \times B} \in \mathbb{R}_+^{|S \times B|}$ . Supplier  $i$ 's total production or quantity sold is  $q_i = \sum_{j \in B: ij \in g} q_{ij}$ . Production and shipping products within any pair takes place against constant marginal costs that depend upon the identity of the suppliers and buyers. Denote  $c_{ij} \geq 0, ij \in g_N$ , as the marginal costs of both production and transportation from supplier  $i$  to buyer  $j$ . Suppliers may sell their products to multiple buyers. Given trades  $Q|g$  on infrastructure  $g \subseteq g_N$ , we define the endogenous trade network  $T(Q|g) \in g$  as all links  $ij \in g$  with  $q_{ij} > 0$ , i.e. all links with positive trade. Supplier  $i$ 's active customer network consists of those buyers with whom this supplier trades positive amounts (and with whom he is linked to).<sup>3</sup> More specific, for supplier  $i$ ,  $T_i(Q|g) \subseteq B$  denotes supplier  $i$ 's set of active buyers  $j$  for which  $q_{ij} > 0$  on infrastructure  $g$ . By definition, this set may be empty if

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<sup>3</sup>The passive or inactive customer network consists of those buyers that are linked to the supplier, and that do not purchase the product.

supplier  $i$  has no customers, a singleton in case he has only one customer or it contains multiple elements in case this supplier has many customers.

Given trades  $Q|g$  on infrastructure  $g \subseteq g_N$ , buyer  $j$ 's total consumption is  $q_j = \sum_{i \in S: ij \in g} q_{ij}$ , and this buyer has the quasi-linear utility function  $u_j(q_j) + m_j$ , where the function  $u_j$  is increasing in  $q_j$  and  $m_j$  is monetary wealth. We consider markets for which the standard demand as a function of the market price satisfies the Law of Demand, i.e. the demand is decreasing in its own price e.g. Mas-Colell, Whinston, and Green (1995). This law holds whenever the utility function is strictly quasi-concave and, by Crouzeix and Lindberg (1986), this is equivalent to strict concavity of the function  $u_j$ . For buyer  $j$ ,  $T_j(Q|g) \subseteq S$  denotes buyer  $j$ 's set of active suppliers  $i$  for which  $q_{ij} > 0$  on infrastructure  $g$ .

Competition in this market takes place through relation-specific prices  $p_{ij} \geq c_{ij}$  and relation-specific fees  $f_{ij} \geq 0$  for all pairs  $ij \in g$ . Joint pair-wise welfare within the pair  $ij \in g$  can be expressed as the sum of  $i$ 's producer surplus  $(p_{ij} - c_{ij})q_{ij} + f_{ij}$  and  $j$ 's consumer surplus  $u_j(q_{ij}) - p_{ij}q_{ij} - f_{ij}$ . The maximal joint welfare for the pair  $ij$  is given by  $\max_{q_{ij} \geq 0} u_j(q_{ij}) - c_{ij}q_{ij}$ . For technical convenience, we assume a unique joint welfare maximum for each possible link, that each buyer can rank such welfare maxima over the suppliers in  $S$  and we impose differentiability.

**Assumption 1** *All  $c_{ij} \geq 0$ ,  $ij \in g_N$ , are mutually different, and for each buyer  $j \in S$ , the function  $u_j$  is increasing, continuously differentiable and strictly concave in  $q_j$ ,  $u_j(0) = 0$ , and  $u'_j(0) > \max_{i \in S} c_{ij}$ .*

Recall that the Law of Demand imposes strict concavity, or  $u'_j$  decreasing, and combined with continuous differentiability of the utility functions implies that  $u'_j(q_{ij}) = c_{ij}$  has at most one solution. Our assumption on the slope of  $u_j$  at  $q_j = 0$  ensures that, in case of exclusive trade in the pair  $ij$ , a solution exists in which buyer  $j$  consumes a positive quantity  $q_{ij}$  and that the maximal joint welfare without building costs is positive for each pair  $ij \in g_N$ .

For the initial non-expandable infrastructure  $g \subseteq g_N$ , the maximal joint welfare associated with exclusive trade by buyer  $j$  with supplier  $i$  on the link  $ij \in g_N$ , denoted  $w_g(ij)$ , is defined

as

$$w_g(ij) = \begin{cases} \max_{q_{ij} \geq 0} [u_j(q_{ij}) - c_{ij}q_{ij}], & \text{if } ij \in g, \\ 0, & \text{if } ij \notin g. \end{cases} \quad (1)$$

For technical convenience, we assume that each buyer and each supplier has an incentive to trade on at least one link (otherwise we could remove such agent from the model). Additionally, for all  $g \subseteq g_N$ , all positive values for  $w_g(ij)$  are mutually different.

**Assumption 2** *Each buyer  $j \in B$  has at least one link  $i'j \in g_N$  such that  $w_g(i'j) > 0$ , each supplier  $i \in S$  has at least one link  $ij' \in g_N$  such that  $w_g(ij') > 0$ , and all positive  $w_g(ij)$ ,  $ij \in g_N$ , are mutually different.*

Finally, we denote the matrices of all prices and fees on  $g \subseteq g_N$  as  $P|g = (p_{ij})_{ij \in S \times B} \in \mathbb{R}_+^{|S \times B|}$ , respectively,  $F|g = (f_{ij})_{ij \in S \times B} \in \mathbb{R}^{|S \times B|}$ . The latter means we allow that links might be subsidized. We set  $p_{ij} = u'_j(0) > c_{ij}$  and  $f_{ij} = 0$  to represent the infeasibility of trade for any pair  $ij \notin g$ , which makes  $q_{ij} = 0$  the optimal trade in  $ij \notin g$  with supplier  $i$ 's producer surplus equal to zero.

### 3 Motivating examples

In this section, we discuss competition in both prices and fees in order to stress that it is a natural extension of the standard oligopolistic competition in prices only.

To set ideas, we first consider the smallest market possible on a non-expandable infrastructure, namely the market that consists of a single supplier that is linked to a single buyer, referred to as supplier 1 and buyer 1 (see the left-hand side of Figure 1). Quantity  $q_{11}$  will be traded against price  $p_{11}$  and fee  $f_{11}$ . Additionally, we suppose that the constant marginal costs of production and transportation are  $c_{11} = 1$ , and buyer 1 has the quasi-linear utility function  $10\sqrt{q_{11}} - p_{11}q_{11} - f_{11}$ . The maximal joint welfare in this market, which consists of the sum of the producer and consumer surplus, is twenty-five and it can be reached by setting the price  $p_{11}$  equal to marginal costs and trading  $q_{11}$  equal to twenty-five units.

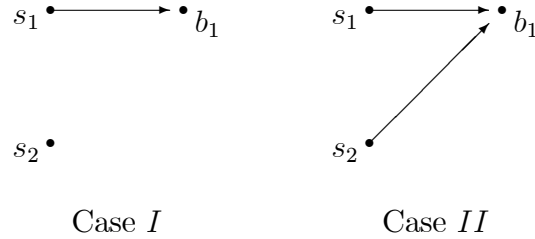


Figure 1: Supplier 1 is the single supplier in Case I, and one of two suppliers in Case II.

Such price implies that the producer surplus equals the fee  $f_{11}$ , and the consumer surplus is  $25 - f_{11}$ . The fee therefore determines how the joint maximal welfare is divided within each pair.

Given that both the supplier and the buyer can act strategically in this market, negotiations will result in marginal-cost pricing  $p_{11} = 1$  and a fee  $f_{11} \in [0, 25]$ . In case of a monopoly, the theory in Oi (1971) predicts that the supplier will extract the entire consumer surplus by setting the price  $p_{11} = 1$  and fee  $f_{11} = 25$ . Hence, the monopoly outcome is Pareto efficient, but also very unfavorable for the consumer. This result differs from the standard monopoly where a price above marginal costs is set to extract consumer surplus and fees are absent. Standard monopoly pricing is Pareto inefficient, but at least the consumer surplus is positive. The monopoly outcome in Oi (1971) can also be seen as the equilibrium outcome of a price-fee-setting game in which the supplier sets a price and fee before the buyer decides how much to buy. By reversing roles in a monopsony, the buyer will set the price  $p_{11} = 1$  and fee  $f_{11} = 0$  and it can be supported as the equilibrium outcome of a price-fee-setting game.

To further illustrate our ideas, we expand the previous situation by introducing a second supplier who is less efficient, called supplier 2, who has constant marginal costs of production and transportation  $c_{21} = 2$ . Supplier 2's price and fee are  $p_{21}$  and  $f_{21}$ . The maximal joint welfare supplier 2 and buyer 1 can attain when linked, which again consists of the sum of the producer and consumer surplus, is twelve-and-a-half. It can be reached by setting

the price  $p_{21}$  equal to supplier 2's marginal costs and trading  $q_{21}$  equal to six-and-a-quarter units. Such price implies that supplier 2's producer surplus equals his fee  $f_{21}$ , and the consumer surplus is  $12.5 - f_{21}$ . Again, the fee redistributes the joint maximal welfare. For a non-expandable infrastructure, Figure 1 illustrates two possible cases. In case the non-expandable infrastructure only links supplier 1 and buyer 1, i.e. Case I, the results for the monopoly case still apply.

We therefore consider the case with both suppliers connected to the buyer, which is Case II of Figure 1. Given that both suppliers and the buyer can act strategically in this market, supplier 1 must take into account the presence of supplier 2 in negotiations on prices and fees.<sup>4</sup> Since supplier 2 and buyer 1 can reach a joint welfare of twelve-and-a-half together, supplier 1 cannot extract more welfare from buyer 1 than twenty-five minus twelve-and-a-half, which is also twelve-and-a-half. So, negotiations will result in supplier 1's price  $p_{11} = c_{11}$  and fee  $f_{11} \in [0, 12.5]$ , and supplier 2's price  $p_{21} = c_{21}$  and fee  $f_{21} = 0$ . The theory in Oi (1971) can be easily extended to competition in prices and fees in this duopoly, if one considers the following price-fee-setting game: Simultaneously and independently suppliers set their price and fee combination, and then the buyer decides how much to demand from each supplier. Then, the unique equilibrium outcome supports the above prices and fees with  $f_{11} = 12.5$ , and the buyer purchases twenty-five units from supplier 1 and nothing from supplier 2.<sup>5</sup> Hence, this equilibrium outcome is Pareto efficient and more favorable for the buyer than the modified price-fee-setting game in the monopoly situation. This result differs from the standard duopoly where supplier 1 sets his price equal to supplier 2's marginal costs to extract consumer surplus and fees are absent.<sup>6</sup> This standard duopoly outcome is Pareto inefficient, supplier 1's producer surplus is six-and-a-quarter, and for this specific numerical example the buyer is equally well off as under competition in prices and fees. Obviously,

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<sup>4</sup>By negotiations, we envision some unmodeled negotiation process that will result in a Core solution. In this case, the two producer surpluses and the consumer surplus sum to twenty-five and the sum of supplier 2's producer surplus and buyer 1's consumer surplus is at least twelve-and-a-half.

