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Insurance Sector Risk

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Abstract

We model and measure simultaneous large losses of the market value of insurers to understand the impact of shocks on the insurance sector. The downside risk of insurers is explicitly modelled by common and idiosyncratic risk factors. Since reinsurance is important for the capacity of insurers, we measure risk dependence among European insurers and reinsurers. The results point to a relatively low insurance sector wide risk. Dependence among insurers is higher than among reinsurers.

1 Introduction

The financial stability of the global insurance sector was a major concern for regulators following the losses of the September 2001 WTC collapse. At the same time, the investment income arising out of the assets of insurers declined, due to low interest rates and a declining stock market during the recession at the time. In this research we study the downside risk dependence of multiple insurers. We measure similarities in sector wide risk exposure using daily stock price returns of European insurers and reinsurers.

We provide an explanation for a similar exposure to very large losses, based on the idea that multiple insurers carry similar risks. Insurance companies can e.g. be exposed to similar insurance risk on the liability side, due

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to reinsurance practices which spread the same risk across companies. This common exposure can also arise because of an exposure to similar macroeconomic variables, like interest rates or inflation on the asset side. We model and estimate the effect of risk diversification on downside risk for individual companies and for the sector as a whole.

Risk diversification may reduce the risk of individual insurance companies, but the risk profiles of multiple insurers becomes more similar due to this diversification. Hence, systemic risk may increase due to risk sharing. For the design of optimal regulation, it matters if regulators have to deal with sector wide risk or firm specific risk. When firms are exposed to the same risks, during a crisis all insurers realize losses on either their assets or liabilities. The capacity of the insurance sector can therefore be at risk and may need to be enhanced. Moreover there is an increasing interest in the effects of a loss of insurance capacity on real economic activity. A better understanding of sector wide downside risk can contribute to this impact assessment.

Since insurers want to limit and diversify their risk exposure they protect themselves by reinsurance contracts. Reinsurers provide insurers protection against major losses and the bankruptcy of a reinsurer might expose insurers to unforseen losses. Reinsurance can be provided by both reinsurance companies, by other insurers and by the capital markets. It is the primary responsibility of insurers to have a sound reinsurance risk management strategy. Regulators are interested in the mutual relations between insurers and reinsurers. The Financial Stability Forum (2002) e.g. is concerned about the impact of the collapse of a major reinsurer on insurance companies. We measure the degree of such dependence between insurers and reinsurers.

The banking sector is also exposed to problems within the insurance sector. Insurers may sell credit protection to banks, via credit default swaps. In practice the reverse occured more frequently and contributed to the woes of the insurance sector during the last recession, while the banking sector was more or less unaffected. The dependence between banks and insurers is investigated in detail in Slijkerman e.a. (2005). Systemic concerns for the banking sector have a higher relevance than for the insurace sector. Bank failures have a public externality because of the maintenance of the payment system by banks. The specificity of the deposit contract also creates a (negative) public externality, due to the possibility of a drain of liquidity due to a bank failure. The stability of the reinsurance sector is nevertheless of a public concern, since the bankruptcy of a major reinsurer may reduce the capacity of the insurance sector and therefore have real consequences. Geluk and De Vries (2005) analyze the asymptotic dependence among reinsurers. This interdependency not withstanding, the insurance sector is less fragile and does not have the same importance to the real economy as the banking sector.

The OECD (2004) writes that the insurance sector has an important role to play in the real economy. For a lot of economic activities insurance is mandatory or necessary to contain the investment risk for economic activities. Airlines, for example, have to insure their airplanes and construction companies want to insure their property. A possible shortage of the capacity to provide insurance is therefore a concern of regulators. In this research we do not explicitly quantify the impact of insufficient insurance cover on real economic activity, but try to quantify the possibility of sector wide losses to the insurance sector. The consequences of a loss in insurance cover is briefly discussed in the following.

Before the collapse of HIH Insurance in March 2001, it was the second biggest insurer in Australia. According to Buchanan et al.(2003), the collapse of HIH made reinsurance premiums rise globally. Moreover, housing construction in Australia was affected, since builders where deprived of insurance cover at HIH Insurance and had to find replacement coverage. (See Vaughan, 2004) Approximately half of the doctors in Australia lost malpractice insurance and thousands of small businesses lost liability coverage.

The WTC attack also caused problems with insurance cover for e.g. property damage, aviation liability, business interruption and life liability. This had the strongest impact on aviation and transport, but also on manufacturing, energy, real estate and construction. The OECD (2004) reports that according to the Bond Market Association, 10 per cent of the commercial mortgage-backed securities market has been suspended or cancelled due to issues of terrorism insurance. These examples highlight the importance of the insurance sector for the broader economy.

