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Consumer search with costly recall*

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Abstract

This paper builds a consumer search model where the cost of going back to stores already searched is explicitly taken into account. We show that the optimal search rule under costly recall is very different from the optimal search rule under perfect recall. Under costly recall, the optimal search behaviour is nonstationary and, moreover, the reservation price is not independent of previously sampled prices. We fully characterize the optimal search rule under costly recall when a finite number of firms draws price quotes from a given distribution.

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Key Words: Search, Costly Recall

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1 Introduction

The main focus of consumer search theory is to analyze how market outcomes are effected if the cost consumers have to make to get information about the prices and/or qualities firms offer is explicitly taken into account. One of the basic results of the extensive literature is that firms have some market power that they can exploit even if there are many firms in the market and that price dispersion emerges as a consequence of the fact that some firms aim at selling to many consumers at low prices, while others make higher margins over fewer customers (see, e.g., Stigler, 1961 and Reinganum, 1979).

Most, if not all, of the consumer literature makes implicitly or explicitly the assumption of perfect or free recall: consumers can always come back to previously sampled firms without making a cost.[1] One of the important consequences of this asumption is that consumers search behavior is characterized by one reservation price that is constant over time (Kohn and Shavell (1974)): for any observed price sequence, consumers stop searching and buy at the firm from which they received a price quote if that price is not larger than this reservation price; otherwise they continue searching.[2]

The assumption of perfect recall is, so we argue, at odds with the general philosophy of the consumer search literature which has search frictions at its core. If consumers have to make a cost to go to a shop in the first place, then in almost any natural environment it also costs something (time, effort, or money) to go back to that shop. Even while searching on the internet, where the costs of search is arguably lower than in nonelectronic markets, it takes some mouse clicks and time to go back to previously visited websites. In other words, in consumer search it is not only important to remember the offers previously received, but one also has to make a cost to activate these offers again.

In this paper we replace the perfect recall assumption by the more natural assumption of costly recall, where the cost of going back to stores previously sampled is explicitly modelled. Under costly recall, we show that consumer search is no longer characterized by a reservation price that is constant over time. Instead, the reservation price at any moment in time depends on (i) the number of firms that are not yet sampled and (ii) the lowest price sampled so

[1] See, e.g., Reinganum (1979), Morgan and Manning (1985), Stahl (1989) and Stahl (1996) for early papers and Janssen et al. (2005), Tse (2006) and Waldeck (2006) for more recent papers explicitly using the perfect recall assumption.

[2] An alternative setting is studied by Weitzman (1979). He considers the interesting case where alternatives differ in the cost of inspection as well as in the distrbution of revenues and he asks the question in which order the alternatives should be explored.

far. In particular, for a given lowest price in the sample the reservation price is (weakly) decreasing in the number of firms that are not yet sampled (increasing over time) and increasing in the minimum price in the sample if this minimum price is not too large. Of course, if no prices are sampled yet, the reservation price is just a constant (depending on the number of firms that quote prices). Only when there are infinitely many price to sample, stationarity re-appears and the reservation price in that case coincides with the reservation price under perfect recall.

These two differences in the characterization of reservation prices have important consequences for the actual search behaviour of consumers. Under costly recall it may very well happen that if consumers observe as part of a price sequence two prices p_t and p_{t+1}, with $p_t < p_{t+1}$, they will rationally decide to accept to buy at p_{t+1} and not at p_t. This behaviour is not possible under perfect recall and rational consumer behaviour. The main reason for the fact that different behaviours are possible is that under costly recall, no matter how small the cost of retrieving previously sampled information, the search process is no longer stationary. In addition, the fewer the number of firms not yet sampled, the worse the chance of observing a low price if one continues searching. Together, this implies that the class of search behaviours that are consistent with rational behaviour on the part of consumers becomes much richer. Obviously, this has important consequences for the literature studying firm behaviour when consumers search sequentially as this literature is entirely based on the idea of a constant reservation price that is represented as a fixed number.[3]

In contrast to the assumption of perfect recall commonly employed in the literature on consumer search, many papers in the literature on job search assume that only current offers can be accepted as previous offers that are not accepted are foregone. Karni and Schwartz (1977) have interpreted these two applications of search theory as making specific assumptions on the probability with which past observations can be successfully retrieved: in consumer search, the probability of successful retrieval is one, in job market search, this probability is zero. They then go on to study situations with "uncertain recall", where the probability that past observations can be successfully retreived is less than one but greater than zero. Our paper interprets the difference between consumer search and job market search differently, namely in terms of the cost one has to make to retrieve information. This cost is either zero or prohibitively high. We study the intermediate case where the cost is positive, but not too high to make it uninteresting to consider the option of going back to previously

[3]An extensive overview of this literature has recently been given by Baye et al. (2006).

sampled firms.[4]

The structure of the rest of this paper is as follows. Section 2 presents the basic framework of analysis. Section 3 analyzes the optimal search behavior of consumers and Section 4 presents an example illustrating the nonstationarity and the the fact that rational consumers may decide to buy later at higher prices. Section 5 concludes.

