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A Real Options Perspective On R&D Portfolio Diversification

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Abstract

This paper shows that the conditionality of investment decisions in R&D has a critical impact on portfolio risk, and implies that traditional diversification strategies should be reevaluated when a portfolio is constructed. Real option theory argues that research projects have conditional or option-like risk and return properties, and are different from unconditional projects. Although the risk of a portfolio always depends on the correlation between projects, a portfolio of conditional R&D projects with real option characteristics has a fundamentally different risk than a portfolio of unconditional projects. When conditional R&D projects are negatively correlated, diversification only slightly reduces portfolio risk. When projects are positively correlated, however, diversification proves more effective than conventional tools predict.

Key words: Real Options; Portfolio Analysis; Research & Development

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1 Introduction

When the future outcomes of a firm's endeavors are unknown, a key strategy for dealing with such risk is betting on more than one horse. Successful research and development (R&D) policy therefore requires careful portfolio analysis to optimise the selection and the development of several concurrent alternatives. Diversification of risk plays a key role in this process. When the portfolio consists of risky R&D projects, however, conventional diversification arguments do not hold since conditionality causes the payoffs to become non-linear. This is a direct result of the option characteristics that many R&D projects display.

Option characteristics follow from managerial flexibility to adjust decisions under uncertainty. Any possibility of altering a project as new information becomes available renders a project conditional. For instance, a project that is started now may be abandoned or expanded in the future, a decision based on a given performance criterion and usually taken when new costs need to be incurred. As the investment decision is conditional, it can be regarded as an 'option' that is acquired by making the prior investment. We will examine the differences between conditional and unconditional investments below, as well as their implications for portfolio analysis.

This paper examines diversification when conditional investment decisions are present in an R&D portfolio, and shows that reliance on traditional diversification strategies can be misleading. Negative correlation makes diversification a less effective instrument for eliminating risk amongst conditionally financed

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projects than for unconditional projects. Positive correlation, however, makes diversification more effective. As compared to unconditional projects, a portfolio of conditionally financed projects is less sensitive to changes in correlation when correlation is not highly positive, and risk is therefore more difficult to diversify. Our findings have implications for diversification strategies in portfolio analysis. This includes (but is not limited to) the strategic choice between a focused or diversified portfolio, diversification over time and risk measurement techniques such as Value at Risk (VaR), often used in risk management regulation.

Real options analysis has become a well-known R&D project valuation technique for intertemporal risky investments in R&D. In their seminal paper, Black and Scholes (1973) consider equity of a real, levered firm as an option on its entity value. Using financial theory, Myers (1977) was the first to describe real options as "the opportunities to purchase real assets on possibly favorable terms". In the strategy literature, Bowman and Hurry (1993) and Bettis and Hitt (1995) propose real options theory as an alternative lens for looking at technology investments that closely resemble the behaviour and characteristics of real options. In the R&D literature, Thomke (1997) indeed shows empirically that flexibility under uncertainty allows firms to continuously adapt to change and improve products. Hartmann and Hassan (2006) find that real options analysis is used as an auxiliary valuation tool in pharmaceutical project valuation [1]. In this context, a basic implementation is pro-

[1] The fundamental difference between real options and traditional discounted cash flow (DCF) valuation lies in the flexibility to adapt when circumstances change. Whereas DCF valuation assumes investments are fixed, an option is the right (not the obligation) to invest in R&D at some future date. If future circumstances are favourable, the option will be exercised; if not, the option will expire without any further cost. Such freedom of choice enables an investor to abandon the project in a timely manner so that further losses are avoided. Therefore, many unfavourable investments (with limited downside risk) can be financed by a few highly profitable

vided by Kellogg and Charnes (2000), and more sophisticated option valuation models for pharmaceutical research have been developed by Loch and Bode-Greuel (2001). Lee and Paxson (2001) view the R&D process and subsequent discoveries as sequential (compound) exchange options. Cassimon et al. (2004) provide an analytical model for valuing the phased development of a pharmaceutical R&D project. The empirical literature also confirms that R&D yields the positively skewed distribution of returns that is typical of options. For instance, Scherer & Harhoff (2000) investigated innovations and show that the top 10% returns captured 48% to 93% of total sample returns. They refer to Nordhaus (1989), who postulates that 99.99% of invention patents submitted per year are worthless, but that the remaining 0.01% have high values.

In concurrence with this literature, we analyse conditionally financed projects as options. R&D typically has a high chance of failure and can be deemed risky. High-risk projects in R&D are generally explorative in nature; examples include basic and fundamental research, or R&D in response to important changes in a firm's strategic environment. We will contrast the option-like projects to unconditional projects that typically behave similarly to equity shares, have a low chance of failure and are subject to less risk. Low-risk projects in R&D are often of an incremental nature; examples include 'me-too' inventions that imitate a successful competitor's invention, or investments into an already commercialized product. We refer to the first group as conditionally financed projects, conditional projects, or real options. We refer to the second as unconditional projects.

Most real options studies have primarily examined projects in isolation. Eng-

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investments (with unlimited upside potential). Profitable investments will account for the majority of returns, so the return distribution becomes positively skewed.

wall (2003), however, argues that every project takes off from, or is executed in, an organizational context. Real options should therefore also be considered as part of a portfolio. Brosch (2001) considers the influence of interacting real options *within* projects. These positive and negative interactions between options make a portfolio's value non-additive. Our focus, however, is on option interactions *between* projects, and we focus on the risk of the portfolio.

Smith and Thompson (2003, 2005) postulate a project selection strategy in sequential petroleum exploration, where the outcome of the prior drillings can be observed before investing in the next drilling. We are also involved with real option selection, but focus on simultaneous (non-sequential) development. Multiple assets have been examined by Wörner et al. (2002, 2003), who describe a firm as a 'basket option' that conducts several R&D projects, or as an option on a set of stochastic variables. Yet, as they focus on the value of a single claim that pertains to many random variables, their analysis does not derive results relevant to portfolio management (which inherently deals with the selection between multiple claims). Such rainbow or basket options are often used to describe R&D projects; Paxson (2003) surveys several real R&D option models and utilises some of them in a case study.

When constructing an R&D portfolio, the selection of candidates comprises many important, non-monetary considerations. For example, Prencipe and Tell (2001) show that firms try to capture synergies that stem from learning processes. Several studies have therefore aimed to integrate risk diversification with expected costs and benefits, inter-project synergies, externalities, R&D quality and overall fit with the business strategy. Taking this angle, Linton et al. (2002) develop a framework that combines both quantitative and qualitative measures to rank and select the projects in a portfolio. In addition, Martino (1995) describes several methods for R&D project selection including cluster analysis, cognitive modeling, simulation, portfolio optimisation, and decision theory. While these sources are suitable for handling technical and physical diversification, they seem less appropriate for allocating financial resources, compared to the Markowitz (1952) diversification principle. Markowitz's objective is to optimise risk given a return, or vice versa. Chien (2002) includes a survey of selection procedures and shows that several originated from Markowitz's work. A recent R&D selection model that is based on that of Markowitz (1952) can be found in Ringuest et al. (2004). Unfortunately, the Markowitz diversification strategy only applies when the distribution of project returns is symmetric, an assumption that does not hold for R&D projects with conditionality. Our argument supplements the Markowitz criterion in that it explicitly considers real option characteristics; we create a skewed distribution by simulating many real options.