<sup>5</sup>Since the buyer decides where to buy, existence of an equilibrium follows from Simon and Zame (1990).

<sup>6</sup>This equilibrium exists for reasons similar as in the previous footnote.

supplier 1 strictly prefers to set a price and fee instead of only a price without a fee. By doing so, supplier 1 extracts both the consumer surplus and the deadweight loss associated with standard duopoly pricing. So, the price-fee-setting game on a non-expandable infrastructure also explains the phenomenon of setting two-part tariffs in practice. By reversing the roles in a monopsony, the buyer will set  $p_{11} = p_{21} = c_{11}$  and  $f_{11} = f_{21} = 0$ , and supplier 1 will exclusively trade twenty-five units with buyer 1. Note that adding a third supplier, called supplier 3, with marginal costs above 2 will not change the above market outcomes. From here on, we consider three suppliers in our example.

The example makes one point clear: From buyer 1's perspective, the presence of the link between supplier 2 is a safeguard against supplier 1's market power, yet this link will never be utilized for trade. The link with the third-efficient supplier 3 is not needed. This is a fundamental tension between the minimal infrastructure that maximizes social welfare from trade and the minimal infrastructure that minimizes the supply side's market power. In this paper, we develop a theory that characterizes both such minimal infrastructures and we show that the former is included in the latter.

## 4 Competition on non-expandable infrastructures

In this section, we analyze oligopolistic competition in prices and fees on a non-expandable infrastructure. As the equilibrium concept, we modify the concept of a deviating (or blocking) coalition to the context of a perfectly divisible good and money on an infrastructure. We characterize the set of stable market outcomes and show that this set has a lattice structure. Then, we analyze strategic negotiation models that yield each side's most preferred stable market outcome as the unique equilibrium outcome. All these issues are treated in separate subsections, and the last one contains two important examples. In essence, this section extends well-known properties of two-sided markets with matching, as surveyed in e.g. Roth and Sotomayor (1990), to oligopolistic markets with a divisible good and money on an initial non-expandable infrastructure.

## 4.1 Competition in prices and fees

In this subsection, we introduce the notion of stability of market outcomes to analyze competition in prices and fees. This notion will be applied in characterizing the set of stable market outcomes in the next subsection.

Economic models of markets are conceived from the fundamental idea that competition between suppliers and buyers is voluntary and trading is decentralized. Bilateral trade within a pair of suppliers and buyers is voluntary only if such trade is at least as good as no trade, because otherwise some agent would be forced to trade against the prevailing conditions. Bilateral trade is not conducted by a pair of suppliers and buyers in isolation, but takes place in a market with a possibility to trade with other suppliers and buyers. The presence of other agents implies outside options for each pair, which presumably affect the conditions of trade within each pair. For the marriage market and markets with indivisible goods and money, pair-wise stability would be the appropriate concept, see e.g. Roth and Sotomayor (1990). In our market, however, the good is perfectly divisible, suppliers may trade with several buyers, and buyers may trade with several suppliers. Since we also assume that suppliers and buyers negotiate prices and fees, the Core concept seems more appropriate to define stability of market outcomes.

Market outcomes consist of prices, fees, and the quantities traded. A market outcome on a non-expandable infrastructure  $g \subseteq g_N$  is defined as the triple  $(P|g, F|g, Q|g)$ . Therefore, each market outcome generates an endogenous trade network  $T(Q|g) \in g$  of positive trades. Recall that supplier  $i$ 's active customer network is given by  $T_i(Q|g) \subseteq B$ . Similar, buyer  $i$ 's active supplier network is denoted as  $T_j(Q|g) \subseteq S$ . The Core concept imposes that any market outcome on a non-expandable infrastructure is stable if no coalition of suppliers and buyers wants to break away and trade on their own. To put it differently, all coalitions of suppliers and buyers weakly prefer the market outcome to trading as a subgroup.

Formally, for coalition  $C \subseteq N$  on infrastructure  $g \subseteq g_N$ , we define the buyer's quantity

purchased from suppliers in coalition  $C$  as

$$q_j(C|g) = \sum_{i \in T_j(Q|g) \cap C} q_{ij}.$$

For  $C = N$ , we write  $q_j(N|g)$ . Any stable market outcome  $(P^*|g, F^*|g, Q^*|g)$  yields each agent a surplus. For supplier  $i \in S$ , the surplus consists of the revenues and fees collected from his active customer network  $T_i(Q^*|g)$ :

$$\sum_{j \in T_i(Q^*|g)} [(p_{ij}^* - c_{ij})q_{ij}^* + f_{ij}^*].$$

For buyer  $j \in B$ , the consumer surplus consists of the difference between his utility from purchasing  $q_j^*(N|g)$  from his active supplier network  $T_j(Q|g)$  and the expenditures and fees paid to his active suppliers:

$$u_j(q_j^*(N|g)) - \sum_{i \in T_j(Q^*|g)} [p_{ij}^*q_{ij}^* + f_{ij}^*].$$

Any market outcome yields each coalition in total a welfare that is equal to the sum of its members' surpluses. Let  $C = C_S \cup C_B$  be the partition of coalition  $C \subseteq N$  into suppliers and buyers. The market outcome  $(P^*|g, F^*|g, Q^*|g)$  yields coalition  $C$  on infrastructure  $g \subseteq g_N$  the joint welfare:

$$\sum_{i \in C_S} \left[ \sum_{j \in T_i(Q^*|g)} [(p_{ij}^* - c_{ij})q_{ij}^* + f_{ij}^*] \right] + \sum_{j \in C_B} \left[ u_j(q_j^*(N|g)) - \sum_{i \in T_j(Q^*|g)} [p_{ij}^*q_{ij}^* + f_{ij}^*] \right]. \quad (2)$$

This is coalition  $C$ 's joint welfare in case it stays and trades according to market outcome  $(P^*|g, F^*|g, Q^*|g)$ . Next, suppose coalition  $C$  considers to deviate from the above market outcome through the alternative market outcome  $(P|g, F|g, Q|g)$ . The surplus for any supplier in coalition  $C$  consists of the revenues and fees collected from his active customer network restricted to the buyers in coalition  $C$ , i.e. for supplier  $i \in C_S$  these are all  $j \in T_i(Q|g) \cap C_B$ . Similarly, the consumer surplus of buyer  $j \in C_B$  consists of the difference between his utility from purchasing  $q_j(C|g)$  from his active suppliers in  $C$ , i.e. all  $i \in T_j(Q|g) \cap C_S$ , and the revenues and fees paid to these active suppliers in  $C$ . Since market outcome  $(P|g, F|g, Q|g)$



yields each coalition a joint welfare that is equal to the sum of its members' surpluses, deviating coalition  $C$ 's joint welfare is given by:

$$\sum_{i \in C_S} \left[ \sum_{j \in T_i(Q|g) \cap C_B} [(p_{ij} - c_{ij})q_{ij} + f_{ij}] \right] + \sum_{j \in C_B} \left[ u_j(q_j(C|g)) - \sum_{i \in T_j(Q|g) \cap C_S} [p_{ij}q_{ij} + f_{ij}] \right]. \quad (3)$$

The main difference between (2) and (3) is that if a coalition stays it will trade internally in  $C$  and externally of its coalition in  $N$ , and if it deviates as a deviating coalition it will only trade internally. In essence, by deviating coalition  $C$  disrupts several external trades. Deviating is profitable only if such disruptions can be compensated by the surpluses from internal trades within the coalition. The market outcome is stable if every conceivable deviation is unprofitable. Given the joint welfare of deviating and non-deviating coalitions, we are ready to define stability of market outcomes.

**Definition 3** *Market outcome  $(P^*|g, F^*|g, Q^*|g)$  on a non-expandable infrastructure  $g \subseteq g_N$  is stable if the following condition holds:*

$$\text{For all coalitions } C \subseteq N \text{ and all } (P|g, F|g, Q|g) : (2) \geq (3). \quad (4)$$

Condition (4) expresses the idea that deviating coalitions are unprofitable, and it is a reformulation of Core stability for cooperative games in characteristic function form. Although our setup refrains from explicit market mechanisms and is therefore not a non-cooperative market game, our stability notion resembles some ideas underlying coalition-proof equilibria, as proposed in Bernheim et al. (1987) and Bernheim and Whinston (1987). Dana (2012) characterizes such equilibria in his study of the formation of buyer groups in the presence of Bertrand price competition. In Section 4.3, we also consider such price competition but without an ex-ante stage where buyers merge into groups. Several important observations can be made. For singleton coalitions, each individual agent is weakly better off by trading than voluntarily refraining from such trade. In any stable market outcome, each pair  $ij \in g$  can at least achieve its bilateral maximal joint welfare  $w_g(ij)$  from exclusive bilateral trade within their pair. For  $C = N$ , (4) imposes Pareto efficiency on the market outcome.

## 4.2 Characterization of stable market outcomes

In this subsection, we provide a full characterization of stable market outcomes on a non-expandable infrastructure. Furthermore, we establish that the set of stable market outcomes has a lattice structure.