2 Framework of Analysis

Consumers are confronted with the following situation. There are N firms selling some product. Each firm makes a specific price-quality offering that can be ranked according to some one-dimensional criterion, denoted by p. For simplicity, one may think of this ranking in terms of price: consumers prefer to buy the good with the lowest price. Each firm chooses a p according to some continuous mixed strategy distribution $F(p)$ with support $[\underline{p}, \overline{p}]$. Each consumer has a unit demand and valuation v for the product. Consumers have search costs c – the price they are paying for visiting a store. Costly recall is modelled by saying that consumers have a cost b of returning back to a store already visited, with $0 \leq b \leq c$. Consumers sequentially sample the prices chosen by firms. Consumers first have to decide whether or not to search, and after the first and each subsequent price offer, whether they want to obtain one more price quote or whether to stop searching, and if they decide to stop searching whether to buy at all and if so whether to buy at the current price or at previously sampled prices. The main issue we are interested in is how the presence of costly recall $(b > 0)$ affects the optimal search rule.[5]

3 Optimal Consumer Search

We start the analysis by considering the optimal stopping rule for consumers. Before searching once, consumers compare the benefits and cost of a first price search, and if the expected benefits exceed cost, which is, if

[4] As far as we are aware, there is no paper studying this most relevant case. Kohn and Shavell (1974) say that some of their results continue to hold if there is no possibility of recall, but they also do not analyze the situation of costly recall.

[5] In later research we intend to investigate this search rule in the context of a specific search model where also the behaviour of firms is explicitly modelled. We do not do that in the context of the present paper as we do not want to mix the very general context in which we analyze the optimal search rule with a specific model of price setting in the market.

$$v - \int_{\underline{p}}^{v} p dF(p) - c \geq 0 \tag{3.1}$$

consumers will search (at least) ones. This is a sufficient condition for searching once. Integrating by parts, this (first-step) condition can be rewritten as follows:

$$\int_{\underline{p}}^{v} F(p)dp \geq c. \tag{3.2}$$

If $F(p)$ satisfies this first-step condition (3.2)[6] we can analyze whether the consumer decides to continue searching or not after having observed a first price.

Since the expected value of continuing to search depends on future period expected values we use backward induction to analyze the optimal stopping rule. To this end, define p_{k-1}^s as the smallest price in a sample of $k-1$ prices previously sampled. We will argue that for each value of p_{k-1}^s there is a unique value of p_k such that an individual consumer is indifferent between buying at p_k and either going back to one of the previously sampled firms and buying there or continue searching. We denote this price by $\rho_k(p_{k-1}^s)$. If $p_k \leq \rho_k(p_{k-1}^s)$, the consumer decides to buy at p_k. Otherwise, he either buys at p_{k-1}^s (if this price is relatively small) or continues to search.

The proof is by induction starting at the last firm. The following lemma introduces the base of induction.

Lemma 3.1. *Let $F(p)$ be a distribution of prices. Then for $k = N-1$ the reservation price ρ_{N-1} is uniquely defined as a function of $p_{N-2}^s \in [\underline{p}, \overline{p}]$ by*

$$\rho_{N-1}(p_{N-2}^s) = \min\left(p_{N-2}^s + b,\ c + p_{N-2}^s + b - \int_{\underline{p}}^{p_{N-2}^s+b} F(p)dp,\ p_{N-1}^*\right)$$

where p_{N-1}^* satisfies the equation

$$p_{N-1}^* = c + E(p_N | p_N < p_{N-1}^* + b)F(p_{N-1}^* + b) + (1 - F(p_{N-1}^* + b))(p_{N-1}^* + b).$$

Moreover, if the consumer decides to continue searching, the continuation cost of search, defined as the additional net expected cost of continuing to search conditional on optimal behaviour after the search is made, is given by

[6] Alternatively, we may follow Stahl (1989) and assume that the first price quotation is give for free.

$$C_{N-1}(p^s_{N-1}) = c + p^s_{N-1} + b - \int_{\underline{p}}^{p^s_{N-1}+b} F(p)dp.$$

Proof. We consider the situation where $N-2$ firms have been sampled and the consumer has decided to make one more search. In this case, the consumer has three options: to buy now at the newly observed price p_{N-1}, to buy now at lowest price among the previously sampled prices p^s_{N-2}, or to continue searching. Knowing the value of $\min(p_{N-1}, p^s_{N-2})$, the last option gives an expected value of

$$\begin{aligned} v - c - E(p_N|p_N &< \min((p_{N-1}, p^s_{N-2}) + b)F(\min(p_{N-1}, p^s_{N-2}) + b) - \\ & (1 - F(\min(p_{N-1}, p^s_{N-2}) + b))(\min(p_{N-1}, p^s_{N-2}) + b). \end{aligned}$$