In this research we study the tail dependence between stock returns of insurers and reinsurers and investigate the extent of sector wide downside risk. We explicitly take into account that the distribution of stock prices for insurers is fat tailed and model the relative importance of market risk. The model helps to understand the impact of adverse shocks which negatively affect multiple insurers. Finally we measure the breadth of downside risk in the insurance sector and in the reinsurance sector. If the downside risk of a loss in market value in the two sectors is the same, this points to similarities in the risks exposure, possibly resulting from similar assets or similar liabilities holdings across insurance firms.

If sector risk is important, this shows as the high mutual dependence between companies. During crisis, multiple insurers may realise losses on their assets or liabilities. Losses in insurance capacity during crises can cause insufficient supply of insurance cover. This lack of insurance capacity may have an adverse impact on economic activity. It is therefore important that new capital can easily enter the insurance sector. Capacity can, for example, be enhanced by increasing the use of the capital markets as a source of insurance cover, or through institutional arrangements in which governments provide part of the insurance cover, when private insurance cover is not available.

In the remainder of the paper we first describe the EU insurance sector. Next, we give an intuitive explanation for the mutual dependence between insurance companies and subsequently model this dependence explicitly. Finally, we estimate the degree of dependence between the different sectors and draw conclusions from the empirical investigation.

2 EU insurance

The European insurance market is the second largest in the world, after the US and accounts for 30% of world premium income. Moreover, the two largest reinsurers (Swiss Re and Munich Re) are European based and we therefore deem it interesting to take a European perspective. The market share of the largest companies is increasing, as the result of consolidation. The five largest insurers in EU countries hold on average close to 50% of life insurance income and close to 40% of non-life insurance income, according to the CEA (2004). Most insurance companies offer both life and non-life products. In 1991 collected premiums for life and non-life were balanced, but nowadays the life insurance sector is relatively larger. Most European companies are mixed insurers and offer both life and non-life insurance. Some companies like Aegon, ING, Zurich Financial Services and Prudential have a sizeable US business, or have a large banking business, like ING and Allianz. This fits in the trend of the emergence of insurance companies with a global presence and insurers with multiple business lines, to make use of economies of scale and scope. The insurance sector is growing due to new products and demand for additional pension products. The expectation is that the life insurance market in most countries will grow, because most countries reform their pension systems. Notably the UK and the Netherlands already have large life insurance markets. It is also likely that there will be more cross border business in the EU, due to the integration of the financial markets because of the introduction of the euro and an increasing harmonization of regulation.

Before September 11 insurance markets were characterized by low prices for reinsurance and excess capacity. Investment income was driving the results of insurers. In 2001 the insurance sector was hit on both the asset side, due to falling stock prices, and the liability side, due to the costs of September 11. According to Swiss Re (2005) European insurers made larger investments in equity than American insurers, but the investments in equity by insurers were lower in 2004 than in 1999. This indicates that insurers have become less willing to take investment risk on their balance sheets.

The importance of the capital markets for the provision of insurance is growing. In some cases it may be difficult for firms to buy insurance cover, since it is hard for insurers to estimate the expected losses. This may be due to a lack of information on the number of accidents and the costs which are incurred by the firms, but also to moral hazard, leading to higher claims than expected. With the help of the capital markets large firms can self insure their risks with the use of insurance captives. Firms pay premiums to a seperate legal entity, owned by the firm (the captive), and create a financial buffer for insurance losses. Capital markets or insurers may provide liquidity if losses occur before the firms made sufficient savings.

Besides the capital market, governments can also help to alleviate the impact of (natural) catastrophes. In a lot of countries governments recognized the difficulties to obtain insurance cover for certain types of risks. They have set up public private insurance schemes to insure e.g. natural catastrophes and terrorism risk, as is described by the OECD (2004).

The EU insurance sector is regulated with seperate directives for life and non-life insurance. The most important elements of this regulation are the requirements on the technical provisions, investment rules, solvency requirements, accounting rules and criteria for home country control and the provision to provide cross border business. There is a proposal for new regulation for the EU reinsurance sector. The Solvency II project aims to modernize insurance regulation, taking into account actual risks of insurers in the calculation of solvency requirements, similar to the revamp of the Basle accords for banks. However, insurance sector regulation does not explicitly take into account the need for sufficient insurance capacity during crisis times. In the following we explain why this capacity may be at risk.