The characterization of stable market outcomes requires the definition of a buyer's most-efficient and second-(most-)efficient supplier on the infrastructure, which will be defined in terms of the maximal joint welfare within pairs of suppliers and buyers. For  $g \subseteq g_N$ , we define buyer  $j$ 's most-efficient supplier  $\alpha_j(g) \in S$  on the infrastructure  $g$  as the supplier on  $g$  with whom  $j$  can attain his largest maximal joint welfare:

$$w_g(\alpha_j(g)j) \geq w_g(ij), \quad \text{for all } i \in S : ij \in g. \quad (5)$$

Since maximal joint welfare is related to the most-efficient supplier in terms of marginal costs, we might alternatively define  $\alpha_j(g)$  as the supplier for whom  $c_{\alpha_j(g)j} = \min_{i \in S : ij \in g} c_{ij}$ , but (5) captures the key insight needed in Section 5. Buyer  $j$ 's most-efficient supplier  $\alpha_j(g)$  is uniquely defined if buyer  $j$  is linked through  $g$  to one or more suppliers in  $S$ . Otherwise, we impose the convention that  $\alpha_j(g) = 0$  denotes supplier "nobody" who has marginal cost  $c_{0j} = u'_j(0)$ , which makes  $q_{0j} = 0$  optimal, and the pair  $0j$  has maximal joint welfare  $w_g(0j) = 0$ . By definition, either  $c_{\alpha_j(g)j} = \min_{i \in S : ij \in g} c_{ij}$  or  $c_{0j}$ . A buyer's second-efficient supplier is defined similarly. For  $g \subseteq g_N$ , buyer  $j$ 's second-efficient supplier  $i \in S$  on the infrastructure  $g$  is the supplier  $\beta_j(g)$  on  $g$  with whom  $j$  can attain his second maximal joint welfare:

$$w_g(\beta_j(g)j) \geq w_g(ij), \quad \text{for all } i \in S \setminus \{\alpha_j(g)\} : ij \in g. \quad (6)$$

Buyer  $j$ 's second-efficient supplier  $\beta_j(g)$  is uniquely defined if buyer  $j$  is linked through  $g$  to two or more suppliers in  $S$ . Otherwise, and similar as before, we impose the convention that  $\beta_j(g) = 0$ , marginal cost  $c_{0j} = u'_j(0)$ , and the pair  $0j$  has maximal joint welfare  $w_g(0j) = 0$ . By definition, either  $c_{\beta_j(g)j} = \min_{i \in S \setminus \{\alpha_j(g)\} : ij \in g} c_{ij}$  or  $c_{0j}$ .

We have the following characterization. All proofs are deferred to the appendix.

**Proposition 4** *Market outcome  $(P^*|g, F^*|g, Q^*|g)$  on a non-expandable infrastructure  $g \subseteq g_N$  is stable if and only if for all  $ij \in g$ :*

$$\begin{aligned}
p_{\alpha_j(g)j}^* &= c_{\alpha_j(g)j}, & p_{ij}^* &\geq c_{ij}, \quad \text{if } i \neq \alpha_j(g), \\
f_{\alpha_j(g)j}^* &\in [0, w_g(\alpha_j(g)j) - w_g(\beta_j(g)j)], & f_{ij}^* &\geq 0, \quad \text{if } i \neq \alpha_j(g), \\
q_{\alpha_j(g)j}^* &= \arg \max_{q_{\alpha_j(g)j} \geq 0} u_j(q_{\alpha_j(g)j}) - c_{\alpha_j(g)j}q_{\alpha_j(g)j}, & q_{ij}^* &= 0, \quad \text{if } i \neq \alpha_j(g).
\end{aligned} \tag{7}$$

For  $i \in S$  and  $j \in B$ , supplier  $i$ 's active customers network  $T_i(Q^*|g) = \{j \in B | i = \alpha_j(g)\}$ , and buyer  $j$ 's active supplier network  $T_j(Q^*|g) = \{\alpha_j(g)\}$ .

From the proof it follows that multiple prices and fees can occur in stable market outcomes on any link  $ij \in g$  that will not be utilized for trade. Suppliers on those links know that even at their lowest acceptable prices and fees, which are  $p_{ij}^* = c_{ij}$  and  $f_{ij}^* = 0$ , their products are too expensive from the perspective of buyer  $j$ . Therefore, their prices and fees do not matter.

Proposition 4 implies a very precise prediction of stable market outcomes. Each buyer exclusively trades with his most-efficient supplier on the infrastructure against a price that equals this supplier's marginal costs and pays a positive fee. This makes buyer  $j$  vulnerable to market power exercised by his most-efficient supplier. Such market power is limited by the buyer's threat to trade with his second-efficient supplier  $\beta_j(g) \in S \cup \{0\}$  on infrastructure  $g$ . Since supplier  $\beta_j(g)$ 's current profit from zero trades with buyer  $j$  is zero, buyer  $j$  can seduce supplier  $\beta_j(g)$  to trade and by doing so guarantee himself a consumer surplus of  $w_g(\beta_j(g)j)$ . This limits the market power of the most-efficient supplier  $\alpha_j(g)$  in extracting consumer surplus from the pair  $\alpha_j(g)j \in g$ , because supplier  $\alpha_j(g)$  must make sure that buyer  $j$  enjoys at least a consumer surplus of  $w_g(\beta_j(g)j)$  through the link  $\beta_j(g)j$ . The most-efficient supplier's fee is therefore bounded from above by the difference between  $w_g(\alpha_j(g)j)$  and  $w_g(\beta_j(g)j)$ .

Under marginal cost of production and transportation, a supplier's producer surplus is equal to the sum of profits per individual link. Supplier  $i$ 's profit that can be attributed to the link  $ij \in g$  for buyer  $j \in T_i(Q^*|g)$  is equal to  $(p_{ij}^* - c_{ij})q_{ij}^* + f_{ij}^* = f_{ij}^* \geq 0$ , with

strict inequality if  $T_i(Q^*|g) \neq \emptyset$  and equality otherwise. Supplier  $i$ 's aggregate profit is equal to the sum of these fees, i.e.  $\sum_{j \in T_i(Q^*|g)} f_{ij}^*$ , and only supplier's who are some buyer's most-efficient supplier make positive profits. Then, it is impossible that trade in a link is subsidized, because  $f_{ij}^* < 0$  would be an incentive for supplier  $i$  to refuse trade on such a link. Supplier  $i$ 's active customer network consists of all buyers for whom this supplier is the most-efficient supplier.

We investigate the upper bound on the most-efficient supplier's fee for several special cases. In case  $j$  is only linked to a single supplier, then  $\beta_j(g) = 0$  and  $w_g(0j) = 0$  imply  $f_{\alpha_j(g)j}^* \leq w_g(\alpha_j(g)j)$ . The absence of a second-efficient supplier imposes no threat to supplier  $\alpha_j(g)$  and hence no limitation to this supplier's market power. In case  $j$  is linked to two or more suppliers, the difference in marginal costs between the most-efficient supplier and the second-efficient supplier matter. If this difference is relatively large, than  $w_g(\beta_j(g)j)$  will be relatively small compared to  $w_g(\alpha_j(g)j)$ , and the limiting effect of the threat to switch suppliers will be relatively small. On the other hand, if both the marginal costs of the most-efficient supplier and the second-efficient supplier are relatively close to each other, then  $w_g(\beta_j(g)j)$  will be relatively close to  $w_g(\alpha_j(g)j)$  and the presence of the second-efficient supplier has a substantial dampening effect on the most-efficient supplier's fee. In case these marginal costs would coincide, which we exclude for convenience, then  $w_g(\beta_j(g)j) = w_g(\alpha_j(g)j)$  and the most-efficient supplier's fee must be zero. This implies a testable hypothesis for industries with almost identical costs structures the equilibrium fees will be small.

The lower bound on all most-efficient suppliers' fees also has an interesting interpretation. In that case the fee  $f_{ij}^* = 0$  for all  $ij \in g$ . It can be interpreted as the generalization of the competitive equilibrium for markets on a non-expandable infrastructure in the sense that all suppliers follow marginal-cost pricing at zero fees. In this equilibrium, the entire maximal joint welfare from trade accrues to the buyers, while the suppliers have zero profits. Given different constant marginal costs, the competitive equilibrium implies relation-specific prices,

and zero profits for all suppliers, whether active or not. Obviously, relation-specific prices are Pareto efficient and a uniform price per supplier would not.

For some particular market structures, we can be more specific. In case of a monopoly, i.e.  $S = \{1\}$ , the monopolist's active customer network is  $T_1(Q^*|g) = \{j \in B | 1j \in g\}$ . By Assumption 1, each connected buyer has the monopolist as his most-efficient supplier and prefers to trade with the monopolist. Since there is no second-efficient supplier, the range of fees is maximal. This means that the only countervailing power for buyers depends upon their negotiation skills. The reverse situation occurs in case of a monopsony, i.e.  $B = \{1\}$ , where only the single link  $\alpha_1(g)1$  will have positive quantities of trade and the fee will fully depend upon supplier  $\alpha_1(g)$  and buyer  $i$ 's negotiation skills. In case of a duopoly, i.e.  $S = \{1, 2\}$ , it depends upon the infrastructure  $g \subseteq g_N$  whether each connected buyer is connected to either supplier 1, or supplier 2 or both suppliers. Those who are connected to both suppliers will be limited in their maximal fees.

For marriage markets and markets for indivisible goods and money, stable market outcomes can be ordered because the preferences of each side of the market in evaluating stable market outcomes are opposed, see e.g. the survey Roth and Sotomayor (1990). In our case this also holds true: The supply side prefers high fees among the set of stable market outcomes and the demand side prefers low fees from this set. Since the maximal joint welfare in the market is constant, an improvement in the profit of any supplier must be at the expense of the consumer surplus of some of the buyers in this supplier's active customer network. Vice versa, an improvement in consumer surplus of some buyer must be at the expense of the profit of his most-efficient supplier. Since we can arbitrarily set the fee in each pair  $(\alpha_j(g)j)$  to any  $f_{\alpha_j(g)j}^* \in [0, w_g(\alpha_j(g)j) - w_g(\beta_j(g)j)]$  independent of how we set fees in other pairs  $ij \in g$ , the lattice structure for markets with a perfectly divisible good and money on a non-expandable infrastructure follows immediately from Proposition 4.

**Proposition 5** *The set of stable market outcomes of Proposition 4 has a lattice structure. The supply side's most-preferred stable market outcome has  $f_{\alpha_j(g)j}^* = w_g(\alpha_j(g)j) -$*

$w_g(\beta_j(g)j)$  for all  $j \in B$ , and the demand side's most-preferred stable market outcome has  $f_{\alpha_j(g)j}^* = 0$  for all  $i \in S$ .

This result is similar to the results for assignment games in e.g. Roth and Sotomayor (1990). In fact, an alternative proof of Proposition 5 can be provided that applies cooperative game theory, and then shows that this game satisfies the conditions that define assignment games. Since the Core of every assignment game has a lattice structure, this structure immediately carries over to our model.

