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New Developments in the Measurement of Welfare and Well-being

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Abstract

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This paper is dating from 1995, when it has been presented at the Ragnar Frisch Centennial Memorial Conference in Oslo. It has never been published before.
In this paper for the first time the Cantril ladder question data have been employed in the way which later has become known as happiness economics. After two introductory sections 1and 2, Section 3 explains the Leyden School methodology to estimate financial satisfaction or in traditional terms a (cardinal) welfare function of money.
In Section 4 the Cantril ladder question is employed to estimate a function of satisfaction with life as a whole. It is found that well-being is quadratic in the number of children, leading to an optimum number of children, given income and given the fact of a one-breadwinner- or two- breadwinners-family. In Section 5 the effects of children on financial satisfaction and on satisfaction with life as a whole are compared. With respect to financial satisfaction it is found that the more children there are the smaller financial satisfaction. Comparison of the two effects makes it possible to distinguish between the monetary cost associated with having children and the non-monetary benefits caused by having children. Part of this paper is based on Plug and Van Praag (1995).

Keywords: happiness economics, Leyden School, Cantril Ladder, family equivalence scales, costs and benefits of children.

JEL-codes: B50, D190, J1, D6.

[1] The paper is unchanged but in December 2011 an abstract, JEL-codes and keywords have been added.

New Developments in the Measurement of Welfare and Well–Being

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1 Introduction

There is hardly any concept in economics with such a mystic spell as the concept of "utility". At various moments in time it was considered to be the "deus ex machina" in order to explain and to predict human behavior; at other times it was ridiculed as a hopeless fake which could only blur the progress of economic thought.

The ambiguity within the profession towards the concept stems from two sources: at one hand it is the excellent vehicle to explain economic behavior, at the other side there is uncertainty, to put it mildly, on how it can be measured and operationalized.

Most economists nowadays, who use the utility concept (or welfare, happiness, ophelimity, etc.) –and that is an increasing number–, begin to ignore the measurement difficulty and simply start as in a fairy tale: "let us assume that an individual has a utility function $U(\cdot)$, which we specify for convenience and without loss of generality as...".

Other economists feel uneasy about using that charming, intuition-based and most useful theoretical tool when it is not clear how it can be measured or still worse whether it is measurable at all.

Frisch (1932) and his fellow Nobel laureate Tinbergen (1972, Geary Lecture) were among the few economists who always believed that utility would prove to be measurable and they themselves tried to estimate utility functions. That their attempts were not accepted widely has not withered their beliefs.

The main alley from which the utility concept is approached is the explanation of choice behavior. It is obvious that any rational choice is based on a criterion function which assigns to any feasible alternative a utility value. Comparing utility values the consumer or more generally the decision maker chooses the alternative with the highest value. Let the alternatives be denoted by $a \in A$ and let the utility function be $U(a)$, then the basic description of the decision problem is

$$\begin{array}{l} \max U(a) \\ a \in A \end{array}$$

The first field where the utility concept enters into economic science is consumer theory. There the feasible set is the budget set

$$p_1x_1 + \dots + p_nx_n \leq y$$

where x is a commodity vector in $(\mathrm{R}^+)^n$ and p is the relevant price vector, while y stands for current income. Assuming a utility function $U(x)$ the consumer problem boils down to

$$\begin{array}{ll} \max & U(x) \\ \text{sub} & p'x \leq y \end{array}$$

Let for a specific (price, income) combination (p, y) the maximum feasible utility be $V(p, y)$, then this indirect utility function may be considered as *the* utility value of a specific money amount y under given prices p. Utility of money was (and is) considered as a basic instrument for the evaluation and comparison of intra– and interpersonal income differences, culminating in basing tax rules and equivalence scales on utility differences. If one believes that an individual's situation is completely characterized by his income we may even use the money utility to compare the situations of well-being of one person in two different situations or between persons.

