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Combination Schemes for Turning Point Predictions*

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Abstract

We propose new forecast combination schemes for predicting turning points of business cycles. The combination schemes deal with the forecasting performance of a given set of models and possibly providing better turning point predictions. We consider turning point predictions generated by autoregressive (AR) and Markov-Switching AR models, which are commonly used for business cycle analysis. In order to account for parameter uncertainty we consider a Bayesian approach to both estimation and prediction and compare, in terms of statistical accuracy, the individual models and the combined turning point predictions for the United States and Euro area business cycles.

JEL codes: C11, C15, C53, E37.
Keywords: Turning Points, Markov-switching, Forecast Combination, Bayesian Model Averaging.

1 Introduction

In recent years, interest has increased in the ability of the business cycle models to forecast economic growth rates and structural breaks in economic activity. The early contributions in this stream of literature consider nonlinear models such as the Markov-switching (MS) models (see for example Goldfeld and Quandt [1973] and Hamilton [1989]) and the threshold autoregressive models (see Tong [1983] and Potter [1995]), both of which are able to capture

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the asymmetry and the turning points in business cycle dynamics. In this paper we focus on the class of MS models. We take the model of Hamilton [1989] as point of departure. For more recent data one needs an adequate business cycle model with more than two regimes (see also Clements and Krolzig [1998]) and a time-varying error variance. For example, Kim and Murray [2002] and Kim and Piger [2000] propose a three-regime (recession, high-growth, and normal-growth) MS model while Krolzig [2000] suggests the use of a model with regime-dependent volatility for the US GDP. In our paper we consider data on US and Euro industrial production, for a period of time including the 2009 recession and find that four regimes (high-recession, contraction, normal-growth, and high-growth) are necessary to capture some important features of the US and EU cycle in the strong-recession phases. As most of the forecast errors are due to shifts to the deterministic factors (see Krolzig [2000]), we consider a model with shifts in the intercept and in the volatility.

The first contribution of this paper is to exploit the time-variations in the forecast performances of linear and nonlinear models to potentially produce better forecasts. More specifically, in some empirical investigations and simulation studies, it has been found that the MS models are superior in in-sample fit, but not always in forecasting and that the relative forecast performances of the MS models depend on the regime present at the time the forecast is made (see Clements and Krolzig [1998]). Thus it seems possible to obtain better forecasts by dynamically combining in a suitable fashion the various model forecasts.

The second main contribution of this paper is to study the relationship between forecast combination and turning point extraction when many points forecasts are available from different models for the same variable of interest. When many models are used for forecasting turning points, one can then alternatively combine the forecasts from the models and detect the turning points on the combined forecasts, or detect the turning points on the model point forecasts and then combine the turning point indicators. We tackle this problem and show that the turning point forecasts are not invariant with respect to the order of the operations of forecast combination and turning point extraction, and that the best combination should be evaluated in the specific case at hand. Our paper is related to Stock and Watson [2010], who consider the issue of dating the turning point for a reference cycle when many series are available. In this context, it is possible to detect clusters of turning points that are cycle-specific, and the problem of aggregating them becomes crucial to determine a reference cycle.

Another relevant contribution of the paper is to propose the use of Bayesian inference to account for both model and parameter uncertainty in combining the turning point forecasts. The combination of the turning point forecasts is based on a Bayesian model averaging (BMA) procedure (see Grunwald et al. [1993] for a review) which accounts for the model forecast performances. The Bayesian approach proposed in this paper is based on a numerical approximation algorithm (the Gibbs sampler) which is general enough to include not only parameter uncertainty but also possible non-normality of the prediction error, as well as nonlinearities of the process. Another advantage of the Gibbs sampling procedures is that they naturally provide approximation of prediction density and forecast intervals for the variable of interest.

Finally, we study different strategies for the specification of the combination weights. More specifically. we compare in terms of forecast performances weighting schemes driven by the prediction errors in predicting alternatively the level or the turning points of the variable of interest.

The paper is structured as follows. Section 2 introduces the Markov-switching model used in the analysis of the cycle. Section 3 presents the Bayesian approach to inference and forecast combination. Section 4 provides a comparison between the forecasting methods for the Euro area and the US business cycles. Section 5 concludes the paper.

2 Predicting with Markov-switching Models

Let y_t, with $t = 1, \dots, T$, be a set of observations for a variable of interest. We assume that y_t follows a Gaussian autoregressive (AR) process of the order p with parameters driven by an MS process with m regimes and denote the resulting process with MS-AR. More specifically we say that y_t follows an MS-AR if

$$y_t = \nu_{s_t} + \phi_{1,s_t} y_{t-1} + \dots + \phi_{p,s_t} y_{t-p} + u_t, \quad u_t \sim \mathcal{N}(0, \sigma^2_{s_t}) \tag{1}$$

where ν_{s_t} is the intercept; $\phi_{l,k}$, with $l = 1, \dots, p$, are the autoregressive coefficients; σ_{s_t} is the volatility; and $\{s_t\}_t$ is a m-states ergodic and aperiodic Markov-chain process. This process is unobservable (latent) and s_t represents the current phase, at time t, of the business cycle (e.g. contraction or expansion). The latent process takes integer values, say $s_t \in \{1, \dots, m\}$, and has transition probabilities $\mathbb{P}(s_t = j | s_{t-1} = j) = p_{ij}$, with $i, j \in \{1, \dots, m\}$. The transition matrix P of the chain is the collection of the transition probabilities; that is,

$$P = \begin{pmatrix} p_{11} & \dots & p_{1m} \\ \vdots & & \vdots \\ p_{m1} & \dots & p_{mm} \end{pmatrix}$$

and has, as a special case, the one-forever-shift model that is widely used in structural-break analysis.

