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High-Speed Rail & Air Transport Competition

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High-Speed Rail & Air Transport Competition: Game Engineering as Tool for Cost-Benefit Analysis

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Abstract

This paper develops a methodology to assess transport infrastructure investments and their effects on a Nash equilibria taking into account competition between multiple privatized transport operator types. The operators, including high-speed rail, hub and spoke legacy airlines and low cost carriers, maximize profit functions via prices, frequency and train/plane sizes, given infrastructure provision and costs and environmental charges. The methodology is subsequently applied to all 27 European Union countries, specifically analyzing four of the prioritized Trans-European Networks.

Keywords: airlines, high-speed rail, networks, applied game theory, infrastructure pricing

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1. Introduction

In this paper we develop a methodology to analyze competition between imperfectly substitutable transport networks in the medium to long distance passenger market. Such a methodology may prove valuable to the development of transport policy. In recent years, the ‘legacy carriers’ have lost ground to the newly formed ‘low-cost’ carriers. Since liberalization, the regional low-cost carriers have performed admirably whereas many legacy-carriers have foundered; a number of carriers are on the verge of bankruptcy, while others (such as KLM) entered alliance agreements or mergers to ensure their long-run existence. The methodology developed in this paper may be used to explain airline performance and to predict the impact of mergers between legacy carriers (Adler and Smilowitz (2007)).

Another recent development in the medium to long haul transport market is the increasing interest in high-speed rail. Whilst air transport demand in the European Union grew at an average annual rate of 5% over the last decade, high-speed rail passenger demand has

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grown by 16% over the same timeframe (Janic 2003). The European Union is considering increasing its financial assistance to these projects by setting up an infrastructure fund with the aim of encouraging the further development of connecting track across countries for purposes of social cohesion. An additional aim, in terms of environmental transportation policy, is to encourage travelers to change modes, namely to move from air to rail transport (European Commission (2001)). An important reason for encouraging mode substitution, in an attempt to reduce the environmental impact of transport, is clearly explained in IPCC (1999) and Givoni (2007). The methodology developed in this paper can be used to predict the likelihood of success of high-speed rail in the face of competition from airlines. We use the case of high-speed rail to illustrate the workings of the model. Specifically, we analyze the potential addition of Trans-European high-speed rail network (TEN) projects in Austria, France, Germany, Italy, Slovenia and Spain on the existing infrastructure in the year 2020 (see Appendix A for a complete description of the proposed networks).

Using a game theoretic setting, the model framework computes equilibria with and without the high-speed rail investments, permitting analyses of the level of rail infrastructure charges on the transport operators' behavior. A social welfare function enables an objective analysis of the potential effects of such changes on producers (privatized companies providing transportation services), consumers (traveling public, split into business and leisure categories), government authorities (local or federal) and the infrastructure manager, accounting for the effects of infrastructure modifications on taxes and subsidies as well as the environment. The model is based on discrete choice theory of product differentiation (Anderson et al., 1996). A representative consumer is assumed to choose the travel alternative (mode and route) which yields the highest utility. The utility depends on the various characteristics of the alternative, including fare, travel time, distance, routing etc. The alternatives have been split into two nests, one air alternative consisting of all hub-spoke and low cost carriers and the second nest including high-speed rail and the no travel / road option. The no-travel / road option is included so that demand for air and rail can increase or decrease following a change in one of the variables explaining the utility of a passenger. Without this option, such a change would only lead to a redistribution of demand over the various air and rail alternatives.

Up until the early 1990's, airline competition and airline network strategies were generally treated as separate subjects in the literature. Ghobrial and Kanafani (1985) did seek to identify equilibrium in an airline network, however they restricted the case to single hub networks. Several papers have since been written in the field of airline competition using hub-spoke networks, including Hansen (1990), Hong and Harker (1992), Dobson and Lederer (1993), Nero (1996), Hendricks et al. (1999), Marianov et al. (1999), Bhaumik (2002) and Adler (2001, 2005). Hansen (1990) developed an *n*-player, non-cooperative game in which the airline's sole strategy set is frequency of service. The set of simplifying assumptions includes fixed airfares, adequate capacity, inelastic demand to price and service level and consideration of nonstop and one-stop services only. Using regression analysis, Hansen could not prove the existence of an equilibrium and his application to the US air transportation industry showed “quasi-equilibrium”. Hong and Harker (1992) developed a two-stage, game-theoretic representation of an air traffic network market mechanism for slot allocation which they solve for a three node example. Dobson and Lederer (1993) developed a mathematical program to study the competitive choice of flight schedules and route prices by airlines operating in a single hub system. Utilizing a sub-game perfect Nash equilibrium for a two-stage game, they found equilibria in a five-node network example. Assumptions in their model include a single aircraft size, one class of customers and that duopolists serve the identical set of spoke cities using the same hub. Marianov, Serra and ReVelle (1999) discuss the relocation of hubs in a competitive environment given changes in the demand matrix over time. Demand, in terms of flow, is captured through a minimum cost breakdown in order to avoid the use of prices. Adler (2001) evaluates airline profits based on profit maximization under deregulation and its connection to hub-and-spoke networks. Through a two-stage Nash best-response game, equilibria in the air-transportation industry are identified. The game is applied to an illustrative example, where profitable hubs are clearly recognizable and monopolistic and duopolistic equilibria are found, the latter requiring sufficient demand. Bhaumik (2002) and Adler (2005) analyze real world industry conditions. Bhaumik (2002) uses non-cooperative game theory to analyze domestic air travel in India based on a non-zero sum game that searches for a focal point amongst Nash equilibria. Bhaumik’s paper studies how a regulator could ensure a reasonable equilibrium outcome by setting airfares, license fees or essential air service requirements. Adler (2005) develops a model framework to identify the most profitable hub-spoke networks, with the aim of classifying airports most likely to remain major hubs in Western Europe.

In the cost-benefit analysis literature discussing high-speed rail infrastructure, several interesting papers have reached different conclusions. Janic (1993) appears to be among the first to develop a model of competition between the two modes concluding that high-speed rail can compete with air transport over a relatively large range of distances (from 400 to over 2,000 km). However, the model assumes that all demand is met and that the aim is to minimize total system costs for both passengers and transport operators. In analyzing a high-speed rail corridor between Los Angeles and San Fransisco, Levinson et al. (1997) utilize an engineering, full-cost approach to argue that high-speed rail infrastructure is significantly more costly than expanding air services and should not be assumed to substitute for air transport. De Rus and Inglada (1997) analyze the Madrid-Sevilla link and reach similar conclusions to Levinson et al. (1997), arguing that an economic valuation of the project suggests that it should not have been constructed due to a negative net present valuation. In analyzing a Canadian high-speed rail corridor, Martin (1997) develops an economic cost-benefit analysis that includes externalities and concluded that an efficient infrastructure project may be rejected due to politically unacceptable inter-regional income transfers, suggesting that the federal government should play an active role in such instances. Van Exel et al. (2002) argue that an accurate cost-benefit analysis of TENs must consider both network effects and European value added. They specify that the high-speed rail link PBKAL (Paris-Brussels-Koln-Amsterdam-London) has an expected economic return 25% higher that the sum of the independent national valuations. Gonzalez-Savignat (2004) develops a stated preference experimental design in order to analyze the potential attraction of a high-speed rail link from Madrid to Barcelona. Gonzalez-Savignat predicts a high substitutability between air services and the rail link, if upgraded, is expected to achieve 40% market shares in the business sector and almost 60% in the leisure sector. de Rus and Nombela (2007) reach the conclusion that "*high-speed rail investment is difficult to justify when the expected first year demand is below 8-10 million passengers for a line of 500 km*" which they demonstrate is unlikely in the majority of transport corridors in Europe. Vickerman (1997) argued that 12 -15 million passengers would be required to ensure a viable rail operator. Martin and Nombela (2007) apply a gravity model to estimate trip demand for the year 2010 in Spain and then compute the parameters of a multinomial logit function. Roman et al. (2007) estimate modal choice based on mixed revealed and stated preference data on the Madrid-Barcelona corridor. Both Martin and Nombela (2007) and Roman et al. (2007) reach the conclusion that after upgrading the infrastructure, a high-speed rail operator will attract approximately 25% of the passenger market share, a very similar conclusion to that of our case study (on average).

The objective of this paper is to further develop the methodological framework analyzing the passenger transport market equilibria. The new elements of the present research comprise the expansion of player types and in-depth analysis of the social welfare function, including the infrastructure manager and government surpluses as well as the standard consumer and producer surpluses. In the game, three main transport operator types are defined: legacy hub-spoke (HS) networks, low cost carriers (LC) and high-speed rail operators (R). We thus include three very different player types, which makes this model more realistic than the earlier studies. Each transport carrier operates in a deregulated market and maximizes profits. The rail infrastructure access charges are exogenous. Scenario based infrastructure pricing rules are used and the results in terms of changes in social welfare analyzed. Hence the major contribution of this research is to offer a new style of cost-benefit analysis that accounts for privatized transport operator behavior over a network, demonstrating their responses to government initiatives in terms of infrastructure provision and charging whilst accounting for both the environment and competition. Sichelschmidt (1999) argues that for reasons of moral hazard, a tax-financed European infrastructure fund should be rejected since the TEN justification is primarily non-economic rather distributional or environmental. He argues that the European Union role should be mainly in encouraging dialogue between relevant member states in order to ensure that spillovers between regions are recognized and positive consumer network externalities taken into account. The model framework developed here permits an analysis of all these relevant elements within an economic framework. Nijkamp (1995), utilizing a Pentagon prism of critical success factors, calls for an evaluation framework for infrastructure appraisal from a European perspective, clearly provided in this paper, which should play a key role in the organization and management of European railway companies.

The paper is organized as follows. The profit functions of the three transport operator types are developed in Section 2. The different transport operators compete for demand, described in Section 3, using a nested multinomial logit (NMNL) model, of the type depicted in Ben-Akiva and Lerman (1985) and Anderson et al. (1996). Section 4 analyzes the European Union transport market under various scenarios for the year 2020. Section 5 describes the outcomes with and without the additional infrastructure being proposed and

Section 6 draws a summary and conclusions. Appendix A specifies the Trans-European networks analyzed in the case study. Appendix B provides a more detailed analysis of the mathematics including derivatives and Appendix C specifies the complete 71 node air network and 54 node rail network with respective connections where applicable.

2. *Airline and High-Speed Rail Characteristics*

This section discusses the three types of profit maximizing transport operators, which entails developing three different best response functions based on the operator types' individual objective functions. The low cost (LC) airlines choose a single aircraft type over all legs and, specify frequencies per leg and a single price per origin-destination market. Most LC airlines have a single aircraft type strategy to reduce maintenance costs and personnel training and do not attempt to distinguish between business and leisure travelers. LC airlines use yield management to maximize revenues by changing ticket prices over time, a strategy designed to capture as much of the consumer surplus as possible. Including this strategy in the model would substantially increase complexity, as the number of decision variables would increase greatly as would the search for an equilibrium outcome in a repeated game. Therefore, this has been considered an interesting potential extension but beyond the scope of this paper. Prices are computed per leg, hence a traveler choosing to fly with a LC carrier over two legs will be required to purchase two separate tickets. The hub-spoke (HS) carriers, based on their hub network decision, are free to choose various aircraft sizes and frequencies over their legs and two sets of prices, one for business and one for leisure, over all origin-destination pairs, whether the flight is direct or not. In this case, prices include tickets which may involve up to three legs, if the traveler is required to pass through one or two hubs, dependent on the network that the airline has chosen a-priori.