However, Pareto (1904) showed quite clearly that the utility function cannot be derived from observing purchase behavior. Let a utility function on the commodity space be written as $U(f(x))$, then we may distinguish two dimensions (see Van Praag (1991)). The first dimension is in the commodity space where we observe the contour lines (or indifference curves) $f(x)=c$; the second dimension is the utility dimension $U(c)$. As Pareto demonstrated, we can only derive the function $f(\cdot)$ but not the translation into the utility dimension $U(c)$. This led to the concept of an ordinal utility function; Hicks and Allen (1934) and Houthakker (1950) showed that (static) consumer theory could dispense with the utility function altogether and that only the concept of a preference ordering described by a preference function was really essential.

Actually the problem discovered by Pareto is an example of a more general question, which is imminent in all situations where we attempt to derive the underlying criterion function from choice behavior only. It is never possible to find a unique criterion function from the observation of choice behavior only. We have to make additional assumptions on the shape of the criterion function. An example is the derivation of the utility function of basic events by looking at the choice between lotteries by making the Von Neumann–Morgenstern (1944) assumption on expected utilities.

Why is choice behavior not sufficient to identify a unique underlying utility function? The answer is that a utility function supplies us with comparative evaluations of alternatives. For making a choice between them not all information in such an evaluation function is necessary; only the ordering is needed. The utility function serves more purposes then choice alone. It says how much better one alternative is than the other. This explains that we have either to add restrictive functional specifications on the utility function to our observation of the choice process or that we have to change the nature of our observations by not looking at choices made only or even to discard the observed choices as our informative observation tool. It is the latter way which we shall follow in this paper.

2 Equivalence scales based on utility

The utility function remained in use even in abstract literature (like Debreu (1959)) but merely as a didactic device. In the late seventies the utility function became respectable again (Deaton and Muellbauer (1980), Jorgenson et al. (1987)) as an instrument to describe and estimate simultaneous demand systems, but again the utility functions were only ordinally specified.

In this literature we find also attempts to estimate family equivalence scales. The idea is that indirect utility depends not only on one's purchase power as given by (p, y) but also on the size of one's family, say fs, or in short $V=V(p, y, fs)$. Comparing two different family sizes fs_1 and fs_2 we may ask which income y_2 is equivalent to a given y_1 in order that utility for the two families is equal. This problem is answered by solving the equation

$$V(p, y_1, fs_1) = V(p, y_2, fs_2) \tag{2.1}$$

for y_2. In general we will find an explicit solution

$$y_2 = g(y_1, fs_1, fs_2; p) \tag{2.2}$$

For a homothetic utility function we find price independence. If we find that

$$\frac{y_2}{y_1} = g(fs_1, fs_2)$$

we call the family equivalence scale Independent of Base (IB) (see Lewbel (1989) and also Blackorby and Donaldson(1991)). However, again we stumble on the problem that $V(p, y, fs)$ cannot be derived from looking at purchase behavior only. Consider the dependency of U on fs. It may be of the type

$$U = U(f(x; fs), fs) \tag{2.3}$$

Or in words, fs may affect $U(\cdot)$ in *two* ways, first by influencing- the shape of the indifference curves reflected in changed purchase behavior, and second by influencing the relation $U(c; fs)$. As our observation of purchase behavior sheds only light on the shape of the indifference curves ,i.e. $f(\cdot)$, we cannot derive an equivalence scale based on $V(\cdot)$. The indirect utility functions derived are coditional upon family size fs. Only if the "purchase" of children at known prices was a part of the observed purchase process, we could find the equivalence scales we looked for.

This important result was first formulated by Pollak and Wales (1979). It follows that all attempts at deriving family equivalence scales from purchase observations are only valid if we assume that there is no additional influence of children on individual utility except via the buying of commodities, (see also for a survey Van Praag and Warnaars (1995)). The same result holds actually for equivalence scales with respect to age, region, and what have you, when the observations do not cover choices on age, region, etc.. Both the tax and the family equivalence problem show that there is an urgent need for knowledge of a cardinal $V(\cdot)$, which admits for unambiguous utility comparisons for one individual in different situations and also between two or more individuals at the same time.

