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When is Quantitative Easing effective?

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When is Quantitative Easing Effective?[1]

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Abstract

We present a simple macroeconomic model with open market operations that allows examining the effects of quantitative and credit easing. The central bank controls the policy rate, i.e. the price of money in open market operations, as well as the amount and the type of assets that are accepted as collateral for money. When the policy rate is sufficiently low, this set-up gives rise to an (il-)liquidity premium on non-eligible assets. Then, a quantitative easing policy, which increases the size of the central bank's balance sheet, can increase real activity and prices, while a credit easing policy, which changes the composition of the balance sheet, can lower interest rate spreads, stimulate real activity, and reduce prices. The effectiveness of quantitative and credit easing is however limited to the extent that eligible assets are scarce. Nevertheless, they can help escaping from the zero lower bound.

JEL classification: E4; E5; E32.
Keywords: Monetary policy, collateralized lending, quantitative easing, credit easing, liquidity premium, zero lower bound

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1 Introduction

Central banks in many industrialized countries have responded to the recent financial crisis with unconventional monetary policy measures. By introducing various newly created lending facilities as well as direct asset purchases, the Federal Reserve for instance doubled its balance sheet in the three months after the climax of the crisis in September 2008. This policy, which has been summarized by the terms "quantitative easing" and "credit easing", has been aimed at ensuring the functioning of the interbank market and at stabilizing stressed credit markets (see Yellen, 2009).[3] However, it has been implemented with only little theoretical or empirical guidance available. The present paper provides an analysis of the effects as well as the limits of quantitative and credit easing in a simple sticky price model. The analysis focuses on monetary policy implementation and the provision of liquidity by the central bank and abstracts from the ability of monetary policy to mitigate financial frictions that were crucial in the financial crisis.[4] We show that quantitative and credit easing can stimulate real activity at the zero lower bound (ZLB) as long as assets eligible for open market operations are scarce, which is reflected by a liquidity premium.

As summarized by Bernanke et al. (2004), quantitative easing involves the purchase of securities, such as government bonds, with central bank reserves. In contrast, qualitative easing refers to changes in the composition of the central bank's balance sheet without creating additional reserves. More recently, Bernanke (2009) introduces the term credit easing which closely relates to qualitative easing: "the Federal Reserve's credit easing approach focuses on the mix of loans and securities that it holds and on how this composition of assets affects credit conditions for households and businesses". According to conventional macroeconomic models, easing money supply at the ZLB should be ineffective since private agents will demand money up to satiation (see Walsh, 2010). Quantitative easing policies should then be irrelevant as long as they do not change expectations about future conduct of monetary and fiscal policy (see Eggertsson and Woodford, 2003), while credit easing has not been considered in standard single interest rate models. Evidence from Federal Reserve policy effects however suggests that quantitative and credit easing have been effective during the recent financial crisis, primarily, via reductions of liquidity premia (see Christensen et al., 2009, Duygan-Bump et al., 2010, Gangon et al., 2010, or Sarkar and Schrader, 2010).

[3] Among the facilities created by the Federal Reserve are for example the Term Auction Facility (TAF), the Commercial Paper Funding Facility (CPFF) and the Treasury Securities Lending Facility (TSLF). TAF gives 28- or 84-day credit to depository institutions, CPFF is a purchase program for 3-month commercial paper, and TSLF provides Treasury securities in exchange for other securities such as mortgage-backed securities and commercial paper. Further facilities include the Primary Dealer Credit Facility, the Term Asset-Backed Securities Loan Facility and the Asset-Backed Commercial Paper Money Market Mutual Fund Liquidity Facility.

[4] Hence, this paper complements the recent literature which analyzes unconventional monetary policy, such as direct lending or asset exchanges, under financial frictions (see Curdia and Woodford, 2010, Del Negro et al., 2010, Gertler and Karadi, 2010, and Gertler and Kiyotaki, 2010).

This paper presents a macroeconomic model which explicitly accounts for the collateral requirements in open market operations. It allows an analysis of quantitative and credit easing policies and their macroeconomic effects. Multiple assets are considered that differ with regard to their ability to serve as collateral for money. The central bank sets the policy rate, i.e. the price of money in terms of eligible assets, and decides on the size and the composition of its balance sheet. Private agents rely on money for goods market purchases, while money is supplied only in exchange for eligible securities, in particular for short-term government bonds. This requirement leads to a spread between the interest rate on non-eligible and eligible assets, i.e. a liquidity premium.[5] It implies that interest rates on non-eligible securities are positive, even if the policy rate is at the ZLB. This accords to the empirical evidence that – as emphasized by Ohanian (2010) – interest rates on non-money market securities tend to be positive even if the policy rate hits the lower bound.

In our model, we consider working capital such that firms demand loans in order to finance production. Due to the associated costs of borrowing, higher loan rates increase the marginal costs of production and thereby exert downward pressure on production. As long as loans are not eligible in open market operations, the loan rate exceeds the interest rate on eligible government bonds by a liquidity premium. By increasing the amount of eligible assets the central bank eases households' access to cash and increases their willingness to spend, which acts like a conventional money injection. Moreover, credit easing can reduce the illiquidity premium on loans and thereby reduce firms' cost of borrowing, which can stimulate the economy. Yet, the effectiveness of both policies is limited. Quantitative and credit easing affect real activity and prices only if eligible assets are scarce, i.e. if the collateral constraint in open market operations is binding. This is the case when eligible assets can be exchanged against money at a price (i.e. policy rate), which is lower than the consumption Euler rate that measures the opportunity costs of money.

Our main results can be summarized as follows. Under flexible prices, monetary policy is non-neutral due to a standard inflation tax and because of its impact on firms' costs of borrowing. A quantitative easing policy in terms of government bonds increases prices and interest rates like an expansionary money supply. The central bank can further increase output and consumption via credit easing, which leads to a lower borrowing rate due to an increased share of eligible loans. For sticky prices, we show that a quantitative easing policy at the ZLB increases output and inflation. We further present numerical results for a calibrated version of the model to explore the limits of quantitative and credit easing at the ZLB. These

[5] In this paper, we disregard default risk and focus on liquidity premia, for which empirical evidence suggest a significant magnitude also in non-crisis times. As summarized by Christensen (2008), the "corporate credit spread puzzle" refers to the empirical observation that the spreads between corporate and government bonds can only partly be attributed to default risk, while the non-default part is typically interpreted as a liquidity premium (see Collin-Dufresne et al. 2001, or Longstaff et al., 2005).

limits are reached when the stimulating policy drives down the Euler rate until it equals the policy rate. We find that a quantitative easing policy (credit easing policy) can substantially reduce interest rate spreads, while it can stimulate output by not more than 1.5% (0.15%) at the ZLB. The inflation responses are much smaller and differ for both policies: Quantitative easing increases inflation, whereas credit easing reduces inflation. Finally, we consider a large liquidity demand shock which drives downs the policy rate to its ZLB and leads to a pronounced output contraction. In this case, a maximum quantitative easing policy can mitigate the output contraction by 50%, which is nevertheless sufficient to escape from the ZLB.[6]

The paper is organized as follows. Section 2 presents the model. In Section 3, we examine the effects of monetary policy in an analytical way. In Section 4, we present quantitative results. Section 5 concludes.

Related literature There is a large literature on monetary policy options at the ZLB. Most of them advocate the possibility of providing monetary stimulus at the ZLB through shaping interest-rate expectations. The basic idea is that a monetary expansion, if perceived as permanent, can stimulate the economy by creating expected inflation and reducing the real rate of interest (see Krugman, 1998). Eggertsson and Woodford (2003) show that a commitment to keep nominal interest rates low in future can provide an effective way of escaping a liquidity trap. Jung et al. (2005) and Eggertsson (2006) derive optimal policy under the non-negativity constraint for the interest rate and obtain the same conclusion. Levin et al. (2009) examine large, persistent shocks and find that a policy relying on shaping interest rate expectations might not be sufficient to stabilize the economy. Auerbach and Obstfeld (2005) analyze open market purchases of government bonds and find that this policy can lift the economy out of the liquidity trap if the monetary base is permanently increased.

According to conventional wisdom, lump-sum injections of money such as helicopter drops are ineffective at the ZLB (see Krugman, 1998, Svensson, 2000, and McCallum, 2006). The reason is that standard macroeconomic models, like the basic New Keynesian model, consider only a single interest rate. Once the policy rate reaches the ZLB, the opportunity costs of holding money fall to zero such that money demand is indetermined or private agents demand money up to satiation. Moreover, open market operations that aim at easing money supply, like a quantitative easing policy, are ineffective at the ZLB as long as they do not change expected future policy paths. Then, neither the size nor the composition of the central bank's balance are relevant as long as financial market are frictionless (see Eggertsson and Woodford, 2003).

[6] In a companion paper, Schabert (2010) applies a closely related model and shows that the additional monetary policy instruments, which are here applied to implement quantitative and credit easing, can help to overcome the well-known monetary policy trade-off between stabilizing prices and closing output-gaps.

Spurred by the recent financial crisis, however, a literature on non-standard monetary policies, like direct lending or asset exchanges by the central bank, under financial frictions is now developing. Gertler and Karadi (2009) analyze direct central bank lending when financial intermediaries need collateral in order to attract deposits. When financial institutions need to deleverage due to a decline in asset prices, central bank interventions such as borrowing directly to firms can be a powerful tool. Applying a purely real framework, which is based on Gertler and Karadi's (2009) model augmented by idiosyncratic investment risks and constraints on the resaleability of assets, Gertler and Kiyotaki (2010) show that direct central bank lending is beneficial in crisis situations when private intermediaries are financially constrained. Del Negro et al. (2010) consider entrepreneurs facing a borrowing and a resaleability constraint (like in Kiyotaki and Moore, 2008) and add these frictions to a medium scale macroeconomic model (see Christiano et al., 2005). They calibrate the model and a negative shock to the resaleability of assets to match the U.S. in late 2008, and show that the Fed's policy interventions prevented a second Great Depression. Curdia and Woodford (2010) apply a model with costly financial intermediation and show that targeted asset purchases (which relate to direct lending) by a central bank can be effective when financial markets are sufficiently disrupted. They further find that quantitative easing is likely to be ineffective. It should be noted that this result is consistent with our conclusion: Given that private agents in their model do not internalize a collateral constraint for money, their case corresponds to a scenario in our model where eligible assets are not scare (i.e. the collateral constraint is not binding).

2 The model

In this section we present a sticky price model where households face a cash-in-advance constraint and firms require working capital, like in Christiano et al. (2005). Money is supplied by the central bank only in exchange for eligible collateral, i.e. government bonds and/or corporate debt.[7] It sets the policy rate and it can further decide on the size and the composition of its balance sheet, which we interpret as quantitative easing and credit easing. Households take these policies into account when they invest in assets, which gives rise to different interest rates due to liquidity premia. Quantitative and credit easing can then lower liquidity premia and stimulate aggregate demand as long as collateral is scare. To present the problems of individual households and firms in a transparent way, we introduce indices even though we do not consider heterogeneity.

[7] Specifically, we augment the model of Reynard and Schabert (2010), which has been applied to explain endogenous liquidity premia, by introducing corporate debt and additional monetary policy instruments.

2.1 Timing of events

Households enter the period with money, government bonds, and household debt, $M^H_{i,t-1} + B_{i,t-1} + D_{i,t-1}$. Households further dispose of a time-invariant time endowment. They supply labor to intermediate goods producing firms, which do not hold any financial wealth. At the beginning of the period aggregate shocks are realized. Then, the central bank sets its instruments, i.e. it announces the fractions of government bonds and corporate loans that are accepted in open market operations, $\kappa^B_t \in (0,1]$ and $\kappa_t \in [0,1]$, and the policy rate $R^m_t \geq 1$. The remainder of the period can be divided into four subperiods.

1. The labor market opens, where a perfectly competitive intermediate goods producing firm j hires workers $n_{j,t}$. We assume that it has to pay workers their wages in cash before goods are sold. Since it does not hold any financial wealth, it has to borrow cash. Firm j thus faces the cash constraint

$$L_{j,t}/R^L_t \geq P_t w_t n_{j,t}, \tag{1}$$

where w_t denotes the real wage rate, P_t denotes the final goods price and $L_{j,t}/R^L_t$ the amount received by the borrowing firm. Firm j commits to repay the amount $L_{j,t}$ at the end of the period, such that R^L_t is the interest rate on the intra-period loan. Lenders sign loan contracts with all firms, taking into account that a fraction κ_t of all loans can be used as collateral for repurchase agreements.

2. The money market opens where the central bank sells or purchases assets outright or under repurchase agreements in exchange for money at the rate R^m_t. In contrast to household debt, corporate loans and government bonds can be eligible, where only the latter can be purchased outright by the central bank. In period t, household i receives new money (injections) from the central bank $I_{i,t}$, which consists of money received from the central bank's outright bond purchases, as well as money received in repos for bonds $M^R_{i,t}$ and loans $M^L_{i,t}$. Specifically, the central bank supplies money against a fraction κ^B_t of randomly selected bonds and a fraction κ_t of randomly selected loan contracts, such that $I_{i,t}$ is constrained by the following collateral contraint, or *money market constraint:*

$$I_{i,t} \leq \kappa^B_t \left(B_{i,t-1}/R^m_t\right) + \kappa_t \left(L_{i,t}/R^m_t\right). \tag{2}$$

After receiving money injections from the central bank, household i delivers the amount $L_{i,t}/R^L_t$ to firms according to the loan contract. Its holdings of money, bonds, and loans are then $M^H_{i,t-1} + I_{i,t} - (L_{i,t}/R^L_t)$, $B_{i,t-1} - \Delta B^c_{i,t}$, and $L_{i,t} - L^R_{i,t}$, where $\Delta B^c_{i,t}$ are bonds received by the central bank and $L^R_{i,t}$ are loans under repos, such that $I_{i,t} = (\Delta B^c_{i,t}/R^m_t) + (L^R_{i,t}/R^m_t)$.

3. Wages are paid, and intermediate as well as final goods are produced. Then, the goods market opens, where purchases of consumption goods require cash holdings. Hence, household i faces the cash-in-advance constraint, or *goods market constraint*:

$$P_t c_{i,t} \leq I_{i,t} + M^H_{i,t-1} - \left(L_{i,t}/R^L_t\right) + P_t w_t n_{i,t}. \tag{3}$$

Final goods producing firms receive cash for their sales, and pay for intermediate goods. Both further pay out dividends to their owners (households), which sum up to $P_t\delta_{i,t}$ for household i, such that its money holdings are $M^H_{i,t-1} + I_{i,t} - (L_{i,t}/R^L_t) + P_t w_t n_{i,t} - P_t c_{i,t} + P_t\delta_{i,t}$.

4. Repurchase agreements are settled, i.e. household i buys back government bonds $B^R_{i,t}$ and corporate debt $L^R_{i,t}$ from the central bank with money. Household i's bond and money holdings are therefore given by $\widetilde{B}_{i,t} = B_{i,t-1} - \Delta B^c_{i,t} + B^R_{i,t}$ and $\widetilde{M}_{i,t} = I_{i,t} + M^H_{i,t-1} - \left(L_{i,t}/R^L_t\right) + P_t w_t n_{i,t} - P_t c_{i,t} + P_t\delta_{i,t} - B^R_{i,t} - L^R_{i,t}$. In the asset market, loans are repaid and households receive payoffs from maturing assets as well as government transfers $P_t\tau_{i,t}$. Further, the government issues new bonds at the price $1/R_t$. Household i can thus carry wealth into period $t+1$ in form of bonds, state-contingent claims, or money, such that its asset market constraint is

$$(B_{i,t}/R_t) + E_t[\varphi_{t,t+1} D_{i,t}] + M^H_{i,t} \leq \widetilde{B}_{i,t} + D_{i,t-1} + \widetilde{M}_{i,t} + L_{i,t} + P_t\tau_{i,t}, \tag{4}$$

where $\varphi_{t,t+1}$ denotes a stochastic discount factor (see section 2.3). The central bank reinvests its payoffs from maturing bonds into new government bonds and leaves money supply unchanged, $\int \widetilde{M}_{i,t} di = \int M^H_{i,t} di$.

2.2 Firms

There are intermediate goods producing firms which are perfectly competitive and sell their goods $y_{j,t}$ to monopolistically competitive retailers. These sell a differentiated good to bundlers who assemble final goods using a Dixit-Stiglitz technology.

There is a continuum of intermediate goods producing firms indexed with $j \in [0,1]$. They are perfectly competitive and owned by the households. In each period a firm j distributes its profits to the owners and rents the production factors, specifically, it hires labor $n_{j,t}$. We assume that wages have to be paid in advance, i.e. before the firm's goods are sold. Firm j therefore borrows cash $L_{j,t}$ from households at the price $1/R^L_t$ and repays the loan at the end of the period. Hence, firm j faces the working capital constraint (1). It then produces an intermediate good according to the production function $IO_{j,t} = n^\alpha_{j,t}$ and sells it to retailers who pay the price Z_t in cash (after these have received the households' money for goods).

With these revenues, it repays intra-period loans. The problem of firm j then reads

$$\max\left(Z_t/P_t\right)n_{j,t}^{\alpha}-w_t n_{j,t}-l_{j,t}\left(R_t^L-1\right)/R_t^L,\ \ s.t.\ (1),$$

where $l_{j,t}=L_{j,t}/P_t$. The first order conditions to this problem are given by

$$\left(Z_t/P_t\right)\alpha n_{j,t}^{1-\alpha}=w_t+\mu_{j,t}w_t,$$
$$R_t^L-1=\mu_{j,t},$$

and $\mu_{j,t}[(l_{j,t}/R_t^L)-w_t n_{j,t}]=0$, where $\mu_{j,t}\geq 0$ is the multiplier on (1). Hence, intermediate goods producing firms do not borrow more then required to pay wages $w_t n_{j,t}$ if $R_t^L>1\Rightarrow \mu_{j,t}>0$. This condition will be satisfied throughout the analysis. The following conditions determine intermediate firms' labour demand as well as the volume of debt they issue for $R_t^L>1:$

$$\left(Z_t/P_t\right)\alpha n_{j,t}^{1-\alpha}=w_t R_t^L, \tag{5}$$
$$l_{j,t}/R_t^L=w_t n_{j,t}. \tag{6}$$

Monopolistically competitive retailers buy intermediate goods $IO_t=\int_0^1 IO_{j,t}dj$ at the common price Z_t. A retailer $k\in[0,1]$ relabels the intermediate good to $y_{k,t}$ and sells it at the price $P_{k,t}$ to perfectly competitive bundlers, who bundle the goods $y_{k,t}$ to the final consumption good y_t with the technology, $y_t^{\frac{\varepsilon-1}{\varepsilon}}=\int_0^1 y_{k,t}^{\frac{\varepsilon-1}{\varepsilon}}dk$. The cost minimizing demand for $y_{k,t}$ is then given by $y_{k,t}=\left(P_{k,t}/P_t\right)^{-\varepsilon}y_t$.

Retailers set their prices to maximize profits. Following Calvo (1983), we assume that each period a measure $1-\phi$ of randomly selected retailers may reset their prices independently of the time elapsed since the last price setting, while a fraction $\phi\in[0,1)$ of retailers do not adjust their prices. Maximizing discounted future profits, a fraction of $1-\phi$ retailers set their price to maximize the expected sum of discounted future. For $\phi>0$, the first order condition for their price $\widetilde{P}_t$ is given by (where we use that Z_t/P_t are real marginal cost, mc_t):

$$\widetilde{P}_t=\frac{\varepsilon}{\varepsilon-1}\frac{\sum_{s=0}^{\infty}\left(\phi\beta\right)^s c_{t+s}^{-\sigma}y_{t+s}P_{t+s}^{\varepsilon}mc_{t+s}}{\sum_{s=0}^{\infty}\left(\phi\beta\right)^s c_{t+s}^{-\sigma}y_{t+s}P_{t+s}^{\varepsilon-1}}.$$

Defining $\tilde{Z}_t=\widetilde{P}_t/P_t$ and writing both the denominator and the numerator in a recursive way, this can be expressed as $\tilde{Z}_t=\frac{\varepsilon}{\varepsilon-1}Z_t^1/Z_t^2$, where $Z_t^1=c_t^{-\sigma}y_t mc_t+\phi\beta E_t\pi_{t+1}^{\varepsilon}Z_{t+1}^1$ and $Z_t^2=c_t^{-\sigma}y_t+\phi\beta E_t\pi_{t+1}^{\varepsilon-1}Z_{t+1}^2$.

With perfectly competitive bundlers and the homogenous bundling technology, the price index P_t for the final consumption good satisfies $P_t^{1-\varepsilon}=\int_0^1 P_{k,t}^{1-\varepsilon}dk$. Using the demand constraint, we obtain a law of motion for inflation depending on the firms' pricing decision $\tilde{Z}_t$, $1=\left(1-\phi\right)\tilde{Z}_t^{1-\varepsilon}+\phi\pi_t^{\varepsilon-1}$. Aggregate intermediate output satisfies $IO_t=n_t^{\alpha}$ where

$\alpha \in (0, 1]$ as every intermediate firm hires an identical amount of labour. However, there is a production inefficiency due to price dispersion across retailers. The market clearing condition in the intermediate goods market, $IO_t = \int_0^1 y_{k,t} dk$, gives $n_t^\alpha = \int_0^1 (P_{k,t}/P_t)^{-\varepsilon} y_t dk \Leftrightarrow$

$$y_t = n_t^\alpha / s_t,$$

where $s_t \equiv \int_0^1 (P_{k,t}/P_t)^{-\varepsilon} dk$ and $s_t = (1-\phi)\tilde{Z}_t^{-\varepsilon} + \phi s_{t-1} \pi_t^\varepsilon$ (see Schmitt-Grohé and Uribe, 2004) given s_{-1}.

2.3 Households

There is a continuum of infinitely lived households indexed with $j \in [0, 1]$. Households have identical asset endowments and identical preferences. Household j maximizes the expected sum of a discounted stream of instantaneous utilities

$$E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left[\left(c_{i,t}^{1-\sigma} - 1 \right) (1-\sigma)^{-1} - \theta n_{i,t}^{1+\sigma_n} (1+\sigma_n)^{-1} \right], \tag{7}$$

where $\theta > 0$, $\sigma \geq 1$, $\sigma_n \geq 0$, and E_0 is the expectation operator conditional on the time 0 information set, and $\beta \in (0, 1)$ is the subjective discount factor. The term ξ_t is a stochastic preference parameter, which has been introduced in several studies on the ZLB. This shock is not relevant for the main results in this paper and will only be considered in section 3.2. A household i is initially endowed with money $M_{i,-1}^H$, government bonds $B_{i,-1}$, and privately issued debt $D_{i,-1}$. In each period it supplies labor, lends out funds to intermediate goods producing firms, trades assets with the central bank in open market operations, and can reinvest.

Before household i enters the goods market where it needs money as the only accepted means of payment, it can get additional money in open market operations in exchange for government bonds. It can further lend cash to firms at the price $1/R_t^L$, which can eventually be refinanced at the central bank. When households lend to firms, they treat all firms in an identical way, since the decision which particular loan contract is eligible is made after loan contracts are signed. The household faces the money market constraint (2), while we will restrict our attention to the case where money is not withdrawn from the private sector $I_{i,t} \geq 0$ by considering a sufficiently large fraction of repos.

In the goods market, household i can use wages, money holdings, and additional cash net of lending from current period open market operations for its consumption expenditures (see 3). Before the asset market opens, it receives repayments from intra-period loans. In the asset market, it further receives pay-offs from maturing assets, it can buy bonds from the government, it can trade all assets with other households, and it can borrow and lend using a full set of nominally state contingent claims. Dividing the period t price of one unit of nominal wealth in a particular state of period $t+1$ by the period t probability of that state gives the stochastic discount factor $\varphi_{t,t+1}$. The period t price of a payoff $D_{i,t}$ in period $t+1$ is then given by $E_t[\varphi_{t,t+1}D_{i,t}]$. Substituting out the stock of bonds and money held before the asset market opens, $\widetilde{B}_{i,t}$ and $\widetilde{M}_{i,t}$, in (4), the asset market constraint of household i can be written as

$$M_{i,t-1}^{H}+B_{i,t-1}+\frac{L_{i,t}}{R_t^L}\left(R_t^L-1\right)+P_tw_tn_{i,t}+D_{i,t-1}+P_t\delta_{i,t}+P_t\tau_{i,t} \tag{8}$$
$$\leq M_{i,t}^{H}+\frac{B_{i,t}}{R_t}+E_t[\varphi_{t,t+1}D_{i,t}]+I_{i,t}\left(R_t^m-1\right)+P_tc_{i,t},$$

where household $i's$ borrowing is restricted by $M_{i,t}^H\geq 0$, $B_{i,t}\geq 0$, and the no-Ponzi game condition $\lim_{s\to\infty}E_t\varphi_{t,t+s}D_{i,t+s}\geq 0$. The term $\left(R_t^m-1\right)I_{i,t}$ in (8) measures the costs of money acquired in open market operations, i.e. household i receives new cash $I_{i,t}$ in exchange for $R_t^mI_{i,t}$ bonds. Maximizing the objective (7) subject to the money market constraint (2), the goods market constraint (3), the asset market constraints (8) and the borrowing constraints, for given initial values $M_{i,-1}$, $B_{i},_{-1}$, and $D_{i,-1}$ leads to the following first order conditions for consumption, working time, additional money, and loans

$$\xi_tc_{i,t}^{-\sigma}=\lambda_{i,t}+\psi_{i,t}, \tag{9}$$
$$\theta\xi_tn_{i,t}^{\sigma_n}=w_t\left(\lambda_{i,t}+\psi_{i,t}\right), \tag{10}$$
$$\psi_{i,t}=\left(R_t^m-1\right)\lambda_{i,t}+R_t^m\eta_{i,t}, \tag{11}$$
$$R_t^m\left(\lambda_{i,t}+\eta_{i,t}\right)=R_t^L\left(\lambda_{i,t}+\eta_{i,t}\kappa_t\right), \tag{12}$$

as well as for investments in contingent claims, government bonds and money,

$$\lambda_{i,t}=\beta R_tE_t\frac{\lambda_{i,t+1}+\kappa_{t+1}^B\eta_{i,t+1}}{\pi_{t+1}}, \tag{13}$$
$$\lambda_{i,t}=\beta E_t\frac{\lambda_{i,t+1}+\psi_{i,t+1}}{\pi_{t+1}}, \tag{14}$$
$$\varphi_{t,t+1}=\frac{\beta}{\pi_{t+1}}\frac{\lambda_{i,t+1}}{\lambda_{i,t}}, \tag{15}$$

where $\lambda_{i,t}\geq 0$ denotes the multiplier on the asset market constraints (8), $\eta_{i,t}\geq 0$ the multiplier on the money market constraints (2), and $\psi_{i,t}\geq 0$ the multiplier on the the goods market constraint (3). Further, (2), (3),

$$\psi_{i,t}[I_{i,t}+M_{i,t-1}^H-\left(L_{i,t}/R_t^L\right)+P_tw_tn_{i,t}-P_tc_{i,t}]\geq 0, \tag{16}$$
$$\eta_{i,t}[\kappa_t^B\left(B_{i,t-1}/R_t^m\right)+\kappa_t\left(L_{i,t}/R_t^m\right)-I_{i,t}]\geq 0, \tag{17}$$

and (8) with equality hold as well as the transversality conditions. The debt rate R_t^D, which slightly differs in the short-run from a standard consumption Euler rate due to the multiplier on the cash-in-advance constraint $\psi_{i,t}$ (see 9), is defined as follows

$$E_t\varphi_{t,t+1} = 1/R_t^D. \tag{18}$$

Condition (12) shows that when the money market constraint is binding, $\eta_{i,t} > 0$, the loan rate depends on the fraction of firm loans eligible as collateral in open market operations, κ_t. As long as loans are not fully eligible $\kappa_t < 0$, there can be a spread between the policy rate and the loan rate, which is a *liquidity premium*. When all intra-period loans are eligible as collateral in open market operations $\kappa_t = 1$, the interest rate on corporate debt compensates exactly for the discount, i.e. $R_t^L = R_t^m$. Combining the optimality conditions (11), (13), and (14) to

$$R_t E_t \left[\left(\lambda_{i,t+1} + \kappa_{t+1}^B \eta_{i,t+1} \right) / \pi_{t+1} \right] = E_t \left[R_{t+1}^m \left(\lambda_{i,t+1} + \eta_{i,t+1} \right) / \pi_{t+1} \right], \tag{19}$$

further shows that households are indifferent between investing in money or investing in government bonds and converting these into cash in the next period at the rate R_{t+1}^m. For $\kappa_t^B = 1$, the interest rate on government bonds is closely linked to next period's expected policy rate, i.e. R_t equals $E_t R_{t+1}^m$ up to first order. If not all bonds are accepted in open market operations, $\kappa_t^B < 1$, bonds are less liquid and get akin to household debt.