Let us first concentrate on the case where $p_{N-1} \geq p^s_{N-2}$. In this case the pay-off of continuing to search does not depend on p_{N-1} so that the reservation price is given by the point where the consumer is either (i) indifferent between buying now at p_{N-1} or buying at p^s_{N-2} (and paying the additional cost of going back b) or (ii) indifferent between buying now at p_{N-1} and continue searching. In the first case $\rho_{N-1}(p^s_{N-2}) = p^s_{N-2} + b$; in the second case

$$\begin{aligned} \rho_{N-1}(p^s_{N-2}) &= c + E(p_N|p_N < p^s_{N-2} + b)F(p^s_{N-2} + b) + \\ &+ (1 - F(p^s_{N-2} + b))(p^s_{N-2} + b) = \\ &= c + \int_{\underline{p}}^{p^s_{N-2}+b} pdF(p) + (1 - F(p^s_{N-2} + b))(p^s_{N-2} + b) = \\ &= c + p^s_{N-2} + b - \int_{\underline{p}}^{p^s_{N-2}+b} F(p)dp. \end{aligned}$$

It is easily seen that the first-order derivative of this expression w.r.t. p^s_{N-2} is positive and strictly smaller than 1. Moreover, it is easily seen that at $p^s_{N-2} = \underline{p}$, this expression equals $p^s_{N-2} + c > p^s_{N-2} + b$. Hence, by continuity, for small values of p^s_{N-2} the reservation price is given by $\rho_{N-1}(p^s_{N-2}) = p^s_{N-2} + b$. For larger values of p^s_{N-2} it is $\rho_{N-1}(p^s_{N-2}) = c + p^s_{N-2} + b - \int_{\underline{p}}^{p^s_{N-2}+b} F(p)dp$, at least when $\rho_{N-1}(p^s_{N-2})$ is still larger than p^s_{N-2}.

Let us next concentrate on the case where $p_{N-1} \leq p^s_{N-2}$. In this case the consumer will never go back to previously sampled prices and thus the reservation price is implicitly characterized by the price that solves

$$p_{N-1} = c + E(p_N|p_N < p_{N-1} + b)F(p_{N-1} + b) + (1 - F(p_{N-1} + b))(p_{N-1} + b).$$

Because of continuity at $p_{N-1} = p^s_{N-2}$, the fact that when $p^s_{N-2} < \rho_{N-1}(p^s_{N-2}) < p^s_{N-2} + b$, the derivative of the reservation price is strictly smaller than 1, and the fact that left differentiability holds at $p_{N-1} = p^s_{N-2}$, we should have that there is exactly one p_{N-1} that solves the above equation. This implies that in the region where $p_{N-1} \leq p^s_{N-2}$, $\rho_{N-1}(p^s_{N-2})$ is independent of p^s_{N-2}. Thus, also in this case $\rho_{N-1}(p^s_{N-2})$ is uniquely defined and non-decreasing in p^s_{N-2}.

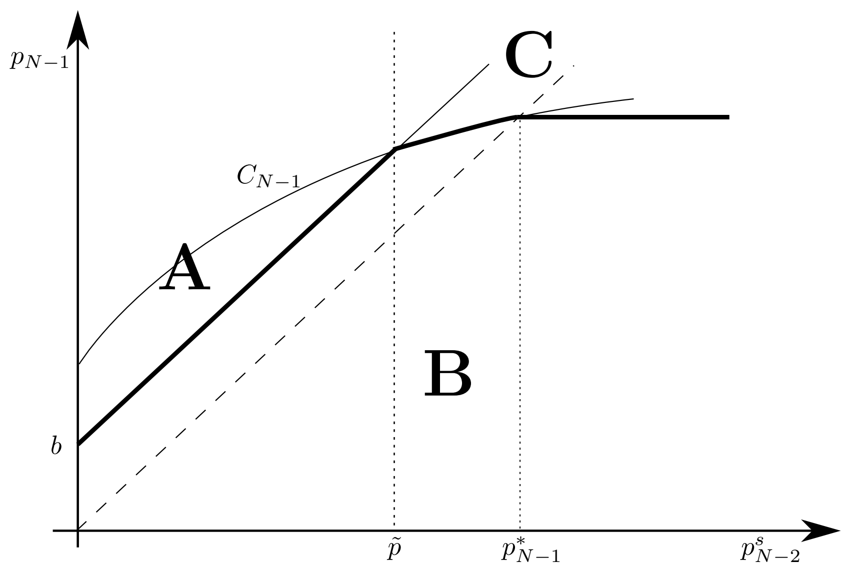
Once price p_{N-1} is observed the continuation costs of search are defined by

$$\begin{aligned} C_{N-1}(p^s_{N-1}) &= c + E(p_N | p_N < p^s_{N-1} + b) F(p^s_{N-1} + b) + \\ &+ (1 - F(p^s_{N-1} + b))(p^s_{N-1} + b) = \\ &= c + \int_{\underline{p}}^{p^s_{N-1}+b} p dF(p) + (1 - F(p^s_{N-1} + b))(p^s_{N-1} + b) = \\ &= c + p^s_{N-1} + b - \int_{\underline{p}}^{p^s_{N-1}+b} F(p) dp. \end{aligned}$$

■

The following picture illustrates the lemma.

