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# Capital Regulation and Tail Risk

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## Abstract

The paper studies risk mitigation associated with capital regulation, in a context when banks may choose tail risk assets. We show that this undermines the traditional result that higher capital reduces excess risk-taking driven by limited liability. Moreover, higher capital may have an unintended effect of enabling banks to take more tail risk without the fear of breaching minimal capital ratio in non-tail risky project realizations. The results are consistent with stylized facts about pre-crisis bank behavior, and suggest implications for the optimal design of capital regulation.

# 1 Introduction

Regulatory reform in the wake of the recent financial crisis has focused on an increase in capital cushions of financial intermediaries. Basel III rules have doubled the minimal capital ratio, and directed banks to hold excess capital as conservation and countercyclical buffers above the minimum (BIS, 2010). These arrangements complement traditional moral suasion and individual targets used by regulators to ensure adequate capital cushions.

There are two key arguments in favor of higher capital. The first is an ex post argument: capital can be seen as a buffer that absorbs losses and hence reduces the risk of insolvency. This risk absorption role helps also reduce systemic risk factors, such as collective uncertainty over counterparty risk, which had a devastating propagation effect during the recent crisis. The second considers the ex ante effects of buffers: more capital reduces risk-shifting incentives for bank shareholders, by increasing their "skin in the game" (potential loss in case of bank failure), and hence reduces limited liability-driven incentives to take excessive risk (Jensen and Meckling 1976, Holmstrom and Tirole 1997).

Yet some recent experience calls for caution. First, banks are increasingly exposed to tail risk, which causes losses only rarely, but when those materialize they often exceed any plausible initial capital. Such risks can result from a number of strategies. A first example are carry trades reliant on short term wholesale funding, which in 2007-2008 produced highly correlated distressed sales (Gorton, 2010). A second example is the reckless underwriting of contingent liabilities on systemic risk, callable at times of collective distress (Acharya and Richardson, 2009). Finally, the combination of higher profits in normal times and massive losses occasionally arises in undiversified industry exposures to inflated housing markets (Shin, 2009). Since under tail risk banks do not internalize losses independently of the level of initial capital, the buffer and incentive effects of capital diminish. Higher capital may become a less effective way of controlling bank risk.

Second, a number of major banks, particularly in the United States, appeared highly capitalized just a couple of years prior to the crisis. Yet these very intermediaries took excessive risks (often tail risk, or highly negatively skewed gambles). In fact, anecdotal evidence suggests that highly capitalized banks were looking for ways to use or put to risk their capital in order to produce returns for shareholders (Berger et al. 2008, Huang and Ratnovski 2009). Therefore higher capital may create incentives for risk-taking instead of mitigating them.

This paper seeks to study these concerns by reviewing the effectiveness of capital regulation, and in particular of excess capital buffers (that is, above minimum ratios), in dealing with tail risk events. We reach two key results.

First, we show that the traditional buffer and incentives effects of capital become less powerful when banks have access to tail risk projects. The reason is that tail risk realizations can wipe out almost any level of capital. Left tails limit the effectiveness of capital as the absorbing buffer and restrict "skin in the game" because a part of the losses is never borne by shareholders. Hence, under tail risk, excess risk-shifting incentives of bank shareholders may exist almost independently of the level of initial or required capital.

Having established that under tail risk the benefits of higher capital are limited, we next consider its possible unintended effects. We note that capital regulation also affects bank risk choices through the threat of capital adjustment costs when banks have to raise equity to comply with minimum capital ratios. (These costs are most commonly associated with equity dilution under asymmetric information on the value of illiquid bank assets, Myers and Majluf, 1984, or reduced managerial incentives for efficiency, Jensen, 1986).[1] Similar to "skin in the game", capital adjustment costs make banks averse to risk, and may discourage risky bank strategies. However, unlike "skin in the game", the incentive effects of capital adjustment costs fall in higher bank capital because the probability of breaching the minimal capital ratio decreases.

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[1]The fact that adjustment costs of bank equity raising are significant was highlighted, for example, in the Basel Committee-FSF (2010) assessment of the impact of the transition to stronger capital requirements.

3

Of course, if highly capitalized banks internalized all losses, they would have taken risk only if that way socially optimal (offered a higher NPV). Yet this result changes dramatically once we introduce tail risk. Then, even banks with high capital never internalize all losses, and may take excess risk. Moreover, the relationship between capital and risk can become not monotonic. The reason is interesting. In the first place, tail risk leads to insolvency whatever the bank capital, so capital does not sufficiently discourage risk taking for well capitalized banks through "skin in the game". At the same time, higher excess capital allows the bank to take riskier project without breaching the minimal capital ratio (and incurring large capital adjustment costs) in the case of low (non tail) returns. So under tail risk, higher capital may create conditions where highly capitalized banks take more excess risk. Further, we show that the negative effect of extra capital on risk taking becomes stronger when banks get access to projects with even higher tail risk.

These results are interesting to consider in historic context. Most sources of tail risk we described are related to recent financial innovations. In the past, tail risks in traditional loan- (or underwriting-) oriented banking were low, hence "skin in the game" effects dominated, and extra capital lead to almost unambiguously lower risk-taking. Yet now, when banks have access to tail risk projects, the buffer and skin in the game effects that are the cornerstone of the traditional approach to capital regulation became weak, while effects where higher capital enables risk-taking became stronger. Therefore, in a context of high financial innovation the beneficial effects of higher capital are reduced, while the scope for undesirable effects increases.

The paper has policy implications relevant for the recent debate. The simpler conclusion is that it is impossible to control all aspects of risk taking using a single instrument. The problem of capital buffers is that they are effective as long as they can minimize not just the chance of default, but also a loss given default. Contractual innovation in finance has enabled intermediaries to manufacture risk profiles which still allow to take maximum advantage of limited liability. The key to contain gambles with skewed return is to either prohibit extreme bets, or to increase their ex ante cost. Leading

policy proposals now emerging are to charge prudential levies on strategies exposed to systemic risk (Acharya et al., 2010), such as extremely mismatched strategies (Perotti and Suarez, 2009), or derivative positions written on highly correlated risks.

A more intricate conclusion relates to implications for capital regulation. The results do not imply that less capital is better: this was not the case in recent years. However, they suggest the following. First, regulators should acknowledge that traditional capital regulation has limitations in dealing with tail risk. This is similar, for example, to an already-accepted understanding that it has limitations in dealing with correlation risk (cf. Acharya, 2009). Second, banks with significant excess capital may be induced to take excess risk (in order to use or put to risk their capital), as amply demonstrated by the crisis experience. Hence, simply relying on higher and "excess" capital of banks as a means of crisis prevention may have ruinous effects if it produces a false sense of comfort. Finally, authorities should introduce complementary measures to target tail risks next to the policy on pro-cyclical and conservation buffers. In this context, enhanced supervision with a focus on capturing tail risks may be essential.

We see our paper as related to two key strands of the banking literature. First are the papers on the unintended effects of bank capital regulation. Early papers (Kahane 1977, Kim and Santomero 1988, Koehn and Santomero 1980) took a portfolio optimization approach to banking and caution that higher capital requirements can lead to an increase in risk of the risky part of the bank's portfolio. Later studies focus on incentive effects. Boot and Greenbaum (1993) show that capital requirements can negatively affect assets quality due to a reduction in monitoring incentives. Blum (1999), Caminal and Matutes (2002), Flannery (1989) and Hellman et al. (2000) argue that higher capital can make banks to take more risk as they attempt to compensate for the cost of capital. Our paper follows this literature, with a distinct and contemporary focus on tail risk.[2] On empirical front, Angora et al. (2009) and Bichsel and Blum (2004) find a positive correlation between levels of capital and bank risk-taking.

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[2]Recent studies develop different measures for banks' tail risks. Acharya and Richardson (2009), Adrian and Brunnermeier (2009), and De Jonghe (2009) compute realized tail risk exposure over a certain period by using historical evidence of tail risk events, while Knaup and Wagner (2010) propose a forward-looking measure for bank tail risk.

Second strand are the recent papers on the regulatory implications of increased so-phistication of financial intermediaries and the recent crisis. These papers generally argue that dealing with new risks (including systemic and tail risks) requires new reg-ulatory tools (Acharya and Yorulmazer 2007, Acharya et al. 2010, Brunnermeier and Pedersen 2008, Huang and Ratnovski 2011, Perotti and Suarez 2009).

The structure of the paper is as follows. Section 2 outlines the theoretical model. Section 3 describes the traditional skin in the game effect of capital on risk-taking. Section 4 shows how higher capital can enable risk-taking when banks have access to tail risk projects. Section 5 concludes. The proofs and extensions are in the Appendix.

## 2    The Model

The model has three main ingredients. First, the bank is managed by an owner-manager (hereafter, the banker) with limited liability, who can opportunistically engage in asset substitution. Second, the bank operates in a prudential framework based on a minimal capital ratio, with a capital adjustment cost if the bank fails to meet the ratio and has to raise extra equity. Finally, the bank has access to tail risk projects. Such a setup is a stylized representation of the key relevant features of the modern banking system. There are three dates $(0, \frac{1}{2}, 1)$, no discounting, and everyone is risk-neutral.

**The bank**    At date 0, the bank has capital $C$ and deposits $D$. For convenience, we normalize $C + D = 1$. Deposits are fully insured at no cost to the bank; they carry a 0 interest rate and need to be repaid at date 1.

