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Advertising for Attention in a Consumer Search Model*

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Abstract

We model the idea that when consumers search for products, they first visit the firm whose advertising is more salient. The gains a firm derives from being visited early increase in search costs, so equilibrium advertising increases as search costs rise. This may result in lower firm profits when search costs increase. We extend the basic model by allowing for firm heterogeneity in advertising costs. Firms whose advertising is more salient and therefore raise attention more easily charge lower prices in equilibrium and obtain higher profits. As advertising cost asymmetries increase, aggregate profits increase, advertising falls and welfare increases.

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1 Introduction

Advertising is an important part of economic activity. In the US, for example, advertising expenditures constitute around 2.2% of GDP. According to PriceWaterhouseCoopers worldwide advertising in 2005 amounted to a staggering \$385 billion (PriceWaterhouseCoopers, 2005). This amount is set to grow to over half-a-trillion dollars in 2010. The average American citizen is exposed to hundreds of commercial messages each day.¹ Not surprisingly, very few of these messages are able to get through the clutter and raise the attention of consumers.²

A significant part of the empirical marketing literature on advertising focuses on the idea that the *primary* challenge for an advertising firm is to raise attention in the first place. Given the (increasing) number of commercial messages that consumers are confronted with every day, the main role of advertising is to create firm *saliency*, that is, the prominence of a brand in consumers' memories. Enhancing the salience of a brand requires fine-tuning on a number of marketing variables. For example, Keller et al. (1998) examine the effects of "suggestive" brand names on saliency;³ Unnava and Burnkrant (1991) study the effects of advertising repetition on brand name memorability;⁴ finally, Burke and Srull (1988) and Alba and Chattopadhyay (1986) study the existence of saliency enhancing advertising externalities and point out that investments in "salience" of one firm *inhibit* the recall of alternative firms.

Economists have also spent quite some effort in trying to understand the role of advertising in product markets (for a survey, see Bagwell, 2007). One branch of the literature

¹Estimates of this number vary widely (see e.g. <http://www.hhcc.com/?p=468> for a discussion). In 1972, Britt et al. (1972) already find between 300 and 600 messages per day. Arens et al. (2007), a popular textbook, even claims that "as a consumer, you are exposed to hundreds and maybe even thousands of commercial messages every day" (pg. 6).

²Franz (1986), for example, reports that out of more than 13,000 individuals questioned in 1985 about the advertisements that were seen, heard or read in the past 30 days, 53 % were unable to remember any specific one.

³They report evidence that product/brand names explicitly conveying a product benefit or characteristic may increase the recall probability of consumers interested in those characteristics. The name of the motorcycle *Triumph Speed Triple* may be a good example; this motorcycle is probably recalled more readily by consumers interested in speedy and agile motorbikes. Another example is *Sharp* televisions.

⁴See also Janiszewski et al. (2003), who investigate the relationship between brand rehearsal, advertising repetition and advertising medium.

considers advertising as a sunk cost firms incur to enhance consumers' willingness-to-pay for their products. This has been coined *persuasive advertising* (see e.g. Braithwaite, 1928; Kaldor, 1950; Galbraith, 1967). It typically enhances perceived product differentiation and thus softens competition. Since Telser (1964) and Nelson (1970, 1974), the view of advertising as a device to transmit information has been gaining support. Through informative advertising, firms can communicate, either directly or through signalling, their existence, location, price, or quality. By increasing the information consumers possess, informative advertising typically enhances competition.⁵

To the best of our knowledge, however, the economics literature has not examined the influence of saliency enhancing advertising on economic outcomes.⁶ To close this gap, this paper proposes a model in which the *main* role for the ads of a firm is to “compete” for the “attention” of consumers. Products are differentiated, and consumers search shops sequentially to find a product to their liking (see Wolinsky, 1986; Anderson and Renault, 1999). As search is costly, a consumer will stop searching if she finds a deal that is sufficiently attractive and visiting more shops is not worth her while. It is thus in the interest of a firm to be visited earlier than the rivals. However, it is hard for a firm to stand out from the crowd, get the attention of consumers, and lure them to their shops (see e.g. Comanor and Wilson, 1974). We assume that the order in which firms are sampled is influenced by how much they advertise. More precisely, at every stage in the search process the probability that a consumer recalls a shop and decides to visit it, is proportional to that shop's share in total industry advertising. Thus, advertising helps a firm to become more salient and to remind consumers of the kind of products it sells. Whenever a consumer needs such a product, the firm hopes consumers will remember its shop more readily than those of its rivals.⁷

⁵For advertising conveying direct information see e.g. Butters (1977), Shapiro (1980), Grossman and Shapiro (1984), Stahl (1994) and Bester and Petrakis (1995). For advertising as a signal of quality see Nelson (1974), Kihlstrom and Riordan (1984) and Milgrom and Roberts (1986).

⁶An exception is Anderson and de Palma (2008), who propose a model where advertising messages vie for consumer attention. Their paper focuses on the existence of congested equilibria (i.e., equilibria where more messages are sent than they are examined) and on the welfare effects of public policies such as subsidies and do-not-call lists among others.

⁷In our model, advertising only provides information about the existence and location of shops, not about prices. The idea is that consumers have limited memory and by the time they go to the market they

We believe that this model provides a suitable framework to understand the effects of search costs on advertising, prices and profits. It also provides a rationale as to why consumers would increase their propensity to buy a product when they see it advertised, without having to make the unsatisfactory assumption that advertising directly increases willingness to pay, as is common in most models of persuasive advertising. Finally, the model helps understand why there is so much advertising that conveys little information about well-known horizontally differentiated brands/products.

We find that both price and advertising expenditures increase in search costs. As search costs increase, consumers are more reluctant to visit several shops. A typical shop then has more market power over each consumer that does pay a visit, hence it will charge a higher price. At the same time, it becomes even more important for a firm to be salient and to be visited early. Once a consumer visits a shop to inspect its product, she is less likely to walk away to sample another shop. Hence firms will advertise more. The effect of an increase in search costs on firm profitability is ambiguous. If search costs are relatively small, the price effect dominates and equilibrium profits increase when search costs increase. However, when search costs are relatively high, the price effect is more than offset by the rent-dissipation effect of increased advertising, so higher search costs imply lower equilibrium profits. This is contrary to the received wisdom in the literature on search costs (see e.g. the classic papers of Reinganum, 1979; Burdett and Judd, 1983 and Stahl, 1989). Our model thus provides an instance in which firms do not necessarily benefit from higher search costs. In fact, when search costs are large enough, we show that firm profits can be lower than in a frictionless world with zero search costs.

In our basic model, firms find themselves in a classic prisoners' dilemma. If a firm advertised less than the rest of the firms, the chance that this firm is pushed to the end of consumers search order would be higher. In equilibrium all firms advertise with the same intensity which implies that consumers end up recalling each firm with the same

are simply unable to remember all the prices they have seen in advertisements. The best an advertiser can hope for is that consumers remember its identity and/or its location. In any case, there are many instances in which advertising prices is impractical. For example, when shops sell many products (like fashion shops, electronics superstores, supermarkets, etc.) it is simply unfeasible to list the prices of all products.

probability. Firms would thus be better off if advertising were banned. From a welfare point of view, advertising is purely wasteful.

To study the effect of asymmetries, we provide an extension with two firms where one firm is more efficient in generating saliency than the other. We find that the more efficient firm advertises more and hence attracts a larger share of consumers on their first visit. This firm also charges a lower price. By choosing to visit a second firm, consumers reveal that they do not particularly like the product the first firm offered. Hence, such consumers are less price-sensitive than consumers who still have the option to visit another shop. As the less efficient firm's pool of visitors has a higher share of these less price-sensitive consumers, it finds it profitable to charge a higher price. Still, equilibrium profits of the more efficient firm are higher. Advertising now has social value as it helps consumers to channel their first-visits towards better deals. As advertising cost asymmetries increase, more consumers end up at the cheapest store first, and fewer consumers incur the cost of searching both firms. Nevertheless, total consumer surplus decreases because of the price increase of the less efficient firm. Savings in advertising and search costs outweigh consumer losses and overall welfare increases as advertising cost asymmetries rise.

As noted, we assume that the order in which firms are visited is influenced by advertising efforts. In Hortacısu and Syverson (2004), sampling probability variation across firms is used to explain price dispersion in the mutual funds industry. The authors use advertising outlays as one proxy for the sampling probability of a fund in the market. That is consistent with our model. Most theoretical papers in the search literature assume that consumers sample firms randomly.⁸ Exceptions include the following. In Arbatskaya (2007), the order in which firms are visited is exogenously given. She finds that prices fall in the order in which firms are visited: a consumer that walks away from a firm reveals that it has low search costs, which gives the next firm an incentive to charge a lower price. In Wilson (2008), a firm can choose the magnitude of the search cost consumers have to incur to visit it. In equilibrium, consumers are more likely to first visit firms with low search cost and

⁸See the seminal contributions of Rob (1985), Bénabou (1993), Burdett and Judd (1983), Carlson and McAfee (1983), Stahl (1989), Reinganum (1979) and Wolinsky (1986). More recent work includes Anderson and Renault (1999), Janssen and Moraga-González (2004, 2007), Janssen et al. (2005) and Rauh (2004, 2007).

prices also fall in the search order. In our model firms sell differentiated products which implies that prices increase in the (expected) order in which they are sampled: a consumer that walks away from a firm has fewer options left and is thus less price sensitive. Finally, and more directly related to our specific model, Armstrong et al. (2009) study a search market with differentiated products where one firm is always visited first, while the other firms are sampled randomly if a consumer decides not to buy from the prominent firm. In equilibrium, the prominent firm charges lower prices and has higher profits than the other firms, for the same reason as in our analysis. Indeed, our model can be interpreted as one in which firms invest in prominence but where prominence can only be imperfect.

Across industries, our model predicts a positive correlation between search costs, advertising expenditures and prices. An industry with higher search costs is able to set higher prices, but will also advertise more. Within an industry, we predict a negative correlation between prices on the one hand, and advertising expenditures and market shares on the other. Firms that are more efficient in generating saliency attract more consumers, set lower prices and make higher profits. Other papers also predict a negative relationship between advertising and prices, but for different reasons. In Robert and Stahl (1993) firms can advertise prices on a search market with homogeneous products, and advertise lower prices more intensively. In Bagwell and Ramey (1994) advertising is used as a coordination device for firms to attract more consumers and hence have lower costs, allowing them to charge lower prices. In our model, firms that advertise more attract a pool of consumers that is more price sensitive, and therefore charge lower prices.

The remainder of this paper is structured as follows. In section 2 we describe the set-up of the model. The equilibrium results for symmetric firms are derived in subsection 3, and the results on the effects of search costs on advertising efforts, prices and profits are given in subsection 3.3. Section 4 presents results for a market with asymmetric firms. Section 5 concludes.

2 The model

There are n firms that sell horizontally differentiated products. Let N denote the set of firms. Marginal costs are constant and normalized to zero. For simplicity and without loss of generality we assume that there is one consumer. She has tastes described by an indirect utility function

$$u^i(p_i) = \varepsilon_i - p_i,$$

if she buys product i at price p_i . The parameter ε_i is a match value between the consumer and product i . Match values are independently distributed across products. We assume that ε_i is the realization of a random variable with distribution F and a continuously differentiable log-concave density f with support normalized to $[0, 1]$. No firm can observe ε_i so practising price discrimination is not feasible. The consumer only learns ε_i upon visiting firm i . We denote the monopoly price by p^m , i.e., $p^m = \arg \max_p \{p(1 - F(p))\}$.

The consumer must incur a search cost s in order to learn the price charged by firm i as well as her match value ε_i for the product sold by that firm. The consumer searches sequentially with costless recall. We assume that search cost s is relatively small so that the first search is always worthwhile, that is:

$$0 \leq s \leq \bar{s} \equiv (1 - F(p^m)) \left(\frac{\int_{p^m}^1 \varepsilon f(\varepsilon) d\varepsilon}{1 - F(p^m)} - p^m \right)$$

Firms engage in an advertising battle to lure consumers to their shops. In particular we assume that at any moment during the search process, a consumer is more likely to go to firm i if she has had more exposure to the ads of that firm (or if the ads of firm i have happened to be relatively more salient than other firms' ads).⁹ The set-up of the model ensures that the equilibrium does not have the consumer necessarily buying in the first shop she enters, as is the case in most search models. Therefore, it is important for firms to be visited early, but it is not crucial for making a sale.