2.4 Public sector

The central bank transfers seigniorage revenues $P_t\tau_t^m$ to the Treasury, which issues one-period bonds and pays a transfer $P_t\tau_t$ to households. Government bonds grow at a constant rate, $B_t^T = \Gamma B_{t-1}^T$, where $\Gamma \geq 1$. The Treasury's budget constraint reads

$$B_t^T/R_t + P_t\tau_t^m = B_{t-1}^T + P_t\tau_t, \tag{20}$$

where government bonds B_t^T are either held by households, B_t, or the central bank, B_t^C : $B_t^T = B_t + B_t^C$. This setup does not require B_t^T to measure total public debt, rather it is a measure of short-term government bonds which are eligible for open market operations. To avoid further effects of fiscal policy, we assume that the government has access to lump-sum taxes, which adjust to balance the budget. Thus, introducing long-term government bonds as a means of financing government expenditures would not have any consequences for the analysis conducted in this paper. In fact, we can easily extend the model by considering longer-term bonds, i.e. two-period, without any further impact on the equilibrium allocation and the associated price system as long as they are not eligible. Accepting long-term bonds as additional collateral would then be equivalent to an increase in the fraction of eligible short-term bonds κ_t^B.

The central bank supplies money outright $M_t^H = \int_0^1 M_{i,t}^H di$, and under repos against bonds, $M_t^R = \int_0^1 M_{i,t}^R di$, and loans, $M_t^L = \int_0^1 M_{i,t}^L di$. It transfers its interest earnings on government bonds held to the Treasury at end of period, $P_t\tau_t^m = B_t^C - B_t^C/R_t +$

$(R_t^m - 1)\left(M_t^R + M_t^L\right)$ and reinvests its wealth exclusively into new government bonds, which accords to central bank practice. Its budget constraint reads

$$\left(B_t^C/R_t\right) - B_{t-1}^C + P_t\tau_t^m = R_t^m\left(M_t^H - M_{t-1}^H\right) + (R_t^m - 1)\left(M_t^R + M_t^L\right)$$

Substituting out transfers, the bond holdings of the central bank evolve according to

$$B_t^C - B_{t-1}^C = R_t^m\left(M_t^H - M_{t-1}^H\right). \tag{21}$$

We assume that the central bank sets the policy rate $R_t^m \geq 1$. It further sets the inflation target π and decides on eligible assets for open market operations by setting $\kappa \in [0,1]$ and $\kappa_t^B \in (0,1]$. Finally, it controls whether money is supplied in exchange for bonds in repos or outright (while loans are only traded under repos). We assume that it controls the share of bond repos $\Omega \geq 0$, defined as

$$M_t^R = \Omega M_t^H. \tag{22}$$

Beside the policy rate and the repo share, the central bank disposes of the instruments κ_t and κ_t^B. The choice of instruments affects both for the size of the monetary base and the eligibility of assets, which has implications for interest rates and liquidity premia.

- *Quantitative easing* is defined as a policy which increases money supply by additionally accepting collateral for open market operations. Hence, we define quantitative easing in terms of public debt or corporate debt as an increase in κ_t or κ_t^B.

- *Credit easing* is a policy that changes the composition of the central bank's balance sheet without affecting its size. We therefore define credit easing as an increase in κ_t, accompanied by reductions in open market operations in government bonds, $\triangle\kappa_t^B < 0$. The sterilization is conducted such that the nominal monetary base ceteris paribus remains unchanged, i.e. $\triangle\kappa_t^B$ and $\triangle\kappa_t$ satisfy $\triangle\kappa_t^B = \frac{l}{b/\pi}\triangle\kappa_t$.

Among the liquidity facilities created by the Federal Reserve during 2008-09, many had elements of both quantitative and credit easing. In particular, the large scale purchases of Treasury securities and the extension of credit to depository institutions through the Term Auction Facility (TAF) were meant to increase liquidity in financial markets rather broadly and thus come closest to a policy of quantitative easing by increasing κ_t^B. In contrast, programs such as the Term Securities Lending Facility (TSLF), the Term Asset-Backed Securities Loan Facility (TALF) and the Commercial Paper Funding Facility (CPFF) were targeted at improving lending conditions in particular credit markets, and relate to credit easing in our model. Participants in the TSLF could for example borrow Treasury securities against a range of collateral including investment grade corporate, municipal, mortgage-backed and asset-backed securities.

2.5 Equilibrium

In equilibrium, there will be no arbitrage opportunities and markets clear, $n_t = \int_0^1 n_{jt}dj = \int_0^1 n_{it}di$, $y_t = \int_0^1 y_{jt}dj = \int_0^1 c_{it}di = c_t$, and $\int_0^1 L_{i,t}di = \int_0^1 L_{j,t}dj$. Households will behave in an identical way and aggregate asset holdings satisfy $\forall t \geq 0 : \int_0^1 D_{i,t}di = 0$, $\int_0^1 M_{i,t}^H di = \int_0^1 \widetilde{M}_{i,t}di = M_t^H$, $\int_0^1 M_{i,t}^R di = M_t^R$, $\int_0^1 B_{i,t}di = B_t$, $\int_0^1 B_{i,t}^C di = B_t^C$, $\int_0^1 I_{i,t}di = I_t = M_t^H - M_{t-1}^H + M_t^R + M_t^L$, and $B_t^T = B_t + B_t^C$. Household household bond holdings further satisfy

$$B_t - B_{t-1} = B_t^T - B_{t-1}^T - R_t^m \left(M_t^H - M_{t-1}^H\right). \tag{23}$$

In a rational expectations (RE) equilibrium all plans and constraints of households and firms are satisfied and consistent with monetary and fiscal policy, for given initial asset endowments. Further details on the RE equilibrium can be found in the appendix A.1, where the cases of binding and non-binding goods and money market constraints are considered.

The goods market constraint, which reads $P_t c_t \leq M_t^H + M_t^R + M_t^L$ in equilibrium, is well-known to be relevant for non-neutraliy of monetary policy. Only if it is binding, changes in money supply can affect prices and the allocation. Further, the money market constraint, which in equilibrium reads

$$M_t^H - M_{t-1}^H + M_t^R + M_t^L \leq \kappa_t^B \left(B_{t-1}/R_t^m\right) + \kappa_t \left(L_t/R_t^m\right), \tag{24}$$

is decisive for the effectiveness of quantitative and credit easing. The instruments κ_t^B and κ_t enter the set of equilibrium conditions (see appendix A.1) only via the money market constraint (24), given that lump-sum taxes are available. Hence, if it is not binding, then quantitative and credit easing will not affect the equilibrium allocation and the associated price system. To see when this is the case, we first use the conditions (9) and (14), which imply $\xi_t c_t^{-\sigma} = \beta E_t \frac{\xi_{t+1} c_{t+1}^{-\sigma}}{\pi_{t+1}} + \psi_t$ and that the multiplier on the goods market constraint ψ_t satisfies

$$\psi_t(c_t^\sigma/\xi_t) = 1 - (1/R_t^{Euler}) \geq 0, \tag{25}$$

in equilibrium, where the Euler rate R_t^{Euler} is defined in the usual way as $1/R_t^{Euler} = \beta E_t \frac{\xi_{t+1} c_{t+1}^{-\sigma}}{\xi_t c_t^{-\sigma} \pi_{t+1}}$. This definition shows that households are indifferent between $1/R_t^{Euler}$ units of the means of payment in period t, which is required for consumption purchases, and one unit in period $t+1$. Put differently, they are willing to pay a price $R_t^{Euler} - 1$ in order to transform one unit of an illiquid asset, i.e. an asset that is not accepted as a means of payment today and delivers one unit of money tomorrow, into one unit of money today. Consequently, a positive Euler rate reflects a positive valuation for money and implies that households will not hold more money than needed for consumption expenditures. Then, $\psi_t > 0$ (see 25) and the goods market constraint is binding (see 16). If, however, the Euler rate equals one, they are indifferent between money and an illiquid asset (which is not explicitly specified in our model), the goods market constraint is not binding, $\psi_t = 0$, and changes in money supply are then irrelevant. Thus, the Euler rate, rather than the policy rate, determines whether the goods market constraint is binding or not.

We further use that the conditions (9), (11), and (14) imply $\xi_t c_t^{-\sigma} = R_t^m(\lambda_t + \eta_t)$ and $\lambda_t = \beta E_t \frac{\xi_{t+1} c_{t+1}^{-\sigma}}{\pi_{t+1}}$. Eliminating λ_t, shows that the multiplier for the money market constraint η_t satisfies

$$\eta_t(c_t^{\sigma}/\xi_t) = (1/R_t^m) - (1/R_t^{Euler}) \geq 0, \qquad (26)$$

in equilibrium and depends on deviations of the policy rate R_t^m from an Euler rate R_t^{Euler}. Condition (26) shows that when the policy rate is smaller than the Euler rate, $R_t^m < R_t^{Euler}$, the multiplier is positive $\eta_t > 0$ and the collateral constraint in the money market is binding (see 17). In this case, given that $R_t^m \geq 1$, the goods market constraint is binding as well, $\psi_t > 0$ (see 25). Households can then get money in exchange for an eligible (more liquid) asset at a price, $R_t^m - 1$, which is below their marginal valuation of money, $R_t^{Euler} - 1$. Hence, they use eligible assets as much as possible to get money in open market operations, such that (24) is binding. In this case, there will be an (il-)liquidity premium on non-eligible assets.

If, however, the policy rate equals the Euler rate, households are indifferent between transforming eligible assets into money or holding them until maturity. Thus, if $R_t^m = R_t^{Euler}$, the money market constraint is not binding, $\eta_t = 0$ (see 26). In this case, the model reduces to a standard model where the real policy rate affects aggregate demand via the consumption Euler equation (see appendix A.1). Then, the policy instruments κ_t and κ_t^B do neither affect the allocation nor the price system, such that quantitative and credit easing are ineffective, which accords to the conventional view on quantitative easing (see e.g. Eggertsson and Woodford, 2003). These results are summarized in the following proposition.

Proposition 1 *Quantitative and credit easing can affect the equilibrium allocation and the associated price system if and only if the policy rate is smaller than the endogenous Euler rate, $R_t^m < R_t^{Euler}$.*

In contrast to macroeconomic models where only a single nominal interest rate is considered (like in most New Keynesian models), money demand can be uniquely determined even if the policy rate is at the ZLB, $R_t^m = 1$. Then, both multiplier are identical $\eta_t = \psi_t$ (see 26 and 25) since eligible assets can costlessly be transformed into money. As long as opportunity costs of money are positive $R_t^{Euler} > 1$, quantitative easing and credit easing, which will be examined in the subsequent sections, can still affect aggregate output and prices. Yet, their effectiveness is not unlimited and relies on the scarcity of liquid assets, i.e. of money and eligible assets. The effectiveness of a quantitative easing policy, which increases the amount of money available (and thus consumption), will reach its limit when the endogenous Euler rate equals the policy rate. Then, eligible assets become abundant and the money market constraint is not binding, $\eta_t = 0$. These limits will quantitatively be examined in section 4.

3 Analytical results

In this section we examine the effects of monetary policy under flexible and sticky prices in an analytical way. In the first subsection, we examine how the different monetary policy instruments effect macroeconomic variables under flexible prices. In the second subsection, we apply a local approximation of the model at a steady state where the money market constraint is binding.

3.1 Macroeconomic effects of monetary policy

Here, we show how changes in the monetary policy instruments R_t^m, κ_t^B, and κ_t affect macroeconomic aggregates and interest rates. To facilitate the derivation of analytical results, we apply a simplified version of the model. In particular, we assume that prices are perfectly flexible, $\phi = 0$, and that the utility function is logarithmic in consumption, $\sigma = 1$. We further assume that money is only supplied temporarily, $\Omega \to \infty$, and is not held outright (see 22). Accordingly, the central bank will hold government bonds only temporarily under repos. Given that this is consistent with initial money holdings and initial central bank bond holdings equal to zero, the total stock of government bonds will be held by households, $B_t = B_t^T$ (see 23). We disregard preference shocks in this subsection, and set $\xi_t = 1$. We focus on the case where the money market constraint is binding, $\eta_t > 0$, which requires the policy rate to be lower than the Euler rate (see 26). A RE equilibrium with a binding money market constraint can be reduced to a set of sequences in output, inflation, household bond holdings, and the loan rate (see appendix A.2).

Definition 1 *For $\sigma = 1$, $\phi = 0$, $\Omega \to \infty$, a RE equilibrium is a set of sequences $\{y_t, \pi_t, R_t^L, b_t\}_{t=0}^{\infty}$ and an initial price level P_0 satisfying*

$$y_t = \left[(\mu/\theta)\left(1/R_t^L\right)\right]^{\alpha/(1+\sigma_n)}, \tag{27}$$

$$\frac{1}{R_t^L} = \kappa_t \frac{1}{R_t^m} + (1-\kappa_t)\beta E_t \frac{y_t}{y_{t+1}\pi_{t+1}}, \tag{28}$$

$$y_t = [\kappa_t^B b_{t-1}\pi_t^{-1} + \kappa_t \mu y_t]/R_t^m, \tag{29}$$

$$b_t = \Gamma b_{t-1}\pi_t^{-1} \forall t \geq 1 \quad \text{and} \quad \Gamma P_0 b_0 = B_{-1}, \tag{30}$$

where $\mu = \frac{\varepsilon - 1}{\varepsilon}\alpha < 1$, for a monetary policy setting $R_t^m < \beta y_t E_t[y_{t+1}^{-1}\pi_{t+1}^{-1}]$, κ_t, and κ_t^B for a given initial stock of bonds $B_{-1} > 0$.

Condition (27) is derived from equating labor supply with labor demand and using the production function as well as goods market clearing. It shows that the costs of loans R_t^L reduces aggregate output. Condition (28), which is based on (9), (12), and (14), shows that the loan price $1/R_t^L$ is a linear combination of the inverses of the policy rate $1/R_t^m$ and of the Euler rate $1/R_t^{Euler} = \beta E_t y_t / \left(y_{t+1}\pi_{t+1}\right)$, where the former is weighted with the fraction of eligible loans κ_t and the latter with $1 - \kappa_t$. If loans are fully eligible, $\kappa_t = 1$, the loan rate equals the policy rate. If they are not eligible, $\kappa_t = 0$, loans cannot be liquidated and the loan rate equals the Euler rate. Hence, by raising the fraction of eligible loans, the central bank can reduce the loan rate and thereby the marginal costs of firms.

Combining the cash-in-advance constraint and the money market constraint leads to (29), which shows that bonds and loans can serve as collateral for money in repos. The evolution of privately held government bonds is further determined by the total supply of bonds, which grow with the rate Γ (see 12). Hence, the long-run inflation rate π can in principle be affected by the supply of eligible assets, when the money market constraint is binding. As the stock of government bonds grows at the rate Γ, the price level would also grow with the same rate when government bonds are eligible. In order to control the long-run supply of money and thus the inflation rate, the central bank can, however, reduce the fraction of accepted bonds κ^B accordingly (see proposition 3 below).

The set of equilibrium conditions (27)-(30) is sufficiently simple to derive effects of exogenous changes in monetary policy in an analytical way. To demonstrate the effects of different monetary policy instruments, we separately consider unexpected permanent changes in each instrument. Initially, the instruments are assumed to be set at follows: The fraction of eligible loans is $\kappa \geq 0$, the fraction of eligible bonds κ_t^B grows initially with a constant rate, $\kappa_t^B = \gamma \kappa_{t-1}^B$, where $\kappa_0^B > 0$ and $\gamma > \beta/\Gamma$ (which allows the central bank to respond to a growing stock of total bonds), and the policy rate is set at $R^m \in [1, \gamma\Gamma/\beta)$. Note that the restrictions $\gamma > \beta/\Gamma$ and $R^m < \gamma\Gamma/\beta$ ensure that the goods market constraint and the money market constraint are binding. The following proposition summarizes the impact effects of permanent changes in the policy instruments.

Proposition 2 *Consider the model given in definition 1. Suppose that the central bank instruments are initially set at $\kappa \geq 0$, $\kappa_t^B = \gamma \kappa_{t-1}^B$, where $\kappa_0^B > 0$ and $\gamma > \beta/\Gamma$, and $R^m \in [1, \gamma\Gamma/\beta)$. Then, the money market constraint is binding and*

1. *a marginal increase in the policy rate to $R_{t+i}^m = \overline{R}^m \in (R^m, \gamma\Gamma/\beta)$ $\forall i \geq 0$ leads to an increase in the loan rate and a decline in output if $\kappa > 0$, and to a decline in the inflation rate.*

2. *a marginal decrease in the growth rate of κ_t^B to $\gamma_{t+i} = \overline{\gamma} \in (R^m \beta/\Gamma, \gamma)$ $\forall i \geq 0$ leads to a decline in the loan rate and an increase in output if $\kappa < 1$, and to an fall in the inflation rate.*

3. *a marginal increase in the fraction of eligible loans κ to $\kappa_{t+i} = \overline{\kappa} > \kappa$ $\forall i \geq 0$ leads to a decline in the loan rate and to an increase in output, while the effect on the inflation rate is ambiguous.*

Proof. See appendix A.2. ■

Hence, all policy instruments can be used by the central bank to affect the equilibrium allocation and the inflation rate. An increase in the policy rate R_t^m leads to a decline in inflation (see part 1 of proposition 2), since a larger amount of nominal bonds are required for a given amount of nominal consumption expenditures. If the fraction of eligible loans is positive, $\kappa > 0$, an increase in the policy rate further induces an increase in the loan rate according to (28). This raises the marginal costs of production such that total output declines. While this effect tends to increase inflation, it is dominated by the first effect.

Part 2 of proposition 2 shows that a decrease in the growth rate of the fraction of eligible bonds κ_t^B affects the inflation rate like a decrease of the money growth rate in a conventional flexible price model. If, further, not all loans are eligible (which would imply that the loan rate equals the policy rate), the fall in inflation reduces the Euler rate and the loan rate according to (28). With smaller costs of borrowing, the marginal costs of firms decrease and output increases. Correspondingly, a policy of a permanent *quantitative easing* in terms of government bonds, where the growth rate γ increases, would impact on the allocation and prices like an increased money growth rate.

As stated in part 3 of proposition 2, the central bank can also directly induce a lower loan rate R_t^L by raising the fraction of eligible loans κ_t if the policy rate is smaller than the Euler rate $R^m < \gamma\Gamma/\beta$ (see 28). The reason is that the central bank can induce households to demand a lower illiquidity premium on the loan rate by increasing the fraction of eligible loans κ. Due to reduced costs of loans, firms increase labor demand which increases output. It should be noted that this effect is independent of the inflation response, which is here ambiguous. Hence, a *credit easing* policy can be expected to stimulate real activity by reducing the costs of borrowing, which will be demonstrated in section 4.

As revealed by proposition 2, the central bank can affect the inflation rate by changing its instruments. In particular, it can control the inflation rate by setting the growth rate γ_t contingent on the growth rate of government bonds Γ.

Proposition 3 *Consider the model given in definition 1. When the central bank sets its instruments according to $\kappa_t \geq 0$, $\gamma_t > \beta/\Gamma$, and $R_t^m \in [1, \gamma\Gamma/\beta)$ it can control the inflation rate by changing the fraction of eligible government bonds κ_t^B. Then, a permament increase in κ_t^B, but not in its growth rate γ_t, leads to a temporary change in inflation. The central bank can further implement long-run price stability by setting $\gamma_t = \Gamma^{-1}$.*

Proof. See appendix A.2. ■

According to proposition 3, the central bank can control the inflation rate under a binding money market constraint. A quantitative easing policy in terms of government bonds that leads to a once-and-for-all increase in the size of the balance sheet cannot lead to a permanent change in the inflation rate, while the central bank can implement its inflation target independent of fiscal policy and can ensure long-run price stability by setting $\gamma_t = \Gamma^{-1}$.

3.2 Quantitative easing under sticky prices

While quantitative easing in terms of government bonds will lead to higher prices and lower output under flexible prices, we expect that it can stimulate real activity under imperfectly flexible prices. In this section, we therefore consider quantitative easing under sticky prices, $\phi > 0$, like in New Keynesian models. We further disregard central bank lending against corporate debt, $\kappa_t = 0$, such that the monetary policy regime is conventional in the sense that only Treasuries securities are eligible. We apply a local analysis of the economy at a steady state where government bonds are not fully eligible, $\kappa^B < 1$, leaving room for a quantitative easing policy. Details on the steady state can be found in appendix A.3. Given that there exists a steady state, we can use that all real endogenous variables are constant. Combining (15) with (18) implies that the steady state debt rate R^D equals the Euler rate and satisfies $R^D = R^{Euler} = \pi/\beta$. The debt rate and the policy rate R^m can differ by an (il-)liquidity premium, as revealed by steady state version of (26) $\eta = c^{-\sigma}[(1/R^m) - (\pi/\beta)] \geq 0$.

We assume that the central bank inflation target is consistent with long-run price stability, $\pi = 1$, which can be justified by the minimization of welfare costs due to long-run price dispersion. Precisely, the central bank can implement long-run price stability by long-run adjustments of κ_t^B contingent on the supply of government bonds (see proposition 3). We disregard a growing supply of bonds $\Gamma > 1$ that can be neutralized by a shrinking fraction of eligible bonds, and assume – without loss of generality – that $\Gamma = 1$. We further assume that the central bank sets the average policy rate below the long-run Euler rate, $R^m < \pi/\beta$, which is consistent with the empirical evidence provided by Canzoneri et al. (2007) for our preference specification. Given that $\pi = 1$ implies $R^{Euler} > 1$ and $R^m < R^{Euler}$, the goods market constraint as well as the money market constraint are binding in the steady state.

To motivate why the central bank sets the policy rate at its ZLB, we consider preference shocks $\widehat{\xi}_t$, which are assumed to be generated by an AR(1) process, $\widehat{\xi}_t = \rho_\xi \widehat{\xi}_t + \varepsilon_t$, where $E_{t-1}\varepsilon_t = 0$ and $\rho_\xi \in [0, 1)$. In a neighborhood of this steady state, the equilibrium sequences are approximated by the solutions to the linearized equilibrium conditions. An equilibrium is then defined as follows, where $\widehat{a}_t$ denotes the percent deviation of a generic variable a_t from its steady state value $\overline{a} : \widehat{a} = \log(a_t) - \log(\overline{a})$.

Definition 2 *For $\Omega \to \infty$, $\Gamma = \pi = 1$, $R^m \in [1, 1/\beta)$, and $\kappa_t = 0$, a RE equilibrium is a set of sequences $\{\widehat{y}_t,\ \pi_t,\ \widehat{b}_t,\ \widehat{R}_t^L\}_{t=0}^{\infty}$ that converge to the steady state and satisfy*

$$\widehat{y}_t = \widehat{b}_{t-1} - \widehat{\pi}_t - \widehat{R_t}^m + \widehat{\kappa}_t^B, \tag{31}$$

$$\sigma \widehat{y}_t = \sigma E_t \widehat{y}_{t+1} - \widehat{R}_t^L + E_t \widehat{\pi}_{t+1} + \left(1 - \rho_\xi\right) \widehat{\xi}_t, \tag{32}$$

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \chi \left(\varpi - 1\right) \widehat{y}_t + \chi \widehat{R}_t^L, \tag{33}$$

$$\widehat{b}_t = \widehat{b}_{t-1} - \widehat{\pi}_t, \tag{34}$$

where $\varpi = \frac{1+\sigma_n}{\alpha} + \sigma$ for monetary policy setting $\{\widehat{\kappa}_t^B, \widehat{R_t}^m\}_{t=0}^{\infty}$ and an initial value $b_{-1} > 0$.

The linear model summarized in definition 2 exhibits some similarities to a New Keynesian model with the "cost channel" (see Ravenna and Walsh, 2006). In particular, the conditions (32) and (33) resemble standard conditions for aggregate demand and for aggregate supply, where the latter is affected by the costs of loans due to the working capital assumption. The crucial difference to a New Keynesian model is, however, that this is not a single interest rate framework. Specifically, the policy rate, which is not identical to the loan rate since $\kappa_t < 1$, neither enters (32) nor (33). Nevertheless, the policy rate affects the equilibrium allocation and prices via the reduced version of the money supply constraint (31). Here, an increase in the policy rate tends – for a given amount of eligible bonds – to reduce the amount of money and thereby aggregate demand. Inflation, output, real bonds, and the loans rate, which equals the Euler rate for $\kappa_t = 0$ (see 28), will simultaneously be determined, given both monetary policy instruments, i.e. the policy rate $\widehat{R_t}^m$ and the fraction of eligible bonds $\widehat{\kappa}_t^B$.

Since the policy rate does not enter the aggregate demand constraint (32), which it typically does in New Keynesian models, the well-known Taylor-principle does not apply to this model. In this model, the range of parameter values which are associated with local equilibrium determinacy differs substantially from the range implied by the Taylor-principle. Specifically, the equilibrium is locally determined for a broad range of values for the feedback parameters of a simple Taylor-rule

$$\widehat{R}_t^m = \rho_\pi \widehat{\pi}_t + \rho_y \widehat{y}_t, \quad \rho_\pi \geq 0, \rho_y \geq 0, \tag{35}$$

including a peg. The following proposition presents the condition for local equilibrium determinacy.

Proposition 4 *Consider the model given in definition 2, where the central bank sets the policy rate according to (35). For an exogenously given fraction* κ_t^B*, the equilibrium is locally determined if and only if* $\frac{\rho_\pi + 1/2}{\rho_y + 1}\left(\sigma - \frac{1-\alpha+\sigma_n}{\alpha}\right) < 1 + \frac{1+\beta}{\chi}$.

Proof. See appendix A.3 ■

We are now prepared to examine monetary policy at the ZLB in a simple way. For this, consider for example a decline in the preference parameter $\widehat{\xi}_t$. This shock leads to a decline in inflation and the policy rate according to (35). If the shock is sufficiently large, the policy rate reaches its ZLB. At this point, the central bank pegs the policy rate at $R_t^m = 1$. The condition given in proposition 4 then reads, $\sigma < \overline{\sigma} = 2\left[1 + (1+\beta)/\chi\right] + (1 - \alpha + \sigma_n)/\alpha$. If σ satisfies this condition, which is hardly restrictive for standard parameter values (e.g. the parameter values applied in section 4 imply $\overline{\sigma} > 50$), the equilibrium is uniquely determined. Hence, the central bank can safely peg the policy rate at the ZLB without inducing local indeterminacy.

The set of equilibrium conditions listed in definition 2 reveal that both policy instruments, $\widehat{\kappa}_t^B$ and $\widehat{R_t}^m$, affect the private sector behavior only via the money supply constraint (31). According to the latter, money supply can be eased by the central bank either by decreasing the policy rate or by increasing the fraction of eligible bonds. Hence, if the policy rate cannot be lowered, because it has reached the ZLB, the central bank can still ease money supply by conducting quantitative easing, $\widehat{\kappa}_t^B > 0$. Specifically, a 1% increase in $\widehat{\kappa}_t^B$ affects the economy in the same way as a reduction of the policy rate by 1%. The following proposition summarizes the effects of an unexpected temporary quantitative easing policy, $\widehat{\kappa}_t^B > 0$, where $\widehat{\kappa}_t^B$ is assumed to follow an AR(1) process, $\widehat{\kappa}_t^B = \rho_\kappa \widehat{\kappa}_{t-1}^B + \varepsilon_t^\kappa$, $\rho_\kappa \in [0,1)$ and $E_{t-1}\varepsilon_t^\kappa = 0$.

Proposition 5 *Consider the model given in definition 2 for $\sigma < \overline{\sigma}$ and $\kappa^B < 1$ and suppose that the central bank sets the policy rate at its ZLB, $R_t^m = 1$. Then, a temporary quantitative easing policy in terms of government bonds, $\widehat{\kappa}_t^B > 0$, leads to an increase in output, $\widehat{y}_t > 0$, and in the inflation rate, $\widehat{\pi}_t > 0$.*

Proof. See appendix A.3 ■

It should be noted that the effectiveness of quantitative easing relies on the scarcity of liquid assets, i.e. on a binding money market constraint. This however implies that there exist limits to the effectiveness of a quantitative easing. These limitations will be examined in the subsequent section.

4 Limits to quantitative and credit easing

In this section, we examine the quantitative impact of monetary policy on macroeconomic aggregates. For this, we disregard preference shocks, $\xi_t = 1$, and assume that the central bank sets its targets according to $\pi < \beta$ and $1 \leq R^m \in [1, \pi/\beta)$, which implies that the money market constraint and the goods market constraints are binding in the steady state (see 25 and 26). We extend the model by considering physical capital and calibrate it to explore the effects and limits of quantitative and credit easing. We further analyze the impact of a large liquidity demand shock on the economy and assess the ability of policy to mitigate the contractionary effects of the shock by quantitative easing. Throughout the analysis, we assume that quantitative and credit easing are implemented when the policy rate is at its ZLB, where their range of effectiveness is particularly large.[8]

4.1 Extension of the model

For a quantitative analysis of monetary policy effects, we extend the model presented in section 2 by introducing physical capital. Households own the stock of capital, $k_t = \int k_{i,t} di$

[8] In a companion paper, Schabert (2010) examines the policy instruments κ_t and κ_t^B when the policy rate is set above its ZLB. He shows that by applying these additional instruments the central bank can enhance welfare compared to the case of a pure interest rate setting.

and rent it to firms at the rate r_t^k. The capital stock of household i evolves according to $k_{i,t+1} = (1-\delta)\,k_{i,t} + x_{i,t}S\,(x_{i,t}/x_{i,t-1})$, where $\delta \in (0,1)$, x_t denotes investment expenditures, $S\,(x_t/x_{t-1}) = 1 - \frac{\vartheta}{2}\,(x_t/x_{t-1} - 1)^2$ is an adjustment cost function, and $\vartheta > 0$ measures the degree of adjustment cost. We assume that households rely on cash for purchases of investment goods up to an exogenous fraction ω_t. We introduce a corresponding parameter ν, which describes the fraction of purchases of consumption goods that require cash. Thus, the cash in advance constraint (3) is replaced by

$$\upsilon P_t c_{i,t} + \omega_t P_t x_{i,t} \leq I_{i,t} + M_{i,t-1}^h - \left(L_{i,t}/R_t^L\right) + P_t w_t n_{i,t}. \tag{36}$$

The parameters $\nu > 0$ and $\omega_t > 0$ govern the liquidity demand of households. These parameters allow relating expenditures to the monetary base in accordance with empirical counterparts, which facilitates the calibration of the model. In section (4.4) we further analyze a shock to ω_t, which captures increased liquidity demand for purchases of investment goods.