Using a portfolio of two investment opportunities, we show that although the risk of an R&D portfolio always depends on the correlation between projects, the dependence differs between conditional projects and unconditional projects. In particular, we find that when projects are positively correlated, the overall portfolio risk for conditional projects is lower than for unconditional projects. Diversification is an important argument to create a portfolio of such projects, because it is more effective than one would expect from unconditional investments. In contrast, when projects are negatively correlated, we find that the overall portfolio risk for conditional projects is higher than for unconditional projects. Moreover, under negative correlation, portfolio risk is less sensitive to changes in correlation as compared to unconditional investment projects. Diversification is therefore less effective than one would initially expect from unconditional investments, and more weight should be placed on non-diversification arguments to motivate a portfolio of such projects, such as synergies and spillovers.

This paper is organized as follows: in Section 2, the theory behind a portfolio of real options is conveyed. In Section 3 we present the model and its results. Section 4 is dedicated to the implications of our findings. In Section 5, we conclude and provide directions for future research. In the Appendices A and B, a proof of our findings is provided, as well as a means to extend our analysis to a more realistic setting.

2 Conceptual Framework

We analyze a portfolio of individual projects that await conditional investments, and represent each project by a simple call. A portfolio of calls is a valid way to describe reality if the portfolio's constituents behave similarly to financial options. This happens if a portfolio consists entirely of conditionally financed projects, as is often found in pharmaceuticals, biotechnology, venture capital and software technology.

It is important to note that we examine a portfolio of multiple contingent claims. This differs from a single claim on several underlying stochastic processes, such as an option on the most valuable R&D project in a portfolio. A single claim does not fit our central goal of selecting and managing multiple claims. Each individual portfolio element may well be an R&D project or a venture facing several uncertainties and aiming for multiple markets, such that each element may be a rainbow or basket option. We study portfolios of these elements, however, and such portfolios consist of several claims (not their underlying values).

The symmetry of a project's value distribution has an impact on portfolio diversification. For unconditionally financed projects, the symmetrical distribution allows for a 'perfect hedge', and a riskless portfolio can be created; when two equity shares are perfectly negatively correlated, one goes down by the amount that the other goes up and vice versa[2], so that all deviation is offset. In line with Markowitz (1952), we call this hedging mechanism the "diversification effect" on the risk of a portfolio.

However, when the projects are conditionally financed, below-average results are no longer offset by above-average returns and Markowitz's (1952) diversification principle is no longer valid. Because the payoff from a call cannot fall below zero, an option already provides insurance against the negative payoffs by nullifying those payoffs that are lower than the exercise price. Hence, the value distribution of a portfolio of real call options becomes skewed from the left and ceases to be symmetrical. The would-be-negative payoffs are no longer available for diversification, and constructing a riskless portfolio is no longer possible. Paradoxically, in a portfolio of options, the option to abandon limits downside risk of the individual project, but complicates diversification and does not limit risk when portfolio correlation is negative. In line with Jensen's Inequality, we call this the 'convexity effect', which affects the diversification effect. In Appendix A, we derive this result as we examine the variance of a conditionally financed portfolio more explicitly.

In the next section, we will develop a Monte Carlo simulation model to show the effect of risky projects on a portfolio of R&D projects. The procedure is straightforward and can easily be used in practice with other portfolio selection criteria. Before we proceed, however, a proper description of our research

[2] That is, when uncertainty is constant and equal for both shares.

subject is appropriate. This paper exclusively focuses on the risk (not the value) of a portfolio of options, and is therefore a supplement to the previously mentioned portfolio selection criteria. Their importance notwithstanding, for the sake of argument we group all these criteria under the heading of "non-diversification criteria". "Uncertainty" in our portfolio is completely determined by how the value of projects varies. Portfolio variance is a well-known measure for this dispersion, used in the financial sector under Basel II regulations. We confine our analysis to the relation between market values of projects, and assume the project costs to be independent and known. We prefer this setup because modeling more than one source of uncertainty would cause our results to become confounded. To more accurately reflect reality, the procedure can be easily extended to accommodate two or more related stochastic processes, such as uncertain costs and benefits.

3 Methodology and Results

3.1 Simulation Model

To find the volatility of an option portfolio, we estimate the volatility of payoffs for each option. We model a portfolio with two projects $i \epsilon \{1, 2\}$. Unless we consider the special cases in Appendix A, it is not possible to determine the risk of an option portfolio analytically since the joint distribution of options is not analytically tractable. Instead, we model the behavior of both end-of-R&D values projects V_i by a simple normal distribution, defined as follows:

$$V_i = \mu_i + \sigma_i \varepsilon_i \tag{1}$$

where μ_i is the project value, σ_i is the standard deviation of project values when the project is completed and ε_i is a random draw from a standard normal distribution. Assuming no dividend payouts for each project i, we calculate the option value OV_i:

$$OV_i = \max[V_i - X_i, 0]e^{-rT} \tag{2}$$

where X_i is the investment needed to start or acquire the project, r is the discount rate and T is project i's time to completion. The value of the project can now be calculated by taking the average value of equation (2) over R simulation rounds, with OV_{ij} representing the result of a single simulation round for OV_i. As the number of rounds increases, this value converges to its true value. To observe how project values are distributed, the volatility of a single option can be found as follows:

$$\sigma_{OV_i} = \sqrt{\frac{1}{R}\sum_j (OV_{ij} - \overline{OV_i})^2} \tag{3}$$