Hicks and Allen (1934) but especially Robbins (1932) extended Pareto's result to mean that cardinal utility did not exist and hence was unmeasurable by definition. Their verdict has done a lot of wrong to our profession, as they declared one way to make our profession more realistic and applicable inadmissible from the beginning. Although the case of "cardinal utility" was a difficult case from after World War II, Frisch and Tinbergen remained to profess their beliefs in the possibility and the necessity to find a measurement method to operationalize cardinal utility. We notice that the utility concept is now quite common in all analysis of decision making under uncertainty (insurance, game theory), decisions over time (saving, growth) and in distributional analysis (income and welfare inequality measurement).

3 A different approach: the Welfare Function of Income (WFI)

As the previous approach based on observing derived behavior appears to fail, in Van Praag (1971) a first attempt was made to establish utility values straightforwardly by a direct questioning approach. The basic instrument is a question module, the so-called Income Evaluation Question, which runs as follows:

> "Which monthly household after tax income would you in your circumstances consider to be very bad? Bad? Insufficient? Sufficient? Good? Very good?
>
> About $.......very bad.
> About $.......bad.
> About $.......insufficient.
> About $.......sufficient.
> About $.......good.
> About $.......very good."

The number of levels distinguished is now mostly fixed at six, although in the first publications we used eight or nine levels; recently five or four levels have been used as well. Let the answers of the IEQ be denoted by c_1, c_2, c_3, c_4, c_5 and c_6, (or $c_1, \cdots, c_k$ for k levels) and let us define

$$\mu = \frac{1}{6}\sum_{i=1}^{6} \ln c_i \tag{3.1}$$

and

$$\sigma^2 = \frac{1}{5}\sum_{i=1}^{6} (\ln c_i - \mu)^2 \tag{3.2}$$

The first basic presumption is that the verbal labels "very bad", "good" and so on, convey the same emotional meaning to all respondents. This assumption is actually the corner stone of a language community, words should mean or are assumed to mean the same to each member of that community. It is well–known from everyday life from semantic and psychological research that this is not exactly true, however deviations are assumed to be corrected by the error term.

For practical purposes, the verbal labels are translated into numbers on a zero–one scale, and more precisely the first (worst label) is identified with 1/12, the following with 3/12...and the sixth label with 11/12. This presupposes that verbal qualification may be translated into a numerical scale and that we use equal intervals. Both assumptions are always used in school evaluations but also in other tests like ice–skating or commodity tests by consumer unions.

In Van Praag and Van der Sar (1988) and Van Praag (1991, 1994a, 1994b) it was shown that the Equal Interval Assumption holds approximately very well by asking people directly whether they would translate words into numerical evaluations. A different more theoretical argument for this assumption has been proposed by Van Praag (1971) and Kapteyn (1977).

For theoretical arguments, (see Van Praag (1968)), we proposed a lognormal specification $U = \Lambda(u; \mu, \sigma)$ where μ and σ are the previously defined parameters, which may differ for individuals. It was found from a multitude of large–scale samples that this lognormal specification was empirically acceptable, although Kapteyn and Van Herwaarden (1979) showed that other functional specifications did also perform fairly well. The logarithmic specification did even slightly better, but it was discarded for the reason that it yields an unbounded function which is psychologically less credible. This paradigm has been developed at Leyden University by Van Praag, Kapteyn, Hagenaars and others. Although none of the authors is still working at Leyden, it is sometimes referred to as the Leyden–School. All these assumptions were heavily criticized by Seidl (1994). In a reaction by Van Praag and Kapteyn (1994) it was shown that Seidl's critique was ill–founded and can be discarded.

The resulting function has been called the (Individual) Welfare Function of Income (WFI). The interesting point of the concept lies in its applications with respect to family equivalence scales, poverty, income inequality and so on.