Competition in prices and fees allows the following interesting reinterpretation of this market that also explains the emergence of the lattice structure. Consider a market in which suppliers and buyers first negotiate a contract that specifies the conditions for trade, which is unlimited trade against the contract's specified fixed price. Obviously, contracts that specify marginal-cost pricing are the most valuable. If we reinterpret the market as a market for the indivisible good "buyer  $j$  has the right to trade with supplier  $i$  against price  $p_{ij}$ ", then the fees  $f_{ij}^*$  represent the standard equilibrium price for markets with indivisible goods such that each buyer purchases at most one such contract. In such markets, each buyer exclusively deals with his most-efficient supplier as if he demands at most one unit of the indivisible good. The insights provide a different intuition why the lattice structure for stable market outcomes holds.

### 4.3 Strategic negotiations

For standard assignment games, the lattice structure of the Core allows to implement each market side's most preferred outcome as the unique equilibrium outcome of a strategic negotiation model, see e.g. Roth and Sotomayor (1990). Since we have also derived a lattice structure, we will also analyze such strategic models in this subsection. In these models, the agents of one side of the market all propose prices and fees once, the other side accepts or rejects, and then the buyers decide from whom to purchase how much against the agreed upon prices and fees. We characterize the unique equilibrium and relate it to the proposing

side's most preferred stable market outcomes. In case the supply side proposes, we relate this equilibrium to the Bertrand equilibrium of standard oligopolistic price competition.

Consider the situation in which the supply side holds all market power. As an important benchmark model, we first introduce the standard Bertrand price competition. In this model, each supplier sets possibly relation-specific prices to all the buyers he is connected with. Then, each buyer decides how much to purchase from each supplier against the prices offered to him.<sup>7</sup> Formally, each supplier  $i \in S$  proposes prices  $(p_{ij})_{j \in B: ij \in g}$ , and then each buyer  $j \in B$  chooses his purchases  $q_{ij} \geq 0$ . This model is a well defined game in extensive form for which the subgame perfect equilibrium concept is the appropriate concept, which we call the Bertrand equilibrium. Dependent upon the infrastructure, buyer  $j$  may face none, one or several suppliers. In case buyer  $j$  is connected to a single supplier, then this supplier can exercise monopoly power on his link with buyer  $j$ . To capture this case on a non-expandable infrastructure  $g \subseteq g_N$ , we denote buyer  $j$ 's set of connected suppliers on  $g$  as  $S_j(g)$ . In case  $|S_j(g)| = 1$ , denote  $p_{\alpha_j(g)j}^M$  as the standard monopoly price that buyer  $j$ 's single supplier would charge. We establish the following equilibrium paths, the supporting strategy profile can be found in the proof.

**Proposition 6** *Consider the Bertrand price competition model on a non-expandable infrastructure  $g \subseteq g_N$ . In any Bertrand equilibrium, the suppliers propose*

$$\begin{aligned} \hat{p}_{\alpha_j(g)j} &= p_{\alpha_j(g)j}^M, & \text{if } |S_j(g)| = 1, \\ \hat{p}_{\alpha_j(g)j} &= p_{\alpha_j(g)j}^M, \quad \hat{p}_{\beta_j(g)j} \geq c_{\beta_j(g)j}, \quad \hat{p}_{ij} \geq c_{ij} \text{ otherwise,} & \text{if } |S_j(g)| \geq 2 \text{ and } p_{\alpha_j(g)j}^M < c_{\beta_j(g)j}, \\ \hat{p}_{\alpha_j(g)j} &= c_{\beta_j(g)j}, \quad \hat{p}_{\beta_j(g)j} = c_{\beta_j(g)j}, \quad \hat{p}_{ij} \geq c_{ij} \text{ otherwise,} & \text{if } |S_j(g)| \geq 2 \text{ and } p_{\alpha_j(g)j}^M \geq c_{\beta_j(g)j}, \end{aligned}$$

and, on the equilibrium path, the buyers purchase

$$\hat{q}_{\alpha_j(g)j} = \arg \max_{q_{\alpha_j(g)j} \geq 0} u_j(q_{\alpha_j(g)j}) - \hat{p}_{\alpha_j(g)j} q_{\alpha_j(g)j}, \quad \hat{q}_{ij} = 0 \text{ otherwise.}$$

---

<sup>7</sup>Endogenous buyers' purchases can be seen as an endogenous tie-breaking rule. As shown in Simon and Zame (1990), such an endogenous rule guarantees existence of Bertrand equilibria. For similar reasons, all our negotiation models have endogenous buyers' purchases.

This result is a straightforward extension of the standard monopoly model and standard Bertrand oligopoly with mutually different marginal costs. The multiplicity of Bertrand equilibria is nonessential in the sense that the multiple equilibrium prices for the third-efficient supplier, the fourth-efficient supplier and so on all result in a unique Bertrand equilibrium outcome: Buyer  $j \in B$  exclusively trades an amount of  $\hat{q}_{\alpha_j(g)j}$  with his most-efficient supplier  $\alpha_j(g) \in S \cup \{0\}$  against the Bertrand equilibrium price  $\hat{p}_{\alpha_j(g)j}$ . In a monopoly or in case the cost advantage of the most-efficient supplier is sufficiently large compared with the second-efficient supplier, then the most-efficient supplier can exercise monopoly power over buyer  $j$  and set the classic monopoly price  $p_{\alpha_j(g)j}^M$ . Otherwise, the presence of at least one competing supplier limits the most-efficient supplier's price to  $c_{\beta_j(g)j}$ . In the last case, buyer  $j$ 's consumer surplus is equal to  $w_g(\beta_j(g)j)$ , and the profit for buyer  $j$ 's most-efficient supplier on their link is given by  $(c_{\beta_j(g)j} - c_{\alpha_j(g)j})\hat{q}_{\alpha_j(g)j}$ . Of course, the Bertrand equilibrium is inefficient.

We now address competition in both prices and fees. Recall our reinterpretation of fees as the price for a contract that allows buyer  $j \in B$  to purchase unlimited amounts of the good from supplier  $i \in S$  against price  $p_{ij}$ . In the negotiation model, we let each supplier propose such contracts to all the buyers he is connected with. So, each supplier sets possibly relation-specific prices and fees to all such buyers. Formally, each supplier  $i \in S$  proposes prices  $(p_{ij})_{j \in B: ij \in g}$  and fees  $(f_{ij})_{j \in B: ij \in g}$ , and then each buyer  $j \in B$  first decides how much to trade with whom, so we interpret  $q_{ij} > 0$  as buyer  $j$ 's acceptance of supplier  $i$ 's contract. Also this negotiation model is a well defined game in extensive form, and by equilibrium we mean subgame perfect equilibrium. We establish the following equilibrium paths, the supporting strategy profile can be found in the proof.

**Proposition 7** *Let  $g \subseteq g_N$  be a non-expandable infrastructure. For the unique equilibrium in the negotiation model where the supply side proposes, and the demand side decides how*



much to trade with whom, it holds that suppliers propose

$$\begin{aligned} p_{\alpha_j(g)j}^* &= c_{\alpha_j(g)j}, & p_{\beta_j(g)j}^* &= c_{\beta_j(g)j}, & p_{ij}^* &\geq c_{ij}, & \text{if } i \neq \alpha_j(g), \beta_j(g), \\ f_{\alpha_j(g)j}^* &= w_g(\alpha_j(g)j) - w_g(\beta_j(g)j), & f_{\beta_j(g)j}^* &= 0, & f_{ij}^* &\geq 0, & \text{if } i \neq \alpha_j(g), \beta_j(g), \end{aligned}$$

and, on the equilibrium path, the buyers purchase

$$q_{\alpha_j(g)j}^* = \arg \max_{q_{\alpha_j(g)j} \geq 0} u_j(q_{\alpha_j(g)j}) - c_{\alpha_j(g)j} q_{\alpha_j(g)j}, \quad q_{\beta_j(g)j}^* = 0, \quad q_{ij}^* = 0, \quad \text{if } i \neq \alpha_j(g).$$

Again, the multiplicity in prices and fees is inessential. In any equilibrium,  $f_{\beta_j(N|g)j}^* > 0$  cannot hold under competition, because supplier  $\alpha_j(N|g)$  is then tempted to charge a fee slightly above the upper bound  $w_g(\alpha_j(N|g)j) - w_g(\beta_j(N|g)j)$  knowing that buyer  $j$  will not switch. Also,  $p_{\beta_j(N|g)j}^* > c_{\beta_j(N|g)j}$  allows additional out-of-equilibrium extraction of consumer surplus.

Proposition 7 states that oligopolistic competition in prices and fees is Pareto efficient, because all suppliers adopt marginal-cost pricing. This result also implies that competition in prices and fees must emerge if suppliers have the possibility to set fees. To see this, consider a buyer with several competing suppliers that limit the most-efficient supplier's Bertrand equilibrium price of Proposition 6 to  $c_{\beta_j(g)j}$ . Recall that buyer  $j$ 's consumer surplus is  $w_g(\beta_j(g)j)$ , and most-efficient supplier  $\alpha_j(g)$ 's profit of this link is  $(c_{\beta_j(g)j} - c_{\alpha_j(g)j})\hat{q}_{\alpha_j(g)j}$ . The sum of this profit plus the positive dead weight loss is equal to  $w_g(\alpha_j(g)j) - w_g(\beta_j(g)j)$ , which implies the upper bound on the most-efficient supplier's fee has a nice graphical interpretation. By adopting marginal-cost pricing and setting a positive fee, the most-efficient supplier is able to extract the Bertrand equilibrium profit and also the dead weight loss through the fee. Hence, each most-efficient supplier will choose the latter and competition in prices and fees must emerge endogenously. Only in case  $w_g(\alpha_j(g)j) = w_g(\beta_j(g)j)$ , which would imply equal marginal costs, fees will be zero.

An important issue is whether consumers are better off under Bertrand competition or competition in prices and fees. For an individual buyer  $j \in B$ , the answer depends upon whether  $\hat{p}_{\alpha_j(g)j} = p_{\alpha_j(g)j}^M$  or  $\hat{p}_{\alpha_j(g)j} = c_{\beta_j(g)j}$  in Proposition 6. In the first case we have

that  $\hat{p}_{\alpha_j(g)j} < c_{\beta_j(g)j}$ , and buyer  $j \in B$  enjoys a higher consumer surplus under Bertrand price competition than under competition in prices and fees. Hence, such buyer is worse off under competition in prices and fees. In the other case, buyer  $j \in B$  is charged  $c_{\beta_j(g)j}$  under both types of competition and enjoy a consumer surplus of  $w_g(\beta_j(g)j)$ . Then, such a buyer is indifferent between Bertrand price competition and competition in prices and fees. The second case will hold for markets in which the differences in marginal costs between most-efficient suppliers and second-efficient suppliers are sufficiently small. To summarize, consumers are weakly worse off under competition in prices and fees than under Bertrand price competition. This generalizes insights of Oi (1971).

