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Abstract

Even under antitrust enforcement, firms may still form a cartel in an infinitely-repeated oligopoly model when the discount factor is sufficiently close to one. We present a linear oligopoly model where the profit-maximizing cartel price converges to the competitive equilibrium price as the discount factor goes to one. We then identify a set of necessary conditions for this seemingly counter-intuitive result.

JEL Classification: L4 Antitrust Policy, C7 Game Theory and Bargaining Theory

Keywords: Antitrust enforcement, Cartel, Oligopoly, Repeated game

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1 Introduction

It is well-known that in an infinitely-repeated oligopoly model, almost all prices, including the monopoly price, can be sustained by trigger strategy profiles as cartel prices for sufficiently patient firms. Since the monopoly price leads to the highest deadweight loss in social welfare, the effects of antitrust enforcement are often confined to the question whether enforcement deters monopoly pricing, see e.g. Block et al. (1981) and Harrington (2005). Antitrust enforcement in practice is regarded as too weak to deter monopoly pricing. We investigate and reexamine the latter conclusion by presenting a simple model where the profit-maximizing cartel price converges to the competitive equilibrium price as the firms become sufficiently patient. Because of its important policy implications for antitrust enforcement, we investigate conditions for such a convergence result in a general infinitely-repeated oligopoly model.

The model studied here is a repeated-game version of the dynamic model in Harrington (2004, 2005), who extensively motivates most of the assumptions adopted in our model. In particular, fines (including other liabilities) and detection probabilities depend upon the current and past prices, which makes Harrington's model a dynamic game. In a steady state, however, the equilibrium analysis in any period will only depend on the cartel price in the current period. For explanatory reasons, we assume that detection probabilities and fines depend upon the current price only.

One crucial feature in our model is that the cartel may reestablish after it is detected and prosecuted. Our approach unifies two extreme treatments in the literature. On one extreme, Harrington (2004, 2005) assumes that a cartel dissolves after it is caught and fined. On other extreme, Motta and Polo (2003) investigate notorious cartels, that will reestablish no matter how many times it has been convicted. Here we assume that a cartel will reestablish probabilistically in every period in which it is detected and fined.

This note makes several contributions. First, the presence of probabilistic reestablishment of the cartel requires different equilibrium conditions on sustainable cartel prices. In Section 2, we set up such a model with probabilistic reestablishment of the cartel and provide the

corresponding equilibrium conditions. Section 3 is devoted to a linear Bertrand oligopoly model with linear detection probabilities. The fine is proportional to the firms' illegal gains, as suggested by the current US sentencing guidelines. We show that the profit-maximizing cartel price converges to the competitive Bertrand equilibrium price as the discount factor goes to one. In Section 4, we establish a set of necessary conditions for such a seemingly counter-intuitive result in a general setup and provide intuition for this convergence.

2 The Model

Consider an infinitely-repeated oligopoly model in the presence of antitrust enforcement. In each period, firms compete in prices and the antitrust authority (AA) investigates the market. If the firms collude, they will be caught and fined with certain probability. Both the probability of detection and the fine depend on how serious the anti-trust violation is in the current period.

Price competition in every period is modelled as a symmetric Bertrand game among $n \geq 2$ firms.¹ Let $\pi(p_1, \dots, p_n)$ be a firm's per-period profit for prices $p_1, \dots, p_n \in \mathbb{R}_+$. For convenience, let $\pi(p) \equiv \pi(p, \dots, p)$ and $\pi^{opt}(p) \equiv \sup_{p'} \pi(p', p, \dots, p)$ be a firm's profit from a unilateral deviation against the cartel price p . Denote the symmetric competitive (Nash) equilibrium price and the maximal collusive (or monopoly) price by p^N and p^M , respectively. As in Harrington (2004, 2005), we assume that $\pi(p)$ is continuous and strictly increasing in $p \in [p^N, p^M)$ with a maximum at p^M , $\pi^{opt}(p)$ is continuous, strictly increasing and $\pi^{opt}(p) > \pi(p) > 0$ for $p \in (p^N, p^M]$. To further simplify the exposition, we normalize the model such that $\pi(p^N) = 0$ and interpret $\pi(p)$ as the net profit above $\pi(p^N)$.