The parameter μ is found to vary over individuals in a very consistent way. For most West–european countries we estimated the following equation

$$\mu = \beta_0 + \beta_1 \ln fs + \beta_2 \ln y_c \tag{3.3}$$

where y_c stand for current income. It has been surprisingly good and stable outcomes of about

$$\mu = \beta_0 + 0.10 \ln fs + 0.60 \ln y_c$$

have been generated. On the contrary the attempts to explain σ have met with only limited success. So in most analyses σ is taken to be randomly varying over individuals. The resulting welfare level U corresponding to an arbitrary income level y is found by standardization to be

$$U = N\left(\frac{\ln y - \mu}{\sigma}; 0, 1\right)$$

resulting into

$$U = N\left(\frac{\ln y - \beta_0 - \beta_1 \ln fs - \beta_2 \ln y_c}{\sigma}; 0, 1\right)$$

Assuming σ to be constant, an alternative (ordinal) welfare index is

$$\ln c = \ln y - \beta_2 \ln y_c - \beta_1 \ln fs - \beta_0 \tag{3.4}$$

It follows that the individual welfare evaluation of any income level y depends of fs and current income y_c, i.e. U=$U(y; y_c, fs)$ that must hold for that income level y. A specific welfare level α is reached by realizing that

$$N\left(\frac{\ln y - \mu}{\sigma}; 0, 1\right) = \alpha \tag{3.5}$$

This corresponds to

$$\ln y = \mu + u_\alpha \sigma \tag{3.6}$$

where u_α is the (normal) α–quantile and where μ varies over individuals. Hence,

$$\ln y = \beta_2 \ln y_c + \beta_1 \ln fs + \beta_0 + u_\alpha \sigma \tag{3.7}$$

may be interpreted as a <u>household</u> <u>cost–function</u> where u_α is an ordinal utility index (see Van Praag and Van der Sar (1988)). In Van Praag (1991) it is shown how this concept can be linked to an ordinary indirect utility functions in which prices appear. Here we only observe that μ must be first order homogeneous in prices and σ zero–homogeneous in prices.

Family equivalence scales can be derived by evaluating current income y_c in welfare terms as

$$U(y_c; y_c, fs) = N\left(\frac{(1-\beta_2)\ln y_c - \beta_0 - \beta_1 \ln fs}{\sigma}; 0, 1\right) \tag{3.8}$$

It follows that a change from fs_1 to fs_2 has to be compensated by

$$\ln y_2 - \ln y_c = \frac{\beta_1}{1-\beta_2}(\ln fs_2 - \ln fs_1) \tag{3.9}$$

We notice that this index is Independent of Base. It follows that

$$\frac{y_2}{y_c} = \left(\frac{fs_2}{fs_1}\right)^{\frac{\beta_1}{1-\beta_2}} = \left(\frac{fs_2}{fs_1}\right)^{0.25} \tag{3.10}$$

if we accept the previously mentioned values for β_1 and β_2. We notice however that the value $\beta_1/(1-\beta_2)$ varies over countries; in America it reaches abouth 1/3 (Dubnoff et. al.(1981) speaks about the "cube law") and in Greece, Portugal, Poland, Czechoslowakia still higher values in the range of 0.40–0.50 have been found. It indicates that scales are different over countries and that they are steeper, the less developed child support in the specific country is. The value of about 0.25 suggests a rather flat scale (see also Buhmann et. al. (1988))

4 A new approach: Welfare and Well–being

The method of direct questioning may have been rather unusual or even suspect for economists at the time (1971) it was originated, in other behavioral sciences this approach is fairly standard. Cantril (1965) devised what we shall call the Cantril–question

> "Here is a picture of a ladder, representing the ladder of life. The bottom of this ladder, step 0, represents the worst possible life while the top of this ladder, step 10, represents the best possible life.
>
> Where on the ladder do you feel you personally stand at the present time?"