Our results differ from the literature on two-part pricing in oligopolistic markets that all have a complete infrastructure in our terminology. Harrison and Kline (2001) assume homogenous agents on both sides, constant marginal costs and competition in quantities and fees. They report marginal-cost pricing and positive homogeneous fees. We attribute their result of positive fees to quantity competition as opposed to the zero fees under price competition in our setting. Kanemoto (2000) studies the first-order conditions for profit maximization of interior Nash equilibria in a general model of competition in prices and fees. Translated to our model, he reports marginal-cost pricing and fees that are related to the buyers' Hicksian expenditure functions. Because the maximal stable fees of Proposition 7 are boundary solutions, his analysis of interior Nash equilibria does not apply. Calem and Spulber (1984) assume two groups of buyers and two suppliers of close, but not necessarily perfect, substitutes, say pasta and rice. They implicitly impose that consumers from both buyer groups pay the same price and fee, i.e., uniform prices and fees. Under all these assumptions, uniform prices exceed marginal costs and the maximal uniform fees are set by the threat of exclusive trade with the other supplier.

In some markets, it is the demand side that has most or all market power. For example, in the airline industry buyers such as Boeing and Airbus appear to be more powerful than their suppliers of particular parts of the aircraft. Obviously, such markets can be captured

by reversing the roles of the agents in the previous negotiation model. In this modified model, we let each buyer propose one contract specifying a price and fee to the suppliers of choice he is connected with. Then, the suppliers who received a contract decide whether to accept. Then, buyers decide how much to trade with whom. Formally, each buyer  $j \in B$  proposes prices  $(p_{ij})_{j \in B: ij \in g}$  and fees  $(f_{ij})_{j \in B: ij \in g}$ , then each supplier  $i \in S$  accepts or rejects, and finally buyers purchase goods. This negotiation model is again a well defined game in extensive form, and by equilibrium we mean subgame perfect equilibrium. Without formal proof, we state the following result.

**Proposition 8** *Let  $g \subseteq g_N$  be a non-expandable infrastructure. For the unique equilibrium in the negotiation model where the demand side proposes, it holds that i) buyer  $j \in B$  always proposes to supplier  $\alpha_j(g)$  the contract  $p_{\alpha_j(g)j}^* = c_{\alpha_j(g)j}$  and  $f_{\alpha_j(g)j}^* = 0$ ; ii) supplier  $\alpha_j(g)$  always accepts, and iii) buyer  $j \in B$  demands  $q_{\alpha_j(g)j}^* = \arg \max_{q_{\alpha_j(g)j} \geq 0} u_j(q_{\alpha_j(g)j}) - c_{\alpha_j(g)j} q_{\alpha_j(g)j}$ .*

This result states that for market power on the demand side, we obtain what we have identified earlier as the competitive equilibrium in relation-specific prices. The demand side cannot do better in any stable market outcome.

## 4.4 Buyer protection

In the analysis thus far, we analyzed one particular infrastructure in isolation. In this section, we compare different non-expandable infrastructures and we pose the question which of these infrastructures provide maximal consumer protection, for which we provide a novel but natural definition. Recall that non-expandable infrastructures have sunk building costs, and we therefore compare different infrastructures by their effects on the set of stable market outcomes on  $w_g$ .

The complete infrastructure  $g = g_N$  is a special case in which each supplier is connected to each buyer. It represents the standard notion of competition in a market in which

everyone can trade with everyone else. Under sunk building costs, the complete infrastructure enables the highest joint welfare from trade, which is  $\max_{g \subseteq g_N} \sum_{j \in B} w_g(\alpha_j(g)j) = \max_{g \subseteq g_N} \sum_{j \in B} \max_{i \in S} w_g(ij)$ . So, the entire market achieves a level of maximal joint welfare of  $\sum_{j \in B} w_g(\alpha_j(g_N)j)$ . Note that the minimal non-expandable infrastructure that achieves the same level of welfare only consists of the links between each buyer and his most-efficient supplier. We define the latter infrastructure as the non-expandable infrastructure  $g_E \subseteq g_N$  given by:

$$g_E = \{ \alpha_j(g_N)j \mid j \in B, \alpha_j(g_N) \in S \}. \quad (8)$$

The issue is that on the non-expandable infrastructure  $g_E$ , buyers are unprotected against market power, because it lacks any of the links between each buyer and his second-efficient supplier. The important question to be answered is what non-expandable infrastructure serves buyers best in protecting their interests? As a criterion, we propose to maximize the worst-case for the buyers's consumer surpluses over all non-expandable infrastructures, which would be their consumer surplus with the second-efficient supplier on such infrastructure. Formally, in non-expandable infrastructure  $g \subseteq g_N$  the buyers can guarantee themselves  $\sum_{j \in B} w_g(\beta_j(g)j)$ . It is this criterion that should be maximized to optimally protect buyers. Obviously, the complete infrastructure guarantees the highest consumer surplus from trade, because  $\max_{g \subseteq g_N} \sum_{j \in B} w_g(\beta_j(g)j) = \sum_{j \in B} \max_{i \in S} w_g(\beta_j(g_N)j)$ . Note that the minimal non-expandable infrastructure  $g \subseteq g_N$  that achieves maximal protection links buyer  $j$  to his most-efficient supplier  $\alpha_j(g_N) \in S$  and his second-efficient supplier  $\beta_j(g_N) \in B$ . Without the link between  $\alpha_j(g_N)$  and  $j$ , both buyer  $j$ 's most-efficient supplier and his second-efficient supplier would change, destroying some joint welfare and some guaranteed consumer surplus. We define the minimal non-expandable infrastructure  $g_M \subseteq g_N$  that achieves maximal buyer protection under sunk building costs as:

$$g_M = \{ \alpha_j(g_N)j, \beta_j(g_N)j \mid j \in B, \alpha_j(g_N), \beta_j(g_N) \in S \}. \quad (9)$$

The set of stable market outcomes under non-expandable infrastructure  $g_M$ , or any non-

expandable infrastructure  $g \subseteq g_N$  containing  $g_M$ , is equal to the set of stable outcomes under the complete infrastructure  $g_N$ . This minimal infrastructure is rather sparse. Without any of the links  $\beta_j(g_N)j$ , the set of stable market outcomes would enlarge for all non-expandable infrastructures  $g \subseteq g_N$  containing  $g_E$ . Formally, every infrastructure  $g \subseteq g_N$  such that  $g \supseteq g_E$  and  $g \not\supseteq g_M$  has a larger set of stable market outcomes than the complete infrastructure  $g_N$ . The reason is that some of the upper bounds on the most-efficient suppliers' fees increase. Removing any of the links  $\alpha_j(g_N)j$  has two negative effect: both the maximal joint welfare on the infrastructure and the maximal attainable consumer surplus drop. To summarize this discussion, we have established:

**Proposition 9** *Non-expandable infrastructure  $g_M$  is the minimal non-expandable infrastructure  $g \subseteq g_N$  that achieves maximal consumer protection. Moreover, all non-expandable infrastructures  $g \subseteq g_N$  that contain  $g_M$  also achieve this, which includes the complete infrastructure  $g_N$ .*

This result also implies that the set of stable market outcomes for all non-expandable infrastructures  $g \subseteq g_N$  that contain  $g_M$  is the smallest set of stable market outcomes over all infrastructures that contain  $g_E$ , which includes the complete infrastructure  $g_N$ . For all infrastructures that contain  $g_E$  but only partly overlap with  $g_M$ , the set of stable market outcomes is larger.

## 4.5 Examples

In this subsection, we discuss two important examples. The first extends the motivating example of Section 3 by having a second buyer. In this example, both buyers have the same most-efficient supplier. In the second example, we consider two geographically differentiated markets, each with a domestic supplier, so that each buyer has a different most-efficient supplier. This model is also a modified version of the spatial competition model in Hotelling (1929). Both interpretations of the model are itself influential models, and our results both of them to competition in prices and fees.

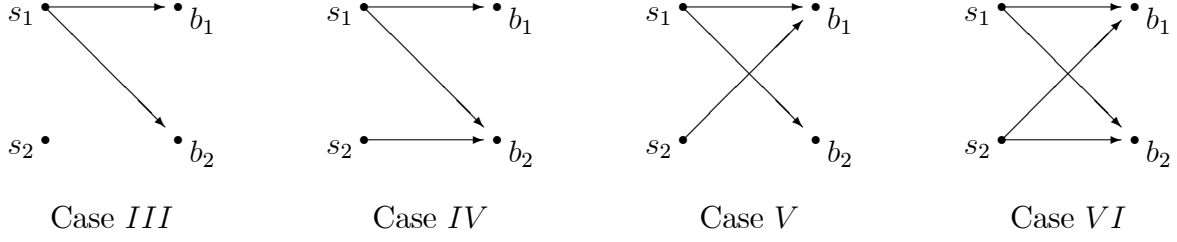
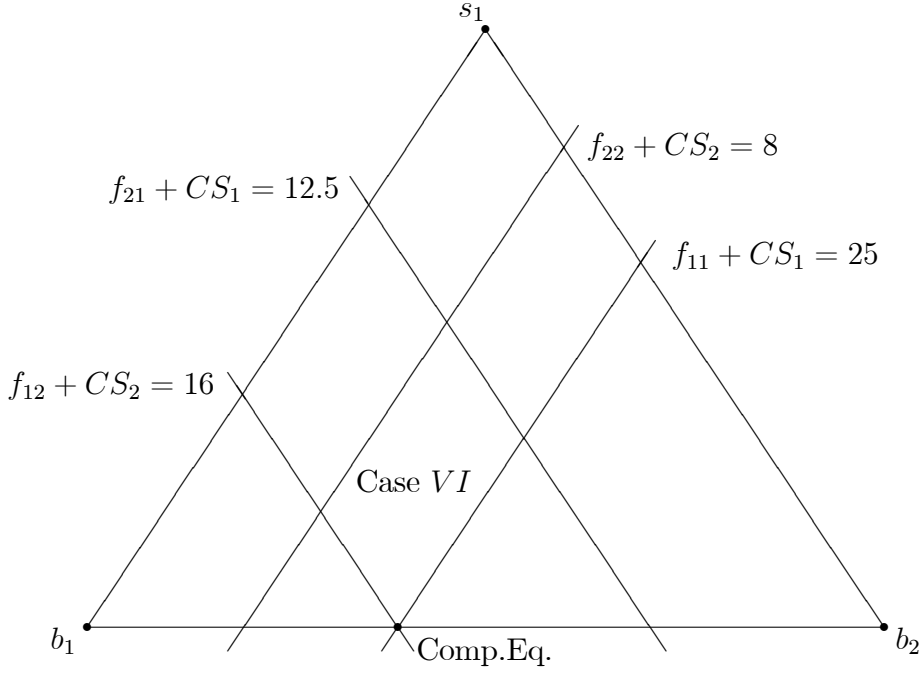


Figure 2: The single supplier infrastructure in Case III is  $g_E$ , Case IV and V represent duopoly markets, and the complete infrastructure of Case VI coincides with  $g_M$ .