3 Risk diversification and financial fragility

In this section we offer an explanation for downside risk dependence between insurers and focus on a reduced form approach to determine this downside risk. Following the Efficient Market Hypothesis, the stock price of insurers should reflect the value of companies. Changes in stock prices are therefore the result of changes in value of assets and liabilities. In a competitive environment it stands to reason that firms with similar assets and liabilities, make similar profits and show similar returns. At the level of our study, these returns are random. Therefore, the degree of dependence one finds between the different returns, depends on the specific nature of the shocks and the economic structure of the assets and liabilities. A presumption of normality gives e.g. quite a different result than the presumption that the returns of assets and liabilities follow a Student-t distribution. But the specific network structure of the insurance sector is also a contributing factor.

3.1 Explaining dependence

We first provide the intuition for the existence of dependence among insurers. The mutual dependence among insurance companies originates from a similar exposure to similar assets and to similar liabilities. Swiss Re (2003) gives an description of the main assets of insurers. These include the following items: shares, investments in bonds and investment funds, real estate and technical reserves held by reinsurers. If companies have similar assets, they will have a similar riks profile. If insurance companies invest e.g. in the same stocks and bonds or (credit) derivates, they are exposed to the same shocks which adversely affect the value of investments. However, little is known about the exact assets insurers invest in. This may indicate a lack of transparency in the European insurance sector. Since insurers started to act as sellers of credit protection, this gained special attention of supervisors and the issue is studied extensively by the Joint Forum (2005).

Insurers do not only invest in similar assets, but may also participate in similar liabilites, insurance risk in particular. It is possible for insurers to provide reinsurance and in this way participate (proportionally) in the insurance portfolio of other insurers. Another important development is the securitisation of insurance risk (IAIS, 2003). Insurers can transfer their insurance portfolio and expected premium income to a legal entity which manages these liabilities. In this way other investors can participate in the same insurance risks. Catastrophe bonds are an example of insurance risk which is transfered to the capital market. Another possible source of risk is the long term interest rate. A particular problem for life insurers is that their profitability depends for an important part on the long term interest rate developments. A decreasing long term interest rates poses problems for the non-hedged liabilities. A low long term interest rate environment poses problems for new production. When the level of the short term interest rate is close to the long term interest rate, the benefits for customers of long term insurance contracts over short term savings are limited. Since the interest rate risk might be similar for all life insurers, we expect that life insurers are affected by an interest rate move in a similar way. On the contrary, if we find that there is a low dependence between insurers, this indicates that the firms invest in different assets and liabilities.

Other risks which may be similar for insurers are the risk of changes in the legal and regulatory environment and fraud. New jurisprudence can increase the unexpected liabilities for insurers and new regulation may increase costs on a sector wide basis.

By modelling the dependence among firms explicitly, we obtain new insights on the degree of similarities in downside risk. Before we present the model and the estimation results, we elaborate on the univariate firm risk. The distributions of the stock price returns of individual insurers exhibit fat-tails. Since we study the downside risk of insurers, we take this characteristic explicitly into account when modelling and estimating dependence among firms.

3.2 Heavy tails and dependence

It is a stylized fact that stock returns are heavy tailed, rather than exhibiting an exponential type tail as under normality. This is e.g. extensively documented by Janssen and De Vries (1991). Moreover there is considerable evidence that the risk exposure of non-life insurers is even heavier tailed, see e.g. Embrechts et al. (1997). We will first elaborate on the heavy tail characteristics of the univariate return series. We do so since the theory for univariate heavy tails is the basis for our multivariate modelling.

Extreme Value Theory (EVT) studies the limit distribution of the maxima or minima of return series. These limit distributions are informative about the tail shape of the underlying distribution. With the help of this limit distibution, one can study the frequency of extreme losses without imposing a particular distribution a priori (like a Student-t or Pareto). Our approach is therefore semi-parametric (as only the tail area of the distrubution is parametrized). For ease of presentation, we work with positive variables and take the negative of the minima. We assume that X_i is an independent and identically distributed random variable with cumulative distribution function $F(x)$. This variable exhibits heavy tails if $F(x)$ far into the tails has a first order term identical to the Pareto distribution, i.e.

$$F(x) = 1 - x^{-\alpha} L(x) \text{ as } x \rightarrow \infty,$$

where $L(x)$ is a slowly varying function such that

$$\underset{t \longrightarrow \infty}{Lim} \frac{L(tx)}{L(t)} = 1, x > 0.$$

It can be shown that the two previous conditions are equivalent to

$$\underset{t \longrightarrow \infty}{Lim} \frac{1 - F(tx)}{1 - F(t)} = x^{-\alpha},\ \alpha > 0,\ t > 0.$$

The coefficient α is known as the tail index and gives the number of bounded moments of the distribution. When a distribution has finite endpoints or exponentially decaying tails (like the normal and lognormal distributions), it does not fit the property of regular variation and all moments are bounded. Because of the Pareto characteristics of the tails of the empirical return distribution, our theoretical model is based on the Pareto law. Before we model dependence, we will first present the measure of dependence which we will use.

