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Federal Reserve Policy viewed through a Money Supply Lens

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Abstract

Federal Reserve nonborrowed reserve supply systematically responded to changes in inflation and in the output gap over the period 1969-2000. While the feedback from output gap is always negative, the response of money supply to changes in inflation varies considerably across time. Nonborrowed reserves decreased with inflation in the post-1979 period and increased in the pre-1979 period. Applying a standard macro-model, the estimated reaction functions are shown to ensure equilibrium determinacy. Viewed through the money supply lens, Federal Reserve policy substantially changed over time, but has never allowed for endogenous fluctuations, which contrasts conclusions drawn from federal funds rate analyses.

Keywords: Money supply, reaction functions, nonborrowed reserves, real-time data, equilibrium determinacy

JEL classification: E51, E52, E32

1. Introduction

This paper re-examines postwar U.S. Federal Reserve policy by looking at the supply of high-powered money, i.e. nonborrowed reserves. The analysis builds upon the fact that the supply of nonborrowed reserves rather than the federal funds rate can be directly controlled by the Federal Reserve (see Meulendyke, 1998). The main objective is to disclose whether the well-established shift in the conduct of monetary policy between the pre-1979 (pre-Volcker)

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and the post-1979 (Volcker-Greenspan) period is mirrored in money supply adjustments.[1]

The empirical findings point to substantial differences in the way the Federal Reserve has adjusted money supply in response to changes in macroeconomic variables between the pre-1979 and the post-1979 period. While this finding is consistent with the empirical results on federal funds rate reaction functions as reported by Clarida et al. (2000), a theoretical analysis of macroeconomic stability in a standard macroeconomic model leads to a novel conclusion. The estimated money supply reaction functions satisfy the requirements for equilibrium determinacy. Hence, the results indicate that high and volatile inflation in the pre-1979 period has not been driven by endogenous fluctuations, as suggested by Clarida et al. (2000).

Following empirical studies on federal funds rate targets, a forward-looking reaction function for the growth rate of nonborrowed reserves is estimated. The results show that money supply has been adjusted to changes in the expected inflation rate and in the output gap. In particular, the growth rate of nonborrowed reserves has always responded negatively to a rise in the output gap. However, adjustments of money supply in response to changes in expected inflation have changed substantially between the pre-1979 and the post-1979 period: Higher expected inflation led to a rise in the growth rate of nonborrowed reserves in the pre-1979 period, but to a decline in the post-1979 period. Thus, money supply in the pre-Volcker period has accommodated inflation and was (according to common view of monetary transmission) less suited to stabilize inflation than in the Volcker-Greenspan period. Notably, the findings are robust to various specifications and different means of modelling expectations, by using ex-post data or real time data, i.e., Greenbook forecasts.

For the theoretical analysis the estimated money supply reaction function is introduced

[1] Federal funds rate adjustments can reasonably well be summarized by reaction functions (also called Taylor-rules) including inflation and the output gap as arguments (see Taylor, 1993 and Woodford, 2003, for an overview), with changes in the feedback coefficients indicating shifts in the conduct of monetary policy. Clarida et al. (2000) have found that the feedback from expected inflation to the federal funds rate has been less pronounced in the pre-1979 period than in the period after 1979.

in a standard sticky price model. The analysis provides characteristics for money supply consistent with welfare maximization and derives requirements for equilibrium determinacy. As the main principle the money growth rate should not rise with (expected) inflation by more than one for one in order to implement a unique equilibrium. This condition, which resembles a "reversed Taylor-principle",[2] thus demands the growth rate of real balances to decrease with inflation. Applying the estimated money supply reaction functions indicates that Federal Reserve policy always ensured equilibrium determinacy. Hence, by viewing through the money supply lens, the pre-Volcker policy, even though it aimed less at stabilizing inflation, nevertheless ruled out endogenous fluctuations.

The remainder of the paper is set out as follows. Section 2 provides the empirical analysis. In Section 3 the model is described, and efficiency and equilibrium determinacy under money supply reaction functions are examined. Section 4 concludes.

2. Postwar Federal Reserve money supply

This Section provides evidence for systematic adjustments of nonborrowed reserves over a period of 40 years. Like in studies on interest rate feedback rules, it will be shown that the supply of money has responded to (expected) movements in core macroeconomic variables, i.e., inflation and the output gap. It is further examined if there has been a change in the Federal Reserve's money supply, similar to the well-established shift in federal funds rate adjustments, which has supported the view that Federal Reserve policy in the pre-1979 period was less stabilizing than in the post-1979 period.

[2] When an interest rate policy satisfies the Taylor-principle, the real interest rate increases with inflation and equilibrium determinacy is ensured (see Woodford, 2003). In contrast, determinacy under money supply policy is ensured when the growth rate of real balances decreases with inflation (see Schabert, 2006).

2.1. Evidence from vector autoregressions

To attain first insight into the Federal Reserve's money supply behavior, a reduced-form VAR is estimated, and impulse responses of nonborrowed reserves to unanticipated changes in prices and real activity are computed. The analysis relates to studies by Eichenbaum (1992), Strongin (1995), and Christiano et al. (1999), who show that unanticipated changes in the supply of nonborrowed reserves affect real activity and aggregate prices. To identify monetary policy shocks, they isolate exogenous policy changes from endogenous reactions of the monetary policy stance.

As to the VAR estimation, we strictly adhere to Christiano et al.'s (1999) analysis. Following their specification, the VAR contains the log of real GDP (Y), the log of the implicit GDP deflator (P), the change in a commodity price index (CP), the federal funds rate (FF), the log of total reserves (TR) and the log of nonborrowed reserves plus extended credit (NBR), respectively.[3] We apply Christiano et al.'s (1999) Wold ordering of the variables, diagonalize the innovations' covariance matrix using a Cholesky decomposition, and compute impulse responses to examine if nonborrowed reserves responded to innovations to CP and Y. The overall sample period covers the time horizon 1960 Q1 to 1999 Q4. To account for the well-documented shift in Federal Reserve policy starting with the year Paul Volcker began his mandate as the Chairman of the Board of Governors, sub-sample estimations are further carried out, covering the period 1960 Q1 to 1979 Q2 (pre-Volcker period) and 1982 Q4 to 1999 Q4 (Volcker-Greenspan period), respectively.[4]

Figure 1 presents impulse responses of nonborrowed reserves to innovations in Y and CP. Overall, nonborrowed reserves tend to decrease in response to positive innovations in real activity and in commodity prices; the latter serving as a proxy for nascent inflation.