Figure 1: Reservation Price ρ_{N-1} as a function of p^s_{N-2}



The reservation price as a function of p^s_{N-2} is presented by the bold curves. It is easy to see that this line consists of three parts:

- for $p^s_{N-2} < \tilde{p}$ the best alternative to buying at p_{N-1} is to go back to the lowest-priced firm in the sample so far. Thus, the reservation price is determined by $\rho_{N-1} = p^s_{N-2} + b$.

- for $\tilde{p} \leq p^s_{N-2} < p^*_{N-1}$ the option to continue searching is always preferred to the option of going back to the lowest-priced firm in the sample so far. Thus, the consumer's optimal choice is based on a comparison between the current price and the expected continuation costs of continuing to search;

- for the region $p^s_{N-2} \geq p^*_{N-1}$ the situation is similar to the previous case, except that the current price is always the lowest price in the sample so far, implying that the continuation cost does not depend on p^s_{N-2}. Therefore, the reservation price is independent of p^s_{N-2} in this case.

Along the bold curve the consumer is indifferent between buying now at the shop he is currently visiting or either continuing to search or to go back to the lowest-priced firm in the sample so far.

Since optimal search behaviour is completely determined by the pair (p_{N-1}, p^s_{N-2}) we can characterize it in the same figure. Indeed,

- in region A which is bounded from below by ρ_{N-1} and from the right by $\tilde{p}$, the consumer always goes back and buys at the lowest-priced firm in the sample so far;

- in region B which is bounded from above by the reservation price, the consumer always buys at the current shop;

- finally in region C, which is bounded from below by reservation price and for which $p^s_{N-2} > \tilde{p}$, the consumer always continues to search.

Now we want to show that on any step $1 < k < N-1$ the reservation price as a function of the lowest price in the sample is uniquely defined and has essentially the same shape as in Figure 3.1. The proof is by induction. Before we give the formal statement of the result and the proof, we have to provide a technical result that turns out to be useful in making the induction step. To this end, assume that y is a random variable with a continuous distribution function $F(y)$. Let for a given search and return cost c and b, the following function be defined

$$
\begin{aligned}
C^*(x) = {} & \mathbb{P}(y < \min(x+b, C(\min(x,y))))\cdot \\
& \cdot \mathbb{E}(y | y < \min(x+b, C(\min(x,y))))+ \\
& + \mathbb{P}(y \geq \min(x+b, C(\min(x,y))))\cdot \\
& \cdot \mathbb{E}(\min(x+b, C(\min(x,y))) | y > \min(x+b, C(\min(x,y)))) + c.
\end{aligned} \quad . \tag{3.3}
$$

The function $C^*(x)$ can be interpreted as a generalized continuation cost of additional search given continuation cost on the next step.

Lemma 3.2. *If* $0 \leq \frac{\partial C(z)}{\partial z} < 1$ *and* $C(\underline{y}) > b$*, where* $\underline{y}$ *is the lower bound of the support of* $F(y)$*, then for any* x *in the support of* $F(y)$ *except the lower bound,* $0 \leq \frac{\partial C^*(x)}{\partial x} < 1$*.*

Proof. Consider the following inequality: $y < \min(x+b, C(\min(x,y)))$. Since $\frac{\partial C(z)}{\partial z} < 1$, this inequality can be rewritten in the form $y < g(x) = \min(x+b, C(x), a)$, where a satisfies equation $a = C(a)$. It is clear that $\frac{\partial g(x)}{\partial x} \leq 1$.

Thus, we can rewrite C^* in the following form:

$$
\begin{aligned}
C^*(x) = {} & \mathbb{P}(y < g(x))\mathbb{E}[y | y < g(x)]+ \\
& + \mathbb{P}(y \geq g(x))\mathbb{E}[\min(x+b, C(\min(x,y))) | y \geq g(x)] + c
\end{aligned}
$$

Now note, that if $x \leq a$ then given that $y \geq g(x)$ we get $\min(x+b, C(\min(x,y)) = \min(x+b, C(x))$ which is just $g(x)$ for $x < a$. Then we get

$$
C^*(x) = \mathbb{P}(y < g(x))\mathbb{E}[y | y < g(x)] + \mathbb{P}(y \geq g(x))\mathbb{E}[g(x) | y \geq g(x)] + c
$$

and therefore

$$
\frac{\partial C^*(x)}{\partial x} = \frac{\partial}{\partial x}\left(\frac{F(g(x))}{F(g(x))}\int_{\underline{y}}^{g(x)} y f(y) dy + (1 - F(g(x)))g(x)\right) =
$$

$$
= [g(x)f(g(x)) + (1 - F(g(x))) - g(x)f(g(x))]\,\frac{\partial g(x)}{\partial x} = [1 - F(g(x))]\frac{\partial g(x)}{\partial x} < 1.
$$

It is also clear that the derivative is non-negative.