Intermediate goods producing firms rent capital from households. Firm j produces with technology $IO_{j,t} = n_{j,t}^{\alpha} k_{j,t}^{1-\alpha}$ and pays the rental rate on capital after their goods are sold. Hence, the working capital condition (1) of firm j is unchanged. Its first order conditions for $R_t^L > 1$ are given by $mc_{j,t}\alpha\,(n_{j,t}/k_{j,t})^{\alpha-1} = w_t R_t^L$, $mc_{j,t}\,(1-\alpha)\,(n_{j,t}/k_{j,t})^{\alpha} = r_t^k$, and (6). To calibrate the model in a consistent way, we further introduce government spending so that goods market clearing requires $y_t = c_t + x_t + g_t$. The full set of equilibrium conditions can be found in Appendix A.4.

4.2 Calibration

We use standard parameter values as far as possible. The parameters of the utility function equal $\sigma = 2$ and $\sigma_n = 1$, the labor share equals $\alpha = 0.66$, the steady state markup $1/mc =$ 11% ($\varepsilon = 10$), steady state working time $n = 1/3$, and the fraction of non-optimally price adjusting firms $\phi = 0.75$. The share of government spending and the long run inflation rate are set to $g/y = 0.19$, the steady state values of ω_t and υ are calibrated to the observed ratios $Px/M0 = 1.15$ and $Pc/M0 = 2.71$, and the depreciation rate is set to $\delta = 0.023$ to match the observed ratio of consumption to investment, $c/x = 2.36$. [9] For the policy rate, we consider two scenarios. The policy rate is either pegged at the ZLB, $R^m = 1$, or set according to a simple Taylor rule (35) with $\rho_\pi = 1.5$, $\rho_y = 0.5$, and a long-run policy rate equal to the Federal Funds Rate's 25-year average $R^m = 1.0133$ (or 5.41% in terms of annualized rates)

[9] Data on (nominal) consumption, investment, government spending and Gross Domestic Product are taken from NIPA Table 1.15, where durable consumption goods are included into investment. Data on the monetary base was taken from the Federal Reserve Board's H3 Statistical Release. Data on inflation was extracted from the series GDPDEF, available from the U.S. Bureau of Economic Analysis. All data except those for the Fed Funds rate are seasonally adjusted series. All data refer to averages for 25 years over the period QI/1981–QIV/2006, except for the Fed Funds rate and inflation, where the sample starts after the pre-Volcker period, QIV/1982–QIII/2008.

and an inflation target equal to its steady state value $\pi = 1.00647$ (or 2.61% at an annual rate). Both policies are consistent with equilibrium determinacy (see proposition A.3). We restrict our attention to the case where the central bank does not trade corporate debt in open market operations in the steady state, i.e. $\kappa = 0$, which accords to the Fed's pre-crisis "Treasuries only" regime. In contrast, government debt is fully eligible for open market operations, $\kappa^B = 1$, where we assume – without explicitly specifying – that long-run growth in government bonds and in κ_t^B is consistent with the long-run inflation rate (see proposition 3). We further set the repo share to $\Omega = 1.5$ to match the observed ratio between total reserves and reserves supplied under repurchase agreements, which was almost constant in the 2000s before the crisis.[10] The value for the adjustment cost parameter $\vartheta = 2.48$ is taken from Christiano et al. (2005).

The spread between the policy rate and the loan rate, which equals the Euler rate $R^L = R^{Euler} = \pi/\beta$ in a steady state with $\kappa = 0$, matters for the size of monetary policy effects. To calibrate this spread, we account for the fact that our model does not account for any kind of default risk and focus on the part of the spread that can be attributed to a liquidity premium. According to the literature on the "corporate bond credit spread puzzle", only a small share of the yield spread between Treasury securities and corporate bonds can actually be explained by default risk. For our calibration, we refer to Longstaff et al.'s (2005) results for the spread between corporate bonds and Treasury securities, which lead to more conservative estimates of the liquidity premium.[11] Specifically, they report that, for AAA rated corporate bonds, 51% of the credit spread can be explained by default risk. Given that the average short-term spread among AAA corporate bonds equals 104 basis points at annualized rates (see Longstaff et al., 2005), we consider a liquidity premium of $(1 + 49\% \cdot 0.0104)^{1/4} - 1 = 13$ basis points (in terms of quarterly rates), which implies the discount factor to equal $\beta = \frac{\pi}{R^m + 13 \cdot 10^{-4}} = 0.992$.

4.3 Isolated effects of quantitative and credit easing

The calibrated model is solved by applying a first-order approximation at the deterministic steady state. Most variables are given in terms of percentage deviations from steady state, $\widehat{a}_t = \log(a_t) - \log(\overline{a})$, as defined earlier. Further, we consider deviations expressed in percentage points, $\widetilde{a}_t = 100\,(a_t - \overline{a})$, for κ, κ^B, ω, and interest rates, e.g., $\tilde{R}_t^m = 1$ denotes an increase in the policy rate by 100 basis points. As stated in proposition 2, quantitative easing can increase output even when the policy rate is at the ZLB. The reason is that the money market constraint binds at $R_t^m = 1$ as long as the Euler rate exceed one (see 26). However, easing money supply will eventually lead to the point where households' and firms' cash de-

[10] See Federal Reserve Bank of New York, Domestic Open Market Operations, various issues, and FRED database.

[11] Collin-Dufresne at al. (2001), for example, can explain only 25% of the variation in credit spread changes across 688 corporate bonds. Huang and Huang (2002) report that around 20% of corporate credit spreads can be explained through default risk.

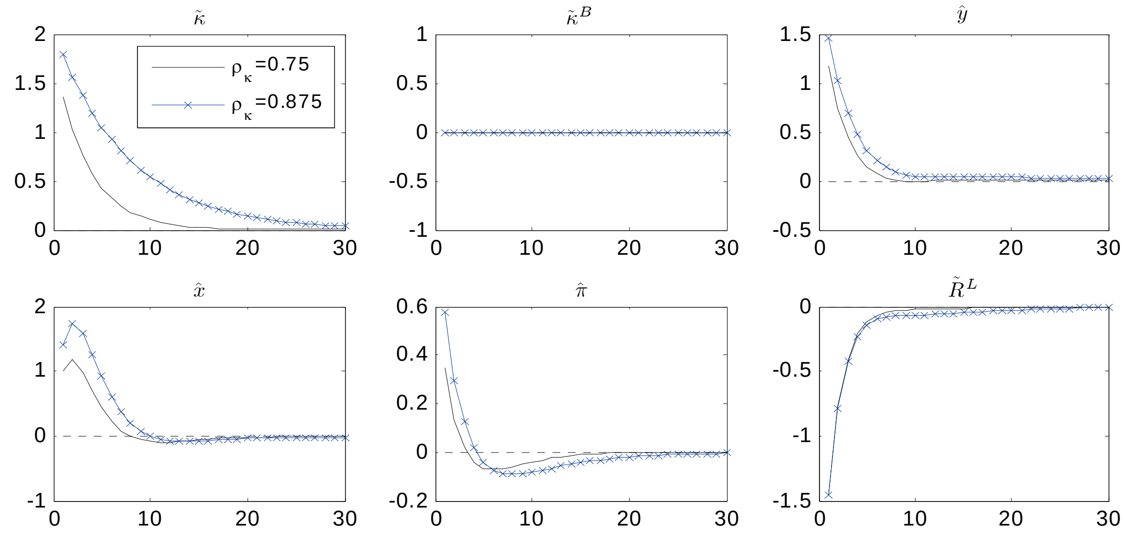

Figure 1: Maximum effects of quantitative easing

mand will not be collateral constrained. Specifically, the multiplier on the money market constraint η_t (see 25 and 26) has to satisfy

$$\eta_t = (c_t^{-\sigma}/R_t^m) - \beta E_t c_{t+1}^{-\sigma} \pi_{t+1}^{-1} > 0, \tag{37}$$

for a quantitative easing policy to be effective. Thus, the range over which the money market constraint is binding is particularly large at its ZLB, $R_t^m = 1$ (see 37). A closer look at (37) shows that the multiplier approaches zero if an increase in consumption is sufficiently large and not too persistent. Beyond this point, quantitative and credit easing can achieve no further stimulus. The purpose of this analysis is to quantify this limit.

We first consider isolated effects of quantitative easing in terms of corporate debt, i.e. an increase in κ_t, where $\widehat{\kappa}_t$ exhibits an autocorrelation of $\rho_\kappa = 0.75$ (0.875) which accords to an expected duration of the policy intervention of one year (two years). Figure 1 shows the impulse response to the maximum intervention, which is defined as the intervention which just lets the collateral constraint bind, $\eta_t > 0$. The maximum intervention with $\rho_\kappa = 0.75$ (see solid line) implies an increase of $\triangle\kappa_t = 0.0136$ (or $\widetilde{\kappa}_t = 1.36$) so that – ceteris paribus – the monetary base, $m_t^H + m_t^R + \kappa_t l_t / R_t^m$, rises by 1.55%. This induces the loan rate to fall to its ZLB and a rise in output by 1.18%, while inflation increases by 35 basis points. When the policy intervention is more persistent, $\rho_\kappa = 0.875$ (see starred line) a larger intervention is possible according to (37). Although, quantitative easing can be conducted at a larger scale ($\triangle\kappa_t = 0.0179$ or $\widetilde{\kappa}_t = 1.79$), it will not achieve substantially higher output responses. Output rises by 1.46%, while inflation increases by 58 basis points. Hence, supplying additional money

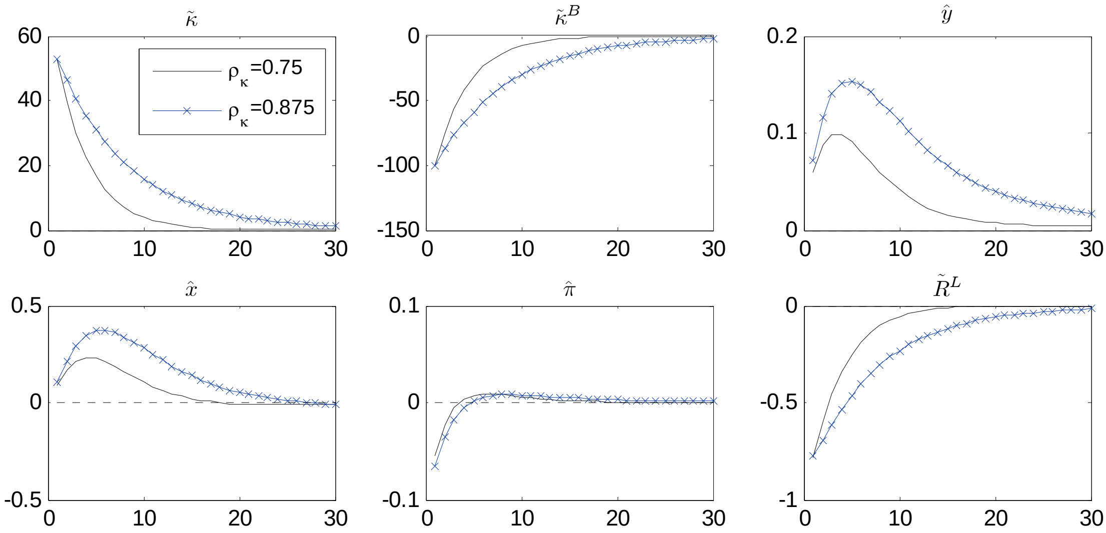

Figure 2: Maximum effects of credit easing

against 1% of all loans in open market operations raises output on impact by 0.87% (0.81%) in case of the less (more) persistent intervention.

Next, we examine isolated effects of credit easing. The extent of credit easing is limited by the size of the central bank's balance sheet and the availability of collateral. In terms of the model, credit easing is limited by $\kappa_t^B \geq 0$ and $\kappa_t \leq 1$. For our calibration, these restrictions are more severe than (37) and a maximum credit easing policy is given by $\triangle\kappa_t^B = -1$ and $\triangle\kappa_t = -\frac{b/\pi}{l}\triangle\kappa_t^B = 0.53$ (or $\widetilde{\kappa}_t = 53$ and $\widetilde{\kappa}_t^B = -100$). Hence, the central bank exchanges more than the half of all loans against government bonds. Like before, we consider a constant policy rate pegged at $R_t^m = 1$. Figure 2 shows the responses to the maximum credit easing. As corporate debt is now eligible in open market operations, R_t^L declines, so that marginal cost and inflation fall. Reduced goods prices and increased real balances allow households to increase consumption and investment spending. For an autocorrelation of 0.75 (see solid line) which corresponds to an expected duration of one year, output exhibits a peak response of 0.1% in the third quarter and inflation declines by five basis points. For a more persistent intervention, $\rho_\kappa = 0.875$ (see starred line), the loan rate declines more persistently, which leads to a more pronounced decline in inflation and an increase in output by a maximum of 0.15%. These numbers demonstrate that credit easing has a relatively small impact. Precisely, exchanging 1% of loans against government bonds leads to a maximum rise in output by 0.0019% (0.0029%) in case of the less (more) persistent intervention.

Finally, comparing the output effects of quantitative easing in terms of corporate debt with the output effects of credit easing implies that the output effects of quantitative easing in terms of government bonds will be virtually identical to those shown in figure 1.

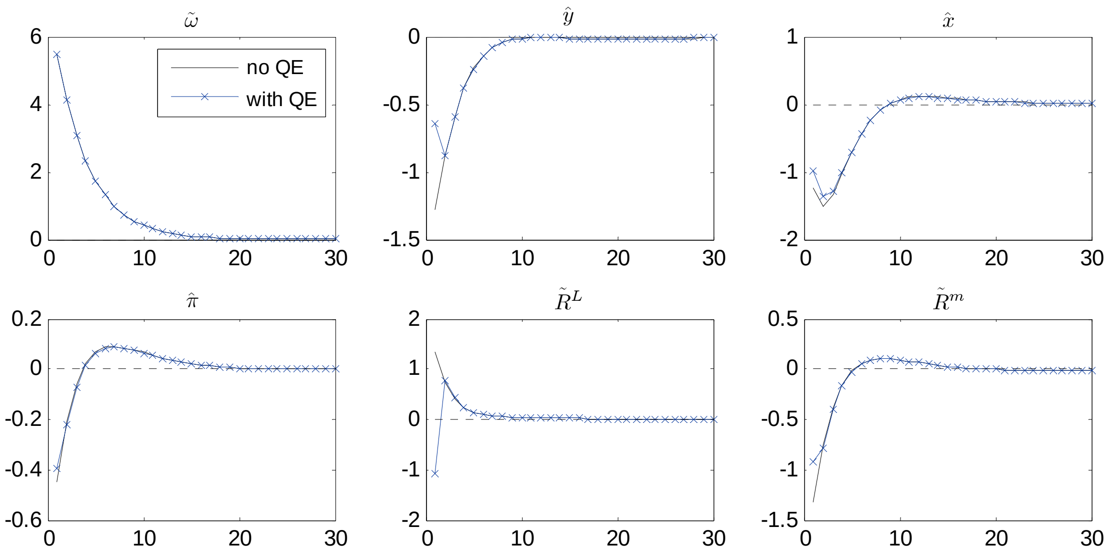

Figure 3: Liquidity demand shock with and without QE intervention

4.4 Response to a liquidity demand shock

We now consider a liquidity demand shock, i.e. an unexpected increase in the fraction ω_t of investment goods that have to be purchased with cash (36). This shock, which implies that less investments can be financed on credit, can for example be interpreted as an increase in financial distress that lowers the extent to which investment goods can be pledged as collateral. We assume an AR(1) process for ω_t, $\omega_t = (1-\rho_\omega)\,\omega + \rho_\omega \omega_{t-1} + \varepsilon_t$, with autocorrelation $\rho_\omega = 0.75$. The shock hits an economy, where the policy rate is initially at its steady state value $R^m = 1.0133$ and otherwise governed by a Taylor rule (35) with $\rho_\pi = 1.5$ and $\rho_y = 0.5$. We consider a shock that drives the policy rate to the ZLB in the impact period, which requires $\triangle\omega_t = 0.0549$ (or $\tilde{w}_t = 5.49$). The solid line in Figure 3 shows the impulse responses to this shock without quantitative easing. Investment and consumption fall, so that output declines by 1.29% despite the reduction of the policy rate. The inflation rate falls, while the spread between the policy rate and the loan rate increases.

The starred line shows the responses for the case where the central bank applies a maximum size of the quantitative easing policy in terms of corporate debt at the ZLB. It should be noted that quantitative easing is only conducted in the first period, since the policy rate increases afterwards. For this policy, the contractionary effects are mitigated and output falls by only 0.65%. The impact output contraction is reduced by 50%. Inflation is 5 basis points larger than without intervention, falling only by 40 basis points. Hence, the central bank can substantially reduce the contractionary output effect of the liquidity demand shock via a quantitative easing policy for one period. The maximum size, $\triangle\kappa_t = 0.0088$ (or $\tilde{\kappa}_t = 0.88$)

is again determined by the multiplier on the money market constraint, which has to satisfy (37). Notably, the intervention is smaller than in section 4.3 (see figure 1). Precisely, the intervention reduces the impact decline in output by 0.64%, compared to 1.46% (1.18%) in section 4.3. The reasons for this is that, first, the policy intervention is less persistent and, second, the policy rate immediately increases in the impact period due to a successful intervention, which both tend reduce the multiplier η_t (see 37). Nevertheless, quantitative easing can help escaping from the ZLB.

5 Conclusion

The recent financial crisis has led several central banks to set the policy rate close to its zero lower bound and to conduct non-standard policies, like quantitative and credit easing. At the same time, the macroeconomic literature on monetary policy has provided little guidance for conducting these policies, and in particular, has left open under which circumstances quantitative easing can be effective. Our analysis is motivated by the observation that eligible securities, like US-Treasury bills, are typically traded at a liquidity premium compared to interest rates on non-eligible assets. Further, financial markets experienced a surge in interest rate spreads during the crisis. This increase, which can to a large extent be attributed to liquidity concerns, was substantially reduced in response to quantitative and credit easing.

In this paper, we apply a simple macroeconomic model where money is supplied by the central bank against collateral. We show that quantitative and credit easing can be effective only if eligible assets are scarce, which is reflected in liquidity premia. A precondition for this is that the central bank supplies money at a price (i.e. the policy rate) below the private sectors nominal savings rate (i.e. the consumption Euler). For this case and in particular at the zero lower bound, we find that quantitative and credit easing can stimulate real activity and reduce interest rate spreads. Notably, quantitative easing increases inflation, whereas credit easing reduces inflation. These policies, however, reach their limits when the liquidity premium approaches zero, indicating that collateral becomes abundant. Taking these limits into account, we find that the maximum effects of quantitative easing are relatively small, though sufficient to help escaping from the zero lower bound.

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A Appendix

A.1 Rational expectations equilibrium

Definition 3 *A RE equilibrium is given by a set of sequences* $\{c_t,\ y_t,\ n_t,\ \lambda_t,\ m_t^R,\ m_t^H,\ b_t,\ b_t^T,\ l_t,\ w_t,\ mc_t,\ \tilde{Z}_t,\ s_t,\ \pi_t,\ R_t,\ R_t^D,\ R_t^L\}_{t=0}^{\infty}$ *satisfying*

$$\theta n_t^{\sigma_n} = w_t c_t^{-\sigma}, \tag{38}$$

$$R_t^L = \left[\lambda_t (1-\kappa_t) + \kappa_t \xi_t c_t^{-\sigma}/R_t^m\right]^{-1} \xi_t c_t^{-\sigma}, \tag{39}$$

$$\lambda_t = \beta E_t \left[\xi_{t+1} c_{t+1}^{-\sigma}/\pi_{t+1}\right], \tag{40}$$

$$\lambda_t = \beta R_t E_t[\left(\lambda_{i,t+1}\left(1-\kappa_{t+1}^B\right) + \kappa_{t+1}^B \xi_{t+1} c_{t+1}^{-\sigma}/R_{t+1}^m\right)\pi_{t+1}^{-1}], \tag{41}$$

$$\lambda_t = \beta R_t^D E_t\left[\lambda_{t+1}/\pi_{t+1}\right], \tag{42}$$

$$c_t - \kappa_t l_t/R_t^m = m_t^H + m_t^R,\ \textit{if}\ \ \psi_t = (R_t^m - 1)\lambda_t + R_t^m\left(\xi_t c_t^{-\sigma} - R_t^m \lambda_t\right) > 0, \tag{43}$$

$$\textit{or}\ c_t - \kappa_t l_t/R_t^m \leq m_t^H + m_t^R,\ \textit{if}\ \ \psi_t = 0,$$

$$\kappa_t^B b_{t-1}/\left(R_t^m \pi_t\right) = m_t^H - m_{t-1}^H \pi_t^{-1} + m_t^R,\ \textit{if}\ \ \eta_t = \xi_t c_t^{-\sigma} - R_t^m \lambda_t > 0, \tag{44}$$

$$\textit{or}\ \kappa_t^B b_{t-1}/\left(R_t^m \pi_t\right) \geq m_t^H - m_{t-1}^H \pi_t^{-1} + m_t^R,\ \textit{if}\ \ \eta_t = 0,$$

$$b_t - b_{t-1}\pi_t^{-1} = (\Gamma - 1) b_{t-1}^T \pi_t^{-1} - R_t^m\left(m_t^H - m_{t-1}^H \pi_t^{-1}\right), \tag{45}$$

$$mc_t \alpha n_t^{\alpha-1} = w_t R_t^L, \tag{46}$$

$$l_t/R_t^L = w_t n_t, \tag{47}$$

$$\tilde{Z}_t(\varepsilon - 1)/\varepsilon = Z_t^1/Z_t^2, \tag{48}$$

$$\textit{where}\ Z_t^1 = c_t^{-\sigma} y_t mc_t + \phi\beta E_t \pi_{t+1}^{\varepsilon} Z_{t+1}^1\ \textit{and}\ Z_t^2 = c_t^{-\sigma} y_t + \phi\beta E_t \pi_{t+1}^{\varepsilon-1} Z_{t+1}^2,$$

$$1 = (1-\phi)(\tilde{Z}_t)^{1-\varepsilon} + \phi \pi_t^{\varepsilon-1}, \tag{49}$$

$$m_t^R = \Omega_t m_t^H, \tag{50}$$

$$b_t^T = \Gamma b_{t-1}^T/\pi_t, \tag{51}$$

$$y_t = n_t^{\alpha}/s_t, \tag{52}$$

$$y_t = c_t, \tag{53}$$

$$s_t = (1-\phi)\tilde{Z}_t^{-\varepsilon} + \phi s_{t-1}\pi_t^{\varepsilon}, \tag{54}$$

the transversality conditions, a monetary policy setting $\{R_t^m \geq 1,\ \kappa_t^B,\ \kappa_t \in [0,1]\}_{t=0}^{\infty}$, $\Omega_t > 0$ *and* $\pi \geq \beta$, *and a fiscal policy setting* $\Gamma \geq 1$, *for a given sequence* $\{\xi_t\}_{t=0}^{\infty}$, *initial values* $M_{-1}^H > 0$, $B_{-1} > 0$, $B_{-1}^T > 0$, *and* $s_{-1} \geq 1$.

When money supply is not effectively rationed due to a *non*-binding money market constraint, the model reduces to a standard sticky price model and a RE equilibrium can be reduced and redefined as follows.

Definition 4 *A RE equilibrium for* $\eta_t = 0$ *is a set of sequences* $\{c_t,\ n_t,\ l_t,\ w_t,\ mc_t,\ \tilde{Z}_t,\ Z_t^1,\ Z_t^2,\ s_t,\ \pi_t,\ R_t^L\}_{t=0}^{\infty}$ *satisfying*

$$\mu_t \theta n_t^{\sigma_n} = w_t c_t^{-\sigma}, \tag{55}$$

$$\xi_t c_t^{-\sigma} = \beta R_t^m E_t[\xi_{t+1} c_{t+1}^{-\sigma} \pi_{t+1}^{-1}], \tag{56}$$

$$R_t^L = R_t^m, \tag{57}$$

(46)-(49), (52)-(54), the transversality conditions, a monetary policy setting $\{R_t^m \geq 1\}_{t=0}^{\infty}$ and the inflation target $\pi \geq \beta$, for a given sequence $\{\xi_t\}_{t=0}^{\infty}$ and $s_{-1} \geq 1$.

Given that the policy instruments κ_t and κ_t^B do not appear in definition 4, we can immediately summarize an irrelevance result.

Corollary 1 *The policy instruments κ_t and κ_t^B are irrelevant for the equilibrium allocation and the associated price system, if the money market constraint is not binding, $\eta_t = 0$.*

A.2 Appendix to the flexible price model

Suppose that prices are fully flexible, $\phi = 0$, the utility function is logarithmic in consumption, $\sigma = 1$, and money is only supplied temporarily, $\Omega \to \infty$, such that money is not held outright, $M^H \to 0$. The set of equilibrium conditions given in appendix A.1 can then be reduced to a system in $\{c_t, y_t, n_t, m_t^R, b_t, l_t, w_t, \pi_t, R_t^L\}_{t=0}^{\infty}$

$$\theta n_t^{\sigma_n} = w_t c_t^{-1}, \tag{58}$$

$$\frac{1}{R_t^L} = \frac{\kappa_t}{R_t^m} + (1-\kappa_t)\beta E_t \frac{\xi_{t+1} c_t}{\xi_t c_{t+1} \pi_{t+1}}, \tag{59}$$

$$m_t^R + \kappa_t l_t / R_t^m \geq c_t, \tag{60}$$

$$\kappa_t^B b_{t-1}/\pi_t \geq R_t^m m_t^R, \tag{61}$$

$$\frac{\varepsilon - 1}{\varepsilon} \alpha n_t^{\alpha-1} = w_t R_t^L, \tag{62}$$

$$l_t = R_t^L w_t n_t, \tag{63}$$

$$b_t = \Gamma b_{t-1} \pi_t^{-1}, \tag{64}$$

$$c_t = n_t^{\alpha}, \tag{65}$$

$$y_t = c_t, \tag{66}$$

where the multiplier ψ_t and η_t satisfy (25) and (26). Since we focus on the case where $\eta_t > 0$, we can disregard ψ_t, given that $\eta_t > 0 \Rightarrow \psi_t > 0$. Eliminating m_t^R, n_t, mc_t, w_t, c_t and l_t and setting $\xi_t = 1$ gives the set of equilibrium conditions listed in definition 1.

Proof of proposition 2. Consider the set of equilibrium conditions given in definition 1. Use (29) or $\kappa_t^B b_{t-1}/\pi_t = (R_t^m - \kappa_t \mu)\, y_t$ as well as its time $t+1$ version and $b_t = \Gamma b_{t-1} \pi_t^{-1}$, to eliminate y_t and y_{t+1} in (28):

$$\frac{\kappa_t^B}{R_t^m - \kappa_t \mu} \beta E_t \frac{1}{\Gamma} \frac{R_{t+1}^m - \kappa_{t+1}\mu}{\kappa_{t+1}^B} (1-\kappa_t) + \frac{\kappa_t}{R_t^m} = \frac{1}{R_t^L}.$$

where $\mu = \frac{\varepsilon-1}{\varepsilon}\alpha \in (0,1)$. We further divide both sides of $\kappa_t^B b_{t-1}/\pi_t = (R_t^m - \mu\kappa_t)\, y_t$ by its period $t-1$ version $\kappa_{t-1}^B b_{t-2}/\pi_{t-1} = \left(R_{t-1}^m - \mu\kappa_{t-1}\right) y_{t-1}$, and use $b_{t-1} = \Gamma b_{t-2} \pi_{t-1}^{-1}$, to express inflation as

$$\pi_t = \Gamma \frac{\kappa_t^B}{\kappa_{t-1}^B} \frac{R_{t-1}^m - \mu\kappa_{t-1}}{R_t^m - \mu\kappa_t} \frac{y_{t-1}}{y_t}. \tag{67}$$

We further replace output (=consumption) in (26) with (29) and real bonds with (30), to get the following set of equilibrium conditions

$$\frac{1}{R_t^L} = \frac{\kappa_t}{R_t^m} + (1-\kappa_t)\frac{\beta}{\Gamma} E_t \left[\frac{R_{t+1}^m - \kappa_{t+1}\mu}{R_t^m - \kappa_t \mu} \frac{\kappa_t^B}{\kappa_{t+1}^B} \right], \tag{68}$$

$$y_t = (\mu/\theta)^{\alpha/(1+\sigma_n)} \left(1/R_t^L\right)^{\alpha/(1+\sigma_n)}, \tag{69}$$

$$\pi_t = \Gamma \frac{\kappa_t^B}{\kappa_{t-1}^B} \frac{R_{t-1}^m - \mu\kappa_{t-1}}{R_t^m - \mu\kappa_t} \left(\frac{R_t^L}{R_{t-1}^L} \right)^{\alpha/(1+\sigma_n)}, \tag{70}$$

$$\frac{\eta_t}{y_t^{-1}} = \frac{1}{R_t^m} - \frac{\beta}{\Gamma} E_t \frac{\kappa_t^B}{\kappa_{t+1}^B} \frac{R_{t+1}^m - \mu\kappa_{t+1}}{R_t^m - \mu\kappa_t} > 0, \tag{71}$$

which can be solved sequentially. For a given solution for output and inflation, we can further solve for real bonds and for the initial price level P_0 using (29) and (30). We now consider the case where the central bank instruments are initially set at $\kappa \geq 0$, $\kappa_t^B = \gamma \kappa_{t-1}^B$, where $\kappa_0^B > 0$ and $\gamma > \beta/\Gamma$, and $R^m \in [1, \gamma\Gamma/\beta)$. Then, $\frac{\eta_t}{y_t^{-1}} = \frac{1}{R^m} - \frac{\beta}{\Gamma}\gamma > 0$, such that the money market constraint is binding.