Extending to a portfolio of two projects, the relation between underlying values (not the option values) is measured by means of a correlation coefficient ρ_{12} between ε_1 and ε_2. Assuming multivariate normality, the correlation between any number of assets can be calculated using the Cholesky decomposition. This process, as well as constructing a consistent variance-covariance matrix for cases where $i > 2$, is described in Appendix A. For the two variable case, independent samples y_1 and y_2 are taken from a univariate standardized normal distribution and the correlated samples ε_1 and ε_2 are calculated as follows:

$$\varepsilon_1 = y_1 \tag{4}$$

$$\varepsilon_2 = \rho_{12} y_1 + y_2 \sqrt{1 - \rho_{12}^2} \tag{5}$$

From one set of independent samples y_1 and y_2, we generate 21 pairs of correlated samples ε_1 and ε_2 (ranging from $\rho_{12} = -1.0$ to $\rho_{12} = 1.0$ with step size 0.10) by inserting the independent sample values into equations (4) and (5). Because, under the assumption of correlation between project values but no interactions between the options, the value of the portfolio is the sum of the project values i,

$$pf = \sum_i OV_i, \tag{6}$$

and the value of the portfolio can be calculated for each correlation. However, we are concerned with portfolio risk (measured by the variance of the summed option values) rather than the value of a portfolio of options. Similar to the case for a single option, the estimate of portfolio variance is based on a simulation of portfolios pf_j for $j = 1, ..., R$ and averaging over R:

$$\hat{\sigma}_{pf=}\sqrt{\frac{1}{R}\sum_j (pf_j - \overline{pf})^2} \tag{7}$$

As a numerical example, we can show a potential simulation round using the numbers from the bivariate base case described in Figure 1. Assume one set of draws from the univariate distribution are the following: $y_1 = 0.5$ and $y_2 = -0.25$. Using equation (5), this independent pair of draws leads to 21 correlated pairs, including (for $\rho_{12} = -0.2$)

$$\varepsilon_1 = 0.5;$$

$$\varepsilon_{2:\rho=-0.2} = -0.2 \times 0.5 - 0.25 \times \sqrt{0.96} \approx -0.34.$$

Using equation (1), if the underlying values are somewhat negatively correlated and uncertainty is 25%, a feasible realisation would be

$$V_1 = 20 + (25\% \times 20) \times 0.5 = 22.5;$$

$$V_2 = 20 + (25\% \times 20) \times -0.34 = 18.3$$

The value of each project and the portfolio is calculated using equation (2):

$$OV_1 = \max[22.5 - 25\,,\, 0]e^{-rT} = 0$$

$$OV_2 = \max[18.3 - 25\,,\, 0]e^{-rT} = 0$$

$$pf = 0 + 0 = 0$$

This procedure is repeated R times for each of the 21 correlations. Of the resulting 21 correlation-specific sets of R-sized portfolio values, we calculate portfolio risk using the variance of the portfolio values and plot it against the correlation. In all graphs, portfolio risk is normalized by dividing over the summed variance of two independent calls. If we treat the two projects as unconditional, we would have used equation (3), leading us to calculate a 'naive' portfolio variance, as described below.

3.2 Simulation Results

The curved, solid line σ^2_{pf} in Figure 1 shows the cumulative variance of 21 ρ-specific option pairs. As this paper is exclusively concerned with portfolio risk and our results cannot be compared with other pricing methods such as the seminal Black–Scholes equation[3], no further option values are reported. Nevertheless, values have been used in equation (7) to construct the *variance*, our measure for dispersion. To illustrate the difference between the actual portfolio risk and the calculated risk when using Markowitz diversification, we add a dashed line $\tilde{\sigma}^2_{pf}$ that shows the variance of the projects if we (wrongly) assume Markowitz diversification to be valid. This would only be appropriate if the separate projects are unconditional and behave as equity shares. The solid line connects the portfolio variance for 21 different correlations and for the dashed line, the following well-known formula to calculate portfolio variance is used:

$$\tilde{\sigma}^2_{pf} = \sigma^2_{OV1} + \sigma^2_{OV2} + 2\rho\sigma_{OV1}\sigma_{OV2} \tag{8}$$

Using equations (2) and (3), this line has the variance of individual option values OV_1 and OV_2 as inputs for σ^2_{OV1} and σ^2_{OV2}. We observe that at $\rho = 0$, the variance of unrelated projects is the same for both σ^2_{pf} and $\tilde{\sigma}^2_{pf}$. A third, dotted line $\sigma^2_0 \equiv \tilde{\sigma}^2_{pf:\rho=0} = \sigma^2_{OV1} + \sigma^2_{OV2}$ shows that the first line σ^2_{pf} is cushioned towards a special case of $\tilde{\sigma}^2_{pf}$, which represents a portfolio of completely unrelated options or options that are both separate and unrelated.

[Insert Figure 1 about here]

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[3] Our results also persist for other stochastic processes such as the geometric Brownian motion, on which the Black & Scholes option pricing model is built. Under this process, the project value's *changes,* instead of *levels*, would be normally distributed. This process is arguably more suitable for modeling skewed R&D project values, but causes an additional asymmetry in the distribution of portfolio values. To isolate the effects of the (also nonlinear) investment option, we choose a symmetrically distributed process; this makes it no longer possible to compare our results with other models.

The difference lies in the interpretation of the correlation coefficient ρ that measures the correlation between projects (the horizontal line σ_0^2 illustrates the degenerate case where ρ is zero). In the case of the naively calculated variance $\tilde{\sigma}_{pf}^2$, the projects are correlated one-to-one with the projects' market values and ρ is a constant. In the case of the correct variance σ_{pf}^2, however, co-movement between real option projects is a function of market value *and* the probability that a project is terminated[4]. This probability, in turn, depends on the moneyness of the call options, the correlation between project values and the volatilities of each project value. A manager who doesn't recognise real option characteristics would end up calculating risk naively, and Figure 1 illustrates how naively calculated risk may differ from correctly simulated risk.

In Figure 1, the naive portfolio variance at $\rho = 0$ is equal to the simulated variance of the portfolio and the separate options. We also see that both $\tilde{\sigma}_{pf}^2$ and σ_{pf}^2 are reduced when projects are less than perfectly positively correlated, and that two perfectly positively correlated projects have a variance of 200% compared to σ_0^2, as proven in Appendix A. When the projects are negatively correlated, both $\tilde{\sigma}_{pf}^2$ and σ_{pf}^2 are less then σ_0^2. All of these diversification effects are in line with the theory proposed by Markovitz.

The 'convexity effect', however, limits the most severe value drops but leaves all positive development intact, so that project payoffs are non-linear and the value distribution becomes skewed. Figure 1 and Appendix A both show that when individual projects can no longer be offset, naively applying Markowitz diversification may lead to significant miscalculations of risk. This is caused

[4] This fact has also been used in the theoretical derivations of our results in Appendix A.

by the interaction between the diversification and convexity effects, which has both positive and negative consequences. When projects are positively correlated, the cushioning of convexity enhances diversification and overall risk becomes lower than under Markowitz diversification. When the projects are negatively correlated, however, the cushioning of convexity hampers the diversification effect, leading to a less effective hedge. As a consequence, options are more complex instruments for diversification than stock. In terms of the effect that correlation has on risk, the sensitivity of unconditional risk to changes in correlation is generally smaller than for unconditional risk, up to a correlation of about $\rho = 0.60$. For negatively correlated projects in particular, diversification changes the portfolio's risk only slightly. Stated more precisely, the variance of a conditionally financed portfolio is compressed towards the cumulative variance of two independent options. The range of a conditionally financed portfolio is smaller than the range of an unconditional portfolio, but the minimum is higher than the unconditional portfolio's minimum. We can formulate the following hypotheses:

H1: For positively correlated project values, conditionally financed projects diversify risk better than unconditional projects.