3.3 A measure of dependence

The most frequently used measure of dependence is the correlation measure. However, regulators are interested in the likelihood of losses and the correlation measure does not provide information on probabilities, without knowledge of the marginal distributions. The correlation measure is only an intermediate step in the calculation of probabilities given a specific multivariate distribution. If two variables e.g. follow a bivariate normal distribution, their joint behavior can be characterized by using the correlation measure. One disadvantage of the correlation measure is that there can be dependence in the data, while the correlation is zero (see Slijkerman et al., 2005). Forbes and Rigobon (2002) moreover, show that changes in the correlation measure over time are difficult to interpret if the variance of variables is not constant over time. The correlation measure is therefore not a very informative variable itself.

We bypass the correlation measure and directly study a measure which is based on the probability of multiple shocks to the financial system. Our indicator is a conditional probability measure. Regulators and risk managers are concerned with a simultaneous loss at multiple insurers, given the losses at one insurer. More specifically, suppose a regulator wants to know the probability that $F_1 > t$, given that $F_2 > t$ and the probability that $F_2 > t$ given that $F_1 > t$, where F_1 and F_2 are the stochastic loss returns and t is the common high loss level. Since we are interested in a crash of an insurer given the crash of another insurer and vice versa, we will condition on either event. Let κ denote the number of insurers which crash. We propose to use the failure measure of Xin (1992) as the measure of systemic risk. In two dimensions it reads

$$E[\kappa|\kappa \geq 1] = \frac{P(F_1 > s) + P(F_2 > s)}{1 - P(F_1 \leq s, F_2 \leq s)}. \tag{1}$$

The failure measure is the conditional expectation of the number of insurance companies that crash, given that there is at least one crash. Hartmann et. al. (2004) give a further motivation for this measure. Note that

$$E[\kappa|\kappa \geq 1] - 1 = \frac{P(F_1 > t, F_2 > t)}{1 - P(F_1 \leq s, F_2 \leq s)} \tag{2}$$

is the conditional probability that both firms fail, given that there is a failure of at least one of the firms. We will use either interpretation, depending on the context.

Unless one is willing to make further assumptions, as in the options based distance to default literature, it is impossible to pin down the exact level at which a firm fails, or at which supervisors consider the institution financially unsound. For this reason we do take limits and consider

$$\lim_{t \longrightarrow \infty} E[\kappa|\kappa \geq 1].$$

Extreme value theory shows that even though the measure is evaluated in the limit, it nevertheless provides a usefull benchmark for the dependency at high but finite levels of t. We also like to note that the measure can be easily adapted in case failure levels at the companies are different. In that case the measure is evaluated along a non 45^o line.

4 Modelling dependence

By making different assumptions regarding the distribution of the returns of the firms, we can study how the conditional failure probability (1) changes as a result of an increasing exposure of firms to common risks. We present a model of tail dependence, assuming that the returns of insurers follow a heavy tailed multivariate distribution. If the returns follow a process with innovations which have a distribution with exponentially declining tails, large shocks occur with very low frequency and a large loss of multiple insurers is highly unlikely. However, if the stochastic process of losses is fat tailed, large losses may occur more often and may be difficult to diversify away. De Vries (2005) provides a detailed analysis of the differences between exponentially distributed returns and returns which follow a fat tailed distribution.

We already provided potential explanations for simultaneous losses at insurance companies. Because of a similar exposure to some risks, we suppose that all insurers carry sector risk. We do not model the individual sources of this risk explicitly, but investigate a reduced form model, which is basic to many models in finance. The returns of individual firms are partly driven by the stochastic variable A, which captures sector risks, both on the asset side as well as on the liability side. Different factors contribute to this sector risk. The variable A is the sum of possible common shocks, such as changes in interest rates and the exchange rate and common insurance risk exposures.

The returns by firms are also driven by a specific firm factor I_i, which is not related to the sector risk. Our model is therefore related to the factor model of Ross (1976). We assume total firm risk to be the sum of industry specific risk, A and firm specific risk, I_i. Firm specific risk arises out of losses on assets and liabilities for firm i, which are not incurred by other firms in the industry. For now, we assume that the downside risk of the common stochastic variable A and firm specific variable I_i are independently Pareto distributed with unitary scale

$$P(A > t) = P(I_i > t) = t^{-\alpha},\ t \in [1, \infty), \tag{3}$$

where t is the loss quantile of interest. In the following we first investigate how the dependence between two firms changes if we change the relative importance of the common component A. We thus analyze the effect of an increase in the importance of common factors. Secondly, a model with multiple firms is analyzed.