**Example 10** Consider a market with two suppliers, supplier 1 being efficient and supplier 2 inefficient, and two buyers, buyer 1 having a higher marginal willingness to pay than buyer 2. Supplier 1's constant joint marginal costs of production and transportation are  $c_{11} = c_{12} = 1$ , and those for supplier 2 are  $c_{21} = c_{22} = 2$ . Buyer 1 has the quasi-linear utility function  $10\sqrt{q_{11} + q_{21}} - p_{11}q_{11} - f_{11} - p_{21}q_{21} - f_{21}$ , and buyer 2 has  $8\sqrt{q_{12} + q_{22}} - p_{12}q_{12} - f_{12} - p_{22}q_{22} - f_{22}$ . Then,  $w_g(11) = 25$ ,  $w_g(21) = 12.5$ ,  $w_g(12) = 16$  and  $w_g(22) = 8$ . In infrastructure III of Figure 2, which is  $g_E$  of (8), both buyers are only connected to their most-efficient suppliers on the complete infrastructure  $g_N$ , which is supplier 1. Then,  $0 \leq f_{11} \leq w_g(11) = 25$  and  $0 \leq f_{12} \leq w_g(12) = 16$ , and the maximal fees correspond to monopoly market power. In contrast, in infrastructure VI of Figure 2, which is  $g_M$  of (9), both buyers are connected to their most-efficient supplier, i.e. 1, and second-most-efficient supplier, i.e. 2, on the complete infrastructure  $g_N$ . Note that for this example  $g_M = g_N$ . Then, under VI the range of fees is smaller  $0 \leq f_{11} \leq w_g(11) - w_g(21) = 12.5$  and  $0 \leq f_{12} \leq w_g(12) - w_g(22) = 8$ , and the maximal fees are limited due to increased competition.

For a graphical illustration of fees and consumer surpluses in relation to non-expandable infrastructures, we can consider all possible infrastructures with two suppliers and two buyers that contain infrastructure  $g_E$ . The most relevant infrastructures are given in Figure 2, the infrastructures  $g_E$  (Case III) and  $g_M$  (Case VI), and both intermediate infrastructures. The graphical representation of the set of stable market outcomes for these non-expandable infrastructures is given in Figure 3. The largest diamond-shaped area represents the set of



For all stable market outcomes: supplier 2's producer surplus is 0

Figure 3: Different areas represent several sets of stable market outcomes of Example 10, where buyer  $i$ 's consumer surplus is denoted  $CS_i$ ,  $i = 1, 2$ . The line  $f_{21} + CS_1 = 12.5$  illustrates the effect of the link 21, and  $f_{22} + CS_2 = 8$  the link 22.

stable market outcomes in case of the single supplier infrastructure  $g_E$ . The effect of having access to second-efficient suppliers, i.e. infrastructure  $g_E$  augmented with one of the links 21 or 22 or both, are illustrated by the two lines that run through the largest diamond-shape area. The link 21 is associated with the line whose sum is 12.5, and the link 22 with 8. In case both these links are present, we are in infrastructure  $g_M$  (Case VI) and the smallest diamond-shaped area corresponds to the smallest set of stable market outcomes on infrastructures that contain  $g_E$ . Although the links with supplier 2 will not be utilized, their presence reduces the maximal fee  $f_{11}$  charged to buyer 1 from 25 to 12.5 and the maximal fee  $f_{12}$  charged to buyer 2 from 16 to 8.

**Example 11** As a second example, we consider two geographically differentiated markets. Supplier 1 and buyer 1 are situated close to each other, i.e. belong to the same geographical market, while supplier 2 and buyer 2 are located in the second market, which is distant

from the market 1. For each supplier, the marginal cost of production and transportation for the home market is 1 and for the distant market equal to 2, i.e.  $c_{11} = c_{22} = 1$  and  $c_{21} = c_{12} = 2$ . Buyers' utility functions are the same as in Example 10. In this setting we have  $w_g(11) = 25$ ,  $w_g(21) = 12.5$ ,  $w_g(12) = 8$  and  $w_g(22) = 16$ . In infrastructure VII of Figure 4, which is  $g_E$ , both buyers are only connected to their most-efficient suppliers on the complete infrastructure  $g_N$ , which is supplier  $i = j$  for buyer  $j = 1, 2$ . In contrast, for infrastructure VI of Figure 4, which is  $g_M$ , both buyers are connected to their most-efficient supplier, i.e.  $i = j$ , and second-most-efficient supplier, i.e.  $i \neq j$ , on the complete infrastructure  $g_N$ . Again,  $g_M = g_N$ . We might reinterpret this market as a discrete version of the spatial competition model in Hotelling (1929) with two buyers (where buyer 1 lives in the proximity of supplier 1 and buyer 2 lives in the proximity of supplier 2) and the differences in marginal costs, i.e.,  $c_{21} - c_{11}$  and  $c_{12} - c_{22}$ , represent buyers travel costs to visit the supplier outside their proximity.

Each supplier has a home market and may compete on his competitor's home market as well. Now, each buyer's most-efficient and second-efficient suppliers switch when compared to Example 10. As a consequence, both suppliers are active only in their regional markets and relation-specific marginal-cost pricing with fees prevails. In particular, in infrastructure VII of Figure 4 both buyers are connected only to their most-efficient suppliers, which are the suppliers on the home market. In that case, maximal fees are the highest on all infrastructures containing  $g_E$  and the ranges of fees are given by  $0 \leq f_{11} \leq w_g(11) = 25$  and  $0 \leq f_{22} \leq w_g(22) = 16$ . On the contrary, in infrastructure V, where both buyers are connected to their most-efficient and second-efficient supplier, maximal fees are limited to  $0 \leq f_{11} \leq w_g(11) - w_g(21) = 12.5$  and  $0 \leq f_{22} \leq w_g(22) - w_g(12) = 8$ . Again, buyer  $j$ 's best protection against excessive fees set by his home (most-efficient) supplier  $i = j$  is to have also access to his second-efficient supplier  $i \neq j$ , who is situated in a different geographical market. The conclusion of our model is similar to Hotelling (1929), where an increase in travel costs allows local suppliers to charge higher prices and, hence, extract a higher



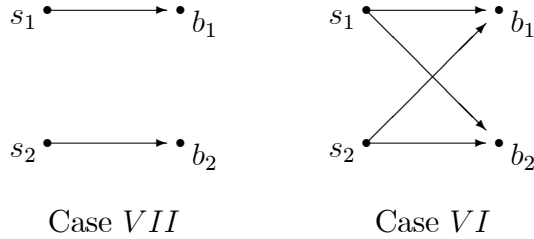


Figure 4: For two geographically differentiated markets, the single supplier infrastructure in Case VII is  $g_E$ , and the complete infrastructure of Case VI coincides with  $g_M$ .

*fraction of consumer surplus from local buyers. A similar conclusion holds in our modified price-fee-setting game: a larger difference in marginal costs would imply lower maximal joint welfare  $w_g$  (12) and  $w_g$  (21) and, hence, a smaller reduction in fees. Finally, not all the links of infrastructure VI are utilized for trade, i.e. links 12 and 21 are not utilized, but their presence prevents suppliers from abuse of market power through setting excessive fees.*

## 5 Concluding Remarks

In this study, we consider price-fee competition in bilateral oligopolies with perfectly-divisible goods, concentrated heterogeneous agents on both sides, non-expandable infrastructures, and constant marginal costs. For such markets, we define and characterize stable market outcomes that reflect that both sides possess market power. For every non-expandable infrastructure, stable market outcomes are both bilaterally and Pareto efficient because suppliers set unit prices equal to the relation-specific marginal costs. The relation-specific fees split the bilateral joint welfare and the fee implicitly reflects the suppliers' market power. In particular, each buyer exclusively trades with his most-efficient supplier on the infrastructure and the maximal relation-specific fee is limited by the buyer's threat to switch to his second-efficient supplier on the infrastructure. Marginal-cost pricing and maximal fees also arise from a negotiation model that extends differentiated Bertrand price competition. Competition in both prices and fees necessarily emerges. Although it improves welfare compared to price competition, the buyers's will be either equally off or worse off.

Our results quantify the countervailing power hypothesis that is first articulated in Galbraith (1952): Buyers have countervailing power that can restrain the market power of suppliers. In our study, buyers have a stronger bargaining position if the threat to switch orders from one supplier to another yields a larger maximal-attainable consumer surplus. We quantify this insight for any non-expandable infrastructure and, generally speaking, the supply side's market power is decreasing in the number of arbitrary links a buyer has. This implies the testable implication that relation-specific fees decrease in the number of such links. We also characterize the minimal infrastructure that protects buyers the most and identify for each buyer two links that are crucial in protecting him from the supply side's market power. Then, the other links become superfluous.

Future research should relax several assumptions made in this study. First of all, every supplier can produce any quantity demanded by the buyers that are linked to him and each link can accommodate such demand. Our setup is flexible enough to include supplier-specific caps on production and limited capacity of links by expanding Definition (1) to groups of buyers and suppliers. Moreover, the stability concept employed can be maintained. Relaxing the assumption of constant marginal costs can be handled the same way. All these model changes will enrich the insights derived from them.