H2: For negatively correlated project values, unconditional projects diversify risk better than conditionally financed projects.

3.3 Robustness Analysis and General Applicability

The base case (Figure 1) shows what happens when two simple and identical options are out of the money. This setting is typical of many R&D projects. Four panels in Figure 2 show results of simulated options that have a lower volatility (Panel A), a different volatility (Panel B), are at the money (Panel C) or in the money (Panel D). In all these situations, the convexity effect persists. Changes in other parameters such as the discount rate have no effect on the results. In Panel A, we halve the volatility so that the project is not in the money until the value is equal to $\mu + 2\sigma$. In R&D, this means that the project is not continued in about 97.5% of the cases and hardly any of these projects are available for risk diversification. As a consequence, the diversification effect is nearly absent and all we see is the convexity effect; we might just as well not diversify at all. In the less extreme case when volatilities differ, Panel B shows that portfolio risk is less sensitive to changes in correlation than in Figure 1 and diversification is still quite ineffective. The unit change on the y-axis indicates that in this case, zero variance cannot be achieved by naive calculation either. When the moneyness increases in Panel C and Panel D, the curves move towards the straight line and our results become less distinct. This reflects the familiar fact that deeply in the money options will behave similarly to the underlying stock. As a consequence, the convexity effect becomes less pronounced and the diversification effect starts to dominate. In R&D, this means that if the value of the project is much higher then its costs, conditional financing doesn't make a large difference because the project will be exercised anyway.

[Insert Figure 2 about here]

Some general remarks can be made on applying our model to practice. Many projects are funded by multiple finance or subsidy rounds and our simple calls represent the last phase. A more complex example involves an R&D project that is split up into stages such that certain requirements must be met before it can enter the next development phase. This project design involves several 'compound' options, as each conditional investment is an option on the next phase. R&D in the pharmaceutical industry, for example, is typically characterized by six stages of development. This means that investing in the sixth phase is conditional upon completion of the fifth phase, which requires investments conditional on the fourth phase, etc.. These more realistic features can easily modeled by using compound options in the simulation. In the compounded case, we are stacking 'effect on effect'. This is not demonstrated here since such simulation results are highly dependent on the success of entering the next round. Arbitrarily chosen input parameters (especially for several stages) will have a critical influence on the portfolio variance and conceal the convexity effect. Compound options can easily be put to practice by means of the closed-form model of the successive phases from R&D to commercialization, developed by Cassimon et al. (2004). Likewise, simulation makes implementing other realistic features such as uncertain costs or time-to-completion straightforward . That, however, would also drive us away from the essential portfolio diversification problem.

For ease of exposition, we have limited the analysis to the smallest portfolio possible- a portfolio of two projects. The effect is also observable when we increase the number of assets. If we introduce a third asset and keep the step size fixed at 0.10, for example, then 21 correlated samples are ranked similarly for every random variable. For the 3-variable case we have a grid of 21 correlation points between variable 1 and 2, 21 between 1 and 3 and 21 between 2 and 3. Appendix B describes how a simulation procedure can be developed for three and more projects by constructing a consistent correlation structure, as in Hull (2006) [5] .

4 Implications

The implications of our results can be readily applied to any research policy that concerns simultaneous development of projects, subject to conditional financing. While various examples may illustrate this, we limit ourselves to three: static diversification, diversification over time and capital reserve regulations that protect an organization from bankruptcy.

4.1 Focus or Diversify?

An example of an application of our framework lies in resource allocation for a geographical area, in order to effectively spur innovation. For instance, a government may want to stimulate economic activity in a certain area. Does a government prefer to focus business activities, in order to create a specialized technology area such as Silicon Valley? Or would it diversify in order to prevent overdependence on a few industries, which has proven problematic in Detroit, an area focused on construction and car manufacturing?

Our results provide an argument based on the risk characteristics of individual firms in both areas. Especially in an innovative field such as information technology, a start-up is often a risky business with option characteristics. This

[5] At the same time, the number of possible correlations is smaller than 63. If, for instance, two projects c_1 and c_2 have a negative correlation of 0.99, the third cannot be highly correlated with both at the same time. In this three-variable case, the correlation between c_1 and c_2 and a third, single option can only be defined on the complete interval [-1, 1] when the correlation of the two projects c_1 and c_2 is held constant at $\rho = 0$.

is not true for construction and manufacturing. We have shown that the risk of a group of positively correlated start-ups is lower than one would expect if conditionality is ignored. Hence, diversification can be a good argument for grouping innovative companies, as risk is more effectively reduced than within industries with a more stable cash flow. Therefore, total risk in Silicon Valley is not easily increased, even when moderate positive correlation exists between the value drivers of the region's companies. In Detroit, however, diversification is an important factor in the region's development.

4.2 Diversification Over Time

An important implication concerns the different effects of diversification as the portfolio matures: when positively correlated projects are still young and in the R&D phase, a portfolio consisting of such projects is less risky than one would expect. But as successful projects mature, uncertainty resolves and option characteristics become less relevant, so that the same correlation between projects leads to more diversification risk. As a result, portfolios need restructuring when projects evolve and policy makers need different diversification criteria over time. Each individual project's milestone will modify the risk characteristics of the portfolio as a whole. To minimize overall portfolio risk, for instance, some of the matured projects may therefore be sold in exchange for more negatively correlated projects with low-risk.

Put differently, Figure 1 indicates that risk first develops as the curvature, and later as the straight, dotted line. The gentle slope of the curve shows that although the risk of positively correlated ventures is still higher than the risk of negatively correlated ventures, the difference doesn't matter as much as standard portfolio theory predicts. Therefore, structuring a portfolio to minimize variance is not as important in the early stages: non-diversification arguments may still provide good reason to combine these projects, but risk reduction isn't one of them. Until the projects mature and risk diminishes, negatively correlated risky projects are less attractive portfolio candidates for risk diversification. When ventures mature, diversification becomes more important and the risk characteristics of positively and negatively correlated ventures become more pronounced.

This can be observed in the pharmaceuticals sector, where many small firms succesfully focus on a few drugs, rather than become part of a portfolio of a large, diversified company. Why is risk diversification not necessary for small research ventures to be successful in such a risky business? One well-known argument is that in the early stages of development, economies of scale (for example in marketing) are not feasible yet. Another is that the R&D process is differently organized for small ventures than for big companies. Our results provide an additional argument for this phenomenon: under conditional financing, a strong focus only marginally increases the risk of the portfolio while it may strongly contribute to non-diversification criteria (such as synergies and spillovers) and preserve the upward potential. Only after several milestones have been completed do the results of these R&D programs become less uncertain; the cushioning by the convexity effect disappears and the projects behave more like stocks. In these later stages, the risk becomes more sensitive to changes in correlation, diversification of risk becomes important and the venture may well be sold to a diversified company.