Let a_i , $i = 1, 2, \dots, n$ denote the number of advertisements of firm i . The cost of

⁹In the marketing and business literatures, the ease with which a brand/shop comes to mind is referred to as "top-of-mind awareness" (see e.g. Kotler, 2000).

producing a_i advertisements is $\phi_i(a_i)$, with $\phi_i' > 0$ and $\phi_i'' \geq 0$.¹⁰ Given an advertising strategy profile (a_1, a_2, \dots, a_n) , suppose that the consumer has already visited v firms. Let V be the set of visited firms. We assume that the probability that she will recall firm $i \in N \setminus V$ in the next search is given by

$$\frac{a_i}{\sum_{j \in N \setminus V} a_j}.$$

This modelling of the recall probability captures the *inhibition* effects that own advertising has on the recall of competing brands (cf. Burke and Srull (1988) and Alba and Chattopadhyay (1986)). For simplicity, we assume that a firm that does not advertise, will not be visited and hence will not sell anything.

This modelling of the consumer recall process is similar to that in the rent-seeking contest described by Tullock (1980).¹¹ Intuitively, one can think of each advertisement of a firm as a ball this firm puts in an urn. Each firm can put as many balls in the urn as it likes. Whenever the consumer needs a product, she draws one ball from the urn and visits the corresponding firm. If, after the first visit, the consumer decides to visit another firm, she proceeds in the same way: again draw a ball from the urn and visit the corresponding firm provided it has not been visited yet; and so forth.¹² We assume that the consumer cannot observe the levels of advertising of firms, i.e. she does not observe how many balls each firm has put in the urn. She only observes which ball she draws from the urn.

The timing in our model is as follows. First, firms simultaneously decide on advertising and prices. Second, consumers sequentially search for a satisfactory deal following the recall process described above. In Section 3.1 we focus on the case where all firms have

¹⁰Alternatively, we may assume that firms decide on the amount A_i to spend on advertising, while the amount of ads of firm i is given by $a_i = \tau_i(A_i)$. It is easy to see that this specification yields the exact same outcomes if we choose $\phi_i(\cdot) = \tau_i^{-1}(\cdot)$. The convexity of the advertising cost function is used only to ensure equilibrium existence and uniqueness (cf. Proposition 1); the other results hold for linear advertising costs.

¹¹Schmalensee (1976) uses a similar idea in the context of advertising, but in his model prices are exogenously given. Hortaçsu and Syverson (2004), in their empirical study of price dispersion in the mutual fund industry, also model the funds' sampling probabilities in a similar way. Chioveanu (2007) also uses this type of advertising technology in the context of persuasive (loyalty-inducing) advertising in a market for homogeneous products.

¹²See Skaperdas (1996) and Kooreman and Schoonbeek (1997) for axiomatizations of Tullock's contest success function. An alternative formulation would be one where firms engage in an advertising race for consumers' attention (akin to the patent race models in the R&D literature). The results in Baye and Hoppe (2003) show that these two formulations are strategically equivalent.

the same advertising technology i.e. where $\phi_i = \phi$, $i = 1, 2, \dots, n$. In Section 4 we study a market where firms differ in their advertising technologies.

3 Symmetric firms

3.1 Analysis

In this section we assume that all firms have the same advertising technology, denoted by ϕ . We look for a symmetric Nash equilibrium. Consider firm i . Suppose that all firms different from i charge price p^* and set advertising level a^* . A symmetric Nash equilibrium then requires that the best reply for firm i is to also set (a^*, p^*) . To calculate firm i 's payoff, we need to take into account the order in which firms may be visited, and the probability to make a sale conditional on being visited.

Suppose that the buyer approaches firm i in her first search. This firm provides her with net utility $\varepsilon_i - p_i$. If $\varepsilon_i - p_i < 0$, the consumer will search again. Suppose $\varepsilon_i - p_i \geq 0$. A visit to some other firm j will yield $\varepsilon_j - p^*$. This is higher than the utility from buying from firm i if $\varepsilon_j > \varepsilon_i - \Delta$, with $\Delta = p_i - p^* \geq 0$. If we define $x \equiv \varepsilon_i - \Delta$, the expected benefit from searching once more thus equals

$$g(x) \equiv \int_x^1 (\varepsilon - x) f(\varepsilon) d\varepsilon.$$

An additional search is worthwhile if and only if these incremental benefits exceed the cost of search s . The buyer is exactly indifferent between an additional search and accepting the offer at hand if $x \geq \hat{x}$, with \hat{x} implicitly defined by

$$g(\hat{x}) = s. \tag{1}$$

The function g is monotonically decreasing. Moreover, $g(0) = \int_0^1 \varepsilon f(\varepsilon) d\varepsilon$ and $g(1) = 0$. It is readily seen that $\bar{s} < \int_0^1 \varepsilon f(\varepsilon) d\varepsilon$. Therefore, for any $s \in [0, \bar{s}]$, there exists a unique $\hat{x} \in [p^m, 1]$ that solves (1).

Since any equilibrium necessarily has $\hat{x} \geq p^*$, the probability that a buyer stops searching at firm i given that firm i is sampled, is equal to¹³

$$\Pr[x > \hat{x} \text{ and } \varepsilon_i > p_i] = \Pr[x > \hat{x}] = (1 - F(\hat{x} + \Delta)).$$

¹³This expression assumes that the deviation price p_i is not too high, that is, $1 - F(\hat{x} + \Delta)$ is strictly

If we denote the probability that a consumer visits firm i in her first search and buys there right away as $\lambda_1^i(a_i, p_i; a^*, p^*)$ we have:

$$\lambda_1^i(a_i, p_i; a^*, p^*) = \frac{a_i}{a_i + (n-1)a^*} (1 - F(\hat{x} + \Delta)) \quad (2)$$

Now consider the case that a consumer goes to firm i in her second search and then decides to buy there. This implies that she has visited some other firm first, say firm j , but decided not to buy there. In the symmetric Nash equilibrium, whenever she walks away from a firm, she expects price to be equal to p^* in the next shop. The probability that she walks away from j is thus given by $\Pr[\varepsilon_j < \hat{x}] = F(\hat{x})$. If we denote by $\lambda_2^i(a_i, p_i; a^*, p^*)$ the probability that firm i is visited in second place and the consumer decides to buy from i right away, we have

$$\lambda_2^i(a_i, p_i; a^*, p^*) = \left(\frac{(n-1)a_i}{a_i + (n-1)a^*} \right) \left(\frac{a_i}{a_i + (n-2)a^*} \right) F(\hat{x})(1 - F(\hat{x} + \Delta)) \quad (3)$$

where we have used the fact that the reservation value \hat{x} is the same no matter how many firms the consumer has already been visited (see Kohn and Shavell, 1974). More generally, the joint probability that a consumer visits firm i in her k^{th} , $k = 3, \dots, n$ search and buys there right away is

$$\lambda_k^i(a_i, p_i; a^*, p^*) = \frac{a_i}{a_i + (n-k)a^*} \prod_{\ell=1}^{k-1} \frac{(n-\ell)a^*}{a_i + (n-\ell)a^*} F(\hat{x})^{k-1} [1 - F(\hat{x} + \Delta)]. \quad (4)$$

There is a probability that the consumer initially decides to walk away from firm i only to find that, after having visited all firms in the industry, firm i offered the best deal after all. Of course, the consumer will then return to firm i to buy there. The probability of this occurring is

$$\Pr[\max\{x, \max_{j \neq i} \{\varepsilon_j\}\} < \hat{x} \text{ and } \varepsilon_i - p_i > \max_{j \neq i} \{\varepsilon_j - p^*\} \text{ and } \varepsilon_i > p_i]$$

This probability is independent of the order in which firms are visited. We will denote it as $R(p_i; p^*)$. Hence

$$R(p_i; p^*) = \int_{p_i}^{\hat{x} + \Delta} F(\varepsilon - \Delta)^{n-1} f(\varepsilon) d\varepsilon. \quad (5)$$

positive. Otherwise, no consumer would ever stop searching at firm i and, as a result, this firm would only sell to consumers who happen to find no acceptable product elsewhere. We come back to this issue in the proof of Proposition 1.

For p_i close to p^* , we can now write firm i 's expected profits as

$$\Pi_i(a_i, p_i; a^*, p^*) = p_i \left[\sum_{k=1}^n \lambda_k^i(a_i, p_i; a^*, p^*) + R(p_i; p^*) \right] - \phi(a_i). \quad (6)$$

It is important to note that this expression is only valid for p close enough to p^* , i.e. for small deviations from the tentative equilibrium (see footnote 13). For large deviations such that $F(\hat{x} + \Delta) = 1$ the profit function looks different as the consumer would then never buy directly at firm i . We take this case into account in the proof of the next result.¹⁴

Proposition 1 *If a symmetric Nash equilibrium exists, advertising levels and prices are given by the following system of equations:*

$$a^* \phi'(a^*) - \frac{p^*}{n} \left(1 - F(\hat{x})^n - \sum_{k=0}^{n-1} \frac{F(\hat{x})^k (1 - F(\hat{x})^{n-k})}{n-k} \right) = 0 \quad (7)$$

$$\frac{1 - F(p^*)^n}{n} + p^* \left(-\frac{f(\hat{x})}{n} \frac{1 - F(\hat{x})^n}{1 - F(\hat{x})} + \int_{p^*}^{\hat{x}} F(\varepsilon)^{n-1} f'(\varepsilon) d\varepsilon \right) = 0 \quad (8)$$

Suppose that F represents the uniform distribution and that ϕ'' is sufficiently large. Then a symmetric equilibrium exists and is unique.

The proof of this result proceeds along the following lines. In step 1 we show that the first order conditions imply the expressions (7) and (8). In step 2, we show that a solution to this system of equations exists, and that it is unique if for example the distribution of match values is uniform. Such a solution is indeed a symmetric Nash equilibrium if a firm cannot profitably deviate when all other firms play (p^*, a^*) . For general distributions functions F , however, the expression $p_i R(p_i; p^*)$ may not be quasi-concave in p_i and so the profit $\Pi_i(\cdot)$ may be maximized at a pair other than (p^*, a^*) . Under log-concavity of the density function f , the expression $p_i R(p_i; p^*)$ is quasi-concave in p_i (see Caplin and Nalebuff, 1991) but this does not guarantee that $\Pi_i(\cdot)$ is also quasi-concave in p_i (since the sum of quasi-concave functions need not be quasi-concave; see also Anderson and Renault, 1999). In step 3, we therefore focus on the case where F is the uniform distribution and

¹⁴Similarly, if a firm were to set an advertising effort arbitrarily close to zero, then the firm would be visited last with a probability arbitrarily close to one and then the payoff would be similar. In the Appendix we show that these cases are not really problematic.

ϕ'' is large. In that case, we show that the payoff function $\Pi_i(\cdot)$ is globally concave, so the solution to the system of equations(7) and (8) is indeed the unique symmetric Nash equilibrium. In step 4, we show that large deviations from (p^*, a^*) , for which the profit function (6) is no longer the relevant one, are not profitable either.

3.2 Comparative Statics

Proposition 2 *The comparative statics of the equilibrium described in Proposition 1 are as follows:*

1. *An increase in the marginal cost of advertising has no effect on equilibrium prices p^* and lowers the equilibrium number of ads a^* .*
2. *If the density of match values is non-decreasing, an increase in search costs s raises equilibrium price p^* and raises the equilibrium number of ads a^* .*
3. *If match values are uniformly distributed, an increase in the number of firms n increases per-firm advertising for n sufficiently low. In general, per-firm advertising goes to zero as the number of firms approaches infinity.*

Since all firms advertise with the same intensity in a symmetric equilibrium, changes in advertising costs have no effect on equilibrium prices; these are only affected by relative differences in advertising levels.¹⁵ Naturally, if advertising is more expensive, firms choose to use less of it.