Clements and Krolzig [1998] found in an empirical study that most forecast errors are due to the constant terms in the prediction models. They suggest considering, for example, MS models with regime-dependent volatility. In the present analysis, we follow Krolzig [2000] and Anas et al. [2008] and assume that both the constant term and the volatility are driven by the regime-switching variable $\{s_t\}_t$. We denote the resulting MS-intercept and MS-heteroscedasticity model with MSIH(m)-AR(p). For inference purposes we follow a data augmentation framework (see Tanner and Wong [1987]) and introduce the allocation variable $\boldsymbol{\xi}_t = (\xi_{1t}, \ldots, \xi_{mt})$, in which $\xi_{kt} = \mathbb{I}_{\{k\}}(s_t)$ indicates the regime associated with the current observation y_t. We can write the random-coefficient dynamic regression model as follows

$$y_t = \sum_{k=1}^{m} \xi_{kt}\nu_k + \phi_1 y_{t-1} + \ldots + \phi_p y_{t-p} + u_t, \quad u_t \sim \mathcal{N}(0, \gamma_t^2) \tag{2}$$

in which $\gamma_t^2 = \sum_{k=1}^{m} \xi_{kt}\sigma_k^2$.

In order to apply a Bayesian approach to estimation we need to complete the description of the model with the specification of the prior distributions of the parameters. We assume uniform prior distributions for all the autoregressive coefficients, the intercept and the precision parameters

$$\begin{aligned}
(\phi_1, \ldots, \phi_p) &\propto \mathbb{I}_{\mathbb{R}_p}(\phi_1, \ldots, \phi_p) \\
(\nu_k, \sigma_k^2) &\propto \frac{1}{\sigma_k^2}\mathbb{I}_{\mathbb{R}}(\nu_k)\mathbb{I}_{\mathbb{R}^+}(\sigma_k^2) \quad k = 1, \ldots, m
\end{aligned}$$

and do not impose stationarity constrains for the autoregressive coefficients.

When estimating an MS model, which is a dynamic mixture model, one needs to deal with the identification issue arising from the invariance of the likelihood function and of the posterior distribution (which follows from the assumption of symmetric prior distributions) to permutations of the allocation variables. Many different ways to solve this problem are discussed, for example, in Frühwirth-Schnatter [2006]. We identify the regimes by imposing some constraints on the parameters, as is standard in business cycle analysis . We consider the following identification constraints on the intercept: $\nu_1 < 0$ and $\nu_1 < \nu_2 < \ldots < \nu_m$, which allow us to interpret the first regime as the one associated with the recession phase. As an alternative, one could introduce the constraints on the volatility or on the transition probability. From a practical point of view, we find in our empirical applications that volatility ordering works as well as the intercept ordering constraint for the regime identification. The ordering on the transition probabilities is not strong enough for the data to identify the regimes.

We assume standard conjugate prior distributions for the transition probabilities. These distributions are independent Dirichlet distributions, one for each row of the transition matrix

$$(p_{i1}, \ldots, p_{im})' \sim \mathcal{D}(\delta_1, \ldots, \delta_m)$$

with $i = 1, \ldots, m$.

Samples from the joint posterior distribution of the parameters and the allocation variables are obtained by iterating a Gibbs sampling algorithm. The joint posterior distribution and the full conditional distributions associated with the Gibbs sampler are given in Krolzig [1997] together with the sampling procedure for the posterior of the allocation variables (and the hidden states). In Krolzig [1997] the multi-move Gibbs sampler (see Carter and Köhn [1994] and Shephard [1994]) is presented for Markov-switching vector autoregressive models as an alternative to the single-move Gibbs sampler given, for example, in Albert and Chib [1993]. The multi-move procedure is particularly useful in our context because the Gibbs sampler makes use of two relevant quantities in order to sample from the full conditional of the allocation variables: the filtering and the smoothing probabilities.

Let $\mathbf{y}_{s:t} = (y_s, \ldots, y_t)'$ be the vector of observation from time s up to time t, with $s \leq t$. The filtering probability at time t is then determined by iterating the prediction step

$$p(\boldsymbol{\xi}_t = \boldsymbol{\iota}_j | \mathbf{y}_{1:t-1}) = \sum_{i=1}^{m} p(\boldsymbol{\xi}_t = \boldsymbol{\iota}_j | \boldsymbol{\xi}_{t-1} = \boldsymbol{\iota}_i) p(\boldsymbol{\xi}_{t-1} = \boldsymbol{\iota}_i | \mathbf{y}_{1:t-1}) \tag{3}$$

and the updating step

$$p(\boldsymbol{\xi}_t | \mathbf{y}_{1:t}) \propto p(\boldsymbol{\xi}_t | \mathbf{y}_{1:t-1}) p(y_t | \mathbf{y}_{t-1-p:t-1}, \boldsymbol{\xi}_t) \tag{4}$$

where $p(\boldsymbol{\xi}_t = \boldsymbol{\iota}_j | \boldsymbol{\xi}_{t-1} = \boldsymbol{\iota}_i) = p(s_t = j | s_{t-1} = i)$, with $\boldsymbol{\iota}_m$ the m-th column of the identity matrix and $p(y_t | \mathbf{y}_{t-p-1:t-1}, \boldsymbol{\xi}_t)$ the conditional distribution of the variable y_t from a MSIH(m)-AR(p).

The prediction step can be used at time T to evaluate the prediction density of $\boldsymbol{\xi}_{T+1}$

$$p(\boldsymbol{\xi}_{T+1} | \mathbf{y}_{1:T}) \propto P' \, p(\boldsymbol{\xi}_T | \mathbf{y}_{1:T}) \tag{5}$$

and the one of y_{T+1}

$$p(y_{T+1} | \mathbf{y}_{1:T}) = \sum_{i=1}^{m} p(\boldsymbol{\xi}_t = \boldsymbol{\iota}_i | \mathbf{y}_{1:T}) p(y_{T+1} | \mathbf{y}_{T+1-p:T}, \boldsymbol{\xi}_{T+1}) \tag{6}$$

which, for a Gaussian AR process, is a mixture of normal distributions.