A single high-speed rail operator serves the entire rail network. Since there is no competition between rail operators in our model (and in practice), and there are no complementary routes in the model, a single operator suffices to capture the general picture. We explicitly do not consider the case where the infrastructure operator is vertically integrated with the rail operator. From an economic perspective one might argue that vertical integration would benefit passengers, given that there is only one rail operator (Economides and Salop, 1992) however vertical unbundling is one of the key elements of the European Union railway policy (de Rus and Nombela (2007)). The rail operator chooses the number of seats on their rolling stock per leg, frequencies per leg and business and leisure prices per origin-destination pair, based on the relevant infrastructure. The number of legs per trip is track dependent and based on the shortest distance between each origin and destination, which consists of a maximum of 15 legs in the case study analyzed.

2.1 Decision Variables

In order to characterize each individual operator, we define their profit functions, which consist of revenues less costs. The revenues depend on market share, which is a function of price p_{ijsa}, frequency f_{ka} and average travel time from origin city center to destination city center of the individual operator and all competitors in the market, whether operating a direct or indirect service. The subscripts are explained below. The cost element is a function of plane or train size, measured in the number of seats S_{ka}, frequency, distance and various other parameters, such as infrastructure charges and taxes, dependent on the scenario to be analyzed. The decision variables that each operator faces include:

p_{ijsa}	price to travel from *i* to *j* via operator *a* per traveler type *s*
S_{ka}	number of seats on aircraft/train per leg *k* for operator *a*
f_{ka}	frequency of flights on leg *k* via operator *a*

The cost and profit functions which are based on these decision variables are explained in Sections 2.2 and 2.3 respectively. The market share model is described in Section 3, the methodology for defining the airline networks in Section 4.2 and the total number of decision variables per operator type in Section 4.3.

2.2 Cost Functions

Swan and Adler (2006) found that great circle distance, GCD_{ij}, and the number of seats on an aircraft, S_{ka}, are the two main factors affecting aircraft trip costs. Two market-based equations were developed based on average length of haul, which incorporate aircraft size. Equation (1) gives the cost function for medium to short haul markets (i.e. less than 5,000 kilometers). Equation (2) provides the cost function for long haul markets (more than 5,000 kilometers).

$$C_{ka}^{short} = \$0.019(GCD_{ij} + 722)(S_{ka} + 104) \qquad (1)$$

$$C_{ka}^{long} = \$0.0115(GCD_{ij} + 2200)(S_{ka} + 211) \qquad (2)$$

The values in equations (1) and (2) have been multiplied by 2.2 in order to translate the dollar values into euros at 2001 prices and to reflect the cost of a return trip. The LC airlines are usually active in short haul regional markets and deliberately purchase or lease a single aircraft type[2], therefore the seat size decision variables are limited (S_{LC}). It has also been shown in Swan and Adler (2006) that low cost airlines save \$50 per flight due to faster turnaround times and lower airport charges due to the use of smaller, secondary airports and lower marketing costs due to greater reliance on online services. Consequently, the HS player type enjoys greater freedom (more decision variables) and serves more markets but suffers from a higher cost structure than its LC competitors.

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[2] One might expect that with the Open Aviation Area, LC airlines may enter transatlantic routes. As there is no agreement yet in the literature whether this will take place, we do not include this option in the analysis. The case study presented here focuses on the results of investments in high-speed rail.

The high-speed rail operator cost function consists of a rolling stock cost, operating cost per train kilometer and access charge for infrastructure use per train kilometer, as defined in equation (3).

$$Total\ Cost\ =\ RS\left(\sum_k \frac{f_{kr}S_{kr}}{2(450)}\right)\ +\ \sum_k \left(\alpha_k^{oc}+\alpha_k^{ac}\right)\left(2\, f_{kr}GCD_{ij}\right) \qquad (3)$$

where:

r high-speed rail operator

RS fixed cost of purchasing a single 450 seat train amortized

α_k^{oc} operating cost per train kilometer per leg k

α_k^{ac} access charge per kilometer per leg k

The first element of the cost function computes the rolling stock capital investment. It is assumed that per 300 kilometer stretch, two round trips a day will require a single train (this is a very conservative estimate, suggesting that the costs may be higher than really necessary). It is also assumed that the cost of purchasing a train is linear in the number of seats, ranging from 15 to 30 million € for a 450 to 900 seat train (de Rus and Nash (2007)). The second element computes the variable costs of running the train as a function of the distance traveled, in terms of operating costs and access charges.

The train size is restricted to lie between 450 and 900 seats, which will appear as a constraint in the model and the plane sizes are restricted to lie between 150 and 401 seats[3], as demonstrated in equation (4).

$$150 \le S_{kHS} \le 401,\ 150 \le S_{LC} \le 401,\ 450 \le S_{kr} \le 900 \qquad (4)$$

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[3] Regional jets have not been considered in this model since their cost functions may be radically different.

2.3 Profit Functions

The generalized profit function for the different operators (*a*) is presented in equation (5).

$$\underset{p_{ijsa},f_{ka},S_{ka}}{Max}\ \pi_a = \left\{ \sum_i \sum_{\substack{j \\ i \neq j}} \sum_s M_{ijsa}\left(f_{ka}, TTT_{ija}, TP_{ijsa}\right) d_{ij}\, p_{ijsa} - \sum_k C_{ka}\left(GCD_{ij}, S_{ka}, f_{ka}, \chi_a\right) \right\} \psi_a \quad (5)$$

where

M_{ijsa}	market share of demand between (*i*,*j*) for traveler type *s* with operator *a*
TTT_{ija}	total trip time from center of city *i* to center of city *j* with operator *a*
TP_{ijsa}	total price to travel from center of city *i* to that of city *j* for traveler type *s* with operator *a*
d_{ij}	maximum potential demand from *i* to *j*
χ_a	environmental charge paid by operator *a* to government
ψ_a	100 - tax % on profits paid to government by operator *a*, if profits are positive

The revenue function depends on market share, the maximal origin-destination demand matrix and relevant prices. In turn, market share is a function of the frequency, generalized trip time and total price of the various alternatives available from origin *i* to destination *j*. The origin-destination demand matrix was computed for the year 2020 based on data received from the SCENES project (SCENES, 2006).

3. Market Share Model

This part of the model will enable passengers to participate in the game by choosing between the available alternatives or not traveling at all. The passengers will choose an alternative based on the total trip time, the total price and the log of frequency (which acts as a proxy for level of service (Hansen (1990), Pels (2000))) on all modes. According to Mandel et al. (1997), a more appropriate Box-Cox logit model would have been appropriate but the data with respect to socio-economic variables was not available.

$a \in \{NT, R, LC, HS\}$ where we assume that the choice set includes several possible airlines in each category (LC and HS), one rail company and a no-travel or road alternative

β_{va}	weight in logit model setting importance of parameters, v = 0,1,2,3 per operator category *a*
U_{ijsa}	deterministic utility of traveler type *s* taking path (*i*,*j*) with operator *a*
μ_s	scale parameter in nested logit per traveler type *s*
m	mode of transport, namely air or non-air (including rail and the no-travel alternative)
N_m	nest of operators belonging to mode *m*

Specifically, passengers choose the alternative that yields the highest utility. Utility consists of a systematic part (equation 6) and a random part. Equation (6) defines the systematic utility of passenger type *s* traveling with operator *a* from *i* to *j*. The utility function includes a constant value per mode, the total price to travel from the center of city *i* to that of city *j* and the log of the minimum frequency along the legs traveled. Hansen (1990) argued that the logarithmic form of service frequency is preferable because "*one would expect diminishing returns with respect to the gain in service attractiveness from adding additional flights*". Since the trip may be indirect, only the leg with the lowest frequency is considered because this represents the bottleneck in the total trip time. An approximation of the minimization function is applied in order to solve the objective function and details are presented in Appendix B (Adler (2005)).

$$U_{ijsa} = \beta_{0a} + \beta_{1a} TTT_{ija} + \beta_{2a} TP_{ijsa} + \beta_{3a} \ln \min_{k \in R_{ija}} \left(f_{ka} \right) \tag{6}$$

Given that the random utility components are assumed to be independently and identically Gumbel distributed, we define the nested multinomial logit model for the individual operators' market share as follows (see Ben-Akiva and Lerman (1985)).

$$M_{ijs}(air) = \frac{e^{\left(\mu_s \ln \sum_{a' \in \{air\}} e^{U_{ijsa'}} \right)}}{\sum_m e^{\left(\mu_s \ln \sum_{a' \in N_m} e^{U_{ijsa'}} \right)}} \tag{7}$$

$$M_{ijs}(a|air) = \frac{e^{U_{ijsa}}}{\sum_{a' \in air} e^{U_{ijsa'}}} \tag{8}$$

The alternatives have been split into two nests, one air nest consisting of all hub-spoke and low cost alternatives and the second nest including high-speed rail and the no travel / road alternatives. Equation (7) defines the probability of a type *s* passenger choosing the air nest, and equation (8) defines the conditional probability of a type *s* traveler choosing operator *a*, given the choice of the 'air' nest. The market share of an alternative is the product of these two equations. The direct elasticity of the market share of a specific alternative, for example in the *air* nest, with respect to the three variables defined in the utility function, x_{ijsa}, is defined in equation (9):

$$\varepsilon_{x_{ijsa}}^{M_{ijs}(a,air))} = \frac{\delta M_{ijs}(a,air)}{\delta x_{ijsa}} \frac{x_{ijsa}}{M_{ijs}(a,air)} = \frac{\delta \{ M_{ijs}(a/air) M_{ijs}(air) \}}{\delta x_{ijsa}} \frac{x_{ijsa}}{M_{ijs}(a/air) M_{ijs}(air)} \tag{9}$$

Using (9), we calculate elasticities for each *ijsa*-combination. Since the combination set is very large, we only report the elasticities per passenger type and per alternative. The market share weighted average is presented in Equation (10).

$$\varepsilon_{x_{sa}}^{M_s(a,air))} = \frac{\sum_{i,j} M_{ijs}(a,air)\varepsilon_{x_{ijsa}}^{M_{ijs}(a,air)}}{\sum_{i,j} M_{ijs}(a,air)} \tag{10}$$

Finally, the welfare function in Equation (11) is defined as the total consumer surplus (maximum expected utility defined in monetary terms), producer surplus (total profits from all operators), government surplus (tax revenues less external costs) and infrastructure manager surplus (revenue from rail operator less maintenance and construction costs). Small and Rosen (1981) provide a detailed methodological account of welfare economic computation with respect to discrete choice modeling.

$$W = \sum_i \sum_j \sum_s \left(d_{ij} \frac{1}{\mu_s} \ln \sum_m {}^1\!/_{\beta_{2a}}\, e^{\left(\mu_s \sum_{a' \in N_m} U_{ijsa'}\right)} \right) + \sum_a \pi_a + \sum_k \sum_a 1.2\{\psi_a + f_{ka}(\chi_a - E_{ka})\} + \sum_k ((\zeta_k - \kappa_k) f_k - FC_k) \tag{11}$$

where

E_{ka}	environmental costs produced per flight/train trip on leg *k* per operator *a*
ζ_k	exogenous access charge paid by rail operator to infrastructure manager per leg *k*
κ_k	maintenance costs to maintain rail track per leg *k*
FC_k	fixed cost of upgrading track *k* to high-speed standards

The assumption in the base scenario is that all track exists at varying speeds based on expected standards by 2020. Consequently, the infrastructure manager's fixed costs consist only of the four TENs to be analyzed. The resulting upgrades, in terms of speed, are specified in Table 7. Government surplus consists of two types of taxes, an environmental charge per flight/train service and a corporate tax on profits. The taxes may be positive or negative, representing either costs to the transport operators (who may then pass on the costs to the passengers) or subsidies. A marginal cost of public funds has been evaluated at 1.2 (Calthrop et al. (2008)). In addition, the externalities caused by the generation of transport have been monetarized (E_{ka}) according to the mode of transport and includes marginal environmental, accident and noise charges (INFRAS/IWW (2004)). In the INFRAS/IWW report, the air transport charge is computed as a function of the journey length, with a 284 km flight (Paris - Brussels) costing €0.048 per passenger/km and a 1,045 km flight (Paris – Vienna) costing €0.029 per passenger/km, since the majority of the environmental cost occurs on landing and take-off. This has been linearized to compute a cost per journey length, dropping to a minimum of €0.01 per passenger/km beyond 1,800 km, equivalent to the cost of a high-speed rail journey, as argued in Janic (2003).