As motivated by Harrington (2004, 2005), the detection probability depends upon the cartel's price setting and this reflects that a higher cartel price might raise more suspicions about cartel abuse and, therefore, makes detection (and conviction) more likely. If firms

¹Our analysis also applies to quantity competition oligopoly model with proper revision of detection probability and fine functions.

collude at price $p > p^N$, they will be detected with probability $\beta(p) \in [0, 1]$, which is non-decreasing and differentiable in p . By default, $\beta(p^N) = 0$. We forego imposing right-continuity of $\beta(p)$ at $p = p^N$ to capture the situation of constant detection probabilities for $p > p^N$. As in Rey (2003), only current period's misconduct is prosecuted. If the firms are found guilty, every firm will have to pay the one-time fine $f(p) \geq 0$, which is a non-decreasing, differentiable function on $p \in (p^N, p^M]$ that may not be right continuous at $p = p^N$ to capture fixed fines for $p > p^N$. In order to avoid triviality of the model, we also assume that $F(p) \equiv \beta(p) f(p) < \pi(p)$ for all $p > p^N$. Note that $\beta'(p) \geq 0$ and $f'(p) \geq 0$ imply that $F'(p) \geq 0$. Also, $0 \leq \lim_{p \downarrow p^N} F(p) \leq \lim_{p \downarrow p^N} \pi(p) = 0$ implies $F(p)$ must be continuous at $p = p^N$ even though $\beta(p)$ or $f(p)$ might not be right continuous.

There are two different treatments in the literature of how firms behave after each conviction. In Harrington (2004, 2005), being caught once is sufficient to deter cartel activity in the future. In Motta and Polo (2003), the economic sector is notorious for cartel activities despite many convictions. To unify these two different treatments, let $\gamma \in [0, 1]$ be the probability that the firms stop illegal activities after each conviction. $\gamma = 0$ implies that cartel is notorious implies, while $\gamma = 1$ means the sector becomes competitive after the first conviction of a cartel.

In the repeated game, every firm has a common discount factor $\delta \in (0, 1)$ per period. It is well-known that an infinitely-repeated game, such as the model studied in this paper, generally admits multiple equilibrium outcomes when the discount factor is sufficiently close to one. We are interested in subgame perfect equilibria with the following modified trigger strategy profile to sustain a cartel price of $p > p^N$: Firms collude at price $p > p^N$ in the first period and continue to collude at price p as long as no firm deviates in setting this price. Any price deviation by any firm leads to perpetual competition at price p^N . Cartel will be detected with probability $\beta(p)$, after which the firms continue to collude with probability $1 - \gamma$ in the following period and switch to perfect competition with probability γ forever. Under such a strategy profile, the present value of a firm's expected profit $V(p; \delta)$ is determined

recursively by

$$V(p; \delta) = \pi(p) + [1 - \beta(p)] \delta V(p; \delta) + \beta(p) [(1 - \gamma) \delta V(p; \delta) - f(p)],$$

which yields

$$V(p; \delta) = \frac{\pi(p) - F(p)}{1 - \delta + \delta \gamma \beta(p)}. \quad (1)$$

Assuming that any price-deviating firm will not be fined, the equilibrium condition becomes $V(p; \delta) \geq \pi^{opt}(p)$.² We assume that $V(p; \delta)$ is strictly log-concave on $[p^N, p^M]$, which ensures that $V(p; \delta)$ has a unique maximum on $[p^N, p^M]$ that is a continuous function of $\delta \in (0, 1)$ and other parameters. The profit-maximizing cartel price is then

$$p^C(\delta) \in \arg \max_p V(p; \delta) \quad \text{subject to } V(p; \delta) \geq \pi^{opt}(p). \quad (2)$$

3 A Linear Bertrand Model

In this section, we show that the profit-maximizing cartel price can converge to the competitive equilibrium price as the discount factor goes to one. Consider a homogeneous Bertrand oligopoly model with linear demand $2 - p$ and constant marginal costs of 0. Then, we have

$$\pi(p) = \frac{1}{n} p(2 - p), \quad p^N = 0, \quad p^M = 1, \quad \text{and } \pi^{opt}(p) = p(2 - p) \quad \text{for all } p \in (p^N, p^M].$$