Another case is if $x > a$. Here, given $y \geq g(x)$we get $\min(x+b, C(\min(x,y)) = C(\min(x,y))$. Then we get

$$
C^*(x) = \mathbb{P}(y < g(x))\mathbb{E}[y | y < g(x)] + \mathbb{P}(y \geq g(x))\mathbb{E}[C(\min(x,y)) | y \geq g(x)] + c
$$

Or

$$C^*(x) = \int_{\underline{y}}^{g(x)} yf(y)dy + \int_{g(x)}^{x} C(y)f(y)d(y) + \int_{x}^{\infty} C(x)f(y)d(y) + c$$

Now, by taking derivative with respect to x we get:

$$\frac{\partial C^*(x)}{\partial x} = [g(x)f(g(x)) - C(g(x))f(g(x))]\frac{\partial g(x)}{\partial x} + \frac{\partial C(x)}{\partial x}(1 - F(x))$$

Now note, that for $x > a$ we have $g(x) = a$ and therefore $C(g(x)) = a$. Thus,

$$\frac{\partial C^*(x)}{\partial x} = \frac{\partial C(x)}{\partial x}(1 - F(x)) < 1$$

which completes the proof since $\frac{\partial C(x)}{\partial x} \geq 0, 1 - F(x) \geq 0$.

■

Given these two lemmas, we are now ready to state and prove the main result of the paper. The result says that the reservation price as a function of p^s_{k-1} is well-defined and unique and a monotone function of p^s_{k-1}. In later results, we prove that the time- and history-dependency of these reservation prices cannot be neglected, unlike the case of costless recall.

Theorem 3.3. *The reservation price $\rho_k(p^s_{k-1})$ is uniquely defined for any k and any p^s_{k-1} from the support of $F(p)$. Moreover, the time- and history-dependent reservation prices $\rho_k(p^s_{k-1})$ are nondecreasing in p^s_{k-1}.*

Proof. Let $C_k(p^s_k)$ be a continuation cost of additional search on the k-th step given realizations of (p^s_{k-1}, p_k) (recall that $p^s_k = \min(p^s_{k-1}, p_k)$). Then, given the optimal search behaviour of the consumer, $C_k(p^s_k)$ is the expected payoff of two events: either the consumer buys at the next firm to be searched or he continues to search onwards or goes back. Thus, we get that

$$\begin{aligned} C_k(p^s_k) = c &+ \mathbb{P}(p_{k+1} < \min(p^s_k + b, C_{k+1}(p^s_{k+1})))\cdot \\ &\cdot \mathbb{E}(p_{k+1} | p_{k+1} < \min(p^s_k + b, C_{k+1}(p^s_{k+1}))) + \\ &+ \mathbb{P}(p_{k+1} \geq \min(p^s_k + b, C_{k+1}(p^s_{k+1})))\cdot \\ &\cdot \mathbb{E}(\min(p^s_k + b, C_{k+1}(p^s_{k+1})) | p_{k+1} \geq \min(p^s_k + b, C_{k+1}(p^s_{k+1}))) \end{aligned} .$$

We prove that $0 \leq \frac{\partial C_k(p_k^s)}{\partial p_k^s} < 1$. The proof is by backward induction. From lemma 3.1 it is easy to see that $0 \leq \frac{\partial C_{N-1}(p_{N-1}^s)}{\partial p_{N-1}^s} < 1$, thus the base of induction is proven. We will now argue that this property also holds for any other period. For proving the induction step we can apply lemma 3.2 by substituting in the equation (3.3) $x = p_k^s$, $y = p_{k+1}$, $C^*(x) = C_k(p_k^s)$, $C(\min(x,y)) = C_{k+1}(p_{k+1}^s)$. Therefore, from $0 \leq \frac{\partial C_{k+1}(p_{k+1}^s)}{\partial p_{k+1}^s} < 1$ it follows that $0 \leq \frac{\partial C_k(p_k^s)}{\partial p_k^s} < 1$ and thus, by induction it follows that for any k $0 \leq \frac{\partial C_k(p_k^s)}{\partial p_k^s} < 1$.

The rest of the proof is straightforward. If $p_k \geq p_{k-1}^s$, then $\rho_k(p_{k-1}^s) = \min(p_{k-1}^s + b, C_k(p_{k-1}^s))$, which is well-defined and unique. Moreover, it is non-decreasing in p_{k-1}^s since both $p_{k-1}^s + b$ and $C_k(p_k^s)$ are non-decreasing in p_{k-1}^s. If, on the other hand, $p_k < p_{k-1}^s$, then the reservation price is a solution to the equation $p_k = C_k(p_k)$, which is unique since $C_k(p_k)$ has a slope strictly smaller than 1. In this case, the reservation price does not depend on p_{k-1}^s and is thus nondecreasing in p_{k-1}^s.