The result on the relationship between prices and search costs is similar to Anderson and Renault (1999), who study a setting where the market is always fully covered in equilibrium, in the sense that every consumer buys a product of one of the shops. Our result extends theirs to a setting where industry demand is elastic. As search costs increase, the probability that a consumer walks away from a firm to sample another one decreases. This confers market power to firms that are visited and hence prices increase. The result

¹⁵The fact that advertising costs do not affect equilibrium prices is an artifact of the symmetry of equilibrium. Later in Section 4 we show how lowering the advertising cost of a firm results in a fall in its equilibrium price.

on the relationship between search costs and advertising is novel. An increase in search costs increases the market power of a firm that is visited. Hence it becomes more desirable for a firm to attract that consumer. As a result, firms advertise with greater intensity as search costs rise.

An increase in the number of firms has two effects on firms' incentive to advertise. First, if there are more firms that put out ads, the marginal effectiveness of an additional ad of firm i decreases. This lowers the incentive to advertise. Second, as the number of firms increases, it becomes more important to attract a consumer early. This raises the incentive to advertise. If the number of firms is small, the second effect dominates. With many firms, the first effect does. Figure 1 shows that advertising intensity is non-monotonic in the number of firms for the uniform distribution case. Prices and profits of the firms also decrease in n .

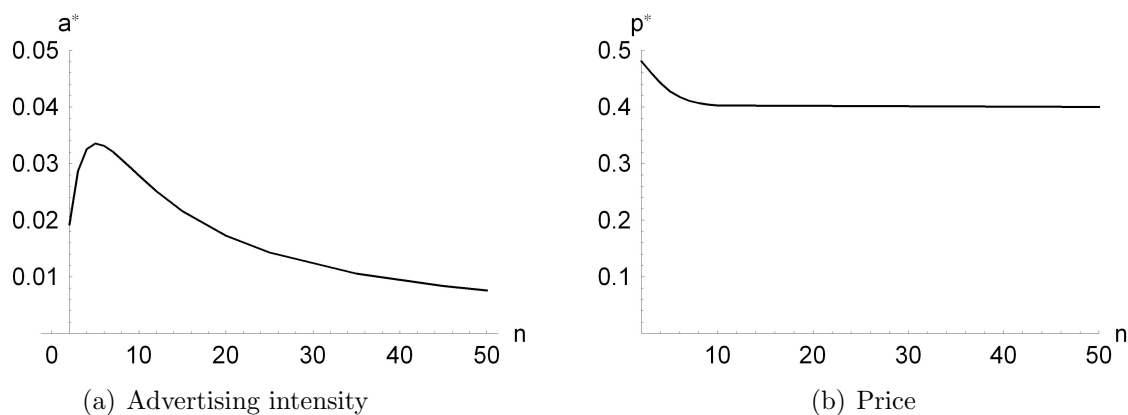


Figure 1: Price, advertising intensity and the number of firms

3.3 Profits and welfare

Search costs are generally seen as a boon to firms. As search costs increase, firms have more market power, which leads to higher profits (see e.g. Reinganum, 1979; Burdett and Judd, 1983; or Stahl, 1989). The following result however shows that that is not necessarily true in our model.

Proposition 3 *Assume the advertising technology is linear. Then, the equilibrium effect of search costs on firm profits is as follows:*

1. *For sufficiently small search costs s , profits increase in s .*
2. *For sufficiently large search costs s , profits may decrease in s and eventually fall below the profits that firms would make in a frictionless world. In particular, this is true with 2 firms and uniformly distributed matching values.*

An increase in search costs s has two opposite effects on firm profits. With an increase in s , firms gain market power over customers that pay them a visit, which allows them to charge a higher price. This tends to increase profits. But this also implies that it becomes more attractive for each individual firm to attract consumers, to invest in saliency and try to beat its rivals in the battle for attention. As a result, firms spend more on advertising, which tends to lower firm profits. In our model, advertising is a rent-seeking activity that leads to a dissipation of the rents generated by greater market power. When search costs are small, the price effect dominates and firms gain from an increase in search costs. When search costs are large, the advertising effect may dominate and profits decrease with higher search costs.¹⁶ Interestingly, we may even have an overdissipation of rents in the sense firms spend more resources to capture the additional rents than those rents are actually worth. This effect can become so severe that firms end up obtaining profits that are lower than those in a world with zero search costs. In Figure 2 we plot equilibrium profits against search costs. The dashed lines show the profits firms would make if advertising were banned ($a = 0$), and the profits firms would make if search costs were zero ($s = 0$).

In our model, lowering search costs always increases welfare. If the market were fully covered, total welfare would be maximized if investments in advertising were minimized. From Proposition 3, we know this to be the case if search costs are zero. Since we consider a case in which industry demand is not completely inelastic, this result is only reinforced,

¹⁶It can be shown that this is not only true in the case described in the Proposition. It can also be shown to be true if the market is fully covered, as in Anderson and Renault (1999), or if the first search is costless, regardless of the number of firms and the distribution of matching values. Details are available from the authors upon request.

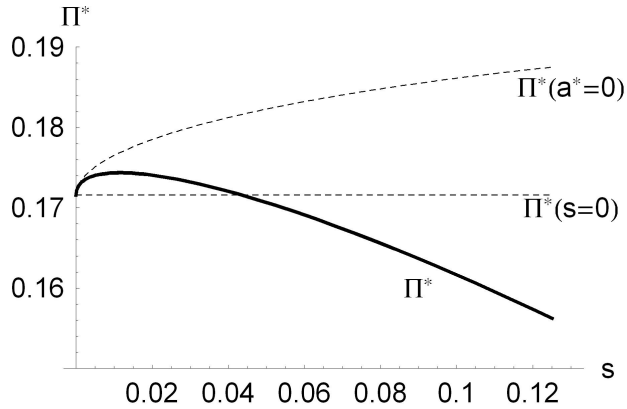


Figure 2: Equilibrium profits ($n = 2$, $f = 1$, $\phi(a) = \alpha a$)

as lower search costs imply lower prices and hence a lower deadweight loss.¹⁷

Note that, in this model, firms find themselves in a prisoners' dilemma. If a firm advertised less than the rest, the chance that this firm is pushed to the end of consumers search order would be higher. In equilibrium all firms advertise with the same intensity, which implies that consumers end up recalling each firm with the same probability. Firms would thus be better off if advertising were banned, while consumers would not be affected. From a welfare point of view, advertising is purely wasteful. That is no longer true if we extend the model to allow for asymmetric advertising technologies. In that case, the equilibrium will see one firm advertising more than the other. This implies that one firm is more likely to be visited than the other, which in turn affects firms' pricing incentives. We study this case in the next section.

4 Asymmetric firms

4.1 Introduction

The analysis in the previous section has ex-ante symmetric firms. This implies that in equilibrium all firms that have not yet been visited by a given consumer, are always equally

¹⁷Interestingly, when search costs are sufficiently high, it would even be a Pareto improvement to have lower search costs. Consumers are better off as equilibrium prices decrease, while firms are better off as equilibrium profits increase. Of course, here we are not taking into consideration the advertising industry. If we did, advertisers would lose as search costs fall (advertising expenditures are just transfers from the product market to the advertising industry).

likely to be visited next. Yet, it would be interesting to see how results are affected if firms are no longer symmetric. Do firms that attract more consumers charge higher or lower prices? More specifically, is higher advertising correlated with higher prices, or with lower ones? How are consumer welfare and firm profits affected if consumers overwhelmingly visit firms in the same order? We address such questions in this section.

To generate asymmetries between firms, we assume that they differ in their advertising technology: for some exogenous reason, one firm is able to raise awareness at lower costs than the other, for example because it runs a more effective advertising campaign, has a more memorable shop's name, or has a higher stock of advertising goodwill inherited from the past.¹⁸ Technically, we assume that advertising cost ϕ differs between firms, so we write $\phi_i(a_i)$, $i \in \{1, 2\}$. Moreover, we assume that this is common knowledge.¹⁹ It turns out that introducing asymmetries greatly complicates the analysis. We therefore have to restrict ourselves to a setting with 2 firms and a uniform distribution of matching values. Even that simple set-up does not allow us to always find analytical results, so we will partly have to resort to a numerical analysis.

One complication has to do with consumer search behavior. Suppose that the equilibrium has one firm charging a low price and one firm charging a high price. Consumers know which prices are set in equilibrium. Suppose moreover that a consumer observes an out-of-equilibrium price at her first visit. Her decision whether to continue searching will then be affected by whether she interprets this out-of-equilibrium price as coming from the low-priced firm or from the high-priced firm. One way to circumvent such complication is to assume that, upon visiting a firm, a consumer can observe its advertising technology.²⁰

¹⁸Admittedly, there are alternative ways to introduce asymmetries across firms, for example the firms could have different marginal costs of production or offer different quality levels. We have chosen differences in advertising costs because in the absence of advertising the equilibrium is still symmetric; this allows us to focus on a case where the asymmetry in equilibrium prices stems only from differences in advertising levels.

¹⁹To retain a priori symmetry between firms, we could have assumed that the parameters of each firm's advertising technology are drawn from the same probability density function $H(\alpha)$. Yet, even in the simplest case of a linear advertising technology, solving for equilibrium advertising levels would then boil down to solving a rent-seeking game between players that have private information about their costs, which is impossible to do. See Fey (2008) for an extensive analysis of such a game.

²⁰For example, from observing the lay-out and the colours in the store, she may realize that she has actually seen more ads from the other store and hence this store must be the one with the more costly advertising technology.

Alternatively, we can invoke an argument akin to Cho and Kreps' (1987) intuitive criterion to argue that for a consumer it is reasonable to believe that a deviation comes always from the high-priced firm.²¹

4.2 Analysis

Let $\omega \in \{1, 2, 12, 21\}$ denote which firms a particular consumer visits, and in what order. Thus $\omega = 12$ implies that the consumer has first visited firm 1, and then firm 2. Let q_i^ω denote total demand for firm i from such consumers. Thus q_1^{12} denotes demand for firm 1 from consumers that visit firm 1 and 2 in that order, while q_1^1 denotes demand for firm 1 from consumers that only visit firm 1. Denote with (a_1^*, p_1^*) and (a_2^*, p_2^*) the equilibrium strategy profile of the firms. In general, to study whether (a_i^*, p_i^*) is a best reply to (a_j^*, p_j^*) , with $i \in \{1, 2\}$, $j \neq i$, we allow firm i to defect to some (a_i, p_i) . Its profits then equal

$$\pi_i = p_i (q_i^i + q_i^{ij} + q_i^{ji}) - \phi_i(a_i), \quad (9)$$

where we have suppressed the arguments of the demand functions for ease of exposition. To evaluate these profits, we first have to derive the relevant demand functions. Consider q_i^i . Suppose a buyer approaches i in her first search. This occurs with probability $a_i/(a_i + a_j^*)$. She then observes ε_i and p_i . In equilibrium, she knows that a visit to j will yield utility $\varepsilon_j - p_j^*$. She benefits from such a visit whenever $\varepsilon_j > \varepsilon_i - (p_i - p_j^*) \equiv x_i$. Hence, her expected benefit is $\int_{x_i}^1 (\varepsilon - x_i) f(\varepsilon) d\varepsilon$. Recall that \hat{x} is the solution to $\int_{\hat{x}}^1 (\varepsilon - \hat{x}) f(\varepsilon) d\varepsilon = s$. The probability that this consumer immediately buys from firm i then equals²² $\Pr[x_i >$

²¹This requires that the consumer attaches zero probability to the event that an out-of-equilibrium action comes from a player that cannot possibly benefit from such a deviation. Suppose that equilibrium prices derived under the assumption that consumers learn the true type of both firms upon visiting one of them are given by (p_1^*, p_2^*) with $p_1^* > p_2^*$. From the point of view of the firms, the most favorable out-of-equilibrium belief a consumer can have is that the defecting firm is the low-priced firm for sure: with that belief, she is least likely to walk away as she believes the other firm to be more expensive. But even if the consumer has such beliefs, the low-priced firm is never willing to defect from p_2^* , since this equilibrium price has already been derived under the assumption that the consumer believes this to be the low-priced firm. Hence, a defection from p_2^* can never be profitable for firm 2. This implies that any defection will be interpreted as coming from the high-priced firm, which in turn implies that this firm never has an incentive to deviate from p_1^* .