The smoothing probabilities given by

$$p(\boldsymbol{\xi}_t = \boldsymbol{\iota}_j|\mathbf{y}_{1:T}) \propto \sum_{i=1}^{m} p(\boldsymbol{\xi}_t = \boldsymbol{\iota}_j|\boldsymbol{\xi}_{t+1} = \boldsymbol{\iota}_i, \mathbf{y}_{1:T})p(\boldsymbol{\xi}_{t+1} = \boldsymbol{\iota}_i|\mathbf{y}_{1:T}) \tag{7}$$

are evaluated recursively and backward in time for $t = T, T-1, \ldots, 1$. These quantities are the posterior probabilities of the observation y_t to be in one of the regimes a time t, given all the information available from the full sample of data. The smoothing probabilities are usually employed to detect the turning points. In this paper, we will not consider the cycle generated by the smoothing probabilities and instead applied a non-parametric approach (see the next section) to extract the turning points from the forecasting values of y_{t+h}.

3 Combining Linear and Non-linear Models

In this section we describe the rules used for combining the forecasts from linear (the AR) and non-linear (MS-AR) models and for predicting the turning points of the business cycle. In both the model combination and turning point forecasts for the variable of interest x_t (e.g. the actual or the forecasted industrial production) we use the Bry and Boschan [1971] (BB) rule and identify a trough (or downturn) at time t if $x_{t-K} < x_t, \ldots, x_{t-1} < x_t$ and $x_t > x_{t+1}, \ldots, x_t > x_{t+K}$ and a peak (or upturn) at time t if $x_{t-K} > x_t, \ldots, x_{t-1} > x_t$ and $x_t < x_t, \ldots, x_t < x_{t+k}$. By applying this rule we get an indicator variable z_t that is equal to 1 in the expansion phases and 0 in the recession phases. This rule is a standard one in business cycle analysis (see for example Chauvet and Piger [2008]) and is also used (with some adjustments) by the NBER institute for building the reference cycle for the US. Our analysis can be extended to include modifications of the BB rule (see for example Mönch and Uhlig [2005]), which account for asymmetries and time-varying duration across business cycle phases.

We propose combining the models through use of two alternative schemes. The first one is a Bayesian Model Averaging (BMA) procedure based on the forecasting performance for the variable of interest. The second one is based on the performance of the models in terms of turning point forecasts.

The BMA procedure gives a combined point forecast $\tilde{y}_t$ for the value y_t using the information available up to time $t-1$, from a set of models M_j, with $j = 1, ..., M$:

$$\tilde{y}_t = \sum_{j=1}^{M} \tilde{y}_{j,t} w_{jt} \tag{8}$$

where w_{jt} is the $(0,1)$-valued weight given to model M_j computed at time $t-1$ and $\tilde{y}_{j,t}$ is the point forecast from the predictive density $p(\tilde{y}_{j,t}|\mathbf{y}_{1:t-1}, M_j)$, which is the density of $\tilde{y}_{j,t}$ conditional on model M_j and on the information available up to time $t-1$.

To assess the forecast accuracy of each model, we follow recent studies in using the predictive likelihood of the model. Sources such as Geweke [1999] and Geweke and Whiteman [2006] emphasize the close relationship between the predictive likelihood and marginal likelihood, previously used in BMA and, more generally, as Bayesian evaluation criterion. As stated in Geweke (1999, p.15), "... the marginal likelihood summarizes the out-of-sample prediction record... as expressed in ... predictive likelihoods." See Bjørnland et al. [2009] and Hoogerheide et al. [2010] for similar recent applications.

The cumulative predictive-likelihood at time t associated to the j-th model is defined as

$$\eta_{jt}^{PL} = \prod_{s=1}^{t} p(\tilde{y}_{j,s}|\mathbf{y}_{1:s-1}, M_j) \tag{9}$$

where $p(\tilde{y}_{j,t}|\mathbf{y}_{1:t-1}, M_j)$ is the (simulated) predictive density for y_t obtained from the model j. The point forecast $\tilde{y}_{j,t}$ is computed as the median of the density $p(\tilde{y}_{j,t}|\mathbf{y}_{1:t-1}, M_j)$. We build the weights for the j-th model, as

$$w_{jt}^{PL} = \frac{\eta_{j,t-1}^{PL}}{\sum_{k=1}^{K} \eta_{j,t-1}^{PL}} \tag{10}$$

with $j =$ AR, MS-AR.

We also suggest combining the forecasts by applying some performance measures that are usually employed in the analysis of the turning points.[1] To take one example, we evaluate through the concordance statistics the ability of the AR and MS-AR to predict turning points with position and frequency similar to those of the turning points in the reference cycle.

Let z_{jt} be the phase indicator built with the forecast from the j-th model. The indicator is built by applying the BB rule described above to the actual values of the variable of interest up to time t and to the one-step-ahead forecast from the j-th model. Let z_{Rt} be the indicator variable of the reference cycle and be determined by applying the rule described above to the actual values of the variable of interest. Then the concordance statistics for

[1] See Clements and Harvey [2011] for a more general analysis on combinations of probability forecasts that are not restricted to be 0 or 1.

the j-th model is given by

$$\eta_{jt}^{CS} = \sum_{t=1}^{T} ((z_{jt} z_{Rt}) - (1 - z_{jt})(1 - z_{Rt})) \tag{11}$$

These statistics are used to built a set of weights for the business cycle indicators from the different models. The phase indicator variable that results from the combination must be a binary variable. Therefore, we propose combining the phased indicators from the different models by using weights that take value 0 or 1. More specifically the model with the highest concordance with the reference cycle has a weight of 1, and the other models have null weights. In formula we have

$$w_{jt}^{CS} = \mathbb{I}_{\{k^*\}}(j) \tag{12}$$

where $k^* = \underset{k \in \{1,\ldots,K\}}{\arg\max} \{\eta_{jt}^{CS}, j = \text{AR}, \text{MS-AR}\}$.

4 Empirical Results

4.1 Data and Reference Cycle

In our study we consider the Industrial Production Index (IPI) from OECD at a monthly frequency for United States (US), from February 1949 to January 2011, and for Euro Area (EU), from January 1971 to January 2011. Data for both US and EU economies are seasonally adjusted and working day adjusted. In order to obtain the IPI at the Euro zone level a back-recalculation has been performed (see Anas et al. [2007a,b] and Caporin and Sartore [2006] for details). Since Phillips-Perron and Dickey-Fuller stationarity tests point out the non-stationarity of the IPI, we considered in our analysis the log-changes of the IPI index. The resulting series (see Fig. 1) are then used to detect and forecast the turning points.