The antitrust regulation is given by $\beta(p) = \beta p$ with $\beta > 0$, and $f(p) = k\pi(p)$, where the fine function reflects the current practice in the US (see Harrington 2004, 2005). Note that $0 \leq F(p) < \pi(p)$ for all $p \in (0, 1]$ if $0 \leq \beta k < 1$. Recall that p can be sustained as a cartel price by our modified grim trigger strategy profile if and only if $V(p; \delta) \geq \pi^{opt}(p)$, which holds as long as $[\beta(k + n\gamma)]p < 1$. Note that this condition is independent of the discount factor δ . This condition ensures that any p that is sufficiently close to the competitive price $p^N = 0$ can be sustained as a cartel price for sufficiently large discount factors. This fact confirms the assertion of Harrington (2004, 2005) that the equilibrium conditions are always non-binding for sufficiently large $\delta < 1$.

²Our analysis is still valid if a price-deviating firm is fined when the cartel is detected.

With our modified trigger strategy profile, a firm's value function is given by

$$V(p; \delta) = \frac{1}{n} \frac{(1 - \beta kp)(2 - p)p}{1 - \delta + \gamma\delta\beta p}. \quad (3)$$

Observe that $V(p; \delta)$ is log concave in $p \in (0, 1]$ because

$$\frac{\partial^2 \ln V(p; \delta)}{\partial p^2} = -\frac{(\beta k)^2}{(1 - \beta kp)^2} - \frac{1}{(2 - p)^2} - \frac{1}{p^2} + \frac{(\gamma\delta\beta)^2}{(1 - \delta + \gamma\delta\beta p)^2} \leq \frac{\partial^2 \ln V(p; 1)}{\partial p^2} < 0.$$

Because any price that is sufficiently close to the competitive price can be supported as a cartel price, and as we will show, the profit-maximizing cartel price converges to the competitive price, the constraint in (2) becomes nonbinding for sufficiently large δ . Consequently, for sufficiently large δ , $p^C(\delta)$ is characterized by $\partial V(p, \delta) / \partial p = 0$, that is,

$$(2 - 2p - 4\beta kp + 3\beta kp^2)(1 - \delta + \gamma\delta\beta p) - \gamma\delta\beta(2p - p^2 - 2\beta kp^2 + \beta kp^3) = 0. \quad (4)$$

Denote $\lim_{\delta \rightarrow 1} p^C(\delta) = \hat{p}$. Taking the limit of (4) as $\delta \rightarrow 1$, and given $\gamma\beta > 0$, we have

$$2\beta k\hat{p}^3 - \hat{p}^2 - 2\beta k\hat{p}^2 = 0 \Rightarrow \hat{p} = 0 \text{ and } \hat{p} = 1 + \frac{1}{2k\beta} > 1.$$

Obviously, $\hat{p} = 1 + \frac{1}{2k\beta} > p^M$ cannot be the limit of profit-maximizing cartel price. Hence, $\lim_{\delta \rightarrow 1} p^C(\delta) = 0 = p^N$, which is the main message of this paper.

To conclude this section, we demonstrate that the profit-maximizing cartel price $p^C(\delta)$ is nonmonotonic in δ . When the equilibrium condition in (2) is binding, $p^C(\delta)$ is the largest price $p \in [0, 1]$ satisfying $V(p; \delta) \geq \pi^{opt}(p)$:

$$\max_{p \in [0, 1]} p, \text{ s.t. } \frac{1 - \beta kp}{n(1 - \delta + \gamma\delta\beta p)} \geq 1 \quad \Rightarrow \quad p^C(\delta) = \min \left\{ \frac{1 - n(1 - \delta)}{n\gamma\delta\beta + k\beta}, 1 \right\},$$

which monotonically nondecreases in the discount factor $\delta \in (0, 1)$. When the equilibrium condition in (2) is nonbinding, then $p^C(\delta)$ is the solution to (4) in $[0, 1]$. Generally speaking, we cannot obtain the analytical solutions to (4). However, when $k\beta = 1/2$, (4) simplifies to

$$(2 - p) \left[-\beta\gamma\delta p^2 - \frac{3}{2}(1 - \delta)p + 5(1 - \delta) \right] = 0 \\ \Rightarrow p^C(\delta) = \frac{\sqrt{\frac{9}{4}(1 - \delta)^2 + 20\beta\gamma\delta(1 - \delta)} - \frac{3}{2}(1 - \delta)}{2\beta\gamma\delta},$$