²²Note that again, we must have $\hat{x} > p_2^*$, which implies that this probability is well-defined. We also assume that $\hat{x} + p_i - p_j^* \in (0, 1)$. In equilibrium, this is indeed the case.

$\hat{x}] = 1 - F(\hat{x} + p_i - p_j^*)$. Hence, using that F is a uniform distribution, we have

$$q_i^i = \frac{a_i}{a_i + a_j^*} (1 - \hat{x} - p_i + p_j^*). \quad (10)$$

Next, q_i^{ij} reflects a consumer that visits i first and finds an acceptable deal there, then decides to also visit j , only to find that j provides her with a worse deal than i . Conditional on visiting i first, the probability of this occurring is $\Pr[x_i < \hat{x} \text{ and } \varepsilon_i - p_i > \varepsilon_j - p_j^* \text{ and } \varepsilon_i > p_i]$, hence

$$q_i^{ij} = \frac{a_i}{a_i + a_j^*} \int_{p_i}^{\hat{x} + p_i - p_j^*} (\varepsilon_i - p_i + p_j^*) d\varepsilon_i. \quad (11)$$

Consider the consumer that visits j first. She observes a deal giving her utility $\varepsilon_j - p_j^*$. At firm i , this consumer expects to see a price equal to p_i^* . If we define $x_j^* \equiv \varepsilon_j - p_j^* + p_i^*$, the probability she also visits i is $\Pr[x_j^* < \hat{x}]$. Conditional on visiting j first, the probability that a consumer buys from i therefore is $\Pr[x_j^* < \hat{x} \text{ and } \varepsilon_i - p_i > \varepsilon_j - p_j^* \text{ and } \varepsilon_i > p_i]$. This implies

$$q_i^{ji} = \frac{a_j^*}{a_i + a_j^*} \left((\hat{x} + p_j^* - p_i^*) (1 - \hat{x} - p_i + p_i^*) + \int_{p_i}^{\hat{x} + p_i - p_i^*} (\varepsilon_i - p_i + p_j^*) d\varepsilon_i \right) \quad (12)$$

Plugging (10), (11) and (12) into profits (9), we have:

$$\begin{aligned} \pi_i &= p_i \frac{a_i}{a_i + a_j^*} \left(1 - \hat{x} - p_i + p_j^* + \frac{1}{2}(\hat{x}^2 - p_j^{*2}) \right) \\ &\quad + p_i \frac{a_j^*}{a_i + a_j^*} \left((\hat{x} + p_j^* - p_i^*)(1 - \hat{x} - p_i + p_i^*) + \frac{1}{2}(\hat{x} - p_i^*)(\hat{x} + 2p_j^* - p_i^*) \right) - \phi_i(a_i). \end{aligned}$$

Taking the first order conditions with respect to own advertising intensity and price, and imposing $p_i = p_i^*$ and $a_i = a_i^*$ we have, respectively:

$$\begin{aligned} 0 &= p_i^* \frac{a_j^*}{(a_i^* + a_j^*)^2} \left(1 - \hat{x} - p_i^* + p_j^* + \frac{1}{2}(\hat{x}^2 - p_j^{*2}) \right) \\ &\quad - p_i^* \frac{a_j^*}{(a_i^* + a_j^*)^2} \left((\hat{x} + p_j^* - p_i^*)(1 - \hat{x}) + \frac{1}{2}(\hat{x} - p_i^*)(\hat{x} + 2p_j^* - p_i^*) \right) - \phi_i'(a_i) \end{aligned} \quad (13)$$

$$\begin{aligned} 0 &= \frac{a_i^*}{a_j^* + a_j^*} \left(1 - \hat{x} - 2p_i^* + p_j^* + \frac{1}{2}(\hat{x}^2 - p_j^{*2}) \right) \\ &\quad + \frac{a_j^*}{a_i^* + a_j^*} \left((\hat{x} + p_j^* - p_i^*)(1 - \hat{x} - p_i^*) + \frac{1}{2}(\hat{x} - p_i^*)(\hat{x} + 2p_j^* - p_i^*) \right) \end{aligned} \quad (14)$$

Writing the conditions (13) and (14) for $i = 1, 2$ and $j \neq i$ yields four nonlinear equalities that can be solved to find equilibrium advertising levels and prices. From these first-order conditions, we can prove the following results:

Proposition 4 *With 2 firms, a uniform distribution of matching values, and asymmetric advertising technologies, we have that the firm that advertises more, sets a lower price: $a_i^* > a_j^*$ necessarily implies $p_i^* < p_j^*$;*

This result can be understood as follows. By choosing to visit a second firm, consumers reveal that they do not particularly like the product the first firm offered. Hence, such consumers are less price-sensitive than consumers who still have the option to visit another shop. The firm with less advertising has a higher share of these less price-sensitive consumers. Therefore, it finds it profitable to charge a higher price. This result is in line with the study of Armstrong et al. (2009) on prominence. In their paper one firm, the prominent one, is visited first for sure. Should the consumer decide to sample more firms, she does so at random. This corresponds to the case in our model with advertising levels exogenously set to $a_1^* > 0$ and $a_2^* = 0$.

4.3 Linear advertising technologies

To put additional structure on the model, we assume that advertising technologies are linear, so $\phi_i(a) = \alpha_i a$. Moreover, we assume that firm 1 is more *advertising-efficient* than firm 2, in the sense that raising additional awareness is always cheaper for firm 1 than it is for firm 2, so $\alpha_1 < \alpha_2$. It is then easy to show that firm 1 will advertise more:

Proposition 5 *In equilibrium, the more advertising-efficient firm will advertise more.*

4.4 Numerical analysis

To do comparative statics we have to resort to a numerical analysis. We again assume linear advertising technologies. Without loss of generality, we assume that firm 1 has the more efficient advertising technology, and normalize α_2 to 1, so $\alpha_1 \leq \alpha_2 < 1$. From the analysis above in Section 4.2 we know that this implies that $a_1^* \geq a_2^*$ and $p_1^* \leq p_2^*$. The

parameter $\alpha \equiv \alpha_1$ now reflects the extent of asymmetry between advertising technologies: as α increases, advertising technologies become more symmetric.

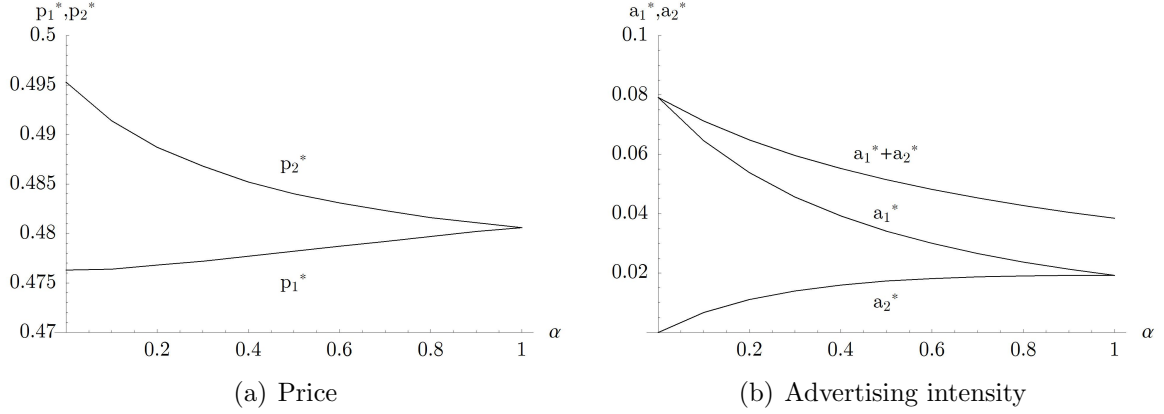


Figure 3: Price, advertising intensity and the number of firms

In Figure 3 we depict equilibrium prices and advertising levels as a function of α . For the level of search costs, we have chosen $s = 0.08$, but changing this value does not affect these graphs qualitatively.

Result 1 *With 2 firms, a uniform distribution of matching values, linear advertising technologies, and firm 1 the more advertising-efficient firm, we have the following:*

1. *if we denote by (a_s^*, p_s^*) equilibrium advertising levels and prices in case of equal advertising technologies, then $p_1^* < p_s^* < p_2^*$.*
2. *an increase in the asymmetry in firm advertising efficiency has the following effects:*
 - (a) *the price of the cheapest firm decreases, that of the most expensive firm increases, while average prices also increase;*
 - (b) *the advertising level of the cheapest firm increases, that of the most expensive firm decreases, while average advertising levels also increase.*

The first result confirms the intuition behind Proposition 4: the cheaper firm also charges a lower price than what it charges with equal advertising, while the more expensive firm charges a price that is also higher than what it charges with equal advertising. Result 2a shows that the price gap becomes more pronounced as the difference in equilibrium advertising levels increases. Result 2b implies that, as the asymmetry in firm advertising costs increases, the difference in advertising efforts will also increase.

Note that a firm that advertises more, is more likely to be visited first by a consumer. As she knows that this firm charges a lower price than the other firm, she is also less likely to walk away from this firm. This suggests that the number of equilibrium searches will be lower when there is more asymmetry between advertising levels of the two firms. The following result establishes that this is indeed the case.²³

Result 2 *With 2 firms, a uniform distribution of matching values, and linear advertising technologies, we have that the number of searches, and hence total search costs incurred by consumers, decreases as the asymmetry in advertising levels increases.*

Hence, advertising now has social value as it helps consumers to channel their first-visits towards better deals.

4.5 Welfare

Consumer welfare will depend on where a consumer buys, and which firms she visits. In Figure 4, we have depicted this in $(\varepsilon_1, \varepsilon_2)$ -space. The left-hand panel gives the analysis for

²³If we take the results in Result 1 as given, we can also establish this result formally. By construction, each consumer searches at least once for sure. If she visits i first, the probability of a second search is $F(\hat{x} + p_i^* - p_j^*)$. If she visits j first, the probability of a second search is $F(\hat{x} + p_j^* - p_i^*)$. Denote $\gamma \equiv a_i^*/(a_i^* + a_j^*)$. We can write the expected number of searches as

$$\begin{aligned} E(\text{searches}) &= 1 + \gamma(\hat{x} + p_i^* - p_j^*) + (1 - \gamma)(\hat{x} + p_j^* - p_i^*) \\ &= 1 + \hat{x} + (1 - 2\gamma)(p_j^* - p_i^*) \end{aligned}$$

The results in Proposition 1 imply that $\partial p_2^*/\partial \gamma > 0$ and $\partial p_1^*/\partial \gamma < 0$, so

$$\frac{\partial E(\text{searches})}{\partial \gamma} = -2(p_j^* - p_i^*) + (1 - 2\gamma) \left(\frac{\partial p_j^*}{\partial \gamma} - \frac{\partial p_i^*}{\partial \gamma} \right) < 0.$$

Hence the number of searches decreases as γ increases, that is, if the asymmetry between equilibrium advertising levels increases.

consumers that first visit firm 1, the right-hand panel reflects consumers that first visit firm 2. In the left-hand panel, the dark-shaded area reflects the consumers that immediately buy from 1. Consumers in the vertically dashed area also buy from 1 – but only after having visited both firms. Consumers in the horizontally dashed area buy from 2 after having visited both firms. Consumers in the horizontally dashed area buy from 2 after having visited both firms. Consumers in the white bottom-left corner do not buy at all.