4.1 Downside risk and dependence

We start by calculating the downside risk of an individual insurer, which is needed in the numerator of our risk measure (1). The returns of the insurance companies follow the sum of the common factor A and the idiosyncratic factor I_i in the following way

$$F_1 = A + I_1 \text{ and } F_2 = A + I_2. \tag{4}$$

The probability of a large loss for a combination of risk factors when these exhibit a power like distribution, is given by Feller's convolution theorem (1971, VIII.8). This theorem holds that if two independent random variables A and I_i satisfy (3), then for large t the convolution has probability

$$P(A + I_i > t) = 2t^{-\alpha}L(t),$$

and where $L(t)$ is slowly varying (i.e. $\lim_{t \longrightarrow \infty} L(at)/L(t) = 1$, for any $a > 0$). The theorem implies that for large failure levels t, the convolution of A and I_i can be approximated by the sum of the univariate distributions of A and I_i. All that counts for the probability of the sum is the (univariate) probability mass which is located along the two axes from the points onward where the line $A + I_i = t$ cuts the axes. The probability that the convolution of A and I_i is larger than t, for large t, is therefore

$$P(A + I_i > t) = 2t^{-\alpha} + o(t^{-\alpha}). \tag{5}$$

If the returns of a firm follow (5), the numerator in (1) is therefore approximately equal to $4t^{-\alpha}$.

To obtain the denominator of (1), we have to determine the probability that the firms do not all have a return smaller than t. The probability that all firms have a return smaller than t is denoted by $P(F_1 \leq t, F_2 \leq t)$. If we examine the complement $1 - P(F_1 \leq t, F_2 \leq t)$, this gives the probability that at least one firm realizes a return exceeding t. Since the returns of firms follow the specification in (4), common or idiosyncratic shocks can cause the returns of one of the firms or both firms to be large. Following the convolution theorem, we can approximate the probability of a large loss by the sum of the probabilities of a common shock or an idiosyncratic shock. Since we have two firms, we can take the sum of the probability that $A > t$, the probability that $I_1 > t$ and the probability that $I_2 > t$ (t being large). For two firms which follow the specification in (4) this sum of probabilities is equal to $3t^{-\alpha} + o(t^{-\alpha})$, or

$$\lim_{t \longrightarrow \infty} \frac{1 - P(F_1 \leq t, F_2 \leq t)}{3t^{-\alpha}} = 1.$$

We can now make our measure of systemic risk (1) operational. The conditional expectation of a crash of two firms, given that one firm crashes, follows once we substitute the theoretical probabilities in the failure measure (1). For large t,

$$\underset{\lim\ t \to \infty}{E[\kappa | \kappa \geq 1]} = \lim_{t \longrightarrow \infty} \frac{P(F_1 > t) + P(F_2 > t)}{1 - P(F_1 \leq t, F_2 \leq t)} = \frac{2t^{-\alpha} + 2t^{-\alpha}}{3t^{-\alpha}} = \frac{4}{3}. \tag{6}$$

As a point of reference, note that the value of the failure measure ranges between 1 and 2. The measure equals 1 under complete independence while it equals 2 in case of complete dependence. The measure therefore gives the expected numer of firms that crash, conditional on at least one crash. In a bivariate setting we can derive the probability of two crashes, given one crash, by substracting 1 of the failure mesaure. Note that one firm crashes for sure, since we condition on this.

The result 4/3 indicates that given a loss of one of the two firms, the expected probability of two crashes is 33% ($= 4/3 - 1$). In this example either one or two firms crash, but if one firm crashes, on average 1 out of 3 times both firms crash. We provide some other stylized examples, before we turn to the estimation of (1).

4.2 Dependence and normality

In the previous section we investigated the tail dependence in case the the common and idiosyncratic risk factors are fat tailed distributed. Under this assumption we find a higher dependence than under the assumption of normally distributed risk factors. In a bivariate normal setting, there is no dependence between extreme losses of two insurers, even if the returns of the two companies are correlated. This is interesting, since the assumption of normality is frequently made. If A and I_i in (4) follow independent standard normal distributions the failure measure will converge to 1, for large t

$$\lim_{t\longrightarrow\infty} E[\kappa|\kappa\geq 1] = \lim_{t\longrightarrow\infty} \frac{P(F_i > t) + P(F_j > t)}{1 - P(F_i \leq t, F_j \leq t)} = 1.$$

The proof is similar to proposition 2 in De Vries (2005). A high correlation between two return series is therefore not equivalent to high tail dependence. Note that the correlation coefficient is often not an useful statistic for financial data. The correlation measure is used as an intermediate step in the calculation of a bivariate conditional probability. If two random variables follow a normal distribution it is sufficient to know the mean, variance and correlation coefficient to characterize their joint behavior. However, the measure is not appropriate to describe the joint behavior of two fat-tailed distributed variables.