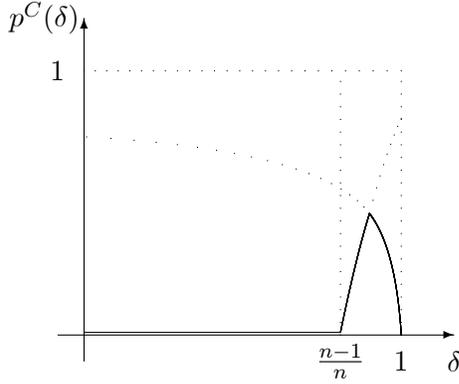


Figure 1: The profit-maximizing cartel price $p^C(\delta)$.

which decreases in $\delta \in (0, 1)$ and converges to the competitive equilibrium price $p^N = 0$ as δ goes to 1. Figure 1 illustrates that the profit-maximizing cartel price is nonmonotonic with respect to the discount factor δ . When δ is small enough, only the competitive price can be the equilibrium price. On the other hand, when the discount factor δ is sufficiently close to 1, the equilibrium condition is nonbinding so (4) characterizes the profit-maximizing cartel price (which decreases in δ). For an intermediate range of discount factors, the profit-maximizing cartel price is determined by the equilibrium condition, and increases in δ . It is worthwhile to note that as the discount factor increases, the set of equilibrium prices grows, yet the profit-maximizing cartel price decreases to the competitive price eventually.

4 Conditions for Convergence

To establish our convergence result, we first need to make sure whether all p that are sufficiently close to the competitive price can be supported by the modified trigger strategy profiles described in Section 2 for sufficiently large $\delta \in (0, 1)$.

Proposition 1 *If there exists $\varepsilon > 0$ such that*

$$\pi(p) > \beta(p) [f(p) + \gamma\pi^{opt}(p)] \quad \text{for all } p \in (p^N, p^N + \varepsilon), \quad (5)$$

then any $p \in [p^N, p^N + \varepsilon)$ can be sustained in equilibrium for all $\delta \in [\delta', 1)$ for some $\delta' < 1$.

Proof. Observe that $p = p^N$ can always be sustained in equilibrium for all $\delta \in (0, 1)$. (5) implies that for all $p \in (p^N, p^N + \varepsilon)$, $\pi(p) - F(p) > \gamma\beta(p)\pi^{opt}(p)$. Because $V(p; \delta)$ is continuous in δ , there exists some $\delta' < 1$ such that for all $\delta \in [\delta', 1)$,

$$V(p; \delta) = \frac{\pi(p) - F(p)}{1 - \delta + \delta\gamma\beta(p)} \geq \pi^{opt}(p),$$

which means that $p \in [p^N, p^N + \varepsilon)$ can be sustained in equilibrium for all $\delta \in [\delta', 1)$. ■

Note that $\beta(p)f(p) < \pi(p)$ implies (5) for notorious cartels, i.e. $\gamma = 0$. For other γ , such as $\gamma = 1$, the more restrictive (5) is needed to guarantee sustainable cartel prices above p^N . It is straightforward to verify that the linear model presented in Section 2 satisfies (5).