The following issue is also left for future research. The minimal non-expandable infrastructure that limits the most the supply side's market power includes the minimal non-expandable infrastructure that maximizes social welfare. Also, the former infrastructure has twice the number of links as the latter one and the link with the second-efficient supplier will never be utilized in any stable outcome. For non-expandable infrastructures this is no issue, the infrastructure is fixed. But for expandable infrastructures with costly building of new links, this poses the intriguing question, when (not) to build costly links that will never be utilized but are needed to reduce maximal fees? We address this issue in the companion paper Funaki et al. (2012).

## 6 Appendix: Proofs

### Proof of Proposition 4.

For any coalition  $C$ , cancelling common terms in (2)-(4) implies

$$\sum_{j \in C_B} \left[ u_j (q_j^* (N|g)) - \sum_{i \in T_j(Q|g)} c_{ij} q_{ij}^* \right] \geq \max_{Q|g} \sum_{j \in C_B} \left[ u_j (q_j (C|g)) - \sum_{i \in T_j(Q|g) \cap C} c_{ij} q_{ij} \right], \quad (10)$$

i.e. for coalition  $C$  the sum of joint welfare in the stable market outcome is at least the maximal sum of joint welfare in the coalition  $C$ . Since all  $c_{ij}$  are mutually different, the maximum is achieved only if each buyer  $j \in C_B$  exclusively deals with his most-efficient supplier  $i \in C_S$  on  $g$ , which we denote  $\alpha_j (C|g)$  in this proof including  $C = N$  for which we already defined  $\alpha_j (g)$ . Combined with (1) for  $ij \in g$ , the right-hand side of (10) is equal to

$$\max_{Q|g} \sum_{j \in C_B} [u_j (q_{\alpha_j(C|g)j}) - c_{\alpha_j(C|g)j} q_{\alpha_j(C|g)j}] = \sum_{j \in C_B} w_g (\alpha_j (C|g) j).$$

So, for each coalition  $C \subseteq N$ , (10) is equivalent to

$$\sum_{j \in C_B} \left[ u_j (q_j^* (N|g)) - \sum_{i \in T_j(Q|g)} c_{ij} q_{ij}^* \right] \geq \sum_{j \in C_B} w_g (\alpha_j (C|g) j),$$

and for  $C = N$ , (10) is equivalent to

$$\sum_{j \in B} \left[ u_j (q_j^* (N|g)) - \sum_{i \in T_j(Q|g)} c_{ij} q_{ij}^* \right] = \sum_{j \in B} w_g (\alpha_j (N|g) j).$$

The previous arguments also imply that the latter equality holds at  $C = N$  if and only if

$$\sum_{j \in B} \left[ u_j (q_j^* (N|g)) - \sum_{i \in T_j(Q|g)} c_{ij} q_{ij}^* \right] = \max_{Q|g} \sum_{j \in B} \left[ u_j (q_j (N|g)) - \sum_{i \in T_j(Q|g)} c_{ij} q_{ij} \right].$$

Hence, for every buyer  $j \in B$  it must hold that  $q_{\alpha_j(N|g)j}^*$  maximizes  $u_j (q_{\alpha_j(N|g)j}) - c_{\alpha_j(N|g)j} q_{\alpha_j(N|g)j}$ , and  $q_{ij}^* = 0$  for all  $i \in S \setminus \{\alpha_j (N|g)\}$ . By assumption 1,  $q_{\alpha_j(N|g)j}^* > 0$ . Then also,  $q_j^* (N|g) = q_{\alpha_j(N|g)j}^*$ , buyer  $i$ 's set of active suppliers is  $T_j (Q|g) = \{\alpha_j (N|g)\}$ , and  $u_j (q_j^* (N|g)) - \sum_{i \in T_j(Q|g)} c_{ij} q_{ij}^* = w_g (\alpha_j (N|g) j)$ . This establishes  $Q^*|g$ .

Next, since trade takes place against prices  $P^*|g$ ,  $Q^*|g$  can be attained through such trade if and only if  $p_{\alpha_j(N|g)j}^* = c_{\alpha_j(N|g)j}$  and  $p_{ij}^* \geq c_{ij}$  for all  $i \in S \setminus \{\alpha_j(N|g)\}$ . The last condition in Definition 3 sets  $p_{ij}^* = c_{ij}$  for every link with  $q_{ij}^* = 0$ , and this is the case for every  $i \neq \alpha_j(N|g)$ . Since all  $c_{ij}$  are mutually different,  $p_{ij}^* > c_{\alpha_j(N|g)j}$  for all  $i \in S \setminus \{\alpha_j(N|g)\}$ .

Finally, we derive  $F^*|g$ . Given  $Q^*|g$ , consider supplier  $i$  and his active customer network  $T_i(Q^*|g)$ , that is  $\{i\} \cup T_i(Q^*|g)$ . Suppose for  $\hat{j} \in T_i(Q^*|g)$ , that supplier  $i$  and part of his trade network want to break away by excluding trade with buyer  $\hat{j}$ , that is consider coalition  $C = \{i\} \cup T_i(Q^*|g) \setminus \{\hat{j}\}$ . Given  $P^*|g$ , supplier  $i$ 's producer surplus is equal to  $f_{ij}^* + \sum_{j \in T_i(Q^*|g) \setminus \{\hat{j}\}} f_{ij}^*$ . This implies that (2) is equivalent to

$$f_{ij}^* + \sum_{j \in T_i(Q^*|g) \setminus \{\hat{j}\}} [u_j(q_{ij}^*) - c_{ij}q_{ij}^*] = f_{ij}^* + \sum_{j \in T_i(Q^*|g) \setminus \{\hat{j}\}} w_g(ij).$$

Hence, (4) imposes

$$f_{ij}^* + \sum_{j \in T_i(Q^*|g) \setminus \{\hat{j}\}} w_g(ij) \geq \sum_{j \in T_i(Q^*|g) \setminus \{\hat{j}\}} w_g(ij) \iff f_{ij}^* \geq 0.$$

Given  $Q^*|g$ , consider buyer  $\hat{j} \in B$ , his second-efficient supplier  $\beta_{\hat{j}}(N|g)$ , and this supplier's active customer network  $T_{\beta_{\hat{j}}(N|g)}(Q^*|g)$ , that is  $C = \{\hat{j}, \beta_{\hat{j}}(N|g)\} \cup T_{\beta_{\hat{j}}(N|g)}(Q^*|g)$ . Then, (4) imposes

$$\begin{aligned} & u_{\hat{j}}(q_{\alpha_j(N|g)\hat{j}}^*) - c_{\alpha_j(N|g)\hat{j}}q_{\alpha_j(N|g)\hat{j}}^* - f_{\alpha_j(N|g)\hat{j}}^* + \sum_{j \in T_{\beta_{\hat{j}}(N|g)}(Q^*|g)} w_g(ij) \\ & \geq w_g(\beta_{\hat{j}}(N|g)\hat{j}) + \sum_{j \in T_{\beta_{\hat{j}}(N|g)}(Q^*|g)} w_g(ij), \end{aligned}$$

Since  $u_{\hat{j}}(q_{\alpha_j(N|g)\hat{j}}^*) - c_{\alpha_j(N|g)\hat{j}}q_{\alpha_j(N|g)\hat{j}}^* = w_g(\alpha_j(N|g)\hat{j})$ , this condition is equivalent to

$$f_{\alpha_j(N|g)\hat{j}}^* \leq w_g(\alpha_j(N|g)\hat{j}) - w_g(\beta_{\hat{j}}(N|g)\hat{j}) \leq w_g(\alpha_j(N|g)\hat{j}).$$

The prices  $p_{ij}^* \geq c_{ij}$  and the fees  $f_{ij}^* \geq 0$  for every link with  $q_{ij}^* = 0$  are unrestricted, and this is the case for every  $i \neq \alpha_j(N|g)$ . ■

### Proof of Proposition 6.

Given the history of proposed prices  $P|g$ , we define for each connected buyer  $j \in B$  the lowest proposed price as  $\bar{p}_{ij}(P|g) = \min_{i \in S: ij \in g} \{p_{ij}\}$ , where  $\hat{i}$  denotes an arbitrary supplier who set such lowest price (which might be  $\alpha_j(g)$ ). Given history  $P|g$ , buyer  $j \in B$  purchases

$$\begin{aligned} q_{\alpha_j(g)j}(P|g) &= \begin{cases} \arg \max_{q_{\alpha_j(g)j} \geq 0} u_j(q_{\alpha_j(g)j}) - p_{\alpha_j(g)j} q_{\alpha_j(g)j}, & \text{if } p_{\alpha_j(g)j} \leq \bar{p}_{ij}(P|g), \\ 0, & \text{otherwise,} \end{cases} \\ q_{\hat{i}j}(P|g) &= \begin{cases} \arg \max_{q_{\hat{i}j} \geq 0} u_j(q_{\hat{i}j}) - p_{\hat{i}j} q_{\hat{i}j}, & \text{if } p_{\alpha_j(g)j} > \bar{p}_{ij}(P|g), \\ 0, & \text{otherwise,} \end{cases} \\ q_{ij}(P|g) &= 0, \quad i \neq \alpha_j(g), \hat{i}. \end{aligned}$$

On the equilibrium path,  $\hat{i} = \alpha_j(g)$  or  $\beta_j(g)$ , buyer  $j$  purchases  $q_{\alpha_j(g)j}(\hat{P}|g) = \hat{q}_{\alpha_j(g)j} > 0$  and  $q_{\beta_j(g)j}(P|g) = 0$ , which is in accordance to the endogenous tie-breaking rule in Simon and Zame (1990). Verification that the suppliers' strategies and the buyers' strategies form a subgame perfect equilibrium strategy profile goes as follows: On and off the equilibrium path, buyer  $j$  always purchases the optimal quantity from a supplier that offers the lowest price, so his strategy is optimal for every history. If  $\hat{p}_{\alpha_j(g)j} = p_{\alpha_j(g)j}^M$ , then supplier  $\alpha_j(g)$ 's producer surplus is maximal and this supplier does not want to deviate. If  $\hat{p}_{\alpha_j(g)j} = c_{\beta_j(g)j}$ , then the deviation  $p_{\alpha_j(g)j} > c_{\beta_j(g)j}$  implies that buyer  $j$  will exclusively trade with  $\beta_j(g)$  against the price  $\hat{p}_{\beta_j(g)j} = c_{\beta_j(g)j}$ , and supplier  $\alpha_j(g)$  loses buyer  $j$  as his customer which reduces his positive equilibrium profits on the link  $\alpha_j(g)j$  to zero. Also, the lower deviating price  $p_{\alpha_j(g)j} < \hat{p}_{\alpha_j(g)j}$  reduces supplier  $\alpha_j(g)$ 's profits. Hence,  $\hat{p}_{\alpha_j(g)j}$  is optimal given the other strategies. Since the other suppliers do not trade whether or not they deviate by setting other prices, they do not have any profitable deviating price. This establishes equilibrium.