4.3 Downside risk and the two factor model

The dependence between firms is higher if the common return component A is more important. To show this, suppose that the returns of individual firms are driven by two different kinds of common factors A_1 and A_2. These two different common risk factors can for example arise out of a similar investment risk and similar insurance liabilities. The return specification for the individual firms now reads $F_1 = A_1 + A_2 + I_1$. The univariate firm risk $\Pr(F_1 > t)$, for large t is equal to

$$\Pr(F_1 > t) = 3t^{-\alpha} + o(t^{-\alpha}), \tag{7}$$

which is the sum of the probability of a loss of one of the three factors in the firm specification, following the convolution theorem. We showed before how we calculate the denominator of the failure measure, $1 - P(F_1 \leq t, F_2 \leq t)$. Since we have 4 elements in the specification of the returns of the individual firms which can cause a loss (A_1, A_2, I_1, I_2), the sum of the probabilities that one of these factors causes a loss will be equal to $4t^{-\alpha} + o(t^{-\alpha})$. When we substitute this and (7) in (1), the dependence measure reads for large t

$$\lim_{t\longrightarrow\infty} E[\kappa|\kappa\geq 1] = \frac{2(3t^{-\alpha})}{4t^{-\alpha}} = \frac{6}{4}. \tag{8}$$

Thus the dependence increases if an extra common risk component is added.

4.4 Downside risk: three firms in a single factor model

To understand the dependence among multiple firms, we give a theoretical exposition of dependence in a setting with three firms. In a setting with more than two firms ($n > 2$) the failure measure can range from 1 to n. When the returns of a firm follow the specification in (4), the firm can crash due to a market shock A or a firm specific shock I_i. The failure measure in a setting with 3 firms then reads

$$\lim_{t \longrightarrow \infty} E[\kappa|\kappa \geq 1] = \lim_{t \longrightarrow \infty} \frac{P(F_1 > t) + P(F_2 > t) + P(F_3 > t)}{1 - P(F_1 \leq t, F_2 \leq t, F_3 \leq t)}$$
$$= \frac{3(2t^{-\alpha})}{4t^{-\alpha}} = \frac{6}{4}, \tag{9}$$

which is in between 1 and 3. The value of the failure measure (9) equals the value in a setting with two firms and three risk drivers as in equation (8). However, since (9) is calculated for a setting with three firms, the measure can be higher than two. This is not possible if there are only two firms. The result of 6/4 in (9) is therefore relatively low, while it is relatively high in (8).

4.5 Downside risk and the market model

So far we have relied on specific examples to show the extent of downside risks among multiple firms, depending on the number of firms and the number of market shocks. We now present a more general form of firm returns in which we let the number of firms approach infinity. Similar to the market model of Sharpe (1964), where the risk of a firm is determined by market risk and idiosyncratic risk, a factor model is specified. The common risk factor A is premultiplied by a constant β, which denotes the relation between the returns of a firm and market risk. Downside risk of a firm is driven by the common risk factor and idiosyncratic risk in the following way

$$F_j = \beta_j A + I_j, \tag{10}$$

where β determines the relative impact of sector risk on the risk of a firm. If β is low or close to zero, this indicates that the risk of a firm is not driven by sector risk but by idiosyncratic risk factors. A high β, indicates that the risk of firms is to a large extend driven by the common factor.

We examine the probability that a firm faces a shock arising out of common risk factors. If the probability of a market shock follows a unit Pareto distribution i.e. $P(A > t) = t^{-\alpha}$, where $t \epsilon [1, \infty)$, the probability that an individual firm is hit by a shock, for large t, is equivalent to

$$\Pr(\beta_j A > t) = \Pr(A > \frac{t}{\beta_j}) = \beta_j^{\alpha} t^{-\alpha}.$$

We assume that the probability of an idiosyncratic shock I_j follows a Pareto distribution scaled with a factor i_j,

$$P(I_j > t) = i_j t^{-\alpha}$$

Since the risk of a large loss for a single firm is the sum of the risk of a market shock and the firm specific shock, following Feller's convolution theorem, this is equal to

$$\Pr(F_j > t) = (i_j + \beta_j^{\alpha}) t^{-\alpha}.$$

This is still pretty much like (5), except that we now have risk factors with non unitary scales. We extend the analysis to n firms, where the returns of each firm is driven by market risk and idiosyncratic risk. In this way the limit behavior of the failure measure can be studied for a large number of firms. We present a general form of the failure measure for multiple firms

$$\lim_{t \longrightarrow \infty} E[\kappa | \kappa \geq 1] = \lim_{t \longrightarrow \infty} \frac{(P(F_1 > t) + ... + P(F_n > t))}{1 - P(F_1 \leq t, ..., F_n \leq t)}. \tag{11}$$

First, we evaluate (11) assuming a stylized form of the firm returns. We assume $\beta_1 = \beta_2 = ... = \beta_n = 1$ and $i_1 = i_2 = ... = i_n = 1$. In this case, the failure measure converges to two, for a large number of firms and for large shocks.