Obviously this question tries also to measure something but "it" is much wider than satisfaction derived from income but it is satisfaction of life as a whole. From now on we call the first (Leyden) concept welfare and the second concept well–being. It is tempting to explain the responses on the Cantril question as well. We refer to Plug and Van Praag (1995) for more details.

It is obvious that both welfare and well–being are metaphysical concepts. Just like other concepts, they can be coupled with a physical (measurable) counterpart by defining a measuring experiment and an operational definition of how to measure the concept. In fact the physical counterpart is thus defined. In the physical sciences a measurement method is accepted as yielding a succes in operationalization of a concept, if the operationalized concept matches our expectations as variation resembles what we expect from the metaphysical pre–scientific concept, (c.f. the definition of an operational temperature concept by a thermometer).

In a similar way we operationalize the metaphysical concepts welfare and well–being. Notice that the Cantril respondent responds with a number W on a zero–ten scale. In order to stretch the explained variable on a $(-\infty, +\infty)$ range we replace the responses by $w = N^{-1}(W; 0, 1)$ where N stands for the standard normal distribution function. In order to test whether W and U as measured by Cantril and the IEQ respectively measure the same or different concepts, Plug and Van Praag (1995) tried the same set of explanatory variables on both variables w and μ and it was found that for the explanation of w we needed a much larger set than μ. Hence, we concluded that Cantril and IEQ were totally different concepts indeed.

In view of the following sections we present here slightly different specifications than in Plug and Van Praag. They have been derived from a rather large sample survey (N=8446) with written anonymous questionnaires carried out in 1991. We refer to Plug and Van Praag for details.

–insert Table 1–

Looking at the μ–equation we see that there is a considerable age–effect as μ rises with increasing age up to a maximum at the age of about 47 year after which income evaluation falls again with age. This implies that need for income increases first and falls later in life. It may also be rewordered as that the same income becomes less satisfactory first and that satisfaction derived after the age of 47 is rising again.

The w–equation depends on a much richer set of variables. First we notice a family size squared and interaction terms of family size with income and the dummy representing two–breadwinner families. It follows that Cantril equivalence scales are not IB as they depend on income and that they differ for two–breadwinners. The fs^2–term indicates that there is an optimal family size which varies with income and the breadwinner variable. Second we see a u-shape relation with age which is just the inverse of that found for μ. We find that well–being is falling with age up to about 37 years and then rising with age. Furthermore in this survey respondents were asked whether they had problems with drugs, the family, how positive they felt about the quality of government, their own health, their neighborhood, their parents, the relation with their partner, whether they had problems with sleeping and their work. The "problem" variables have been rescaled by applying the inverse normal distribution function on the bounded scales, differentiated into subgroups according to age, gender, employment status and education. Finally we include dummies for having a job or not and gender of the respondent. Nearly all these variables were significant and had the "right" sign. In this paper we shall not consider all variables in detail, but we shall focus on the fs–variable. It is noticed that

$$\frac{\partial w}{\partial \ln fs} = \gamma_0 + 2\gamma_1 \ln fs + \gamma_2 \ln y_c + \gamma_3 D_{tw} \tag{4.1}$$

where the γ's read as respectively $-1.526, -0.131, 0.175$ and -0.0327. It follows that the sign of the derivative is not unambiguous. It rises with income. In Figure 1 its shape as a function of $\ln fs$ is sketched for various rising income levels y_1, y_2 and y_3.

–insert Figure 1–

It follows that there is an optimal family size for the various income levels reached. For low income y_1 it is always negative, implying that that family would prefer to be childless. The more income you receive, the higher the optimal family size.

–insert Table 2–

5 Costs and benefits of children: An combination of both welfare concepts

In Section 3 we demonstrated how to derive family equivalence scales in a simple and elegant way from the IEQ results. In this Section we shall compare this with results from a similar analysis on the basis of the Cantril–module. The analysis in this Section is mostly based on Van Praag and Plug (1993). Given the results for the Cantril equation of well–being it is also possible to derive family equivalence scales for well–being, but the results will be rather unusual in the sense that their sign is not unique.