The bank has access to two alternative investment projects. Both require an outlay of 1 at date 0 (all resources available to the bank), and produce return at date 1. The return of the *safe* project is certain: $R_S > 1$. The return of the *risky* project is probabilistic: high, $R_H > R_S$, with probability $p$; low, $0 < R_L < 1$, with probability $1 - p - \mu$; or extremely low, $R_0 = 0$, with probability $\mu$. We consider the risky project with three outcomes in order to capture both the second (variance) and the third (skewness, or

6

"left tail", driven by the $R_0$ realization) moments of the project's payoff.

In the spirit of asset substitution literature, we assume that the net present value (NPV) of the safe project is higher than that of the risky project:

$$R_S > pR_H + (1 - p - \mu)R_L, \tag{1}$$

and yet the return on the safe asset, $R_S$, is not too high so that the banker has incentives to choose the risky project at least for low levels of initial capital:

$$R_S - 1 < p(R_H - 1). \tag{2}$$

The left-hand side of (2) is the banker's expected payoff from investing in the safe project, and the right-hand side is the expected payoff from shifting to the risky project, conditional on bank having no initial capital (i.e., $C = 0$ and $D = 1$). We consequently study conditions under which the bank's leverage creates incentives for it to opportunistically choose the suboptimal, risky project.

The bank's project choice is unobservable and unverifiable. However, the return of the project chosen by the bank becomes observable and verifiable before final returns realize, at date $\frac{1}{2}$.[3] The bank chooses the safe project when indifferent. The bank has no continuation value beyond date 1. (We discuss the impact of a positive continuation value in the Appendix; it reduces bank risk-taking but does not affect our results.)

**Capital regulation**  Capital regulation is based on the minimal capital (leverage) ratio. We take this regulatory design as exogenous, since it is the key feature of Basel regulation. (The paper limits itself to positive analysis and does not intend to rationalize or suggest optimal design for capital regulation.)

We define the bank's capital ratio, $c = (A - D)/A$, where $A$ is the value of bank assets, $D$ is the face value of deposits, and $A - D$ is economic capital. At date 0, before

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[3] The assumption that project choice is unobservable while project returns are, is a standard approach to modelling (Hellman et al. 2000, Rochet 2004).

the investment is undertaken, the capital ratio is $c = C/(C + D) = C$. At dates $\frac{1}{2}$ and 1 the capital ratio is $c_i = (R_i - D)/R_i$, with $i \in \{S, H, L, 0\}$ reflecting project choice and realization. The fact that the date $\frac{1}{2}$ capital position is defined in a forward-looking way is consistent with the practice of banks recognizing known future gains or losses.

At any point in time, the bank's capital ratio $c$ must exceed a certain $c_{\min}$, $c_{\min} > 0$. We assume that the minimal ratio is satisfied at date 0: $c > c_{\min}$. Consequently, the minimal ratio is also satisfied for realizations $R_S$ (when the bank chooses the safe project) and $R_H$ (when the bank chooses the risky project and is successful): $c_H > c_S > c > c_{\min}$, since $R_H > R_S > 1$. The minimal capital ratio is never satisfied for $R_0$ (in the extreme low outcome of the risky project), since the bank's capital is negative, $c_0 = -\infty < 0 < c_{\min}$. The banks' capital sufficiency under a low realization of the risky project $R_L$ is ambiguous. As we will show below, depending on the bank's initial capital, it can be positive and sufficient, $c_L > c_{\min}$, or positive but insufficient, $0 < c_L < c_{\min}$, or negative, $c_L < 0$.

At date $\frac{1}{2}$, once the bank's future returns and capital position become known, the regulator imposes corrective action if a bank does not satisfy the minimal capital ratio. Recall that it happens under $R_0$ and possibly $R_L$ realizations of the risky project. Specifically, the banker is given two options. One is to surrender the bank to the regulator. Then, banker's stake (equity value) is wiped out and the banker receives a zero payoff. Alternatively, the bank can attract additional capital to bring its capital ratio to the regulatory minimum, $c_{\min}$. Attracting capital carries a fixed cost $T$ (which is exogenous to the model and reflects agency costs of raising new equity and asymmetric information, or it can be associated with a necessary dilution of equity). A more general specification of the cost is discussed in the Appendix; we show that it does not affect our results. The banker chooses to abandon rather than recapitalize the bank when indifferent.

**Timeline**  The model outcomes and the sequence of events are depicted in Figure 1.

<< Figure 1 here >>

# 3    "Skin in the game" and tail risk

In this section we study how tail risk affects the traditional "skin in the game" incentive effects of higher capital on risk-taking. We show that these effects become weaker when the bank has access to tail risk projects, because the banker does not internalize some cost of risk-taking for any value of initial capital. This brings us to the first policy result, that capital regulation may have limited effectiveness in dealing with tail risk.

Throughout the section, we abstract from the effects of capital adjustment costs (and hence from the effects of the minimal capital ratio); we introduce these in the next section. We solve the model backwards: first deriving the payoffs depending on bank project choice, and then the project choice itself. The solution is followed by comparative statics.

## 3.1    Payoff and project choice

Consider the bank with access to tail risk projects ($\mu \geq 0$), but in a setup without capital adjustment costs ($T = 0$). The banker's payoff to choosing the safe project is:

$$\Pi_S^{T=0} = R_S - D = R_S - (1 - c). \tag{3}$$

The banker's payoff to choosing the risky project is:

$$\Pi_R^{T=0} = p \cdot [R_H - (1 - c)] + (1 - p - \mu) \cdot \max\{R_L - (1 - c); 0\}, \tag{4}$$

where on right hand side $p \cdot [R_H - (1 - c)]$ is the probability of and the payoff in $R_H$ realization, and $(1 - p - \mu) \cdot \max\{R_L - (1 - c); 0\}$ is the probability of and the payoff in $R_L$ realization. The third, $R_0$ realization occurs with probability $\mu$ but carries a zero payoff.

The bank chooses the safe project over the risky project when:

$$\Pi_S^{T=0} \geq \Pi_R^{T=0},$$

which is equivalent to:

$$R_S - (1-c) \geq p \cdot [R_H - (1-c)] + (1-p-\mu) \cdot \max\{[R_L - (1-c)]; 0\}. \tag{5}$$

The following Proposition describes the bank's investment decision.

**Proposition 1** *The bank's project choice is characterized by a threshold for initial capital $c$ as follows:*

*(a) For*

$$R_S < pR_H + (1-p)R_L, \tag{6}$$

*the bank chooses the safe project for $c \geq 1 - \frac{R_S - pR_H - (1-p-\mu)R_L}{\mu}$, and the risky project otherwise;*

*(b) For*

$$R_S \geq pR_H + (1-p)R_L, \tag{7}$$

*the bank chooses the safe project for $c \geq 1 - \frac{R_S - pR_H}{1-p}$, and the risky project otherwise.*

**Proof.** In Appendix. ■

The intuition for Case (b) is that when $R_S$ is high enough, the bank's risk-shifting incentives are so low, that the bank will only take a risky project when it has negative capital under $R_L$ realization, allowing the bank to shift more of the downside to the creditors. Then, the bank gets the same payoff in $R_0$ and $R_L$ realizations (i.e., zero) and its project choice is not affected by the probability of the tail risk realization, $\mu$. Note also that when $\mu$ is low enough, by (1), $R_S$ has to be high enough, so that for $\mu \to 0$, Case (a) converges to Case (b). The result depicted in Case (b) validates the traditional

10

intuition that, in absence of tail risk projects, the bank's incentives to opportunistically choose the risky investment monotonically decrease in its initial capital. The reason, a standard one, is that the bank with low capital does not internalize the losses in the downside $R_L$ realization of the risky project (its limited liability is binding).

Since in Case (b) the distinction between tail and non-tail risk does not play a role in bank's choice, we focus on Case (a) which captures the key points of our model. This case allows us to study the impact of tail risk on bank's project choice. We denote:

$$c^{T=0} = 1 - \frac{R_S - pR_H - (1-p-\mu)R_L}{\mu},$$ (8)

with $c^{T=0}$ being the threshold for risk-shifting incentives under (6).

## 3.2  Comparative statics

We study how the threshold $c^{T=0}$, the initial capital necessary to prevent the bank from risk-shifting, is affected by the project's tail risk $\mu$. To maintain comparability, we consider transformations of the risky project that increase $\mu$ but preserve its expected value, denoted by $E(R)$. There are various ways to alter model parameters to achieve that, but we highlight the two with the best interpretations, which we analyze in turn.

**Case 1**  One of the sources of tail risk in the run-up to the recent crisis was engaging in carry trades or taking undiversified exposures e.g. to housing markets. Such activities shift the distribution of the risky project to extreme outcomes: within the confines of our models we can interpret that as a shift in the probability mass from $R_L$ to $R_0$ and $R_H$. Formally, that implies an increase in $\mu$ and $p$, at the expense of $(1-p-\mu)$. To keep $E(R)$ constant, following to an increase in $\mu$ by $\Delta\mu$, $p$ should increase by $\frac{R_L}{R_H-R_L}\Delta\mu$.

Using (8),
$$\frac{\partial c^{T=0}}{\partial \mu}\Big|_{E(R)=constant} = \frac{R_S - E(R)}{\mu^2} > 0.$$ (9)

So that the amount of capital necessary to prevent risk-shifting increases in tail risk.

**Case 2** Another source of tail risk was the underwriting of contingent liabilities on market risk, when the bank is compensated through premia. Formally, this can be interpreted as that a higher $\mu$ is compensated by higher $R_L$ and $R_H$ (other parameters being equal) such that $E(R)$ is kept constant. In order to achieve this, following to an increase in $\mu$ by $\Delta\mu$, both $R_L$ and $R_H$ should increase by $R_L\frac{\Delta\mu}{1-\mu-\Delta\mu}$.[4]

Deriving (8) with respect to $\mu$ we get the same first derivative as in the previous case,

$$\frac{\partial c^{T=0}}{\partial\mu}\Big|_{E(R)=constant} = \frac{R_S - E(R)}{\mu^2} > 0.$$

Hence, again, the amount of capital necessary to prevent risk-shifting increases in tail risk.