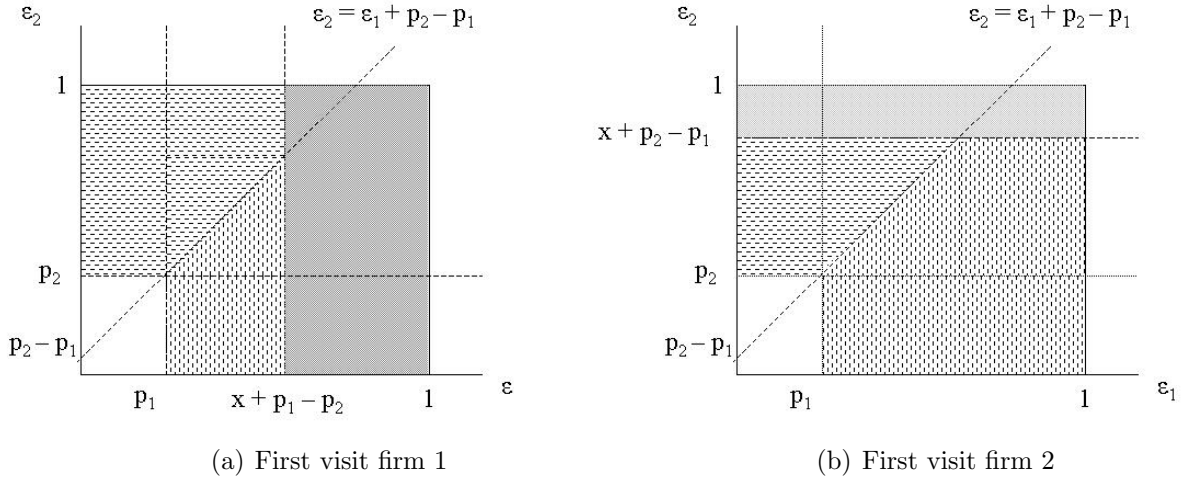


Figure 4: Consumer purchasing behavior

In the right-hand panel, consumers in the vertically dashed area again buy from 1, and consumers in the horizontally dashed area from 2, both after having visited both firms. Consumers in the lightly shaded area buy from 2, consumers in the white area do not buy at all.

Consider an increase in advertising asymmetry. The first effect of this is that total advertising of firm 1 increases, hence the left-hand panel of firm 1 will become relevant for more consumers. This effect is beneficial for consumer welfare, as more consumers now visit the cheaper firm first. Next, p_1^* decreases while p_2^* increases. This implies for the left-hand panel that the lines $\varepsilon_1 = p_1^*$ and $\varepsilon_1 = \hat{x} + p_1^* - p_2^*$ move to the left, while the lines $\varepsilon_2 = \varepsilon_1 + p_2^* - p_1^*$ and $\varepsilon_1 = p_2^*$ move upwards. Consumers that already bought from

1, or switch their choice to 1, benefit. The total number of searches decreases. Consumers that still buy from 2, however, are hurt, while numerical simulations show that the total number of non-buyers also increases. The effects in the right-hand panel are similar.

In sum, as asymmetry between advertising technologies increases (so α_1 decreases), the price of 1 decreases while that of 2 increases. The first effect is good news for consumers, also as they visit 1 more frequently than 2. However, as 2's price is higher, consumers who fail to find a satisfactory product at 1 are forced to accept a (much) higher price more often. On average consumers search less, which lowers their search costs but also makes them less exposed to variety. The total number of consumers who buy decreases, which is obviously a source of inefficiency. The aggregate effect on consumer welfare is therefore complex.

To calculate consumer surplus, we use the same notation as above: we let $\omega \in \{1, 2, 12, 21\}$ denote which firms a consumer has visited, and in what order. Thus $\omega = 12$ implies that this consumer has first visited firm 1, and then firm 2. Let CS_i^ω denote the total surplus of such consumers who buy from firm i . Consider, for example, consumers that buy from i that have only visited i . These consumers each incur total search costs s . Their net surplus thus is $\varepsilon_i - p_i^* - s$. Moreover, they have $\varepsilon_i > \hat{x} - p_j^* + p_i^*$. Hence

$$CS_i^i = \int_0^1 \int_{\hat{x} - p_j^* + p_i^*}^1 (\varepsilon_i - p_i^* - s) d\varepsilon_i d\varepsilon_j$$

Similarly,

$$CS_i^{ij} = \int_{p_i^*}^{\hat{x} - p_j^* + p_i^*} \int_0^{\varepsilon_i + p_j^* - p_i^*} (\varepsilon_i - p_i^* - 2s) d\varepsilon_j d\varepsilon_i$$

$$CS_j^{ij} = \int_{\hat{x}}^1 \int_0^{\hat{x} - p_j^* + p_i^*} (\varepsilon_j - p_j^* - 2s) d\varepsilon_i d\varepsilon_j + \int_{p_j^*}^{\hat{x}} \int_0^{\varepsilon_j - p_j^* + p_i^*} (\varepsilon_j - p_j^* - 2s) d\varepsilon_i d\varepsilon_j$$

Ex-ante expected consumer surplus then equals:

$$CS = \frac{a_1^*}{a_1^* + a_2} (CS_1^1 + CS_1^{12} + CS_2^{12}) + \frac{a_2^*}{a_1^* + a_2} (CS_2^2 + CS_2^{21} + CS_1^{21})$$

and total welfare is $W = CS + \Pi_1^* + \Pi_2^*$ as usual.

To fully appreciate the effect of a change in α on welfare, we have to resort to numerical analysis. In figure 5, we depict the components of total welfare equilibrium profits of firm

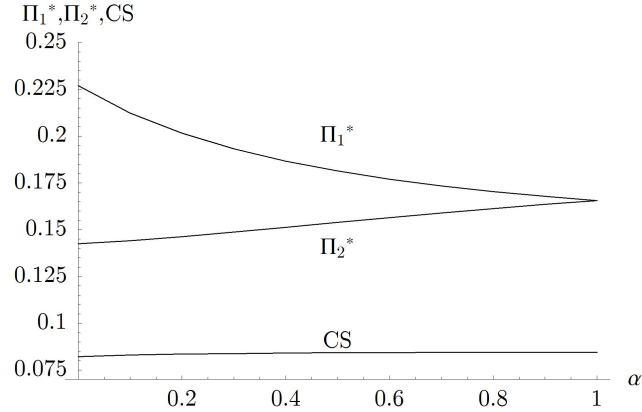


Figure 5: Welfare

1 and 2, as a function of α , for the case that $s = 0.08$. For different levels of s , the picture looks qualitatively the same. We can see that profits of firm 1 decrease as firms become more symmetric, while profits of firm 2 increase – but less so. Total profits thus decrease. From the figure, it is hardly discernible that consumer welfare increases as firms become more symmetric. Thus, the net effect of a decrease in α is for consumer welfare to go down, but this effect is very small.²⁴ Total welfare goes up as firms become more asymmetric. This is driven by the increase in the profits of firm 1 along with savings in search and advertising costs.²⁵

Simulations show that the comparative statics with respect to search costs are qualitatively unaffected by the asymmetry of advertising technologies.²⁶ For given advertising asymmetry α , total advertising is still increasing in search costs s . Equilibrium advertising levels of both firms increase in s , as do prices. Profits of firm 1, the most advertising efficient firm, are non-monotonic in s : initially they increase, but for high enough s they decrease. The same is true for firm 2. Total welfare decreases in search costs, as does consumer welfare.

²⁴For this particular parametrization, total consumer welfare falls by less than 1% as α changes from 1 to 0.1. If we set search costs equal to 0.02 rather than 0.08, exactly the same is true.

²⁵The welfare result is not only driven by the fact that firm 1 has access to a more efficient advertising technology. In fact we obtain also a welfare gain if we introduce asymmetries by increasing rather than decreasing the marginal cost of advertising of firm 1.

²⁶Details are available from the authors upon request.

5 Conclusion

In this paper we modelled the idea that firms engage themselves in a battle for attention in an attempt to being visited as early as possible in the course of search of a consumer. Through investments in more appealing advertising, a firm can achieve a salient place in consumer memories so that consumers will visit this firm sooner when searching for a product they need. We modelled this in the framework of a model of search with differentiated products. In such a framework, advertising is not a winner-takes-all contest: after a consumer has visited a firm, she may still decide to go to a different one if she does not sufficiently like the product of the current particular firm.

We found that prices and advertising levels are increasing in consumers' search costs. Yet, the effect on profits is ambiguous. If search costs are small to start with, then firms are better off if search costs increase. Instead, when search costs are already high a further increase in search costs may lower firm profits. In the latter case, getting the attention of a consumer becomes so important that firms over-dissipate the rents generated by being visited earlier than rival firms. This highlights the importance of looking at the interaction of advertising and search costs, rather than only looking at search costs or advertising in isolation. We believe this to be a general phenomenon, that applies beyond the scope of this particular model.

Another interesting finding is that firms with more efficient advertising technologies advertise more, charge lower prices and obtain greater profits than less efficient rivals. Moreover, an increase in advertising cost asymmetries leads to a fall in consumer surplus. Even though advertising serves to direct consumers to better deals on average, less advertising-efficient rivals increase their prices by so much that ultimately fewer consumers purchase a product in the market equilibrium. Asymmetries in advertising cost weaken the advertising competition between firms. This cut in advertising outlays along with search cost savings imply that total welfare increases.

Traditionally, persuasive advertising has been modelled as advertising that increases a consumer's utility from buying the product. This interpretation is problematic, as it makes it difficult to perform welfare analysis (see Bagwell, 2007). By combining saliency

enhancing advertising and search costs, our modelling approach may provide a natural way to think of persuasive advertising. In our model, advertising also increases demand for a product that is heavily advertised; however, this is not because consumers derive higher utility from advertised products but simply because they are more likely to visit shops for which they see many ads earlier than other shops, and, hence, because search costs are non-negligible, they are also more likely to buy from such shops. This difference has an implication on the relationship between prices and advertising outlays. Our model predicts that a firm that has more persuasive advertising than its competitor, will charge a lower price, as opposed to earlier work. Intuitively, consumers that only visit a shop because of its persuasive ads will be more price elastic than consumers that were already interested in the shop without seeing the ads. This price effect vanishes if both firms have the same level of persuasive advertising.

Appendix

Proof of Proposition 1

This proof consists of four steps. First, we show that the first-order conditions for profit maximization indeed imply (7) and (8). Second, we show that there exists a pair (p^*, a^*) that satisfies (7) and (8), and that it is unique if $f' \geq 0$, a property that is also satisfied by the uniform distribution. Third, we show that (p^*, a^*) is indeed a Nash equilibrium if we restrict attention to a uniform distribution of matching values, relatively small price deflections such that profits are given by (6), and sufficiently convex advertising cost functions. Fourth, we show that large deflections from (p^*, a^*) are never profitable either.