[3] All data are obtained from the Federal Reserve Bank of St. Louis. Standard lag selection criteria recommended a lag length of 4 quarters for all VAR estimations.

[4] Although Volcker was appointed Chairman in 1979, we refrain from including the first three years of this period, where the Fed pursued a policy of nonborrowed reserves targeting (see Meulendyke, 1998).

Figure 1 further shows that the responses are most pronounced in the Volcker-Greenspan period, while the responses in the Pre-Volcker period are insignificant. The dotted lines represent a two standard error band, computed with the Monte Carlo method, spanning a 95% confidence interval. Thus, these reactions of nonborrowed reserves to unanticipated changes in prices and real activity provide first evidence that the Fed's adjustments of high powered money aimed to lean against the wind.

2.2. Money supply reaction functions

To further unveil how U.S. money supply has been systematically adjusted, a reaction function for the growth rate of nonborrowed reserves is estimated, which closely relates to the empirical specification of the reaction function for the federal funds rate in Clarida et al. (2000). In particular, it is assumed that the growth rate of nonborrowed reserves responds to expected inflation and the output gap in the following way:

$$\mu_t = \rho\mu_{t-1} + \mu_\pi E_t \pi_{t+n} + \mu_y x_t + \varepsilon_t, \tag{1}$$

where μ_t denotes the annualized growth rate of nonborrowed reserves, x_t the output gap measure, and $E_t\{\pi_{t+n}\}$ is the expected inflation rate in $t+n$. The error term ε_t is assumed to be independently and identically distributed (i.i.d.) Gaussian.

This specification has three advantages compared to the VAR-based specification. First, the parsimonious single-equation approach facilitates the interpretation of the estimated coefficients. Second, under specification (1) expected inflation is now modelled explicitly and, unlike in the VAR approach, is not approximated by a commodity price index. Finally, in line with the interest rate feedback rules, the output gap instead of output is considered as an explanatory variable.

All data are obtained from the Federal Reserve Bank of St. Louis and are of quarterly frequency, spanning the time horizon 1960 Q1 to 1999 Q4. Our benchmark inflation measure is based on the GDP deflator and is defined as the annualized percentage change in the price level between two subsequent quarters. Alternatively, consumer price (CPI) inflation is also considered. The output gap is defined as the percent deviation between actual GDP and potential GDP as constructed by the Congressional Budget Office (CBO). An inflation horizon of one quarter ($n = 1$) serves as the benchmark. Additionally, longer inflation horizons (4 quarters, $n = 4$) are also considered.

A widely used technique for estimating an equation of above nature is the Generalized Method of Moments (see e.g. Clarida et al., 2000), which is applied for estimating the parameter vector (ρ, μ_π, μ_y).[5] The estimation results for the full sample are summarized in Table 1. A widening output gap leads to a negative and statistically significant response of the supply of nonborrowed reserves. In contrast, there is no statistically significant relationship between the growth rate of nonborrowed reserves and expected inflation in any of the estimated specifications, including the real-time data specification "Greenbook" (which is discussed in Section 2.3.). Notably, the estimated coefficient μ_π on expected inflation, while not statistically significant, reveals a positive sign.

Based on the common view on monetary transmission, which predicts prices and real activity to increase with money injections, a stabilizing monetary policy should reduce the supply of nonborrowed reserves in response to a widening output gap as well as to higher expected inflation (see also Section 3.). Yet, the above reported estimation results are not fully consistent with this view. An explanation for this finding might relate to the considered sample period, which does not account for the change in the conduct of Fed policy. To this end, sub-sample estimations are carried out.

For the pre-Volcker period the feedback from changes in the output gap is again negative and significant (see Table 2, Panel A). However, the estimated coefficient $\widehat{\mu}_\pi$ on forward-

[5] For all estimations the vector of instruments includes four lags of the growth rate of nonborrowed reserves, output gap, and inflation. Since not all current information may be available to the public at the time they form expectations, contemporary variables are not used as instruments.

looking inflation is significantly positive, indicating that monetary policy during the pre-Volcker period was accommodating, i.e., higher expected inflation led to an increase in money supply. These findings are not sensitive to the chosen inflation measure with $\widehat{\mu}_\pi$ varying between 0.11 and 0.24. The feedback from expected future inflation on the growth rate of the monetary aggregate appears to be less pronounced for longer horizons ($n = 4$).

For the Volcker-Greenspan period the estimates disclose a remarkable difference with respect to the response to changes in inflation. The estimated inflation elasticity is now found to be significantly negative for all inflation measures and inflation target horizons with estimates of μ_π ranging between -0.35 and -0.62. Thus, our findings lend support to the view that the Fed pursued a stronger anti-inflationary policy in the Volcker-Greenspan period than in the pre-Volcker period. The estimates of μ_y further indicate that the responses of nonborrowed reserves to the cyclical variable in both sub-periods are of comparable size. The average value for $\widehat{\mu}_y$ equals -0.51 (-0.34) for the GDP deflator (CPI) in the pre-Volcker period, which compares to a value of -0.60 (-0.27) for the Volcker-Greenspan period.[6]

In general, the goodness-of-fit statistics are satisfactory for both sub-samples, with the coefficient of determination ranging from 0.52 for the pre-Volcker period to 0.73 for the Volcker-Greenspan era. Hansen's J-test, which tests the validity of overidentifying restrictions, indicates that overall the null hypothesis that the overidentifying restrictions are satisfied could not be rejected.

2.3. Real-Time Estimates

To further assess the robustness of the results real-time data are used. Boivin (2006) and Orphanides (2001, 2002) have argued that the assessment of monetary policy based on

[6] Additionally, sub-sample estimations are conducted with two alternative output gap measures based on the deviation of either (log) GDP from a fitted quadratic function of time or the unemployment rate from a similar time trend. The results, which are available upon request, confirm the robustness of the main findings: The output-gap feedback is always negative and of similar size, while the inflation feedback is positive (negative) for the pre-Volcker (Volcker-Greenspan) period.

ex-post data produces a blurry picture, as central bankers are constrained by real-time information. Orphanides (2002, 2004), estimating interest rate reaction functions with real-time data, offers evidence that monetary policy during the pre-Volcker era was not accommodative, but responded strongly to inflation forecasts, contrasting results reported by Clarida et al. (2000) based on ex-post data.