■

The proof of the theorem basically shows that the function $\rho_{k+1}(p_k^s)$ is defined over three separate intervals and essentially looks like the reservation price for the last step (see Figure 3.1). When p_{k-1}^s is relatively small $\rho_k(p_{k-1}^s) = p_{k-1}^s + b$. Then for intermediate values of p_{k-1}^s, $\rho_k(p_{k-1}^s) = C_k(p_{k-1}^s)$ and for higher values $\rho_k(p_{k-1}^s)$ is independent of p_{k-1}^s. One can thus, define the price $\tilde{p}_k$ as the price such that the consumer is indifferent between going back to the shop charging this price and continuing to search, i.e., $\tilde{p}_k + b = C_k(\tilde{p}_k)$.

We are now in the position to prove some special properties of the reservation price function. To this end, define ρ^{pr} as the reservation price under perfect recall, i.e., as noted, e.g., by Stahl (1989),

$$c = \int_{\underline{p}}^{\rho^{pr}} F(p)dp.$$

By considering the limiting case where the cost of recall is zero we provide more insight into the reason why the cases of perfect recall and costly recall are so different from one another. Moreover, the reservation price under perfect recall turns out to play an important role in further characterizing the optimal search behaviour under costly recall.

Proposition 3.4. [7] *Let $b = 0$. Then for any k the reservation price is defined*

[7] As this fact is intuitively obvious the proof is available upon request.

by:

$$\rho_k = \min(p^s_{k-1}, \rho^{pr}).$$

Under perfect recall, the search rule is stationary, but (interestingly) slightly different from what is commonly thought as in any period the reservation price is still dependent on the lowest of previously sampled prices. When the current price is smaller than any of the previously sampled prices, then the consumer simply compares the current price with ρ^{pr} and decides whether or not to buy. If the current price is larger, the consumer simply forgets about the current price. Because of stationarity, previously sampled prices are in a full model including price setting behaviour of the firms, irrelevant. Either these previously sampled prices are below ρ^{pr}, but then the consumer simply does not continue to search, or they are above ρ^{pr}, but then the consumer never considers buying there unless he has visited all the stores and knows for sure that there are no lower prices in the sample.[8]

To further characterize the optimal search rule, under costly recall we show that the price $\tilde{p}_k$ is intimately related to the price ρ^{pr} under perfect recall.

Proposition 3.5. *For all k, $\tilde{p}_k = \tilde{p} = \rho^{pr} - b$.*

Proof. Note that the price $\tilde{p}_k$ is defined such that after visiting k stores, the consumer is indifferent between continuing searching and going back to the lowest-priced store in the sample so far. Therefore, at $\tilde{p}_k$ the reservation price $\rho_k(\tilde{p}_k) = \tilde{p}_k + b$. The expected costs of continuing to search are:

$$c + F(\tilde{p}_k + b)\mathbb{E}(p_{k+1}|p_{k+1} < \tilde{p}_k + b) + (1 - F(\tilde{p}_k + b))(\tilde{p}_k + b)$$

By equating it to the best current option $(\tilde{p}_k + b)$ and some simplifications we have also used in previous proofs, we get

$$c = \int_{\underline{p}}^{\tilde{p}_k + b} F(p)dp.$$

It follows therefore that $\tilde{p}_k$ does not depend on k and that (by comparing this equation to the definition of ρ^{pr} it is actually just equal to $\rho^{pr} - b$).

■

Next, we show that rational consumers never use the option of going back to previously sampled stores, unless they have visited every store available.

[8] However, in equilibrium even this could not be the case with $b = 0$ as then the traditional argument kicks in that no firm wants to charge the highest price above ρ^{pr} as no consumer will ever buy at this price, implying that no firm will want to choose a price above ρ^{pr}.

Corollary 3.6. *Assume the consumer behaved optimally on all steps $1 \leq k \leq K$. Then if $K < N$, it is never optimal for this consumer to go back.*

Proof. Note, that the option of going back is preferred to continue searching or stopping only if $p^s_K < \tilde{p}$. On the first step any price $p_1 \leq \rho^{pr}$ would be accepted immediately. So, if the consumer continued his search it must be the case that $p_1 > \rho^{pr}$. Given $p^s_1 > \rho^{pr}$ on the second step any price $p_2 \leq \rho^{pr}$ also would be accepted immediately. Thus, if consumer continued his search it must be the case that $p_2 > \rho^{pr}$. Then by induction if customer reached step K it must be the case that for any $1 \leq k \leq K$ it was the case that $p_k > \rho^{pr}$. Therefore $p^s_K > \rho^{pr} > \tilde{p}$ and it is never optimal to go back, except possibly at the last step.

■

Next, we show that reservation prices are non-decreasing over time. In particular, if a price smaller than $\tilde{p} = \rho^{pr} - b$ is sampled before, then the reservation price is simply $\rho_k(p^s_{k-1}) = p^s_{k-1} + b$ and therefore if $p^s_k = p^s_{k-1}$, then $\rho_{k+1}(p^s_k) = \rho_k(p^s_{k-1})$. However, if a price strictly larger than $\tilde{p} = \rho^{pr} - b$ is the lowest price in the sample so far, then $\rho_{k+1}(p^s_k) > \rho_k(p^s_{k-1})$. Thus, under costly recall reservation prices are essentially nonstationary.