4. European Network Case Study

This section discusses the demand zones to be analyzed and the general parameters of the European case study. Subsequently, the air and rail transport networks are discussed in detail as well as the decision variables involved. Section 5 describes the results drawing from this case.

4.1 Demand Zones and General Parameters of the Model

The model requires maximum potential demand flows between zones as input. The network to be analyzed includes 71 zones, three of which represent traffic flow to America, Africa and the Far East. All 27 E.U. countries are represented, some more disaggregated than others in order to cover the train network in greater detail. Table 1 presents the breakdown of countries into zones and Appendix C specifies all zone descriptions (based on territorial units for statistics (NUTS) regions 1 and 2 aggregation levels) and the complete set of rail connections.

Table 1: Breakdown of Zones

Country	Number of Zones	Country	Number of Zones
Austria	7	Norway	1
Belgium	1	Poland	1
Switzerland	1	Portugal	1
Czech	1	Sweden	1
Germany	16	Slovenia	1
Denmark	1	Slovakia	1
Spain	8	Turkey	1
Finland	1	United Kingdom	1
France	12	Baltics	1
Greece	1	Russia	1
Hungary	3	Balkans	1
Ireland	1	Cyprus-Malta	1
Italy	12	Far East	1
Luxembourg	1	Middle East & Africa	1
Netherlands	1	America	1

Vickerman (1997) argues that a key issue for competitive analysis requires inclusion of the pattern of total trip times. Gonzalez-Savignat (2004) goes so far as to argue that separate parameter values should be computed for access times, however this has proven difficult empirically, hence we have summed the total trip time. The calculation of the total trip time for each origin-destination pair is split into the net trip time, based on the distance between two directly linked nodes divided by the velocity of the mode, with an additional takeoff/landing time, time spent at the airport/train station and time required to access and egress the airport/train station and layover time spent at a hub if necessary. The trip times summarized in Table 3 may be somewhat arbitrary, but they reflect the difference between business and leisure passengers (business passengers place a higher value on their time), and the fact that LC airlines usually choose to fly from secondary airports that are often located further from the city center. When the origin-destination is a direct link, the net trip time and the extra constants are simply summed to compute the total trip time. If the trip is indirect, the total trip time is computed by summing the net trip times of each direct leg that would be taken in order to arrive at the destination, with additional time constants computed at each end. This is to ensure that each passenger only accesses the airport or train station from which s/he departs and arrives.

The assumptions with regard to average velocity, access times, airport times and takeoff/landing times are summarized in Table 3, per passenger type and transport mode and are relevant to the specific European case study analyzed in this paper.

Table 3: Trip Time Computation in Hours

	Hub-Spoke	**Low Cost**	**Train**
Takeoff/Landing time	0.25	0.25	0
Access Time	1	2	0.5
Airport Processing Time-Business	0.5	0.5	0
Airport Processing Time-Leisure	1.5	0.5	0
Airport Processing Time-International	1 – business 2 - leisure		
Switching time at hub/station	1.5	2	0.25
Average Velocity	740 km/h	740 km/h	depends on route infrastructure

Table 4 provides summary data on average great circle distances for direct trips and the respective maximum demand per day based on expected values for the year 2020 (SCENES (2006)).

Table 4: Average Distances and Maximum Demand (with number of relevant routes in brackets)

	Distance (km)	**Demand (pax per day)**	
		Business	Leisure
Europe	1,103 (2701)	207 (2591)	323 (2607)
Non-European	5,015 (213)	436 (205)	104 (213)

Since rail speeds are substantially lower than air (lying between 130 and 280 km/h compared to 740 km/h for air travel), we expect the real competition between the two modes to exist in the 300 to 750 kilometer market.

The parameter values in the logit function per traveler type *s*, dependent on whether the destinations are intercontinental or international, are presented in Table 5 and are based on Pels et al. (2000).

Table 5: Logit Parameters

	Europe	Business	Leisure	Intn.	Business	Leisure
ln (log frequency)		1.16	0.89		0.928	0.356
Total Price		-0.004	-0.01		-0.0016	-0.004
Total Trip time		-0.15	-0.02		-0.01	-0.004
Inter-nest Heterogeneity		0.77	0.68			

4.2 Air & Rail Networks

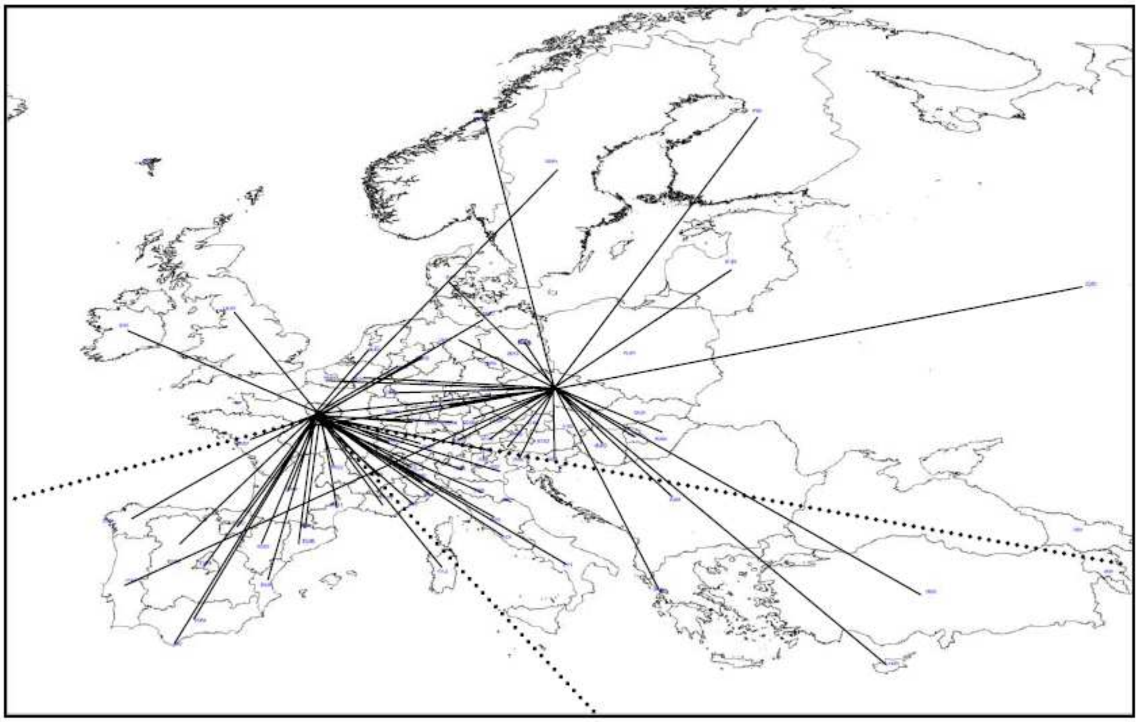
In the case study, three hub-spoke internationals, two low cost regionals and one high-speed rail operator have been defined. The hub-spoke networks roughly represent the three alliances currently growing around the world, namely Oneworld, Star Alliance and Skyteam. It is assumed that each alliance will organize two hubs within Europe and use one of them as the international gateway, and one as a regional hub as presented in Table 6.

Table 6: Airline Hubs

	Hub 1	**Hub 2**
Hub-Spoke 1	Paris	Prague
Hub-Spoke 2	London	Budapest
Hub-Spoke 3	Frankfurt	Poland
Low Cost 1	London	
Low Cost 2	Berlin	

For example, as depicted in Figure 2, the Skyteam alliance is assumed to utilize Paris as the international gateway (dotted lines represent international flights) and Prague as the regional hub.

Figure 2: Paris-Prague Hub-Spoke International Network



Partially balanced, demand weighted distance was defined in the objective function of an allocation, integer linear program in order to develop a basic network for each of the HS airlines. There are many possible methods of producing a connected HS network, the most direct of which is to simply connect spoke nodes to a chosen set of hub nodes according to minimum distance. Alternatively a more balanced solution could be sought as presented in the integer linear program in equation (12).

$$\underset{z_{ij}}{Min}\ (\omega_1+\omega_2)\phi + \sum_{\substack{j=1\\ j\neq i_1,i_2}}^{n} \left[GCD_{i_1 j}\, z_{i_1 j} + GCD_{i_2 j}\, z_{i_2 j} \right]$$

subject to

$$\begin{aligned}
& z_{i_1 j} + z_{i_2 j} = 1 && ,\forall j\,,\, j \neq i_1\,, i_2 \\
& \sum_{\substack{j=1\\ j\neq i_1,i_2}}^{n} z_{i_1 j} - \sum_{\substack{j=1\\ j\neq i_1,i_2}}^{n} z_{i_2 j} = \omega_1 - \omega_2 && \\
& z_{i_1 j}, z_{i_2 j} \in \{0,1\} && ,\forall j, j \neq i_1, i_2 \\
& \omega_i \geq 0 && ,\ i = 1,2
\end{aligned} \tag{12}$$

where i_1 hub number 1

i_2 hub number 2

ω_1-ω_2 variable measuring level of balance of solution

If the balance parameter, ϕ, equals zero, model (12) minimizes distance and may result in an almost pure HS system i.e. a single hub. Were one of the hubs to be geographically further away from other nodes, for example London, almost all spokes may be attached to the secondary hub. Since the hubs are supposed to represent the “center” of the network, with all other nodes acting as spokes, it was determined that a second solution, whereby both hubs have a reasonable number of connections, could also be considered. In addition, it may be true that no single hub could carry all the demand, since large airports around the world suffer severe congestion at present. Thus the integer linear program included the balance parameter which, if large enough, would ensure a completely balanced network, such that approximately half the nodes are connected to one hub and the remainder to the second hub. For ϕ between this value and zero, we may attain various different solutions. An alternative formulation could, for example, minimize the total passenger kilometers traveled or the total number of travelers required to fly over more than one-leg journeys. The distances between the nodes could also be included in the objective function in order to minimize the total number of passenger kilometers traveled.

The two low cost airlines, assumed to fly within Europe only utilizing a pure, star network, are based in London and Berlin, in order to represent the likely number of regional airlines expected to survive by 2020. The high-speed railway network is depicted in Figure 3. The basic assumption of this case study is that the entire rail network will exist by 2020, but the four TENs will consist of conventional rail only, unless the projects are undertaken. Data from a railway network database used for the modelling work was supplied by Büro für Raumforschung, Raumplanung und Geoinformation (fRRG, 2006).