Step 1 We first derive the expressions (7) and (8) given in the Proposition. Maximizing (6) with respect to a_i and p_i yields the following first-order conditions:

$$p_i \sum_{k=1}^n \frac{\partial \lambda_k^i(a_i, p_i; a^*, p^*)}{\partial a_i} - \phi'(a_i) = 0 \quad (15)$$

$$\sum_{k=1}^n \lambda_k^i(a_i, p_i; a^*, p^*) + R(p_i; p^*) + p_i \left[\sum_{k=1}^n \frac{\partial \lambda_k^i(a_i, p_i; a^*, p^*)}{\partial p_i} + \frac{\partial R(p_i; p^*)}{\partial p_i} \right] = 0. \quad (16)$$

Using the expressions (2)-(4), we can compute

$$\begin{aligned} \frac{\partial \lambda_1^i}{\partial a_i} &= \frac{(n-1)a^*}{(a_i + (n-1)a^*)^2} (1 - F(\hat{x} + \Delta)) \\ &\dots \\ \frac{\partial \lambda_k^i}{\partial a_i} &= \left[\frac{a_i}{a_i + (n-k)a^*} \sum_{\ell=1}^{k-1} \left[\frac{-(n-\ell)a^*}{(a_i + (n-\ell)a^*)^2} \prod_{m \neq \ell}^{k-1} \frac{(n-m)a^*}{a_i + (n-m)a^*} \right] \right. \\ &\quad \left. + \frac{(n-k)a^*}{(a_i + (n-k)a^*)^2} \prod_{\ell=1}^{k-1} \frac{(n-\ell)a^*}{a_i + (n-\ell)a^*} \right] F(\hat{x})^{k-1} (1 - F(\hat{x} + \Delta)) \\ &\dots \\ \frac{\partial \lambda_n^i}{\partial a_i} &= \sum_{\ell=1}^{n-1} \left[\frac{-(n-\ell)a^*}{(a_i + (n-\ell)a^*)^2} \prod_{m \neq \ell}^{n-1} \frac{(n-m)a^*}{a_i + (n-m)a^*} \right] F(\hat{x})^{n-1} (1 - F(\hat{x} + \Delta)) \end{aligned}$$

In symmetric equilibrium we have

$$\begin{aligned}
\frac{\partial \lambda_1^i}{\partial a_i} &= \frac{n-1}{n^2 a^*} (1 - F(\hat{x})) \\
&\dots \\
\frac{\partial \lambda_k^i}{\partial a_i} &= \left[\frac{1}{n-k+1} \sum_{\ell=1}^{k-1} \left[\frac{-(n-\ell)}{(n-\ell+1)^2 a^*} \prod_{m \neq \ell}^{k-1} \frac{n-m}{n-m+1} \right] \right. \\
&\quad \left. + \frac{n-i}{(n-i+1)^2 a^*} \prod_{\ell=1}^{k-1} \frac{n-\ell}{n-\ell+1} \right] F(\hat{x})^{k-1} (1 - F(\hat{x})) \\
&\dots \\
\frac{\partial \lambda_n^i}{\partial a_i} &= \sum_{\ell=1}^{n-1} \left[\frac{-(n-\ell)}{(n-\ell+1)^2 a^*} \prod_{m \neq \ell}^{n-1} \frac{n-m}{n-m+1} \right] F(\hat{x})^{n-1} (1 - F(\hat{x})).
\end{aligned}$$

Note that

$$\prod_{\ell=1}^{k-1} \frac{n-\ell}{n+1-\ell} = \frac{n-1}{n} \cdot \frac{n-2}{n-1} \cdot \dots \cdot \frac{n+1-k}{n+2-k} = \frac{n+1-k}{n}.$$

This allows us to simplify some expressions, in particular:

$$\begin{aligned}
\frac{\partial \lambda_k^i}{\partial a_i} &= \left[\frac{1}{n-k+1} \sum_{\ell=1}^{k-1} \left[\frac{-1}{(n-\ell+1)^2 a^*} \left(\frac{n+1-k}{n} \right) (n-\ell+1) \right] \right. \\
&\quad \left. + \frac{n-k}{(n-k+1) n a^*} \right] F(\hat{x})^{k-1} (1 - F(\hat{x})) \\
&= \frac{1}{n a^*} \left[\frac{n-k}{n-k+1} - \sum_{\ell=1}^{k-1} \frac{1}{(n-\ell+1)} \right] F(\hat{x})^{k-1} (1 - F(\hat{x})), \text{ for all } k = 1, 2, \dots, n.
\end{aligned}$$

Moreover

$$\sum_{k=1}^n \lambda_k(a^*, p^*) = \frac{1}{n} (1 - F(\hat{x})^n).$$

Using these derivations and the expression for $R(p^*)$ in (5) above, the first order conditions in (15) and (16) can be rewritten as:

$$p^* \sum_{k=1}^n \frac{1}{n a^*} \left(\frac{n-k}{n-k+1} - \sum_{\ell=1}^{k-1} \frac{1}{(n-\ell+1)} \right) F(\hat{x})^{k-1} (1 - F(\hat{x})) - \phi'(a^*) = 0, \quad (17)$$

$$\frac{1 - F(\hat{x})^n}{n} + \int_{p^*}^{\hat{x}} F(\varepsilon)^{n-1} f(\varepsilon) d\varepsilon \quad (18)$$

$$+ p^* \left(-\frac{f(\hat{x})}{n} \frac{1 - F(\hat{x})^n}{1 - F(\hat{x})} - \int_{p^*}^{\hat{x}} (n-1) F(\varepsilon)^{n-2} f(\varepsilon)^2 d\varepsilon - F(p^*)^{n-1} f(p^*) + F(\hat{x})^{n-1} f(\hat{x}) \right) = 0.$$

Integration by parts of (18) yields (8). To see that (17) implies (7), denote

$$C_k \equiv \frac{n-k}{n-k+1} - \sum_{\ell=1}^{k-1} \frac{1}{n-\ell+1}. \quad (19)$$

so we can write (17) as

$$a^* \phi'(a^*) = \frac{p^*}{n} (1 - F(\hat{x})) \sum_{k=1}^n C_k \cdot F(\hat{x})^{k-1}. \quad (20)$$

Note that

$$\begin{aligned} C_k - C_{k-1} &= \left(\frac{n-k}{n-k+1} - \sum_{\ell=1}^{k-1} \frac{1}{n-\ell+1} \right) - \left(\frac{n-k+1}{n-k+2} - \sum_{\ell=1}^{k-2} \frac{1}{n-\ell+1} \right) \\ &= \frac{n-k}{n-k+1} - \frac{1}{n-k+2} - \frac{n-k+1}{n-k+2} = \frac{-1}{n-k+1}. \end{aligned}$$

From (19), we have $C_1 = (n-1)/n$ so by induction

$$C_k = \frac{n-1}{n} - \sum_{\ell=1}^{k-1} \frac{1}{n-\ell}.$$

Plugging this back into (20), we have

$$\begin{aligned} a^* \phi'(a^*) &= \frac{p^*}{n} (1 - F(\hat{x})) \sum_{k=1}^n \left[\frac{n-1}{n} - \sum_{\ell=1}^{k-1} \frac{1}{n-\ell} \right] \cdot F(\hat{x})^{k-1} \\ &= \frac{p^*}{n} (1 - F(\hat{x})) \left[\frac{n-1}{n} \sum_{k=1}^n F(\hat{x})^{k-1} - \sum_{k=1}^n \sum_{\ell=1}^{k-1} \frac{1}{n-\ell} \cdot F(\hat{x})^{k-1} \right] \\ &= \frac{p^*}{n} (1 - F(\hat{x})) \left[\frac{n-1}{n} \sum_{k=1}^n F(\hat{x})^{k-1} - \sum_{\ell=1}^{n-1} \left(\frac{1}{n-\ell} \sum_{k=\ell+1}^n F(\hat{x})^{k-1} \right) \right] \\ &= \frac{p^*}{n} (1 - F(\hat{x})) \left[\frac{n-1}{n} \sum_{k=0}^{n-1} F(\hat{x})^k - \sum_{\ell=1}^{n-1} \left(\frac{1}{n-\ell} \sum_{k=\ell}^{n-1} F(\hat{x})^k \right) \right], \end{aligned}$$

which can be further simplified to

$$\begin{aligned} a^* \phi'(a^*) &= \frac{p^*}{n} (1 - F(\hat{x})) \left[\frac{n-1}{n} \cdot \frac{1 - F(\hat{x})^n}{1 - F(\hat{x})} - \sum_{\ell=1}^{n-1} \left(\frac{1}{n-\ell} \right) \frac{F(\hat{x})^\ell - F(\hat{x})^n}{1 - F(\hat{x})} \right] \\ &= \frac{p^*}{n} \left[\frac{n-1}{n} \cdot (1 - F(\hat{x})^n) - \sum_{\ell=1}^{n-1} \left(\frac{1}{n-\ell} \right) F(\hat{x})^\ell (1 - F(\hat{x})^{n-\ell}) \right] \\ &= \frac{p^*}{n} \left[1 - F(\hat{x})^n - \sum_{k=0}^{n-1} \frac{F(\hat{x})^k (1 - F(\hat{x})^{n-k})}{n-k} \right], \end{aligned}$$

which is exactly (7).

Step 2. We now show that there exists a pair (p^*, a^*) that satisfies (7) and (8). By inspection of (7), it is immediately clear that for any p^* there is a unique a^* that accompanies p^* . We therefore focus on equation (8). To study the existence of a solution in p^* , it is useful to rewrite it as follows:

$$\frac{1 - F(p^*)^n}{np^*} = \frac{f(\hat{x})}{n} \frac{1 - F(\hat{x})^n}{1 - F(\hat{x})} - \int_{p^*}^{\hat{x}} F(\varepsilon)^{n-1} f'(\varepsilon) d\varepsilon. \quad (21)$$

Note that the RHS of (21) is finite when $p^* \rightarrow 0$. The LHS is a positive-valued function that decreases monotonically in p^* . Moreover, when $p^* \rightarrow 0$ the LHS goes to ∞ . Hence, for $p^* \rightarrow 0$ the LHS is larger than the RHS. If $p^* \rightarrow \hat{x}$, the LHS is smaller than the RHS if and only if $1 - F(\hat{x}) < \hat{x}f(\hat{x})$. Since $\hat{x} > p^m > p^*$ and by definition $1 - F(p^m) - p^m f(p^m) = 0$, concavity of monopoly profits implies that this condition always holds. With the LHS larger than the RHS at $p^* \rightarrow 0$, but smaller at $p^* \rightarrow \hat{x}$, continuity implies that there must be at least one $p^* \in (0, \hat{x})$ such that (21) is satisfied. If also $f' \geq 0$, we have that the RHS is strictly increasing in p^* . With the LHS strictly increasing in p^* , this implies that the solution to (21) is unique.

Step 3 In step 2, we established that there is an (a^*, p^*) that solves equations (7) and (8). Yet, that does not immediately imply that such an (a^*, p^*) is a Nash equilibrium. For this to be the case, we need that the payoff function of a firm i is globally quasi-concave on its domain. The domain of the payoff function is the set $D \equiv \{(a_i, p_i) \in [0, \infty) \times (0, p^m)\}$ but it is convenient to split it as follows: $D = D_1 \cup D_2 \cup D_3$ where $D_1 \equiv \{(a_i, p_i) \in (0, \infty) \times (0, F^{-1}(1) - \hat{x} + p^*)\}$, $D_2 = \{(a_i, p_i) \in [0, \infty) \times [F^{-1}(1) - \hat{x} + p^*, p^m)\}$ and $D_3 \equiv \{(a_i, p_i) \in \{0\} \times (0, p^m)\}$. On the set D_1 , the deviating payoff $\Pi_i(a_i, p_i; a^*, p^*)$ is given by (6).²⁷

Claim 1 *On D_1 , the function $\Pi_i(a_i, p_i; a^*, p^*)$ is strictly concave in a_i .*

To see this, define the function

$$y_n(a_i, a^*) \equiv \frac{a_i}{a_i + (n-1)a^*} + \left(\frac{(n-1)a^*}{a_i + (n-1)a^*} \right) \frac{a_i}{a_i + (n-2)a^*} F(\hat{x}) \\ + \sum_{k=3}^n \frac{a_i}{a_i + (n-k)a^*} \prod_{\ell=1}^{k-1} \frac{(n-\ell)a^*}{a_i + (n-\ell)a^*} F(\hat{x})^{k-1},$$

so we can write

$$\Pi_i(a_i, p_i; a^*, p^*) = p_i y_n(a_i, a^*) (1 - F(\hat{x} + \Delta)) + p_i R(p_i; p^*) - \phi(a_i).$$

Note that y_n reflects the probability that firm i will be visited given that all other firms stick to the candidate SNE price and advertising level. Also note that

$$y_2 = \frac{a_i}{a_i + a^*} + \frac{a^*}{a_i + a^*} F(\hat{x})$$

²⁷Deviations for which $p_i \geq F^{-1}(1) - \hat{x} + p^*$ are special because in those situations firm i would only sell to consumers who have walked away from all other rivals; we treat these cases later in step 4.

and, moreover

$$y_{k+1} = \frac{a_i}{a_i + ka^*} + \frac{ka^*}{a_i + ka^*} F(\hat{x}) y_k$$

for any $k > 2$. Taking the derivative of y_2 with respect to a_i :

$$y'_2 = \frac{a^*}{(a_i + a^*)^2} (1 - F(\hat{x})) > 0.$$

Hence y_2 is strictly increasing in a_i . For y'_{k+1} , we can write

$$y'_{k+1} = \frac{ka^*}{(a_i + ka^*)^2} (1 - F(\hat{x}) y_k) + F(\hat{x}) \left(\frac{ka^*}{a_i + ka^*} \right) y'_k.$$

Note that $F(\hat{x}) y_k < 1$. Hence, sufficient for this expression to be positive is that $y'_k > 0$. But we already know that this holds for $k = 2$. Hence, from this expression, it also holds for $k = 3$. Induction then implies that it holds for any k .

