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On The Relevance Of Irrelevant Information

Parakhonyak Alexei

Erasmus University Rotterdam, and Tinbergen Institute.

Tinbergen Institute

The Tinbergen Institute is the institute for economic research of the Erasmus Universiteit Rotterdam, Universiteit van Amsterdam, and Vrije Universiteit Amsterdam.

Tinbergen Institute Amsterdam
Roetersstraat 31
1018 WB Amsterdam
The Netherlands
Tel.: +31(0)20 551 3500
Fax: +31(0)20 551 3555

Tinbergen Institute Rotterdam
Burg. Oudlaan 50
3062 PA Rotterdam
The Netherlands
Tel.: +31(0)10 408 8900
Fax: +31(0)10 408 9031

On the relevance of irrelevant information *

Parakhonyak Alexei[†]

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Abstract

This paper analyses the role of information in the search process. I build a simple model of a good with two random attributes with some joint probability distribution. I consider seemingly unimportant changes in this distribution, i.e. changes which neither affect expected utility nor its variance. These changes have a great impact on the search behaviour: the customer may start to search the characteristics and buy the good even if she did not do so before. The optimal search rule is derived and the model is generalized to multiple objects.

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†Erasmus University Rotterdam and Tinbergen Institute. E-mail: parakhonyak@ese.eur.nl

1 Introduction

The purpose of this paper is to explore the role of information and interdependencies in the search process.

Consider a situation where you are planning to buy a book. You can read reviews on two different websites, say amazon.co.uk and amazon.com, and you are only interested in a book if it has positive reviews on both websites. Of course, reading reviews costs you some time. If you know that English readers are generally more skeptical than American readers, which website should you search first? Can it be optimal to stop and buy a book after searching just one site?

One can also think about an investment decision problem. Suppose the research department of a large company has to find out whether to buy a particular firm or to launch a new technology. The true value of the firm (the benefits of the technology) is unknown, and depends on two unknown parameters. For example, to calculate the value of the firm it is necessary to estimate future cash flows, and therefore it is necessary both to assess how the firm's costs will evolve and to predict demand fluctuations. These two factors are obviously somewhat correlated, at least due to mutual macroeconomic factors in prediction, such as the GDP growth rate or inflation. An accurate forecast can be quite costly and require both internal and external resources (such as marketing forecasts). Which of the parameters should the company research? What is the optimal order? Or perhaps some of the parameters do not require detailed investigation? For example, in an economy with high inflation both costs and revenues should follow the same pattern which is predetermined by the inflation rate.

What these examples have in common is that a researcher investigates some object with unknown attributes which she values. The researcher can investigate the attributes, and after learning one or more of their values she can either accept the object (buy, launch investment, etc.) or decline. Then the decision about the optimal investigation procedure is based on two factors: *(i)* utility concern, which is the value of a particular attribute for the researcher; *(ii)* informational concern, which is how knowing the true value of one attribute may provide information about the other.

If one attribute functionally determines the value of the other, it might be optimal to research the former first to save on further investigation costs, even if it is not particularly important from a utility point of view. More generally, if knowing the value of one attribute generates some information about another attribute, then an interesting search issue appears.

One of the obvious particular cases of this situation is when a customer wants to buy a bundle of goods and the price of each good is unknown (here prices play the role of attributes in the indirect utility function). This situation is quite well-studied in the literature. Burdett and Malueg (1981) and Carlson and McAfee (1984) studied optimal search rules for several commodities given various recall assumptions. However, in their setup the customer observes the entire price vector once she enters the store. Anglin (1990) pointed out that though the goods are consumed jointly, the customer can make a disjoint search for prices. Unfortunately all the results in the multicommodity search literature are based on the assumption of independent price distributions which makes it impossible to study the informational aspect of the problem. In reality searching for prices within a store is costly, and prices of different goods can be correlated. The customer's optimal search decision then can be based on informational concerns. This paper attempts to cover this gap in the literature.