Figure 3: (Mostly) High-Speed Rail Network within Europe 2020

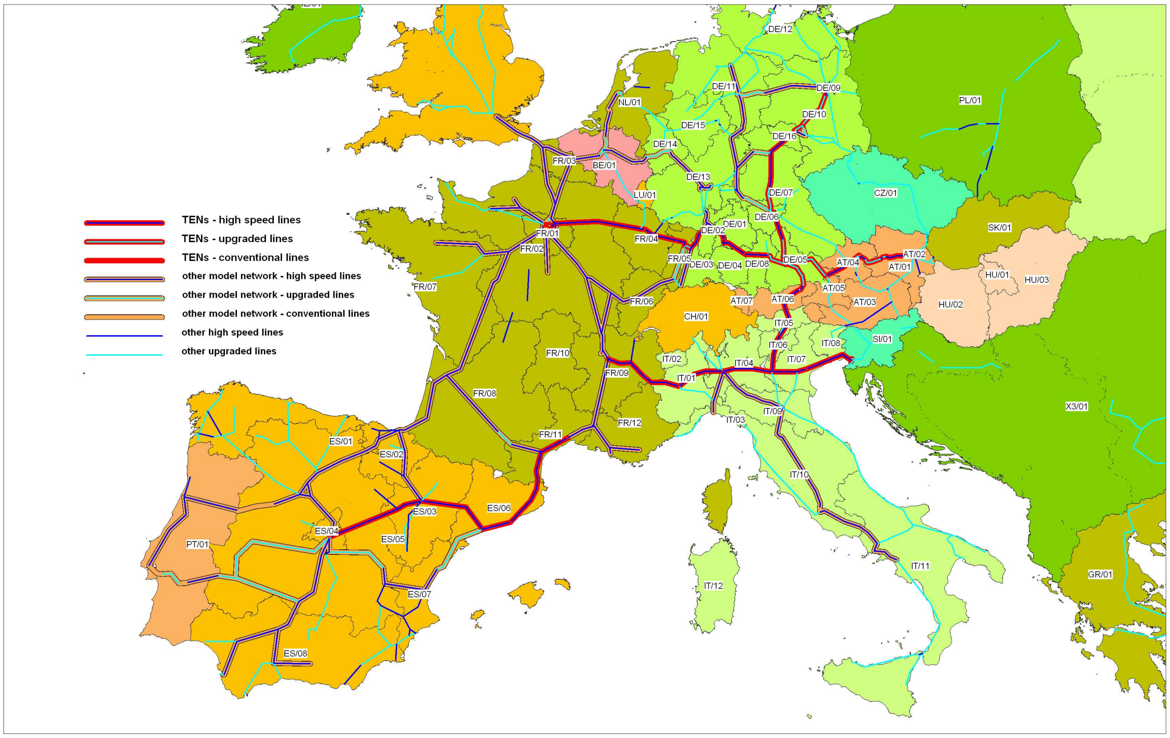


Table 7 identifies which parts of the TEN links under scrutiny exist in the base scenario and their presumed speeds after the improvements. Consequently, the upgrading of track covers Germany and Austria in TEN 1, France and Spain in TEN 3, the French-Italian connection to Slovenia in TEN 6 and the French-German-Austrian links in TEN 17.

Table 7: TENs Upgrades

					Speed in km	
TENs	**From**		**To**		**without TENs**	**with Tens**
1	DE/09	Berlin	DE/10	Brandenburg and Saxony	130	211
	DE/10	Brandenburg and Saxony	DE/16	Halle	130	200
	DE/16	Halle	DE/06	Mittelfranken	130	249
	DE/06	Mittelfranken	DE/05	Oberbayern	130	235
	DE/05	Oberbayern	AT/06	Tirol and Vorarlberg	130	215
	AT/06	Tirol and Vorarlberg	IT/04	Bolzano-Bozen	130	280
	IT/04	Bolzano-Bozen	IT/05	Trento	130	280
	IT/05	Trento	IT/07	Veneto	130	280
3	FR/09	Rhône-Alpes and Auvergne	ES/04	Central Spain	130	280
	ES/04	Central Spain	ES/02	Aragón	130	280
	ES/02	Aragón	ES/03	Madrid	130	280
6	FR/09	Rhône-Alpes and Auvergne	IT/01	Piemonte and Valle d'Aosta/Vallée d'Aoste	130	223
	IT/01	Piemonte and Valle d'Aosta/Vallée d'Aoste	IT/03	Lombardia	130	280

TENs	From		To		Speed in km without TENs	 with Tens
	IT/03	Lombardia	IT/06	Veneto	130	280
	IT/06	Veneto	IT/07	Friuli-Venezia Giulia	130	280
	IT/07	Friuli-Venezia Giulia	SI/01	Slovenija	130	189
17	FR/01	Île de France	FR/04	Lorraine and Luxembourg (Grand-Duché)	130	258
	FR/04	Lorraine and Luxembourg (Grand-Duché)	FR/05	Alsace	130	280
	FR/05	Alsace	DE/02	Karlsruhe	130	257
	DE/02	Karlsruhe	DE/01	Stuttgart	130	280
	DE/01	Stuttgart	DE/04	Tübingen	130	280
	DE/04	Tübingen	DE/08	Schwaben	130	280
	DE/08	Schwaben	DE/05	Oberbayern	130	200
	DE/05	Oberbayern	AT/05	Salzburg	130	202
	AT/05	Salzburg	AT/04	Oberöterreich	130	233
	AT/04	Oberöterreich	AT/02	Wien	130	232

Another issue that is high on the policy agenda is the question of whether or not an EU infrastructure fund is required to finance (rail) infrastructure, and, if the answer is positive, the most appropriate source for such financing. Infrastructure costs are usually very high, so it is proving difficult to finance the TENs privately; private operators of high-speed or standard rail often find it difficult to break even, even without covering the infrastructure costs (in fact, in some scenarios with a relatively high access charge analyzed in Section 5, the net operating result of the rail operator is negative, indicating that the private operator may not be able to bear the infrastructure cost). It is implicitly assumed in the paper that the infrastructure operator may receive a subsidy. We distinguish scenarios with a low, marginal cost access charge and scenarios with a higher, average cost access charge. When the access charge is close to the marginal cost, the rail operator does not pay the full cost of the infrastructure, in which case the authorities must cover part of the infrastructure cost with the remainder left to the rail operator. When the access charge is high, the rail operator pays for a large share of the infrastructure cost. Furthermore, in the modeling exercise, the fixed cost of the TENs (Table 8) is known, so we can compare this to the revenue drawn from taxes and tolls. In the model we consider scenarios with taxes on corporate profits and environmental tolls. All of these revenues may be seen as a source of money for subsidies, alongside an EU infrastructure fund, although it should be noted that ideally the objective of the environmental toll is not to generate revenues, but to optimize the level of environmental damage i.e. reduce the damage to a level which is consistent with welfare maximization. Therefore, we can evaluate whether the infrastructure cost is covered by i) the access charge and/or ii) the taxes and toll revenues. Another source of capital for the EU infrastructure fund might be the monopoly rents, if any, of the rail operator, although there are two complications. Firstly, the existence of monopoly rents means that economic inefficiency exists and welfare is not being maximized. When we use these rents to finance capacity, we more or less accept the fact that welfare is not maximized and finance a level of capacity that also may not be optimal. Secondly, it is likely that in practice the high-speed rail operator will be regulated thus reducing the level of monopoly rents.

Table 8 specifies the net present value infrastructure investment costs of each of the four projects under analysis assuming an expected economic life of 40 years and a discount rate of 5%, as recommended by the European Commission (1997).

Table 8: Cost of TENs Upgrading per Project

TENs	Total Cost (M€)	Cost per day (NPV, M€)
1	31,925	5.015
3	12,506	1.964
6	32,839	5.158
17	8,190	1.286

4.3 Decision Variables

Table 9 describes the number of variables and constraints involved in the mathematical analysis per player type. The objective function is highly non-linear but all constraints are linear. The LC carrier constraints require ticket prices on indirect links to be the sum of the two relevant ticket prices, reflecting the fact that LC airlines do not offer indirect tickets. The rail operator constraints require business tickets to be at least as expensive as leisure tickets for the same origin-destination combination. Finally, all plane and train sizes are limited to upper and lower bounds (equation (4)).

Table 9: Decision Variables for 71 Zone Network

	Hub-Spokes **71 nodes**	**Low Cost** **68 nodes**	**Rail** **54 nodes & 68 arcs**
Variables			
Price (p)	4970	2278	2862
Frequency (f)	70	67	68
Plane/Train Size (S)	70	1	68
Total	5110	2346	2998
Constraints			
Plane/Train Bounds	140	2	136
Sum of Prices		2211	
Business Prices ≥ Leisure Prices			1431
Total	140	2213	1567

The problem has been solved using KNITRO, having programmed the first derivatives for all variables. Clearly, the solution found may only be locally optimal, hence the multi-start command has been applied, increasing the probability of finding one of the global solutions. There are many potential equilibria solution outcomes to this case study, depending on the order of the players when computing the solution, and given the non-linearity of the mathematical model and the simple assumptions as to the high-speed rail operator’s structure. We therefore provide here generalizations and averages rather than suggest that we can specifically identify which operators are likely to be more successful than others. It should be noted that solution outcomes were always found, though cannot be guaranteed. All solutions, depending on the order of players, proved to be of very similar magnitudes.

4. Scenarios and Social Welfare Function

In this section, we present four scenario solutions, with and without the upgraded TENs routes for a relatively low rail access charge of €2 per kilometer and a relatively high charge of 10 € per kilometer. These numbers draw on results from two European funded projects, GRACE (2005) and UNITE (2002). After an analysis of these results, we then present the social welfare computations drawing on these solutions and discuss in greater detail the differences between the scenarios. Finally, we will discuss the effects of environmental charging on the potential transport equilibrium.

The results presented here consist of a series of tables specifying averages over all the networks and have been computed based on weighted market shares (equation (10)). Therefore, occasionally business and leisure prices for low cost airlines appear to be different despite the fact that for each origin-destination pair, these airlines have been restricted to offering a single price. Clearly the airlines have taken advantage of the fact that certain links carry a higher percentage of business travelers resulting in a higher tariff on these origin-destination markets. The airlines are free to choose an aircraft with a minimum of 150 seats and a maximum of 401. From the solutions presented in tables 10, it is clear that the larger aircraft are chosen on the international links. Finally, it should be noted that the rail operator pricing policy was restricted to ensure that business prices are at least as large as leisure prices. Without this set of constraints, in some instances, the rail operator attempted to improve market share by offering the business traveler a lower price (the business traveler is more sensitive to time and frequency). Finally, for lack of space, we have been forced to use weighted averages, and given the very subtle differences between the scenarios, this may appear to produce rather similar results. However, the extended detail in tables 11 to 14 highlight some of the larger differences that averages tend to smooth. Indeed, there would appear to be substantial competition in the German region between a Hub-Spoke (Lufthansa), Low Cost carrier (Air Berlin) and the HSR operator, leading to only partial use of the upgraded high-speed rail infrastructure along TEN 17.