There do not exist other Bertrand equilibrium outcomes. To see this, in any Bertrand equilibrium buyers always purchase from suppliers who set the lowest price. For  $|S_j(g)| = 1$ ,  $\alpha_j(g) = 1$  and standard monopoly pricing implies  $\hat{p}_{1j} = p_{1j}^M$  is the unique price. Next, consider  $|S_j(g)| = 2$ . Renumber the suppliers in  $S$  such that  $S_j(g) = \{1, 2\}$ ,  $\alpha_j(g) = 1$  and  $\beta_j(g) = 2$ . If  $p_{1j}^M < c_{2j}$ , it is optimal for supplier 1 to act as a standard monopolist, and supplier 2 would make negative profits by undercutting this price. Then, only  $\hat{p}_{1j} = p_{1j}^M$  and any  $\hat{p}_{2j} \geq c_{2j}$  can be equilibrium prices. Next,  $p_{1j}^M \geq c_{2j}$ . Then, modification of the

arguments for the first-price auction with mutually different valuations in Simon and Zame (1990), establishes that in any Bertrand equilibrium we must have  $\hat{p}_{1j} = \hat{p}_{2j} = c_{2j}$  and buyer  $j$  exclusively trades with supplier 1, i.e.  $q_{1j}(\hat{P}|g) = \hat{q}_{1j}$  and  $q_{2j}(\hat{P}|g) = 0$ . Finally, for  $|S_j(g)| \geq 3$ , it is straightforward to show that both  $\hat{p}_{\alpha_j(g)j} = \hat{p}_{\beta_j(g)j} \leq \min_{i \in S \setminus \{\alpha_j(g), \beta_j(g)\}} c_{ij}$ , and then the arguments for  $|S_j(g)| = 2$  apply to suppliers  $\alpha_j(g)$  and  $\beta_j(g)$ . The other  $p_{ij} \geq c_{ij}$  are unrestricted.  $\blacksquare$

### Proof of Proposition 7.

Given the history  $h$  of proposed prices  $P|g$  and fees  $F|g$ , for each connected buyer  $j \in B$  define the proposed price and fee combination from which this buyer can achieve the largest consumer surplus as  $\bar{p}_{ij}(P|g)$  and  $\bar{f}_{ij}(P|g)$ , where  $\hat{i}$  denotes an arbitrary supplier who set such combination (which might be  $\alpha_j(g)$ ). Given history  $h$ , buyer  $j \in B$  exclusively purchases from supplier  $\alpha_j(g)$  the quantity  $q_{\alpha_j(g)j}(h) = \arg \max_{q_{\alpha_j(g)j} \geq 0} u_j(q_{\alpha_j(g)j}) - p_{\alpha_j(g)j} q_{\alpha_j(g)j} - f_{\alpha_j(g)j}$  if

$$\max_{q_{\alpha_j(g)j} \geq 0} u_j(q_{\alpha_j(g)j}) - p_{\alpha_j(g)j} q_{\alpha_j(g)j} - f_{\alpha_j(g)j} \geq \max_{i \in S: ij \in g} \max_{q_{ij} \geq 0} [u_j(q_{ij}) - p_{ij} q_{ij} - f_{ij}],$$

and  $q_{\alpha_j(g)j}(P|g) = 0$  otherwise. Buyer  $j$  exclusively purchases from supplier  $\hat{i}$  the quantity  $q_{\hat{i}j}(h) = \arg \max_{q_{\hat{i}j} \geq 0} u_j(q_{\hat{i}j}) - p_{\hat{i}j} q_{\hat{i}j}$  if

$$\max_{q_{\alpha_j(g)j} \geq 0} u_j(q_{\alpha_j(g)j}) - p_{\alpha_j(g)j} q_{\alpha_j(g)j} - f_{\alpha_j(g)j} < \max_{i \in S: ij \in g} \max_{q_{ij} \geq 0} [u_j(q_{ij}) - p_{ij} q_{ij} - f_{ij}],$$

and  $q_{\hat{i}j}(P|g) = 0$  otherwise. Buyer  $j$  always purchase  $q_{ij}(P|g) = 0$  from the other suppliers. On the equilibrium path, buyer  $j$  purchases  $q_{\alpha_j(g)j}^* \equiv q_{\alpha_j(g)j}(h^*)$ , where history  $h^*$  denotes the proposed  $P^*|g$  and  $F^*|g$ . This is in accordance to the endogenous tie-breaking rule in Simon and Zame (1990). Verification that the suppliers' strategies and the buyers' strategies form a subgame perfect equilibrium strategy profile is as follows: For any history  $h$  on and off the equilibrium path, buyer  $j$  always purchases the optimal quantity from one of the suppliers from which he achieves the maximal consumer surplus dependent upon the history  $h$ , so his strategy is optimal. Since  $p_{\alpha_j(g)j}^* = c_{\alpha_j(g)j}$  and  $f_{\alpha_j(g)j}^* = w_g(\alpha_j(g)j) - w_g(\beta_j(g)j)$ ,

then any deviation  $p_{\alpha_j(g)j} \geq c_{\alpha_j(g)j}$  and  $f_{\alpha_j(g)j} \geq f_{\alpha_j(g)j}^*$  with at least one strict inequality implies that buyer  $j$  will exclusively trade with  $\beta_j(g)$  against  $p_{\beta_j(g)j}^* = c_{\beta_j(g)j}$  and  $f_{\beta_j(g)j}^* = 0$ , and supplier  $\alpha_j(g)$  loses buyer  $j$  as his customer which reduces his equilibrium profit on the link  $\alpha_j(g)j$  from  $f_{\alpha_j(g)j}^* > 0$  to zero. Also, the lower deviating fee  $f_{\alpha_j(g)j} < f_{\alpha_j(g)j}^*$  reduces supplier  $\alpha_j(g)$ 's profits. Hence,  $p_{\alpha_j(g)j}^*$  and  $f_{\alpha_j(g)j}^*$  are optimal given the other strategies. Since the other suppliers do not trade independent of the prices and fees they set, there does not exist any profitable deviating price and fee combination for any of them. This establishes equilibrium.

There do not exist other equilibrium outcomes. To see this, in any equilibrium buyers always purchase from suppliers who set a price and fee combination from which buyers can achieve a maximal consumer surplus. For  $|S_j(g)| = 1$ , we remember such that  $S_j(g) = \{1\}$ . Since  $\beta_j(g) = 0$  and  $w_g(0j) = 0$ , we have that  $p_{1j}^* = c_{1j}$  and  $f_{1j}^* = w_g(1j)$  are optimal by the results in Oi (1971). Next, consider  $|S_j(g)| = 2$ . Remember the suppliers in  $S$  such that  $S_j(g) = \{1, 2\}$ ,  $\alpha_j(g) = 1$  and  $\beta_j(g) = 2$ . There cannot exist an equilibrium in which buyer  $j$  exclusively trades with supplier 1 against  $p_{1j}^* > c_{1j}$  and

$$f_{1j}^* = \max_{q_{1j} \geq 0} [u_j(q_{1j}) - p_{1j}^* q_{1j}] - w_g(2j) < w_g(1j) - w_g(2j),$$

because then supplier 1 can increase his profits arbitrarily close to  $w_g(1j) - w_g(2j)$  by setting the deviating price  $p_{1j} = c_{1j}$  and  $f_{1j} = w_g(1j) - w_g(2j) - \varepsilon$ , for sufficiently small  $\varepsilon > 0$ . So, in any equilibrium, supplier 1 can secure  $w_g(1j) - w_g(2j)$ . Also, an equilibrium with  $p_{1j}^* > c_{1j}$  and  $f_{1j}^* > \max_{q_{1j} \geq 0} [u_j(q_{1j}) - p_{1j}^* q_{1j}] - w_g(2j)$  is impossible. So,  $p_{1j}^* = c_{1j}$  in any equilibrium. Next, there cannot exist an equilibrium with  $p_{2j}^* > c_{2j}$  and  $f_{2j}^* = 0$  either. To see this, first note that then buyer  $j$  would exclusively trade with supplier 1 against  $p_{1j}^* = c_{1j}$  and

$$f_{1j}^* = w_g(1j) - \max_{q_{2j} \geq 0} [u_j(q_{2j}) - p_{2j}^* q_{2j}] > w_g(1j) - w_g(2j).$$

Exclusive trade is dictated by the insights in Simon and Zame (1990). Supplier 2 will have a profit of zero, and could obtain a positive profit by setting  $p_{2j} = c_{2j}$  and  $f_{2j} =$

$\frac{1}{2} (f_{1j}^* - [w_g(1j) - w_g(2j)]) > 0$ , a contradiction. Finally, there cannot exist an equilibrium with  $p_{2j}^* = c_{2j}$  and  $f_{2j}^* > 0$ , because then buyer  $j$  would exclusively trade with supplier 1 at the fee  $f_{1j}^* = w_g(1j) - w_g(2j) + f_{2j}^*$  (exclusive trade again by Simon and Zame (1990)). Supplier 2 will have a profit of zero, and could obtain a positive profit by decreasing his fee to  $f_{\beta_j(g)j} = \frac{1}{2}f_{\beta_j(g)j}^* > 0$ . So, in any equilibrium  $p_{2j}^* = c_{2j}$  and  $f_{2j}^* = 0$ . Since also  $p_{1j}^* = c_{1j}$  is necessary, and supplier 1 can secure a profit of  $w_g(1j) - w_g(2j)$ , the unique equilibrium fee must be  $f_{1j}^* = w_g(1j) - w_g(2j)$ . By Simon and Zame (1990), this can only be supported by exclusive trade between buyer  $j$  and supplier 1. Finally, for  $|S_j(g)| \geq 3$ , only suppliers  $\alpha_j(g)$  and  $\beta_j(g)$  matter in the argument, the other suppliers'  $p_{ij} \geq c_{ij}$  and  $f_{ij} \geq 0$  are unrestricted.

■

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