In this context, it may be useful to provide examples of positively and negatively correlated risk. Positively correlated risk can be partially ascribed to non-diversifiable market risk. Another part may be ascribed to the medical context, if projects develop drugs for 'complementary treatment'. An example is the treatment of HIV, where a combination of three drugs is prescribed; if the side effects of one drug become less severe or effectiveness improves, the value of all three drugs will increase, since the quality of the treatment increases. Another example are drugs that treat closely related disorders such as lung cancer and cardiovascular diseases. Often, both have a common cause, such as an unhealthy lifestyle. When a patient can be treated for one illness, he or she will live longer and odds increase that he or she will suffer from the second illness. Ironically, this is good news for investors as the market value of both drugs increases. An example of negatively correlated risk are two development programs that aim to cure similar diseases; if one program yields a major discovery, the value of the other program automatically goes down.

The problem in both examples can be described by a trade-off between focus and diversification; when focusing, $\rho > 0$, and conditional portfolio risk is lower than than standard portfolio theory might suggest because the diversification effect is cushioned by the convex nature of options. When diversifying, $\rho < 0$, and the cushioning of convexity causes diversification to be less effective than would be expected from standard diversification arguments.

The implications of diversification over time can be summarized by the effectiveness of risk management; risk diversification is not important until other uncertainties (that justify conditional financing) have been resolved. In other words, when an investor's risk is minimized by a milestone financing requirement, choosing negatively correlated projects will not diversify the risk much further. Rather, having a strong focus is not as risky as one would expect, and non-diversification arguments are more important selection criteria. Until the correlation becomes more than moderate, diversification has an insignificant effect on total risk as long as other factors justify conditional investment decisions.

4.3 Capital Reserves

Using variance as a measure of risk lies at the basis of common financial risk measures such as Value at Risk (VaR). Over the period that a portfolio is held, it measures the value of the portfolio at risk that for a given confidence interval. Similar to our 'naively' constructed portfolio, the most common VaR model assumes portfolio value to be linearly dependent on the value of the underlying assets. In the R&D setting, however, this relationship is linear only between portfolio value and the value of the options.

When the number of conditional projects is sufficiently large, the value of the portfolio becomes normally distributed. If the number of conditional projects is not sufficiently large, our results imply that the linearity assumption is inappropriate. To determine the VaR correctly, the variance of a portfolio should be simulated to contruct the confidence interval. We show that the standard deviation is significantly higher if projects are negatively correlated, and naively calculated variance leads to an underestimation of VaR. As an international standard for creating capital reserves regulations, the Basel Accords may recommend simulation as a tool for risk assessment. This is true particularly for industries in which conditional investment decisions are common practice.

5 Conclusion and Suggestions for Future Research Directions

In this article we have shown that the presence of conditional financing in R&D may invalidate diversification strategies for portfolio construction. Under negative correlation, emphasis should be placed on other (non-diversification) arguments when constructing a portfolio. Under positive correlation, by contrast, the advantages of diversification are larger than one may expect using Markowitz diversification. We have also demonstrated that due to the convexity of high-risk projects, the sensitivity of portfolio risk to correlation is smaller for high-risk projects than for low-risk projects.

The difference in risk between high-risk and low-risk projects can be quite substantial; for two negatively correlated risky projects of about $\rho = -0.5$, the uncertainty is reduced by only $10\%/50\% = 20\%$ as compared to low-risk uncertainty reduction. For $\rho = +0.5$, the uncertainty is increased by only $30\%/50\% = 60\%$ as compared to low-risk uncertainty. These differences can easily become more dramatic (Figure 2.A shows that diversification may become impossible for negative correlations), and our findings are robust to changes in the parameter structure of the model. We have provided examples to show why this is important for R&D portfolio analysis.

An important implication of our work is that when evaluating the risk of a portfolio of risky R&D opportunities, it is not sufficient to merely examine the risk-return properties between projects. It is also important to determine the presence of conditional investment decisions before drawing conclusions on how effective a project will be at reducing the risk of the portfolio. Furthermore, policy makers may need to change their selection criteria over time.

As companies mature and the need for conditional financing disappears along with uncertainty, diversification of succesful projects becomes more important in the future.

Extending the model in several ways facilitates the analysis of portfolio risk under more specific circumstances. As we have shown, one can easily construct a portfolio with projects that differ in volatility, time to maturity and moneyness. It is also possible to compound several options when additional parameters (such as success probabilities) are known. Using Appendix A, it is easy to extend the analysis to a large portfolio, with each project having its own distinct features such as the required investment outlay, estimated date of completion and volatility of market value.

Applying our model in real-world case studies may yield interesting results in the future. The simulation procedure remains the same for several underlying stochastic processes and may include other case-specific properties such as mean reversion, barriers or autocorrelation. It is also possible to account for synergies on the cost side. To explore these directions, however, and to compare empirical results with our framework, real-life data is needed to provide realistic input parameters. Our study demonstrates the complexity of options in a portfolio context, but when additional information on project parameters is available to tailor the model to a specific problem, our framework can be helpful in formulating and assessing research and development policy by public and private parties.

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Appendix A: Explicit Derivation of Main Results

To examine the variance of a risky R&D portfolio more closely, we will present an analytical treatment of our theoretical framework to convey what happens when the correlation is perfectly positive, negative or absent. Because of the nature of options (caused by the the max operator), the variance of a single call option consists of two properly weighted variances: one variance for the case in which the call value is positive – which we will denote by $Var(c^+)$ – and one for the case in which the outcome is zero:

$$Var(\max[V - X, 0]) = w_1 Var(V - X) + w_2 Var(0) = w_1 Var(c^+) \qquad (9)$$

where w_1 and w_2 are the appropriate weights, V is the project's value and X the cost of investment. The key to an analytical derivation of the variances is recognizing the outcome possibilities that exist in each of the three correlation scenarios, and constructing a single variance from there, using a variance decomposition formula that is defined as

$$Var(A) = E[Var(A|B)] + Var(E[A|B]) \qquad (10)$$

We will consider a portfolio of two simple investment opportunities (calls) that are exactly equal to each other. Both require an investment X that is assumed to be equal to the expected value of the project (for ease of notation, we drop the subscript i that we introduced in Section 3.1):

$$X_1 = X_2 = X = E[V_T] \qquad (11)$$

As a consequence, for at the money options, each call will be distributed around $E[V_T]$:

$$\Pr(V_T > X | X = E[V_T]) = \Pr(\varepsilon > 0) = 0.5; \varepsilon \sim N(0,1) \tag{12}$$

Furthermore, since both calls are identical, we know that the probability of being in the money is equal for both calls i, j:

$$\Pr(V_{i,T} > X) = \Pr(V_{j,T} > X) \tag{13}$$

The cases of perfectly positive, negative or absent correlation differ only in the correlation that exist between two projects, and each will yield a different expression for the portfolio variance, as expressed in terms of the option components' variance in equation (9).