1. Consider a marginal increase in the policy rate in period t from R^m to $R_{t+i}^m = \overline{R}^m$ $\forall i \geq 0$ where $\overline{R}^m \in [R^m, \gamma\Gamma/\beta)$, while $\kappa_t = \kappa > 0$ and $\kappa_t^B = \gamma \kappa_{t-1}^B$. Condition (68) then reads $\frac{1}{R_t^L} = \frac{\kappa}{R_t^m} + (1-\kappa)\frac{\beta}{\Gamma}\frac{1}{\gamma}$, such that

$$\frac{\partial(1/R_t^L)}{\partial R_t^m} = -\frac{1}{(R_t^m)^2}\kappa < 0 \text{ and } \frac{\partial y_t}{\partial R_t^m} = -\frac{\alpha}{1+\sigma_n} y_t \frac{R_t^L}{(R_t^m)^2}\kappa < 0,$$

where we used $\partial y_t / \partial\left(1/R_t^L\right) = [\alpha/\left(1+\sigma_n\right)] y_t R_t^L [\partial(1/R_t^L)/\partial R_t^m]$ (see 69). Inflation satisfies $\pi_t = \Gamma\gamma \frac{R^m - \mu\kappa}{R_t^m - \mu\kappa} \frac{\left(1/R^L\right)^{\alpha/(1+\sigma_n)}}{\left(1/R_t^L\right)^{\alpha/(1+\sigma_n)}}$ (see 70), such that

$$\frac{\partial \pi_t}{\partial R_t^m} = -\Gamma\gamma \frac{\left(1/R^L\right)^{\alpha/(1+\sigma_n)}}{\left(1/R_t^L\right)^{\alpha/(1+\sigma_n)}} \frac{R^m - \mu\kappa}{R_t^m - \mu\kappa} \left(\frac{\kappa}{R_t^m} + (1-\kappa)\frac{\beta}{\Gamma}\frac{1}{\gamma} \right)^{-1}$$
$$\cdot \frac{1}{(R_t^m)^2} \left(\left[\frac{R_t^m}{R_t^m - \mu\kappa} - \frac{\alpha}{1+\sigma_n} \right] \kappa + \frac{(R_t^m)^2}{R_t^m - \mu\kappa}(1-\kappa)\frac{\beta}{\Gamma}\frac{1}{\gamma} \right) < 0$$

where we used that the term in the square bracket is non-negative. There are two effects: First, a higher policy rate lowers the amount of cash for each unit of collateral. Second, it raises the loan rate for $\kappa > 0$, such that output contracts. The second effect on inflation is smaller such that a higher policy rate reduces inflation. Since $\frac{\eta_t}{c_t^{-1}} = \frac{1}{R_t^m} - \frac{\beta}{\Gamma}\gamma$ further holds, the money market constraint is binding, $\eta_t > 0$, given that $\overline{R}^m < \gamma\Gamma/\beta$.

2. Consider a marginal decrease in the growth rate of κ^B from γ to $\gamma_{t+i} = \overline{\gamma} < \gamma \; \forall i \geq 0$, while $R_t^m = R^m$ and $\kappa_t = \kappa$. Then, (68) and (70) reduce to $\frac{1}{R_t^L} = \frac{\kappa}{R^m} + (1-\kappa)\frac{\beta}{\Gamma}\frac{1}{\gamma_t}$ and $\pi_t = \Gamma\gamma_t \frac{y_{t-1}}{y_t}$, such that

$$\frac{\partial(1/R_t^L)}{\partial\gamma_t} = -(1-\kappa)\frac{\beta}{\Gamma}\frac{1}{\gamma_t^2} < 0,$$

$$\frac{\partial y_t}{\partial\gamma_t} = \frac{\partial y_t}{\partial(1/R_t^L)}\frac{\partial(1/R_t^L)}{\partial\gamma_t} = -\frac{\alpha}{1+\sigma_n} y_t R_t^L (1-\kappa)\frac{\beta}{\Gamma}\frac{1}{\gamma_t^2} < 0 \quad \text{and} \quad \frac{\partial\pi_t}{\partial\gamma_t} > 0.$$

where we used $\frac{\partial\pi_t}{\partial y_t} < 0$ for the last inequality. Hence, a lower γ_t reduces the loan rate R_t^L and increases output if $\kappa < 1$, reduces inflation. Since $\frac{\eta_t}{c_t^{-1}} = \frac{1}{R_t^m} - \frac{\beta}{\Gamma\gamma_t}$, further holds, the money market constraint remains binding, $\eta_t > 0$, as long as $\gamma_t > \frac{R^m\beta}{\Gamma}$.

3. Consider a marginal increase in the fraction of eligible loans κ from κ to $\kappa_{t+i} = \overline{\kappa} > \kappa$ $\forall i \geq 0$, while $R_t^m = R^m$ and $\gamma_t = \gamma$. Then, (68) reduces to $\frac{1}{R_t^L} = \frac{\kappa_t}{R^m} + (1-\kappa_t)\frac{\beta}{\Gamma\gamma}$, such that

$$\frac{\partial(1/R_t^L)}{\partial\kappa_t} = \frac{1}{R^m} - \frac{\beta}{\Gamma\gamma} > 0,$$

$$\frac{\partial y_t}{\partial\kappa_t} = y_t[\alpha/(1+\sigma_n)]\left(1/R_t^L\right)^{-1}\left(\frac{1}{R^m} - \frac{\beta}{\Gamma\gamma}\right) > 0.$$

Hence, a higher κ_t unambiguously reduces the loan rate and raises output. The impact on inflation (see 67), is $\frac{\partial\pi_t}{\partial\kappa_t} = \Gamma\gamma\frac{y_{t-1}}{y_t}\frac{R^m - \kappa_{t-1}\mu}{R^m - \kappa_t\mu}\left(\frac{\mu}{R^m - \kappa_t\mu} - \frac{\alpha}{1+\sigma_n}R_t^L\left(\frac{1}{R^m} - \frac{\beta}{\Gamma\gamma}\right)\right)$ and therefore ambiguous due to changes in κ_t and y_t. Since $\frac{\eta_t}{c_t^{-1}} = \frac{1}{R^m} - \frac{\beta}{\Gamma\gamma} > 0$, the money market constraint remains binding.

■

Proof of proposition 3. Condition (68) shows that the growth rate γ_t can change the level but not the growth rate of the loan rate. Condition (70) therefore implies that a permament increase in κ_t^B, but not in its growth rate, leads to a temporary change in inflation. Further, inflation in a long-run equilibrium, where output is constant, only depends on the long-run growth rate γ and $\Gamma : \pi = \Gamma\gamma$. Hence, the central bank can control the inflation rate by setting γ contingent on Γ. Specifically, for $\gamma = \Gamma^{-1}$, prices are stable $\pi = 1$ in the long-run.
■

A.3 Appendix to the sticky price model

In this appendix we first examine the deterministic steady state of the model (steady state variables will not be indexed with a time index) and then present the linear approximation of the model at this steady state.

Steady state The central bank determines $\kappa \in [0,1]$ and target values for the inflation rate $\pi \geq \beta$ and the policy rate $R^m \geq 1$. In a steady state, all endogenous variables grow with a constant rate. Thus, to be consistent with a long-run equilibrium, the time-invariant policy targets have to be consistent with the steady state. In what follows we examine properties of all other endogenous variables in a steady state.

Given the steady state inflation rate π, the equilibrium condition (49) implies $\tilde{Z} = \left(\left(1-\phi\pi^{\varepsilon-1}\right)/\left(1-\phi\right)\right)^{1/(1-\varepsilon)}$, and (48) and Z^1/Z^2 is also constant. The price dispersion term s_t satisfying (54), thus converges in the long run to $s = \frac{1-\phi}{1-\phi\pi^{\varepsilon}}\tilde{Z}^{-\varepsilon}$, given that $\phi\pi^{\varepsilon} < 1 \Leftrightarrow \pi < (1/\phi)^{1/\varepsilon}$. Since s is bounded from below and neither productivity nor labor supply exhibit trend growth, real resources and therefore working time, output, capital and consumption cannot permanently grow with a non-zero rate in the steady state, $y = c - \delta k = k^{1-\alpha}n^{\alpha}/s$. Then, Z_t^2 converges to $Z^2 = yc^{-\sigma}/\left(1-\phi\beta\pi^{\varepsilon-1}\right)$ if $\phi\beta\pi^{\varepsilon-1} < 1 \Leftrightarrow \pi < \left[1/\left(\phi\beta\right)\right]^{1/(\varepsilon-1)}$. Given that Z^1/Z^2 and $\tilde{Z}$ are constant, and $Z_t^1 = Z^1 = \frac{yc^{-\sigma}mc}{1-\phi\beta\pi^{\varepsilon}}$, since $Z_t^1/Z_t^2 = Z^{1,2}$, such that real marginal costs are also constant and given by $mc = \tilde{Z}\left(\varepsilon-1\right)\varepsilon^{-1}\left(1-\phi\beta\pi^{\varepsilon}\right)/\left(1-\phi\beta\pi^{\varepsilon-1}\right)$. Since steady state consumption is constant, (40) and (42) determine the long-run debt rate in the usual way, $R^D = \pi/\beta$. Condition (39) and (40) further imply the steady state loan rate to satisfy

$$\frac{1}{R^L} = \kappa\frac{1}{R^m} + (1-\kappa)\frac{1}{\pi/\beta}. \tag{72}$$

Given that the loan rate, marginal cost, and working time are constant, (46) implies a constant steady state wage rate, $w = mc\alpha n^{\alpha-1}/R^L$. Moreover, the steady state is characterized by $\theta n^{\sigma_n} = wc^{-\sigma}$, $c = n^{\alpha}$, and $l = R^L wn = R^L\theta c^{\sigma+(1+\sigma_n)/\alpha} = \frac{\varepsilon-1}{\varepsilon}\alpha c$.

The linearized model We consider the simplified case, where $\Omega \to \infty$ and $\Gamma = \pi = 1$. Log-linearizing the set of conditions (see appendix A.1) at the steady state gives

$$\alpha^{-1}\widehat{y}_t = \widehat{n}_t, \tag{73}$$

$$\left(\sigma + \sigma_n/\alpha\right)\widehat{c}_t = \widehat{w}_t, \tag{74}$$

$$\widehat{mc}_t - \frac{1-\alpha}{\alpha}\widehat{c}_t = \widehat{w}_t + \widehat{R}_t^L, \tag{75}$$

$$\log\widehat{\pi}_t = \beta E_t\log\widehat{\pi}_{t+1} + \chi\log\widehat{mc}_t, \tag{76}$$

$$\widehat{b}_t = \widehat{b}_{t-1} - \widehat{\pi}_t, \tag{77}$$

$$\widehat{c}_t = \widehat{y}_t, \tag{78}$$

where $\chi = (1-\phi)(1-\beta\phi)/\phi$. Further, log-linearizing (59) leads to

$$\sigma E_t\widehat{c}_{t+1} + E_t\widehat{\pi}_{t+1} + \frac{\kappa}{1-\kappa}\left(1-\frac{\pi/\beta}{R^m}\right)\widehat{\kappa}_t + \frac{\kappa}{1-\kappa}\frac{\pi/\beta}{R^m}\widehat{R}_t^m$$
$$= \sigma\widehat{c}_t + \left(E_t\widehat{\xi}_{t+1} - \widehat{\xi}_t\right) + \left(1 + \frac{\kappa}{1-\kappa}\frac{\pi/\beta}{R^m}\right)\widehat{R}_t^L,$$

where we used (72). Defining $\varkappa = \frac{\kappa}{1-\kappa}\frac{\pi/\beta}{R^m}$, the last condition can be simplified to

$$\sigma E_t \widehat{c}_{t+1} - \sigma \widehat{c}_t + \left(1-\rho_\xi\right)\widehat{\xi}_t + E_t \widehat{\pi}_{t+1} - \left(\varkappa - \frac{\kappa}{1-\kappa}\right)\widehat{\kappa}_t + \varkappa \widehat{R}_t^m = \left(1+\varkappa\right)\widehat{R}_t^L, \tag{79}$$

where we assumed that the preference shock is autocorrelated: $E_t\widehat{\xi}_{t+1} = \rho_\xi \widehat{\xi}_t$. Further, linearizing $\kappa_t R_t^L w_t n_t + \kappa_t^B b_{t-1}/\pi_t = R_t^m m_t^R$ yields

$$\left(\varpi - \varsigma\right)\widehat{c}_t + \widehat{\kappa}_t + \widehat{R}_t^L + \left(\varsigma - 1\right)\widehat{\kappa}_t^B + \left(\varsigma - 1\right)\widehat{b}_{t-1} - \left(\varsigma - 1\right)\widehat{\pi}_t = \varsigma \widehat{R_t^m}, \tag{80}$$

where $\varsigma = \frac{R^m}{\kappa\frac{\varepsilon-1}{\varepsilon}\alpha} > 1$ and $\varpi = \frac{1+\sigma_n}{\alpha} + \sigma > 1 + \sigma$. Eliminating $\widehat{c}_t$, $\widehat{w}_t$, $\widehat{n}_t$,and $\widehat{mc}_t$, the system (73)-(80) in $\widehat{R}_t^L$, $\widehat{y}_t$, π_t, and $\widehat{b}_t$ is given by

$$\left(\varsigma - \varpi\right)\widehat{y}_t = \left(\varsigma - 1\right)\widehat{b}_{t-1} - \varsigma\widehat{R_t^m} - \left(\varsigma - 1\right)\widehat{\pi}_t + \left(\varsigma - 1\right)\widehat{\kappa}_t^B + \widehat{\kappa}_t + \widehat{R}_t^L, \tag{81}$$

$$\left(1+\varkappa\right)\widehat{R}_t^L = \sigma E_t \widehat{y}_{t+1} - \sigma\widehat{y}_t + E_t\widehat{\pi}_{t+1} - \left(\varkappa - \frac{\kappa}{1-\kappa}\right)\widehat{\kappa}_t + \varkappa\widehat{R}_t^m + \left(1-\rho_\xi\right)\widehat{\xi}_t, \tag{82}$$

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \chi\left(\varpi - 1\right)\widehat{y}_t + \chi\widehat{R}_t^L, \tag{83}$$

$$\widehat{b}_t = \widehat{b}_{t-1} - \widehat{\pi}_t, \tag{84}$$

where $\widehat{\kappa}_t$, $\widehat{\kappa}_t^B$, and $\widehat{R}_t^m$ are set by the central bank. At a steady state where $\kappa = 0$, the conditions (81)-(84) reduce to (31)-(34).

Proof of proposition 4. Consider the equilibrium defined in definition 2. We further assume that the central bank follows the Taylor-rule (35) and set κ_t^B in an exogenous way. Eliminating the policy rate and the loan rate with (31) gives

$$\left(1+\rho_y\right)\widehat{y}_t = \widehat{b}_t - \rho_\pi\widehat{\pi}_t + \widehat{\kappa}_t^B, \tag{85}$$

$$\widehat{\pi}_t = \left(\beta + \chi\right)E_t\widehat{\pi}_{t+1} + \chi\left(\varpi - 1 - \sigma\right)\widehat{y}_t + \chi\sigma E_t\widehat{y}_{t+1} + \chi\left(1-\rho_\xi\right)\widehat{\xi}_t, \tag{86}$$

$$\widehat{b}_t = \widehat{b}_{t-1} - \widehat{\pi}_t. \tag{87}$$

Further substituting out output in (86) with (85) and neglecting terms in $\widehat{\kappa}_t^B$ for simplicity leads to

$$\zeta_2 E_t\widehat{\pi}_{t+1} + \zeta_3\widehat{b}_t = \zeta_1\widehat{\pi}_t - \chi\left(1-\rho_\xi\right)\widehat{\xi}_t, \tag{88}$$

where $\zeta_1 = 1 + \frac{\chi(\varpi-1-\sigma)}{(1+\rho_y)}\rho_\pi > 0$, $\zeta_2 = \beta + \chi - \chi\sigma\frac{1+\rho_\pi}{1+\rho_y}$, and $\zeta_3 = \frac{\chi(\varpi-1-\sigma)}{(1+\rho_y)} + \frac{\chi\sigma}{1+\rho_y} > 0$. Hence, the conditions (87) and (88) can be written as

$$\begin{pmatrix} E_t\widehat{\pi}_{t+1} \\ \widehat{b}_t \end{pmatrix} = A\begin{pmatrix} \widehat{\pi}_t \\ \widehat{b}_{t-1} \end{pmatrix} + \begin{pmatrix} \zeta_2 & \zeta_3 \\ 0 & 1 \end{pmatrix}^{-1}\begin{pmatrix} -\chi\left(1-\rho_\xi\right) \\ 0 \end{pmatrix}\widehat{\xi}_t,$$

$$\text{where } A = \begin{pmatrix} \zeta_2 & \zeta_3 \\ 0 & 1 \end{pmatrix}^{-1}\begin{pmatrix} \zeta_1 & 0 \\ -1 & 1 \end{pmatrix}.$$

Given that there exists exactly one predetermined variable, $\widehat{b}_{t-1}$, local determinacy requires one stable and one unstable eigenvalue. The characteristic polynomial of A is given by $F(X) = X^2 + \left(-\frac{\zeta_1}{\zeta_2} - \frac{1}{\zeta_2}\zeta_3 - 1\right) X + \frac{\zeta_1}{\zeta_2}$, where $F(0) = \zeta_1/\zeta_2$ and $F(1) = -\zeta_3/\zeta_2$ and $signF(0) = -signF(1)$. Hence, there exists at least one real stable eigenvalue between zero and one. Further, $F(X)$ at $X = -1$ is given by

$$F(-1) = \left[-\chi\frac{1+2\rho_\pi}{1+\rho_y}\left(\sigma - \frac{1-\alpha+\sigma_n}{\alpha}\right) + 2\left(1+\beta+\chi\right)\right]/\zeta_2,$$

where the term in the square bracket is strictly positive, such that $signF(0) = signF(-1)$, if and only if

$$\frac{\rho_\pi + 1/2}{\rho_y + 1}\left(\sigma - \frac{1-\alpha+\sigma_n}{\alpha}\right) < 1 + \frac{1+\beta}{\chi}.$$

Then, there exists exactly one stable and one unstable eigenvalue, indicating local determinacy. ■

Proof of proposition 5. Consider the equilibrium defined in definition 2 and suppose that the central bank pegs the policy rate $\widehat{R}_t^m = 0$ (e.g. at the ZLB) and sets κ_t^B according to an AR1 process, $\widehat{\kappa}_t^B = \rho_\kappa \widehat{\kappa}_{t-1}^B + \varepsilon_t$. Disregarding preference shocks and eliminating the policy rate and the loan rate with (31) gives

$$\widehat{y}_t = \widehat{b}_t + \widehat{\kappa}_t^B, \tag{89}$$

$$\widehat{\pi}_t = \left(\beta+\chi\right) E_t\widehat{\pi}_{t+1} + \chi\widetilde{\varpi}\widehat{y}_t + \chi\sigma E_t\widehat{y}_{t+1}, \tag{90}$$

where $\widetilde{\varpi} = \varpi - 1 - \sigma > 0$, and (84). Now eliminate $\widehat{y}_t$ in (90) with (89), use $E_t\widehat{\kappa}_{t+1}^B = \rho_\kappa\widehat{\kappa}_t^B$ and that (84) implies $E_t\widehat{b}_{t+1} = \widehat{b}_t - \widehat{\pi}_t$, to get

$$\left(1+\chi\sigma\right)\widehat{\pi}_t = \left(\beta+\chi\right) E_t\widehat{\pi}_{t+1} + \chi\left(\widetilde{\varpi}+\sigma\right)\widehat{b}_t + \chi\left(\widetilde{\varpi}+\sigma\rho_\kappa\right)\widehat{\kappa}_t^B, \tag{91}$$

Condition (91) together with (84) determine the solution for $\widehat{\pi}_t$ and $\widehat{b}_t$. Given that real bonds $\widehat{b}_{t-1}$ are predetermined, the solution takes the form $\widehat{b}_t = u_1\widehat{b}_{t-1} + u_2\widehat{\kappa}_t^B$ and $\widehat{\pi}_t = u_3\widehat{b}_{t-1} + u_4\widehat{\kappa}_t^B$, where the coefficients $u_{1,2,3,4}$ are undetermined. It should be noted that the eigenvalue u_1 has already been show in the proof of proposition 4 to lie between zero and one, $u_1 \in (0,1)$. Plugging the solution forms into (84) and (91) and using that $\widehat{b}_{t-1}$ and $\widehat{\kappa}_t^B$ are uncorrelated, leads to the following conditions

$$u_3 = 1 - u_1 \in (0,1), \quad u_4 = -u_2, \quad 0 = \left[\left(\beta+\chi\right)u_3 + \chi\left(\widetilde{\varpi}+\sigma\right)\right]u_1 - \left(1+\chi\sigma\right)u_3,$$
$$0 = \left(\beta+\chi\right)u_3u_2 + \chi\left(\widetilde{\varpi}+\sigma\right)u_2 + \left(\beta+\chi\right)u_4\rho_\kappa - \left(1+\chi\sigma\right)u_4 + \chi\left(\widetilde{\varpi}+\sigma\rho_\kappa\right),$$

Eliminating u_3 and u_2 in the last condition and isolating u_4, leads to the following expression for $\partial\widehat{\pi}_t/\partial\widehat{\kappa}_t^B = u_4$:

$$\frac{\partial\widehat{\pi}_t}{\partial\widehat{\kappa}_t^B} = \frac{\chi\left(\widetilde{\varpi}+\sigma\rho_\kappa\right)}{\left(\beta+\chi\right)\left(1-u_1\right)+\chi\left(\widetilde{\varpi}+\sigma\right)+\left(1-\beta\rho_\kappa\right)+\chi\left(\sigma-\rho_\kappa\right)} > 0,$$

which is positive given that $\sigma \geq 1$. Hence, inflation increases in response to quantitative easing. Further, we use that output satisfies $\widehat{y}_t = \widehat{b}_t + \widehat{\kappa}_t^B$ and thus $\widehat{y}_t = u_1\widehat{b}_{t-1} + u_5\widehat{\kappa}_t^B$, where $u_5 = 1 - u_4$ and $\partial\widehat{y}_t/\partial\widehat{\kappa}_t^B = u_5$ such that

$$\frac{\partial\widehat{y}_t}{\partial\widehat{\kappa}_t^B} = \frac{\left(\beta+\chi\right)\left(1-u_1\right)+\left(1-\beta\rho_\kappa\right)+\chi\left(\sigma-\rho_\kappa\right)+\chi\sigma\left(1-\rho_\kappa\right)}{\left(\beta+\chi\right)\left(1-u_1\right)+\chi\left(\widetilde{\varpi}+\sigma\right)+\left(1-\beta\rho_\kappa\right)+\chi\left(\sigma-\rho_\kappa\right)} > 0$$

Hence, output also increases in response to quantitative easing at the ZLB. ■

A.4 A model version with physical capital

A RE equilibrium is given by a set of sequences $\{c_t,\ y_t,\ k_t,\ x_t,\ n_t,\ \lambda_t,\ \psi_t,\ \eta_t,\ q_t,\ m_t^R,\ m_t^H,\ b_t,\ b_t^T,\ l_t,\ w_t,\ r_t^k,\ mc_t,\ \tilde{Z}_t,\ s_t,\ \pi_t,\ R_t,\ R_t^D,\ R_t^L\}_{t=0}^{\infty}$ satisfying (42), (44)-(45), (47)-(51), as well as

$$c_t^{-\sigma} = \lambda_t + \upsilon\psi_t, \tag{92}$$

$$\theta n_t^{\sigma_n} = \left(\lambda_t + \psi_t\right) w_t, \tag{93}$$

$$\lambda_t + \psi_t = R_t^m\left(\lambda_t + \eta_t\right), \tag{94}$$

$$\lambda_t + \psi_t = R_t^L\left(\lambda_t + \eta_t\kappa_t\right), \tag{95}$$

$$\begin{aligned}\lambda_t + \omega_t\psi_t = {} & c_t^{-\sigma}q_t\left[S\left(x_t/x_{t-1}\right) + \left(x_t/x_{t-1}\right)S'\left(x_t/x_{t-1}\right)\right] \\ & - \beta E_t c_{t+1}^{-\sigma}q_{t+1}\left[\left(x_{t+1}/x_t\right)^2 S'\left(x_{t+1}/x_t\right)\right],\end{aligned} \tag{96}$$

$$c_{i,t}^{-\sigma}q_t = \beta E_t\left[\lambda_{t+1}r_{t+1}^k + \left(1-\delta\right)q_{i,t+1}c_{i,t+1}^{-\sigma}\right], \tag{97}$$

$$\lambda_t = \beta E_t\frac{\lambda_{t+1}+\psi_{t+1}}{\pi_{t+1}}, \tag{98}$$

$$\lambda_t = \beta E_t\frac{\lambda_{t+1}+\eta_{t+1}\kappa_{t+1}^B}{\pi_{t+1}}R_t, \tag{99}$$

$$\begin{aligned}\upsilon c_t + \omega_t x_t - \kappa_t l_t/R_t^m = m_t^H + m_t^R, & \text{ if } \ \psi_t > 0, \\ \text{or } \upsilon c_t + \omega_t x_t - \kappa_t l_t/R_t^m \leq m_t^H + m_t^R, & \text{ if } \ \psi_t = 0,\end{aligned} \tag{100}$$

$$w_t R_t^L = mc_t\alpha\left(n_t/k_t\right)^{\alpha-1}, \tag{101}$$

$$r_t^k = mc_t\left(1-\alpha\right)\left(n_t/k_t\right)^{\alpha}, \tag{102}$$

$$k_{i,t+1} = \left(1-\delta\right)k_{i,t} + x_{i,t}S\left(x_{i,t}/x_{i,t-1}\right), \tag{103}$$

$$y_t = n_t^{\alpha}k_t^{1-\alpha}/s_t, \tag{104}$$

$$y_t\left(1-g/y\right) = c_t + x_t, \tag{105}$$

(where q_t denotes the value of installed capital relative to consumption goods and the adjustment cost function is given by $S\left(x_t/x_{t-1}\right) = 1 - \frac{\vartheta}{2}\left(x_t/x_{t-1}-1\right)^2$) as well as the transversality conditions, a monetary policy setting $\{R_t^m \geq 1,\ \kappa_t^B,\ \kappa_t \in [0,1]\}_{t=0}^{\infty}$, $\Omega_t > 0$, and $\pi \geq \beta$, and a fiscal policy setting $\Gamma \geq 1$ and $g/y > 0$, for a given sequence $\{\omega_t\}_{t=0}^{\infty}$ and initial values $M_{-1}^H > 0$, $B_{-1} > 0$, $B_{-1}^T > 0$, and $s_{-1} \geq 1$.

A.5 Parameter values

Table A1 Benchmark parameter values	
Subjective discount factor	$\beta = 0.992$
Inverse of intertemporal substitution elasticity	$\sigma = 2$
Inverse of Frisch elasticity of labour supply	$\sigma_n = 1$
Substitution elasticity	$\varepsilon = 10$
Steady state working time	$n = 0.33$
Labour share	$\alpha = 0.66$
Investment adjustment cost	$\vartheta = 2.48$
Rate of depreciation of capital stock	$\delta = 0.03$
Government expenditure share (constant)	$g = 0.19$
Calvo price stickiness	$\phi = 0.75$
Steady state interest rate	$R^m = 1.0105$
Taylor rule inflation coefficient	$w_\pi = 1.5$
Taylor rule output coefficient	$w_y = 0.5$
Steady state share of repos to outright purchases	$\Omega = 1.5$
Steady state share of loans eligible in open market operations	$\kappa = 0$
Steady state share of gov. bonds eligible in open market operations	$\kappa^B = 1$
Steady state inflation	$\pi = 1.00575$
Steady state cash requirement for consumption	$\nu = 0.7399$
Steady state cash requirement for investment	$\omega = 0.4292$

When is Quantitative Easing Effective?[1]

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Abstract

We present a simple macroeconomic model with open market operations that allows examining the effects of quantitative and credit easing. The central bank controls the policy rate, i.e. the price of money in open market operations, as well as the amount and the type of assets that are accepted as collateral for money. When the policy rate is sufficiently low, this set-up gives rise to an (il-)liquidity premium on non-eligible assets. Then, a quantitative easing policy, which increases the size of the central bank's balance sheet, can increase real activity and prices, while a credit easing policy, which changes the composition of the balance sheet, can lower interest rate spreads, stimulate real activity, and reduce prices. The effectiveness of quantitative and credit easing is however limited to the extent that eligible assets are scarce. Nevertheless, they can help escaping from the zero lower bound.