In both cases, observe that $c^{T=0}$ grows logarithmically in $\mu$.[5] Therefore, capital becomes progressively a less effective incentive tool for controlling bank risk-taking when the bank has access to tail risk projects. The impact is more pronounced for low values of $\mu$. As an implication, tail risk limits the effectiveness of capital regulation in dealing with bank risk-taking incentives.

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[4] We assume for expositional simplicity that $R_L$ and $R_H$ increase by the same amount $\Delta$, the premium obtained by the bank. However, our results will not change if we assume that following an increase in $\mu$ by $\Delta\mu$, $R_L$ and $R_H$ increase by $\Delta R_L$ and $\Delta R_H$, as long as the identity $p \cdot \Delta R_H + (1 - p - \mu) \cdot \Delta R_L = \Delta\mu \cdot (R_L + \Delta R_L)$ holds, with $\Delta R_L \neq \Delta R_H$.

[5] From (8), keeping $E(R)$ constant for any value of $\mu$, we see that $c^{T=0}(\mu) = 1 - \frac{const}{\mu}$, with $const$ being a positive number measuring by how much the expected value of the safe project is higher than that of the risky project. To find the degree of polynomial $c^{T=0}(\mu)$ we need to compute $\lim_{\mu\to\infty}\frac{\mu \cdot [c^{T=0}(\mu)]'}{c^{T=0}(\mu)}$. This equals $\lim_{\mu\to\infty}\frac{const}{\mu-const} = 0$, which is the degree of the logarithm function.

# 4 Tail risk and the unintended effects of capital

In the previous section, we showed that capital becomes a less effective tool for controlling bank risk-taking when there is tail risk. We will now introduce an additional feature – capital adjustment costs – to obtain a stronger result. In addition to being a less powerful tool, higher capital may have unintended effects of *enabling* banks to take higher risk. Specifically, we show that marginally capitalized banks do not take risk because they are averse to breaching the minimal capital ratio in case of mildly negative realizations of the risky project $(R_L)$. Yet banks with higher capital can take more risk because their chance of breaching the ratio in such realizations is lower. Further, in comparative statics, we demonstrate that the unintended effects of higher capital become stronger when banks get access to projects with higher tail risk.

To put differently, this section will outline a horse race of two opposite effects of higher capital. On the positive side, higher capital increases "skin in the game" and reduces risk-shifting incentives. Yet this effect becomes weaker in higher tail risk. On the negative side, higher capital increases the distance to the minimal capital ratio and allows the banker to take more risk without the fear of breaching the ratio. This effect becomes stronger in tail risk. Therefore, under high tail risk, negative unintended effects of higher capital may dominate, so that well-capitalized banks choose riskier projects in equilibrium.

As before, we solve the model backwards: first we derive the payoffs depending on bank project choice, then the project choice itself. The solution is followed by comparative statics.

## 4.1 Payoffs and the recapitalization decision

The banker's payoff to choosing the safe project is:

$$\Pi_S = R_S - D = R_S - (1 - c).$$

Now consider the banker's payoff to the risky project. When the projects' return is $R_H$, the banker obtains $R_H - (1-c)$. When the project's return is $R_0$, the banker obtains zero.

The case when the risky project's return is $R_L$ is more complex, because depending on the relative values of $c$ and $R_L$, the bank's capital may be positive and sufficient, positive but insufficient, or negative. Consider these in turn.

Under $R_L$, the bank has positive and sufficient capital ($c_L \geq c_{\min}$) when:

$$\frac{R_L - (1-c)}{R_L} \geq c_{\min},$$

which gives:

$$c \geq c^{Sufficient} = 1 - (1 - c_{\min})R_L. \tag{10}$$

Then, the bank continues to date 1, repays depositors, and obtains $R_L - (1-c)$.

When $c < c^{Sufficient}$, the bank has insufficient capital in the $R_L$ realization ($c_L < c_{\min}$), and has to be either abandoned or recapitalized at cost $T$. The banker chooses to recapitalize the bank for:

$$R_L - (1-c) - T > 0, \tag{11}$$

where the left-hand side is the banker's return after repaying depositors net off the recapitalization cost, and the right hand side is the zero return in case the bank is abandoned. Expression (11) can be re-written as:

$$c > c^{\mathrm{Re}\,capitalize} = 1 + T - R_L. \tag{12}$$

We focus our analysis on the case when $c^{\mathrm{Re}\,capitalize} < c^{Sufficient}$, corresponding to:

$$T < c_{\min}R_L, \tag{13}$$

so that there exist values of $c$ such that $c^{\mathrm{Re}\,capitalize} < c < c^{Sufficient}$ where the banker chooses to recapitalize the bank following the $R_L$ realization instead of abandoning it.

When $T$ is larger than $c_{\min} R_L$, the banker always abandons a bank with insufficient capital. Note that both thresholds $c^{\text{Re} capitalize}$ and $c^{Sufficient} \in (0,1)$.

Figure 2 illustrates the bank's recapitalization decision.

$$<< \text{Figure 2 here} >>$$

Overall, the banker's payoff to the $R_L$ realization of the risky project is:

$$
\Pi_L = \begin{cases} R_L - (1-c), \text{if } c \geq c^{Sufficient} \\ R_L - (1-c) - T, \text{ if } c^{\text{Re} capitalize} < c < c^{Sufficient} \\ 0, \text{ if } c \leq c^{\text{Re} capitalize} \end{cases} , \tag{14}
$$

and the overall payoff to the risky project is:

$$
\Pi_R = p \cdot [R_H - (1-c)] + (1 - p - \mu) \cdot \Pi_L. \tag{15}
$$

## 4.2 Project choice

We now consider bank project choice at date 0, depending on its initial capital ratio $c$.

The bank chooses the safe project over the risky project for:

$$
\Pi_S \geq \Pi_R,
$$

which is equivalent to:

$$
R_S - (1-c) \geq p \cdot [R_H - (1-c)] + (1 - p - \mu) \cdot \Pi_L. \tag{16}
$$

To describe the results we introduce two thresholds:

$$
W = p R_H + (1-p) R_L - \mu c_{\min} R_L, \tag{17}
$$

and

$$Z = pR_H + (1-p)(R_L - T) + \mu(T - c_{\min}R_L). \tag{18}$$

$W$ is a threshold point for the existence of risk-shifting: for $R_S < W$ there exist values of initial capital such that a well-capitalized bank which remains with sufficient capital upon the $R_L$ realization of the risky project (i.e., $c \geq c^{Sufficient}$) chooses a risky project due to risk-shifting. $Z$ is a threshold point for the existence of binding capital adjustment cost effect: for $R_S \geq Z$ there exist values of initial capital such that a less capitalized bank ($c^{\text{Re}\,capitalize} < c < c^{Sufficient}$) chooses a safe project to prevent recapitalization costs upon the $R_L$ realization of the risky project. The derivation of the thresholds is in the Appendix; the Appendix also verifies that $Z < W$.

Then, the risk-shifting and capital adjustment effect of bank project choice interact with each other as follows:

**Proposition 2** *The bank's project choice is characterized by thresholds $c^*$ and $c^{**}$:*

*(a) For $Z \leq R_S < W$, there exist $c^* < c^{Sufficient}$, and $c^{**} > c^{Sufficient}$, such that*

- *For $c < c^*$ the bank chooses the risky project and may abandon or recapitalize it upon the $R_L$ realization;*

- *For $c^* \leq c < c^{Sufficient}$ the bank chooses the safe project to avoid abandonment or recapitalization upon the $R_L$ realization; the choice of the safe project here represents the capital adjustment cost effect;*

- *For $c^{Sufficient} \leq c < c^{**}$ the bank chooses the risky project because its capital is high enough to avoid breaching the minimal capital ratio in the $R_L$ realization; the choice of the risky project here represents the risk-shifting effect driven by higher capital;*

- *For $c \geq c^{**}$ the bank chooses the safe project because its capital is high enough to prevent risk-shifting;*

*(b) For $R_S < Z$, there exist $c^{**} > c^{Sufficient}$ such that for $c < c^{**}$ the bank chooses the risky project, and for $c \geq c^{**}$ the safe project; there is only a risk-shifting effect, a bank with $c < c^{Sufficient}$ never chooses a safe project to avoid recapitalization cost;*

*(c) for $R_S \geq W$, there exist $c^* < c^{Sufficient}$ such that for initial capital $c < c^*$ the bank chooses a risky project, and for $c \geq c^*$ the safe project ; there is only a capital adjustment cost effect, a bank with $c > c^{Sufficient}$ never engages in risk-shifting.*

**Proof.** *In Appendix.* ■

The thresholds

$$c^* = 1 - \frac{R_S - pR_H - (1 - p - \mu)(R_L - T)}{\mu}, \tag{19}$$

and

$$c^{**} = 1 - \frac{R_S - pR_H - (1 - p - \mu)R_L}{\mu}, \tag{20}$$

are also derived in Appendix.