Defining a shadow price Δy for the $(fs+1)$st family member by solving

$$U(y, fs) = U(y + \Delta y, fs + 1) \tag{5.1}$$

we get shadow prices for additional family members. We restricted the sample to two–adult household. Hence fs=3 indicates one child. For the IEQ it is easily seen that the shadow price is always negative. Individuals have to get a <u>positive</u> additional income Δy to feel equal welfare after the "$fs+1$"st has arrived. If we replace U by V and do the same exercise, we find the corresponding shadow prices for the well–being concept and they turn out to be rising with fs and rising with income. A negative shadow price implies that a child is desired, but the price of a marginal child falls. Both types of shadow prices are tabulated in Table 3.

–insert Table 3–

At first sight this difference is striking, until we realize that both question modules and the concepts derived from it measure different metaphysical concepts.

The first focuses on <u>income</u> and hence on monetary costs. The second (Cantril) concept focuses on the ladder of <u>life</u> (as a whole). For life as a whole an additional child means less purchase power but also family blessing. In short these are non–cost aspects to be derived from having a child which are beneficial for most families. The Cantril shadow prices cover the benefits <u>and</u> the pure cost aspects. More precisely a child may cause

A. a monetary cost increase

B. a non–monetary benefit

The non–monetary benefit may also be negatively valued for those who are child–haters. Referring to the Greek $\mu\eta\delta\epsilon\nu\ \alpha\gamma\alpha\nu$, "of nothing too much", any family will finally at the arrival of the n^{th} child exclaim "no more". For families with a very low income even the non–monetary benefit of the first child may be negative. In the case of a negative non–monetary benefit we speak of non–monetary costs.

It follows that the Cantril price tabulated in Table 3 may be identified as *B–A* while we assume that the Leyden price stands for *A* only. It follows that subtracting the first part of Table 3 from the second and third part we calculate the counter monetary value of the non–monetary benefits (or costs). Using an infinitesimal approach we define the shadow prices as

$$\begin{array}{lll} A: & \Delta y = -\left(\frac{U_{fs}}{U_{yc}}\right)\Delta fs & \text{monetary cost} \\ B: & \Delta y = -\left(\frac{V_{fs}}{V_{yc}}\right)\Delta fs & \text{total shadow price} \\ B-A: & \Delta y = -\left(\frac{U_{fs}}{U_{yc}} - \frac{V_{fs}}{V_{yc}}\right)\Delta fs & \text{non–monetary benefits} \end{array}$$

In Table 4 we tabulate the value of those non–monetary benefits as the result of this subtraction.

–insert Table 4–

We notice that the pure benefit value of a child after correction for monetary costs is not extremely large except for the first two children in the well–to–do families. On the other hand this is an annual benefit, which when discounted for say at 10% yields capital value of ten times as much. These amounts conform pretty well to adoption costs where children are "imported" into the household from the own country or from abroad.

6 Conclusions

Just like Frisch we are convinced that cardinal utility is a necessary ingredient for a major part of economic analysis. In this paper we outlined first that

1. there are more welfare concepts than one.
2. that after suitable operationalization these concepts are measurable by means of questioning methods.
3. that the concepts of welfare and well–being functions mostly depend on the own situation as a point of reference. For the IEQ the evidence is that the evaluation of arbitrary income levels depends on own current income. (This phenomenon is called preference drift (Van Praag (1971)).

4. that it may be useful to exploit two or more welfare concepts simultaneously to split up balances of two or more effects into their components.

Obviously our analysis in this paper is not authoritative in the sense that no other operationalizations or explanations of welfare concepts are possible. However, it is not true that there is only one all–embracing concept. Concepts may compete in the sense that both claim to reflect the same metaphysical concept. For the coming time it does not seem a first priority to defend claims for different concepts. That is a luxury problem. The first priority is to exploit this way further, because as the list of variables in Table 1 demonstrates the Cantril concept covers a multitude of aspects.