Moreover, most of consumer search literature (see, for example, Kohn and Shavell (1974)) assumes that the consumer who searches learns the exact value of the good. This is not a very realistic assumption: quite often the true value of a good is revealed to the customer after the purchase, because a thorough investigation of all the characteristics of the good is too time consuming and costly. Hey and McKenna (1981) study such a model with two characteristics: price and quality. However, in their model the search costs for the quality are infinite, it cannot be explored prior to purchase. In this paper decision whether to investigate the value of "quality" prior to purchase or not is endogenous.

To focus on the value of information, I develop a simple model. First, I study a research process with one object. The object possesses two random attributes with a known joint distribution[1]. The researcher can investigate these attributes sequentially at some cost. The paper studies how the dependency between the attributes affects the optimal search (investigation) decision. To facilitate the analysis I consider a situation where each attribute can take either an acceptable or an unacceptable value and the researcher accepts the object only if both attributes are acceptable. Moreover, the utility function is symmetric between the attributes to eliminate the utility concern and so are the research costs. Symmetry of the utility function allows me to concentrate on the informational concern mentioned above. Finally, I consider a specific class of probability measures, such that the expectation and the variance of utility are constant for all measures in the class.

I characterize the optimal investigation rule, which is quite counterintuitive: it is optimal to first investigate the attribute with the lowest probability of taking on an acceptable value. After that I show that any probability distribution which is asymmetric between the attributes is preferred by the searcher to the symmetric one. Moreover, changes in seemingly absolutely irrelevant information can be crucial for search behaviour, i.e. probabilities of unacceptable values of the attributes (given constant utility and risk), in other words probabilities of various outcomes when the searcher would not accept the good anyway. In particular, the researcher may switch from non-researching to research, which in the case of consumer search may result in a purchase.

I also consider what would happen if there are multiple objects. Say Mr. Abramovich wants to buy a football club, he considers a few clubs in Premier League and each club varies in players and fan base. Weitzman (1979) derived

[1]The study of objects with n attributes is also possible. However, it requires an optimization over all possible search sequences which is a problem of combinatorial complexity. Analysis of the $n = 2$ case shows the main ideas without obscuring them with algebraic complications.

the optimal search rule for objects with different distributions. The results of a single object search problem can be generalized using the same logic as Weitzman. The optimal search rule between the objects is characterized by reservation values, while within the object the researcher should follow the results developed for the single object. Moreover, once the researcher starts to investigate one object she never switches to another one unless she discover an unacceptable value of some attribute.

The structure of the paper is as follows. Section 2 presents the model of the object with two attributes. Analysis the of single object case is presented in section 3, the investigation process with multiple objects is considered in section 4. Section 5 concludes. An interested reader can find the analysis of the continuous case of the model in the appendix.

2 Single Object: A Model

The object has two attributes: $a \in \{A, \overline{A}\}$ and $b \in \{B, \overline{B}\}$ which affect the researcher's utility. Thus, there are four possible types of the object. Each type can appear with a certain probability. The researcher is only interested in the object of type AB, i.e. then both the attributes take acceptable values. The utility function is symmetric, so the researcher values objects of type $\overline{A}B$ and $A\overline{B}$ equally, and has no utility grounds to prefer investigation of attribute a to attribute b.

The structure of the model is represented by the following two matrices.

Utilities

	B	$\overline{B}$
A	u_2	u_1
$\overline{A}$	u_1	u_0

Probabilities

	B	$\overline{B}$
A	α	γ
$\overline{A}$	δ	β

Specification of the utility function is summarized by the following assumption.

Assumption 2.1. $u(AB) = u_2, u(\overline{A}B) = u(\overline{B}A) = u_1, u(\overline{A}, \overline{B}) = u_0$ *with* $u_2 > 0 > u_1 \geq u_0$.

Utilities in the matrix are net of price (investments, etc.) After at least one attribute has been investigated the researcher can either accept or reject the object. If the object is rejected, the researcher gets reservation utility which I assume to be equal zero, if it is accepted she gets the utility corresponding to the type of the object.

Probabilities of the outcomes are presented in the above probability matrix.

At cost c the researcher can investigate whether $\omega \in A$ (if not then obviously $\omega \in \overline{A}$), and then I say that she investigates attribute a. For the same cost she can investigate whether $\omega \in B$, and then I say that she investigates attribute b.