Case (a) of Proposition 2 contains the main result of our paper: that the relationship between bank capital and risk-taking can be non-monotonic in the presence of tail risk and capital adjustment cost. When capital is very low, $c < c^*$, the banker faces strong risk-shifting incentives and a low cost of abandoning the bank, hence chooses high risk. For intermediate initial capital, $c^* \leq c < c^{Sufficient}$, the banker's equity value is higher, and the banker chooses a safe project to avoid abandoning or recapitalizing the bank in the $R_L$ realization. The choice of the safe project is driven by capital adjustment cost - a novel effect highlighted in this paper. Yet as soon as the bank has initial capital high enough to satisfy the minimal ratio in the $R_L$ realization, for $c^{Sufficient} \leq c < c^{**}$, the capital adjustment cost stops being binding and the banker again switches to the risky project, driven by the risk-shifting effect. Finally, for very high levels of capital, $c \geq c^{**}$,

the banker has so much skin in the game that risk-shifting incentives are not binding. This is the traditional effect of capital regulation; recall that under tails risk the effect might not kick in until $c$ is very high. The bank's project choice is depicted in Figure 3.

$$<< \text{Figure 3 here} >>$$

## 4.3 Comparative statics

In this section we repeat the comparative statics exercise of Section 3.2, in the presence of capital adjustment costs – with respect to the Case (a) of Proposition 2. We show that when tail risk increases (the risky project has a heavier left tail), highly capitalized banks get stronger incentives to take excess risk. We use the two transformations of the risky project highlighted in Section 3.2.

**Case 1**  When a higher $\mu$ is compensated by a higher $p$, keeping $E(R)$ constant (illustrating the case when tail risk is a result if carry trades or undiversified exposures), that affects both thresholds $c^*$ and $c^{**}$. To focus on banker's incentives to take excessive risk, we consider the area $[c^{Sufficient}, c^{**})$, corresponding to levels of initial bank capital for which the bank undertakes the risky project. Note that $c^{Sufficient}$ is determined only by $c_{\min}$ and $R_L$ (see (10)), and hence is unaffected by a change in the probability distribution of the risky project. The critical threshold for the discussion is therefore $c^{**}$. Note from (8) that $c^{**} = c^{T=0}$. The impact of the change in probability distribution of the risky project on $c^{**}$ is then the same as in (9):

$$\frac{\partial c^{**}}{\partial \mu}\big|_{E(R)=constant} = \frac{R_S - E(R)}{\mu^2} > 0.$$

This means that when tail risk increases, the interval $[c^{Sufficient}, c^{**})$ on which a well-capitalized bank chooses the risky project expands. Interestingly, the interval expands because banks with *higher* capital start taking more risk. This highlights the relationship between tail risk and the unintended effects of higher bank capital. The intuition is that when investment returns become more polarized, they enable well-capitalized banks to

18

earn higher profits in good time, while at the same time reducing the expected cost of recapitalization since the low return $R_L$ (which triggers the recapitalization decision) is less frequent. Unintended effects of bank capital appear specifically in well-capitalized banks.

Figure 4a depicts the impact of increased tail risk on bank project choice, when there is a shift in the mass of the probability distribution from $R_L$ to $R_0$ and $R_H$.

<< Figure 4a here >>

**Case 2** When a higher $\mu$ is compensated by higher $R_L$ and $R_H$ (illustrating the case when a bank underwrites contingent liabilities on market risk and is compensated through collected premia), this change in the return profile of the risky asset affects thresholds $c^*$ and $c^{**}$, and $c^{Sufficient}$ as well. To focus on bank's incentives to take excessive risk, we consider the area $[c^{Sufficient}, c^{**})$, corresponding to levels of initial bank capital for which the bank undertakes the risky project. Note that $c^{Sufficient}$ is decreasing in $R_L$ (see (10)) and hence in $\mu$. At the same time, from (9), $c^{**}$ in increasing in $\mu$. Hence, under higher tail risk, the interval $[c^{Sufficient}, c^{**})$ widens even more than in Case 1, and both better and less capitalized banks start choosing the risky project.

Figure 4b depicts the impact of increased tail risk on bank project choice, when a higher $\mu$ is compensated by higher premia (i.e., higher $R_L$ and $R_H$).

<< Figure 4b here >>

19

# 5    Conclusion

This paper examined the relationship between bank capital and risk-taking when banks have access to tail risk projects. We showed that traditional capital regulation becomes less effective in controlling bank risk because banks never internalize the negative realizations of tail risk projects. Moreover, we have suggested novel channels for unintended effects of higher capital: it enables banks to take higher tail risk without the fear of breaching the minimal capital requirement in mildly bad (i.e., non-tail) project realizations. The results are consistent with stylized facts about pre-crisis bank behavior, and have implications for the design of bank regulation.

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# A    Proofs

## A.1    Proof of Proposition 1

Consider first the case when $c > 1 - R_L$. The relevant incentive compatibility condition derived from (5) becomes $R_S - (1-c) \geq p \cdot [R_H - (1-c)] + (1-p-\mu) \cdot [R_L - (1-c)]$, which can be rewritten as $c \geq 1 - \frac{R_S - pR_H - (1-p-\mu)}{\mu}$. We denote $c^{T=0} = 1 - \frac{R_S - pR_H - (1-p-\mu)R_L}{\mu}$. The threshold exists and is strictly larger than $1 - R_L$ for values of $R_S$ satisfying (6). Hence, for $R_S < pR_H + (1-p)R_L$, $\exists \ c^{T=0} \in (1-R_L, 1]$ such that $\forall \ c \in (1-R_L, c^{T=0})$ the risky project is selected and $\forall \ c \in [c^{T=0}, 1]$ the safe project is chosen. Otherwise, if (6) is not fulfilled, the bank selects the safe project $\forall \ c \in (1-R_L, 1]$.

Consider now the case $c \leq 1 - R_L$. The relevant incentive compatibility condition derived from (5) is $R_S - (1-c) \geq p \cdot [R_H - (1-c)]$. The condition is equivalent with $c \geq 1 - \frac{R_S - pR_H}{1-p}$. We denote $c_{Traditional}^{T=0} = 1 - \frac{R_S - pR_H}{1-p}$. The threshold exists and is below or equal to $1 - R_L$ for values of $R_S$ satisfying (7). Thus, for $R_S \geq pR_H + (1-p)R_L$, $\exists \ c_{Traditional}^{T=0} \in [0, 1-R_L]$ such that $\forall \ c \in [0, c_{Traditional}^{T=0})$ the risky project is selected and $\forall \ c \in [c_{Traditional}^{T=0}, 1-R_L]$ the safe project is chosen. Otherwise, if (7) is not fulfilled, the bank selects the risky project $\forall \ c \in [0, 1-R_L]$.

To sum up, when (6) is fulfilled, $\exists \ c^{T=0} \in (1-R_L, 1]$ such that $\forall \ c \in [0, c^{T=0})$ the risky project is selected and $\forall \ c \in [c^{T=0}, 1]$ the safe project is chosen. Likewise, when (6) is not fulfilled, $\exists \ c_{Traditional}^{T=0} \in [0, 1-R_L]$ such that $\forall \ c \in [0, c_{Traditional}^{T=0})$ the risky project is selected and $\forall \ c \in [c_{Traditional}^{T=0}, 1]$ the safe project is chosen.

## A.2    Proof of Proposition 2

We consider two scenarios in turn. We start by analyzing a scenario in which the cost of recapitalization is such that $\frac{\mu}{1-p} c_{\min} R_L < T < c_{\min} R_L$. Subsequently we show that our results are similar for $T \leq \frac{\mu}{1-p} c_{\min} R_L$. Both cases follow from assumption (13) of low adjustment cost.

### A.2.1 $\quad \frac{\mu}{1-p} c_{\min} R_L < T < c_{\min} R_L$

We study bank's behavior for three levels of initial capital: low (i.e., $c \leq c^{\text{Re}\,capitalize}$), intermediate (i.e., $c^{\text{Re}\,capitalize} < c < c^{Sufficient}$) and high (i.e., $c \geq c^{Sufficient}$).

Consider first the case when $c \in [0, c^{\text{Re}\,capitalize}]$. The banker never finds optimal to recapitalize for low realization of the risky project. The relevant incentive compatibility condition derived from (16) is $R_S - (1-c) \geq p \cdot [R_H - (1-c)]$, where the left-hand side is the return on investing in the safe project and the right-hand side is the expected return on selecting the risky project. The condition can be rewritten as $c \geq 1 - \frac{R_S - pR_H}{1-p}$. We denote $c_1^* = 1 - \frac{R_S - pR_H}{1-p}$. The threshold $c_1^*$ exists if and only if the next two constraints are jointly satisfied:

$$R_S < 1 - p + pR_H, \tag{T1a'}$$

$$R_S > pR_H + (1-p)(R_L - T). \tag{T1a}$$

The former condition guarantees a positive $c_1^*$, while the latter forces the threshold to be lower than $c^{\text{Re}\,capitalize}$, the upper limit for the interval we analyze. If (T1a') is not fulfilled, then $c_1^* < 0$ and $\forall\, c \in [0, c^{\text{Re}\,capitalize}]$, the bank prefers the safe project. If (T1a) is not fulfilled, then $c_1^* > c^{\text{Re}\,capitalize}$ and $\forall\, c \in [0, c^{\text{Re}\,capitalize}]$ the bank invests risky. When both constraints are simultaneously satisfied, $\exists\, c_1^* \in [0, c^{\text{Re}\,capitalize}]$ such that $\forall\, c \in [0, c_1^*)$ the risky project is selected and $\forall\, c \in [c_1^*, c^{\text{Re}\,capitalize}]$ the safe project is chosen. Assumption (2) implies that (T1a') is always fulfilled.