A set of inflation forecasts suited for analyzing the nature of real-time U.S. monetary policy is drawn from the so-called Greenbook, which contains forecasts prepared for FOMC meetings. These forecasts are generated using information that is actually available at the time monetary policy decisions are made, thus providing a more accurate view on monetary policy decisions. The Greenbook forecasts are published with a five year lag and were first published in 1965. One shortcoming of the early forecasts is that observations were not consistently available and forecasts for longer horizons were not produced. Hence, for practical reasons the sample period covers the time horizon from the first quarter of 1968 to the last quarter of 1999. Using the Greenbook forecasts we estimate the structural relationship described by (1) for the pre-Volcker and Volcker-Greenspan era. The set of instruments is identical to the set of previously used instruments.

Consistent with the previous findings, the growth rate of nonborrowed reserves responds positively to rising inflation forecasts in the pre-Volcker era and negatively in the Volcker-Greenspan period, while the feedback from the real-time output gap measure is always negative (see Table 3). Overall, these results confirm the shift in the conduct of U.S. monetary policy. The estimates further indicate that using ex-post data tends to overstate the responses to inflation in absolute terms. On the basis of the real-time estimates, the pre-Volcker policy was in fact less accommodating and the Volcker-Greenspan policy was not as reactive as evidenced by the ex-post data.

Regarding the real-time output gap measure, money supply has reacted in a more pronounced way during the pre-Volcker period. Yet, for both periods we find that money supply exhibits a stronger feedback from real-time output gap data than from ex-post output gap data.[7] To conclude, the estimates based on real-time data support the view that there exists a change in U.S. post-war monetary policy. Overall, our results correspond more with Clarida et al.'s (2000) evidence on a substantial policy shift based on the analysis of the federal funds rate using ex-post data, than with Orphanides' (2002, 2004) findings.

3. Money supply and macroeconomic stability

In this Section a standard New Keynesian model is applied to assess the implementation of optimal monetary policy and equilibrium determinacy under a money supply reaction function as specified in (1). First, the model is described and the relation between the interest rate and money growth is discussed. Second, conditions for optimal money supply and the requirements for equilibrium determinacy are examined, and finally is it shown that the estimated reaction functions lead to a unique rational expectations equilibrium (REE).

3.1. Interest rates and money in a New Keynesian model

To facilitate comparisons with studies on macroeconomic stability under interest rate reaction functions, a standard New Keynesian model is used with staggered price setting, endogenous labor supply, and an additively separable CES utility function. Money demand is induced by (end-of-period) real balances entering the utility function, where the elasticities of intertemporal substitution for consumption and real balances, $1/\sigma$, are identical. Uncertainty is due to cost push (mark-up) shocks φ_t, i.e., shocks to the elasticity of substitution between monopolistically produced intermediate goods.

The model is locally approximated by log-linearizing the equilibrium conditions at the

[7] This difference in the estimated output gap coefficients might be due to a distorted estimation of the trend component of output, leading to a mis-measurement of the output gap, as argued by Orphanides (2004). He further points out that misperceptions concerning potential output were only much later understood, which may explain the striking difference in magnitude in the estimated coefficient for the real-time output gap.

zero inflation steady state ($R = 1/\beta > 1$), where the support of aggregate shocks is sufficiently small.[8] Long-run distortions are disregarded and it is assumed that a lump-sum financed subsidy eliminates average distortions due to monopolistic competition.[9] In a neighborhood of the steady state the equilibrium sequences are then approximated by the convergent solutions to the linearized equilibrium conditions, where $\widehat{z}_t$ denotes the percent deviation of a generic variable z_t from its steady state value z, $\widehat{z}_t = (z_t - z)/z$.

A REE consists of a set of sequences for inflation $\widehat{\pi}_t$ (where $\pi_t = P_t/P_{t-1}$), the output gap $\widehat{x}_t$, (end-of-period) real balances $\widehat{m}_t$ (where $m_t = M_t/P_t$), and the gross nominal interest rate $\widehat{R}_t$, $\{\widehat{\pi}_t, \widehat{x}_t, \widehat{m}_t, \widehat{R}_t\}_{t=0}^{\infty}$, that converge to the steady state and satisfy

$$\widehat{\pi}_t = \omega\widehat{x}_t + \beta E_t\widehat{\pi}_{t+1} + \widehat{\varphi}_t, \quad \text{where } \beta \in (0,1), \omega > 0, \tag{2}$$

$$\widehat{x}_t = E_t\widehat{x}_{t+1} - (1/\sigma)(\widehat{R}_t - E_t\widehat{\pi}_{t+1}), \quad \text{where } \sigma > 0, \tag{3}$$

$$\widehat{m}_t = \widehat{x}_t - \gamma\widehat{R}_t, \quad \text{where } \gamma = \beta[\sigma\,(1-\beta)]^{-1}, \tag{4}$$

and a monetary policy reaction function, for a sequence of i.i.d. shocks $\{\widehat{\varphi}_t\}_{t=0}^{\infty}$ satisfying $E_{t-1}\widehat{\varphi}_t = 0$, and given $m_{-1} = M_{-1}/P_{-1} > 0$, where E denotes the expectations operator, M nominal balances and P the aggregate price level.

It should be noted that there exists no unique mapping between a standard interest rate reaction function and a money supply reaction function like (1). Given a particular REE, which is, for example, implemented by an inertial interest rate rule $\widehat{R}_t(\widehat{R}_{t-1}, E_t\widehat{\pi}_{t+1}, \widehat{x}_t)$, the equilibrium money growth rate can be expressed in several alternative ways as functions of state variables and other indicators.[10] In fact, as long as the RHS of (1) contains more

[8] The derivation is available upon request and can also be found in Schabert (2005).

[9] Output deviations from the steady state then represent output gaps, since any deviation of current output from its steady state value is induced by a distortionary shock.

[10] For a given REE the money growth rate can, for example, be expressed as a function of exogenous states only, or as a function of (expected) endogenous variables and exogenous states. This property of "instrument rules" is well established (see Woodford, 2003, chapter 8), and underlies Evans and Honkapohja's (2003) classification of policy rules as "fundamentals based policy rules" and "expectations based policy rules".

arguments than there are state variables (which for an inertial interest rate rule are given by $\widehat{R}_{t-1}$ and $\widehat{\varphi}_t$), there are infinitely many money supply reaction functions of the form (1) that are consistent with the same REE.