Table 10a: No TENs Upgrades and a Rail Access Charge of €2 per kilometer

Europe	HS1	HS2	HS3	LC1	LC2	TN	Int	HS1	HS2	HS3
Primary hub	Paris	England	Frankfurt	England	Berlin			Paris	England	Frankfurt
Secondary hub	Prague	Hungary	Poland					Prague	Hungary	Poland
Profit	11,058,822	18,801,365	7,560,270	7,374,525	5,494,666	13,818,531				
Business Price	527	577	527	360	381	240		1195	1197	1196
Leisure Price	262	266	259	269	375	134		697	668	691
Frequency	19	21	20	9	10	15		18	21	23
Plane/Train Size	167	176	164	199	207	479		285	330	220
Business market share	0.175	0.160	0.174	0.160	0.155	0.176		0.298	0.321	0.323
Leisure market share	0.197	0.196	0.203	0.119	0.072	0.214		0.253	0.285	0.262
Load Factor	0.8560	0.7942	0.8559	0.8191	0.7381	0.6147		0.9609	0.9148	0.9485
Business Frequency Elasticity	0.4631	0.4919	0.4630	0.4622	0.4124	0.3401		0.3274	0.3190	0.3004
Leisure Frequency Elasticity	0.3603	0.3676	0.3584	0.3754	0.3494	0.2521		0.1357	0.1348	0.1313
Business Price Elasticity	-0.7973	-0.9075	-0.7968	-0.7594	-0.7317	-0.2464		-0.6014	-0.5885	-0.5796
Leisure Price Elasticity	-1.1179	-1.2255	-1.1122	-1.9429	-1.9110	-0.2888		-2.6653	-2.3661	-2.6169

Table 10b: TENs Upgrades and a Rail Access Charge of €2 per kilometer

Europe	HS1	HS2	HS3	LC1	LC2	TN	Int	HS1	HS2	HS3
Primary hub	Paris	England	Frankfurt	England	Berlin			Paris	England	Frankfurt
Secondary hub	Prague	Hungary	Poland					Prague	Hungary	Poland
Profit	10,268,976	18,359,735	8,679,529	7,177,828	5,452,489	28,674,513				
Business Price	528	584	528	359	382	382		1196	1197	1196
Leisure Price	266	269	265	270	379	211		706	670	701
Frequency	19	21	19	12	11	21		21	21	21
Plane/Train Size	164	169	165	174	198	528		224	325	243
Business market share	0.173	0.154	0.164	0.172	0.151	0.187		0.313	0.314	0.315
Leisure market share	0.197	0.196	0.197	0.130	0.072	0.209		0.257	0.282	0.259
Load Factor	0.8446	0.7945	0.8585	0.8184	0.7424	0.5009		0.9638	0.9182	0.9525
Business Frequency Elasticity	0.4617	0.4949	0.4659	0.4496	0.4217	0.3148		0.3170	0.3192	0.3103
Leisure Frequency Elasticity	0.3577	0.3664	0.3582	0.3658	0.3516	0.2380		0.1344	0.1346	0.1323
Business Price Elasticity	-0.7983	-0.9274	-0.8032	-0.7527	-0.7392	-0.2336		-0.5910	-0.5918	-0.5872
Leisure Price Elasticity	-1.1207	-1.2712	-1.1234	-1.9319	-1.9168	-0.3001		-2.6688	-2.3834	-2.6481

Table 10c: No TENs Upgrades and a Rail Access Charge of €10 per kilometer

Europe	HS1	HS2	HS3	LC1	LC2	TN	Int	HS1	HS2	HS3
Primary hub	Paris	England	Frankfurt	England	Berlin			Paris	England	Frankfurt
Secondary hub	Prague	Hungary	Poland					Prague	Hungary	Poland
Profit	11,653,553	21,661,062	8,883,522	8,822,376	6,300,958	-56,795				
Business Price	531	584	531	361	381	3		1195	1197	1196
Leisure Price	265	269	263	270	376	2		700	668	695
Frequency	21	23	21	12	12	0		21	22	22
Plane/Train Size	165	173	165	184	204	452		225	328	236
Business market share	0.202	0.188	0.193	0.204	0.183	0.029		0.306	0.321	0.317
Leisure market share	0.225	0.228	0.228	0.148	0.085	0.086		0.255	0.285	0.260
Load Factor	0.8787	0.8012	0.8811	0.8252	0.7476	0.9793		0.9648	0.9138	0.9553
Business Frequency Elasticity	0.4538	0.4802	0.4495	0.4316	0.3913	0.5096		0.3178	0.3196	0.3094
Leisure Frequency Elasticity	0.3546	0.3593	0.3486	0.3606	0.3372	0.3525		0.1346	0.1346	0.1323
Business Price Elasticity	-0.7869	-0.8958	-0.7852	-0.7380	-0.7186	-0.0270		-0.5963	-0.5873	-0.5841
Leisure Price Elasticity	-1.1040	-1.2148	-1.0974	-1.9192	-1.8970	-0.0281		-2.6643	-2.3674	-2.6309

Table 10d: TENs Upgrades and a Rail Access Charge of €10 per kilometer

Europe	HS1	HS2	HS3	LC1	LC2	TN	Int	HS1	HS2	HS3
Primary hub	Paris	England	Frankfurt	England	Berlin			Paris	England	Frankfurt
Secondary hub	Prague	Hungary	Poland					Prague	Hungary	Poland
Profit	11,157,891	21,139,502	8,138,777	8,838,369	5,889,899	-55,453				
Business Price	532	580	532	363	380	2		1195	1197	1196
Leisure Price	261	267	260	271	373	1		692	666	689
Frequency	21	24	22	12	12	0		21	22	24
Plane/Train Size	165	171	165	182	200	452		227	325	231
Business market share	0.2	0.189	0.197	0.201	0.183	0.03		0.307	0.318	0.32
Leisure market share	0.226	0.228	0.231	0.145	0.085	0.085		0.258	0.286	0.262
Load Factor	0.8686	0.8064	0.8626	0.8216	0.7584	0.9843		0.9668	0.9163	0.9437
Business Frequency Elasticity	0.4536	0.4798	0.4481	0.4333	0.3933	0.5098		0.3206	0.3201	0.306
Leisure Frequency Elasticity	0.3542	0.3594	0.3483	0.3624	0.3383	0.3534		0.135	0.1349	0.132
Business Price Elasticity	-0.7841	-0.8912	-0.7839	-0.7408	-0.7213	-0.0264		-0.5959	-0.5901	-0.5804
Leisure Price Elasticity	-1.0949	-1.1834	-1.0925	-1.9257	-1.9001	-0.0278		-2.6471	-2.3555	-2.612

These results suggest that five airlines can be supported by forecasted 2020 demand given the current expected cost structures. Indeed, the existence or lack thereof of the rail option has little effect ***on average*** on the air transport operators' decision variables, including frequencies and tariffs. The major points to be drawn from tables 10 include the following:

1) The strongest competitors of the high-speed rail operator include the second LC airline, operating out of Berlin, and the third HS airline, based in Frankfurt. As the

program cycles between the operators, these three require the most time to converge.

2) The high-speed rail operator achieves higher profits with the TENs upgrades under the €2 per kilometer access charge scenario, through higher frequencies on average which permit higher tariffs without the loss of market share. Furthermore, the lower trip times enable the operator to charge as much as the LC airlines, at least for the business travelers (the rail operator splits the pricing between the two core markets, whereas the LC airlines charge a single fare). The rail operator cannot achieve profitability under the €10 per kilometer access charge scenario, indicating a private operator cannot bear relatively high infrastructure costs and will utilize very few of the links.

Under further scrutiny, some substantial differences appear when the rail operator is running with or without the upgrades under the lower access charge. For example, many of the trips to Madrid with very low frequencies prior to the upgrade, increase dramatically afterwards (to 2 trains an hour). On the other hand, some of the frequencies to Stuttgart and Karlsruhe have been reduced to almost nothing, from ten trains per day. Prices have also increased in certain areas and decreased in others, for example from Slovenia to France and the United Kingdom to Southern Italy, rail tariffs dropped in order to encourage longer distance demand on services that have already been justified by shorter origin-destination markets. In Table 11, increases in tariffs are documented prior to the upgrade and afterwards, identifying multiple markets in which the train can better challenge air transport in terms of trip times, hence the HSR alternative becomes more competitive and is able to charge higher fares.

Table 11: 15 Largest Rail Price Increases as a Result of TENs Upgrades

Business		**Price (€)**			**Leisure**		**Price (€)**		
Origin	**Destination**	**no upgrade**	**with TENs**	**% Change**	**Origin**	**Destination**	**no upgrade**	**with TENs**	**% Change**
Central Italy	Liguria	113	590	420.19%	Aragon	Schwaben	10	63	525.82%
Provence-Alpes-Côte d'Azur	Central Spain	59	296	405.99%	Aragon	Bremen and Lower Saxony	32	177	445.31%
Portugal	Provence-Alpes-Côte d'Azur	93	376	306.35%	Central Spain	Schwaben	23	114	399.97%
Liguria	Central Spain	109	412	278.28%	Central Italy	Liguria	65	284	337.67%
Portugal	Madrid	69	249	262.48%	Rhône-Alpes and Auvergne	Alsace	45	190	327.12%
Rhône-Alpes and Auvergne	Alsace	164	578	253.27%	Catalonia	Bremen and Lower Saxony	70	273	289.19%
Bolzano-Bozen	Catalonia	128	432	239.07%	Portugal	Madrid	43	157	263.88%
Rhône-Alpes and Auvergne	Schwaben	32	108	235.23%	Catalonia	Madrid	9	29	238.49%
Liguria	Rheinland and Westphalia	77	253	227.23%	Friuli-Venezia Giulia	Rheinland and Westphalia	36	120	235.89%
Provence-Alpes-Côte d'Azur	Aragon	57	184	221.78%	United Kingdom	Madrid	11	32	200.52%

Business		**Price (€)**			**Leisure**		**Price (€)**		
Origin	**Destination**	**no upgrade**	**with TENs**	**% Change**	**Origin**	**Destination**	**no upgrade**	**with TENs**	**% Change**
Piemonte and Valle d'Aosta/Vallée d'Aoste	Rheinland and Westphalia	130	412	215.95%	Madrid	North East and North West Spain	11	30	167.55%
West France	Schwaben	23	73	214.07%	South Spain	Brandenburg and Saxony	62	161	158.15%
Madrid	North East and North West Spain	45	137	204.63%	Franche-Comté	Oberöterreich	82	212	157.24%
Friuli-Venezia Giulia	Île de France	95	288	203.49%	Provence-Alpes-Côte d'Azur	Central Spain	58	148	153.99%
Aragon	Bremen and Lower Saxony and	82	246	199.90%	Lorraine and Luxembourg (Grand-Duché)	Oberöterreich	57	144	151.67%

The literature on competition between air and high-speed rail is rather limited. Steer Davies Gleave (2006) concluded that high-speed rail captures a large market share in markets where passengers would have traveled by air, if the high-speed alternative was not available, and that in certain markets, e.g. Germany, aviation prices may drop below high-speed rail prices, all of which proved true in this case study. When travel time is significantly reduced due to the opening of a high-speed rail link, the rail operator increases prices to maximize profits, without significantly compromising its competitive position. Steer Davies Gleave come to a similar conclusion, stating that journey time is the most important determinant of market share. Bel (1997) and Gonzalez-Savignat (2004) also emphasize the importance of travel time in explaining inter-urban rail demand.

A further check as to the likelihood of such a solution outcome to occur requires computations of load factors and elasticities, based on the decision variables computed, namely prices, frequency and plane/train seat size. The elasticities of frequency and price with respect to market share represent reasonable expectations according to the relevant literature (Mandel et al. (1997), Brons et al. (2002), Gonzalez-Savignat (2004)). The leisure price elasticities are above -1 and business price elasticities are below -1, as expected in the literature. Air transport business frequency elasticity with respect to market share is higher (approximately 0.45) than leisure fare price elasticity (approximately 0.35). It is frequently argued in the air transport literature that load factors above 60% cover the cost break-even point, but most airlines strive to achieve more than 80%, solutions achieved in tables 10. Rail fare elasticities with respect to market share in the scenarios whereby rail remains a competitor are in the -0.25 range, slightly lower than cited in Wardman et al. (2002), but within the approximate values computed in Mandel et al. (1997) and Gonzalez-Savignat (2004). In absolute terms, rail elasticities computed in the literature appear to be lower than their air counterparts, as appears in the results of this model. The load factors for high-speed rail are also slightly low (in the 55 to 60% range) and the reason for this lies in the utility function of passengers. Higher frequency and lower prices attract travelers and since high-speed rail is at an initial disadvantage due to the longer trip times (at least for those over 500 kilometers), the rail operator compensates with lower prices and higher frequency, leading to lower load factors and relatively lower elasticities in comparison to air. Potentially, this model should be analyzed with different logit parameters for rail and air transport, however no comparable values were available at the time of computation hence we preferred to use those that have been validated, if only for air transport.