Consider now the case when $c \in (c^{\text{Re}\,capitalize}, c^{Sufficient})$. The banker finds optimal to recapitalize for low realization $R_L$. The relevant incentive compatibility condition is $R_S - (1-c) \geq p \cdot [R_H - (1-c)] + (1-p-\mu) \cdot [R_L - (1-c) - T]$, with the certain return on choosing the safe project depicted on the left-hand side, and expected return on investing in the risky project depicted on the right-hand side. Rearranging terms the condition can be rewritten as $c \geq 1 - \frac{R_S - pR_H - (1-p-\mu)(R_L - T)}{\mu}$. We denote $c_2^* = 1 - \frac{R_S - pR_H - (1-p-\mu)(R_L - T)}{\mu}$. Similarly with the previous case, the threshold $c_2^*$ exists if

and only if it is simultaneously higher and lower than the lower and the higher boundary of the analyzed interval, respectively. The conditions are as follows:

$$R_S < pR_H + (1-p)(R_L - T), \tag{T2a}$$

$$R_S > pR_H + (1-p)(R_L - T) + \mu(T - c_{\min}R_L). \tag{T2b}$$

Condition (T2a) is the opposite of (T1a). Thus, a satisfied condition (T1a) implies that (T2a) is not fulfilled. Condition (T2a) not satisfied implies that $c_2^* < c^{\mathrm{Re}\,capitalize}$ and $\forall\ c \in (c^{\mathrm{Re}\,capitalize}, c^{Sufficient})$, the bank prefers the safe project. If the second condition is not fulfilled, then $c_2^* > c^{Sufficient}$ and $\forall\ c \in (c^{\mathrm{Re}\,capitalize}, c^{Sufficient})$ the bank invests risky. When both constraints are simultaneously satisfied, $\exists\ c_2^* \in (c^{\mathrm{Re}\,capitalize}, c^{Sufficient})$ such that $\forall\ c \in (c^{\mathrm{Re}\,capitalize}, c_2^*)$ the risky project is selected. The safe project is preferred $\forall\ c \in [c_2^*, c^{Sufficient})$.

Consider now the final interval, when $c \in [c^{Sufficient}, 1]$. For low realization $R_L$ the bank always complies with the regulatory requirements. No additional capital is needed. The relevant incentive compatibility condition is $R_S - (1-c) \geq p \cdot [R_H - (1-c)] + (1-p-\mu) \cdot [R_L - (1-c)]$. Rearranging terms the condition becomes $c \geq 1 - \frac{R_S - pR_H - (1-p-\mu)R_L}{\mu}$. We denote $c^{**} = 1 - \frac{R_S - pR_H - (1-p-\mu)R_L}{\mu}$. The threshold $c^{**}$ exists if and only if $c^{**} > c^{Sufficient}$ and $c^{**} < 1$. The later is always fulfilled following from the assumption (1) of higher NPV for the safe project. The former condition is depicted in (T3a) below. When (T3a) is not satisfied, the bank prefers the safe project for any level of initial capital larger than $c^{Sufficient}$. Otherwise, $\forall\ c \in [c^{Sufficient}, c^{**})$ the risky project is selected, while the safe project is preferred $\forall\ c \in [c^{**}, 1]$.

$$R_S < pR_H + (1-p)R_L - \mu c_{\min}R_L. \tag{T3a}$$

Next we discuss the process of project selection. Recall that $Z = pR_H + (1-p)(R_L - T) + \mu(T - c_{\min}R_L)$ and $W = pR_H + (1-p)R_L - \mu c_{\min}R_L$, from (18) and (17), respectively.

27

We also denote $B = pR_H + (1-p)(R_L - T)$. Under assumption (13), $Z < B < W$. We distinguish among four possible scenarios.

Scenario S1: $R_S < Z$. As a consequence, condition (T2b) is not satisfied and $\forall$ $c \in (c^{\mathrm{Re}\,capitalize}, c^{Sufficient})$ the bank selects the risky project. $R_S < Z$ also implies that $R_S < B$ and $R_S < W$. Condition (T1a) is not satisfied but (T3a) is. As a result, the bank invests risky $\forall c \in [0, c^{\mathrm{Re}\,capitalize}] \cup [c^{Sufficient}, c^{**})$, and the bank invests safe $\forall c \in [c^{**}, 1]$.

Scenario S2: $Z \leq R_S \leq B$. The right-hand side implies that condition (T1a) is not satisfied. For initial capital $c$ lower than $c^{\mathrm{Re}\,capitalize}$ the bank prefers the risky project. The left hand side implies that condition (T2b) is fulfilled. Also condition (T2a) is satisfied being the opposite of (T1a). Hence, we can conclude that $\exists \, c^* \in (c^{\mathrm{Re}\,capitalize}, c^{Sufficient})$ with $c^* = c_2^*$, such that $\forall c \in (c^{\mathrm{Re}\,capitalize}, c^*)$ the risky project is selected, while the safe project is preferred $\forall c \in [c^*, c^{Sufficient})$. Condition (T3a) is also satisfied. Similarly with the previous scenario, the bank invests risky $\forall c \in [c^{Sufficient}, c^{**})$, and safe $\forall c \in [c^{**}, 1]$.

Scenario S3: $B < R_S < W$. The left hand-side implies that condition (T1a) is satisfied. We can argue that $\exists \, c^* \in (0, c^{\mathrm{Re}\,capitalize})$ with $c^* = c_1^*$, such that $\forall c \in [0, c^*)$ the risky project is selected, while the safe project is preferred $\forall c \in [c^*, c^{\mathrm{Re}\,capitalize}]$. Condition (T1a) implies that (T2a) is not satisfied. Thus, $\forall c \in (c^{\mathrm{Re}\,capitalize}, c^{Sufficient})$ the safe project will be selected. The bank investment decision is identical with the one from previous scenarios when the level of capital is high enough (i.e., $c$ larger than $c^{Sufficient}$).

Scenario S4: $R_S \geq W$. Neither condition (T3a) nor condition (T2a) are satisfied anymore. The bank selects the safe project $\forall c \in (c^{\mathrm{Re}\,capitalize}, 1]$. However, condition (T1a) is fulfilled. Hence, $\exists \, c^* \in [0, c^{\mathrm{Re}\,capitalize}]$ with $c^* = c_1^*$, such that $\forall c \in [0, c^*)$ the risky project is selected, while the safe project is preferred $\forall c \in [c^*, c^{\mathrm{Re}\,capitalize}]$.

The values for thresholds $c^*$ and $c^{**}$ for Case (a) of Proposition 2, are derived under Scenario S2 above, for $Z \leq R_S \leq B$.

28

## A.2.2 $T \leq \frac{\mu}{1-p} c_{\min} R_L$

We consider now the scenario under which the cost of recapitalization is very low. Lowering $T$ has no quantitative impact on $c^{Sufficient}$ and $c^{\text{Re}\,capitalize}$, the thresholds in initial capital which trigger bank's decision between raising additional capital or letting the regulator to overtake the bank. Their relative position is unchanged: $c^{Sufficient}$ is larger than $c^{\text{Re}\,capitalize}$ following from easily verifiable identity $\frac{\mu}{1-p} c_{\min} R_L < c_{\min} R_L$ combined with our restriction on $T$. However, the process of project selection under assumption (13) is marginally affected. In this scenario $Z < W < B$, as a consequence of lower $T$. As discussed before, we distinguish among four possible scenarios (S1') $R_S < Z$, (S2') $Z \leq R_S < W$, (S3') $W \leq R_S < B$ and (S4') $R_S > B$. Discussions for scenarios S1', S2' and S4' are identical with our previous discussion for scenarios S1, S2, and S4. We discuss scenario S3' next. $W \leq R_S$ implies that condition (T3a) is not satisfied. Hence, the bank prefers the safe project for any level of initial capital larger than $c^{Sufficient}$. $R_S < B$ implies that condition (T1a) is not satisfied. For initial capital $c$ lower than $c^{\text{Re}\,capitalize}$ the bank prefers the risky project. However, condition (T2a) is satisfied being the opposite of (T1a), and also condition (T2b) is implied by the fact that $W > Z$. Hence, we can conclude that $\exists\ c^* \in (c^{\text{Re}\,capitalize}, c^{Sufficient})$ with $c^* = c_2^*$, such that $\forall$ $c \in (c^{\text{Re}\,capitalize}, c^*)$ the risky project is selected, while the safe project is preferred $\forall$ $c \in [c^*, c^{Sufficient})$.

## A.3   Analysis of the robustness of results of Proposition 2

We offer here a discussion for the results of Proposition 2. We analyze bank's project choice for the case of high cost of recapitalization: $T > c_{\min} R_L$; we show that our results are robust to this specification. Recall that under assumption (13), there exist values of $c$ such that $c^{\text{Re}\,capitalize} < c < c^{Sufficient}$ where the banker chooses to recapitalize the bank following the $R_L$ realization instead of abandoning it. We show next that when $T$ is larger than $c_{\min} R_L$, the banker always abandons a bank with insufficient capital. Although the main results from Proposition 2 are not qualitatively affected, higher

recapitalization cost has a quantitative impact on our results. Therefore, we start by deriving the new conditions which drive these results. It is optimal for the bank to raise additional capital (if this was demanded by the regulator) when conditions $c_L < c_{\min}$ and (11) are simultaneously satisfied. The former condition implies that $c < 1 - R_L(1 - c_{\min})$, while from the latter $c > 1 + T - R_L$. Under our modified assumption of high cost of recapitalization $T$, these conditions can not be satisfied simultaneously. For any levels of initial capital $c$ below $1 - R_L(1 - c_{\min})$, the bank receives a request for adding extra capital but she never finds optimal to do so because such an action will not generate positive payoffs. As a result, the bank is closed and the shareholder expropriated. Conversely, when the level of initial capital is above $1 - R_L(1 - c_{\min})$ the banking authority doesn't take any corrective action against the bank since returns $R_L$ are above the critical level $R_{\min}$. We denote:

$$c_{NEW}^{\mathrm{Re}\,capitalize} = 1 - R_L(1 - c_{\min}), \tag{21}$$

where $c_{NEW}^{\mathrm{Re}\,capitalize} \in (0, 1)$. Next, we explore the bank's project choice for levels of initial capital below and above this critical threshold.