Moreover, there might not even exist a unique solution for the money growth rate under one particular interest rate rule, if it fails to pin down a REE uniquely, which Clarida et al. (2000) claim to be the case in the pre-Volcker period. Nonetheless, the equilibrium conditions (2)-(4) imply a structural relation between money growth rates and interest rates, which applies in any REE: Suppose for example that the central bank follows an inertial interest rate rule $\widehat{R}_t = \overline{\rho}_R \widehat{R}_{t-1} + \overline{\rho}_\pi E_t \widehat{\pi}_{t+1} + \overline{\rho}_y \widehat{x}_t$, where $\overline{\rho}_R \in (0,1)$ and $\overline{\rho}_\pi, \overline{\rho}_y > 0$. Then, eliminating the output gap in (3) with (4), gives $\sigma E_t \widehat{\mu}_{t+1} = \sigma E_t \left(\widehat{m}_{t+1} - \widehat{m}_t + \widehat{\pi}_{t+1}\right) = (1+\sigma\gamma)\widehat{R}_t - \sigma\gamma E_t \widehat{R}_{t+1} + (\sigma - 1) E_t \widehat{\pi}_{t+1}$, and substituting out $E_t \widehat{R}_{t+1}$ with the interest rate rule and $E_{t+1}\widehat{\pi}_{t+2}$ with (2) for $t+1$, leads to

$$E_t \widehat{\mu}_{t+1} = \left(\sigma^{-1} + \gamma\left(1 - \overline{\rho}_R\right)\right)\widehat{R}_t - \left(\sigma^{-1} + \gamma\beta^{-1}\overline{\rho}_\pi - 1\right) E_t \widehat{\pi}_{t+1} - \gamma\left(\overline{\rho}_y - \omega\beta^{-1}\overline{\rho}_\pi\right) E_t \widehat{x}_{t+1}.$$

The first part of the RHS reveals a positive relation between money growth and the nominal interest rate, which accords to the well-known liquidity puzzle in this class of models.[11] The other two terms on the RHS show that the central bank's response to inflation and the output gap, measured by $\overline{\rho}_\pi$ and $\overline{\rho}_y$, can reverse the previous effect. The money growth rate then tends to decrease with inflation and with the output gap, given that the gross interest rate elasticity of money demand γ and the Phillips curve coefficient ω are typically sufficiently large and small, respectively.

Whether the money growth rate actually increases or decreases after a rise in the nominal interest rate thus depends on the feedback coefficients ($\overline{\rho}_R$, $\overline{\rho}_\pi$, $\overline{\rho}_y$) and the structural

[11] Dynamic general equilibrium models with frictionless financial markets typically fail to produce a liquidity effect, and rather predict that a rise in the nominal interest rate (which ought to be contractionary) is associated with a rise in the money growth rate (see e.g. Christiano et al. 1997).

parameter (σ, γ, ω). Concisely, larger values for the feedback coefficients $\overline{\rho}_\pi$ and $\overline{\rho}_y$ increase the likelihood for the money growth rate to decline when the central bank raises the nominal interest rate in response to higher inflation or to a widening output gap. Applying Clarida et al.'s (2000) estimates, money growth in the Volcker-Greenwood is more likely to decrease with inflation than in the Pre-Volcker period, which is consistent with our empirical results.

3.2. Efficiency and determinacy under money supply reaction functions

In this Section the estimated money supply reaction functions are examined. For simplicity, it is assumed that the stock of money that provides transactions services in the goods market is tied to the stock of high-powered money,[12] so that the central bank is able to control the growth rate $\widehat{\mu}_t$ (where $\mu_t = M_t/M_{t-1}$). Thus, the description of the equilibrium is completed by assuming that the money growth rate, $\widehat{\mu}_t = \widehat{m}_t - \widehat{m}_{t-1} + \widehat{\pi}_t$, satisfies $\forall t \geq 0$

$$\widehat{\mu}_t = \overline{\rho}\widehat{\mu}_{t-1} + \overline{\mu}_\pi E_t\widehat{\pi}_{t+1} + \overline{\mu}_y\widehat{x}_t + \Xi_t, \tag{5}$$

where $\overline{\rho} \geq 0$ and $\Xi_t = \Xi(\widehat{\varphi}_t)$ is some linear function of the exogenous state.

To describe the characteristics of optimal money supply, Woodford's (2003) second-order approximation of household welfare at the steady state is adopted. It is assumed that the distortions due to transactions frictions are negligible, such that a second-order Taylor-expansion of household welfare at the undistorted steady state leads to the following standard central bank objective (see Woodford, 2003): $\min E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2}\left(\hat{\pi}_t^2 + \frac{\omega}{\epsilon}\widehat{x}_t^2\right)$, where $\epsilon > 1$ denotes the elasticity of substitution between differentiated intermediate goods.

The optimal plan of the central bank acting under commitment in a timeless perspective is then characterized by (2)-(4) and the central bank's first order condition $\widehat{x}_t - \widehat{x}_{t-1} = -\epsilon\widehat{\pi}_t$ $\forall t \geq 0$ (see Woodford, 2003). While it is well established that the plan can be implemented

[12] Endogenous shifts and exogenous shocks to the money multiplier are disregarded and it is assumed that high-powered money is held as a constant fraction of M (e.g. due to reserve requirements), which is sufficient to model the structural relations between money supply and macroeconomic aggregates.

by an interest rate reaction function (including lagged output gap), it can easily be shown that the plan can also be implemented by a money supply reaction function (5).[13]

Proposition 1 *The central bank can implement its optimal commitment plan under a timeless perspective if it supplies money according to (5) with* $\overline{\rho} = 0$, $\overline{\mu}_\pi = (\sigma - 1)\beta/\sigma < 1$, $\overline{\mu}_y = -[(\sigma - 1)\beta + \sigma(\epsilon - 1)]\omega/[(1-\beta)\sigma]$, *and* $\Xi_t = -\gamma\sigma(\widehat{x}_t - E_{t-1}\widehat{x}_t) - \gamma(\widehat{\pi}_t - E_{t-1}\widehat{\pi}_t) - [(\epsilon - 1) + \gamma(\sigma\epsilon - 1)]\widehat{\varphi}_t$.