Proposition 1 asserts that the equilibrium condition will not be binding for sufficiently large $\delta \in (0, 1)$ whenever $p^C(\delta)$ converges to p^N as δ goes to one. Consequently, $p^C(\delta)$ maximizes $V(p; \delta)$ for sufficiently large $\delta \in (0, 1)$. The next proposition provides necessary conditions for such convergence:

Proposition 2 *Under (5) and $\beta'(p) > 0$ for all $p \in (p^N, p^N + \varepsilon)$,*

$$\text{if } \lim_{\delta \rightarrow 1} p^C(\delta) = p^N, \text{ then either } \pi'(p^N) = \lim_{p \downarrow p^N} F'(p) \text{ or } \lim_{p \downarrow p^N} \beta(p) = 0. \quad (6)$$

Proof. If $\lim_{\delta \rightarrow 1} p^C(\delta) = p^N$, then Proposition 1 implies that the equilibrium condition in (2) is not binding when δ is sufficiently close to 1. Consequently, there exists a $\delta'' \in [\delta', 1)$ such that for all $\delta \in [\delta'', 1)$, we have $V'(p^C(\delta)) = 0$, or

$$(1 - \delta + \gamma\delta\beta(p^C(\delta))) [\pi'(p^C(\delta)) - F'(p^C(\delta))] = \gamma\delta [\pi(p^C(\delta)) - F(p^C(\delta))] \beta'(p^C(\delta)), \quad (7)$$

where the first term on the left-hand side and the last two terms on the right-hand side are positive. If $\gamma\beta'(p^C(\delta)) = 0$, then (7) reduces to $\pi'(p^C(\delta)) = F'(p^C(\delta))$, which implies that $p^C(\delta)$ is independent of $\delta \in [\delta'', 1)$. So for $\lim_{\delta \rightarrow 1} p^C(\delta) = p^N$, it is necessary that $p^C(\delta) = p^N$ for all $\delta \in [\delta'', 1)$, i.e. $\pi'(p^N) = \lim_{p \downarrow p^N} F'(p)$. Otherwise, i.e. $\gamma\beta'(p^C(\delta)) > 0$, by $\lim_{p \downarrow p^N} [\pi(p) - F(p)] = 0$, taking the limit of (7) as $\delta \rightarrow 1$ yields

$$\gamma \cdot \lim_{\delta \rightarrow 1} \beta(p^C(\delta)) \cdot \lim_{\delta \rightarrow 1} [\pi'(p^C(\delta)) - F'(p^C(\delta))] = 0$$

which implies that either $\pi'(p^N) = \lim_{p \downarrow p^N} F'(p)$ or $\lim_{\delta \rightarrow 1} \beta(p^C(\delta)) = 0$. ■

In case either $\gamma = 0$ or $\beta'(p) = 0$ for all $p \in (p^N, p^N + \varepsilon)$, only the first condition under (6) is necessary for convergence. This condition implies that penalties $F(p)$ such that $\lim_{p \downarrow p^N} F'(p) = \pi'(p^N)$ lead to the competitive outcome independent of $\gamma \in [0, 1]$ and $\beta'(\cdot)$. If the first condition fails, convergence is impossible for notorious cartels ($\gamma = 0$) or detection probabilities that are constant. For less notorious cartels ($\gamma > 0$) under increasing detection probabilities ($\beta'(p) > 0$), convergence to the competitive outcome requires the second condition $\lim_{p \downarrow p^N} \beta(p) = 0$, as in the linear model. We interpret $\lim_{p \downarrow p^N} \beta(p) = 0$ as putting relatively less effort on prosecuting mild abuses just above the competitive equilibrium price, and $\beta'(p) > 0$ as monitoring price fluctuations. So, such antitrust enforcement can be effective in reducing the cartel price to the competitive equilibrium price.

References

- [1] Block, M., F. Nold, and J. Sidak (1981). The Deterrent Effect of Antitrust Enforcement. *Journal of Political Economy*, 89, 429-445.
- [2] Harrington, J. (2004). Cartel pricing dynamics in the presence of an antitrust authority. *The Rand Journal of Economics* 35, 651-673.
- [3] Harrington, J. (2005). Optimal cartel pricing in the presence of an antitrust authority. *International Economic Review* 46, 145-170.
- [4] Motta, M. and M. Polo (2003). Leniency programs and cartel prosecution. *International Journal of Industrial Organization* 21, 347-379
- [5] Rey, P. (2003). Towards a theory of competition policy. In M. Dewatripont, L. Hansen, and S. Turnovsky (Eds.), *Advances in Economics and Econometrics: Theory and Applications*. Cambridge University Press.