Perfectly Positively Correlated Projects

For $\rho = 1$, either both calls are in the money or both calls are out of the money. This means that the portfolio consists of two possible outcomes:

$$Pf = (c_1^+ + c_2^+ | V_1 > X, V_2 > X) + (0 | V_1 < X, V_2 < X)$$

Because of equation (12) and equation (13), each outcome is equally likely. In this case (denoting the positive part of the portfolio as pf^+ and the negative as pf^-), the variance composites on the right-hand side are:

$$Var_{pf^+} = Var(2c^+|V > X) = 4 \times Var(c^+|V > X)$$
$$Var_{pf^-} = 0$$

Furthermore, we know that $E[pf^+] = 2E[c^+]$ since both projects are identical. From equation (10), it follows that the portfolio variance is:

$$\begin{aligned} Var(pf|\rho = 1) &= \frac{4Var(c^+) + 0}{2} + \frac{(2E[c^+] - E[c^+])^2 + (0 - E[c^+])^2}{2} \\ &= 2 \times Var(c^+) + E[c^+]^2 \end{aligned}$$

Perfectly Independent Projects

For $\rho = 0$, we know from equation (12) and equation (13) that each option can be in the money or out of the money with equal probability. In this case, we can therefore distinguish four possible outcomes :

$$\begin{aligned} Pf = &(V_1 - X|V_1 > X, V_2 < X) \\ &+ (V_2 - X|V_1 < X, V_2 > X) \\ &+ (V_1 - X + V_2 - X|V_1 > X, V_2 > X) \\ &+ (0|V_1 < X, V_2 < X) \end{aligned}$$

The variance of the first two terms on the right hand side is equal to $Var(c^+)$, and the expected value for both is $E[c^+]$. Since the non-linear payoff is accounted for in the last term, we can use Markowitz's equation to find the variance of the third term, which is simply the sum of the variances $Var(c_1^+)$ and $Var(c_2^+)$ because $\rho = 0$. Furthermore, we know that the expected value of this term is equal to the sum of the expected values $E[c_1^+]$and $E[c_2^+]$. It follows from equation (10) that

$$
\begin{aligned}
Var(Pf|\rho=0) = & \frac{Var(c^+) + Var(c^+) + 2var(c^+) + 0}{4} \\
& + \frac{0 + 0 + (2E[c^+] - E[c^+])^2 + (0 - E[c^+])^2}{4} \\
& = Var(c^+) + 0.5(E[c^+])^2
\end{aligned}
$$

This is exactly half of the variance found at $\rho = +1$, a finding that corresponds with the simulation results.

Perfectly negatively correlated projects

For $\rho = -1$ and at the money options, we know that either one call or the other is in the money. Yet since both projects can never jointly be in or out of the money at $\rho = -1$, this simply means that the variance is equal to either the variance of one call, or that of the other. More precisely, we can state that:

$$
\begin{aligned}
Pf = & (c_1^+|V_1 > X, V_2 < X) + (c_2^+|V_1 < X, V_2 > X) \\
= & c_1^+ = c_2^+ = c^+
\end{aligned}
$$

The last line follows from the observation that the calls are identical under the given conditions. It follows directly that

$$
Var(Pf|\rho = -1) = Var(c^+)
$$

This demonstrates why in our results, the variance of a perfectly negatively correlated portfolio doesn't go to 0% in the limit but is of a magnitude between zero and the variance at $\rho = 0$. Indeed, diversification under these circumstances does not permit risk to be diversified away.

Appendix B: How to Generate Random Samples from a Multivariate Normal Distribution

When a third stock enters our model, a third sample is drawn; ρ_{13} and ρ_{23} need to be defined in such a manner that the variances and covariance are consistent. For instance, if asset 1 and asset 2 strongly move together as well as asset 1 and 3 (that is, the correlations ρ_{12} and ρ_{13} are highly positive), then the relation between asset 2 and 3 needs to be positive (that is, ρ_{23} needs to have a high positive value) as well. If we require 3 correlated samples from normal distributions, the required samples are defined as follows:

$$\begin{aligned} \varepsilon_1 &= \alpha_{11}x_1 \\ \varepsilon_2 &= \alpha_{21}x_1 + \alpha_{22}x_1 \\ \varepsilon_3 &= \alpha_{31}x_1 + \alpha_{32}x_1 + \alpha_{33}x_1 \end{aligned} \tag{14}$$

The Choleski decomposition procedure is used to create a consistent correlation structure, and is appropriate since we assume the disturbances to be multivariate normally distributed. The procedure starts by setting $\alpha_{11} = 1$ and requires α_{21} to be chosen such that $\alpha_{21}\alpha_{11} = \rho_{21}$ and $\alpha_{21}^2 + \alpha_{22}^2 = 1$. This yields

$$\alpha_{21} = \rho_{21} \tag{15}$$

and

$$\alpha_{22} = \sqrt{1 - \rho_{21}^2} \tag{16}$$

For the third sample, α_{31} is to be chosen such that $\alpha_{31}\alpha_{11} = \rho_{31}$, yielding

$\alpha_{31} = \rho_{31}$. α_{32} is then to be chosen such that

$$\alpha_{31}\alpha_{21} + \alpha_{32}\alpha_{22} = \rho_{32}, \tag{17}$$

leading to

$$\alpha_{32} = \frac{\rho_{32} - \rho_{12}\rho_{13}}{\sqrt{1-\rho_{12}^2}} \tag{18}$$

We conclude by the requirement that

$$\alpha_{31}^2 + \alpha_{32}^2 + \alpha_{33}^2 = 1, \tag{19}$$

leading to

$$\alpha_{33} = \sqrt{1 - \rho_{13}^2 - (\frac{\rho_{23} - \rho_{12}^2\rho_{13}^2}{\sqrt{1-\rho_{12}^2}})^2} \tag{20}$$

We can simply generalize this case to n by expanding the Choleski matrix in equation (14), for example to

$$\varepsilon_4 = \alpha_{41}x_1 + \alpha_{42}x_2 + \alpha_{43}x_3 + \alpha_{44}x_4 \tag{21}$$

and repeat this procedure. As the number of projects increases, however, correlations need to be chosen with more and more care. Take the example of a single pharmaceutical drug. If we want to simulate two additional projects that both are correlated to this drug $\rho_{12} = \rho_{13} = -0.9$, then the additional projects need to be positively correlated. More specifically, if we let the third variable enter the simulation, it must satisfy

$$\alpha_{31}^2 + \alpha_{32}^2 + \alpha_{33}^2 = 1 \tag{22}$$

or

$$\alpha_{33}^2 = \sqrt{1 - \alpha_{31}^2 - \alpha_{32}^2} = \sqrt{1 - 0.9^2 - \alpha_{32}^2} > 0. \tag{23}$$