According to proposition 1, a non-inertial money supply reaction function ($\overline{\rho} = 0$) can be used to implement the optimal policy plan. The money growth rate should decrease with the output gap ($\overline{\mu}_y < 0$) for reasonable parameter values (e.g. for $\sigma \geq 1$ and a mark-up $\frac{\epsilon}{\epsilon-1} - 1$ smaller than 100%). The inflation feedback $\overline{\mu}_\pi$ can either be positive or negative, depending on the relative risk aversion σ. When households exhibit a higher degree of risk aversion (large σ), their valuation of a smooth consumption sequence increases and optimal policy shifts towards the stabilization of output. Consequently, $\overline{\mu}_\pi$ is positive for $\sigma > 1$, and negative if households are less risk averse, $\sigma < 1$. Hence, a positive or a negative inflation feedback can be consistent with optimal money supply, though the money growth rate should not increase by more than one for one with inflation, $\overline{\mu}_\pi < 1$.[14]

Based on the conditions in proposition 1, one can roughly assess the efficiency of the Fed's money supply: The estimated reaction functions exhibit inertia, $\rho > 0$ (see Section 2.), which is not required by optimal policy. The long-run feedback coefficients show that in both sub-periods – the pre-Volcker as well as the Volcker-Greenspan period – the long-run response of money supply to inflation $\widehat{\mu}_\pi/(1-\widehat{\rho})$ and to the output gap $\widehat{\mu}_y/(1-\widehat{\rho})$ are

[13] Eliminating $\widehat{R}_t$ with (4) in (3) leads to $\widehat{m}_t = (1+\gamma\sigma)\widehat{x}_t - \gamma\sigma E_t\widehat{x}_{t+1} - \gamma E_t\widehat{\pi}_{t+1}$. The equilibrium money growth rate $\widehat{\mu}_t = \widehat{m}_t - \widehat{m}_{t-1} + \widehat{\pi}_t$ thus satisfies $\widehat{\mu}_t = \widehat{\pi}_t + (1+\gamma\sigma)(\widehat{x}_t - \widehat{x}_{t-1}) - \gamma\sigma(E_t\widehat{x}_{t+1} - E_{t-1}\widehat{x}_t) - \gamma(E_t\widehat{\pi}_{t+1} - E_{t-1}\widehat{\pi}_t)$. Using the central bank's first order condition for t, $\widehat{x}_t - \widehat{x}_{t-1} = -\epsilon\widehat{\pi}_t$, as well as for $t+1$ and taking expectations, leads to $\widehat{\mu}_t = -[(\epsilon-1) + \gamma(\sigma\epsilon - 1)]\widehat{\pi}_t + \gamma(\sigma\epsilon - 1)E_t\widehat{\pi}_{t+1} - \gamma\sigma(\widehat{x}_t - E_{t-1}\widehat{x}_t) - \gamma(\widehat{\pi}_t - E_{t-1}\widehat{\pi}_t)$. Eliminating $\widehat{\pi}_t$ with (2), finally gives $\widehat{\mu}_t = \frac{\sigma-1}{\sigma}\beta E_t\widehat{\pi}_{t+1} - \frac{(\sigma-1)\beta+\sigma(\epsilon-1)}{(1-\beta)\sigma}\omega\widehat{x}_t + \Xi_t$.

[14] Applying the solution for the REE under the optimal plan, which is uniquely determined, one can easily solve for Ξ_t as a linear function of $\widehat{\varphi}_t$. Details are available upon request.

smaller than one and negative, respectively (see Table 2 and 3). Thus, the Fed's adjustment of money supply to changes in inflation and the output gap has been needlessly delayed, but has in principle been consistent with the characteristics of optimal money supply.

Next, equilibrium determinacy under a money supply reaction function (5) is examined using Blanchard and Kahn's (1980) criterion. Given that the model (2)-(5) exhibits two backward-looking elements, i.e., the predetermined state variables $\widehat{m}_{t-1}$ and $\widehat{m}_{t-2}$ (or $\widehat{\mu}_{t-1}$), equilibrium determinacy requires exactly two stable eigenvalues. It can be shown (see below) that the following condition is necessary for the model to exhibit exactly two stable and positive eigenvalues:[15]

$$\overline{\mu}_{\pi} + \overline{\mu}_{y}\frac{1-\beta}{\omega} < 1 - \overline{\rho}. \tag{6}$$

Condition (6) demands the weighted sum of the long-run feedback coefficients in (5) to be smaller than one, $\frac{\overline{\mu}_{\pi}}{1-\overline{\rho}} + \frac{\overline{\mu}_{y}}{1-\overline{\rho}}\frac{1-\beta}{\omega} < 1$. This condition evidently corresponds to the well-known Taylor principle, which requires the weighted sum of the long-run coefficients of an interest rate feedback rule to be larger than one (see e.g. Woodford, 2003).

To get an intuition for this "reversed Taylor principle", consider the simple case $\overline{\mu}_{y} = \overline{\rho} = 0$. If the economy is hit by a cost push shock $\widehat{\varphi}_{t} > 0$, current and expected inflation tend to rise, leading to decreasing real balances for $\overline{\mu}_{\pi} < 1$. In this case, the output gap tends to get negative (see 4), which stabilizes inflation by (2). Otherwise, $\overline{\mu}_{\pi} > 1$, money supply is accommodative such that real balances grow. This tends to reduce the interest rate and to raise aggregate demand, such that the upward pressure on prices further increases. Due to this mechanism money supply can render multiplicity or non-existence of convergent equilibria possible. Yet, (6) is only a necessary condition and does not suffice to ensure equilibrium determinacy. To check if the estimated reaction functions implement a REE

[15] A condition similar to (6) is shown in Schabert (2006) to be necessary and sufficient for equilibrium determinacy under non-inertial money growth rules in a continuous time sticky price model. Black (1974) has shown that the price level can be indetermined when the money growth rate satisfies $\overline{\mu}_{\pi} > 1$.

uniquely, a set of sufficient conditions are applied, which are presented in the following proposition.[16]

Proposition 2 *Suppose that the central bank adjusts money supply according to (5).*

1. *Then, the REE is uniquely determined and exhibits a set of non-oscillatory equilibrium sequences only if (6) is satisfied.*
2. *Then, the REE is uniquely determined if but not only if i.) (6), ii.)* $\rho < \beta^2$*,*
   iii.) $\sigma\omega(1-\beta)(1+\rho+\mu_\pi) - \sigma\mu_y(1-\beta^2) + (1+\rho)2(\sigma+2\sigma\beta+\beta\omega+\sigma\beta^2) > 0$*, and*
   iv.) $\beta(\omega(1+\beta)-\sigma\mu_y)\rho - \sigma(1+\beta)(\beta-\rho)^2 - \sigma\omega(\beta^2-\mu_\pi)\rho > 0$ *are satisfied.*

Given the conditions in proposition 2 we can easily check the stability implications of the Fed's money supply. Since the output gap feedback is always negative, condition (6) can only be violated if the long-run inflation feedback $\widehat{\mu}_\pi/(1-\widehat{\rho})$ exceeds one. This, however, is never the case, since the values for all specifications in Section 2 are always strictly smaller than one. Thus, (6) is satisfied by all reactions functions estimated for both periods.