To avoid utility-specific effects I consider a specific class of probability measures, which is characterized by constant probabilities of outcomes AB and $\overline{AB}$.

Definition 2.2. *Let $M(\alpha, \beta)$ be the class of probability measures such that $P(AB) = \alpha, P(\overline{AB}) = \beta$, $\alpha + \beta < 1$.*

Lemma 2.3. *For any probability measure $\mu \in M(\alpha, \beta)$*

$$\mathbb{E}u = \alpha u_2 + (1 - \alpha - \beta)u_1 + \beta u_0 \tag{1}$$

$$Var(u) = \alpha(u_2 - \mathbb{E}u)^2 + (1 - \alpha - \beta)(u_1 - \mathbb{E}u)^2 + \beta(u_0 - \mathbb{E}u)^2 \tag{2}$$

Therefore, all $\mu \in M(\alpha, \beta)$ are characterized by the same expected utility and risk. Thus, focusing on symmetric utility functions and $\mu \in M(\alpha, \beta)$ allows me to study the impact of information and dependencies on the investigator's behaviour.

3 Single Object: Analysis

The expectation and the variance of utility do not depend on (γ, δ). If (α, β) are fixed, the model is characterized by one degree of freedom γ (since $\delta = 1 - \alpha - \beta - \gamma$).

Intuitively, it seems that the value of γ should not play a relevant role in the model, since:

1. whatever the value of γ is, both expected utility and risk are constant;
2. changing the value of γ corresponds to changing the probabilities of two events with the same level of utility;
3. changing the value of γ corresponds to changing the probabilities of two events with negative utility, i.e. events when the searcher does not accept the object.

However, given that investigation of the attributes is possible, the intuition that the value of γ is irrelevant is incorrect. To illustrate this I derive an optimal investigation rule, and show that it heavily depends on γ, namely on how far the value of γ stays form the symmetric case $\gamma_0 = \delta_0 = \frac{1-\alpha-\beta}{2}$. The value of γ drives not only the optimal investigation order but sometimes can influence the decision whether to start research at all.

Consider a rational research process. Suppose the researcher decides first to investigate an attribute a. She pays c and observes whether the object is of type A or $\overline{A}$. In the latter case it is optimal to terminate the investigation and reject the object, because the best she can get is $u_1 < 0$. In the former case she can either accept the object immediately or investigate an attribute b. If she decides to proceed, she pays c and observes b. If $b = B$ she accepts, and if $b = \overline{B}$ she rejects. Thus, given the optimal behaviour, the value of the investigation process started with the attribute a denoted as V_a can be written as

$$V_a = -c + P(A)\max(P(B|A)u_2 + P(\overline{B}|A)u_1, P(B|A)(u_2 - c) + P(\overline{B}|A)(-c)) \tag{3}$$

Or equivalently

$$V_a = -c + \max(\alpha u_2 + \gamma u_1, \alpha(u_2 - c) - c\gamma) \tag{4}$$

If the investigation order is to first investigate b and then a, the value of the investigation process denoted as V_b is:

$$V_b = -c + \max(\alpha u_2 + (1 - \alpha - \beta - \gamma)u_1, \alpha(u_2 - c) - c(1 - \alpha - \beta - \gamma)) \tag{5}$$

This allows me to formulate the following result:

Theorem 3.1 (Optimal investigation rule). *The optimal investigation rule is:*

- *if* $\max(\alpha u_2 + \gamma u_1, \alpha(u_2 - c) - c\gamma, \alpha u_2 + (1 - \alpha - \beta - \gamma)u_1, \alpha(u_2 - c) - c(1 - \alpha - \beta - \gamma)) < c$ *then do not search;*
- *otherwise:*
  - *if* $\gamma < \frac{1-\alpha-\beta}{2}$ *it is optimal to investigate* a *first;*
  - *if* $\gamma > \frac{1-\alpha-\beta}{2}$ *it is optimal to investigate* b *first;*
  - *if* $\gamma = \frac{1-\alpha-\beta}{2}$ *both investigation orders yield the same expected utility;*
  - *if the researcher knows that the object is of type* A *then it is optimal to investigate the object further if* $u_1 < \frac{-c(\alpha+\gamma)}{\gamma}$*, and to terminate the investigation and accept the object otherwise;*
  - *if the researcher knows the object is of type* B *then it is optimal to invetigate the object further if* $u_1 < \frac{-c(1-\beta-\gamma)}{1-\alpha-\beta-\gamma}$ *and to terminate the investigation and accept the object otherwise;*
  - *if after the first investigation the researcher realizes that the object is either of type* $\overline{A}$ *or* $\overline{B}$ *then it is optimal to terminate the investigation and refrain from accepting.*