Fig. 1 shows the reference cycle used in our analysis. The cycle is obtained by applying a BB rule to the US and EU IPI series. For comparison purposes, we show for the US economy the NBER official turning points, which are obtained by applying the BB rule with some adjustments on the whole series. The application of this rule allows for detection of the following contraction phases (from peak to trough) for the US economy since 1980M01:

- 1980 recession (1982M04-1982M12) which is within the NBER references dates;
- 1990 recession (1989M08-1991M01) which is within the NBER references dates;

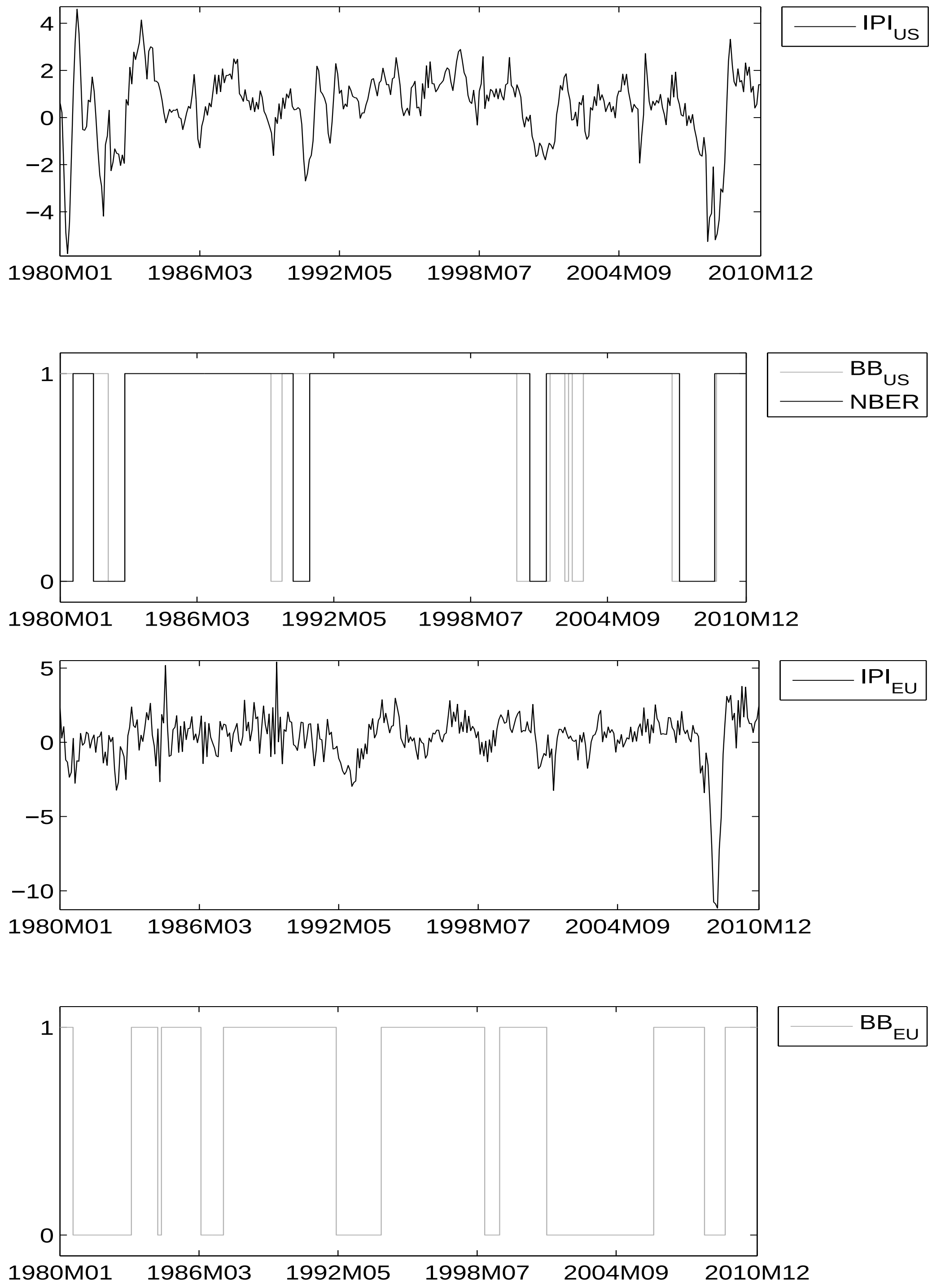

Figure 1: First and third chart: log-changes in the Industrial Production Index (IPI) for US and EU at monthly frequency for the period: January 1980 to January 2010. Second and fourth chart: the reference cycles (BB) for US and EU. Second chart: the NBER reference cycle (light gray).

- short contraction (2000M09-2002M02) which is not within the NBER dates;
- Internet bubble burst and 9/11 dates (2002M11-2002M12);
- Slaggish recovery of the US economy and EU industrial recession. This made Greenspan and FED to keep rates very low (2003M03-2003M08);
- the 2007-2009 recession (2007M09-2009M08) which is within the NBER reference dates.

Following the results of the BB algorithm, the Euro area has experienced the follow contraction phases since January 1980M01:

- the second oil shock and US double dip recession (1980M09-1984M07);
- the 1986-87 recession (1986M06-1987M04);
- the 1992-94 recession (1992M05-1994M04);
- the Asian-crises related recession (1998M12-1999M07);
- the 2001 and 2003 industrial recessions (2001M09-2006M05);
- the 2007-09 recession (2008M09-2009M07).

4.2 Estimation and Forecasting

In the following we show the results of the sequential estimation and forecast of the AR and MS-AR models. The estimation results are based on 10,000 Gibbs iterations. The number of iterations has been chosen on the basis of both a graphical inspection of the Markov Chain Monte Carlo averages and on the application of the convergence diagnostic (CD) statistics proposed in Geweke [1992]. An initial set of 5,000 samples has been discarded to loose the dependence on the initial conditions of the sampler and the remaining samples were thinned down by a factor of 10 to have reasonably less-dependent posterior samples.