In order to compare the results of the different scenarios, Table 12 presents a social welfare comparison that accounts for consumer, producer, infrastructure manager and government surplus less environmental externalities generated. Maximizing social welfare may be achieved by upgrading the links of the four TENs under discussion, provided the high-speed rail operator pays a low access charge per kilometer (€2) and the cost of the infrastructure has been computed reasonably accurately. This implies that subsidies may be necessary, however only passenger markets beyond 300 km have been considered yet the effect of the upgrades will also apply to the freight market and regional services, potentially alleviating congestion on existing lines. Finally, given past experience in the high-speed rail markets in England, France and Spain, it has been shown that approximately half the travelers on the new links represent new market niches and the other half were drawn from existing operators (de Rus and Nash (2007)). SCENES (2006) produced a single demand matrix for the current study and were unable to produce demand matrices dependent on the existence of upgraded infrastructure. It is therefore entirely possible that the demand should have expanded (or contracted) according to the scenario, but this has proven beyond the scope of the current paper.

The worst scenario in terms of social welfare would be to upgrade the TEN links and charge the rail operator a €10 per km access charge i.e. the full average cost, since producer surplus drops (costs increase) and consumer surplus drops (prices increase). The rail operator does not achieve profitability, hence no rail tax revenues are collected. If the rail operator is expected to cover the entire infrastructure cost through access charges, society would be better off without the upgraded links. However, the results in Table 12 suggest that the rail operator's tax revenues would be sufficient to cover the fixed costs of infrastructure under the lower rail access charge scenario. Note that the external costs in Europe are higher when the rail access charge is relatively high because of a modal shift to air. With the TEN investments, external costs in Europe are slightly higher than in the scenario without the investments because the external cost of high-speed rail exceeds that of standard rail and the LC airlines increase frequency to better compete with the improved rail competitor. There seems to be little environmental benefit from the TEN investments because the rail operator absorbs the benefits of the higher quality by increasing fares rather than market share, so transport policy stimulating a modal shift from aviation to rail for environmental reasons would require on-track competition, price regulation and/or substantial environmental taxes.

Table 12: Social Welfare Comparison across Scenarios

Infrastructure Type Rail Access Charge (€/km)	basic 2	basic 10	with TENs 2	with TENs 10
Consumer Surplus	83,485,405	33,364,426	113,082,572	46,028,103
Producer Surplus	64,108,178	57,264,677	78,613,071	55,108,986
Environmental Charge	21,015,200	21,660,113	22,285,323	22,095,678
Air Taxes	25,863,247	29,479,614	25,682,687	28,370,283
Rail Taxes	7,106,673	0	14,746,893	0
Government Surplus	64,782,145	61,367,672	75,257,882	60,559,153
Externalities: Europe	-42,437,503	-44,584,257	-42,711,291	-44,748,975
Externalities: International	-4,348,313	-4,269,366	-4,309,195	-4,285,381
Fixed cost of TENs	0	0	-13,423,589	-13,423,589
Infrastructure Manager Surplus	0	33,490	-13,423,589	-13,390,997
Social Welfare	165,589,912	103,176,642	206,509,450	99,270,889

The airlines appear able to produce flights at reasonable levels and pay an environmental charge of €100 per flight. The high-speed rail operator has been charged €50 per train service in all four base scenarios. The lower environmental high-speed rail charge represents the lower environmental externalities caused by this mode according to INFRAS/IWW (2004) and Janic (2003). This argument is in line with Levinson et al. (1997), although the values of the externalities are priced substantially higher in the INFRAS/IWW report.

As part of a series of sensitivity analyses, not all of which are presented here for reasons of brevity, we tested the effect of the environmental charging policy on the behavior of the transport operators. According to Givoni (2007), it is important to analyze the environmental impact of transport in monetary terms as opposed to the United Kingdom's Environmental Audit Committee (2003), in which it is argued that monetarization should not be emphasized due to the intrinsic difficulties of calculation. From Table 13a it is clear that environmental charges in the range of €400 per flight and €200 per train service do not substantially affect the overall solution and simply collect almost twice the value of the externalities produced. Consumer and producer surplus decreases as the environmental charges increase because the transport companies pass on at least part of the charges to the passengers hence reducing demand. As a result, external costs are reduced.

Table 13a: Environmental Charging Policies

Air Transport Charge (€/flight) Rail Charge (€/train service)	0 0	100 50	400 200
Consumer Surplus	125,262,118	113,082,572	100,275,053
Producer Surplus	78,998,146	78,613,071	76,187,586
Environmental Charge	0	22,285,323	84,651,728
Air Taxes	25,679,383	25,682,687	24,409,064
Rail Taxes	14,948,235	14,746,893	14,773,123
Government Surplus	48,753,141	75,257,882	148,600,698
Externalities: Europe	-42,630,734	-42,711,291	-42,547,629
Externalities: International	-4,296,704	-4,309,195	-4,250,980
Fixed cost of TENs	-13,423,589	- 13,423,589	-13,423,589
Infrastructure Manager Surplus	-13,423,589	-13,423,589	-13,423,589
Social Welfare	192,662,378	206,509,450	264,841,139

Table 13b shows that demand is lightly affected. Frequencies drop slightly as the charges increase and approximately 2,000 less travelers are carried per day across the network with each doubling in the charge. However, to dramatically reduce the production of transport, sums of substantially greater magnitude would need to be charged.

Table 13b: Changes in Demand with Environmental Charging

Air Transport Charge (€/flight)			0		100		400	
Rail Transport Charge (€/train service)			0		50		200	
	Primary hub	Secondary hub	Business (000's)	Leisure (000s)	Business (000s)	Leisure (000s)	Business (000s)	Leisure (000s)
HS1	Paris	Prague	76.898	129.03	77.411	129.1	77.047	131.118
HS2	England	Hungary	67.642	126.298	68.811	128.263	69.797	129.943
HS3	Frankfurt	Poland	73.286	129.999	73.405	128.734	73.659	131.151
LC1	England		76.757	84.582	76.9	85.1	75.589	85.093
LC2	Berlin		68.24	47.353	67.573	46.977	68.619	47.713
TN			85.546	138.785	83.745	136.849	82.338	127.121
No travel			36.641	94.238	37.144	95.213	37.8	97.899
% air			0.75	0.69	0.75	0.69	0.75	0.7
% rail			0.18	0.18	0.17	0.18	0.17	0.17
Total Traveling (000s)			448.369	656.049	447.845	655.023	447.049	652.139
Sum Total (000s)				1104.417		1102.868		1099.188

Table 14 presents greater detail with respect to the total number of passengers traveling with each alternative within Europe over the four scenarios tested. Approximately 2,300 more passengers travel with the high-speed rail operator on the upgraded system per day, which amounts to over 800,000 in the course of a year. The rail operator manages to increase business demand at the expense of the third HS airline and slightly reduces the number of leisure passengers, as it has the opportunity to sell its capacity at higher prices. The second LC airline attracts the leisure passengers instead. The balance draws on the trade-off between a good scheduled service (represented in the model as the minimum log of frequency), total trip time and tariffs, in order to attract passengers in the business and leisure markets. The benefits of high-speed rail over LC airlines for business travelers can be explained by the fact that stations are usually located in the city center whilst LC carriers generally use secondary airports requiring relatively longer trips to the city center.

Table 14: Travel Summary

Infrastructure			basic		basic		with TENs		with TENs	
Access charge (€/km)			2		10		2		10	
	Primary hub	Secondary hub	Business (000's)	Leisure (000s)	Business (000s)	Leisure (000s)	Business (000s)	Leisure (000s)	Business (000s)	Leisure (000s)
HS1	Paris	Prague	78.131	128.832	88.346	142.664	77.411	129.1	87.297	143.209
HS2	England	Hungary	71.444	128.434	82.097	144.231	68.811	128.263	82.759	144.926
HS3	Frankfurt	Poland	77.37	132.713	84.159	144.027	73.405	128.734	85.892	146.713
LC1	England		71.161	78.265	89.057	93.501	76.9	85.1	87.895	91.721
LC2	Berlin		68.845	46.871	80.092	53.79	67.573	46.977	79.877	53.947
TN			78.275	140.082	12.748	54.676	83.745	136.849	13.241	53.741
No travel			39.672	94.965	48.585	117.502	37.144	95.213	48.16	116.464
% air			0.76	0.69	0.87	0.77	0.75	0.69	0.87	0.77
% rail			0.16	0.19	0.03	0.07	0.17	0.18	0.03	0.07
Traveling Total (000s)			445.226	655.197	436.499	632.889	447.845	655.023	436.962	634.257
Sum Total (000s)				1,100.42		1,069.39		1,102.87		1,071.22

Furthermore, if we breakdown the rail market share over distance, it is noticeable that the rail system attracts almost 25% in the 750 kilometer or less origin-destination markets but this drops to 9% in the longer haul markets, in line with the literature e.g. Janic (2003). In analyzing the TENs upgraded routes more specifically, it becomes apparent that frequencies may drop without the rail operator losing market share, due to the improved trip times.

5. Summary and Conclusions

In this paper we analyze a high-speed rail system in order to investigate the implications of changes to the network on social welfare. This type of analysis, based on game theory, attempts to explore the effects of infrastructure provision and charging on the best response function of all competitors in the relevant market. In this context, we have modeled the reactions of hub-spoke international alliances and low cost regional airlines on the survivability and profitability of a high-speed rail operator free to utilize the entire European rail network in the year 2020. The results of this work suggest that it is possible to justify some of the Trans-European high-speed rail projects despite their vast fixed costs. The difference between the analysis presented here and the cost-benefit analyses that were undertaken previously (Leveinson et al. (1997) and de Rus and Inglada (1997)) derives from the fact that we have developed a network based model, whereas many cost-benefit analyses undertaken generally look at distinct parts of each of the projects individually and therefore have ignored to some extent the aggregate network effects. Secondly, we have directly modeled the competitive reactions of private transport companies to the responses of other operators in the market, taking into account schedule quality and price endogenously.