JEL classification: E4; E5; E32.
Keywords: Monetary policy, collateralized lending, quantitative easing, credit easing, liquidity premium, zero lower bound

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1 Introduction

Central banks in many industrialized countries have responded to the recent financial crisis with unconventional monetary policy measures. By introducing various newly created lending facilities as well as direct asset purchases, the Federal Reserve for instance doubled its balance sheet in the three months after the climax of the crisis in September 2008. This policy, which has been summarized by the terms "quantitative easing" and "credit easing", has been aimed at ensuring the functioning of the interbank market and at stabilizing stressed credit markets (see Yellen, 2009).[3] However, it has been implemented with only little theoretical or empirical guidance available. The present paper provides an analysis of the effects as well as the limits of quantitative and credit easing in a simple sticky price model. The analysis focuses on monetary policy implementation and the provision of liquidity by the central bank and abstracts from the ability of monetary policy to mitigate financial frictions that were crucial in the financial crisis.[4] We show that quantitative and credit easing can stimulate real activity at the zero lower bound (ZLB) as long as assets eligible for open market operations are scarce, which is reflected by a liquidity premium.

As summarized by Bernanke et al. (2004), quantitative easing involves the purchase of securities, such as government bonds, with central bank reserves. In contrast, qualitative easing refers to changes in the composition of the central bank's balance sheet without creating additional reserves. More recently, Bernanke (2009) introduces the term credit easing which closely relates to qualitative easing: "the Federal Reserve's credit easing approach focuses on the mix of loans and securities that it holds and on how this composition of assets affects credit conditions for households and businesses". According to conventional macroeconomic models, easing money supply at the ZLB should be ineffective since private agents will demand money up to satiation (see Walsh, 2010). Quantitative easing policies should then be irrelevant as long as they do not change expectations about future conduct of monetary and fiscal policy (see Eggertsson and Woodford, 2003), while credit easing has not been considered in standard single interest rate models. Evidence from Federal Reserve policy effects however suggests that quantitative and credit easing have been effective during the recent financial crisis, primarily, via reductions of liquidity premia (see Christensen et al., 2009, Duygan-Bump et al., 2010, Gangon et al., 2010, or Sarkar and Schrader, 2010).

[3] Among the facilities created by the Federal Reserve are for example the Term Auction Facility (TAF), the Commercial Paper Funding Facility (CPFF) and the Treasury Securities Lending Facility (TSLF). TAF gives 28- or 84-day credit to depository institutions, CPFF is a purchase program for 3-month commercial paper, and TSLF provides Treasury securities in exchange for other securities such as mortgage-backed securities and commercial paper. Further facilities include the Primary Dealer Credit Facility, the Term Asset-Backed Securities Loan Facility and the Asset-Backed Commercial Paper Money Market Mutual Fund Liquidity Facility.

[4] Hence, this paper complements the recent literature which analyzes unconventional monetary policy, such as direct lending or asset exchanges, under financial frictions (see Curdia and Woodford, 2010, Del Negro et al., 2010, Gertler and Karadi, 2010, and Gertler and Kiyotaki, 2010).

This paper presents a macroeconomic model which explicitly accounts for the collateral requirements in open market operations. It allows an analysis of quantitative and credit easing policies and their macroeconomic effects. Multiple assets are considered that differ with regard to their ability to serve as collateral for money. The central bank sets the policy rate, i.e. the price of money in terms of eligible assets, and decides on the size and the composition of its balance sheet. Private agents rely on money for goods market purchases, while money is supplied only in exchange for eligible securities, in particular for short-term government bonds. This requirement leads to a spread between the interest rate on non-eligible and eligible assets, i.e. a liquidity premium.[5] It implies that interest rates on non-eligible securities are positive, even if the policy rate is at the ZLB. This accords to the empirical evidence that – as emphasized by Ohanian (2010) – interest rates on non-money market securities tend to be positive even if the policy rate hits the lower bound.

In our model, we consider working capital such that firms demand loans in order to finance production. Due to the associated costs of borrowing, higher loan rates increase the marginal costs of production and thereby exert downward pressure on production. As long as loans are not eligible in open market operations, the loan rate exceeds the interest rate on eligible government bonds by a liquidity premium. By increasing the amount of eligible assets the central bank eases households' access to cash and increases their willingness to spend, which acts like a conventional money injection. Moreover, credit easing can reduce the illiquidity premium on loans and thereby reduce firms' cost of borrowing, which can stimulate the economy. Yet, the effectiveness of both policies is limited. Quantitative and credit easing affect real activity and prices only if eligible assets are scarce, i.e. if the collateral constraint in open market operations is binding. This is the case when eligible assets can be exchanged against money at a price (i.e. policy rate), which is lower than the consumption Euler rate that measures the opportunity costs of money.

Our main results can be summarized as follows. Under flexible prices, monetary policy is non-neutral due to a standard inflation tax and because of its impact on firms' costs of borrowing. A quantitative easing policy in terms of government bonds increases prices and interest rates like an expansionary money supply. The central bank can further increase output and consumption via credit easing, which leads to a lower borrowing rate due to an increased share of eligible loans. For sticky prices, we show that a quantitative easing policy at the ZLB increases output and inflation. We further present numerical results for a calibrated version of the model to explore the limits of quantitative and credit easing at the ZLB. These

[5] In this paper, we disregard default risk and focus on liquidity premia, for which empirical evidence suggest a significant magnitude also in non-crisis times. As summarized by Christensen (2008), the "corporate credit spread puzzle" refers to the empirical observation that the spreads between corporate and government bonds can only partly be attributed to default risk, while the non-default part is typically interpreted as a liquidity premium (see Collin-Dufresne et al. 2001, or Longstaff et al., 2005).

limits are reached when the stimulating policy drives down the Euler rate until it equals the policy rate. We find that a quantitative easing policy (credit easing policy) can substantially reduce interest rate spreads, while it can stimulate output by not more than 1.5% (0.15%) at the ZLB. The inflation responses are much smaller and differ for both policies: Quantitative easing increases inflation, whereas credit easing reduces inflation. Finally, we consider a large liquidity demand shock which drives downs the policy rate to its ZLB and leads to a pronounced output contraction. In this case, a maximum quantitative easing policy can mitigate the output contraction by 50%, which is nevertheless sufficient to escape from the ZLB.[6]

The paper is organized as follows. Section 2 presents the model. In Section 3, we examine the effects of monetary policy in an analytical way. In Section 4, we present quantitative results. Section 5 concludes.

Related literature There is a large literature on monetary policy options at the ZLB. Most of them advocate the possibility of providing monetary stimulus at the ZLB through shaping interest-rate expectations. The basic idea is that a monetary expansion, if perceived as permanent, can stimulate the economy by creating expected inflation and reducing the real rate of interest (see Krugman, 1998). Eggertsson and Woodford (2003) show that a commitment to keep nominal interest rates low in future can provide an effective way of escaping a liquidity trap. Jung et al. (2005) and Eggertsson (2006) derive optimal policy under the non-negativity constraint for the interest rate and obtain the same conclusion. Levin et al. (2009) examine large, persistent shocks and find that a policy relying on shaping interest rate expectations might not be sufficient to stabilize the economy. Auerbach and Obstfeld (2005) analyze open market purchases of government bonds and find that this policy can lift the economy out of the liquidity trap if the monetary base is permanently increased.

According to conventional wisdom, lump-sum injections of money such as helicopter drops are ineffective at the ZLB (see Krugman, 1998, Svensson, 2000, and McCallum, 2006). The reason is that standard macroeconomic models, like the basic New Keynesian model, consider only a single interest rate. Once the policy rate reaches the ZLB, the opportunity costs of holding money fall to zero such that money demand is indetermined or private agents demand money up to satiation. Moreover, open market operations that aim at easing money supply, like a quantitative easing policy, are ineffective at the ZLB as long as they do not change expected future policy paths. Then, neither the size nor the composition of the central bank's balance are relevant as long as financial market are frictionless (see Eggertsson and Woodford, 2003).

[6] In a companion paper, Schabert (2010) applies a closely related model and shows that the additional monetary policy instruments, which are here applied to implement quantitative and credit easing, can help to overcome the well-known monetary policy trade-off between stabilizing prices and closing output-gaps.

Spurred by the recent financial crisis, however, a literature on non-standard monetary policies, like direct lending or asset exchanges by the central bank, under financial frictions is now developing. Gertler and Karadi (2009) analyze direct central bank lending when financial intermediaries need collateral in order to attract deposits. When financial institutions need to deleverage due to a decline in asset prices, central bank interventions such as borrowing directly to firms can be a powerful tool. Applying a purely real framework, which is based on Gertler and Karadi's (2009) model augmented by idiosyncratic investment risks and constraints on the resaleability of assets, Gertler and Kiyotaki (2010) show that direct central bank lending is beneficial in crisis situations when private intermediaries are financially constrained. Del Negro et al. (2010) consider entrepreneurs facing a borrowing and a resaleability constraint (like in Kiyotaki and Moore, 2008) and add these frictions to a medium scale macroeconomic model (see Christiano et al., 2005). They calibrate the model and a negative shock to the resaleability of assets to match the U.S. in late 2008, and show that the Fed's policy interventions prevented a second Great Depression. Curdia and Woodford (2010) apply a model with costly financial intermediation and show that targeted asset purchases (which relate to direct lending) by a central bank can be effective when financial markets are sufficiently disrupted. They further find that quantitative easing is likely to be ineffective. It should be noted that this result is consistent with our conclusion: Given that private agents in their model do not internalize a collateral constraint for money, their case corresponds to a scenario in our model where eligible assets are not scare (i.e. the collateral constraint is not binding).

2 The model

In this section we present a sticky price model where households face a cash-in-advance constraint and firms require working capital, like in Christiano et al. (2005). Money is supplied by the central bank only in exchange for eligible collateral, i.e. government bonds and/or corporate debt.[7] It sets the policy rate and it can further decide on the size and the composition of its balance sheet, which we interpret as quantitative easing and credit easing. Households take these policies into account when they invest in assets, which gives rise to different interest rates due to liquidity premia. Quantitative and credit easing can then lower liquidity premia and stimulate aggregate demand as long as collateral is scare. To present the problems of individual households and firms in a transparent way, we introduce indices even though we do not consider heterogeneity.

[7] Specifically, we augment the model of Reynard and Schabert (2010), which has been applied to explain endogenous liquidity premia, by introducing corporate debt and additional monetary policy instruments.

2.1 Timing of events

Households enter the period with money, government bonds, and household debt, $M^H_{i,t-1} + B_{i,t-1} + D_{i,t-1}$. Households further dispose of a time-invariant time endowment. They supply labor to intermediate goods producing firms, which do not hold any financial wealth. At the beginning of the period aggregate shocks are realized. Then, the central bank sets its instruments, i.e. it announces the fractions of government bonds and corporate loans that are accepted in open market operations, $\kappa^B_t \in (0,1]$ and $\kappa_t \in [0,1]$, and the policy rate $R^m_t \geq 1$. The remainder of the period can be divided into four subperiods.

1. The labor market opens, where a perfectly competitive intermediate goods producing firm j hires workers $n_{j,t}$. We assume that it has to pay workers their wages in cash before goods are sold. Since it does not hold any financial wealth, it has to borrow cash. Firm j thus faces the cash constraint

   $$L_{j,t}/R^L_t \geq P_t w_t n_{j,t}, \tag{1}$$

   where w_t denotes the real wage rate, P_t denotes the final goods price and $L_{j,t}/R^L_t$ the amount received by the borrowing firm. Firm j commits to repay the amount $L_{j,t}$ at the end of the period, such that R^L_t is the interest rate on the intra-period loan. Lenders sign loan contracts with all firms, taking into account that a fraction κ_t of all loans can be used as collateral for repurchase agreements.

2. The money market opens where the central bank sells or purchases assets outright or under repurchase agreements in exchange for money at the rate R^m_t. In contrast to household debt, corporate loans and government bonds can be eligible, where only the latter can be purchased outright by the central bank. In period t, household i receives new money (injections) from the central bank $I_{i,t}$, which consists of money received from the central bank's outright bond purchases, as well as money received in repos for bonds $M^R_{i,t}$ and loans $M^L_{i,t}$. Specifically, the central bank supplies money against a fraction κ^B_t of randomly selected bonds and a fraction κ_t of randomly selected loan contracts, such that $I_{i,t}$ is constrained by the following collateral contraint, or *money market constraint:*

   $$I_{i,t} \leq \kappa^B_t \left(B_{i,t-1}/R^m_t\right) + \kappa_t \left(L_{i,t}/R^m_t\right). \tag{2}$$

   After receiving money injections from the central bank, household i delivers the amount $L_{i,t}/R^L_t$ to firms according to the loan contract. Its holdings of money, bonds, and loans are then $M^H_{i,t-1} + I_{i,t} - (L_{i,t}/R^L_t)$, $B_{i,t-1} - \Delta B^c_{i,t}$, and $L_{i,t} - L^R_{i,t}$, where $\Delta B^c_{i,t}$ are bonds received by the central bank and $L^R_{i,t}$ are loans under repos, such that $I_{i,t} = (\Delta B^c_{i,t}/R^m_t) + (L^R_{i,t}/R^m_t)$.

3. Wages are paid, and intermediate as well as final goods are produced. Then, the goods market opens, where purchases of consumption goods require cash holdings. Hence, household i faces the cash-in-advance constraint, or *goods market constraint*:

$$P_t c_{i,t} \leq I_{i,t} + M^H_{i,t-1} - \left(L_{i,t}/R^L_t\right) + P_t w_t n_{i,t}. \tag{3}$$

Final goods producing firms receive cash for their sales, and pay for intermediate goods. Both further pay out dividends to their owners (households), which sum up to $P_t\delta_{i,t}$ for household i, such that its money holdings are $M^H_{i,t-1} + I_{i,t} - (L_{i,t}/R^L_t) + P_t w_t n_{i,t} - P_t c_{i,t} + P_t \delta_{i,t}$.

4. Repurchase agreements are settled, i.e. household i buys back government bonds $B^R_{i,t}$ and corporate debt $L^R_{i,t}$ from the central bank with money. Household i's bond and money holdings are therefore given by $\widetilde{B}_{i,t} = B_{i,t-1} - \Delta B^c_{i,t} + B^R_{i,t}$ and $\widetilde{M}_{i,t} = I_{i,t} + M^H_{i,t-1} - \left(L_{i,t}/R^L_t\right) + P_t w_t n_{i,t} - P_t c_{i,t} + P_t\delta_{i,t} - B^R_{i,t} - L^R_{i,t}$. In the asset market, loans are repaid and households receive payoffs from maturing assets as well as government transfers $P_t\tau_{i,t}$. Further, the government issues new bonds at the price $1/R_t$. Household i can thus carry wealth into period $t+1$ in form of bonds, state-contingent claims, or money, such that its asset market constraint is

$$(B_{i,t}/R_t) + E_t[\varphi_{t,t+1} D_{i,t}] + M^H_{i,t} \leq \widetilde{B}_{i,t} + D_{i,t-1} + \widetilde{M}_{i,t} + L_{i,t} + P_t\tau_{i,t}, \tag{4}$$

where $\varphi_{t,t+1}$ denotes a stochastic discount factor (see section 2.3). The central bank reinvests its payoffs from maturing bonds into new government bonds and leaves money supply unchanged, $\int \widetilde{M}_{i,t} di = \int M^H_{i,t} di$.

2.2 Firms

There are intermediate goods producing firms which are perfectly competitive and sell their goods $y_{j,t}$ to monopolistically competitive retailers. These sell a differentiated good to bundlers who assemble final goods using a Dixit-Stiglitz technology.

There is a continuum of intermediate goods producing firms indexed with $j \in [0,1]$. They are perfectly competitive and owned by the households. In each period a firm j distributes its profits to the owners and rents the production factors, specifically, it hires labor $n_{j,t}$. We assume that wages have to be paid in advance, i.e. before the firm's goods are sold. Firm j therefore borrows cash $L_{j,t}$ from households at the price $1/R^L_t$ and repays the loan at the end of the period. Hence, firm j faces the working capital constraint (1). It then produces an intermediate good according to the production function $IO_{j,t} = n^\alpha_{j,t}$ and sells it to retailers who pay the price Z_t in cash (after these have received the households' money for goods).

With these revenues, it repays intra-period loans. The problem of firm j then reads

$$\max\left(Z_t/P_t\right)n_{j,t}^{\alpha}-w_tn_{j,t}-l_{j,t}\left(R_t^L-1\right)/R_t^L,\ \ s.t.\ (1),$$

where $l_{j,t}=L_{j,t}/P_t$. The first order conditions to this problem are given by

$$\left(Z_t/P_t\right)\alpha n_{j,t}^{1-\alpha}=w_t+\mu_{j,t}w_t,$$
$$R_t^L-1=\mu_{j,t},$$

and $\mu_{j,t}[(l_{j,t}/R_t^L)-w_tn_{j,t}]=0$, where $\mu_{j,t}\geq 0$ is the multiplier on (1). Hence, intermediate goods producing firms do not borrow more then required to pay wages $w_tn_{j,t}$ if $R_t^L>1\Rightarrow\mu_{j,t}>0$. This condition will be satisfied throughout the analysis. The following conditions determine intermediate firms' labour demand as well as the volume of debt they issue for $R_t^L>1$:

$$\left(Z_t/P_t\right)\alpha n_{j,t}^{1-\alpha}=w_tR_t^L, \tag{5}$$
$$l_{j,t}/R_t^L=w_tn_{j,t}. \tag{6}$$

Monopolistically competitive retailers buy intermediate goods $IO_t=\int_0^1 IO_{j,t}dj$ at the common price Z_t. A retailer $k\in[0,1]$ relabels the intermediate good to $y_{k,t}$ and sells it at the price $P_{k,t}$ to perfectly competitive bundlers, who bundle the goods $y_{k,t}$ to the final consumption good y_t with the technology, $y_t^{\frac{\varepsilon-1}{\varepsilon}}=\int_0^1 y_{k,t}^{\frac{\varepsilon-1}{\varepsilon}}dk$. The cost minimizing demand for $y_{k,t}$ is then given by $y_{k,t}=\left(P_{k,t}/P_t\right)^{-\varepsilon}y_t$.

Retailers set their prices to maximize profits. Following Calvo (1983), we assume that each period a measure $1-\phi$ of randomly selected retailers may reset their prices independently of the time elapsed since the last price setting, while a fraction $\phi\in[0,1)$ of retailers do not adjust their prices. Maximizing discounted future profits, a fraction of $1-\phi$ retailers set their price to maximize the expected sum of discounted future. For $\phi>0$, the first order condition for their price $\widetilde{P}_t$ is given by (where we use that Z_t/P_t are real marginal cost, mc_t):

$$\widetilde{P}_t=\frac{\varepsilon}{\varepsilon-1}\frac{\sum_{s=0}^{\infty}\left(\phi\beta\right)^s c_{t+s}^{-\sigma}y_{t+s}P_{t+s}^{\varepsilon}mc_{t+s}}{\sum_{s=0}^{\infty}\left(\phi\beta\right)^s c_{t+s}^{-\sigma}y_{t+s}P_{t+s}^{\varepsilon-1}}.$$

Defining $\tilde{Z}_t=\widetilde{P}_t/P_t$ and writing both the denominator and the numerator in a recursive way, this can be expressed as $\tilde{Z}_t=\frac{\varepsilon}{\varepsilon-1}Z_t^1/Z_t^2$, where $Z_t^1=c_t^{-\sigma}y_tmc_t+\phi\beta E_t\pi_{t+1}^{\varepsilon}Z_{t+1}^1$ and $Z_t^2=c_t^{-\sigma}y_t+\phi\beta E_t\pi_{t+1}^{\varepsilon-1}Z_{t+1}^2$.

With perfectly competitive bundlers and the homogenous bundling technology, the price index P_t for the final consumption good satisfies $P_t^{1-\varepsilon}=\int_0^1 P_{k,t}^{1-\varepsilon}dk$. Using the demand constraint, we obtain a law of motion for inflation depending on the firms' pricing decision $\tilde{Z}_t$, $1=\left(1-\phi\right)\tilde{Z}_t^{1-\varepsilon}+\phi\pi_t^{\varepsilon-1}$. Aggregate intermediate output satisfies $IO_t=n_t^{\alpha}$ where

$\alpha \in (0, 1]$ as every intermediate firm hires an identical amount of labour. However, there is a production inefficiency due to price dispersion across retailers. The market clearing condition in the intermediate goods market, $IO_t = \int_0^1 y_{k,t}dk$, gives $n_t^\alpha = \int_0^1 (P_{k,t}/P_t)^{-\varepsilon} y_t dk \Leftrightarrow$

$$y_t = n_t^\alpha / s_t,$$

where $s_t \equiv \int_0^1 (P_{k,t}/P_t)^{-\varepsilon} dk$ and $s_t = (1-\phi)\tilde{Z}_t^{-\varepsilon} + \phi s_{t-1}\pi_t^\varepsilon$ (see Schmitt-Grohé and Uribe, 2004) given s_{-1}.

2.3 Households

There is a continuum of infinitely lived households indexed with $j \in [0, 1]$. Households have identical asset endowments and identical preferences. Household j maximizes the expected sum of a discounted stream of instantaneous utilities

$$E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left[\left(c_{i,t}^{1-\sigma} - 1 \right) (1-\sigma)^{-1} - \theta n_{i,t}^{1+\sigma_n} (1+\sigma_n)^{-1} \right], \tag{7}$$

where $\theta > 0$, $\sigma \geq 1$, $\sigma_n \geq 0$, and E_0 is the expectation operator conditional on the time 0 information set, and $\beta \in (0, 1)$ is the subjective discount factor. The term ξ_t is a stochastic preference parameter, which has been introduced in several studies on the ZLB. This shock is not relevant for the main results in this paper and will only be considered in section 3.2. A household i is initially endowed with money $M_{i,-1}^H$, government bonds $B_{i,-1}$, and privately issued debt $D_{i,-1}$. In each period it supplies labor, lends out funds to intermediate goods producing firms, trades assets with the central bank in open market operations, and can reinvest.

Before household i enters the goods market where it needs money as the only accepted means of payment, it can get additional money in open market operations in exchange for government bonds. It can further lend cash to firms at the price $1/R_t^L$, which can eventually be refinanced at the central bank. When households lend to firms, they treat all firms in an identical way, since the decision which particular loan contract is eligible is made after loan contracts are signed. The household faces the money market constraint (2), while we will restrict our attention to the case where money is not withdrawn from the private sector $I_{i,t} \geq 0$ by considering a sufficiently large fraction of repos.

In the goods market, household i can use wages, money holdings, and additional cash net of lending from current period open market operations for its consumption expenditures (see 3). Before the asset market opens, it receives repayments from intra-period loans. In the asset market, it further receives pay-offs from maturing assets, it can buy bonds from the government, it can trade all assets with other households, and it can borrow and lend using a full set of nominally state contingent claims. Dividing the period t price of one unit of nominal wealth in a particular state of period $t+1$ by the period t probability of that state gives the stochastic discount factor $\varphi_{t,t+1}$. The period t price of a payoff $D_{i,t}$ in period $t+1$ is then given by $E_t[\varphi_{t,t+1}D_{i,t}]$. Substituting out the stock of bonds and money held before the asset market opens, $\widetilde{B}_{i,t}$ and $\widetilde{M}_{i,t}$, in (4), the asset market constraint of household i can be written as

$$\begin{aligned} & M^H_{i,t-1} + B_{i,t-1} + \frac{L_{i,t}}{R^L_t}\left(R^L_t - 1\right) + P_t w_t n_{i,t} + D_{i,t-1} + P_t \delta_{i,t} + P_t \tau_{i,t} \\ & \leq M^H_{i,t} + \frac{B_{i,t}}{R_t} + E_t[\varphi_{t,t+1} D_{i,t}] + I_{i,t}\left(R^m_t - 1\right) + P_t c_{i,t}, \end{aligned} \tag{8}$$

where household $i's$ borrowing is restricted by $M^H_{i,t} \geq 0$, $B_{i,t} \geq 0$, and the no-Ponzi game condition $\lim_{s\to\infty} E_t \varphi_{t,t+s} D_{i,t+s} \geq 0$. The term $(R^m_t - 1)\, I_{i,t}$ in (8) measures the costs of money acquired in open market operations, i.e. household i receives new cash $I_{i,t}$ in exchange for $R^m_t I_{i,t}$ bonds. Maximizing the objective (7) subject to the money market constraint (2), the goods market constraint (3), the asset market constraints (8) and the borrowing constraints, for given initial values $M_{i,-1}$, $B_{i},_{-1}$, and $D_{i,-1}$ leads to the following first order conditions for consumption, working time, additional money, and loans

$$\xi_t c^{-\sigma}_{i,t} = \lambda_{i,t} + \psi_{i,t}, \tag{9}$$

$$\theta \xi_t n^{\sigma_n}_{i,t} = w_t\left(\lambda_{i,t} + \psi_{i,t}\right), \tag{10}$$

$$\psi_{i,t} = \left(R^m_t - 1\right)\lambda_{i,t} + R^m_t \eta_{i,t}, \tag{11}$$

$$R^m_t\left(\lambda_{i,t} + \eta_{i,t}\right) = R^L_t\left(\lambda_{i,t} + \eta_{i,t}\kappa_t\right), \tag{12}$$

as well as for investments in contingent claims, government bonds and money,

$$\lambda_{i,t} = \beta R_t E_t \frac{\lambda_{i,t+1} + \kappa^B_{t+1}\eta_{i,t+1}}{\pi_{t+1}}, \tag{13}$$

$$\lambda_{i,t} = \beta E_t \frac{\lambda_{i,t+1} + \psi_{i,t+1}}{\pi_{t+1}}, \tag{14}$$

$$\varphi_{t,t+1} = \frac{\beta}{\pi_{t+1}}\frac{\lambda_{i,t+1}}{\lambda_{i,t}}, \tag{15}$$

where $\lambda_{i,t} \geq 0$ denotes the multiplier on the asset market constraints (8), $\eta_{i,t} \geq 0$ the multiplier on the money market constraints (2), and $\psi_{i,t} \geq 0$ the multiplier on the the goods market constraint (3). Further, (2), (3),

$$\psi_{i,t}[I_{i,t} + M^H_{i,t-1} - \left(L_{i,t}/R^L_t\right) + P_t w_t n_{i,t} - P_t c_{i,t}] \geq 0, \tag{16}$$

$$\eta_{i,t}[\kappa^B_t\left(B_{i,t-1}/R^m_t\right) + \kappa_t\left(L_{i,t}/R^m_t\right) - I_{i,t}] \geq 0, \tag{17}$$

and (8) with equality hold as well as the transversality conditions. The debt rate R^D_t, which slightly differs in the short-run from a standard consumption Euler rate due to the multiplier on the cash-in-advance constraint $\psi_{i,t}$ (see 9), is defined as follows

$$E_t\varphi_{t,t+1} = 1/R_t^D. \tag{18}$$

Condition (12) shows that when the money market constraint is binding, $\eta_{i,t} > 0$, the loan rate depends on the fraction of firm loans eligible as collateral in open market operations, κ_t. As long as loans are not fully eligible $\kappa_t < 0$, there can be a spread between the policy rate and the loan rate, which is a *liquidity premium*. When all intra-period loans are eligible as collateral in open market operations $\kappa_t = 1$, the interest rate on corporate debt compensates exactly for the discount, i.e. $R_t^L = R_t^m$. Combining the optimality conditions (11), (13), and (14) to

$$R_t E_t\left[\left(\lambda_{i,t+1} + \kappa_{t+1}^B\eta_{i,t+1}\right)/\pi_{t+1}\right] = E_t\left[R_{t+1}^m\left(\lambda_{i,t+1} + \eta_{i,t+1}\right)/\pi_{t+1}\right], \tag{19}$$

further shows that households are indifferent between investing in money or investing in government bonds and converting these into cash in the next period at the rate R_{t+1}^m. For $\kappa_t^B = 1$, the interest rate on government bonds is closely linked to next period's expected policy rate, i.e. R_t equals $E_tR_{t+1}^m$ up to first order. If not all bonds are accepted in open market operations, $\kappa_t^B < 1$, bonds are less liquid and get akin to household debt.