Proof. The proof comes naturally from comparing the benefits in different cases. If $\max(V_a, V_b) < 0$ then it is optimal not to search. By expanding this inequality we get $\max(\alpha u_2 + \gamma u_1, \alpha(u_2 - c) - c\gamma, \alpha u_2 + (1 - \alpha - \beta - \gamma)u_1, \alpha(u_2 - c) - c(1 - \alpha - \beta - \gamma)) < c$.

If $\max(V_a, V_b) > 0$ then it is optimal to investigate the attributes in the order which gives the highest expected value. Thus, if $V_a > V_b$ it is optimal to start with a. $V_a > V_b$ if and only if $\gamma < \frac{1-\alpha-\beta}{2}$. If an attribute a was investigated and $a = \overline{A}$ then it is optimal to terminate the search because $u_0 < u_1 < 0$. The same holds for b. If $a = A$, then it is optimal to continue the research if $\alpha u_2 + \gamma u_1 < \alpha(u_2 - c) - c\gamma$, or equivalently $u_1 < \frac{-c(\alpha+\gamma)}{\gamma}$. The analysis for $b = B$ is exactly the same. □

Note that the optimal investigation rule possesses quite a counterintuitive property: it is optimal to investigate an attribute with the lowest probability of taking on a positive value. Indeed, the probability of $a = A$ is $\alpha + \gamma$, and the probability $b = B$ is $1-\beta-\gamma$. It is optimal to investigate a first when $\gamma < \frac{1-\alpha-\beta}{2}$ which implies that $\alpha + \gamma < 1 - \beta - \gamma$. The idea behind this fact is that the researcher sacrifices the high probability of a positive outcome in the first step for a more favorable probability distribution in the next step. In the case when it is optimal to explore both attributes before accepting the object the idea which drives the result is quite clear: the researcher tries to minimize the expected costs of investigation and therefore maximizes the probability to stop after the first investigation round. In the case when it is optimal to stop just after the first round (in some cases the second search can be simply prohibited due to lack of time, etc.) pure informational concerns play role: the researcher faces more favourable distribution in the second round (maximizes the probability of correct choice) at the cost of lower a probability in the first round.

It is important to emphasize that the value of γ affects the optimal investigation rule. Thus this information is not irrelevant given that investigation of the object is possible. Theorem 3.1 shows how the value of γ affects the optimal investigation order. The following proposition illustrates the importance of this information for the customer's preferences over distributions and welfare. Let's denote the value of the search process by $V(\gamma) = \max(V_a(\gamma), V_b(\gamma), 0)$, and $\gamma_0 = \frac{1-\alpha-\beta}{2}$

Proposition 3.2. *For any $\mu \in M(\alpha, \beta)$ $V(\gamma) \geq V(\gamma_0)$. If $\gamma_2 \leq \gamma_1 \leq \gamma_0$ or $\gamma_2 \geq \gamma_1 \geq \gamma_0$ then $V(\gamma_2) \geq V(\gamma_1)$.*

Proof. Note that $V_a(\gamma_0) = V_b(\gamma_0)$ and V_a is a decreasing function of γ, while V_b is an increasing. Therefore $\max(V_a(\gamma), V_b(\gamma)) \geq \max(V_a(\gamma_0), V_b(\gamma_0))$. In the same way, since either $V_a(\gamma_2) \geq V_a(\gamma_1)$ or $V_b(\gamma_2) \geq V_b(\gamma_1)$, the value of search $V(\gamma_2) \geq V(\gamma_1)$. □

Thus the researcher would prefer to have an asymmetric distribution to a symmetric one, and the more asymmetric the better. The reason is that given a positive outcome of investigation of the first attribute, the quality of information increases and the conditional probability distribution becomes more favorable. If the researcher prefers say extremely low values of γ to moderate ones it is reasonable to assume that for some γ close to γ_0 it is optimal not to search and to get reservation utility of zero, while for extreme values of γ it is optimal to start research and probably accept the object (buy the good, launch the investments). Thus, the seemingly irrelevant γ can dramatically affect the researcher's behaviour. Formally this result is shown in the following proposition.

