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World Equity Premium based Risk Aversion Estimates[†]

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January 4, 2010

Abstract

The equity premium puzzle holds that the coefficient of relative risk aversion estimated from the consumption based CAPM under power utility is excessively high. Moreover, estimates in the literature vary considerably across countries. We gauge the uncertainty pertaining to the country risk aversion estimates by means of jackknife resampling and pooling. The confidence band for the world risk aversion estimate from the pooled country data is much tighter and the pooled point estimate presents less of a puzzle than the individual country estimates.

Keywords: Equity premium puzzle; Jackknife; Pooling
JEL: E21, G12

1 Introduction

The equity premium puzzle is the empirical observation due to Mehra and Prescott (1985) that the coefficient of relative risk aversion estimated from the macro consumption based CAPM with power utility is excessively high on US data. This observation has fascinated many in the economics profession over the past two decades. Progress has been made towards understanding the puzzle by looking into the consequences of limited participation in the stock market (Mankiw and Zeldes, 1991; Attanasio et al., 2002; Vissing-Jorgensen, 2002), habit formation of investors (Campbell and Cochrane, 1999) and decoupling risk aversion from the elasticity of intertemporal substitution (Epstein and Zin, 1989). But the final verdict is not out.

The point of this paper is not to come up with yet another theoretical explanation for the high coefficient of relative risk aversion. Rather, we ask to which extent the equity premium puzzle is a statistical phenomenon due to lack of macro data reliability. In his authoritative review Campbell (1998) already showed that the estimates vary widely across a number of countries, but gave no statistical analysis of this uncertainty. We replicate Campbell's results, extend his data set and use the jackknife resampling method to gauge the uncertainty. Furthermore, we show that by pooling the country data one obtains a much tighter confidence band on the world coefficient of relative risk aversion than an analysis based on an individual country.

The remainder of this paper is organized as follows. In the next section the model is outlined. Our methodology is introduced in the third section. Next, the data used in our empirical investigation are described. Section 5 discusses the results and the final section concludes.

2 Model

In the canonical framework where a representative investor has a time-separable power utility function over aggregate consumption, the Euler equation reads

$$1 = E_t[R_{i,t+1}M_{t+1}]. \tag{1}$$

Here R_i is the gross return on some asset i and M denotes the stochastic discount factor. The discount factor is $M_{t+1} = \delta(C_{t+1}/C_t)^{-\gamma}$, where δ is the subjective rate of time preference, C is aggregate consumption and γ is the coefficient of relative risk aversion. Supose that asset returns and aggregate consumption are conditionally lognormally distributed with constant variance. Take logs of equation (1) and rewrite to get

$$E_t r_{i,t+1} = -\log \delta + \gamma E_t \Delta c_{t+1} - \frac{\sigma_i^2 + (\gamma \sigma_c)^2 - 2\gamma \sigma_{ic}}{2}.$$

The small case letters denote logarithms; σ_i^2 and σ_c^2 are, respectively, the unconditional variance of the log return innovations and consumption innovations and σ_{ic} is their unconditional covariance. For a riskless asset, return innova-

tions and the covariance with consumption are both zero, implying that the log risk premium for asset i can be conveniently expressed as

$$\mathrm{E}_t[r_{i,t+1} - r_{f,t+1}] + \frac{\sigma_i^2}{2} = \gamma\sigma_{ic}. \tag{2}$$

In words, equation (2) says that the expected excess returns of equity are equal to the amount of risk aversion times the correlation between consumption growth and excess returns minus a correction factor (i.e. a Jensen inequality term arising as we are using expectations of log returns). The expression permits a calibration of the risk aversion parameter γ from estimates of the moments in equation (2). In most cases US data on these moments are used. As the US has historically been blessed with high equity returns and rather smooth consumption growth, typical γ estimates are high.

Using moment estimates given in equation (2) to back out the parameter of relative risk aversion gives a point estimate, but not a confidence interval. For this reason confidence bands are mostly not reported[1]. Given that estimates in the literature vary quite dramatically it is of interest to provide for a confidence interval. The next section uses the jackknife resampling scheme to provide confidence intervals.