2.4 Public sector

The central bank transfers seigniorage revenues $P_t\tau_t^m$ to the Treasury, which issues one-period bonds and pays a transfer $P_t\tau_t$ to households. Government bonds grow at a constant rate, $B_t^T = \Gamma B_{t-1}^T$, where $\Gamma \geq 1$. The Treasury's budget constraint reads

$$B_t^T/R_t + P_t\tau_t^m = B_{t-1}^T + P_t\tau_t, \tag{20}$$

where government bonds B_t^T are either held by households, B_t, or the central bank, B_t^C : $B_t^T = B_t + B_t^C$. This setup does not require B_t^T to measure total public debt, rather it is a measure of short-term government bonds which are eligible for open market operations. To avoid further effects of fiscal policy, we assume that the government has access to lump-sum taxes, which adjust to balance the budget. Thus, introducing long-term government bonds as a means of financing government expenditures would not have any consequences for the analysis conducted in this paper. In fact, we can easily extend the model by considering longer-term bonds, i.e. two-period, without any further impact on the equilibrium allocation and the associated price system as long as they are not eligible. Accepting long-term bonds as additional collateral would then be equivalent to an increase in the fraction of eligible short-term bonds κ_t^B.

The central bank supplies money outright $M_t^H = \int_0^1 M_{i,t}^H di$, and under repos against bonds, $M_t^R = \int_0^1 M_{i,t}^R di$, and loans, $M_t^L = \int_0^1 M_{i,t}^L di$. It transfers its interest earnings on government bonds held to the Treasury at end of period, $P_t\tau_t^m = B_t^C - B_t^C/R_t +$

$(R_t^m - 1)\left(M_t^R + M_t^L\right)$ and reinvests its wealth exclusively into new government bonds, which accords to central bank practice. Its budget constraint reads

$$\left(B_t^C / R_t\right) - B_{t-1}^C + P_t \tau_t^m = R_t^m \left(M_t^H - M_{t-1}^H\right) + (R_t^m - 1)\left(M_t^R + M_t^L\right)$$

Substituting out transfers, the bond holdings of the central bank evolve according to

$$B_t^C - B_{t-1}^C = R_t^m \left(M_t^H - M_{t-1}^H\right). \tag{21}$$

We assume that the central bank sets the policy rate $R_t^m \geq 1$. It further sets the inflation target π and decides on eligible assets for open market operations by setting $\kappa \in [0, 1]$ and $\kappa_t^B \in (0, 1]$. Finally, it controls whether money is supplied in exchange for bonds in repos or outright (while loans are only traded under repos). We assume that it controls the share of bond repos $\Omega \geq 0$, defined as

$$M_t^R = \Omega M_t^H. \tag{22}$$

Beside the policy rate and the repo share, the central bank disposes of the instruments κ_t and κ_t^B. The choice of instruments affects both for the size of the monetary base and the eligibility of assets, which has implications for interest rates and liquidity premia.

- *Quantitative easing* is defined as a policy which increases money supply by additionally accepting collateral for open market operations. Hence, we define quantitative easing in terms of public debt or corporate debt as an increase in κ_t or κ_t^B.

- *Credit easing* is a policy that changes the composition of the central bank's balance sheet without affecting its size. We therefore define credit easing as an increase in κ_t, accompanied by reductions in open market operations in government bonds, $\triangle\kappa_t^B < 0$. The sterilization is conducted such that the nominal monetary base ceteris paribus remains unchanged, i.e. $\triangle\kappa_t^B$ and $\triangle\kappa_t$ satisfy $\triangle\kappa_t^B = \frac{l}{b/\pi}\triangle\kappa_t$.

Among the liquidity facilities created by the Federal Reserve during 2008-09, many had elements of both quantitative and credit easing. In particular, the large scale purchases of Treasury securities and the extension of credit to depository institutions through the Term Auction Facility (TAF) were meant to increase liquidity in financial markets rather broadly and thus come closest to a policy of quantitative easing by increasing κ_t^B. In contrast, programs such as the Term Securities Lending Facility (TSLF), the Term Asset-Backed Securities Loan Facility (TALF) and the Commercial Paper Funding Facility (CPFF) were targeted at improving lending conditions in particular credit markets, and relate to credit easing in our model. Participants in the TSLF could for example borrow Treasury securities against a range of collateral including investment grade corporate, municipal, mortgage-backed and asset-backed securities.

2.5 Equilibrium

In equilibrium, there will be no arbitrage opportunities and markets clear, $n_t = \int_0^1 n_{jt}dj = \int_0^1 n_{it}di$, $y_t = \int_0^1 y_{jt}dj = \int_0^1 c_{it}di = c_t$, and $\int_0^1 L_{i,t}di = \int_0^1 L_{j,t}dj$. Households will behave in an identical way and aggregate asset holdings satisfy $\forall t \geq 0 : \int_0^1 D_{i,t}di = 0$, $\int_0^1 M_{i,t}^H di = \int_0^1 \widetilde{M}_{i,t}di = M_t^H$, $\int_0^1 M_{i,t}^R di = M_t^R$, $\int_0^1 B_{i,t}di = B_t$, $\int_0^1 B_{i,t}^C di = B_t^C$, $\int_0^1 I_{i,t}di = I_t = M_t^H - M_{t-1}^H + M_t^R + M_t^L$, and $B_t^T = B_t + B_t^C$. Household household bond holdings further satisfy

$$B_t - B_{t-1} = B_t^T - B_{t-1}^T - R_t^m \left(M_t^H - M_{t-1}^H \right). \tag{23}$$

In a rational expectations (RE) equilibrium all plans and constraints of households and firms are satisfied and consistent with monetary and fiscal policy, for given initial asset endowments. Further details on the RE equilibrium can be found in the appendix A.1, where the cases of binding and non-binding goods and money market constraints are considered.

The goods market constraint, which reads $P_t c_t \leq M_t^H + M_t^R + M_t^L$ in equilibrium, is well-known to be relevant for non-neutraliy of monetary policy. Only if it is binding, changes in money supply can affect prices and the allocation. Further, the money market constraint, which in equilibrium reads

$$M_t^H - M_{t-1}^H + M_t^R + M_t^L \leq \kappa_t^B \left(B_{t-1}/R_t^m \right) + \kappa_t \left(L_t/R_t^m \right), \tag{24}$$

is decisive for the effectiveness of quantitative and credit easing. The instruments κ_t^B and κ_t enter the set of equilibrium conditions (see appendix A.1) only via the money market constraint (24), given that lump-sum taxes are available. Hence, if it is not binding, then quantitative and credit easing will not affect the equilibrium allocation and the associated price system. To see when this is the case, we first use the conditions (9) and (14), which imply $\xi_t c_t^{-\sigma} = \beta E_t \frac{\xi_{t+1} c_{t+1}^{-\sigma}}{\pi_{t+1}} + \psi_t$ and that the multiplier on the goods market constraint ψ_t satisfies

$$\psi_t (c_t^\sigma / \xi_t) = 1 - (1/R_t^{Euler}) \geq 0, \tag{25}$$

in equilibrium, where the Euler rate R_t^{Euler} is defined in the usual way as $1/R_t^{Euler} = \beta E_t \frac{\xi_{t+1} c_{t+1}^{-\sigma}}{\xi_t c_t^{-\sigma} \pi_{t+1}}$. This definition shows that households are indifferent between $1/R_t^{Euler}$ units of the means of payment in period t, which is required for consumption purchases, and one unit in period $t+1$. Put differently, they are willing to pay a price $R_t^{Euler} - 1$ in order to transform one unit of an illiquid asset, i.e. an asset that is not accepted as a means of payment today and delivers one unit of money tomorrow, into one unit of money today. Consequently, a positive Euler rate reflects a positive valuation for money and implies that households will not hold more money than needed for consumption expenditures. Then, $\psi_t > 0$ (see 25) and the goods market constraint is binding (see 16). If, however, the Euler rate equals one, they are indifferent between money and an illiquid asset (which is not explicitly specified in our model), the goods market constraint is not binding, $\psi_t = 0$, and changes in money supply are then irrelevant. Thus, the Euler rate, rather than the policy rate, determines whether the goods market constraint is binding or not.

We further use that the conditions (9), (11), and (14) imply $\xi_t c_t^{-\sigma} = R_t^m(\lambda_t + \eta_t)$ and $\lambda_t = \beta E_t \frac{\xi_{t+1} c_{t+1}^{-\sigma}}{\pi_{t+1}}$. Eliminating λ_t, shows that the multiplier for the money market constraint η_t satisfies

$$\eta_t(c_t^\sigma/\xi_t) = (1/R_t^m) - (1/R_t^{Euler}) \geq 0, \tag{26}$$

in equilibrium and depends on deviations of the policy rate R_t^m from an Euler rate R_t^{Euler}. Condition (26) shows that when the policy rate is smaller than the Euler rate, $R_t^m < R_t^{Euler}$, the multiplier is positive $\eta_t > 0$ and the collateral constraint in the money market is binding (see 17). In this case, given that $R_t^m \geq 1$, the goods market constraint is binding as well, $\psi_t > 0$ (see 25). Households can then get money in exchange for an eligible (more liquid) asset at a price, $R_t^m - 1$, which is below their marginal valuation of money, $R_t^{Euler} - 1$. Hence, they use eligible assets as much as possible to get money in open market operations, such that (24) is binding. In this case, there will be an (il-)liquidity premium on non-eligible assets.

If, however, the policy rate equals the Euler rate, households are indifferent between transforming eligible assets into money or holding them until maturity. Thus, if $R_t^m = R_t^{Euler}$, the money market constraint is not binding, $\eta_t = 0$ (see 26). In this case, the model reduces to a standard model where the real policy rate affects aggregate demand via the consumption Euler equation (see appendix A.1). Then, the policy instruments κ_t and κ_t^B do neither affect the allocation nor the price system, such that quantitative and credit easing are ineffective, which accords to the conventional view on quantitative easing (see e.g. Eggertsson and Woodford, 2003). These results are summarized in the following proposition.

Proposition 1 *Quantitative and credit easing can affect the equilibrium allocation and the associated price system if and only if the policy rate is smaller than the endogenous Euler rate,* $R_t^m < R_t^{Euler}$.

In contrast to macroeconomic models where only a single nominal interest rate is considered (like in most New Keynesian models), money demand can be uniquely determined even if the policy rate is at the ZLB, $R_t^m = 1$. Then, both multiplier are identical $\eta_t = \psi_t$ (see 26 and 25) since eligible assets can costlessly be transformed into money. As long as opportunity costs of money are positive $R_t^{Euler} > 1$, quantitative easing and credit easing, which will be examined in the subsequent sections, can still affect aggregate output and prices. Yet, their effectiveness is not unlimited and relies on the scarcity of liquid assets, i.e. of money and eligible assets. The effectiveness of a quantitative easing policy, which increases the amount of money available (and thus consumption), will reach its limit when the endogenous Euler rate equals the policy rate. Then, eligible assets become abundant and the money market constraint is not binding, $\eta_t = 0$. These limits will quantitatively be examined in section 4.

3 Analytical results

In this section we examine the effects of monetary policy under flexible and sticky prices in an analytical way. In the first subsection, we examine how the different monetary policy instruments effect macroeconomic variables under flexible prices. In the second subsection, we apply a local approximation of the model at a steady state where the money market constraint is binding.

3.1 Macroeconomic effects of monetary policy

Here, we show how changes in the monetary policy instruments R_t^m, κ_t^B, and κ_t affect macroeconomic aggregates and interest rates. To facilitate the derivation of analytical results, we apply a simplified version of the model. In particular, we assume that prices are perfectly flexible, $\phi = 0$, and that the utility function is logarithmic in consumption, $\sigma = 1$. We further assume that money is only supplied temporarily, $\Omega \to \infty$, and is not held outright (see 22). Accordingly, the central bank will hold government bonds only temporarily under repos. Given that this is consistent with initial money holdings and initial central bank bond holdings equal to zero, the total stock of government bonds will be held by households, $B_t = B_t^T$ (see 23). We disregard preference shocks in this subsection, and set $\xi_t = 1$. We focus on the case where the money market constraint is binding, $\eta_t > 0$, which requires the policy rate to be lower than the Euler rate (see 26). A RE equilibrium with a binding money market constraint can be reduced to a set of sequences in output, inflation, household bond holdings, and the loan rate (see appendix A.2).

Definition 1 *For $\sigma = 1$, $\phi = 0$, $\Omega \to \infty$, a RE equilibrium is a set of sequences $\{y_t, \pi_t, R_t^L, b_t\}_{t=0}^{\infty}$ and an initial price level P_0 satisfying*

$$y_t = \left[(\mu/\theta)\left(1/R_t^L\right)\right]^{\alpha/(1+\sigma_n)}, \tag{27}$$

$$\frac{1}{R_t^L} = \kappa_t \frac{1}{R_t^m} + (1-\kappa_t)\beta E_t \frac{y_t}{y_{t+1}\pi_{t+1}}, \tag{28}$$

$$y_t = [\kappa_t^B b_{t-1}\pi_t^{-1} + \kappa_t \mu y_t]/R_t^m, \tag{29}$$

$$b_t = \Gamma b_{t-1}\pi_t^{-1} \forall t \geq 1 \quad and \quad \Gamma P_0 b_0 = B_{-1}, \tag{30}$$

where $\mu = \frac{\varepsilon-1}{\varepsilon}\alpha < 1$, for a monetary policy setting $R_t^m < \beta y_t E_t[y_{t+1}^{-1}\pi_{t+1}^{-1}]$, κ_t, and κ_t^B for a given initial stock of bonds $B_{-1} > 0$.

Condition (27) is derived from equating labor supply with labor demand and using the production function as well as goods market clearing. It shows that the costs of loans R_t^L reduces aggregate output. Condition (28), which is based on (9), (12), and (14), shows that the loan price $1/R_t^L$ is a linear combination of the inverses of the policy rate $1/R_t^m$ and of the Euler rate $1/R_t^{Euler} = \beta E_t y_t / (y_{t+1}\pi_{t+1})$, where the former is weighted with the fraction of eligible loans κ_t and the latter with $1 - \kappa_t$. If loans are fully eligible, $\kappa_t = 1$, the loan rate equals the policy rate. If they are not eligible, $\kappa_t = 0$, loans cannot be liquidated and the loan rate equals the Euler rate. Hence, by raising the fraction of eligible loans, the central bank can reduce the loan rate and thereby the marginal costs of firms.

Combining the cash-in-advance constraint and the money market constraint leads to (29), which shows that bonds and loans can serve as collateral for money in repos. The evolution of privately held government bonds is further determined by the total supply of bonds, which grow with the rate Γ (see 12). Hence, the long-run inflation rate π can in principle be affected by the supply of eligible assets, when the money market constraint is binding. As the stock of government bonds grows at the rate Γ, the price level would also grow with the same rate when government bonds are eligible. In order to control the long-run supply of money and thus the inflation rate, the central bank can, however, reduce the fraction of accepted bonds κ^B accordingly (see proposition 3 below).

The set of equilibrium conditions (27)-(30) is sufficiently simple to derive effects of exogenous changes in monetary policy in an analytical way. To demonstrate the effects of different monetary policy instruments, we separately consider unexpected permanent changes in each instrument. Initially, the instruments are assumed to be set at follows: The fraction of eligible loans is $\kappa \geq 0$, the fraction of eligible bonds κ_t^B grows initially with a constant rate, $\kappa_t^B = \gamma\kappa_{t-1}^B$, where $\kappa_0^B > 0$ and $\gamma > \beta/\Gamma$ (which allows the central bank to respond to a growing stock of total bonds), and the policy rate is set at $R^m \in [1, \gamma\Gamma/\beta)$. Note that the restrictions $\gamma > \beta/\Gamma$ and $R^m < \gamma\Gamma/\beta$ ensure that the goods market constraint and the money market constraint are binding. The following proposition summarizes the impact effects of permanent changes in the policy instruments.

Proposition 2 *Consider the model given in definition 1. Suppose that the central bank instruments are initially set at* $\kappa \geq 0$, $\kappa_t^B = \gamma\kappa_{t-1}^B$, *where* $\kappa_0^B > 0$ *and* $\gamma > \beta/\Gamma$, *and* $R^m \in [1, \gamma\Gamma/\beta)$. *Then, the money market constraint is binding and*

1. *a marginal increase in the policy rate to* $R_{t+i}^m = \overline{R}^m \in (R^m, \gamma\Gamma/\beta)$ $\forall i \geq 0$ *leads to an increase in the loan rate and a decline in output if* $\kappa > 0$, *and to a decline in the inflation rate.*

2. *a marginal decrease in the growth rate of* κ_t^B *to* $\gamma_{t+i} = \overline{\gamma} \in (R^m\beta/\Gamma, \gamma)$ $\forall i \geq 0$ *leads to a decline in the loan rate and an increase in output if* $\kappa < 1$, *and to an fall in the inflation rate.*

3. *a marginal increase in the fraction of eligible loans* κ *to* $\kappa_{t+i} = \overline{\kappa} > \kappa$ $\forall i \geq 0$ *leads to a decline in the loan rate and to an increase in output, while the effect on the inflation rate is ambiguous.*

Proof. See appendix A.2. ■

Hence, all policy instruments can be used by the central bank to affect the equilibrium allocation and the inflation rate. An increase in the policy rate R_t^m leads to a decline in inflation (see part 1 of proposition 2), since a larger amount of nominal bonds are required for a given amount of nominal consumption expenditures. If the fraction of eligible loans is positive, $\kappa > 0$, an increase in the policy rate further induces an increase in the loan rate according to (28). This raises the marginal costs of production such that total output declines. While this effect tends to increase inflation, it is dominated by the first effect.

Part 2 of proposition 2 shows that a decrease in the growth rate of the fraction of eligible bonds κ_t^B affects the inflation rate like a decrease of the money growth rate in a conventional flexible price model. If, further, not all loans are eligible (which would imply that the loan rate equals the policy rate), the fall in inflation reduces the Euler rate and the loan rate according to (28). With smaller costs of borrowing, the marginal costs of firms decrease and output increases. Correspondingly, a policy of a permanent *quantitative easing* in terms of government bonds, where the growth rate γ increases, would impact on the allocation and prices like an increased money growth rate.

As stated in part 3 of proposition 2, the central bank can also directly induce a lower loan rate R_t^L by raising the fraction of eligible loans κ_t if the policy rate is smaller than the Euler rate $R^m < \gamma\Gamma/\beta$ (see 28). The reason is that the central bank can induce households to demand a lower illiquidity premium on the loan rate by increasing the fraction of eligible loans κ. Due to reduced costs of loans, firms increase labor demand which increases output. It should be noted that this effect is independent of the inflation response, which is here ambiguous. Hence, a *credit easing* policy can be expected to stimulate real activity by reducing the costs of borrowing, which will be demonstrated in section 4.

As revealed by proposition 2, the central bank can affect the inflation rate by changing its instruments. In particular, it can control the inflation rate by setting the growth rate γ_t contingent on the growth rate of government bonds Γ.

Proposition 3 *Consider the model given in definition 1. When the central bank sets its instruments according to $\kappa_t \geq 0$, $\gamma_t > \beta/\Gamma$, and $R_t^m \in [1, \gamma\Gamma/\beta)$ it can control the inflation rate by changing the fraction of eligible government bonds κ_t^B. Then, a permament increase in κ_t^B, but not in its growth rate γ_t, leads to a temporary change in inflation. The central bank can further implement long-run price stability by setting $\gamma_t = \Gamma^{-1}$.*

Proof. See appendix A.2. ■

According to proposition 3, the central bank can control the inflation rate under a binding money market constraint. A quantitative easing policy in terms of government bonds that leads to a once-and-for-all increase in the size of the balance sheet cannot lead to a permanent change in the inflation rate, while the central bank can implement its inflation target independent of fiscal policy and can ensure long-run price stability by setting $\gamma_t = \Gamma^{-1}$.

3.2 Quantitative easing under sticky prices

While quantitative easing in terms of government bonds will lead to higher prices and lower output under flexible prices, we expect that it can stimulate real activity under imperfectly flexible prices. In this section, we therefore consider quantitative easing under sticky prices, $\phi > 0$, like in New Keynesian models. We further disregard central bank lending against corporate debt, $\kappa_t = 0$, such that the monetary policy regime is conventional in the sense that only Treasuries securities are eligible. We apply a local analysis of the economy at a steady state where government bonds are not fully eligible, $\kappa^B < 1$, leaving room for a quantitative easing policy. Details on the steady state can be found in appendix A.3. Given that there exists a steady state, we can use that all real endogenous variables are constant. Combining (15) with (18) implies that the steady state debt rate R^D equals the Euler rate and satisfies $R^D = R^{Euler} = \pi/\beta$. The debt rate and the policy rate R^m can differ by an (il-)liquidity premium, as revealed by steady state version of (26) $\eta = c^{-\sigma}[(1/R^m) - (\pi/\beta)] \geq 0$.

We assume that the central bank inflation target is consistent with long-run price stability, $\pi = 1$, which can be justified by the minimization of welfare costs due to long-run price dispersion. Precisely, the central bank can implement long-run price stability by long-run adjustments of κ_t^B contingent on the supply of government bonds (see proposition 3). We disregard a growing supply of bonds $\Gamma > 1$ that can be neutralized by a shrinking fraction of eligible bonds, and assume – without loss of generality – that $\Gamma = 1$. We further assume that the central bank sets the average policy rate below the long-run Euler rate, $R^m < \pi/\beta$, which is consistent with the empirical evidence provided by Canzoneri et al. (2007) for our preference specification. Given that $\pi = 1$ implies $R^{Euler} > 1$ and $R^m < R^{Euler}$, the goods market constraint as well as the money market constraint are binding in the steady state.

To motivate why the central bank sets the policy rate at its ZLB, we consider preference shocks $\widehat{\xi}_t$, which are assumed to be generated by an AR(1) process, $\widehat{\xi}_t = \rho_\xi \widehat{\xi}_t + \varepsilon_t$, where $E_{t-1}\varepsilon_t = 0$ and $\rho_\xi \in [0, 1)$. In a neighborhood of this steady state, the equilibrium sequences are approximated by the solutions to the linearized equilibrium conditions. An equilibrium is then defined as follows, where $\widehat{a}_t$ denotes the percent deviation of a generic variable a_t from its steady state value $\overline{a} : \widehat{a} = \log(a_t) - \log(\overline{a})$.

Definition 2 *For $\Omega \to \infty$, $\Gamma = \pi = 1$, $R^m \in [1, 1/\beta)$, and $\kappa_t = 0$, a RE equilibrium is a set of sequences $\{\widehat{y}_t,\ \pi_t,\ \widehat{b}_t,\ \widehat{R}_t^L\}_{t=0}^{\infty}$ that converge to the steady state and satisfy*

$$\widehat{y}_t = \widehat{b}_{t-1} - \widehat{\pi}_t - \widehat{R_t}^m + \widehat{\kappa}_t^B, \tag{31}$$

$$\sigma\widehat{y}_t = \sigma E_t\widehat{y}_{t+1} - \widehat{R}_t^L + E_t\widehat{\pi}_{t+1} + \left(1 - \rho_\xi\right)\widehat{\xi}_t, \tag{32}$$

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} + \chi\left(\varpi - 1\right)\widehat{y}_t + \chi\widehat{R}_t^L, \tag{33}$$

$$\widehat{b}_t = \widehat{b}_{t-1} - \widehat{\pi}_t, \tag{34}$$

where $\varpi = \frac{1+\sigma_n}{\alpha} + \sigma$ for monetary policy setting $\{\widehat{\kappa}_t^B, \widehat{R_t}^m\}_{t=0}^{\infty}$ and an initial value $b_{-1} > 0$.

The linear model summarized in definition 2 exhibits some similarities to a New Keynesian model with the "cost channel" (see Ravenna and Walsh, 2006). In particular, the conditions (32) and (33) resemble standard conditions for aggregate demand and for aggregate supply, where the latter is affected by the costs of loans due to the working capital assumption. The crucial difference to a New Keynesian model is, however, that this is not a single interest rate framework. Specifically, the policy rate, which is not identical to the loan rate since $\kappa_t < 1$, neither enters (32) nor (33). Nevertheless, the policy rate affects the equilibrium allocation and prices via the reduced version of the money supply constraint (31). Here, an increase in the policy rate tends – for a given amount of eligible bonds – to reduce the amount of money and thereby aggregate demand. Inflation, output, real bonds, and the loans rate, which equals the Euler rate for $\kappa_t = 0$ (see 28), will simultaneously be determined, given both monetary policy instruments, i.e. the policy rate $\widehat{R_t}^m$ and the fraction of eligible bonds $\widehat{\kappa}_t^B$.

Since the policy rate does not enter the aggregate demand constraint (32), which it typically does in New Keynesian models, the well-known Taylor-principle does not apply to this model. In this model, the range of parameter values which are associated with local equilibrium determinacy differs substantially from the range implied by the Taylor-principle. Specifically, the equilibrium is locally determined for a broad range of values for the feedback parameters of a simple Taylor-rule

$$\widehat{R}_t^m = \rho_\pi \widehat{\pi}_t + \rho_y \widehat{y}_t, \quad \rho_\pi \geq 0, \rho_y \geq 0, \tag{35}$$

including a peg. The following proposition presents the condition for local equilibrium determinacy.

Proposition 4 *Consider the model given in definition 2, where the central bank sets the policy rate according to (35). For an exogenously given fraction κ_t^B, the equilibrium is locally determined if and only if* $\frac{\rho_\pi+1/2}{\rho_y+1}\left(\sigma - \frac{1-\alpha+\sigma_n}{\alpha}\right) < 1 + \frac{1+\beta}{\chi}$.

Proof. See appendix A.3 ■

We are now prepared to examine monetary policy at the ZLB in a simple way. For this, consider for example a decline in the preference parameter $\widehat{\xi}_t$. This shock leads to a decline in inflation and the policy rate according to (35). If the shock is sufficiently large, the policy rate reaches its ZLB. At this point, the central bank pegs the policy rate at $R_t^m = 1$. The condition given in proposition 4 then reads, $\sigma < \overline{\sigma} = 2\left[1 + (1+\beta)/\chi\right] + (1 - \alpha + \sigma_n)/\alpha$. If σ satisfies this condition, which is hardly restrictive for standard parameter values (e.g. the parameter values applied in section 4 imply $\overline{\sigma} > 50$), the equilibrium is uniquely determined. Hence, the central bank can safely peg the policy rate at the ZLB without inducing local indeterminacy.

The set of equilibrium conditions listed in definition 2 reveal that both policy instruments, $\widehat{\kappa}_t^B$ and $\widehat{R_t}^m$, affect the private sector behavior only via the money supply constraint (31). According to the latter, money supply can be eased by the central bank either by decreasing the policy rate or by increasing the fraction of eligible bonds. Hence, if the policy rate cannot be lowered, because it has reached the ZLB, the central bank can still ease money supply by conducting quantitative easing, $\widehat{\kappa}_t^B > 0$. Specifically, a 1% increase in $\widehat{\kappa}_t^B$ affects the economy in the same way as a reduction of the policy rate by 1%. The following proposition summarizes the effects of an unexpected temporary quantitative easing policy, $\widehat{\kappa}_t^B > 0$, where $\widehat{\kappa}_t^B$ is assumed to follow an AR(1) process, $\widehat{\kappa}_t^B = \rho_\kappa \widehat{\kappa}_{t-1}^B + \varepsilon_t^\kappa$, $\rho_\kappa \in [0, 1)$ and $E_{t-1}\varepsilon_t^\kappa = 0$.

Proposition 5 *Consider the model given in definition 2 for $\sigma < \overline{\sigma}$ and $\kappa^B < 1$ and suppose that the central bank sets the policy rate at its ZLB, $R_t^m = 1$. Then, a temporary quantitative easing policy in terms of government bonds, $\widehat{\kappa}_t^B > 0$, leads to an increase in output, $\widehat{y}_t > 0$, and in the inflation rate, $\widehat{\pi}_t > 0$.*

Proof. See appendix A.3 ■

It should be noted that the effectiveness of quantitative easing relies on the scarcity of liquid assets, i.e. on a binding money market constraint. This however implies that there exist limits to the effectiveness of a quantitative easing. These limitations will be examined in the subsequent section.

4 Limits to quantitative and credit easing

In this section, we examine the quantitative impact of monetary policy on macroeconomic aggregates. For this, we disregard preference shocks, $\xi_t = 1$, and assume that the central bank sets its targets according to $\pi < \beta$ and $1 \leq R^m \in [1, \pi/\beta)$, which implies that the money market constraint and the goods market constraints are binding in the steady state (see 25 and 26). We extend the model by considering physical capital and calibrate it to explore the effects and limits of quantitative and credit easing. We further analyze the impact of a large liquidity demand shock on the economy and assess the ability of policy to mitigate the contractionary effects of the shock by quantitative easing. Throughout the analysis, we assume that quantitative and credit easing are implemented when the policy rate is at its ZLB, where their range of effectiveness is particularly large.[8]

4.1 Extension of the model

For a quantitative analysis of monetary policy effects, we extend the model presented in section 2 by introducing physical capital. Households own the stock of capital, $k_t = \int k_{i,t} di$

[8] In a companion paper, Schabert (2010) examines the policy instruments κ_t and κ_t^B when the policy rate is set above its ZLB. He shows that by applying these additional instruments the central bank can enhance welfare compared to the case of a pure interest rate setting.

and rent it to firms at the rate r_t^k. The capital stock of household i evolves according to $k_{i,t+1} = (1-\delta) k_{i,t} + x_{i,t} S(x_{i,t}/x_{i,t-1})$, where $\delta \in (0,1)$, x_t denotes investment expenditures, $S(x_t/x_{t-1}) = 1 - \frac{\vartheta}{2}(x_t/x_{t-1}-1)^2$ is an adjustment cost function, and $\vartheta > 0$ measures the degree of adjustment cost. We assume that households rely on cash for purchases of investment goods up to an exogenous fraction ω_t. We introduce a corresponding parameter ν, which describes the fraction of purchases of consumption goods that require cash. Thus, the cash in advance constraint (3) is replaced by

$$\upsilon P_t c_{i,t} + \omega_t P_t x_{i,t} \leq I_{i,t} + M_{i,t-1}^h - \left(L_{i,t}/R_t^L\right) + P_t w_t n_{i,t}. \tag{36}$$

The parameters $\nu > 0$ and $\omega_t > 0$ govern the liquidity demand of households. These parameters allow relating expenditures to the monetary base in accordance with empirical counterparts, which facilitates the calibration of the model. In section (4.4) we further analyze a shock to ω_t, which captures increased liquidity demand for purchases of investment goods.