For the second derivative, we have

$$y''_2 = \frac{-2a^* (1 - F(\hat{x}))}{(a_i + a^*)^3} < 0.$$

and

$$y''_{k+1} = \frac{-2ka^*}{(a_i + ka^*)^3} (1 - F(\hat{x}) y_k) + F(\hat{x}) \left(\frac{ka^*}{a_i + ka^*} y''_k - 2 \frac{ka^*}{(a_i + ka^*)^2} y'_k \right)$$

Note that $F(\hat{x}) y_k < 1$. Hence, sufficient for this expression to be negative is that $y''_k < 0$ and $y'_k > 0$. But we already know that this holds for $k = 2$. Hence, from this expression, it also holds for $k = 3$. Induction then implies that it holds for any k . With $y''_{k+1} < 0$, we immediately have that $\partial^2 \Pi(\cdot) / \partial a_i^2 < 0$ for any (weakly) convex function $\phi(a_i)$.

Claim 2 *On D_1 , the function $\Pi_i(a_i, p_i; a^*, p^*)$ is not necessarily quasi-concave in p_i . However, when F represents the uniform distribution, then $\Pi_i(a_i, p_i; a^*, p^*)$ is strictly concave in p_i .*

To see that $\Pi_i(a_i, p_i; a^*, p^*)$ is not generally quasi-concave in p_i , consider the case in which $\hat{x} \rightarrow 1$, so search costs s go to zero). In that case our model converges to that of Perloff and Salop (1985). From Caplin and Nalebuff (1991), we know that the payoff function

$$p_i \int_{p_i}^1 F(\varepsilon - \Delta)^{n-1} f(\varepsilon) d\varepsilon$$

is quasi-concave if the density f is log-concave. However, with strictly positive search costs ($\hat{x} < 1$), our payoff function equals a summation of functions of p_i . This sum may not be quasi-concave in p_i , even if every summand is. In fact, if one sets $a_i = a^*$ above, our model approaches that of Anderson and Renault (1999) and, as they show, with positive search costs stronger conditions are needed for the payoff to be quasi-concave (see their appendix

B). We therefore focus on the case where F is the uniform distribution. In that case we have

$$\frac{\partial^2 \Pi_i(a_i, p_i; a^*, p^*)}{\partial p_i^2} = -2 \sum_{k=1}^n \frac{a_i}{a_i + (n-k)a^*} \prod_{\ell=1}^{k-1} \frac{(n-\ell)a^*}{a_i + (n-\ell)a^*} \hat{x}^{k-1} < 0,$$

which implies that $\Pi_i(a_i, p_i; a^*, p^*)$ is strictly concave in p_i

Claim 3 *When F is the uniform distribution, and when ϕ'' is sufficiently large, the function $\Pi_i(a_i, p_i; a^*, p^*)$ (defined on D_1) is strictly globally concave.*

The Hessian of Π_i is given by the matrix

$$H \equiv \begin{pmatrix} \frac{\partial^2 \Pi_i}{\partial p_i^2} & \frac{\partial^2 \Pi_i}{\partial p_i \partial a_i} \\ \frac{\partial^2 \Pi_i}{\partial p_i \partial a_i} & \frac{\partial^2 \Pi_i}{\partial a_i^2} \end{pmatrix}.$$

We already know that $\partial^2 \Pi(\cdot)/\partial p_i^2 < 0$ and $\partial^2 \Pi(\cdot)/\partial a_i^2 < 0$. Therefore, it suffices to show that the determinant of H is strictly positive. That is $(\partial^2 \Pi(\cdot)/\partial p_i^2)(\partial^2 \Pi(\cdot)/\partial a_i^2) - (\partial^2 \Pi(\cdot)/\partial p_i \partial a_i)^2 > 0$, which holds whenever $\partial^2 \Pi(\cdot)/\partial a_i^2$ is sufficiently negative.

From Claims 1, 2, and 3, we conclude that there do not exist any profitable deviation in the set D_1 if matching valuations are uniformly distributed and the advertising cost function is sufficiently convex. To complete the proof, we now study deviations outside the set D_1 .

Step 4 Consider now deviations to pairs (a_i, p_i) in the sets D_2 and D_3 defined above, i.e., we need to make sure that a firm i has no interest in deviating by charging a price such that $1 - F(\hat{x} + \Delta) = 0$. In that case no consumer would ever stop searching at firm i and the deviant would only sell to the consumers who happen to find no acceptable product somewhere else. Then, deviating profits would be:

$$\Pi_i(a_i, p_i; a^*, p^*) = p_i \int_{p_i}^1 F(\varepsilon - \Delta)^{n-1} f(\varepsilon) d\varepsilon - \phi(a_i). \quad (22)$$

By monotonicity, it is clear that the deviant would find it optimal to accompany the deviating price with an advertising effort that is vanishingly small.²⁸ Because of log-concavity of f , this profits expression is quasi-concave in p_i (see Caplin and Nalebuff, 1991). Taking the derivative with respect to p_i yields:

$$\int_{p_i}^1 F(\varepsilon - \Delta)^{n-1} f(\varepsilon) d\varepsilon - p_i \left[F(p^*)^{n-1} f(p_i) + (n-1) \int_{p_i}^1 F(\varepsilon - \Delta)^{n-2} f(\varepsilon - \Delta) f(\varepsilon) d\varepsilon \right]$$

²⁸Likewise, notice that if the deviating firm sets an advertising effort that is vanishingly small, the firm would be visited last and the profit expression would be similar to that in (22).

Setting $p_i = p^*$ in this expression we have:

$$\int_{p^*}^1 F(\varepsilon)^{n-1} f(\varepsilon) d\varepsilon - p^* \left[F(p^*)^{n-1} f(p^*) + (n-1) \int_{p^*}^1 F(\varepsilon)^{n-2} f(\varepsilon)^2 d\varepsilon \right] = \frac{1 - F(p^*)^n}{n} - p^* \left(f(1) - \int_{p^*}^1 F(\varepsilon)^{n-1} f'(\varepsilon) d\varepsilon \right). \quad (23)$$

where the last equality follows from integration by parts. This expression is exactly the limit of the first order condition in (8) when $\hat{x} \rightarrow 1$. We will show in the proof of Proposition 2 that the expression

$$\frac{f(\hat{x})}{n} \frac{1 - F(\hat{x})^n}{1 - F(\hat{x})} - \int_{p^*}^{\hat{x}} F(\varepsilon)^{n-1} f'(\varepsilon) d\varepsilon. \quad (24)$$

is decreasing in \hat{x} . This implies that (23) is negative and therefore the profits expression in (22) is decreasing at $p_i = p^*$. This fact along with the quasi-concavity of the expression in (22) implies that deviating profits are monotonically decreasing in p_i , for all $p_i \in [\tilde{p}_i, p^m]$, with \tilde{p}_i solving $1 - F(\hat{x} + \tilde{p}_i - p^*) = 0$. As a result, deviating to a price above p^* is not profitable.

Taken together, these steps establish the Proposition.

Proof of Proposition 2.

1. The result on prices follows straightforwardly from the equilibrium condition on prices (8), which does not depend on advertising costs. From the equilibrium condition on advertising (7), we have that a change in advertising costs should leave $a^* \phi'(a^*)$ constant. Consider two advertising cost functions ϕ_1 and ϕ_2 , with $\phi_1'(a) > \phi_2'(a) \forall a$, hence the marginal cost of advertising are higher in case 1 than in case 2. Equilibrium requires $a_1^* \phi_1'(a_1^*) = a_2^* \phi_2'(a_2^*)$. As $\phi_1' > \phi_2'$, we require $a_1^* \phi_1'(a_1^*) < a_2^* \phi_1'(a_2^*)$. Convexity of ϕ_1 implies that $a \phi_1'(a)$ is strictly increasing in a , hence equilibrium requires $a_1^* < a_2^*$.
2. (a) For the part on prices, we build on the proof of Proposition 1. The equilibrium price is given by the solution of the following equation:

$$\frac{1 - F(p^*)^n}{np^*} = \frac{f(\hat{x})}{n} \frac{1 - F(\hat{x})^n}{1 - F(\hat{x})} - \int_{p^*}^{\hat{x}} F(\varepsilon)^{n-1} f'(\varepsilon) d\varepsilon. \quad (25)$$

In this equation the effects of higher search costs are manifested only through changes in \hat{x} . The LHS of (25) decreases in p^* and does not depend on \hat{x} . The RHS is nondecreasing in p^* for any distribution that has $f' \geq 0$, and this includes the uniform. Taking the derivative of the RHS of (25) with respect to \hat{x} yields:

$$\frac{[f'(\hat{x})(1 - F(\hat{x})^n) - nF(\hat{x})^{n-1} f^2(\hat{x})](1 - F(\hat{x})) + f(\hat{x})^2(1 - F(\hat{x})^n)}{n(1 - F(\hat{x}))^2} - F(\hat{x})^{n-1} f'(\hat{x}) \quad (26)$$

Collecting terms the expression in (26) can be written as:

$$\frac{1}{n} \left(f'(\hat{x}) + \frac{f^2(\hat{x})}{(1-F(\hat{x}))} \right) \left[\frac{1-F(\hat{x})^n}{1-F(\hat{x})} - nF(\hat{x})^{n-1} \right] \quad (27)$$

The first term is positive because of log concavity of $1-F$. The second term is also positive because it equals $\sum_{k=0}^{n-1} [F(\hat{x})^k - F(\hat{x})^{n-1}]$ and F is a distribution function. We thus have that the RHS of (25) is increasing in \hat{x} . From (1), we have that \hat{x} is decreasing in s so the result holds.

(b) For the result on advertising intensities, rewrite a^* as

$$a^* \phi'(a^*) = \frac{p^* A}{n}, \quad (28)$$

with

$$A \equiv 1 - F(\hat{x})^n - \sum_{k=0}^{n-1} \frac{F(\hat{x})^k (1 - F(\hat{x})^{n-k})}{n-k}.$$

We take the derivative of $a^* \phi'(a^*)$ with respect to \hat{x} . Recall that for a^* to be increasing in s , we need it to be decreasing in \hat{x} . Convexity of ϕ then implies that we need $a^* \phi'(a^*)$ to be increasing in s . We thus require:

$$\frac{d}{d\hat{x}}(a^* \phi'(a^*)) = \frac{A dp^*}{n d\hat{x}} + \frac{p^* dA}{n d\hat{x}} < 0,$$

From the discussion in (a) we know that $dp^*/d\hat{x} < 0$. Therefore, if we show that $dA/d\hat{x} < 0$, the result follows. Dropping subscripts, we have

$$\begin{aligned} \frac{dA}{d\hat{x}} &= -nF^{n-1}f + F^{n-1}f - \sum_{k=1}^{n-1} \frac{kF^{k-1}(1-F^{n-k}) + (n-k)F^kF^{n-k-1}}{n-k} f \\ &= -nF^{n-1}f + F^{n-1}f - \sum_{k=1}^{n-1} \frac{kF^{k-1}(1-F^{n-k})}{n-k} f + \sum_{k=1}^{n-1} F^{n-1}f \\ &= -\sum_{k=1}^{n-1} \frac{kF^{k-1}(1-F^{n-k})}{n-k} f < 0. \end{aligned} \quad (29)$$

3. (a) Let (a_n, p_n) be the solution to the first order conditions (7) and (8) when the number of firms is n . Setting $n = 2$ in (7) yields $a_2 \phi'(a_2) = p_2(1-F(\hat{x}))^2/4$ while setting $n = 3$ in the same first order condition yields $a_3 \phi'(a_3) = p_3(1-F(\hat{x}))^2(4+5F(\hat{x}))/18$. Since $a\phi'(a)$ is increasing in a , we have that $a_2 > a_3$ provided that $p_2(1-F(\hat{x}))^2/2 > p_3(1-F(\hat{x}))^2(4+5F(\hat{x}))/9$. For $n = 2$ and $n = 3$, it is still possible to solve for equilibrium prices with a uniform distribution of match values. Doing so, some particularly tedious calculations reveal that the required inequality is indeed satisfied.