Tab. 1 shows the estimation results for the AR(p) based on the full sample. We use the Bayesian information criteria for selecting the order of the autoregressive processes and find that for the US IPI log-changes an AR(8) should be used while an AR(4) should be considered for modelling the Euro area business cycle. For both of the cycles the AR(p) has a positive intercept value that is statistically close to 0.1, which underestimates the mean value of the IPI log-changes during an expansion phase and overestimate it during a recession phases. The HPD region for the volatility is $(1.124, 6.634)$ for the US and

Country Sample Period	US 1949M1-2011M1			EU 1971M1-2011M1		
θ	$\hat{\theta}_T$	$q_{0.05}$	$q_{0.95}$	$\hat{\theta}_T$	$q_{0.05}$	$q_{0.95}$
ν	0.111	0.042	0.181	0.074	-0.039	0.189
ϕ_1	1.162	1.083	1.242	0.651	0.560	0.743
ϕ_2	-0.105	-0.222	0.012	0.339	0.237	0.441
ϕ_3	-0.692	-0.807	0.576	-0.415	-0.517	-0.313
ϕ_4	0.795	0.670	0.920	0.187	0.096	0.278
ϕ_5	-0.281	-0.405	0.156			
ϕ_6	-0.326	-0.441	0.211			
ϕ_7	0.459	0.343	0.575			
ϕ_8	-0.165	-0.240	0.089			
σ	3.891	1.124	6.634	2.357	1.379	6.011

Table 1: Estimated parameters of the AR(p) model for the log-change of the US (with $p = 8$) and EU (with $p = 4$) Industrial Production Indexes. For each country: parameter estimates (first column) and the 0.05 and 0.95 quantiles (second and third columns).

$(1.379, 6.011)$ for the EU which are quite high and tend to overestimate volatility during the normal growth and the expansion periods.

We compare the AR(p) model with the MSIH(m)-AR(p) and as we expected the MSIH(m)-AR(p) are able to give a better description of the features of the cycles and to capture different phases in the IPI growth level and volatility. Tab. 2 shows the estimation results for the MSIH(m)-AR(p) based on the whole sample period. We consider here a flexible model by considering $p = 4$ lags as in Hamilton [1989] and Krolzig [2000] for the US gross domestic product and $m = 4$ regimes, extending the three-regimes model used in Krolzig [2000].

We find in our comparisons that the four-regimes model is necessary in order to capture the last recession. The interpretation of two of the four regimes will be similar to the one given in Krolzig [2000], i.e. normal growth and high growth, and two regimes are used to describe the recession phases. Thus in our model the fourth regime characterizes high-growth episodes, the third regime normal-growth phases, the second regime a normal slowdown in economic activity. The first regime may indicate strong-recession periods. We find evidence of the four regimes in both the US and the EU economies (see the first graph in both the US and the EU panels of Fig. 1). The graphs in the rows from two to four of Fig. 1 US and EU panels show the smoothing probabilities of the MSIH(m)-AR(p) model estimated on the full sample. The smoothing probabilities for the first regime, $P(s_t = 1|y_{1:T})$, show that some strong recession periods are present in the sample with a high probability. In particular, in the 1976 and 2009 crises for both the EU and US cycles there are some periods where the smoothing probabilities of the first regime are greater than the probabilities of the other regimes.

From Fig. 4 one can see that the regimes have different degrees of persistence. The analysis of the transition probabilities brings us to the following conclusions. The first regime is moderately persistent with transition probabilities $\hat{p}_{11} = 0.641$ for the US and $\hat{p}_{11} = 0.709$ for the EU (see Tab. 2). It is less persistent than the third regime (normal growth), which has estimated transition probabilities (see Tab. 2) $\hat{p}_{33} = 0.886$ for US and $\hat{p}_{33} = 0.775$ for EU. The second regime (normal recession) is less persistent than the other regimes, for US, with probability $\hat{p}_{22} = 0.675$ to stay in the regime, and more persistent, for EU, with transition $\hat{p}_{22} = 0.841$. The fourth regime is more persistent than the first regime, for the US, with probability $\hat{p}_{44} = 0.777$ to stay in the regime, while the opposite is the case for the EU, which has the probability of staying in a strong recession regime of $\hat{p}_{44} = 0.676$.

The four regimes have substantially different values for the intercept and scale parameters (see Tab. 2). The differences between the constant terms in the first and in the fourth regime are similar for the US and the EU, i.e. $(\hat{\nu}_4 - \hat{\nu}_1) = 3.616$ for the US and $(\hat{\nu}_4 - \hat{\nu}_1) = 3.996$ for the EU. The volatility gap between the first and fourth regimes is instead different in the two cycles: $\hat{\sigma}_4^2 - \hat{\sigma}_1^2 = -1.836$ for US and $\hat{\sigma}_4^2 - \hat{\sigma}_1^2 = -0.967$ for the EU. More generally the volatility of the EU cycle associated with regimes of strong recession and high growth is larger than the volatility of the US cycle. For both cycles the MS model results show that volatility significantly changes across the four regimes. For this reason, the use in this context of a AR model with constant volatility may be inappropriate. Accordingly, one could expect that the MS-AR models have superior forecasting ability than the AR models.

Fig. 4 shows the combination weights obtained from the sequential evaluation of the forecasting abilities of the different models for the US and the EU IPI log-changes. From the first and third chart in Fig. 4 it can be seen that the combination weights, $w_{MS-AR,US}^{PL}$ and $w_{MS-AR,EU}^{PL}$, increase in the last part of the sample, starting at September 2008. This corresponds to an increase in the forecasting ability, in terms of predictive likelihood, of the MS-AR with respect to the AR models. From our experiments we find that the good performance of the MS-AR models in the last part of the sample cannot be obtained with three regimes and that four-regime models are necessary to have an adequate description, in terms of expected growth-rate and volatility, of both the US and EU cycles during a strong recession phase.