To examine if the conditions in part 2 of proposition 2 are satisfied, a standard set of parameter values ($\sigma = 2$, $\beta = 0.99$, and $\omega = 0.1$) is used. Applying the significant point estimates for the full sample, the pre-Volcker period, and the Volcker-Greenspan periods (see Table 2 and 3), shows that the conditions *i.)*-*iv.)* in part 2 of proposition 2 are always jointly satisfied. Hence, equilibrium determinacy is ensured by the estimated money supply reaction functions. Further numerical checks for alternative parameter values show that the

[16] Details of the proof can be downloaded at Science Direct or is available from the authors. To sketch the proof, note that the characteristic polynomial is quartic $H(X) = X^4 + X^3a_1 + X^2a_2 + Xa_3 + a_4$, with $a_4 = \rho/\beta^2 \geq 0$. Thus, there exist either zero, two or four eigenvalues with a positive real part and at least one eigenvalue inside the unit circle if $\rho < \beta^2$. Further, $H(1) > 0$ if (6) is satisfied, which is required for two stable and positive eigenvalues (see part 1), and $H(1) < 0$ if *iii.)* in part 2 is satisfied, indicating that there exists either two or four stable (real or complex) eigenvalues. Finally, Schur's theorem is applied, which states that all eigenvalues are stable if, inter alia, $(a_3 + a_4^2 - a_1a_4 - 1)(-a_3 + a_4^2 + a_1a_4 - 1) > 0$, which is violated when *iv.)* holds. Thus, if *i.)*-*iv.)* are satisfied, there are exactly two stable eigenvalues.

estimated reaction functions always lead to two stable eigenvalues for reasonable values for σ, β and ω , even if some of the conditions *ii.)*-*iv.)* are violated.

Thus, our analysis of macroeconomic stability leads to a different conclusion than previous studies based on interest rate rules: Pre-Volcker monetary policy did not allow for multiple equilibria. Notably, this conclusion is in fact not inconsistent with empirical evidence on interest rate rules violating the Taylor-principle. The simple reason for different determinacy results is that by controlling the money supply path, monetary policy links (even for $\rho = 0$) the equilibrium allocation to an initial condition that is predetermined, m_{t-1}. The latter thus serves as an equilibrium selection criterion under a money supply policy (ruling out endogenous fluctuations), while it is irrelevant under an interest rate rule.[17]

Nevertheless, equilibrium determinacy is not guaranteed when a central bank adjusts money supply. In particular, macroeconomic stability requires (6) and therefore the growth rate of real balances to decrease with inflation. According to our findings, Federal Reserve policy always satisfied this requirement and thus never allowed for macroeconomic instability.

4. Conclusion

This paper provides empirical evidence that the Federal Reserve's money supply has responded to changes in expected inflation and the output gap during the past four decades. Estimates of forward-looking money supply reaction functions reveal that money supply has always responded negatively to a widening output gap. Conversely, money supply responses to changes in expected inflation exhibit considerable differences between the pre-Volcker and Volcker-Greenspan era. During the latter regime, the Fed's monetary policy was characterized by an anti-inflationary stance, while money supply has accommodated inflation in

[17] This difference in equilibrium determinacy corresponds to the difference in price level (in)determinacy under money growth and interest rate policy, stressed by Sargent and Wallace (1975). While a constant money growth policy facilitates nominal determinacy under perfectly flexible prices, it causes beginning-of-period real balances to be relevant for equilibrium determination when prices are not perfectly flexible.

the former regime. These findings, which are robust to changes in the way expectations are modelled, confirm evidence from federal funds rate analyses on the shift in Federal Reserve policy.

Further, the stabilization implications of the estimated money supply reaction functions are examined, applying a standard sticky-price model. The key requirement for equilibrium determinacy is that the money growth rate should not increase by more than one for one with inflation. Money supply in both periods satisfies this condition and ensures equilibrium determinacy. Thus, we cannot confirm the hypothesis of Clarida et al. (2000) that pre-Volcker policy has contributed to high and volatile inflation rates in the 1970's by allowing for endogenous fluctuations. Viewed through a money supply lens, Federal Reserve policy in the pre-1979 has been less anti-inflationary, though is was sufficiently reactive to ensure macroeconomic stability.

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Figure 1: Responses of Nonborrowed Reserves

Response to One S.D. innovations ±2 S.E.