Proposition 3.3. *There is a pair (c, γ) such that $V(\gamma) > 0 > V(\gamma_0)$.*

Proof. First, note that $V_a(\gamma_0) = V_b(\gamma_0)$. By expanding the expression of the value of the search we get that $V(\gamma_0) < 0$ if $c > \max\left(\alpha u_2 + \gamma_0 u_1, \frac{u_2}{1+\alpha+\gamma_0}\right)$.

Note, that the right hand side of this expression is a decreasing continuous function of γ. Therefore it is possible to choose c and γ in such a way that

$$\max\left(\alpha u_2 + \gamma u_1, \frac{u_2}{1+\alpha+\gamma}\right) > c > \max\left(\alpha u_2 + \gamma_0 u_1, \frac{u_2}{1+\alpha+\gamma_0}\right)$$

with $V(\gamma) > 0$. □

Thus, the value of γ affects the optimal research order and sometimes can affect the decision to investigate itself. The researcher prefers probability distributions which are further from symmetric ones.

4 Multiple Objects

A search problem with multiple objects can be quite interesting. For example, in times of crisis a major financial company decides to take over one of the banks which has experienced some trouble. Each bank can be characterized by multiple attributes which affect the final decision. What is the optimal investigation rule? Weitzman (1979) showed that it is optimal to explore objects in order of reservation values. However, if each of the objects has a complex structure and there is a possibility to investigate the object itself, the reservation value has to be redefined. A situation when one object is partially explored (one attribute is known) deserves a particular interest. I show that it is never optimal to stop investigation and switch to another object if the attribute explored has a positive value. This section provides a study of the attribute search problem with multiple objects.

Assume there are multiple objects which satisfy the assumptions described above, i.e. they posses symmetric attributes (which in principle can be different in nature between objects). Then each object is characterized by a set $(u_{1i}, u_{2i}, u_{3i}, \alpha_i, \beta_i, \gamma_i, c_i)$, $i = \overline{1, n}$ is the object index. Attributes are independent between objects. Let $V_i = \max(V_{ai}, V_{bi})$ be an expected value given the optimal order of search of the attributes. For simplicity of notation assume that it is optimal to investigate a first for all the objects. and thus $V_i = V_{ia}$. Of course, attributes can be renamed in such a way that this holds for each object.

Assume that the researcher has some "sure thing" as a result of previous search, and denote it by z_i. Assume by now that only object i is left unexplored. Of course, if $z_i \neq 0$ the value of investigation changes in part which deals with reservation object. To take this into account let's denote

$$\tilde{V}_{ai}(z_i) = -c_i + \max(\alpha_i u_{2i} + \gamma_i u_{1i}, \alpha_i(u_{2i} - c_i) + (z_i - c_i)\gamma_i) \tag{6}$$

$$\tilde{V}_{bi}(z_i) = -c_i + \max(\alpha_i u_{2i} + \delta_i u_{1i}, \alpha_i(u_{2i} - c_i) + (z_i - c_i)\delta_i) \tag{7}$$

where $\delta_i = 1 - \alpha_i - \beta_i - \gamma_i$. It is clear that if $V_{ai} \geq V_{bi}$ then $\tilde{V}_{ai}(z_i) \geq \tilde{V}_{bi}(z_i)$ for all z_i and vice versa. If she searches object i in the optimal way the expected benefits equal

$$\max(\tilde{V}_{ai}(z_i + P(\overline{A})_i z_i, \tilde{V}_{bi} + P(\overline{B})_i z_i) \tag{8}$$