The general conclusions of the paper are as follows. Firstly, it appears worthwhile upgrading the TENs modeled (TENs 1, 3, 6 and part of 17), if the authorities are interested in maximizing social welfare and encouraging travelers to move from air to rail transport modes. However, this conclusion is dependent on the real infrastructure costs which, were they to be severely underestimated, may change the equilibria outcomes. Secondly, it is worthwhile upgrading to high-speed rail infrastructure if the rail operator has access to the entire European network and is charged a marginal cost access charge. The increases in consumer, producer and government surpluses are sufficient to cover the daily cost of these four TEN projects, estimated at approximately €13.4 million per day (for 40 years at a 5% discount rate). Indeed under the low access charge, the high-speed rail operator's corporate taxes would be sufficient to cover the infrastructure cost. This would also be the case if a two-part tariff is charged rather than a higher per km access charge. However, whilst a two-part tariff is compatible with a franchised monopoly, it may prove more problematic if there is on track competition. Consequently, the local and/or federal governments would need to be willing to subsidize the construction costs to some extent and given the high degree of transit traffic, an EU subsidy scheme could be justified. Thirdly, the least appropriate scenario tested would be to upgrade the TENs and charge the rail operator an average cost access charge (in the region of €10 per km) since the infrastructure would not be utilized efficiently. Finally, it is possible to set an environmental charge of €100 per flight and €50 per train service without dramatically changing the transport equilibrium, thus collecting approximately half the estimated environmental damage, possibly for the purpose of increasing research and development in the field and helping the operators to reduce emissions in the longer term. An environmental charge of €200 per flight and €100 per train service would cover the estimated environmental costs generated, by slightly reducing frequencies and the total number of traveling public by approximately 2,000 people per day within Europe. In general, the aim is to provide the right incentives for operators to look for cleaner operations and for passengers to opt for the more environmentally friendly mode of transport.

In this paper we have shown that a network competition model with different types of operators (high-speed rail, low cost airlines and conventional airlines) can be formulated, yielding results that may be surprising when compared to some of the cost benefit analyses. Such results depend on the parameterization of the model but the contribution of this paper is the methodology to analyze different policy options in a network setting, taking into account the reactions of relevant competitors. In the air and rail industry, this is crucial.

Future directions consist of extensive options as this work represents a first attempt to model multiple player types in a competitive, network setting. It may prove interesting to extend the game over time, allowing an analysis of yield management techniques and permitting greater uncertainty as to demand ranges, which would require a stochastic modeling framework and computation of equilibria in a repeated game. If the zones or nodes were to represent a single set of one major and one minor airport and a train station, the demand and cost functions could be adapted to study congestion, slot allocation policies and scarcity charges where relevant. With this level of input disaggregation, it would also be possible to analyze multi-modal choice alternatives so that passengers could choose two or more modes of transport e.g. purchasing a rail ticket from Brussels to Paris and then flying to their final destination. In the current model, high-speed rail is viewed purely as a competitor to air transport, however it would appear to be not only a substitute but also a likely potential complement to air networks and therefore expanding the model to consider such multi-model trips is likely to further improve a rail operator’s likelihood of success (Vickerman (1997), Givoni (2007)). In addition, the conclusions drawn from the results of the case study are dependent on the accuracy of the infrastructure cost evaluation and there appears to be evidence of systematic bias in such estimates with regard to large infrastructure projects. Flyvbjerg et al. (2003) found that 90% of such projects suffer cost overruns. It may therefore be of interest to adapt the model to consider these risk. Finally, expanding the player types to include local and federal governments may generate new solution outcomes of interest and would require additional objective functions such as minimizing subsidies or maximizing social welfare.

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Appendix A: Trans-European Networks

Trans European Network	Region	Country
1	Berlin	Germany
	Brandenburg and Saxony	
	Halle	
	Mittelfranken	
	Oberbayern	
	Tirol and Vorarlberg	Austria
	Provincia Autonoma Bolzano/Bozen	Italy
	Provincia Autonoma Trento	
	Veneto	
3	Rhône-Alpes and Auvergne	France
	Central Spain	Spain
	Aragón	
	Comunidad de Madrid	
6	Rhône-Alpes	France
	Piemonte and Valle d'Aosta/Vallée d'Aoste	Italy
	Lombardia	
	Veneto	
	Friuli-Venezia Giulia	
	Slovenija	Slovenia
17	Île de France	France
	Lorraine	
	Alsace	
	Karlsruhe	Germany
	Stuttgart	
	Tübingen	
	Schwaben	
	Oberbayern	
	Salzburg	Austria
	Oberöterreich	
	Wien	

Appendix B: Mathematical Model and Partial Differentials

A	set of operators
a	index that represents a specific operator
m	mode of transport (air, non-air)
$l(m)$	set of alternatives in mode m
N	set of all existing nodes in the network configuration
$i,j,y,z \in N$	node indices
R_{ija}	set of legs in route connecting origin i to destination j for operator a
$Arc(a)$	set of all existing legs in operator a's specific network configuration
$k \in Arc(a)$	leg index
$s \in \{b, \ell\}$	s represents the type of traveler, either business (b) or non-business (ℓ)
β_{us}	parameter in utility function reflecting importance of variable u to type s traveler
d_{ijs}	type s traveler O-D demand from node i to node j (passengers per week)
C_{ka}	cost of leg k to operator a
c_{la}	parameters in cost function for operator a, l=1,2,3
f_{ka}	service frequency over leg k for operator a
p_{ijsa}	tariff from node i to node j for passenger type s with operator a
S_{ka}	plane or train size in seats per operator a on leg k
M_{ijsa}	market share of operator a over O-D trip (i,j) for passenger type s
F_{ija}	reduction factor on path (i,j) for operator a (load factor $\leq$ to 1)

The objective function for the hub-spoke airlines is defined in (1a) and the partial differentiation with respect to price in (2a), plane sizes in (3a) and frequency in (4a).

$$\underset{f_{ka},p_{ijsa},S_{ka}}{Max}\ \pi_a = \sum_{\substack{i,j,s\\ i>j}} F_{ija} d_{ijs} M_{ijsa} p_{ijsa} - \sum_{k'\in R_{ija}} \left(\gamma_1 S_{k'a} + \gamma_2\right) f_{k'a}$$

where $\forall i, j \in N, i > j, \forall s, \forall a \in A, \forall k \in Arc(a)$

$$M_{ijsa} = \left(1 + \chi_{ijsa}\left(\sum_{a'\in l(m)} e^{U_{ijsa'}}\right)^{-\mu}\right)^{-1} [1 + \psi_{ijsa} e^{-U_{ijsa}}]^{-1}$$

$$U_{ijsa} = \beta_0 - \beta_1 TTT_{ija} - \beta_2 p_{ijsa} + \beta_3 \ln\left(\min_{k'\in R_{ija}} f_{k'a}\right)$$

$$\chi_{ijsa} = \sum_{\substack{m'\neq m\\ a\in l(m)}} \left(\sum_{a'\in l(m')} e^{U_{ijsa'}}\right)^{\mu} \quad , \quad \psi_{ijsa} = \left(\sum_{a'\in l(m)\setminus\{a\}} e^{U_{ijsa'}}\right)$$

$$D_{k'a} = \sum_{\substack{y,z,s' \mid y>z\\ k'\in R_{yza}}} d_{yzs'} M_{yzs'a}$$

$$F_{ija} = \left[1 + \sum_{k'\in R_{ija}} \left(\frac{f_{k'a} S_{k'a}}{D_{k'a}}\right)^{\tau}\right]^{\frac{1}{\tau}} \tag{1a}$$

$$C_k = c_{1a}\left(c_{2a} + GCD_k\right)\left(c_{3a} + S_{ka}\right) f_{ka}$$

thus $\gamma_1 = c_{1a} GCD_k + c_{1a} c_{2a}$ and $\gamma_2 = \left(c_{1a} c_{3a} GCD_k + c_{1a} c_{2a} c_{3a}\right)$

$$\frac{\partial \Pi_a}{\partial p_{ijsa}} = \sum_{yzt \mid y>z, R_{ija}\cap R_{yza}\neq\varphi} \left\{\frac{\partial F_{yza}}{\partial p_{ijsa}} d_{yzt} M_{yzta} p_{yzta}\right\} + F_{ija} d_{ijs} \frac{\partial M_{ijsa}}{\partial p_{ijsa}} p_{ijsa} + F_{ija} d_{ijs} M_{ijsa}$$

where

$$\frac{\partial D_{k'a}}{\partial p_{ijsa}} = d_{ijs} \frac{\partial M_{ijsa}}{\partial p_{ijsa}} \quad for \ \ k' \in R_{ija}$$

$$\begin{aligned}\frac{\partial M_{yzta}}{\partial p_{ijsa}} = {} & -\beta_2 \mu \chi_{yzta} e^{U_{yzta}} [1 + \psi_{yzta} e^{-U_{yzta}}]^{-1} \left(1 + \chi_{yzta}\left(\sum_{a'\in l(m)} e^{U_{yzta'}}\right)^{-\mu}\right)^{-2} \left(\sum_{a'\in l(m)} e^{U_{yzta'}}\right)^{-\mu-1} \\ & - \beta_2 \psi_{yzta} e^{-U_{yzta}} \left[1 + \psi_{yzta} e^{-U_{yzta}}\right]^{-2} \left(1 + \chi_{yzta}\left(\sum_{a'\in l(m)} e^{U_{yzta'}}\right)^{-\mu}\right)^{-1}\end{aligned} \tag{2a}$$

$$\frac{\partial F_{yza}}{\partial p_{ijsa}} = F_{yza}^{\frac{1}{\tau}-1} \sum_{k'\in R_{ija}\cap R_{yza}\neq\phi} \left\{-\left(\frac{f_{k'a} S_{k'a}}{D_{k'a}}\right)^{\tau-1} \frac{f_{k'a} S_{k'a}}{{D_{k'a}}^2} \frac{\partial D_{k'a}}{\partial p_{ijsa}}\right\}$$

$$\frac{\partial \Pi_a}{\partial S_{ka}} = \sum_{\substack{i,j,s|i>j \\ k\in R_{ija}}} \frac{\partial F_{ija}}{\partial S_{ka}} d_{ijs} M_{ijsa} p_{ijsa} - \sum_{k'\in R_{ija}} \gamma_1 f_{k'a}$$

where (3a)

$$\frac{\partial F_{ija}}{\partial S_{ka}} = F_{ija}^{\frac{1}{\tau}-1} \left(\frac{f_{ka} S_{ka}}{D_{ka}} \right)^{\tau-1} \frac{f_{ka}}{D_{ka}} \quad for\ k \in R_{ija}$$

$$\frac{\partial \Pi_a}{\partial f_{ka}} = \sum_{\substack{i,j,s|i>j \\ k\in R_{ija}}} \left\{ F_{ija} d_{ijs} \frac{\partial M_{ijsa}}{\partial f_{ka}} p_{ijsa} \right\} + \sum_{\substack{i,j,s|i>j,\exists k'\in R_{ija} \\ \exists (y,z)|k,k'\in R_{yza}}} \left\{ \frac{\partial F_{ija}}{\partial f_{ka}} d_{ijs} M_{ijsa} p_{ijsa} \right\} - (\gamma_1 S_{ka} + \gamma_2)$$

where

$$\begin{aligned} \frac{\partial M_{yzta}}{\partial f_{ka}} = {} & \mu \chi_{yzta} e^{U_{yzta}} \left[1 + \chi_{yzta} \left(\sum_{a'\in l(m)} e^{U_{yzta'}} \right)^{-\mu} \right]^{-2} \left(\sum_{a'\in l(m)} e^{U_{yzta'}} \right)^{-\mu-1} \frac{\partial U_{yzta}}{\partial f_{ka}} \left[1 + \psi_{yzta} e^{-U_{yzta}} \right]^{-1} \\ & + \psi_{yzta} e^{-U_{yzta}} \left[1 + \psi_{yzta} e^{-U_{yzta}} \right]^{-2} \frac{\partial U_{yzta}}{\partial f_{ka}} \left[1 + \chi_{yzta} \left(\sum_{a'\in l(m)} e^{U_{yzta'}} \right)^{-\mu} \right]^{-1} \quad for\ k \in R_{yza} \end{aligned}$$

$$\frac{\partial D_{k'a}}{\partial f_{ka}} = \sum_{\substack{yzt,y>z \\ k,k'\in R_{yza}}} d_{yzt} \frac{\partial M_{yzta}}{\partial f_{ka}}$$

$$\frac{\partial U_{yzta}}{\partial f_{ka}} = \beta_3 \left(\sum_{k'\in R_{yza}} f_{k'a}^{\tau} \right)^{-1} f_{ka}^{\tau-1} \quad for\ k \in R_{yza}$$

$$\frac{\partial F_{yza}}{\partial f_{ka}} = F_{yza}^{\frac{1}{\tau}-1} \sum_{k'\in R_{yza}|\exists (i',j');k,k'\in R_{i'j'a}} \left(\frac{f_{k'a} S_{k'a}}{D_{k'a}} \right)^{\tau-1} \left[\frac{\underset{(k=k')}{I} \cdot (S_{k'a} D_{ka}) - f_{k'a} S_{k'a} \frac{\partial D_{k'a}}{\partial f_{ka}}}{(D_{k'a})^2} \right] \tag{4a}$$

$$if\ \exists k' \in R_{yza,}, \exists (i',j') | k,k' \in R_{i'j'a}$$

The objective function for the low-cost airlines is defined in (5a) and the partial differentiation with respect to price and frequency is the same as that of the hub-spoke airline. The derivative with respect to plane size (the low-cost carriers are assumed to use only one plane size throughout their network) is defined in (6a) and the set of constraints relevant to low-cost airlines in (7a).