3   Methodology

We use the block-jackknife procedure, see e.g. Shao and Tu (1995), as it is easy to implement and because it can deal with the serial correlation that is present in the consumption series. Let n be the size of the sample, let m denote the number of omitted observations in a resample and let N denote the number of resamples. Note that the total number of resamples is $N = n - m + 1$. To estimate the variance of $\hat{\gamma}_n$, the estimate of γ based on all observations, one first deletes m subsequent observations at a time and denotes the new estimate of γ by $\hat{\gamma}^{(i)}$. Then the i-th pseudovalue of $\hat{\gamma}_n$, denoted by $\tilde{\gamma}^i$, is defined as $\left[n\hat{\gamma}_n - (n-m)\hat{\gamma}^{(i)} \right]/m$. The resulting vector of pseudovalues across the resamples is used to estimate the variance of $\hat{\gamma}_n$, i.e.

$$S_{\hat{\gamma}_n}^2 = \frac{m}{nN} \sum_{i=1}^{N} \left(\tilde{\gamma}^{(i)} - \frac{1}{N} \sum_{i=1}^{N} \tilde{\gamma}^{(i)} \right)^2. \tag{3}$$

These estimates can then be used to form a country-specific confidence interval (CI) for $\hat{\gamma}_n$. More specifically, the Quenouille-Tukey mean of the pseudovalues, see Shao and Tu (1995, p. 6), is the bias-corrected version of the estimate of γ_n and the required critical value is taken from a t-distribution, with $N - 1$ degrees of freedom (Miller, 1974).

Our second contribution is to use the cross-sectional dimension of our data set to get a CI for the pooled estimator, to which we refer as the world coefficient

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[1]Cecchetti et al. (1993) use GMM to construct standard errors of estimated parameters. Their results are difficult to compare with our outcomes, since they employ a Markov switching model.

of relative risk aversion (γ_w). It is highly likely that the information contained in the estimates of γ_n differs between countries, so some kind of weighted average seems to be a natural choice. As the block-jackknife procedure gives us the country-specific sample variances, we can use the optimal weights from Graybill and Deal (1959), defined as

$$w_j = \frac{1/S^2_{\hat{\gamma}_{n,j}}}{\sum_{j=1}^{k} 1/S^2_{\hat{\gamma}_{n,j}}}, \tag{4}$$

where k denotes the number of countries. Confidence intervals are then formed by weighting the country-specific averages of the pseudovalues to get $\hat{\gamma}_w$. It is straightforward to show that the variance of this estimate is given by

$$S^2_{\hat{\gamma}_w} = \frac{1}{\sum_{j=1}^{k} 1/S^2_{\hat{\gamma}_{n,j}}}. \tag{5}$$

The critical values are based on a t-distribution with $k-1$ degrees of freedom.

Besides the optimal weighting scheme, we also employ two economic based weighting schemes. For these we use GDP and stock market capitalization. The variance of the world coefficient of risk aversion is then given by

$$S^2_{\hat{\gamma}_w} = \sum_{j=1}^{k} (w_j S_{\hat{\gamma}_{n,j}})^2, \tag{6}$$

where w_j is now defined as the GDP (market capitalization) of country j divided by the sum of GDP (market capitalization) of all countries.

4 Data

The methodology described in the previous section is applied to data for OECD countries. Of the 30 countries, 9 had to be dropped for the following reasons. Iceland, Luxembourg and the Slovak Republic are excluded since we do not have three-month interest rates for these countries. Too few observations has led to the exclusion of Greece, Hungary, Ireland, Mexico, Portugal and Turkey. For all 21 remaining countries, data are taken from three different sources. First, private final consumption expenditure (Quarterly National Accounts, in constant prices)[2] and CPI (Main Economic Indicators, all items) are from the OECD. Second, equity returns (in local currency units) are from MSCI. Finally, the risk-free rate and population figures are from the International Financial Statistics provided by the IMF. The sample period is country-specific, as can be seen in Table 1. Thus the panel is unbalanced.

For the economic based weighting schemes we need two additional variables. The first is GDP (Annual National Accounts, in US dollars, current prices, current exchange rates; OECD) and the second is market capitalization in US dollars (World Federation of Exchanges).

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[2]Note that this implies that our results are independent of the measure of inflation.

5 Results

Estimates of γ_n based on equation (2) are shown in Table 1. The number of omitted observations in each block, m, is set to 4 so that we leave out one year of observations at a time. Four of the 21 coefficients of relative risk aversion are negative, implying that agents are risk loving. The remaining estimates are positive and rather high, with the exception of Korea and Poland. Yet the results are in line with those reported by Campbell (1998). In the Appendix we replicated the results of Campbell, but add our estimates for the pseudovalues and the confidence bands[3].

The averages of the pseudovalues, i.e. $\sum_{i=1}^{N} \tilde{\gamma}_i$, are reported in the next column. Note that these differ considerably from the overall estimates $\hat{\gamma}_n$. Given the small value of m, this is indicative for the uncertainty in the estimates of γ. More than half of these values are negative, perhaps implying that most estimates are severely biased. These average pseudovalues are the basis of the symmetric 95% confidence interval. From this, it is evident that there is a lot of uncertainty in the estimate of the coefficient of relative risk aversion. Only the confidence band for Korea is rather tight, the others are fairly wide. The averages of the pseudovalues may look fluke, but if one considers the CI, or realizes that the CI for Campbell (1998) also comprise a very wide range, the point estimates of Table 1 look more in line with the generally smaller point estimates from the Campbell subset (see Appendix).