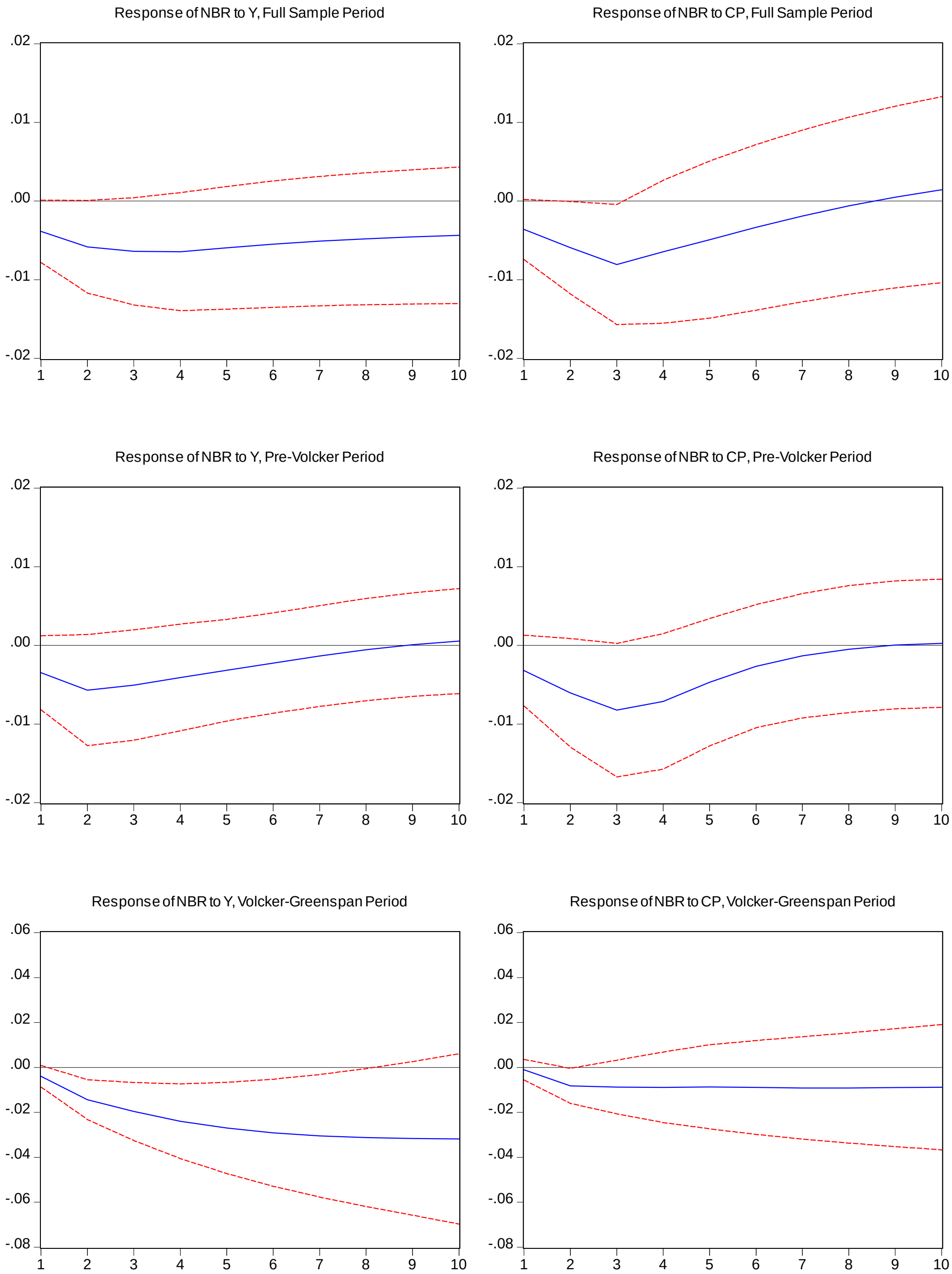

Table 1. Estimation Results: Full Sample

	GDP Deflator		CPI		Greenbook	
	n=1	n=4	n=1	n=4	n=1	n=4
$\widehat{\rho}$	0.88* (0.03)	0.89* (0.03)	0.88* (0.03)	0.88* (0.03)	0.90* (0.03)	0.91* (0.03)
$\widehat{\mu}_\pi$	0.07 (0.06)	0.06 (0.06)	0.04 (0.04)	0.04 (0.05)	0.03 (0.06)	−0.04 (0.08)
$\widehat{\mu}_y$	−0.25* (0.11)	−0.26* (0.11)	−0.29* (0.11)	−0.29* (0.11)	−0.75* (0.28)	−0.49 (0.32)
R^2	0.70	0.70	0.70	0.70	0.44	0.60
J	0.62	0.61	0.58	0.57	0.69	0.69

Notes: Figures in parentheses below coefficient estimates denote standard errors. Coefficients which are significant at the 5 percent level are marked with "*". R^2 denotes the coefficient of determination; J is a test statistic for the null hypothesis that the overidentifying restrictions are satisfied. For the latter we only report p-values.

Table 2. Estimation Results: Split Sample

	Panel A: Pre-Volcker Period				Panel B: Volcker-Greenspan Period			
	GDP Deflator		CPI		GDP Deflator		CPI	
	n=1	n=4	n=1	n=4	n=1	n=4	n=1	n=4
$\widehat{\rho}$	0.72* (0.04)	0.72* (0.04)	0.74* (0.06)	0.75* (0.06)	0.91* (0.02)	0.92* (0.03)	0.95* (0.07)	0.98* (0.05)
$\widehat{\mu}_\pi$	0.24* (0.06)	0.22* (0.06)	0.11* (0.05)	0.08 (0.06)	−0.35* (0.15)	−0.39* (0.14)	−0.55* (0.05)	−0.62* (0.05)
$\widehat{\mu}_y$	−0.49* (0.18)	−0.52* (0.19)	−0.37* (0.20)	−0.32 (0.22)	−0.62* (0.14)	−0.59* (0.13)	−0.15* (0.04)	−0.39* (0.08)
$\widehat{\mu}_\pi/(1-\widehat{\rho})$	0.85* (0.02)	0.78* (0.03)	0.42* (0.07)	−	−3.89* (0.01)	−4.87* (0.01)	−11.0* (0.01)	−31.0* (0.01)
$\widehat{\mu}_y/(1-\widehat{\rho})$	−1.75* (0.02)	−1.85* (0.02)	−1.42* (0.03)	−	−6.89* (0.01)	−7.37* (0.01)	−3.0* (0.01)	−19.5* (0.01)
R^2	0.52	0.52	0.54	0.53	0.73	0.73	0.69	0.71
J	0.77	0.77	0.65	0.61	0.92	0.95	0.80	0.76

Notes: Standard errors of the long-run coefficients were computed with the delta-method as described in Papke and Wooldridge (2005) and are only reported for significant "short-run" coefficients. See notes to Table 1.

Table 3. Estimation Results: Greenbook Forecasts

	Pre-Volcker		Volcker-Greenspan	
	n=1	n=4	n=1	n=4
$\widehat{\rho}$	0.56* (0.03)	0.30* (0.02)	0.87* (0.03)	0.85* (0.05)
$\widehat{\mu}_\pi$	0.12* (0.05)	0.03 (0.05)	−0.20* (0.08)	−0.25* (0.06)
$\widehat{\mu}_y$	−1.79* (0.14)	−1.82* (0.15)	−0.84* (0.18)	−0.63* (0.14)
$\widehat{\mu}_\pi/(1-\widehat{\rho})$	0.27* (0.32)	–	−1.54* (0.01)	−1.67* (0.03)
$\widehat{\mu}_y/(1-\widehat{\rho})$	−4.07* (0.01)	−2.60* (0.02)	−6.46* (0.01)	−4.20* (0.01)
R^2	0.62	0.48	0.70	0.71
J	0.94	0.96	0.76	0.76

Notes: See notes to Table 1 and Table 2.