Note that from theorem 3.1 it follows that if $V_{ai} \geq V_{bi}$ then $P(\overline{A})_i \geq P(\overline{B})_i$. Therefore, since $z_i \geq 0$

$$\max(\tilde{V}_{ai}(z_i) + P(\overline{A})_i z_i, \tilde{V}_{bi}(z_i) + P(\overline{B})_i z_i) = \tilde{V}_{ai}(z_i) + P(\overline{A})_i z_i$$

The researcher is indifferent between researching and non-researching if $\tilde{V}_{ai}(z_i) + P(\overline{A})_i z_i = z_i$. Therefore each object is characterized by the reservation value

$$z_i = \max\left(\frac{\alpha_i u_{2i} + \gamma_i u_{1i} - c_i}{\alpha_i + \gamma_i}, \frac{\alpha_i u_{2i} - (1 + \alpha_i + \gamma_i)c_i}{\alpha_i}\right) \tag{9}$$

Weitzman (1979) proved that it is optimal to search objects in the order of reservation values: from the highest z_i in descending order. Note that the search order in general is different from the order of expected values given the optimal search $\{V_i\}_{i=1}^n$, so it can be that $V_i > V_j$ but it is still optimal to search j before i.

It is clear that if object i is searched and an unacceptable value of an attribute is discovered, it is optimal to search another object or to terminate search if no object are left. But what if the first attribute takes the acceptable value? Let's denote a reservation value after a positive outcome in the first round ($a = A$) by $\tilde{z}_i$. If it is optimal to terminate the search and buy without looking at attribute b then $\tilde{z}_{i\text{stop}} = \frac{\alpha_i u_{2i} + \gamma_i u_{1i} - c_i}{\alpha_i + \gamma_i}$. If it is optimal to search for b before buying, object i is characterized by new reservation value $\tilde{z}_i$ is defined by:

$$\frac{\alpha_i u_{2i}}{\alpha_i + \gamma_i} + \frac{\gamma_i \tilde{z}_i}{\alpha_i + \gamma_i} - c_i = \tilde{z}_i \tag{10}$$

and then

$$\tilde{z}_{ib} = \frac{\alpha_i u_{2i} - c_i(\alpha_i + \gamma_i)}{\alpha_i} \tag{11}$$

Therefore

$$\tilde{z}_i = \max\left(\frac{\alpha_i u_{2i} + \gamma_i u_{1i}}{\alpha_i + \gamma_i}, \frac{\alpha_i u_{2i} - (\alpha_i + \gamma_i)c_i}{\alpha_i}\right) \tag{12}$$

Lemma 4.1. *If $a = A$, it is optimal either to terminate the search and accept the object, or search for another attribute of the same object.*

Proof. The lemma holds if $\tilde{z}_i > z_i$, which is true because

$$\frac{\alpha_i u_{2i} + \gamma_i u_{1i}}{\alpha_i + \gamma_i} > \frac{\alpha_i u_{2i} + \gamma_i u_{1i} - c_i}{\alpha_i + \gamma_i}$$

and

$$\frac{\alpha_i u_{2i} - (\alpha_i + \gamma_i)c_i}{\alpha_i} > \frac{\alpha_i u_{2i} - (1 + \alpha_i + \gamma_i)c_i}{\alpha_i}$$

□

The lemma shows that once the searcher has got a positive search result about characteristic a, the reservation value of the object is not decreasing and therefore it is optimal to search this object further. This allows me to formulate the optimal search rule for multicommodity research.

Theorem 4.2 (Optimal investigation rule with multiple objects). *The optimal investigation rule is:*