The confidence intervals also show that the null hypothesis of a coefficient of relative risk aversion between one and ten, a range most economists (see, e.g. Mehra and Prescott (1985)) consider to be a reasonable guess, is never rejected on the basis of the individual country jackknifed confidence bands.

Some preliminary results from pooling can already be deducted from Table 1. In particular, consider the means and standard errors of the third and fourth columns. The mean of $\hat{\gamma}_n$ is 97, with a standard error of 50; implying that a confidence interval based on these numbers would be rather wide. When using the bias-corrected estimates, the results are even worse. With a mean of -517 and a standard error of 338, the resulting confidence interval would be even wider. However, note that these results are obtained under equal weighting. Hence the estimate of Switzerland would have a weight equal to that of Korea, although the latter is estimated much more precisely. Now we turn to some results in which some alternative weighting schemes are used.

Table 2 presents the results based on pooling of country-specific data, which gives an estimate of the world risk aversion parameter. The top panel presents the outcomes obtained while using the optimal variance weights. The optimally weighted mean of the pseudovalues is just over 4.5 (hence within this plausible range) and the 95% CI is rather tight. Note that it is less wide than its counterpart for the US, hence pooling of information is clearly beneficial. Moreover, the resultant CI contains the often hypothesized reasonable values of below 10. Again equality to one is not rejected for the world risk aversion parameter.

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[3]Note that all estimates of γ_n are identical, except the one for the US. The reason is that the riskfree rate we employ differs slightly from the one used by Campbell.

Table 1: **Country-specific results**

Notes: The estimate of the coefficient of relative risk aversion is denoted by $\hat{\gamma}_n$, $\overline{PS}$ is the mean of the pseudovalues, defined in Section 3. The confidence interval is based on $\overline{PS}$, with standard error equal to the square root of the variance estimated by equation (3) and critical value from a t-distribution with $N-1$ degrees of freedom. Mnemonics are as follows: Australia (AU), Austria (OE), Belgium (BG), Canada (CN), Czech Republic (CZ), Denmark (DK), Finland (FN), France (FR), Germany (BD), Italy (IT), Japan (JP), Korea (KO), the Netherlands (NL), New Zealand (NZ), Norway (NW), Poland (PO), Spain (ES), Sweden (SD), Switzerland (SW), the United Kingdom (UK) and the United States (US).

Country	Sample period	$\hat{\gamma}_n$	$\overline{PS}$	95% CI	
AU	1970Q1 - 2003Q3	128.137	-649.743	-6436.591	5137.106
OE	1988Q2 - 2003Q3	91.785	105.830	-164.519	376.178
BG	1995Q2 - 2003Q3	194.012	-890.138	-3253.383	1473.107
CN	1970Q1 - 2003Q3	45.283	31.026	-63.631	125.683
CZ	1996Q2 - 2003Q3	-192.062	-1186.110	-2549.966	177.745
DK	1990Q2 - 2003Q3	97.100	-1148.469	-3548.994	1252.056
FN	1990Q2 - 2003Q3	205.859	-0.029	-824.942	824.884
FR	1978Q2 - 2003Q3	165.192	126.980	-99.475	353.436
BD	1975Q4 - 2003Q3	165.699	408.698	-1242.742	2060.138
IT	1981Q2 - 2003Q3	109.933	57.053	-831.561	945.666
JP	1994Q2 - 2003Q3	-36.888	-140.967	-576.125	294.190
KO	1988Q1 - 2003Q3	4.370	-4.678	-43.445	34.090
NL	1988Q2 - 2003Q3	230.626	-141.995	-1208.822	924.832
NZ	1988Q1 - 2003Q3	-108.278	-6922.594	-28175.744	14330.556
NW	1978Q2 - 2003Q3	-395.137	578.187	-5001.918	6158.293
PO	1995Q2 - 2003Q3	4.880	-22.444	-117.167	72.280
ES	1995Q2 - 2003Q3	155.836	98.382	-182.405	379.168
SD	1993Q2 - 2003Q3	178.290	-240.757	-1160.223	678.709
SW	1980Q2 - 2003Q3	852.612	-987.567	-55476.582	53501.449
UK	1970Q1 - 2003Q3	69.220	24.059	-139.946	188.063
US	1970Q1 - 2003Q3	67.561	47.147	-74.843	169.137

The other two rows hold the results after, respectively, disregarding Korea and Switzerland. The reason for Korea is that its variance of γ is estimated much more precisely than any other country. As a result its weight is much larger. In contrast, the estimated variance of Switzerland is the highest. As shown in the top panel of Table 2, although Korea has a rather large weight, our results are quite robust to its inclusion. The point estimate is now outside the a priori plausible range, but the confidence interval is still tight. As expected, excluding Switzerland has only a negligible effect.