The results for the performance abilities change if we consider the concordance with

Country Sample Period	US 1949M1-2011M1			EU 1971M1-2011M1		
θ	$\hat{\theta}_T$	$q_{0.05}$	$q_{0.95}$	$\hat{\theta}_T$	$q_{0.05}$	$q_{0.95}$
ν_1	-2.436	-5.868	-0.539	-1.981	-4.225	-0.423
ν_2	-0.524	-1.542	0.156	-0.152	-1.145	0.335
ν_3	0.132	-0.290	0.511	0.482	-0.040	1.433
ν_4	1.180	0.121	3.410	2.015	0.435	4.771
σ_1	2.783	7.743	1.350	4.051	9.395	1.999
σ_2	1.567	4.085	0.725	1.337	5.735	0.800
σ_3	0.552	2.496	0.358	1.450	6.356	0.794
σ_4	0.947	4.493	0.424	3.354	7.116	1.524
ϕ_1	0.935	0.650	1.204	0.555	0.330	0.774
ϕ_2	0.050	-0.311	0.404	0.331	0.099	0.562
ϕ_3	-0.473	-0.815	-0.135	-0.374	-0.610	-0.140
ϕ_4	0.271	0.033	0.516	0.185	-0.021	0.398
p_{11}	0.641	0.429	0.832	0.709	0.519	0.870
p_{12}	0.186	0.039	0.389	0.130	0.025	0.286
p_{13}	0.082	0.004	0.240	0.079	0.004	0.223
p_{14}	0.089	0.005	0.248	0.080	0.005	0.217
p_{21}	0.041	0.002	0.136	0.032	0.001	0.130
p_{22}	0.675	0.468	0.862	0.841	0.544	0.981
p_{23}	0.165	0.031	0.359	0.090	0.005	0.281
p_{24}	0.116	0.004	0.301	0.034	0.000	0.156
p_{31}	0.014	0.000	0.048	0.051	0.000	0.207
p_{32}	0.053	0.008	0.202	0.135	0.008	0.400
p_{33}	0.886	0.596	0.976	0.775	0.414	0.980
p_{34}	0.046	0.000	0.212	0.037	0.000	0.159
p_{41}	0.033	0.000	0.139	0.096	0.005	0.272
p_{42}	0.060	0.001	0.214	0.111	0.007	0.293
p_{43}	0.128	0.005	0.319	0.115	0.008	0.288
p_{44}	0.777	0.515	0.977	0.676	0.441	0.868

Table 2: Estimated parameters of the MSIH(4)-AR(4) model for the log-change of the US and EU Industrial Production Indexes. For each country: parameter estimates (first column) and the 0.05 and 0.95 quantiles (second and third columns).

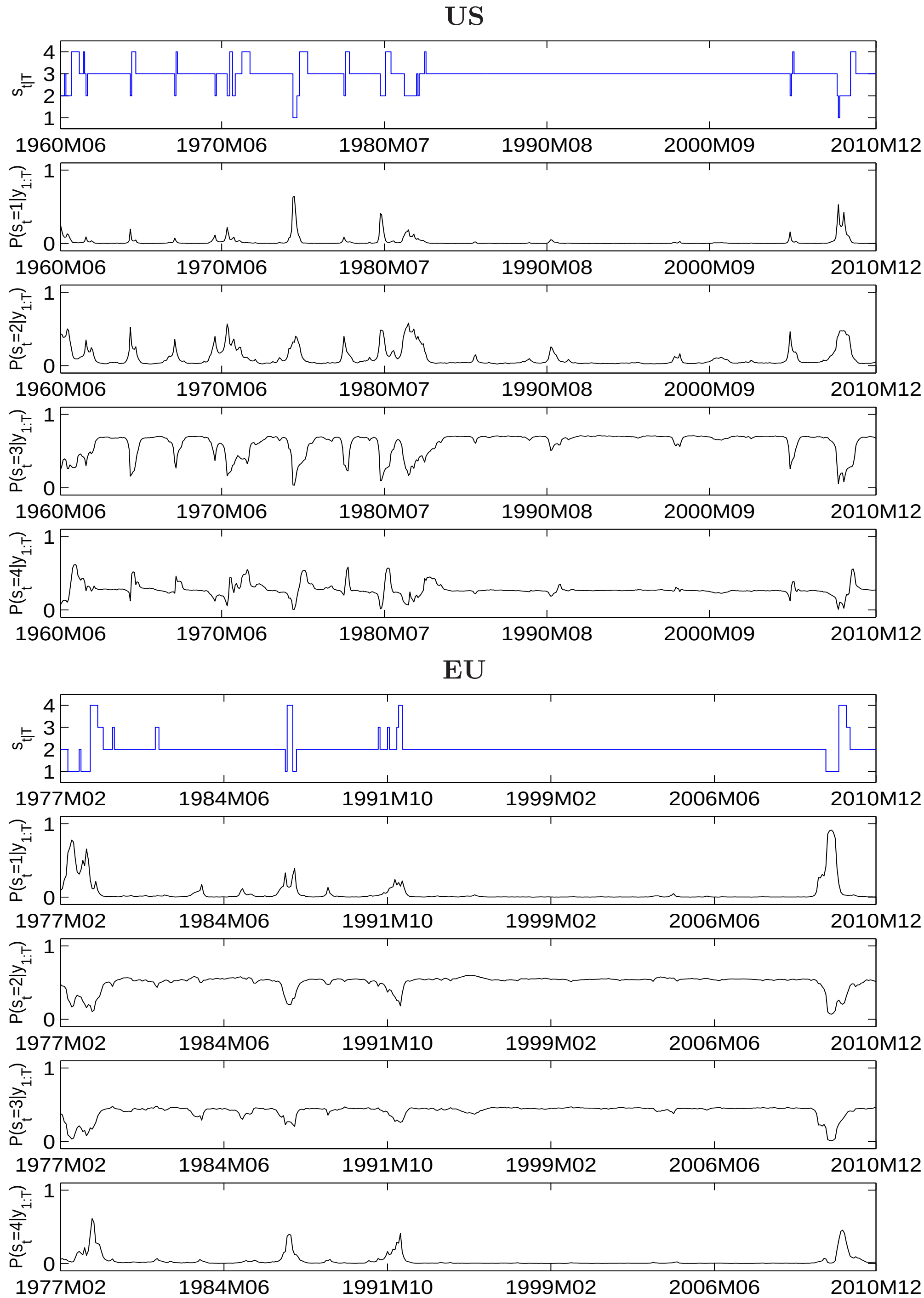

Figure 2: Hidden state estimates $s_{t|T}$ and smoothing probabilities $P(s_t|y_{1:T})$, for $t = 1, \ldots, T$, for US (upper panel) and EU (lower panel) data.