- *start with the object with the highest z_i;*
- *if $\max(\alpha_i u_{2i} + \gamma_i u_{1i}, \alpha_i(u_{2i} - c_i) - c_i\gamma_i, \alpha_i u_{2i} + (1 - \alpha_i - \beta_i - \gamma_i)u_{1i}, \alpha_i(u_{2i} - c_i) - c_i(1 - \alpha_i - \beta_i - \gamma_i)) < c_i$, then terminate the investigation;*
- *otherwise:*
  - *if $\gamma_i < \frac{1-\alpha_i-\beta_i}{2}$ it is optimal to investigate a first;*
  - *if $\gamma_i > \frac{1-\alpha_i-\beta_i}{2}$ it is optimal to investigate b first;*
  - *if $\gamma_i = \frac{1-\alpha_i-\beta_i}{2}$ both research orders yield the same expected utility;*
  - *if the researcher knows that the object is of type A, then it is optimal to investigate further if $u_{1i} < \frac{-c_i(\alpha_i+\gamma_i)}{\gamma_i}$ and to terminate the investigation and accept the object otherwise;*
  - *if the searcher knows that object is of type B, then it is optimal to investigate further if $u_{1i} < \frac{-c_i(1-\beta_i-\gamma_i)}{1-\alpha_i-\beta_i-\gamma_i}$ and to terminate the investigation and accept the object otherwise;*
  - *if after the first investigation the researcher realizes that the object is either of type $\overline{A}$ or $\overline{B}$ then it is optimal proceed with the object with the highest z_i among the objects left or to terminate the investigation if there are no more objects.*

5 Conclusions

This papers illustrates the importance of information contained in one attribute of a good about another one. Even if two probability distributions provide the same expected utility and risk, the researcher might prefer one over another if she possesses the possibility of investigation of the attributes. Moreover, a change in the probability distribution which preserves the mean and variance of utility and only affects probabilities of outcomes when the researcher does not accept the object can dramatically affect her behaviour. She may switch from passive non-investigating (and hence rejection of the object) behaviour to active investigation, which might result in the acceptence of the object: purchase of a good, launching an investment project. The results can be illustrated for different distributions and utility functions, and the interested reader can find the continuous case with a uniform distribution in the appendix. The model allows one to consider the investigation problem with several heterogeneous objects, and the results preserve all essential properties of the single object solution. The results of the paper can potentially be embedded in an equilibrium setup, when the firms make decisions about characteristics of the products (say, price and quality) or prices of two goods.

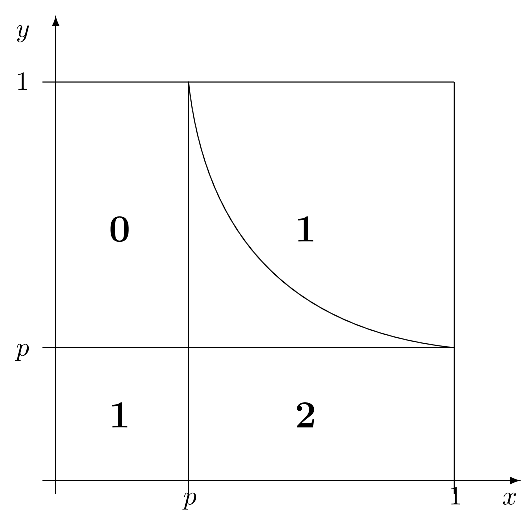
Appendix

For the sake of simplicity, in the main body of the paper I restricted myself to the case where characteristics take binary values. However A and $\overline{A}$ can be considered as two regions where the utility function takes different signs. Here I show that though utility may not be constant over these regions, the analysis and results stay essentially the same. To do this I look at a continuous case: each attribute is a continuous variable bounded between zero and one.

Consider an example where attributes can take continuous values. Let $(x, y) \in [0, 1]^2$ be a characteristic space and $u = xy - p$ be a utility function, where $p < 1/2$ is the price of the good. Initially we assume that (x, y) are uniformly distributed.

It is obvious that if one of the characteristics is less then p, the researcher is never going to accept the object. Let us transform the probability distribution in this area in the following way: we set $\mathbb{P}(x \leq p, y \geq p) = 0$ and double density for $x \geq p, y \leq p$. See the figure for details.

Figure 1: Values of density function and reservation utility level



As before, assume that the researcher is allowed to investigate only one attribute, and then he has to make a decision.

1. x is investigated first, then y.