As this paper gives evidence for, welfare or well–being is not an exclusively economic concept. It is a concept which has to do with income, but also with a lot of traditionally non–economic variables as well. May be, the Cantril–concept and the IEQ may serve as a linking–pin between economic science and their social sister–sciences.

Anyway, this analysis is certainly inspired by the unorthodox non–dogmatic way in which Professor Frisch, whom we commemorate today, made his contribution to our science. I am happy to notice that I sent my first paper (as a manuscript) to Professor Frisch and that Prof. Frisch according to a (dictated) letter expressed his appreciation for my work as a valuable addition to his own life–long quest for measurable utility.

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Table 1[a]

THE μ EQUATION

	ESTIMATE	T-VALUE
Constant	1.807	4.311
$\ln fs$	0.054	7.714
$\ln y$	0.601	95.194
$\ln a$	1.115	4.781
$\ln^2 a$	−0.144	−4.547
N	8447	
R^2	0.529	

Table 1[b]

THE CANTRIL EQUATION

	ESTIMATE	T-VALUE
Constant	4.224	2.212
$\ln fs$	−1.526	−1.928
$\ln y$	0.278	3.181
$\ln y \ln fs$	0.175	2.438
$\ln^2 fs$	−0.131	−1.907
$D_{tw} \ln fs$	−0.032	−1.931
$\ln a$	−4.066	−4.098
$\ln^2 a$	0.564	4.162
D_{work}	−0.074	−2.563
Female	0.117	5.448
Religion	0.040	3.875
Problems related to		
Health	0.148	11.889
Family	0.118	7.852
Work	0.114	8.275
Partner	0.177	11.589
Sleep	0.077	5.615
Drugs and Alcohol	0.084	4.326
Neighborhood	0.180	17.134
Parents	0.016	1.118
Government	0.074	6.679
N	8447	
R^2	0.191	

Table 2

OPTIMUM FAMILY SIZES
AND CORRESPONDING INCOME LEVELS

fs	3	4	5	6
	33,345	50,712	70,436	92,269
	39,946	60,890	84,677	111,007

Table 3

SHADOW PRICES ACCORDING TO IEQ

Income	1^{th} child	2^{th} child	3^{th} child	4^{th} child
20,000	906	680	544	453
30,000	1360	1020	816	680
40,000	1813	1360	1088	906
50,000	2266	1700	1360	1133
60,000	2720	2040	1632	1360

SHADOW PRICES ACCORDING TO CANTRIL
ONE BREADWINNER

Income	1^{th} child	2^{th} child	3^{th} child	4^{th} child
20,000	1686	1518	1549	1492
30,000	245	1256	1564	1639
40,000	−1098	709	1367	1619
50,000	−2756	−49	1012	1474
60,000	−4662	−978	513	1230

SHADOW PRICES ACCORDING TO CANTRIL
TWO BREADWINNERS

Income	1^{th} child	2^{th} child	3^{th} child	4^{th} child
20,000	1632	1833	1784	1676
30,000	941	1728	1916	1916
40,000	−170	1338	1836	1988
50,000	−1595	736	1597	1935
60,000	−3270	−34	1233	1783

Table 4

MONEY VALUE OF NON-MONETARY CHILD BENEFITS
ONE BREADWINNER

Income	1^{th} child	2^{th} child	3^{th} child	4^{th} child
20,000	−262	−838	−1005	−1039
30,000	1114	−236	−748	−959
40,000	2911	651	−279	−713
50,000	5023	1749	348	−341
60,000	7383	3018	1100	130

TWO BREADWINNERS

Income	1^{th} child	2^{th} child	3^{th} child	4^{th} child
Income	1^{th} child	2^{th} child	3^{th} child	4^{th} child
20,000	−726	−1153	−1240	−1223
30,000	419	−708	−1100	−1236
40,000	1983	22	−747	−1082
50,000	3871	964	−237	−802
60,000	5990	2074	399	−423