Proposition 3.7. *If $p^s_k = p^s_{k-1}$, then $\rho_{k+1}(p^s_k) \geq \rho_k(p^s_{k-1})$, i.e., reservation prices are non-decreasing over time. Moreover, $\rho_{k+1}(p^s_k) > \rho_k(p^s_{k-1})$ for all p^s_k and p^s_{k-1} such that $p^s_k = p^s_{k-1} > \tilde{p} = \rho^{pr} - b$.*

Proof. Note, that the reservation price essentially represents the cost of the next-best available alternative to buying now at the shop the consumer is currently visiting. If the next-best available alternative is to go back to the lowest-priced firm in the sample before visiting this shop, i.e., $p^s_{k-1} < \tilde{p}$ the reservation price is simply independent of the periods, i.e., $\rho_{k+1}(p^s_{k-1}) = \rho_k(p^s_{k-1}) = p^s_{k-1} + b$.

Now consider the case where the next-best available alternative is to continue searching. Let $\{\rho_k(p^s_{k-1})\}_{k=1}^N$ be the sequence of the reservation price functions. Consider the following suboptimal strategy. If on step k the consumer makes a decision to visit one more firm he either buys at the firm he visits at step $k+1$ or continues his search but *forgets* about this firm later on (thus, he never comes back to that firm). Let us denote a reservation price under this suboptimal strategy by $\rho'_k(p^s_{k-1})$. Then $\rho_k(p^s_{k-1}) \leq \rho'_k(p^s_{k-1})$. On the other hand for any $p^s_{k-1} > \tilde{p}$ we get

$$
\begin{aligned}
\rho'_k(p^s_{k-1}) &= F(\rho_{k+1}(p^s_{k-1}))\mathbb{E}(p_{k+1}|p_{k+1} < \rho_{k+1}(p^s_{k-1}) + \\
&+ (1 - F(\rho_{k+1}(p^s_{k-1})))\rho_{k+1}(p^s_{k-1}) < \rho_{k+1}(p^s_{k-1})
\end{aligned}
$$

which completes the proof.

■

We finally consider the limiting case (of perfect competition) where there are potentially infinitely many prices to sample. As the time dependency of the reservation prices disappears due to the fact that now the cost of continuing to search is independent of time, i.e., $\rho_k(p^s_{k-1}) = \rho_{k+1}(p^s_k)$. For prices below $\tilde{p}$, we knew already that this equality holds. Interestingly, with infinitely many firms and previously sampled prices above $\tilde{p}$, the reservation prices becomes independent of previously sampled prices and equal to the reservation price under perfect recall. Thus, the cost of going back to previously sampled firms does not play an important role under perfect competition.

Proposition 3.8. *Let $K \in \mathbb{N}$. Then for any $p \geq \tilde{p}$ $\lim_{N\to\infty} \rho_K(p) = \rho^{pr}$.*

Proof. Note, that for any $p \geq \tilde{p}$, $C_{N-1}(p)$ is fixed and does not depend on N. On the other hand for any $p \geq \tilde{p}$ we have

$$
\begin{aligned}
C_k(p) &= F(\rho_{k+1}(p))\mathbb{E}(p_{k+1}|p_{k+1}\rho_{k+1}(p)) + \\
&+ (1 - F(\rho_{k+1}(p))\mathbb{E}(C_{k+1}(p_{k+1})|p_{k+1} < \rho_{k+1}(p)) \leq \\
&\leq C'_k(p) = F(\rho_{k+1}(p))\mathbb{E}(p_{k+1}|p_{k+1}\rho_{k+1}(p)) + (1 - F(\rho_{k+1}(p))\rho_{k+1}(p)
\end{aligned}
$$

Note, that $C'_k(p)$ can be rewritten in the form:

$$C'_k(p) = \rho_{k+1}(p) + c - \int_{\underline{p}}^{\rho_{k+1}(p)} F(p)dp$$

Therefore

$$\frac{\partial C'_k(p)}{\partial p} = \frac{\partial \rho_{k+1}(p)}{\partial p}(1 - F(\rho_{k+1}(p))) \leq \frac{\partial C'_{k+1}(p)}{\partial p}(1 - F(\rho_{k+1}(p)))$$

Then

$$\frac{\partial C'_K(p)}{\partial p} \leq \prod_{i=K+1}^{N-K} \frac{\partial C'_{i+1}(p)}{\partial p}(1 - \rho_{i+1}(p)))$$

As $1 - F(\rho_{i+1}(p)) < 1$ for any $p > \tilde{p}$ and $i > K$ (note, that $\rho_{i+1}(p) < \rho_{i+2}(p) \Rightarrow 1 - F(\rho_{i+1}(p)) > 1 - F(\rho_{i+2}(p)))$ we get

$$\lim_{N \to \infty} \frac{\partial C'_K(p)}{\partial p} = 0.$$

.