Consider first the case when $c \in (0, c_{NEW}^{\mathrm{Re}\,capitalize})$. The bank never recapitalizes for the low realization of the risky project. The bank would have incentive to select the safe project when $R_S - (1 - c) \geq p[R_H - (1 - c)]$, which implies that initial capital $c$ to be larger than $1 - \frac{R_S - pR_H}{1 - p}$. We previously denoted $c_1^* = 1 - \frac{R_S - pR_H}{1 - p}$. This threshold exists if and only if (T1a') and the following condition are jointly satisfied:

$$R_S > pR_H + (1 - p)R_L(1 - c_{\min}). \tag{T1a NEW}$$

The second condition guarantees that $c_1^*$ is lower than $c_{NEW}^{\mathrm{Re}\,capitalize}$, the upper boundary for the interval we analyze. For large returns on the safe project (i.e., condition (T1a') is not fulfilled), $\forall\, c \in (0, c_{NEW}^{\mathrm{Re}\,capitalize})$, the bank prefers the safe project. If (T1a NEW) is not fulfilled, then $\forall\, c \in (0, c_{NEW}^{\mathrm{Re}\,capitalize})$ the bank invests risky. Otherwise,

when both constraints are simultaneously satisfied, $\forall\, c \in (0, c_1^*)$ the risky project is selected and $\forall\, c \in (c_1^*, c_{NEW}^{\mathrm{Re}\,capitalize})$ the safe project is chosen. Our assumption (2) implies that (T1a') is always fulfilled.

Consider now the second case when $c \in (c_{NEW}^{\mathrm{Re}\,capitalize}, 1)$. The bank always complies with the regulatory requirements when $R_L$ is obtained due to high initial capital. No additional capital is needed. The bank would have incentive to select the safe project when $R_S - (1-c) \geq p[R_H - (1-c)] + (1 - p - \mu)[R_L - (1-c)]$, which implies $c \geq 1 - \frac{R_S - pR_H - (1-p-\mu)R_L}{\mu}$. We previously denoted $c^{**} = 1 - \frac{R_S - pR_H - (1-p-\mu)R_L}{\mu}$. The threshold $c^{**}$ exists if and only if condition (T3a) is satisfied. The safe project is preferred for any level of initial capital larger than $c_{NEW}^{\mathrm{Re}\,capitalize}$ whenever (T3a) is not satisfied. Otherwise, $\forall\, c \in (c_{NEW}^{\mathrm{Re}\,capitalize}, c^{**})$ the risky project is selected, while the safe project is preferred $\forall\, c \in (c^{**}, 1)$.

Recall that $W = pR_H + (1-p)R_L - \mu c_{\min}R_L$. We also denote $Q = pR_H + (1-p)R_L(1 - c_{\min})$. It is easy to show that $Q < W$ due to the identity $1 - p - \mu > 0$. We distinguish among only three possible scenarios.

Scenario S1": $R_S \leq Q$. As a consequence, condition (T1a NEW) is not satisfied and $\forall\, c \in (0, c_{NEW}^{\mathrm{Re}\,capitalize})$ the bank selects the risky project. $R_S < Q$ implies that $R_S < W$. Condition (T3a) is satisfied. As a result, the bank invests risky $\forall\, c \in (c_{NEW}^{\mathrm{Re}\,capitalize}, c^{**})$, while she prefers the safe project $\forall\, c \in (c^{**}, 1)$.

Scenario S2": $Q < R_S < W$. The left hand-side implies that condition (T1a NEW) is satisfied. This implies that $\exists\, c^* \in (0, c_{NEW}^{\mathrm{Re}\,capitalize})$ with $c^* = c_1^*$, such that $\forall\, c \in (0, c^*)$ the risky project is selected, while the safe project is preferred $\forall\, c \in (c^*, c_{NEW}^{\mathrm{Re}\,capitalize})$. Similarly with the previous scenario, the bank invests risky $\forall\, c \in (c_{NEW}^{\mathrm{Re}\,capitalize}, c^{**})$, and safe $\forall\, c \in (c^{**}, 1)$. This result is implied by $R_S$ being lower than $W$.

Scenario S3": $R_S \geq W$. Condition (T1a NEW) is satisfied while condition (T3a) is not. Hence, the bank selects the risky projects $\forall\, c \in (0, c^*)$, with $c^* = c_1^*$, and she selects the safe project $\forall\, c \in (c^*, 1)$.

To conclude, we can argue that the qualitative results of Proposition 2 are valid

under the relaxed assumption. Nevertheless, condition (T2b) has to be replaced by the relevant condition (T1a NEW).

## A.4   Bank's choice when the return on safe project is large

Let us consider here that the return on the safe asset is large, that is $R_S > 1 - p + pR_H$. This drives the following results under Assumption (13): (1) condition (T1a') is not satisfied, implying that $\forall\ c \in [0, c^{\mathrm{Re}\,capitalize}]$, the bank prefers the safe project; (2) condition (T1a) is satisfied, which implies that condition (T2a) is not and as a result $\forall\ c \in (c^{\mathrm{Re}\,capitalize}, c^{Sufficient})$, the bank prefers the safe project; (3) condition (T3a) is not satisfied and as a consequence $\forall\ c \in [c^{Sufficient}, 1]$, the bank invests in the safe project. Summing up, for any levels of initial capital $c$, the bank prefers the safe project when the certain return $R_S$ is high enough.

# B   Extensions

We offer here two extensions for our model and examine the implications of charter value and of different specification for recapitalization costs. We show that our results are robust to these generalizations.

## B.1   Charter value

In Section 2 we have assumed that there is no charter value for the continuation of bank's activity. In this section we introduce a positive charter value $V > 0$ and show that our results are robust to this extension. Our model suggests that low competition in banking, which provides a high charter value, leads to investment in the efficient safe project even by well-capitalized banks.

The role of banks' franchise values have been shown relevant in other studies. Hellmann et al. (2000) and Repullo (2004) argue that prudent behavior can be facilitated by increasing banks' charter value. They study the links between capital requirements,

competition for deposits, charter value and risk-taking incentives, and point out that banks are more likely to gamble and to take more risk in a competitive banking system, since competition erodes profits and implicitly the franchise value. A similar idea is put forward by Matutes and Vives (2000). They argue that capital regulation should be complemented by deposit rate regulation and direct asset restrictions in order to efficiently keep risk-taking under control. Acharya (2002) explores how continuation value affects risk preferences in the context of optimal regulation, and demonstrates the disciplinating effect of charter value on bank risk-taking. Finally, Keeley (1990) and Furlong and Kwan (2005) explore empirically the relation between charter value and different measures of bank risk, and find strong evidence that bank charter value disciplined bank risk-taking.

In the new setting, the banker's payoff to the safe project after repaying depositors becomes $\Pi_S^V = R_S - (1 - c) + V$. The banker's payoff to the risky project is as follows: when $R_H$ is realized, the banker gets $\Pi_H^V = R_H - (1 - c) + V$, while the payoff is 0 for extremely low realization $R_0$. When the low return $R_L$ is realized and capital is positive but insufficient ex-post, the banker prefers to recapitalize at a cost $T$ for lower levels of initial capital $c$. The reason for this is that banker's expected payoff increases by $V$ if bank is not closed by the regulator. Hence, the bank raises additional capital when initial capital $c$ is higher than $c_V^{\text{Re}\,capitalize}$, where:

$$c_V^{\text{Re}\,capitalize} = 1 + T - R_L - V, \tag{22}$$

and $c_V^{\text{Re}\,capitalize} < c^{\text{Re}\,capitalize}$. On the other hand, the threshold point $c^{Sufficient}$ does not change since it is given by the exogenous regulation.

We make the simplifying assumption that the charter value is not larger than a certain threshold:

$$V < 1 + T - R_L. \tag{23}$$

This makes threshold $c_V^{\text{Re}\,capitalize}$ positive and assures the existence for the area $[0, c_V^{\text{Re}\,capitalize}]$ where the bank is abandoned for low realization of the risky asset. Consider the area $(c_V^{\text{Re}\,capitalize}, c^{Sufficient})$. When initial capital $c$ is in this range, a bank which is subject to regulator's corrective action prefers to raise additional capital. Since $c_V^{\text{Re}\,capitalize} < c^{\text{Re}\,capitalize}$, while right boundary of the interval is left unchanged by any increase in $V$, we can argue that any reduction in banking competition, which increases bank charter value, makes the decision to raise fresh capital more likely.