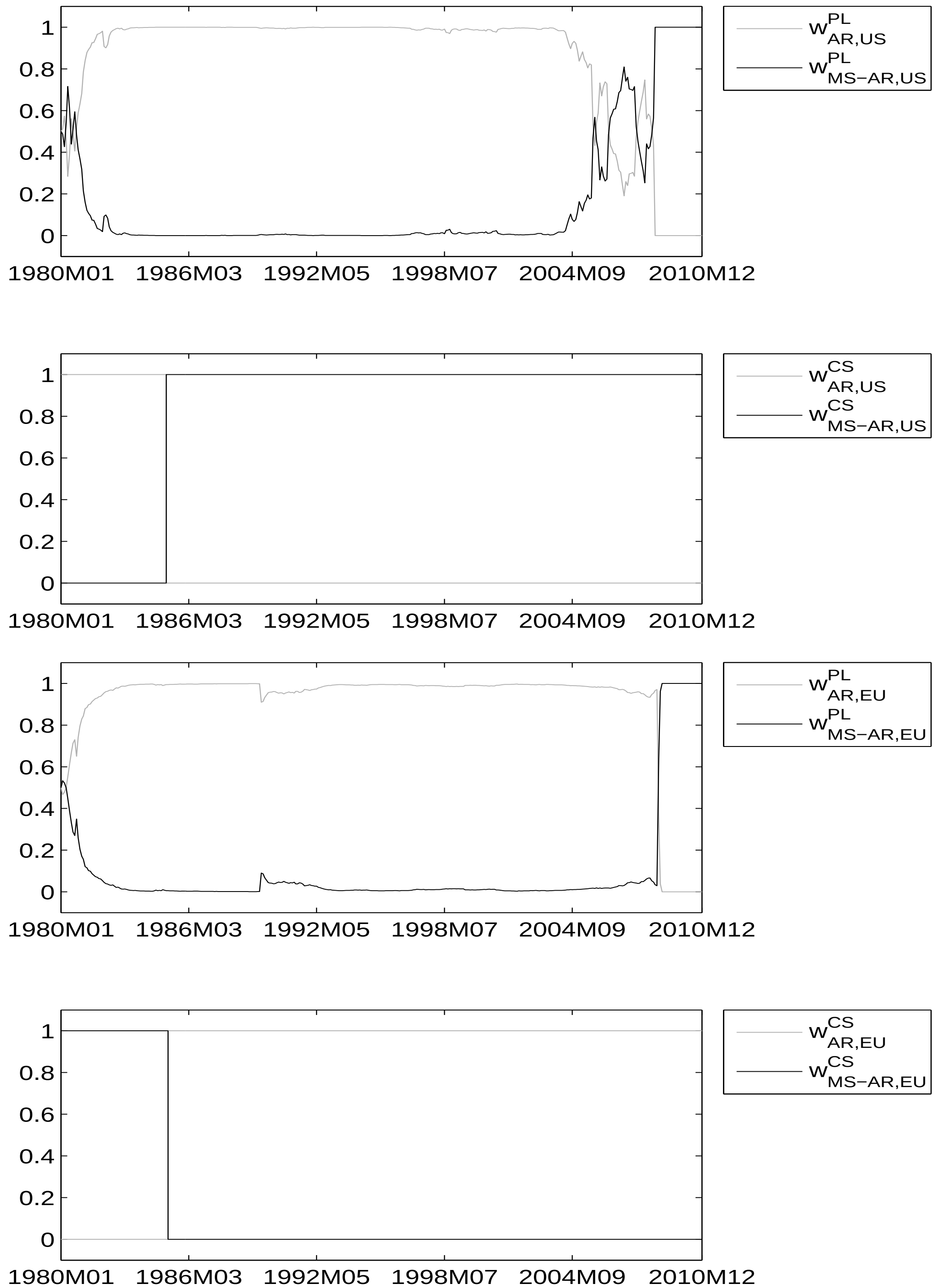

Figure 3: Combination weights for the AR and MS-AR forecasts by using predictive-likelihood (PL) and concordance statistics (CS) for US and EU data.