Hence, the Choleski-variable α_{32}^2 must not be larger than (1 - 0.81 =) 0.19 and

$$-\sqrt{0.19} \leqq \alpha_{32} \leqq \sqrt{0.19}. \tag{24}$$

Using this condition in Eq. (17), we find the following range:

$$0.62 = 0.90 \times 0.90 - 0.19 \times 0.19 \leq \rho_{23} \leq 0.90 \times 0.90 + 0.19 \times 0.19 = 0.88$$

If a fourth project enters the story and $\rho_{14} = \rho_{12} = \rho_{13} = -0.9$, it is required that

$$\alpha_{44}^2 = \sqrt{1 - \alpha_{41}^2 - \alpha_{42}^2 - \alpha_{43}^2} = \sqrt{1 - 0.9^2 - \alpha_{42}^2 - \alpha_{43}^2} > 0$$

and, similarly to equation (24), that

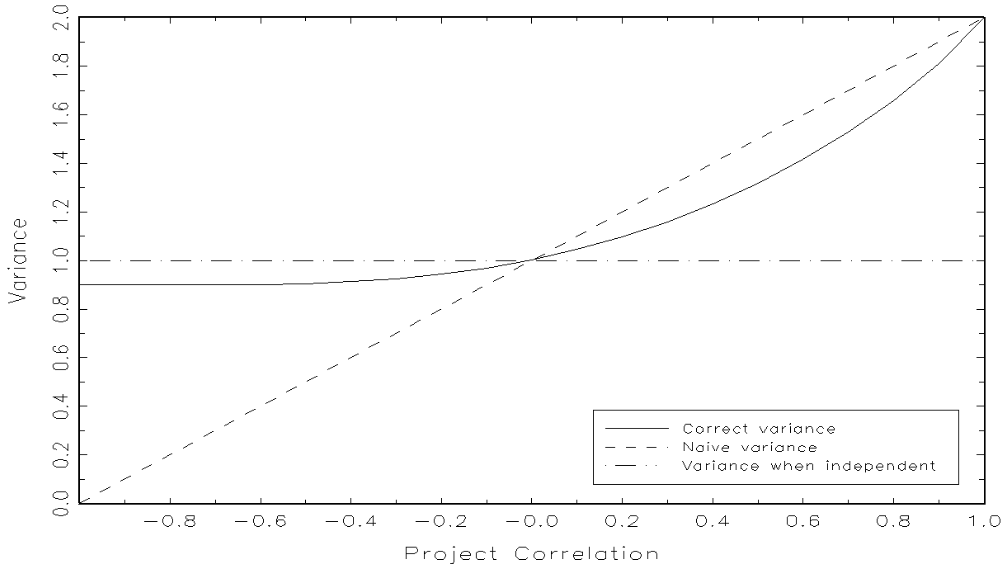
$$-\alpha_{22} \leq \alpha_{42} + \alpha_{43} \leq \alpha_{22},$$

meaning that $\alpha_{42} + \alpha_{43}$ are subject to the same constraint as was α_{32}. Thus, any new simulation variable is subject to all previous constraints, plus a new constraint. For instance, if we choose $\rho_{42} = \rho_{32}$(so that $\alpha_{42} = \alpha_{32}$ and $\alpha_{41}, \alpha_{42}, \alpha_{43} = \alpha_{31}, \alpha_{32}, \alpha_{33}$), it must be true that

$$\alpha_{44} = \sqrt{1 - \alpha_{41}^2 - \alpha_{42}^2 - \alpha_{43}^2} = \sqrt{1 - 0.81 - 0.19 - \alpha_{43}^2} > 0$$

and the fourth project needs to be uncorrelated with the others for consistency.

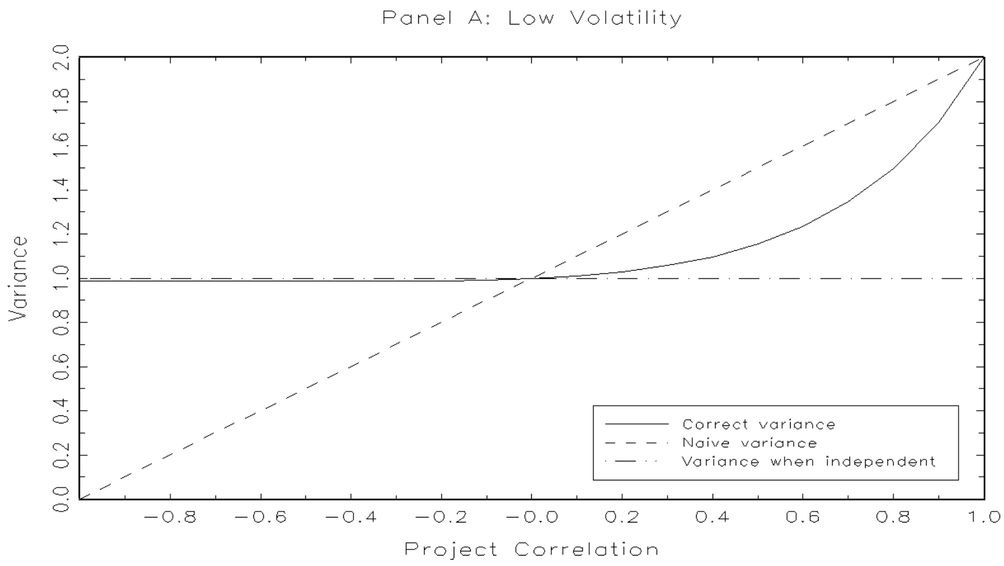
Figure 1. Simulated Risk for Two Identical Investment Opportunities



Using portfolio variance as a measure for risk, the cumulative variance of two simulated call options changes along with the correlation between the underlying values. Using Equation (8) for calculating portfolio risk would be naive, as it neglects option characteristics. The correct portfolio risk is graphed by the variance of the sum of option values (values are obtained through simulation and not reported); portfolio risk is normalized by dividing over the summed variance of two independent calls. The correct variance develops similar to naive variance, but is compressed towards the horizontal line, i.e. portfolio variance when projects are independent ($\rho = 0$). Simulation parameters (no dividend payments are made) are set as follows:

Number of trials: $R = 50,000$	Number of options: $n = 2$
Investment: $X_1 = X_2 = 25$	Project volatility: $\sigma_1 = \sigma_2 = 25\%$
Project market value: $V_1 = V_2 = 20$	Discount rate: $r = 20\%$
Time to completion: $T = 18$ months	