(a) Given value of x density functions for y are:

- $x \leq p$

$$f(y) = \begin{cases} 1/p & \text{if} \quad y \leq p \\ 0 & \text{if} \quad y > p \end{cases} \tag{13}$$

- $x > p$

$$f(y) = \begin{cases} \frac{2}{1+p} & \text{if} \quad y \leq p \\ \frac{1}{1+p} & \text{if} \quad y > p \end{cases} \tag{14}$$

(b) The expected value of utility function conditional on x is:

- $x \leq p$

$$Eu = \frac{1}{p} \int_0^p (xy - p)\, dy = \frac{p}{2}x - p < 0 \tag{15}$$

- $x > p$

$$\begin{aligned} Eu = & \frac{1}{1+p} \int_0^1 (xy - p)\, dy + \frac{1}{1-p} \int_0^p (xy - p)\, dy \\ = & \frac{1}{1+p} \left[\frac{1+p^2}{2} x - (p + p^2) \right] \end{aligned} \tag{16}$$

Note, that the object is accepted only if expected utility is greater than 0, therefore

$$x > \frac{2(p+p^2)}{1+p^2} \tag{17}$$

and the RHS is greater than p.

(c) The density function of x is

$$f(x) = \begin{cases} p & \text{if} \quad x \leq p \\ 1+p & \text{if} \quad x > p \end{cases} \tag{18}$$

(d) Expected utility of research equals to

$$\begin{aligned} Eu = & (1+p) \int_{\frac{2(p+p^2)}{1+p^2}}^1 \frac{1}{1+p} \left[\frac{1+p^2}{2} x - (p+p^2) \right] dx \\ = & \frac{(1 - 2p - p^2)^2}{4(1+p^2)} \end{aligned} \tag{19}$$

2. y is investigated first, then x.

   (a) Given the value of y the density functions for x are:

- $y \leq p$

$$f(x) = \begin{cases} \frac{1}{2-p} & \text{if} \quad x \leq p \\ \frac{2}{2-p} & \text{if} \quad x > p \end{cases} \tag{20}$$

- $y > p$

$$f(x) = \begin{cases} 0 & \text{if} \quad x \leq p \\ \frac{1}{1-p} & \text{if} \quad x > p \end{cases} \tag{21}$$

(b) The expected value of utility function conditional on y is:

- $y \leq p$

$$\begin{aligned} Eu = & \frac{1}{2-p}\int_0^1 (xy-p)\,dx + \frac{1}{2-p}\int_p^1 (xy-p)\,dx \\ = & \frac{1}{2-p}\left[\frac{2-p^2}{2}y - (2p-p^2)\right] < 0 \end{aligned} \tag{22}$$

- $y > p$

$$Eu = \frac{1}{1-p}\int_p^1 (xy-p)\,dx = \frac{1+p}{2}y - p \tag{23}$$

Note, that the object is accepted only if expected utility is greater than 0, therefore

$$y > \frac{2p}{1+p} \tag{24}$$

and the RHS is greater than p.

(c) The density function of y is

$$f(y) = \begin{cases} 2-p & \text{if} \quad y \leq p \\ 1-p & \text{if} \quad y > p \end{cases} \tag{25}$$

(d) Expected utility of search equals to

$$\begin{aligned} Eu = & \frac{1}{1+2p}\int_{\frac{2p}{1-p}}^1 \left[\frac{1-p}{2}y - p\right] dy \\ = & \frac{(1-p)^3}{4(1+p)} \end{aligned} \tag{26}$$

3. It is easy to check that

$$\frac{(1-p)^3}{4(1+p)} > \frac{(1-2p-p^2)^2}{4(1+p^2)} \tag{27}$$

Thus, the key properties of the optimal investigation rule are preserved in the continuous setup. The investigator still prefers to look at the attribute with the lowest probability of an acceptable outcome first.

References

P.M Anglin. Disjoint search for the prices of two goods consumed jointly. *International Economic Review*, 31:383–408, 1990.

K. Burdett and D.A. Malueg. The theory of search for several goods. *Journal of Economic Theory*, 24:362–376, 1981.

J. Carlson and P. McAfee. Joint search for several goods. *Journal of Economic Theory*, 32:337–345, 1984.

J. D. Hey and C. J. McKenna. Consumer search with uncertain product quality. *The Journal of Political Economy*, 89(1):54–66, 1981.

M. Kohn and S. Shavell. The theory of search. *Journal of Economic Theory*, 9: 93–123, 1974.

M. Weitzman. Optimal search for the best alternative. *Econometrica*, 47:541–554, 1979.