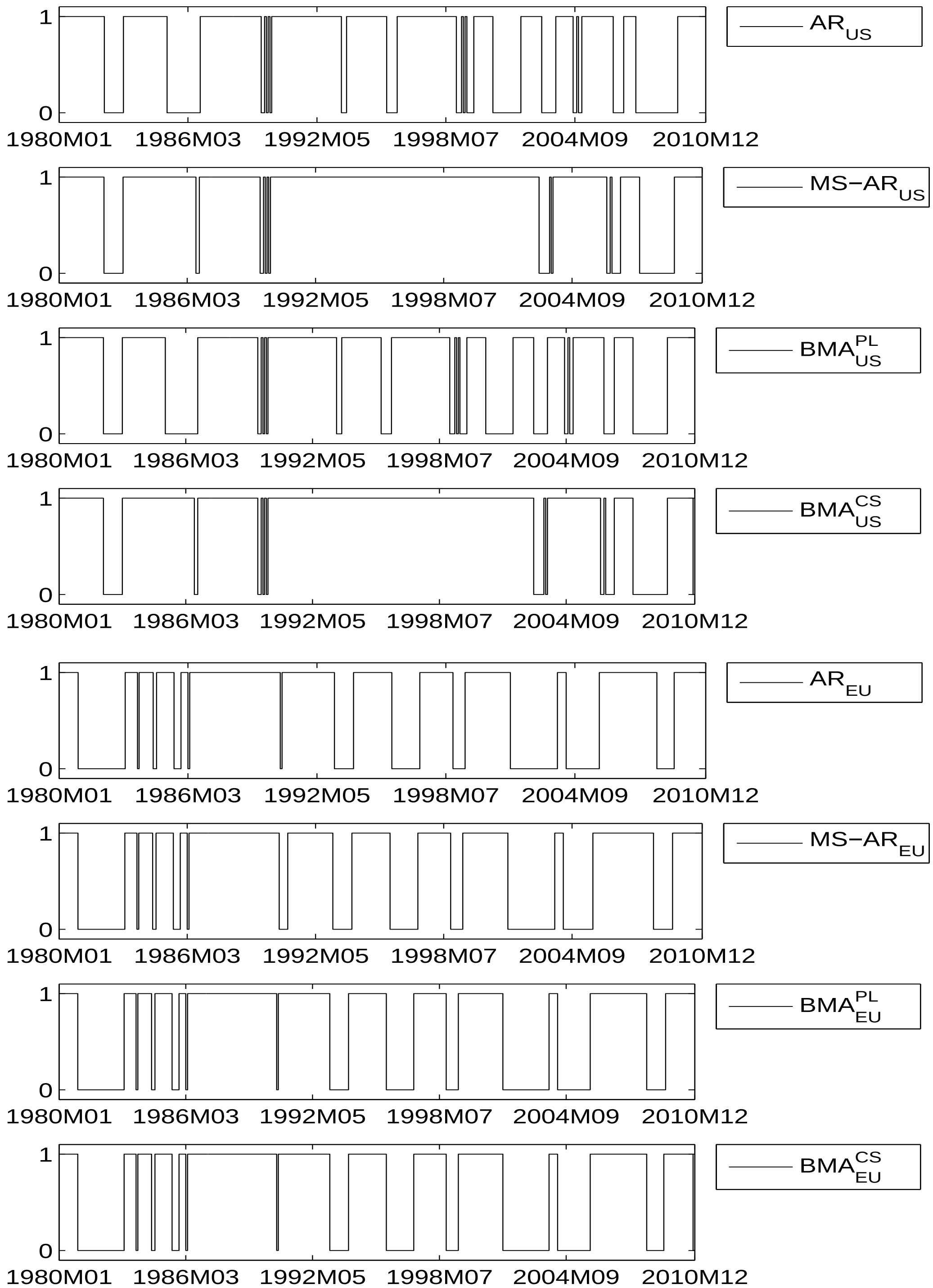

Figure 4: Turning point forecasts for US and EU IPI obtained from different models (AR and MS-AR) and their BMA combinations based on the predictive likelihood (PL) and the concordance statistics (CS).

a reference cycle as a performance measure (see the combination weights $w^{CS}_{MS-AR,US}$ and $w^{CS}_{MS-AR,EU}$ in the second and fourth graph of Fig. 4). More specifically, for the US cycle (second chart in Fig. 4) the MS-AR model is superior to the AR model starting at the beginning of 1985. Conversely, the turning point forecast abilities of the MS-AR are worse than those of the AR model for the EU cycle, starting at the beginning of 1985. These results are all in line with the results in Clements and Krolzig [1998] about the time-varying performance of the MS models. MS models behave in a different way depending on the value of the regime present when the forecast performances are evaluated.

4.3 Sequential Turning Points Detection

Turning point prediction with different models (AR and MS-AR) and model combinations (predictive likelihood BMA and concordance statistics BMA) are given in Fig. 4. Fig. 4 (charts 3 and 4) shows that the two combination strategies for the US cycle give two sequences of turning point forecasts that exhibit substantial differences. Charts seven and eight of the same figures show that the two strategies give similar turning points for the EU cycle.

In order to evaluate, at the end of the sample period T, the forecast abilities of the two combination strategies we consider the Mean Square Prediction Error (MSPE)

$$MSPE = \frac{1}{T}\sum_{t=1}^{T}(y_t - \tilde{y}_{t+1})^2 \tag{13}$$

and the Logarithmic Score (LS)

$$LS = -\frac{1}{T}\sum_{t=1}^{T}\ln p(\tilde{y}_{t+1}|\mathbf{y}_{1:t}) \tag{14}$$

Tab. 3 shows that one of the two models performs better for both the US and EU, in terms of MSPE, than the two combination strategies. When considering the LS, then the BMA based on the concordance statistics that correspond to the combination of the turning point indicators is the best strategy to use for the US cycle. For the EU cycle the BMA based on predictive likelihood performs petter than the BMA based on concordance statistics. This leads to the conclusion that, for the EU it is better to combine first the growth-rate forecasts and then apply the BB rule for the detection of the turning points.

	AR	MS-AR	BMA-PL	BMA-CS
	US			
MSPE	0.489	0.556	0.519	0.523
LS	-1.200	-1.144	-1.209	-1.121
	EU			
MSPE	1.323	1.299	1.299	1.331
LS	-1.683	-1.541	-1.552	-1.697

Table 3: Mean square prediction error (MSPE), Log-score (LS) for the AR(p), MSIH(m)-AR(p) models and for the model combinations based on predictive likelihood (BMA-PL) and on the concordance statistics (BMA-CS).

5 Conclusion

In this paper we analyze empirically the relationship between forecast combination and turning point detection, when many forecast models are available for a variable of interest. We propose a Bayesian inference approach to both model estimation and model combination, which accounts for parameter and model uncertainty.

We consider linear (AR) and nonlinear (MS-AR) models and different combination strategies to forecast the turning points. It should be noted that our analysis could be extended up to include some generalisations of the model of Hamilton [1989] such as MS latent factor models (Kim [1994] and Kim and Nelson [1999]), MS models with time-varying transition probability (Sichel [1991], Watson [1994], Diebold and Rudebusch [1996], Durland and McCurdy [1994], and Filardo [1994]), time-varying and stochastic duration models (Billio and Casarin [2010], Billio and Casarin [2011] and Chib and Dueker [2004]), and finally multivariate MS models (Diebold and Rudebusch [1996] and Krolzig [1997, 2004]). We leave the analysis of the combination of predictions from these models as a topic for further research.

We mainly find that the forecast abilities of the models change across different phases of the cycle and that the performances of the different combination strategies are cycle-specific and need to be evaluated for the problem at hand.

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