Intermediate goods producing firms rent capital from households. Firm j produces with technology $IO_{j,t} = n_{j,t}^{\alpha} k_{j,t}^{1-\alpha}$ and pays the rental rate on capital after their goods are sold. Hence, the working capital condition (1) of firm j is unchanged. Its first order conditions for $R_t^L > 1$ are given by $mc_{j,t}\alpha(n_{j,t}/k_{j,t})^{\alpha-1} = w_t R_t^L$, $mc_{j,t}(1-\alpha)(n_{j,t}/k_{j,t})^{\alpha} = r_t^k$, and (6). To calibrate the model in a consistent way, we further introduce government spending so that goods market clearing requires $y_t = c_t + x_t + g_t$. The full set of equilibrium conditions can be found in Appendix A.4.

4.2 Calibration

We use standard parameter values as far as possible. The parameters of the utility function equal $\sigma = 2$ and $\sigma_n = 1$, the labor share equals $\alpha = 0.66$, the steady state markup $1/mc = 11\%$ ($\varepsilon = 10$), steady state working time $n = 1/3$, and the fraction of non-optimally price adjusting firms $\phi = 0.75$. The share of government spending and the long run inflation rate are set to $g/y = 0.19$, the steady state values of ω_t and υ are calibrated to the observed ratios $Px/M0 = 1.15$ and $Pc/M0 = 2.71$, and the depreciation rate is set to $\delta = 0.023$ to match the observed ratio of consumption to investment, $c/x = 2.36$. [9] For the policy rate, we consider two scenarios. The policy rate is either pegged at the ZLB, $R^m = 1$, or set according to a simple Taylor rule (35) with $\rho_\pi = 1.5$, $\rho_y = 0.5$, and a long-run policy rate equal to the Federal Funds Rate's 25-year average $R^m = 1.0133$ (or 5.41% in terms of annualized rates)

[9] Data on (nominal) consumption, investment, government spending and Gross Domestic Product are taken from NIPA Table 1.15, where durable consumption goods are included into investment. Data on the monetary base was taken from the Federal Reserve Board's H3 Statistical Release. Data on inflation was extracted from the series GDPDEF, available from the U.S. Bureau of Economic Analysis. All data except those for the Fed Funds rate are seasonally adjusted series. All data refer to averages for 25 years over the period QI/1981–QIV/2006, except for the Fed Funds rate and inflation, where the sample starts after the pre-Volcker period, QIV/1982–QIII/2008.

and an inflation target equal to its steady state value $\pi = 1.00647$ (or 2.61% at an annual rate). Both policies are consistent with equilibrium determinacy (see proposition A.3). We restrict our attention to the case where the central bank does not trade corporate debt in open market operations in the steady state, i.e. $\kappa = 0$, which accords to the Fed's pre-crisis "Treasuries only" regime. In contrast, government debt is fully eligible for open market operations, $\kappa^B = 1$, where we assume – without explicitly specifying – that long-run growth in government bonds and in κ_t^B is consistent with the long-run inflation rate (see proposition 3). We further set the repo share to $\Omega = 1.5$ to match the observed ratio between total reserves and reserves supplied under repurchase agreements, which was almost constant in the 2000s before the crisis.[10] The value for the adjustment cost parameter $\vartheta = 2.48$ is taken from Christiano et al. (2005).

The spread between the policy rate and the loan rate, which equals the Euler rate $R^L = R^{Euler} = \pi/\beta$ in a steady state with $\kappa = 0$, matters for the size of monetary policy effects. To calibrate this spread, we account for the fact that our model does not account for any kind of default risk and focus on the part of the spread that can be attributed to a liquidity premium. According to the literature on the "corporate bond credit spread puzzle", only a small share of the yield spread between Treasury securities and corporate bonds can actually be explained by default risk. For our calibration, we refer to Longstaff et al.'s (2005) results for the spread between corporate bonds and Treasury securities, which lead to more conservative estimates of the liquidity premium.[11] Specifically, they report that, for AAA rated corporate bonds, 51% of the credit spread can be explained by default risk. Given that the average short-term spread among AAA corporate bonds equals 104 basis points at annualized rates (see Longstaff et al., 2005), we consider a liquidity premium of $(1 + 49\% \cdot 0.0104)^{1/4} - 1 = 13$ basis points (in terms of quarterly rates), which implies the discount factor to equal $\beta = \frac{\pi}{R^m + 13 \cdot 10^{-4}} = 0.992$.

4.3 Isolated effects of quantitative and credit easing

The calibrated model is solved by applying a first-order approximation at the deterministic steady state. Most variables are given in terms of percentage deviations from steady state, $\widehat{a}_t = \log(a_t) - \log(\overline{a})$, as defined earlier. Further, we consider deviations expressed in percentage points, $\widetilde{a}_t = 100\,(a_t - \overline{a})$, for κ, κ^B, ω, and interest rates, e.g., $\tilde{R}_t^m = 1$ denotes an increase in the policy rate by 100 basis points. As stated in proposition 2, quantitative easing can increase output even when the policy rate is at the ZLB. The reason is that the money market constraint binds at $R_t^m = 1$ as long as the Euler rate exceed one (see 26). However, easing money supply will eventually lead to the point where households' and firms' cash de-

[10] See Federal Reserve Bank of New York, Domestic Open Market Operations, various issues, and FRED database.

[11] Collin-Dufresne at al. (2001), for example, can explain only 25% of the variation in credit spread changes across 688 corporate bonds. Huang and Huang (2002) report that around 20% of corporate credit spreads can be explained through default risk.

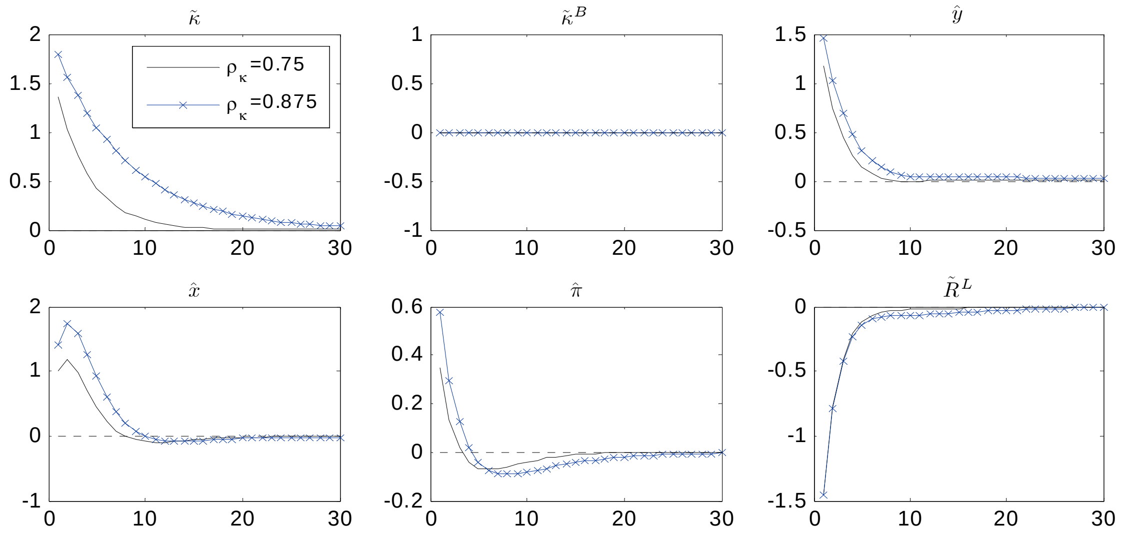

Figure 1: Maximum effects of quantitative easing

mand will not be collateral constrained. Specifically, the multiplier on the money market constraint η_t (see 25 and 26) has to satisfy

$$\eta_t = (c_t^{-\sigma}/R_t^m) - \beta E_t c_{t+1}^{-\sigma} \pi_{t+1}^{-1} > 0, \tag{37}$$

for a quantitative easing policy to be effective. Thus, the range over which the money market constraint is binding is particularly large at its ZLB, $R_t^m = 1$ (see 37). A closer look at (37) shows that the multiplier approaches zero if an increase in consumption is sufficiently large and not too persistent. Beyond this point, quantitative and credit easing can achieve no further stimulus. The purpose of this analysis is to quantify this limit.

We first consider isolated effects of quantitative easing in terms of corporate debt, i.e. an increase in κ_t, where $\widehat{\kappa}_t$ exhibits an autocorrelation of $\rho_\kappa = 0.75$ (0.875) which accords to an expected duration of the policy intervention of one year (two years). Figure 1 shows the impulse response to the maximum intervention, which is defined as the intervention which just lets the collateral constraint bind, $\eta_t > 0$. The maximum intervention with $\rho_\kappa = 0.75$ (see solid line) implies an increase of $\triangle\kappa_t = 0.0136$ (or $\widetilde{\kappa}_t = 1.36$) so that – ceteris paribus – the monetary base, $m_t^H + m_t^R + \kappa_t l_t / R_t^m$, rises by 1.55%. This induces the loan rate to fall to its ZLB and a rise in output by 1.18%, while inflation increases by 35 basis points. When the policy intervention is more persistent, $\rho_\kappa = 0.875$ (see starred line) a larger intervention is possible according to (37). Although, quantitative easing can be conducted at a larger scale ($\triangle\kappa_t = 0.0179$ or $\widetilde{\kappa}_t = 1.79$), it will not achieve substantially higher output responses. Output rises by 1.46%, while inflation increases by 58 basis points. Hence, supplying additional money

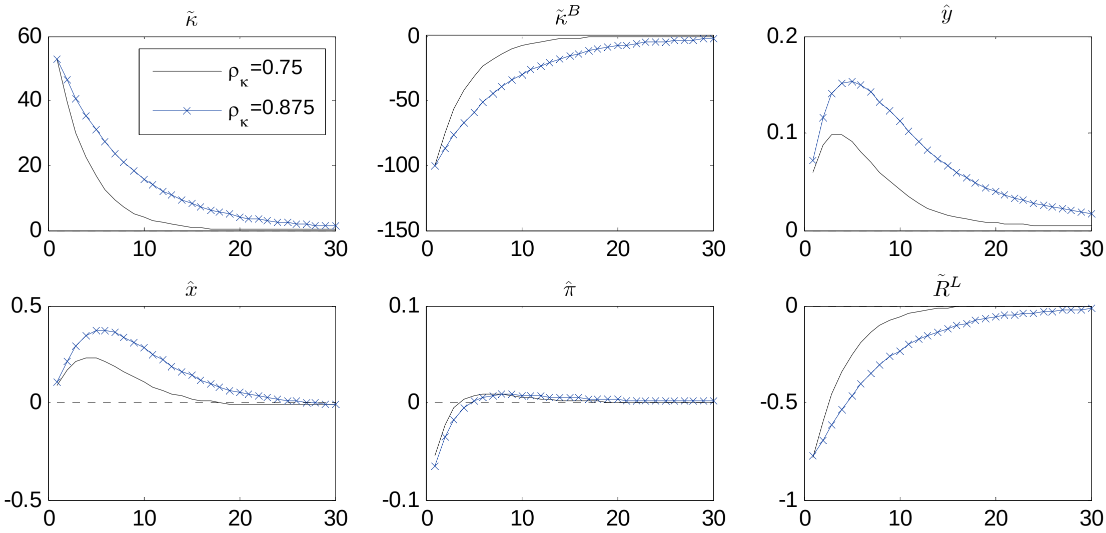

Figure 2: Maximum effects of credit easing

against 1% of all loans in open market operations raises output on impact by 0.87% (0.81%) in case of the less (more) persistent intervention.

Next, we examine isolated effects of credit easing. The extent of credit easing is limited by the size of the central bank's balance sheet and the availability of collateral. In terms of the model, credit easing is limited by $\kappa_t^B \geq 0$ and $\kappa_t \leq 1$. For our calibration, these restrictions are more severe than (37) and a maximum credit easing policy is given by $\triangle\kappa_t^B = -1$ and $\triangle\kappa_t = -\frac{b/\pi}{l}\triangle\kappa_t^B = 0.53$ (or $\widetilde{\kappa}_t = 53$ and $\widetilde{\kappa}_t^B = -100$). Hence, the central bank exchanges more than the half of all loans against government bonds. Like before, we consider a constant policy rate pegged at $R_t^m = 1$. Figure 2 shows the responses to the maximum credit easing. As corporate debt is now eligible in open market operations, R_t^L declines, so that marginal cost and inflation fall. Reduced goods prices and increased real balances allow households to increase consumption and investment spending. For an autocorrelation of 0.75 (see solid line) which corresponds to an expected duration of one year, output exhibits a peak response of 0.1% in the third quarter and inflation declines by five basis points. For a more persistent intervention, $\rho_\kappa = 0.875$ (see starred line), the loan rate declines more persistently, which leads to a more pronounced decline in inflation and an increase in output by a maximum of 0.15%. These numbers demonstrate that credit easing has a relatively small impact. Precisely, exchanging 1% of loans against government bonds leads to a maximum rise in output by 0.0019% (0.0029%) in case of the less (more) persistent intervention.

Finally, comparing the output effects of quantitative easing in terms of corporate debt with the output effects of credit easing implies that the output effects of quantitative easing in terms of government bonds will be virtually identical to those shown in figure 1.

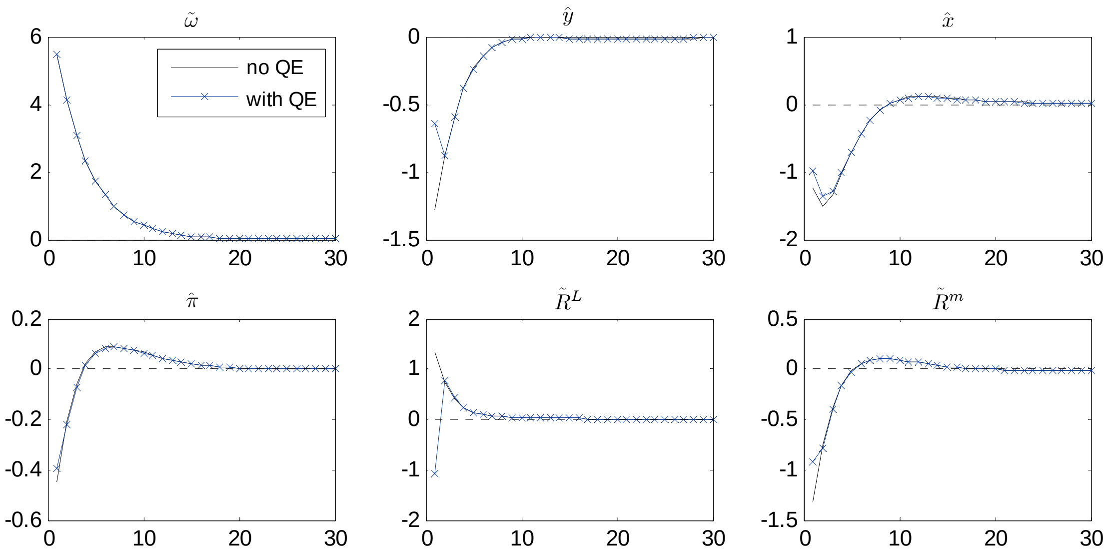

Figure 3: Liquidity demand shock with and without QE intervention

4.4 Response to a liquidity demand shock

We now consider a liquidity demand shock, i.e. an unexpected increase in the fraction ω_t of investment goods that have to be purchased with cash (36). This shock, which implies that less investments can be financed on credit, can for example be interpreted as an increase in financial distress that lowers the extent to which investment goods can be pledged as collateral. We assume an AR(1) process for ω_t, $\omega_t = (1 - \rho_\omega)\,\omega + \rho_\omega \omega_{t-1} + \varepsilon_t$, with autocorrelation $\rho_\omega = 0.75$. The shock hits an economy, where the policy rate is initially at its steady state value $R^m = 1.0133$ and otherwise governed by a Taylor rule (35) with $\rho_\pi = 1.5$ and $\rho_y = 0.5$. We consider a shock that drives the policy rate to the ZLB in the impact period, which requires $\triangle\omega_t = 0.0549$ (or $\tilde{w}_t = 5.49$). The solid line in Figure 3 shows the impulse responses to this shock without quantitative easing. Investment and consumption fall, so that output declines by 1.29% despite the reduction of the policy rate. The inflation rate falls, while the spread between the policy rate and the loan rate increases.

The starred line shows the responses for the case where the central bank applies a maximum size of the quantitative easing policy in terms of corporate debt at the ZLB. It should be noted that quantitative easing is only conducted in the first period, since the policy rate increases afterwards. For this policy, the contractionary effects are mitigated and output falls by only 0.65%. The impact output contraction is reduced by 50%. Inflation is 5 basis points larger than without intervention, falling only by 40 basis points. Hence, the central bank can substantially reduce the contractionary output effect of the liquidity demand shock via a quantitative easing policy for one period. The maximum size, $\triangle\kappa_t = 0.0088$ (or $\tilde{\kappa}_t = 0.88$)

is again determined by the multiplier on the money market constraint, which has to satisfy (37). Notably, the intervention is smaller than in section 4.3 (see figure 1). Precisely, the intervention reduces the impact decline in output by 0.64%, compared to 1.46% (1.18%) in section 4.3. The reasons for this is that, first, the policy intervention is less persistent and, second, the policy rate immediately increases in the impact period due to a successful intervention, which both tend reduce the multiplier η_t (see 37). Nevertheless, quantitative easing can help escaping from the ZLB.

5 Conclusion

The recent financial crisis has led several central banks to set the policy rate close to its zero lower bound and to conduct non-standard policies, like quantitative and credit easing. At the same time, the macroeconomic literature on monetary policy has provided little guidance for conducting these policies, and in particular, has left open under which circumstances quantitative easing can be effective. Our analysis is motivated by the observation that eligible securities, like US-Treasury bills, are typically traded at a liquidity premium compared to interest rates on non-eligible assets. Further, financial markets experienced a surge in interest rate spreads during the crisis. This increase, which can to a large extent be attributed to liquidity concerns, was substantially reduced in response to quantitative and credit easing.

In this paper, we apply a simple macroeconomic model where money is supplied by the central bank against collateral. We show that quantitative and credit easing can be effective only if eligible assets are scarce, which is reflected in liquidity premia. A precondition for this is that the central bank supplies money at a price (i.e. the policy rate) below the private sectors nominal savings rate (i.e. the consumption Euler). For this case and in particular at the zero lower bound, we find that quantitative and credit easing can stimulate real activity and reduce interest rate spreads. Notably, quantitative easing increases inflation, whereas credit easing reduces inflation. These policies, however, reach their limits when the liquidity premium approaches zero, indicating that collateral becomes abundant. Taking these limits into account, we find that the maximum effects of quantitative easing are relatively small, though sufficient to help escaping from the zero lower bound.

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A Appendix

A.1 Rational expectations equilibrium

Definition 3 *A RE equilibrium is given by a set of sequences* $\{c_t,\, y_t,\, n_t,\, \lambda_t,\, m_t^R,\, m_t^H,\, b_t,\, b_t^T,\, l_t,\, w_t,\, mc_t,\, \tilde{Z}_t,\, s_t,\, \pi_t,\, R_t,\, R_t^D,\, R_t^L\}_{t=0}^{\infty}$ *satisfying*

$$\theta n_t^{\sigma_n} = w_t c_t^{-\sigma}, \tag{38}$$

$$R_t^L = \left[\lambda_t (1-\kappa_t) + \kappa_t \xi_t c_t^{-\sigma}/R_t^m\right]^{-1} \xi_t c_t^{-\sigma}, \tag{39}$$

$$\lambda_t = \beta E_t \left[\xi_{t+1} c_{t+1}^{-\sigma}/\pi_{t+1}\right], \tag{40}$$

$$\lambda_t = \beta R_t E_t [\left(\lambda_{i,t+1}\left(1-\kappa_{t+1}^B\right) + \kappa_{t+1}^B \xi_{t+1} c_{t+1}^{-\sigma}/R_{t+1}^m\right) \pi_{t+1}^{-1}], \tag{41}$$

$$\lambda_t = \beta R_t^D E_t \left[\lambda_{t+1}/\pi_{t+1}\right], \tag{42}$$

$$c_t - \kappa_t l_t/R_t^m = m_t^H + m_t^R, \text{ if } \psi_t = (R_t^m - 1)\lambda_t + R_t^m\left(\xi_t c_t^{-\sigma} - R_t^m \lambda_t\right) > 0, \tag{43}$$

$$\text{or } c_t - \kappa_t l_t/R_t^m \leq m_t^H + m_t^R, \text{ if } \psi_t = 0,$$

$$\kappa_t^B b_{t-1}/\left(R_t^m \pi_t\right) = m_t^H - m_{t-1}^H \pi_t^{-1} + m_t^R, \text{ if } \eta_t = \xi_t c_t^{-\sigma} - R_t^m \lambda_t > 0, \tag{44}$$

$$\text{or } \kappa_t^B b_{t-1}/\left(R_t^m \pi_t\right) \geq m_t^H - m_{t-1}^H \pi_t^{-1} + m_t^R, \text{ if } \eta_t = 0,$$

$$b_t - b_{t-1}\pi_t^{-1} = (\Gamma - 1) b_{t-1}^T \pi_t^{-1} - R_t^m \left(m_t^H - m_{t-1}^H \pi_t^{-1}\right), \tag{45}$$

$$mc_t \alpha n_t^{\alpha-1} = w_t R_t^L, \tag{46}$$

$$l_t/R_t^L = w_t n_t, \tag{47}$$

$$\tilde{Z}_t (\varepsilon - 1)/\varepsilon = Z_t^1/Z_t^2, \tag{48}$$

$$\text{where } Z_t^1 = c_t^{-\sigma} y_t mc_t + \phi\beta E_t \pi_{t+1}^{\varepsilon} Z_{t+1}^1 \text{ and } Z_t^2 = c_t^{-\sigma} y_t + \phi\beta E_t \pi_{t+1}^{\varepsilon-1} Z_{t+1}^2,$$

$$1 = (1-\phi)(\tilde{Z}_t)^{1-\varepsilon} + \phi \pi_t^{\varepsilon-1}, \tag{49}$$

$$m_t^R = \Omega_t m_t^H, \tag{50}$$

$$b_t^T = \Gamma b_{t-1}^T/\pi_t, \tag{51}$$

$$y_t = n_t^{\alpha}/s_t, \tag{52}$$

$$y_t = c_t, \tag{53}$$

$$s_t = (1-\phi)\tilde{Z}_t^{-\varepsilon} + \phi s_{t-1}\pi_t^{\varepsilon}, \tag{54}$$

the transversality conditions, a monetary policy setting $\{R_t^m \geq 1,\, \kappa_t^B,\, \kappa_t \in [0,1]\}_{t=0}^{\infty}$, $\Omega_t > 0$ *and* $\pi \geq \beta$, *and a fiscal policy setting* $\Gamma \geq 1$, *for a given sequence* $\{\xi_t\}_{t=0}^{\infty}$, *initial values* $M_{-1}^H > 0$, $B_{-1} > 0$, $B_{-1}^T > 0$, *and* $s_{-1} \geq 1$.

When money supply is not effectively rationed due to a *non*-binding money market constraint, the model reduces to a standard sticky price model and a RE equilibrium can be reduced and redefined as follows.

Definition 4 *A RE equilibrium for* $\eta_t = 0$ *is a set of sequences* $\{c_t,\, n_t,\, l_t,\, w_t,\, mc_t,\, \tilde{Z}_t,\, Z_t^1,\, Z_t^2,\, s_t,\, \pi_t,\, R_t^L\}_{t=0}^{\infty}$ *satisfying*

$$\mu_t \theta n_t^{\sigma_n} = w_t c_t^{-\sigma}, \tag{55}$$

$$\xi_t c_t^{-\sigma} = \beta R_t^m E_t[\xi_{t+1} c_{t+1}^{-\sigma} \pi_{t+1}^{-1}], \tag{56}$$

$$R_t^L = R_t^m, \tag{57}$$

(46)-(49), (52)-(54), the transversality conditions, a monetary policy setting $\{R_t^m \geq 1\}_{t=0}^{\infty}$ *and the inflation target* $\pi \geq \beta$*, for a given sequence* $\{\xi_t\}_{t=0}^{\infty}$ *and* $s_{-1} \geq 1$*.*

Given that the policy instruments κ_t and κ_t^B do not appear in definition 4, we can immediately summarize an irrelevance result.

Corollary 1 *The policy instruments* κ_t *and* κ_t^B *are irrelevant for the equilibrium allocation and the associated price system, if the money market constraint is not binding,* $\eta_t = 0$*.*

A.2 Appendix to the flexible price model

Suppose that prices are fully flexible, $\phi = 0$, the utility function is logarithmic in consumption, $\sigma = 1$, and money is only supplied temporarily, $\Omega \to \infty$, such that money is not held outright, $M^H \to 0$. The set of equilibrium conditions given in appendix A.1 can then be reduced to a system in $\{c_t, y_t, n_t, m_t^R, b_t, l_t, w_t, \pi_t, R_t^L\}_{t=0}^{\infty}$

$$\theta n_t^{\sigma_n} = w_t c_t^{-1}, \tag{58}$$

$$\frac{1}{R_t^L} = \frac{\kappa_t}{R_t^m} + (1-\kappa_t)\beta E_t \frac{\xi_{t+1} c_t}{\xi_t c_{t+1} \pi_{t+1}}, \tag{59}$$

$$m_t^R + \kappa_t l_t / R_t^m \geq c_t, \tag{60}$$

$$\kappa_t^B b_{t-1}/\pi_t \geq R_t^m m_t^R, \tag{61}$$

$$\frac{\varepsilon - 1}{\varepsilon} \alpha n_t^{\alpha-1} = w_t R_t^L, \tag{62}$$

$$l_t = R_t^L w_t n_t, \tag{63}$$

$$b_t = \Gamma b_{t-1} \pi_t^{-1}, \tag{64}$$

$$c_t = n_t^{\alpha}, \tag{65}$$

$$y_t = c_t, \tag{66}$$

where the multiplier ψ_t and η_t satisfy (25) and (26). Since we focus on the case where $\eta_t > 0$, we can disregard ψ_t, given that $\eta_t > 0 \Rightarrow \psi_t > 0$. Eliminating m_t^R, n_t, mc_t, w_t, c_t and l_t and setting $\xi_t = 1$ gives the set of equilibrium conditions listed in definition 1.

Proof of proposition 2. Consider the set of equilibrium conditions given in definition 1. Use (29) or $\kappa_t^B b_{t-1}/\pi_t = (R_t^m - \kappa_t \mu)\, y_t$ as well as its time $t+1$ version and $b_t = \Gamma b_{t-1} \pi_t^{-1}$, to eliminate y_t and y_{t+1} in (28):

$$\frac{\kappa_t^B}{R_t^m - \kappa_t \mu} \beta E_t \frac{1}{\Gamma} \frac{R_{t+1}^m - \kappa_{t+1}\mu}{\kappa_{t+1}^B} (1-\kappa_t) + \frac{\kappa_t}{R_t^m} = \frac{1}{R_t^L}.$$

where $\mu = \frac{\varepsilon-1}{\varepsilon}\alpha \in (0,1)$. We further divide both sides of $\kappa_t^B b_{t-1}/\pi_t = (R_t^m - \mu\kappa_t)\, y_t$ by its period $t-1$ version $\kappa_{t-1}^B b_{t-2}/\pi_{t-1} = \left(R_{t-1}^m - \mu\kappa_{t-1}\right) y_{t-1}$, and use $b_{t-1} = \Gamma b_{t-2} \pi_{t-1}^{-1}$, to express inflation as

$$\pi_t = \Gamma \frac{\kappa_t^B}{\kappa_{t-1}^B} \frac{R_{t-1}^m - \mu\kappa_{t-1}}{R_t^m - \mu\kappa_t} \frac{y_{t-1}}{y_t}. \tag{67}$$

We further replace output (=consumption) in (26) with (29) and real bonds with (30), to get the following set of equilibrium conditions

$$\frac{1}{R_t^L} = \frac{\kappa_t}{R_t^m} + (1-\kappa_t)\frac{\beta}{\Gamma} E_t \left[\frac{R_{t+1}^m - \kappa_{t+1}\mu}{R_t^m - \kappa_t\mu} \frac{\kappa_t^B}{\kappa_{t+1}^B} \right], \tag{68}$$

$$y_t = (\mu/\theta)^{\alpha/(1+\sigma_n)} \left(1/R_t^L\right)^{\alpha/(1+\sigma_n)}, \tag{69}$$

$$\pi_t = \Gamma \frac{\kappa_t^B}{\kappa_{t-1}^B} \frac{R_{t-1}^m - \mu\kappa_{t-1}}{R_t^m - \mu\kappa_t} \left(\frac{R_t^L}{R_{t-1}^L} \right)^{\alpha/(1+\sigma_n)}, \tag{70}$$

$$\frac{\eta_t}{y_t^{-1}} = \frac{1}{R_t^m} - \frac{\beta}{\Gamma} E_t \frac{\kappa_t^B}{\kappa_{t+1}^B} \frac{R_{t+1}^m - \mu\kappa_{t+1}}{R_t^m - \mu\kappa_t} > 0, \tag{71}$$

which can be solved sequentially. For a given solution for output and inflation, we can further solve for real bonds and for the initial price level P_0 using (29) and (30). We now consider the case where the central bank instruments are initially set at $\kappa \geq 0$, $\kappa_t^B = \gamma\kappa_{t-1}^B$, where $\kappa_0^B > 0$ and $\gamma > \beta/\Gamma$, and $R^m \in [1, \gamma\Gamma/\beta)$. Then, $\frac{\eta_t}{y_t^{-1}} = \frac{1}{R^m} - \frac{\beta}{\Gamma}\gamma > 0$, such that the money market constraint is binding.