We introduce the following two thresholds:

$$Z_V = pR_H + (1-p)(R_L - T) + \mu(T - V - c_{\min}R_L), \tag{24}$$

as the new threshold for the binding impact of the prompt corrective action (with $Z_V < Z$), and

$$B = pR_H + (1-p)(R_L - T). \tag{25}$$

Following a similar proof as for Proposition 2, we can show that there exist two thresholds $c_V^*$ and $c_V^{**}$ for the level of initial bank capital such that under assumption (13) and for levels of return on the safe project satisfying $Z_V < R_S < B$ , with $Z_v$ and $B$ defined in (24) and (25), respectively, the bank's investment preference is as follows:

(a) the bank prefers the risky project for $0 \le c < c_V^*$, while for $c_V^* \le c < c^{Sufficient}$ the safe project is preferred, with $c_V^* \in (c_V^{\text{Re}\,capitalize}, c^{Sufficient})$, where $c_V^{\text{Re}\,capitalize}$ and $c^{Sufficient}$ are defined in (22) and (10), respectively, and

$$c_V^* = 1 - V - \frac{R_S - pR_H - (1-p-\mu)(R_L - T)}{\mu}, \tag{26}$$

(b) the bank prefers the risky project for $c^{Sufficient} \le c < c_V^{**}$, and the safe project for $c \ge c_V^{**}$, where $c_V^{**} \in (c^{Sufficient}, 1)$ and

$$c_V^{**} = 1 - V - \frac{R_S - pR_H - (1 - p - \mu)R_L}{\mu}. \tag{27}$$

Observe that a positive charter value has a negative impact on all relevant thresholds which drive bank's preferences (i.e., $c_V^{\mathrm{Re}\,capitalize}$, $c_V^*$, and $c_V^{**}$), except for $c^{Sufficient}$. Hence, we can argue that higher charter value plays the role of a counterbalancing force to the risk-taking incentives generated by the presence of risky projects with heavy left tails. This means that when the continuation value of bank's activity is high enough, both intervals $(0, c_V^*)$ and $(c^{Sufficient}, c_V^{**})$ shrink. This suggests that low competition in the banking industry induces banks with larger capital buffers to take less risk.

In summary, the results of our basic model are therefore robust to the introduction of charter value, conditional on the fact that this value is not too large. For large values of franchise value $V$, there are no risk-taking problems in banks, regardless the level of initial capital.

## B.2 Concave capital adjustment cost

In Section 2 we considered a simple fixed cost of recapitalization. We now show that results are robust to a more general specification of this cost function.

In this section we discuss a variation of the model in which the cost of recapitalization has a fix and a variable component. The variable component is proportional to the amount of new capital that the bank has to raise in order to comply with the minimal capital ratio. Specifically, the bank has to raise a capital level $R_{\min} - R_L$, where $R_{\min}$ equals:

$$R_{\min} = \frac{1 - c}{1 - c_{\min}}. \tag{28}$$

The above threshold is derived from the condition of a minimal capital ratio of $c_{\min}$ (i.e., $c \leq c_{\min} = [R_{\min} - (1 - c)]/R_{\min}$), by solving for the value of bank's assets (i.e.,

$R_{\min}$).[6] In this new setting, the recapitalization cost is concave in capital level, and has the following specification:

$$Cost(c, R_L) = T + \beta(\frac{1-c}{1-c_{\min}} - R_L). \tag{29}$$

We assume that variable cost of recapitalization (i.e., $\beta$) is positive and not as low as to make the banker abandon the bank regardless the level of initial capital:

$$T < R_L(1 + \beta). \tag{30}$$

The banker's payoff from the safe project, as well as the realizations of the risky project are the same as in the basic model. However, when the low realization $R_L$ is obtained, the bank is abandoned more often than in the basic model due to higher cost of recapitalization. The bank is closed when $c < c_{CC}^{\mathrm{Re}\,capitalize}$, where:

$$c_{CC}^{\mathrm{Re}\,capitalize} = 1 + \frac{T - R_L(1+\beta)}{1 + \frac{\beta}{1-c_{\min}}}, \tag{31}$$

(with $CC$ for concave cost).

Under assumption (13), $c_{CC}^{\mathrm{Re}\,capitalize} > c^{\mathrm{Re}\,capitalize}$. On the other hand, the level of capital which guarantees that the bank satisfies ex-post the regulatory minimal upon realization of $R_L$ (i.e., $c^{Sufficient}$) remains unchanged. Hence, the interval $(c_{CC}^{\mathrm{Re}\,capitalize}, c^{Sufficient})$ shrinks, suggesting that the bank is less likely to raise additional capital if required to do so.

We denote:

$$B_{CC} = pR_H + (1-p)\frac{R_L(1+\beta) - T}{1 + \frac{\beta}{1-c_{\min}}}. \tag{32}$$

---

[6]Consider the following example. Assume that the bank has to raise $\delta$ units of capital to satisfy the regulatory minimum when $R_L$ is realized. Hence, $c_{\min} = \frac{R_L - (1-c) + \delta}{R_L + \delta}$. This implies that $R_L + \delta = \frac{1-c}{1-c_{\min}}$, which equals $R_{\min}$ according to (28). We can conclude that $\delta = R_{\min} - R_L$.

36

Following the lines of proof for Proposition 2 we can show that exist two thresholds $c_{CC}^{*}$ and $c_{CC}^{**}$ for the level of initial bank capital such that under assumption (13) and for level of return on the safe project satisfying $Z < R_S < B_{CC}$, with $Z$ and $B_{CC}$ defined in (18) and (32), respectively, the bank's investment preference is as follows:

(a) the bank prefers the risky project for $0 \leq c < c_{CC}^{*}$, while for $c_{CC}^{*} \leq c < c^{Sufficient}$ the safe project is preferred, with $c_{CC}^{*} \in (c_{CC}^{\text{Re}\,capitalize}, c^{Sufficient})$, where $c_{CC}^{\text{Re}\,capitalize}$ and $c^{Sufficient}$ are defined in (31) and (10), respectively, and

$$c_{CC}^{*} = 1 - \frac{R_S - pR_H - (1 - p - \mu)[R_L(1 + \beta) - T]}{\mu - \frac{\beta}{1 - c_{\min}}(1 - p - \mu)}; \tag{33}$$

(b) the bank prefers the risky project for $c^{Sufficient} \leq c < c_{CC}^{**}$, and the safe project for $c \geq c_{CC}^{**}$, where $c_{CC}^{**} \in (c^{Sufficient}, 1)$ and $c_{CC}^{**} = c^{**}$, with $c^{**}$ defined in (20).

Observe that the introduction of a variable component for recapitalization cost leaves both boundaries of the interval $(c^{Sufficient}, c_{CC}^{**})$ unchanged. Thus, our model is robust and a concave cost of recapitalization does not affect the risk-taking preferences of well-capitalized banks, when they are allowed to invest in projects exhibiting heavier left tails.

**Figure 1.**

**The timeline**

$R_S$; Sufficient capital

Safe

High, $R_H$;
Sufficient capital

Risky

Sufficient capital

Recapitalize

Low, $R_L$

Insufficient

Depending on
the levels of
initial capital,
minimal capital
ratio, and the
recapitalization
cost

Very low, $0$
Negative capital

Abandon

No incentive to recapitalize
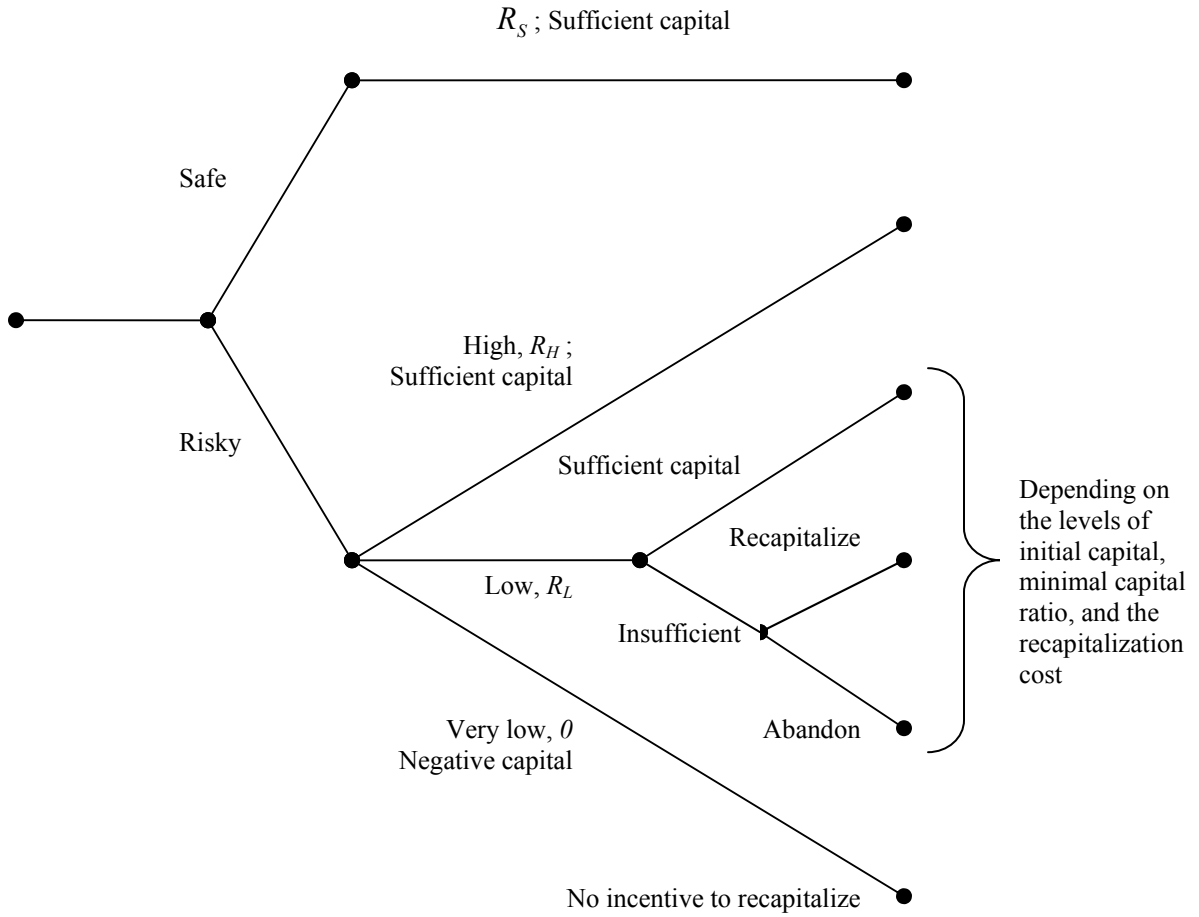
**Date 0**

- Bank has capital $C$
  and deposits $D$
- Bank selects one
  project

**Date 1/2**

- Information about future
  returns
- Regulator may take
  corrective action
- Bank decides whether to
  recapitalize

**Date 1**

- Returns are
  realized and
  distributed

**Figure 2.**

**Bank's recapitalization decision and payoffs**

The figure illustrates the bank's recapitalization decision and banker's payoffs as a function of initial capital $c$, upon the realization of low return $R_L$. For $c \geq c^{Sufficient}$, the bank has positive and sufficient capital at date ½. The bank continues to date 1, repays depositors and obtains a positive payoff. For $c < c^{Sufficient}$, the bank has positive and insufficient or negative capital. The bank can be either abandoned or recapitalized. The bank is abandoned for $c \leq c^{\mathrm{Re}\,capitalize}$. As a result the bank is closed and the banker gets a zero payoff. The bank is recapitalized at a cost for $c^{\mathrm{Re}\,capitalize} < c < c^{Sufficient}$. The bank continues to date 1, repays depositors, pays the recapitalization cost, and obtains a positive payoff.