(b) We finally prove that $a_n \rightarrow 0$ as $n \rightarrow \infty$. First note that $a_n \rightarrow 0$ if and only if $a_n \phi'(a_n) \rightarrow 0$. From equation (28), we have

$$\lim_{n \rightarrow \infty} a_n \phi'(a_n) = \lim_{n \rightarrow \infty} p_n \lim_{n \rightarrow \infty} \frac{A}{n}$$

It is easy to see that $\lim_{n \rightarrow \infty} p_n = (1 - F(\hat{x}))/f(\hat{x})$, which is strictly positive (Wolinsky, 1986). Therefore we need to show that $\lim_{n \rightarrow \infty} A/n = 0$. We have

$$\begin{aligned} & \lim_{n \rightarrow \infty} \left(\frac{1}{n} - \frac{F(\hat{x})^n}{n} - \sum_{k=0}^{n-1} \frac{F(\hat{x})^k (1 - F(\hat{x})^{n-k})}{n(n-k)} \right) \\ &= - \lim_{n \rightarrow \infty} \sum_{k=0}^{n-1} \frac{F(\hat{x})^k (1 - F(\hat{x})^{n-k})}{n(n-k)} = - \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=0}^{n-1} \frac{1}{(n-k)} \end{aligned}$$

where the last equality follows from the fact that $F(\hat{x})^k (1 - F(\hat{x})^{n-k})$ is strictly positive and bounded by 1. Consider the sum $\sum_{k=0}^{n-1} \frac{1}{(n-k)}$, which can be rewritten as $\sum_{k=1}^n \frac{1}{k}$. It is known that the Euler number γ is given by

$$\gamma \equiv \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{k} - \ln n$$

Therefore

$$- \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=0}^{n-1} \frac{1}{(n-k)} = - \lim_{n \rightarrow \infty} \frac{\gamma + \ln n}{n} = 0$$

which completes the proof.

Proof of Proposition 3

First note that the payoff of a typical firm in symmetric equilibrium is:

$$\Pi_i(a^*, p^*) = \frac{1}{n} p^* (1 - F(p^*)^n) - \phi(a^*)$$

We are interested in the derivative of Π_i with respect to search cost s . We then have

$$\frac{d\Pi(\cdot)}{ds} = \left(\frac{\partial \Pi}{\partial p^*} \frac{dp^*}{d\hat{x}} + \frac{\partial \Pi}{\partial a^*} \frac{da^*}{d\hat{x}} \right) \frac{d\hat{x}}{ds} \quad (30)$$

where

$$\frac{d\hat{x}}{ds} = - \frac{1}{1 - F(\hat{x})}$$

Equation (30) shows that search costs do not affect profits directly but via price and advertising efforts. From Proposition 2, we know that $dp^*/d\hat{x} < 0$ and $da^*/d\hat{x} < 0$. In

equilibrium it is obvious that all firms gain if they all raise their prices, i.e., $\partial\Pi/\partial p^* > 0$. This implies that an increase in search costs tends to raise profits because prices increase; however, since $\partial\Pi/\partial a^* = -\phi'(a^*) < 0$, an increase in search costs tends to lower profits because advertising efforts go up. As a result, an increase in search costs operates on profits in two ways that go in opposite directions.

1. To prove (1), we first use the first order condition (7), to rewrite (30) as

$$\frac{d\Pi(\cdot)}{ds} = \left(\frac{\partial\Pi}{\partial p^*} - \phi'(a^*) \frac{\partial a^*}{\partial p^*} \right) \frac{dp^*}{d\hat{x}} \frac{d\hat{x}}{ds} - \phi'(a^*) \frac{\partial a^*}{\partial \hat{x}} \frac{d\hat{x}}{ds}. \quad (31)$$

Second we note that, from (28), we have

$$\frac{\partial a^*}{\partial p^*} = \frac{A}{n(\phi'(a^*) + a^*\phi''(a^*))}$$

Moreover, from Proposition 1, we have that

$$\frac{dp^*}{d\hat{x}} = -\frac{\frac{1}{n} \left(f'(\hat{x}) + \frac{f^2(\hat{x})}{(1-F(\hat{x}))} \right) \left[\frac{1-F(\hat{x})^n}{1-F(\hat{x})} - nF(\hat{x})^{n-1} \right]}{-\frac{nF(p^*)f(p^*)p^*+(1-F(p^*)^n)}{np^{*2}} - F(p^*)^{n-1}f'(p^*)}.$$

Consider the case where search costs are very small, that is $s \rightarrow 0$, which implies that $\hat{x} \rightarrow 1$ and so $F(\hat{x}) \rightarrow 1$. In this case, since $\lim_{F \rightarrow 1} (1 - F^n)/(1 - F) = n$, we have that

$$\lim_{s \rightarrow 0} \frac{dp^*}{ds} = \lim_{s \rightarrow 0} \frac{dp^*}{d\hat{x}} \frac{d\hat{x}}{ds} = \infty \text{ and that } \lim_{s \rightarrow 0} \frac{\partial a^*}{\partial p^*} = 0$$

As a result the first term in the RHS of (31) goes to ∞ as $s \rightarrow 0$. Therefore, if the second term is finite, then we are sure the result follows. Using (28) again, we have

$$\frac{\partial a^*}{\partial \hat{x}} = \frac{p^*(\partial A/\partial \hat{x})}{n(\phi'(a^*) + a^*\phi''(a^*))}$$

Therefore, from (29), we get that

$$\lim_{s \rightarrow 0} \frac{\partial a^*}{\partial \hat{x}} \frac{d\hat{x}}{ds} = \lim_{s \rightarrow 0} \frac{p^*}{n(\phi'(a^*) + a^*\phi''(a^*))} \lim_{F \rightarrow 1} \sum_{k=1}^{n-1} \frac{kF^{k-1} (1 - F^{n-k})}{(n-k)(1-F)} f.$$

For linear advertising costs, this is clearly finite by the L'Hopital's rule.

2. Profits need not be increasing in search costs. Here we provide a counter-example. Suppose match values are uniformly distributed and two firms operate in the industry. In this case, the equilibrium of the model is given by:

$$\begin{aligned} p^* &= \frac{1}{2} \left(\sqrt{2s} - 2 + \sqrt{8 - 4\sqrt{2s} + 2s} \right), \\ a^*\phi'(a^*) &= \frac{sp^*}{2}, \\ \Pi^* &= \frac{1}{2}p^*(1 - p^{*2}) - \phi(a^*), \end{aligned}$$

where s ranges from 0 to $1/8$ in this case. It is straightforward to verify that when the advertising function is linear Π^* is a strictly concave function reaching a maximum at $s = 0.0115631$. Moreover, equilibrium profits are always lower than profits in a frictionless world (zero search cost) as long as search cost is sufficiently large. ■

Proof of Proposition 4

Define the probability that i is visited first as γ . Thus $\gamma \equiv a_i^*/(a_i^* + a_j^*)$. Rewrite the equilibrium condition on p_i^* , (14), as

$$h_1(\gamma, p_1^*, p_2^*) \equiv \gamma \left(1 - \hat{x} - 2p_1^* + p_2^* + \frac{1}{2}(\hat{x}^2 - p_2^{*2}) \right) \\ + (1 - \gamma) \left(\frac{1}{2}(2 - \hat{x} - 3p_1^*)(\hat{x} - p_1^* + p_2^*) + \frac{1}{2}(\hat{x} - p_1^*)p_2^* \right) = 0.$$

For the other firm we have

$$h_2(\gamma, p_1^*, p_2^*) \equiv (1 - \gamma) \left(1 - \hat{x} - 2p_2^* + p_1^* + \frac{1}{2}(\hat{x}^2 - p_1^{*2}) \right) \\ + \gamma \left(\frac{1}{2}(2 - \hat{x} - 3p_2^*)(\hat{x} - p_2^* + p_1^*) + \frac{1}{2}(\hat{x} - p_2^*)p_1^* \right) = 0$$

This implies that equilibrium also requires that

$$h_1(\gamma, p_1^*, p_2^*) = h_2(\gamma, p_1^*, p_2^*)$$

which we can rewrite as

$$2\gamma = \frac{4\hat{x} - 4p_1^* + 6p_2^* - 2\hat{x}p_1^* + 4p_1^{*2} - 4p_1^*p_2^* - 2\hat{x}^2 - 2}{4\hat{x} + p_1^* + p_2^* - \hat{x}p_1^* - \hat{x}p_2^* + 2p_1^{*2} + 2p_2^{*2} - 4p_1^*p_2^* - 2\hat{x}^2 - 2}.$$

Now suppose that $2\gamma > 1$ and $p_1^* > p_2^*$. This implies that we can write $p_1^* = p_2^* + \Delta$, for some $\Delta > 0$. Then

$$2\gamma = \frac{4x - 4(p_2 + \Delta) + 6p_2 - 2x(p_2 + \Delta) + 4(p_2 + \Delta)^2 - 4(p_2 + \Delta)p_2 - 2x^2 - 2}{4x + (p_2 + \Delta) + p_2 - x(p_2 + \Delta) - xp_2 + 2(p_2 + \Delta)^2 + 2p_2^2 - 4(p_2 + \Delta)p_2 - 2x^2 - 2}.$$

This can only be consistent with equilibrium if the numerator is larger than the denominator, i.e.

$$-5(p_2 + \Delta) + 5p_2 - x(p_2 + \Delta) + 2(p_2 + \Delta)^2 > -xp_2 + 2p_2^2$$

or

$$-5\Delta - x\Delta + 2(p_2 + \Delta)^2 > 2p_2^2$$

hence

$$\Delta(-x + 2\Delta + 4p_2 - 5) > 0$$

This requires $-x + 2\Delta + 4p_2 - 5 > 0$ thus

$$\Delta > \frac{1}{2}(1-x) + 2(1-p_2)$$

But as $p_2 < 1/2$, the right-hand side is larger than 1, which is infeasible. Hence we have a contradiction. Thus, we have established that $\gamma > 1/2$ (and thus $a_1^* > a_2^*$) necessarily requires $p_1^* < p_2^*$. ■

Proof of Proposition 5

We can rewrite the equilibrium condition for advertising levels (13) to read

$$\begin{aligned} 0 = & p_i^* \frac{a_j^*}{(a_i^* + a_j^*)^2} \left[1 - \hat{x} - p_i^* + p_j^* + \frac{1}{2}(\hat{x}^2 - p_j^{*2}) \right. \\ & \left. - (\hat{x} + p_j^* - p_i^*)(1 - \hat{x}) - \frac{1}{2}(\hat{x} - p_i^*)(\hat{x} + 2p_j^* - p_i^*) \right] - \alpha_i \end{aligned}$$

Suppose that firm 1 is the more advertising-efficient firm, so $\alpha_1 < \alpha_2$. From the condition above, we then require

$$\begin{aligned} & p_1^* a_2^* \left[1 - \hat{x} - p_1^* + p_2^* + \frac{1}{2}(\hat{x}^2 - p_2^{*2}) \right. \\ & \quad \left. - (\hat{x} + p_2^* - p_1^*)(1 - \hat{x}) - \frac{1}{2}(\hat{x} - p_1^*)(\hat{x} + 2p_2^* - p_1^*) \right] \\ & < p_2^* a_1^* \left[1 - \hat{x} - p_2^* + p_1^* + \frac{1}{2}(\hat{x}^2 - p_1^{*2}) \right. \\ & \quad \left. - (\hat{x} + p_1^* - p_2^*)(1 - \hat{x}) - \frac{1}{2}(\hat{x} - p_2^*)(\hat{x} + 2p_1^* - p_2^*) \right] \end{aligned}$$

Close inspection of the bracketed terms reveals that they are exactly equal. Hence the inequality simplifies to

$$p_1^* a_2^* < p_2^* a_1^*.$$

Suppose that $a_1^* < a_2^*$. This, by Proposition 4, necessarily implies $p_1^* > p_2^*$, hence $p_1^* a_2^* > p_2^* a_1^*$. But this contradicts the inequality derived above, thus establishing the result. ■

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