$$\underset{f_{ka},p_{ijsa},S_{ka}}{Max}\ \pi_a = \sum_{\substack{i,j,s\\ i>j}} F_{ija} d_{ijs} M_{ijsa} p_{ijsa} - \sum_{k'\in R_{ija}} \left(\overline{\gamma_1} S_a + \overline{\gamma_2}\right) f_{k'a}$$

$$where\ \forall i, j \in N, i > j, \forall s, \forall a \in A, \forall k \in Arc(a)$$

$$M_{ijsa} = \left(1 + \chi_{ijsa}\left(\sum_{a'\in l(m)} e^{U_{ijsa'}}\right)^{-\mu}\right)^{-1} [1 + \psi_{ijsa} e^{-U_{ijsa}}]^{-1}$$

$$U_{ijsa} = \beta_0 - \beta_1 TTT_{ija} - \beta_2 p_{ijsa} + \beta_3 \ln\left(\min_{k'\in R_{ija}} f_{k'a}\right)$$

$$\chi_{ijsa} = \sum_{\substack{m'\neq m\\ a\in l(m)}} \left(\sum_{a'\in l(m')} e^{U_{ijsa'}}\right)^{\mu} \ , \ \psi_{ijsa} = \left(\sum_{a'\in l(m)\setminus\{a\}} e^{U_{ijsa'}}\right)$$

$$D_{k'a} = \sum_{\substack{y,z,s'|y>z\\ k'\in R_{yza}}} d_{yzs'} M_{yzs'a}$$

$$F_{ija} = \left[1 + \sum_{k'\in R_{ija}} \left(\frac{f_{k'a} S_a}{D_{k'a}}\right)^{\tau}\right]^{\frac{1}{\tau}}$$

(5a)

$$C_k = \overline{c_{1a}}\left(\overline{c_{2a}} + GCD_k\right)\left(\overline{c_{3a}} + S_a\right) f_{ka}$$

thus $\overline{\gamma}_1 = \overline{c_{1a}} GCD_k + \overline{c_{1a} c_{2a}}$ and $\overline{\gamma}_2 = \left(\overline{c_{1a} c_{3a}} GCD_k + \overline{c_{1a} c_{2a} c_{3a}}\right)$

$$\frac{\partial \Pi_a}{\partial S_a} = \sum_{\substack{i,j,s|i>j\\ k\in R_{ija}}} \frac{\partial F_{ija}}{\partial S_a} d_{ijs} M_{ijsa} p_{ijsa} - \sum_{k'\in R_{ija}} \overline{\gamma_1} f_{k'a}$$

where (6a)

$$\frac{\partial F_{ija}}{\partial S_a} = F_{ija}^{\frac{1}{\tau}-1} \left(\frac{f_{ka} S_a}{D_{ka}}\right)^{\tau-1} \frac{f_{ka}}{D_{ka}} \quad for\ k \in R_{ija}$$

Constraints whereby the passenger must purchase two tickets to arrive at their final destination (only two are necessary since the assumption is a pure star network) are defined in equation (7a).

$$p_{ijsa} = p_{ihsa} + p_{hjsa} \quad for\ \{i,j\} \notin Arc(a) \qquad (7a)$$

The objective function for the rail operator is defined in (8a) and the partial differentials with respect to price, train size and frequency are the same as that of the hub-spoke airline. The set of constraints relevant to high-speed rail is defined in (9a).

$$\underset{f_{ka},p_{ijsa},S_{ka}}{Max}\ \pi_a = \sum_{\substack{i,j,s\\ i>j}} F_{ija} d_{ijs} M_{ijsa} p_{ijsa} - \sum_{k'\in R_{ija}} \left(\overline{\overline{\gamma_1}} S_{k'a} + \overline{\overline{\gamma_2}} \right) f_{k'a}$$

$$where\ \forall i,\ j \in N, i > j, \forall s, \forall a \in A, \forall k \in Arc(a)$$

$$M_{ijsa} = \left(1 + \chi_{ijsa} \left(\sum_{a'\in l(m)} e^{U_{ijsa'}} \right)^{-\mu} \right)^{-1} [1 + \psi_{ijsa} e^{-U_{ijsa}}]^{-1}$$

$$U_{ijsa} = \beta_0 - \beta_1 TTT_{ija} - \beta_2 p_{ijsa} + \beta_3 \ln\left(\min_{k'\in R_{ija}} f_{k'a} \right)$$

$$\chi_{ijsa} = \sum_{\substack{m'\neq m\\ a\in l(m)}} \left(\sum_{a'\in l(m')} e^{U_{ijsa'}} \right)^{\mu} \ ,\ \ \psi_{ijsa} = \left(\sum_{a'\in l(m)\setminus\{a\}} e^{U_{ijsa'}} \right)$$

$$D_{k'a} = \sum_{\substack{y,z,s' | y>z\\ k'\in R_{yza}}} d_{yzs'} M_{yzs'a}$$

$$F_{ija} = \left[1 + \sum_{k'\in R_{ija}} \left(\frac{f_{k'a} S_{k'a}}{D_{k'a}} \right)^{\tau} \right]^{\frac{1}{\tau}}$$

$$C_k = \left(\overline{\overline{c_{1a}}} \left(\frac{S_{ka}}{\overline{\overline{c_{2a}}}} \right) + \left(2\overline{\overline{c_{3a}}} GCD_k \right) \right) f_{ka} \tag{8a}$$

$$\text{thus}\ \ \overline{\overline{\gamma_1}} = \frac{\overline{\overline{c_{1a}}}}{\overline{\overline{c_{2a}}}} + c_{1a}\ \text{and}\ \ \overline{\overline{\gamma_2}} = 2\overline{\overline{c_{3a}}} GCD_k$$

$$p_{ijba} \geq p_{ijla}\ \forall i,j,a \tag{9a}$$

Appendix C: List of Zones and Connecting Train Links

Zone	Zone Identity	Name of Zone Region	Direct Rail Connections						
1	AT/01	East Austria	AT/02	AT/04					
2	AT/02	Vienna	AT/01	AT/04					
3	AT/03	South Austria	IT/07						
4	AT/04	Oberöterreich	AT/01	AT/05	AT/02				
5	AT/05	Salzburg	AT/04	DE/05					
6	AT/06	Tirol and Vorarlberg	IT/04	DE/05					
7	BE/01	Belgium	DE/13	FR/03	NL/01				
8	CH/01	Switzerland							
9	CZ/01	Czech Republic							
10	DE/01	Stuttgart	DE/02	DE/04					
11	DE/02	Karlsruhe	DE/01	DE/03	FR/05				
12	DE/03	Freiburg	DE/02	FR/05					
13	DE/04	Tübingen	DE/01	DE/08					
14	DE/05	Oberbayern	AT/05	AT/06	DE/06	DE/08			
15	DE/06	Mittelfranken	DE/05	DE/07	DE/14				
16	DE/07	Bayern and Thüringen	DE/06	DE/12	DE/14				
17	DE/08	Schwaben	DE/04	DE/05					
18	DE/09	Berlin	DE/10						
19	DE/10	Brandenburg and Saxony	DE/09	DE/11	DE/14				
20	DE/11	Bremen and Lower Saxony	DE/10	DE/12					
21	DE/12	South West Germany	DE/07	DE/11	DE/13				
22	DE/13	Rheinland and Westphalia	BE/01	DE/12					
23	DE/14	Halle	DE/07	DE/06	DE/10				
24	DK/01	Denmark							
25	ES/01	North East Spain and North West Spain	ES/02	ES/04	FR/08				
26	ES/02	Aragon	ES/01	ES/03	ES/04	ES/05			
27	ES/03	Madrid	ES/02	ES/04					
28	ES/04	Central Spain	ES/01	ES/02	ES/03	ES/06	ES/07	FR/09	PT/01
29	ES/05	Catalonia	ES/02	ES/06	FR/10				
30	ES/06	East Spain	ES/04	ES/05					
31	ES/07	South Spain	ES/04						
32	FR/01	Île de France	FR/02	FR/04					
33	FR/02	Paris Basin	FR/01	FR/03	FR/04	FR/06	FR/07	FR/09	
34	FR/03	Nord - Pas-de-Calais	BE/01	FR/02	UK/01				
35	FR/04	Lorraine and Luxembourg (Grand-Duché)	FR/01	FR/02	FR/05				
36	FR/05	Alsace	DE/02	DE/03	FR/04	FR/06			
37	FR/06	Franche-Comté	FR/02	FR/05					
38	FR/07	West France	FR/02	FR/08					
39	FR/08	South West France	ES/01	FR/07	FR/10				

40	FR/09	Rhône-Alpes and Auvergne	ES/04	FR/02	FR/10	IT/01
41	FR/10	Languedoc-Roussillon	ES/05	FR/08	FR/09	FR/11
42	FR/11	Provence-Alpes-Côte d'Azur	FR/10			
43	GR/01	Greece				
44	HU/01	Hungary	SI/01			
45	IE/01	Ireland				
46	IT/01	Piemonte and Valle d'Aosta/Vallée d'Aoste	FR/09	IT/03		
47	IT/02	Liguria	IT/03			
48	IT/03	Lombardia	IT/01	IT/02	IT/06	IT/08
49	IT/04	Bolzano-Bozen	AT/06	IT/05		
50	IT/05	Trento	IT/04	IT/06		
51	IT/06	Veneto	IT/03	IT/05	IT/07	IT/08
52	IT/07	Friuli-Venezia Giulia	AT/03	IT/06	SI/01	
53	IT/08	Emilia-Romagna	IT/03	IT/06	IT/09	
54	IT/09	Central Italy	IT/08	IT/10		
55	IT/10	South Italy	IT/09			
56	IT/11	Sardegna				
57	NL/01	Netherlands	BE/01			
58	PL/01	Poland				
59	PT/01	Portugal	ES/04			
60	SE/01	Sweden, Norway and Finland				
61	SI/01	Slovenia	HU/01	IT/07		
62	SK/01	Slovakia				
63	TR/01	Turkey				
64	UK/01	United Kingdom	FR/03			
65	X1/01	Baltics				
66	X2/01	Russia				
67	X3/01	Balkans				
68	X4/01	Cyprus and Malta				
69	X5/01	Far East				
70	X6/01	Middle East				
71	X7/01	America				