Proposition 1 *Suppose the returns of the firms follow $F_j = \beta_j A + I_j$, where $P(\beta_j A > t) = P(I_j > t) = t^{-\alpha}$, $t \epsilon [1, \infty)$ and $n \to \infty$. If $\beta_j = 1$ $\forall$ j, $i_j = 1$ $\forall$ j, then the failure measure (11) converges to 2, for t large.*

Proof. To derive the numerator of the failure measure, the individual firm risk can be premultiplied by the number of firms. The denominator can be decomposed as the sum of the different factors which determine the returns of the firms, according to Feller (1971). Examine $1 - P(F_1 \leq t, ..., F_n \leq t)$, which is the probability that at least one firm realizes a return larger than t. The factors which can cause a loss at a firm are the idiosyncratic risk factors and the common risk factor. The probability on at least one firm with a large loss can be approximated by the sum of the probabilities of a large idiosyncratic shock (which gives the probability that at least one firm experiences such a shock), and the market shock. An individual firm faces a large market shock with probability $\beta_j^\alpha t^{-\alpha}$ and a large idiosyncratic shock with probability $i_j t^{-\alpha}$. The probability of the market shock inducing one of the firms to fail is equal to $(\max[\beta_1, ..., \beta_n])^\alpha) t^{-\alpha}$, since the market risk of the firm with the highest β has the most effect on violation of $F_i \leq t$, $i = 1, .., n$. The probability that one of the firms is affected by an idiosyncratic shock is equal to $(i_1 + ... + i_n) t^{-\alpha}$. The denominator of (1) thus reads

$$1 - P(F_1 \leq t, ..., F_n \leq t) = ((i_1 + ... + i_n) + (\max[\beta_1, ..., \beta_n])^\alpha) t^{-\alpha}.$$

If we substitute these probabilities in the failure measure (1) we find that for large t

$$\lim_{t \longrightarrow \infty} E[\kappa | \kappa \geq 1] = \frac{(i_1 + ... + i_n) + (\beta_1^\alpha + ... + \beta_j^\alpha)}{(i_1 + ... + i_n) + \max[\beta_1^\alpha, ..., \beta_n^\alpha]}. \tag{12}$$

Now use the assumption that $\beta_j = 1 \; \forall \; j$ and $i_j = 1 \; \forall \; j$ and let the number of firms become unbounded to conclude the proof

$$\begin{aligned} \lim_{n \longrightarrow \infty} \left(\underset{\lim t \to \infty}{E[\kappa | \kappa \geq t]} \right) &= 1 + \lim_{n \longrightarrow \infty} \frac{\beta_1^\alpha + ... + \beta_n^\alpha - \max[\beta_1^\alpha, ..., \beta_n^\alpha]}{(i_1 + ... + i_n) + \max[\beta_1^\alpha, ..., \beta_n^\alpha]} \\ &= 1 + \lim_{n \longrightarrow \infty} \frac{n-1}{n+1} = 2. \end{aligned} \tag{13}$$

■

We have thus proved that if one firm realizes a large loss, at most the loss of one other firm is expected, even though the number of firms becomes unbounded. The failure measure will only be equal to the number of firms in the absence of idiosyncratic risk. If $\beta_j \; \forall \; j$ is equal to 1 and there is no idiosyncratic risk, i.e. $i_j = 0 \; \forall \; j$, the measure equals n. If the parameter i_j denoting the idiosyncratic risk of the firms is equal to a 1 for all firms and there is no market risk, *i.e.* $\beta_j = 0$, the measure converges to 1. A value of 1 for the failure measure indicates that the returns of the insurers are asymptotically independent. In these last, stylized examples there is either no dependence or full dependence.

The focus of this paper is the degree of sector risk in the insurance sector. The dependence is determined by the relative impact of common and idiosyncratic shocks. The previous assumption of β_j and i_j being equal to 1 is very stylized. We therefore rewrite (12) for the average of the betas raised to the power α, $\bar{\beta}^{\alpha} = \frac{1}{n}\sum_{j=1}^{n}\beta_j^{\alpha}$ and average idiosyncratic risk, $\bar{i} = \frac{1}{n}\sum_{j=1}^{n} i_j$). In the following proposition we investigate the degree of dependence after rewriting (12) for average market risk and average idiosyncratic risk.