Now note that from proposition 3.5 it follows that $\rho_K(\tilde{p}) = \rho^{pr}$ and therefore $C_K(\tilde{p}) = \rho^{pr}$. Therefore, since $C'_K(p)$ is a continuous function we get that for any $p \geq \tilde{p}$,

$$\lim_{N \to \infty} C'_K(p) = \rho^{pr}.$$

Therefore

$$\lim_{N \to \infty} C_K(p) = \rho^{pr}.$$

■

Thus, under perfect competition the reservation price under costly recall is exactly identical to the case where consumers have perfect recall.

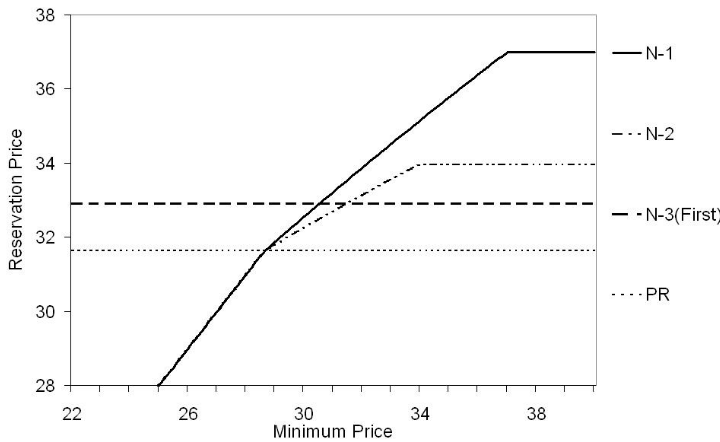
4 Example

In the previous section, we have provided a general characterization of the time- and history-dependency of the reservation price. In this Section we provide an example to illustrate the features of these reservation prices. The example clearly shows that it can be rational to accept a price in a future period even if a lower price has been observed in the past.

Consider the uniform distribution of prices on $[0, 100]$. Assume there are 4 firms in the market, search costs c are equal to 5 and the costs of going back to a previously sampled firm b equals 3. The reservation prices after visiting no, one and two firms as well as the reservation price under perfect recall are presented in Figure 2. In this case, the reservation price under perfect recall equals approximately 31.62, while the reservation price before visiting any shop under costly recall equals 32.90. Thus, if a consumer faces, say, a price of 33 in the first period he decides to continue searching. From Figure 2 it is clear, however, that if the third price the consumer encounters is say 34 it is optimal for him to stop.

The figure also illustrates most of the results we proved in the previous section. In particular, it is easy to observe that:

Figure 2: Simulation Results for Uniform Distribution .



Parameters of simulation: $N = 4, a = 100, b = 3, c = 5$.

- all reservation price functions are non-decreasing in p_k^s (Theorem 3.3);
- all the reservation price functions have a kink at the same $\tilde{p} = \rho^{pr} - b \approx 28.62$ (Proposition 3.5);
- the sequence of reservation prices is non-decreasing in the number of firms left, and strictly increasing for all prices above $\tilde{p}$ (Proposition 3.7);

5 Conclusions

In this paper, we built a consumer search model where we explicitly model the cost of going back to stores already searched. We show that in general the optimal search rule under costly recall is very different from the optimal search rule under perfect recall. Under costly recall, the optimal search behaviour is non-stationary and, moreover, the reservation price is not independent of previously sampled prices. Consequently, it may happen that the optimal search strategy tells consumers to reject relatively low prices early on in the search process and accept higher prices later on. Stationarity is obtained only in the special case of perfect competition where there are infinitely many firms.

Future work should incorporate the optimal search rule under costly recall characterized here, in a full consumer search model where the pricing behaviour of firms is explicitly modelled. Due to the assumption of costless recall, consumers in most search models actually do not search very much in any symmetric equilibrium. This is because in a symmetric equilibrium where firms set prices above the reservation price under free recall, the firm charging the highest price in the market does not make any sales as there is always a firm present with a lower price and consumers will continue searching until they find this price. So, no firm would want to charge the highest price above the reservation price and, therefore, in equilibrium no prices above the reservation price will be charged. Future research should inquire whether this result continues to hold under costly recall. Under costly recall, it may well be possible that for example in a duopoly market all firms charging with some positive probability prices above the reservation price (for the first price observation) is part of an equilibrium as by doing so these firms still have two potential sources of revenue: (i) from consumers who first searched firm A, next search firm B and want to stop there even if B charges higher price than A, due to the costs of going back to firm A and (ii) from consumers who first search firm A, after that B and go back to firm A if it had sufficiently lower prices.

We have not analyzed a full model including price setting behaviour of firms in this paper as this would require a choice of a specific market set-up. Here, we have characterized the optimal search rule under costly recall in a general form that could be applied to any specific market environment.

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