Figure 2. Sensitivity Analysis of Simulation Results

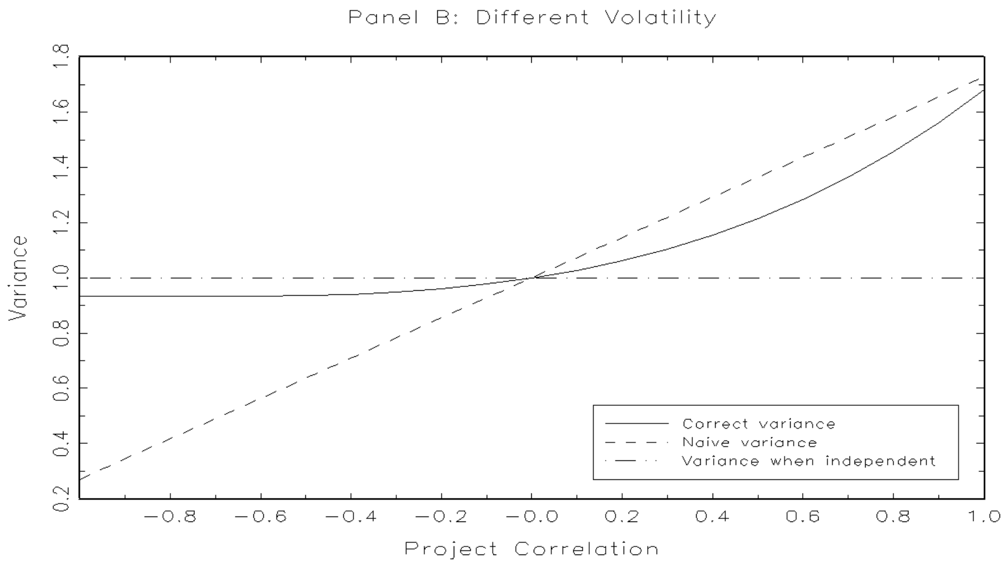


Number of trials: $R = 50,000$ Number of options: $n = 2$

Investment: $X_1 = X_2 = 25$ Project volatility: $\sigma_1 = \sigma_2 = 12.5\%$

Project market value:$V_1 = V_2 = 20$ Discount rate: $r = 20\%$

Time to completion: $T = 18$ months

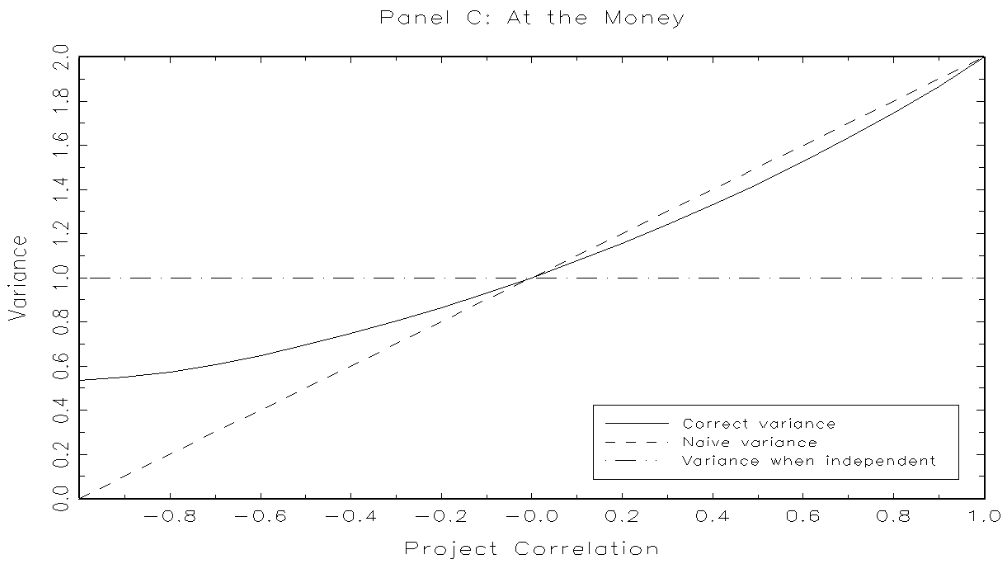


Number of trials: $R = 50,000$ Number of options: $n = 2$

Investment: $X_1 = X_2 = 25$ Project volatility: $\sigma_1 = 20\%$, $\sigma_2 = 30\%$

Project market value:$V_1 = V_2 = 20$ Discount rate: $r = 20\%$

Time to completion: $T = 18$ months

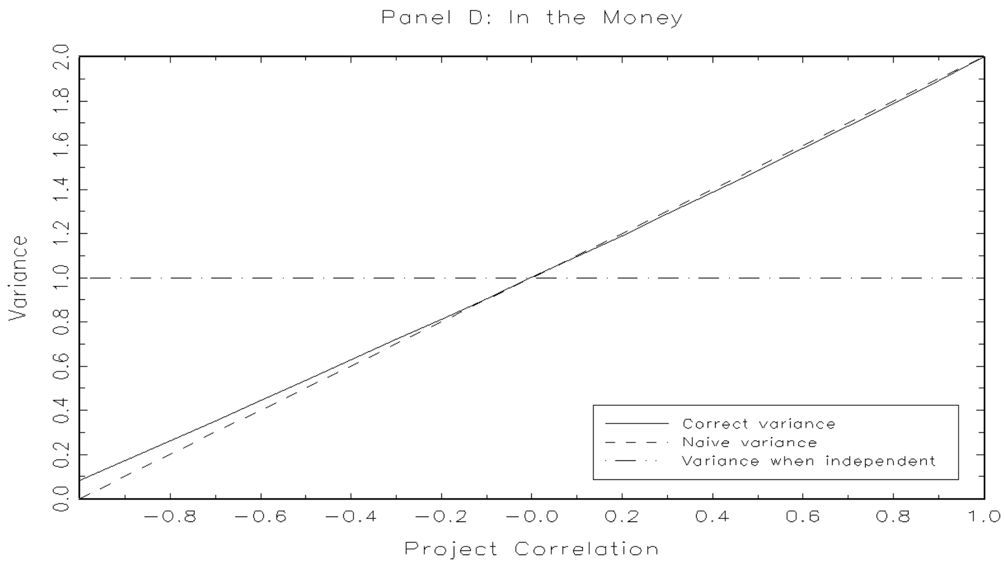


Number of trials: $R = 50,000$ Number of options: $n = 2$

Investment: $X_1 = X_2 = 25$ Project volatility: $\sigma_1 = \sigma_2 = 25\%$

Project market value:$V_1 = V_2 = 25$ Discount rate: $r = 20\%$

Time to completion: $T = 18$ months



Number of trials: $R = 50,000$ Number of options: $n = 2$

Investment: $X_1 = X_2 = 25$ Project volatility: $\sigma_1 = \sigma_2 = 25\%$

Project market value: $V_1 = V_2 = 30$ Discount rate: $r = 20\%$

Time to completion: $T = 18$ months