1. Consider a marginal increase in the policy rate in period t from R^m to $R_{t+i}^m = \overline{R}^m$ $\forall i \geq 0$ where $\overline{R}^m \in [R^m, \gamma\Gamma/\beta)$, while $\kappa_t = \kappa > 0$ and $\kappa_t^B = \gamma\kappa_{t-1}^B$. Condition (68) then reads $\frac{1}{R_t^L} = \frac{\kappa}{R_t^m} + (1-\kappa)\frac{\beta}{\Gamma}\frac{1}{\gamma}$, such that

$$\frac{\partial(1/R_t^L)}{\partial R_t^m} = -\frac{1}{(R_t^m)^2}\kappa < 0 \text{ and } \frac{\partial y_t}{\partial R_t^m} = -\frac{\alpha}{1+\sigma_n} y_t \frac{R_t^L}{(R_t^m)^2}\kappa < 0,$$

where we used $\partial y_t / \partial\left(1/R_t^L\right) = [\alpha/\left(1+\sigma_n\right)] y_t R_t^L [\partial(1/R_t^L)/\partial R_t^m]$ (see 69). Inflation satisfies $\pi_t = \Gamma\gamma \frac{R^m - \mu\kappa}{R_t^m - \mu\kappa} \frac{\left(1/R^L\right)^{\alpha/(1+\sigma_n)}}{\left(1/R_t^L\right)^{\alpha/(1+\sigma_n)}}$ (see 70), such that

$$\frac{\partial \pi_t}{\partial R_t^m} = -\Gamma\gamma \frac{\left(1/R^L\right)^{\alpha/(1+\sigma_n)}}{\left(1/R_t^L\right)^{\alpha/(1+\sigma_n)}} \frac{R^m - \mu\kappa}{R_t^m - \mu\kappa} \left(\frac{\kappa}{R_t^m} + (1-\kappa)\frac{\beta}{\Gamma}\frac{1}{\gamma} \right)^{-1}$$
$$\cdot \frac{1}{(R_t^m)^2} \left(\left[\frac{R_t^m}{R_t^m - \mu\kappa} - \frac{\alpha}{1+\sigma_n} \right] \kappa + \frac{(R_t^m)^2}{R_t^m - \mu\kappa}(1-\kappa)\frac{\beta}{\Gamma}\frac{1}{\gamma} \right) < 0$$

where we used that the term in the square bracket is non-negative. There are two effects: First, a higher policy rate lowers the amount of cash for each unit of collateral. Second, it raises the loan rate for $\kappa > 0$, such that output contracts. The second effect on inflation is smaller such that a higher policy rate reduces inflation. Since $\frac{\eta_t}{c_t^{-1}} = \frac{1}{R_t^m} - \frac{\beta}{\Gamma}\gamma$ further holds, the money market constraint is binding, $\eta_t > 0$, given that $\overline{R}^m < \gamma\Gamma/\beta$.

2. Consider a marginal decrease in the growth rate of κ^B from γ to $\gamma_{t+i} = \overline{\gamma} < \gamma \; \forall i \geq 0$, while $R_t^m = R^m$ and $\kappa_t = \kappa$. Then, (68) and (70) reduce to $\frac{1}{R_t^L} = \frac{\kappa}{R^m} + (1-\kappa)\frac{\beta}{\Gamma}\frac{1}{\gamma_t}$ and $\pi_t = \Gamma\gamma_t \frac{y_{t-1}}{y_t}$, such that

$$\frac{\partial(1/R_t^L)}{\partial\gamma_t} = -(1-\kappa)\frac{\beta}{\Gamma}\frac{1}{\gamma_t^2} < 0,$$
$$\frac{\partial y_t}{\partial\gamma_t} = \frac{\partial y_t}{\partial(1/R_t^L)}\frac{\partial(1/R_t^L)}{\partial\gamma_t} = -\frac{\alpha}{1+\sigma_n}y_t R_t^L(1-\kappa)\frac{\beta}{\Gamma}\frac{1}{\gamma_t^2} < 0 \quad \text{and} \quad \frac{\partial\pi_t}{\partial\gamma_t} > 0.$$

where we used $\frac{\partial\pi_t}{\partial y_t} < 0$ for the last inequality. Hence, a lower γ_t reduces the loan rate R_t^L and increases output if $\kappa < 1$, reduces inflation. Since $\frac{\eta_t}{c_t^{-1}} = \frac{1}{R_t^m} - \frac{\beta}{\Gamma\gamma_t}$, further holds, the money market constraint remains binding, $\eta_t > 0$, as long as $\gamma_t > \frac{R^m\beta}{\Gamma}$.

3. Consider a marginal increase in the fraction of eligible loans κ from κ to $\kappa_{t+i} = \overline{\kappa} > \kappa$ $\forall i \geq 0$, while $R_t^m = R^m$ and $\gamma_t = \gamma$. Then, (68) reduces to $\frac{1}{R_t^L} = \frac{\kappa_t}{R^m} + (1-\kappa_t)\frac{\beta}{\Gamma\gamma}$, such that

$$\frac{\partial(1/R_t^L)}{\partial\kappa_t} = \frac{1}{R^m} - \frac{\beta}{\Gamma\gamma} > 0,$$
$$\frac{\partial y_t}{\partial\kappa_t} = y_t[\alpha/(1+\sigma_n)]\left(1/R_t^L\right)^{-1}\left(\frac{1}{R^m} - \frac{\beta}{\Gamma\gamma}\right) > 0.$$

Hence, a higher κ_t unambiguously reduces the loan rate and raises output. The impact on inflation (see 67), is $\frac{\partial\pi_t}{\partial\kappa_t} = \Gamma\gamma\frac{y_{t-1}}{y_t}\frac{R^m-\kappa_{t-1}\mu}{R^m-\kappa_t\mu}\left(\frac{\mu}{R^m-\kappa_t\mu} - \frac{\alpha}{1+\sigma_n}R_t^L\left(\frac{1}{R^m} - \frac{\beta}{\Gamma\gamma}\right)\right)$ and therefore ambiguous due to changes in κ_t and y_t. Since $\frac{\eta_t}{c_t^{-1}} = \frac{1}{R^m} - \frac{\beta}{\Gamma\gamma} > 0$, the money market constraint remains binding.

■

Proof of proposition 3. Condition (68) shows that the growth rate γ_t can change the level but not the growth rate of the loan rate. Condition (70) therefore implies that a permament increase in κ_t^B, but not in its growth rate, leads to a temporary change in inflation. Further, inflation in a long-run equilibrium, where output is constant, only depends on the long-run growth rate γ and $\Gamma : \pi = \Gamma\gamma$. Hence, the central bank can control the inflation rate by setting γ contingent on Γ. Specifically, for $\gamma = \Gamma^{-1}$, prices are stable $\pi = 1$ in the long-run.
■

A.3 Appendix to the sticky price model

In this appendix we first examine the deterministic steady state of the model (steady state variables will not be indexed with a time index) and then present the linear approximation of the model at this steady state.

Steady state The central bank determines $\kappa \in [0,1]$ and target values for the inflation rate $\pi \geq \beta$ and the policy rate $R^m \geq 1$. In a steady state, all endogenous variables grow with a constant rate. Thus, to be consistent with a long-run equilibrium, the time-invariant policy targets have to be consistent with the steady state. In what follows we examine properties of all other endogenous variables in a steady state.

Given the steady state inflation rate π, the equilibrium condition (49) implies $\tilde{Z} = \left(\left(1-\phi\pi^{\varepsilon-1}\right)/\left(1-\phi\right)\right)^{1/(1-\varepsilon)}$, and (48) and Z^1/Z^2 is also constant. The price dispersion term s_t satisfying (54), thus converges in the long run to $s = \frac{1-\phi}{1-\phi\pi^{\varepsilon}}\tilde{Z}^{-\varepsilon}$, given that $\phi\pi^{\varepsilon} < 1 \Leftrightarrow \pi < (1/\phi)^{1/\varepsilon}$. Since s is bounded from below and neither productivity nor labor supply exhibit trend growth, real resources and therefore working time, output, capital and consumption cannot permanently grow with a non-zero rate in the steady state, $y = c - \delta k = k^{1-\alpha}n^{\alpha}/s$. Then, Z_t^2 converges to $Z^2 = yc^{-\sigma}/\left(1-\phi\beta\pi^{\varepsilon-1}\right)$ if $\phi\beta\pi^{\varepsilon-1} < 1 \Leftrightarrow \pi < \left[1/\left(\phi\beta\right)\right]^{1/(\varepsilon-1)}$. Given that Z^1/Z^2 and $\tilde{Z}$ are constant, and $Z_t^1 = Z^1 = \frac{yc^{-\sigma}mc}{1-\phi\beta\pi^{\varepsilon}}$, since $Z_t^1/Z_t^2 = Z^{1,2}$, such that real marginal costs are also constant and given by $mc = \tilde{Z}\left(\varepsilon-1\right)\varepsilon^{-1}\left(1-\phi\beta\pi^{\varepsilon}\right)/\left(1-\phi\beta\pi^{\varepsilon-1}\right)$. Since steady state consumption is constant, (40) and (42) determine the long-run debt rate in the usual way, $R^D = \pi/\beta$. Condition (39) and (40) further imply the steady state loan rate to satisfy

$$\frac{1}{R^L} = \kappa\frac{1}{R^m} + (1-\kappa)\frac{1}{\pi/\beta}. \tag{72}$$

Given that the loan rate, marginal cost, and working time are constant, (46) implies a constant steady state wage rate, $w = mc\alpha n^{\alpha-1}/R^L$. Moreover, the steady state is characterized by $\theta n^{\sigma_n} = wc^{-\sigma}$, $c = n^{\alpha}$, and $l = R^L wn = R^L\theta c^{\sigma+(1+\sigma_n)/\alpha} = \frac{\varepsilon-1}{\varepsilon}\alpha c$.

The linearized model We consider the simplified case, where $\Omega \to \infty$ and $\Gamma = \pi = 1$. Log-linearizing the set of conditions (see appendix A.1) at the steady state gives

$$\alpha^{-1}\widehat{y}_t = \widehat{n}_t, \tag{73}$$

$$\left(\sigma + \sigma_n/\alpha\right)\widehat{c}_t = \widehat{w}_t, \tag{74}$$

$$\widehat{mc}_t - \frac{1-\alpha}{\alpha}\widehat{c}_t = \widehat{w}_t + \widehat{R}_t^L, \tag{75}$$

$$\log\widehat{\pi}_t = \beta E_t \log\widehat{\pi}_{t+1} + \chi\log\widehat{mc}_t, \tag{76}$$

$$\widehat{b}_t = \widehat{b}_{t-1} - \widehat{\pi}_t, \tag{77}$$

$$\widehat{c}_t = \widehat{y}_t, \tag{78}$$

where $\chi = (1-\phi)(1-\beta\phi)/\phi$. Further, log-linearizing (59) leads to

$$\sigma E_t\widehat{c}_{t+1} + E_t\widehat{\pi}_{t+1} + \frac{\kappa}{1-\kappa}\left(1 - \frac{\pi/\beta}{R^m}\right)\widehat{\kappa}_t + \frac{\kappa}{1-\kappa}\frac{\pi/\beta}{R^m}\widehat{R}_t^m$$
$$= \sigma\widehat{c}_t + \left(E_t\widehat{\xi}_{t+1} - \widehat{\xi}_t\right) + \left(1 + \frac{\kappa}{1-\kappa}\frac{\pi/\beta}{R^m}\right)\widehat{R}_t^L,$$

where we used (72). Defining $\varkappa = \frac{\kappa}{1-\kappa}\frac{\pi/\beta}{R^m}$, the last condition can be simplified to

$$\sigma E_t \widehat{c}_{t+1} - \sigma \widehat{c}_t + \left(1-\rho_\xi\right)\widehat{\xi}_t + E_t \widehat{\pi}_{t+1} - \left(\varkappa - \frac{\kappa}{1-\kappa}\right)\widehat{\kappa}_t + \varkappa \widehat{R}^m_t = \left(1+\varkappa\right)\widehat{R}^L_t, \tag{79}$$

where we assumed that the preference shock is autocorrelated: $E_t\widehat{\xi}_{t+1} = \rho_\xi \widehat{\xi}_t$. Further, linearizing $\kappa_t R^L_t w_t n_t + \kappa^B_t b_{t-1}/\pi_t = R^m_t m^R_t$ yields

$$\left(\varpi - \varsigma\right)\widehat{c}_t + \widehat{\kappa}_t + \widehat{R}^L_t + \left(\varsigma - 1\right)\widehat{\kappa}^B_t + \left(\varsigma-1\right)\widehat{b}_{t-1} - \left(\varsigma - 1\right)\widehat{\pi}_t = \varsigma \widehat{R_t}^m, \tag{80}$$

where $\varsigma = \frac{R^m}{\kappa\frac{\varepsilon-1}{\varepsilon}\alpha} > 1$ and $\varpi = \frac{1+\sigma_n}{\alpha} + \sigma > 1+\sigma$. Eliminating $\widehat{c}_t$, $\widehat{w}_t$, $\widehat{n}_t$,and $\widehat{mc}_t$, the system (73)-(80) in $\widehat{R}^L_t$, $\widehat{y}_t$, π_t, and $\widehat{b}_t$ is given by

$$\left(\varsigma - \varpi\right)\widehat{y}_t = \left(\varsigma-1\right)\widehat{b}_{t-1} - \varsigma\widehat{R_t}^m - \left(\varsigma-1\right)\widehat{\pi}_t + \left(\varsigma-1\right)\widehat{\kappa}^B_t + \widehat{\kappa}_t + \widehat{R}^L_t, \tag{81}$$

$$\left(1+\varkappa\right)\widehat{R}^L_t = \sigma E_t\widehat{y}_{t+1} - \sigma\widehat{y}_t + E_t\widehat{\pi}_{t+1} - \left(\varkappa - \frac{\kappa}{1-\kappa}\right)\widehat{\kappa}_t + \varkappa\widehat{R}^m_t + \left(1-\rho_\xi\right)\widehat{\xi}_t, \tag{82}$$

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} + \chi\left(\varpi - 1\right)\widehat{y}_t + \chi\widehat{R}^L_t, \tag{83}$$

$$\widehat{b}_t = \widehat{b}_{t-1} - \widehat{\pi}_t, \tag{84}$$

where $\widehat{\kappa}_t$, $\widehat{\kappa}^B_t$, and $\widehat{R}^m_t$ are set by the central bank. At a steady state where $\kappa = 0$, the conditions (81)-(84) reduce to (31)-(34).

Proof of proposition 4. Consider the equilibrium defined in definition 2. We further assume that the central bank follows the Taylor-rule (35) and set κ^B_t in an exogenous way. Eliminating the policy rate and the loan rate with (31) gives

$$\left(1+\rho_y\right)\widehat{y}_t = \widehat{b}_t - \rho_\pi\widehat{\pi}_t + \widehat{\kappa}^B_t, \tag{85}$$

$$\widehat{\pi}_t = \left(\beta+\chi\right)E_t\widehat{\pi}_{t+1} + \chi\left(\varpi - 1 - \sigma\right)\widehat{y}_t + \chi\sigma E_t\widehat{y}_{t+1} + \chi\left(1-\rho_\xi\right)\widehat{\xi}_t, \tag{86}$$

$$\widehat{b}_t = \widehat{b}_{t-1} - \widehat{\pi}_t. \tag{87}$$

Further substituting out output in (86) with (85) and neglecting terms in $\widehat{\kappa}^B_t$ for simplicity leads to

$$\zeta_2 E_t\widehat{\pi}_{t+1} + \zeta_3\widehat{b}_t = \zeta_1\widehat{\pi}_t - \chi\left(1-\rho_\xi\right)\widehat{\xi}_t, \tag{88}$$

where $\zeta_1 = 1 + \frac{\chi(\varpi-1-\sigma)}{\left(1+\rho_y\right)}\rho_\pi > 0$, $\zeta_2 = \beta + \chi - \chi\sigma\frac{1+\rho_\pi}{1+\rho_y}$, and $\zeta_3 = \frac{\chi(\varpi-1-\sigma)}{\left(1+\rho_y\right)} + \frac{\chi\sigma}{1+\rho_y} > 0$. Hence, the conditions (87) and (88) can be written as

$$\begin{pmatrix} E_t\widehat{\pi}_{t+1} \\ \widehat{b}_t \end{pmatrix} = A\begin{pmatrix} \widehat{\pi}_t \\ \widehat{b}_{t-1} \end{pmatrix} + \begin{pmatrix} \zeta_2 & \zeta_3 \\ 0 & 1 \end{pmatrix}^{-1}\begin{pmatrix} -\chi\left(1-\rho_\xi\right) \\ 0 \end{pmatrix}\widehat{\xi}_t,$$

$$\text{where } A = \begin{pmatrix} \zeta_2 & \zeta_3 \\ 0 & 1 \end{pmatrix}^{-1}\begin{pmatrix} \zeta_1 & 0 \\ -1 & 1 \end{pmatrix}.$$

Given that there exists exactly one predetermined variable, $\widehat{b}_{t-1}$, local determinacy requires one stable and one unstable eigenvalue. The characteristic polynomial of A is given by $F(X) = X^2 + \left(-\frac{\zeta_1}{\zeta_2} - \frac{1}{\zeta_2}\zeta_3 - 1\right) X + \frac{\zeta_1}{\zeta_2}$, where $F(0) = \zeta_1/\zeta_2$ and $F(1) = -\zeta_3/\zeta_2$ and $signF(0) = -signF(1)$. Hence, there exists at least one real stable eigenvalue between zero and one. Further, $F(X)$ at $X = -1$ is given by

$$F(-1) = \left[-\chi\frac{1+2\rho_\pi}{1+\rho_y}\left(\sigma - \frac{1-\alpha+\sigma_n}{\alpha}\right) + 2\left(1+\beta+\chi\right)\right]/\zeta_2,$$

where the term in the square bracket is strictly positive, such that $signF(0) = signF(-1)$, if and only if

$$\frac{\rho_\pi + 1/2}{\rho_y + 1}\left(\sigma - \frac{1-\alpha+\sigma_n}{\alpha}\right) < 1 + \frac{1+\beta}{\chi}.$$

Then, there exists exactly one stable and one unstable eigenvalue, indicating local determinacy. ■

Proof of proposition 5. Consider the equilibrium defined in definition 2 and suppose that the central bank pegs the policy rate $\widehat{R}_t^m = 0$ (e.g. at the ZLB) and sets κ_t^B according to an AR1 process, $\widehat{\kappa}_t^B = \rho_\kappa \widehat{\kappa}_{t-1}^B + \varepsilon_t$. Disregarding preference shocks and eliminating the policy rate and the loan rate with (31) gives

$$\widehat{y}_t = \widehat{b}_t + \widehat{\kappa}_t^B, \tag{89}$$

$$\widehat{\pi}_t = \left(\beta + \chi\right) E_t\widehat{\pi}_{t+1} + \chi\widetilde{\varpi}\widehat{y}_t + \chi\sigma E_t\widehat{y}_{t+1}, \tag{90}$$

where $\widetilde{\varpi} = \varpi - 1 - \sigma > 0$, and (84). Now eliminate $\widehat{y}_t$ in (90) with (89), use $E_t\widehat{\kappa}_{t+1}^B = \rho_\kappa\widehat{\kappa}_t^B$ and that (84) implies $E_t\widehat{b}_{t+1} = \widehat{b}_t - \widehat{\pi}_t$, to get

$$\left(1+\chi\sigma\right)\widehat{\pi}_t = \left(\beta+\chi\right)E_t\widehat{\pi}_{t+1} + \chi\left(\widetilde{\varpi}+\sigma\right)\widehat{b}_t + \chi\left(\widetilde{\varpi}+\sigma\rho_\kappa\right)\widehat{\kappa}_t^B, \tag{91}$$

Condition (91) together with (84) determine the solution for $\widehat{\pi}_t$ and $\widehat{b}_t$. Given that real bonds $\widehat{b}_{t-1}$ are predetermined, the solution takes the form $\widehat{b}_t = u_1\widehat{b}_{t-1} + u_2\widehat{\kappa}_t^B$ and $\widehat{\pi}_t = u_3\widehat{b}_{t-1} + u_4\widehat{\kappa}_t^B$, where the coefficients $u_{1,2,3,4}$ are undetermined. It should be noted that the eigenvalue u_1 has already been show in the proof of proposition 4 to lie between zero and one, $u_1 \in (0,1)$. Plugging the solution forms into (84) and (91) and using that $\widehat{b}_{t-1}$ and $\widehat{\kappa}_t^B$ are uncorrelated, leads to the following conditions

$$u_3 = 1 - u_1 \in (0,1), \quad u_4 = -u_2, \quad 0 = \left[\left(\beta+\chi\right)u_3 + \chi\left(\widetilde{\varpi}+\sigma\right)\right]u_1 - \left(1+\chi\sigma\right)u_3,$$
$$0 = \left(\beta+\chi\right)u_3u_2 + \chi\left(\widetilde{\varpi}+\sigma\right)u_2 + \left(\beta+\chi\right)u_4\rho_\kappa - \left(1+\chi\sigma\right)u_4 + \chi\left(\widetilde{\varpi}+\sigma\rho_\kappa\right),$$

Eliminating u_3 and u_2 in the last condition and isolating u_4, leads to the following expression for $\partial\widehat{\pi}_t/\partial\widehat{\kappa}_t^B = u_4$:

$$\frac{\partial\widehat{\pi}_t}{\partial\widehat{\kappa}_t^B} = \frac{\chi\left(\widetilde{\varpi}+\sigma\rho_\kappa\right)}{\left(\beta+\chi\right)\left(1-u_1\right)+\chi\left(\widetilde{\varpi}+\sigma\right)+\left(1-\beta\rho_\kappa\right)+\chi\left(\sigma-\rho_\kappa\right)} > 0,$$

which is positive given that $\sigma \geq 1$. Hence, inflation increases in response to quantitative easing. Further, we use that output satisfies $\widehat{y}_t = \widehat{b}_t + \widehat{\kappa}_t^B$ and thus $\widehat{y}_t = u_1\widehat{b}_{t-1} + u_5\widehat{\kappa}_t^B$, where $u_5 = 1 - u_4$ and $\partial\widehat{y}_t/\partial\widehat{\kappa}_t^B = u_5$ such that

$$\frac{\partial\widehat{y}_t}{\partial\widehat{\kappa}_t^B} = \frac{\left(\beta+\chi\right)\left(1-u_1\right)+\left(1-\beta\rho_\kappa\right)+\chi\left(\sigma-\rho_\kappa\right)+\chi\sigma\left(1-\rho_\kappa\right)}{\left(\beta+\chi\right)\left(1-u_1\right)+\chi\left(\widetilde{\varpi}+\sigma\right)+\left(1-\beta\rho_\kappa\right)+\chi\left(\sigma-\rho_\kappa\right)} > 0$$

Hence, output also increases in response to quantitative easing at the ZLB. ■

A.4 A model version with physical capital

A RE equilibrium is given by a set of sequences $\{c_t,\, y_t,\, k_t,\, x_t,\, n_t,\, \lambda_t,\, \psi_t,\, \eta_t,\, q_t,\, m_t^R,\, m_t^H,\, b_t,\, b_t^T,\, l_t,\, w_t,\, r_t^k,\, mc_t,\, \tilde{Z}_t,\, s_t,\, \pi_t,\, R_t,\, R_t^D,\, R_t^L\}_{t=0}^{\infty}$ satisfying (42), (44)-(45), (47)-(51), as well as

$$c_t^{-\sigma} = \lambda_t + \upsilon\psi_t, \tag{92}$$

$$\theta n_t^{\sigma_n} = \left(\lambda_t + \psi_t\right) w_t, \tag{93}$$

$$\lambda_t + \psi_t = R_t^m\left(\lambda_t + \eta_t\right), \tag{94}$$

$$\lambda_t + \psi_t = R_t^L\left(\lambda_t + \eta_t\kappa_t\right), \tag{95}$$

$$\begin{aligned}\lambda_t + \omega_t\psi_t = {} & c_t^{-\sigma}q_t\left[S\left(x_t/x_{t-1}\right) + \left(x_t/x_{t-1}\right)S'\left(x_t/x_{t-1}\right)\right] \\ & - \beta E_t c_{t+1}^{-\sigma}q_{t+1}\left[\left(x_{t+1}/x_t\right)^2 S'\left(x_{t+1}/x_t\right)\right],\end{aligned} \tag{96}$$

$$c_{i,t}^{-\sigma}q_t = \beta E_t\left[\lambda_{t+1}r_{t+1}^k + \left(1-\delta\right)q_{i,t+1}c_{i,t+1}^{-\sigma}\right], \tag{97}$$

$$\lambda_t = \beta E_t\frac{\lambda_{t+1}+\psi_{t+1}}{\pi_{t+1}}, \tag{98}$$

$$\lambda_t = \beta E_t\frac{\lambda_{t+1}+\eta_{t+1}\kappa_{t+1}^B}{\pi_{t+1}}R_t, \tag{99}$$

$$\begin{aligned}\upsilon c_t + \omega_t x_t - \kappa_t l_t/R_t^m &= m_t^H + m_t^R, \text{ if } \ \psi_t > 0, \\ \text{or } \upsilon c_t + \omega_t x_t - \kappa_t l_t/R_t^m &\leq m_t^H + m_t^R, \text{ if } \ \psi_t = 0,\end{aligned} \tag{100}$$

$$w_t R_t^L = mc_t\alpha\left(n_t/k_t\right)^{\alpha-1}, \tag{101}$$

$$r_t^k = mc_t\left(1-\alpha\right)\left(n_t/k_t\right)^{\alpha}, \tag{102}$$

$$k_{i,t+1} = \left(1-\delta\right)k_{i,t} + x_{i,t}S\left(x_{i,t}/x_{i,t-1}\right), \tag{103}$$

$$y_t = n_t^{\alpha}k_t^{1-\alpha}/s_t, \tag{104}$$

$$y_t\left(1-g/y\right) = c_t + x_t, \tag{105}$$

(where q_t denotes the value of installed capital relative to consumption goods and the adjustment cost function is given by $S\left(x_t/x_{t-1}\right) = 1 - \frac{\vartheta}{2}\left(x_t/x_{t-1}-1\right)^2$) as well as the transversality conditions, a monetary policy setting $\{R_t^m \geq 1,\ \kappa_t^B,\ \kappa_t \in [0,1]\}_{t=0}^{\infty}$, $\Omega_t > 0$, and $\pi \geq \beta$, and a fiscal policy setting $\Gamma \geq 1$ and $g/y > 0$, for a given sequence $\{\omega_t\}_{t=0}^{\infty}$ and initial values $M_{-1}^H > 0$, $B_{-1} > 0$, $B_{-1}^T > 0$, and $s_{-1} \geq 1$.

A.5 Parameter values

Table A1 Benchmark parameter values	
Subjective discount factor	$\beta = 0.992$
Inverse of intertemporal substitution elasticity	$\sigma = 2$
Inverse of Frisch elasticity of labour supply	$\sigma_n = 1$
Substitution elasticity	$\varepsilon = 10$
Steady state working time	$n = 0.33$
Labour share	$\alpha = 0.66$
Investment adjustment cost	$\vartheta = 2.48$
Rate of depreciation of capital stock	$\delta = 0.03$
Government expenditure share (constant)	$g = 0.19$
Calvo price stickiness	$\phi = 0.75$
Steady state interest rate	$R^m = 1.0105$
Taylor rule inflation coefficient	$w_\pi = 1.5$
Taylor rule output coefficient	$w_y = 0.5$
Steady state share of repos to outright purchases	$\Omega = 1.5$
Steady state share of loans eligible in open market operations	$\kappa = 0$
Steady state share of gov. bonds eligible in open market operations	$\kappa^B = 1$
Steady state inflation	$\pi = 1.00575$
Steady state cash requirement for consumption	$\nu = 0.7399$
Steady state cash requirement for investment	$\omega = 0.4292$