Initial capital $c$

0                $c^{\mathrm{Re}\,capitalize}$                $c^{Sufficient}$                1

- No recapitalization;
- Bank is abandoned;

- Banker gets zero payoff.

- The bank is recapitalized at cost $T$;

- Banker gets a positive payoff
$$R_L - (1-c) - T$$

- Capital is sufficient;

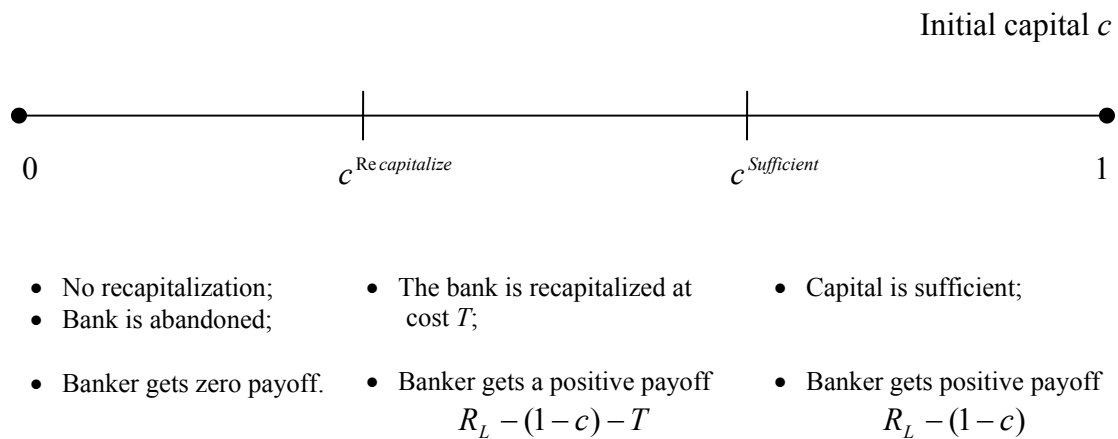- Banker gets positive payoff
$$R_L - (1-c)$$

**Figure 3.**

**Bank's project choice**

The figure depicts the bank's project choice depending on the level of initial capital, in Case (a) of Proposition 2. The relationship between bank capital and risk-taking is non-linear and is characterized by two thresholds as follows. When the level of capital is low ($c<c^*$), the bank prefers the risky project, while for high level of capital ($c\geq c^{**}$) the safe project is chosen. For intermediate level of capital ($c^*\leq c<c^{**}$), the bank prefers either the safe project (for $c^*\leq c<c^{Sufficient}$) or the risky one (for $c^{Sufficient}\leq c<c^{**}$).

Initial capital $c$

0            $c^{Re\,capitalize}$           $c^{Sufficient}$          1

$c^*$             $c^{**}$

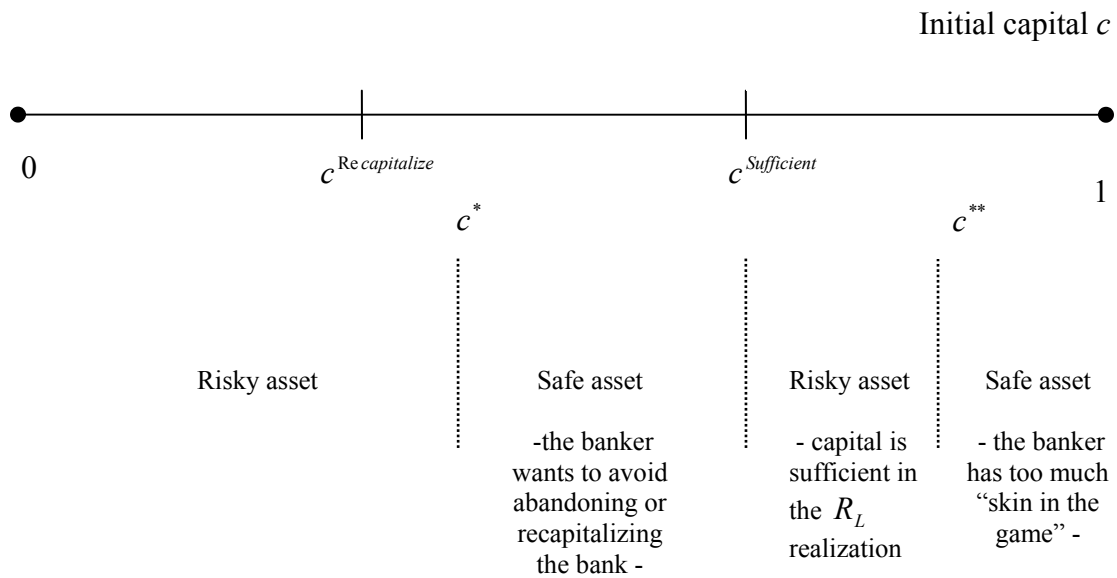| Risky asset | Safe asset | Risky asset | Safe asset |
|---|---|---|---|
| | -the banker wants to avoid abandoning or recapitalizing the bank - | - capital is sufficient in the $R_L$ realization | - the banker has too much "skin in the game" - |

**Figure 4a.**

**Bank's project choice when the risky project has a heavier left tail. Case 1.**

A heavier left tail is characterized by a higher probability for the extremely low outcome (i.e., a higher $\mu$). A change in the return profile of the risky project following a change in probability distribution (i.e., both $p$ and $\mu$ are increased, other else equal, such that the expected value of the risky project remains the same), affects both thresholds $c^*$ and $c^{**}$. The interval $[c^{Sufficient}, c^{**})$ widens, suggesting that well-capitalized banks which behave prudently in absence of tail risk projects, have a strong incentive to undertake more risk, if projects with heavier left tail are available in economy.
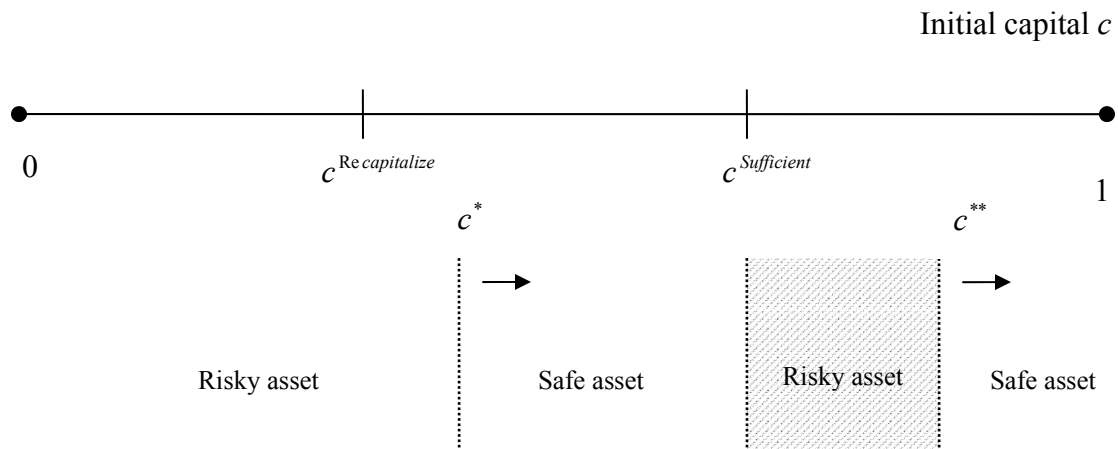
Initial capital $c$



0        $c^{Re\,capitalize}$        $c^{Sufficient}$        1

$c^*$        $c^{**}$

Risky asset        Safe asset        Risky asset        Safe asset

**Figure 4b.**

**Bank's project choice when the risky project has a heavier left tail. Case 2.**

A heavier left tail is characterized by a higher probability for the extremely low outcome (i.e., a higher $\mu$ ). A change in the return profile of the risky project following a change in probability distribution (i.e., $\mu$ is increased), compensated by higher $R_L$ and $R_H$ , other else equal, such that the expected value of the risky project remains the same, affects thresholds $c^*$ and $c^{**}$ , and $c^{Sufficient}$ as well. The interval $[c^{Sufficient}, c^{**})$ widens even more, suggesting that both well and less capitalized banks will start choosing the risky project.

Initial capital $c$

0

$c^{Re\,capitalize}$

$c^{Sufficient}$

$c^*$

1

$c^{**}$

Risky asset          Safe asset          Risky asset          Safe asset