APPENDIX

Proof of proposition 2: To examine equilibrium determinacy, the interest rate in $\widehat{x}_t = E_t\widehat{x}_{t+1} - (1/\sigma)(\widehat{R}_t - E_t\widehat{\pi}_{t+1})$ is eliminated with $\widehat{m}_t = \widehat{x}_t - \gamma\widehat{R}_t$, to give $(\sigma + \frac{1}{\gamma})\widehat{x}_t = \sigma E_t\widehat{x}_{t+1} + \frac{1}{\gamma}\widehat{m}_t + E_t\widehat{\pi}_{t+1}$. The set of equilibrium conditions then consists of the latter equation, the aggregate supply constraint $\widehat{\pi}_t = \omega\widehat{x}_t + \beta E_t\widehat{\pi}_{t+1} + \widehat{\varphi}_t$, monetary policy $\widehat{\mu}_t = \overline{\rho}\widehat{\mu}_{t-1} + \overline{\mu}_\pi E_t\widehat{\pi}_{t+1} + \overline{\mu}_y\widehat{x}_t + \Xi_t$, and $\widehat{\mu}_t = \widehat{m}_t + \widehat{\pi}_t - \widehat{m}_{t-1}$. In matrix form, the model reads (where shocks are disregarded, for convenience)

$$\begin{pmatrix} E_t\widehat{\pi}_{t+1} \\ E_t\widehat{x}_{t+1} \\ \widehat{m}_t \\ \widehat{\mu}_t \end{pmatrix} = \mathbf{A} \begin{pmatrix} \widehat{\pi}_t \\ \widehat{x}_t \\ \widehat{m}_{t-1} \\ \widehat{\mu}_{t-1} \end{pmatrix} \text{ where } \mathbf{A} = \begin{pmatrix} \beta & 0 & 0 & 0 \\ 1 & \sigma & \gamma^{-1} & 0 \\ 0 & 0 & -1 & 1 \\ -\mu_\pi & 0 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1 & -\omega & 0 & 0 \\ 0 & \sigma + \gamma^{-1} & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & \mu_y & 0 & \rho \end{pmatrix}$$

Using $\gamma = \frac{\beta}{\sigma(1-\beta)}$, the characteristic polynomial of $\mathbf{A}$ is given by

$$\begin{aligned} H(X) &= \left(\rho/\beta^2\right) - X\left(\sigma + \sigma\rho + 2\sigma\beta\rho + \sigma\omega\rho + \beta\omega\rho - \sigma\beta\omega\rho\right)\left(\sigma\beta^2\right)^{-1} \\ &+X^2\left(\sigma + 2\sigma\beta + \sigma\omega + \beta\omega - \sigma\beta\omega - \sigma\mu_y + \sigma\beta\mu_y + 2\sigma\beta\rho + \beta\omega\rho + \sigma\beta^2\rho\right)\left(\sigma\beta^2\right)^{-1} \\ &-X^3\left(2\sigma\beta + \beta\omega + \sigma\omega\mu_\pi - \sigma\beta\mu_y - \sigma\beta\omega\mu_\pi + \sigma\beta^2 + \sigma\beta^2\mu_y + \sigma\beta^2\rho\right)\left(\sigma\beta^2\right)^{-1} + X^4 \end{aligned}$$

Given that two variables are predetermined $(\widehat{m}_{t-1}, \widehat{\mu}_{t-1})$ and two can jump $(\widehat{\pi}_t, \widehat{x}_t)$ a unique and convergent solution of the REE requires exactly two stable eigenvalues.

The value of H at $X = 0$ (which equals the product of the roots of $H(X) = 0$), $H(0) = \rho/\beta^2 \geq 0$, reveals that there exists either zero, two or four eigenvalues with a positive real part and that at least one eigenvalue lies inside the unit circle if

$$\rho < \beta^2 \tag{1}$$

At $X = 1$, the value of H is given by $H(1) = \left(\omega\left(1 - \rho - \mu_\pi\right) - \left(1 - \beta\right)\mu_y\right) / \left(\sigma\beta\gamma\right)$ such that

$H(1) > 0 \Leftrightarrow$

$$\frac{1-\beta}{\omega}\frac{\mu_y}{1-\rho} + \frac{\mu_\pi}{1-\rho} < 1 \tag{2}$$

Thus, (2) is necessary for $\mathbf{A}$ to exhibit two positive stable eigenvalues, which establishes the claim in the first part of the proposition 2. At $X = -1$, the value of H is given by

$$H(-1) = [\sigma\omega(1-\beta)(1+\rho+\mu_\pi) - \sigma\mu_y\left(1-\beta^2\right) + (1+\rho)\,2\left(\sigma + 2\sigma\beta + \beta\omega + \sigma\beta^2\right)]/(\beta^2\sigma)$$

such that $H(-1) > 0 \Leftrightarrow$

$$\sigma\omega(1-\beta)(1+\rho+\mu_\pi) - \sigma\mu_y\left(1-\beta^2\right) + (1+\rho)\,2\left(\sigma + 2\sigma\beta + \beta\omega + \sigma\beta^2\right) > 0 \tag{3}$$

Hence, if (1)-(3) are satisfied, there exists either two or four stable (real or complex) eigenvalues. To establish the existence of exactly two stable roots, we just need to show that at least one eigenvalue is unstable. For this, Schur's theorem is applied, which states that all eigenvalues of $\mathbf{A}$ are stable if and only if four conditions are jointly satisfied. It is thus sufficient to show that one of the four conditions is violated. For this, the following condition from Schur's theorem for a quartic polynomial $X^4 + X^3a_1 + X^2a_2 + Xa_3 + a_4$ is used:

$$\begin{vmatrix} 1 & 0 & a_4 & a_3 \\ a_1 & 1 & 0 & a_4 \\ a_4 & 0 & 1 & a_1 \\ a_3 & a_4 & 0 & 1 \end{vmatrix} = \left(a_3 + a_4^2 - a_1a_4 - 1\right)\left(-a_3 + a_4^2 + a_1a_4 - 1\right) > 0$$

For $H(X)$ this condition reads

$$[\sigma(1+\beta)(\beta-\rho)^2 + \sigma\omega\left(\beta^2 - \mu_\pi\right)\rho - \beta\left(\omega(1+\beta) - \sigma\mu_y\right)\rho]/[\sigma(1-\beta)] > 0,$$

such that the following condition is sufficient to ensure that at least one eigenvalue is unstable

$$\beta\left(\omega\left(1+\beta\right)-\sigma\mu_y\right)\rho-\sigma\left(1+\beta\right)\left(\beta-\rho\right)^2-\sigma\omega\left(\beta^2-\mu_\pi\right)\rho>0 \tag{4}$$

Thus, if but not only if (1)-(4) are satisfied, **A** exhibits exactly two stable eigenvalues. This establishes the claim in the second part of the proposition 2.