Looking at the middle panel clearly shows that Switzerland is a bit of an

outsider. As it has a large GDP weight, it considerably affects the point estimate. Omitting Switzerland increases $\hat{\gamma}_w$ and the estimate then lies well within the plausible range. However, due to the different weights, the standard error of this estimate is higher than before. Consequently, the confidence interval is wide and uninformative. Comparing the results with and without Korea shows that its large weight has only a minor influence on the results.

Finally, the results based on market capitalization convey more or less the same message as those based on GDP. Korea's influence is minimal, while Switzerland seems to drive the outcomes to a rather large extent. Now the uncertainty in the world coefficient of risk aversion (not excluding Switzerland) is even higher than in the middle panel, as can be inferred from the very wide confidence interval. In contrast, the result excluding Switzerland is more (yet marginally) informative compared to the middle panel. This can be easily explained from the notion that the weight of Switzerland is larger in the bottom panel.

Table 2: **Pooled results**

Notes: The world coefficient of relative risk aversion is denoted by $\hat{\gamma}_w$, the standard error for the confidence interval is the square root of equation (5) for the top panel and the square root of equation (6) for the middle and bottom panels. The critical value is taken from a t-distribution with $k-1$ degrees of freedom. The top panel uses the optimal weights described by equation (4), the middle and bottom panels use economic weights based on, respectively, GDP and market capitalization at the end of 2003. In each panel, the top row holds the outcomes using all countries, the second row disregards Korea while the third row excludes Switzerland from the analysis.

Description	$\hat{\gamma}_w$	95 % CI	
IV, A	4.569	-27.597	36.736
IV, KO	20.556	-32.760	73.871
IV, SW	4.570	-27.706	36.845
GDP, A	-16.897	-712.005	678.211
GDP, KO	-17.165	-729.899	695.569
GDP, SW	-5.650	-240.947	229.646
MC, A	-32.023	-1737.959	1673.913
MC, KO	-32.359	-1765.127	1700.408
MC, SW	-2.849	-202.368	196.669

6   Conclusion

The equity premium puzzle holds that the coefficient of relative risk aversion estimated from the consumption based CAPM with power utility is excessively high. Moreover, estimates in the literature vary considerably. We employ the jackknife resampling method in order to estimate the uncertainty asso-

ciated with the coefficient of relative risk aversion. Our results show that the country-specific confidence intervals are fairly wide. We never reject equality to one. However, when the data of countries are pooled and a single, optimally weighted point estimate is constructed, the resulting confidence band is tighter and presents less of a puzzle than the individual country estimates.

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Appendix

Table 3: **Replication of Campbell (1998)**

Notes: The estimate of the coefficient of relative risk aversion is denoted by $\hat{\gamma}_n$, $\overline{PS}$ is the mean of the pseudovalues, defined in Section 3. The confidence interval is based on $\overline{PS}$, with standard error equal to the square root of the variance estimated by equation (3) and critical value from a t-distribution with $N-1$ degrees of freedom. The estimates in the third column, with the exception of the US, match those reported in Campbell (1998), the final three columns are our contribution. Mnemonics are as follows: Australia (AU), Canada (CN), France (FR), Germany (BD), Italy (IT), Japan (JP), the Netherlands (NL), Sweden (SD), Switzerland (SW), the United Kingdom (UK) and the United States (US).

Country	Sample period	$\hat{\gamma}_n$	$\overline{PS}$	95 % CI	
AU	1970Q1 - 1996Q2	45.704	7.107	-162.627	187.906
CN	1970Q1 - 1996Q2	56.434	8.965	-135.341	185.899
FR	1973Q2 - 1996Q2	-310.315	14.634	-2431.378	7700.344
BD	1978Q4 - 1996Q2	343.133	13.327	-45863.656	80054.164
IT	1971Q2 - 1995Q2	2465.323	4.703	35243.952	80566.083
JP	1970Q2 - 1996Q2	134.118	13.44	-923.569	562.810
NL	1977Q2 - 1996Q1	1050.925	23.97	-33070.353	69140.341
SD	1970Q1 - 1994Q4	7215.176	20.705	-2035151.375	3726567.765
SW	1982Q2 - 1996Q2	-207.291	26.785	-19693.370	15176.756
UK	1970Q1 - 1996Q2	156.308	14.858	-5425.253	2239.238
US	1970Q1 - 1996Q3	150.822	37.446	-253.480	408.725