Proposition 2 *Suppose the returns of the firms follow $F_j = \beta_j A + I_j$, where $P(\beta_j A > t) = \beta_j^{\alpha} t^{-\alpha}$, $P(I_j > t) = i_j t^{-\alpha}$, $t \epsilon [1, \infty)$ and $n \to \infty$, then the failure measure (11) converges to $1 + \frac{\bar{\beta}^{\alpha}}{\bar{i}}$ for t large , where $\bar{\beta}^{\alpha} = \frac{1}{n}\sum_{j=1}^{n}\beta_j^{\alpha}$ and $\bar{i} = \frac{1}{n}\sum_{j=1}^{n} i_j$ (assuming the β_j and i_j are bounded).*

Proof. Rewrite the failure measure (12) for the average common factor, β_j, and average idiosyncratic risk factor, $\bar{i}$. For a large number of firms and for large shocks this reads

$$\lim_{n\longrightarrow\infty}\left(\underset{\lim t\to\infty}{E[\kappa|\kappa \geq t]}\right) = \lim_{n\longrightarrow\infty}\frac{\bar{i} + \bar{\beta}^{\alpha}}{\bar{i} + \max[\beta_1^{\alpha}, ..., \beta_n^{\alpha}]/n} = 1 + \frac{\bar{\beta}^{\alpha}}{\bar{i}}. \tag{14}$$

■

The ratio of $\bar{\beta}^{\alpha}/\bar{i}$ gives the relative importance of market risk and idiosyncratic risk for the occurance of multiple large losses. If the average idiosyncratic risk is relatively small, the dependence among firms will be high. If the average beta is small, the failure measure will converge to a 1, which implies asymptotic independence. If the failure measure returns a 1, the returns of the firms are asymptotically independent. A value for the failure measure around 2 (when estimated for a large number of firms) implies that a firm is exposed to idiosyncratic shocks and common shock with roughly the same probability, i.e. for large n and t, $E[\kappa|\kappa \geq t] = 1 + \bar{\beta}^{\alpha}/\bar{i} \approx 2$ and therefore $\bar{\beta}^{\alpha} = \bar{i}$, following (14). If the returns of the firms are highly dependent, the failure measure is larger than 2. This corresponds with a large $\bar{\beta}^{\alpha}$ and a comparatively small $\bar{i}$ in (14). A value between 1 and 2 implies that idiosyncratic shocks are more frequent than common shocks. A value larger than 2 implies that common shocks are more frequent.

We show in the following section that the measure provides information on the relative importance of common risk factors and idiosyncratic risk factors for the riskiness of the sector.

4.6 Model interpretation

Regulators of the insurance sector are concerned with the stability of the financial sector. They are therefore interested in the degree of dependence among insurers. If insurers carry the same risks, it is likely that multiple insurers are affected by large losses and that losses in their market value are dependent. We discuss how the model relates to high or low dependence between the losses of the insurers.

If insurers hold different assets and liabilities, the dependence among insurers will be low, since it is unlikely that multiple firms are hit by a loss on shared liabilities or assets. In this case idiosyncratic risk is the most important source of risk. If the idiosyncratic risk dominates over market risk, the failure measure will be between 1 and 2.

Vice versa, if most of the assets and liabilities of insurers are the same, the dependence among insurers is high. The exposure to market risk is high and the common risk factor is the most important source of risk. The failure measure will therefore be larger than 2, as was shown in (14). A high dependence may be the result of risk diversification. Because of risk diversification, insurers are exposed to the same risks. Implicitly, the failure measure gives information on similarities in the risk exposure between insurers. This is interesting, since the risk exposure can be regulated to reduce the industry wide dependence.

If the estimation of the dependence indicates that the failure measure (11) is smaller than two, this implies that idiosyncratic risk is the most important source of risk. If the failure measure estimates indicate that dependence is high, i.e. larger than two, market risk is the dominant risk factor. This framework can therefore help to understand the impact of market risk on the downside risk of multiple insurers. An improved understanding of the impact of market risk can help to design appropriate regulatory policy, as is argued in the following.

If regulators deem the dependence among insurers being to high, they may consider to reduce the common exposures of insurers. The probability that an individual firm faces a large loss may be increased by this policy. However, losses will be more isolated to single firms and the probability of multiple simultaneous losses of insurers is reduced. If an individual insurers goes bankrupt, other insurers may remain solvent. They can take over the clients of the bankrupt insurer, reducing the economic impact of a bankruptcy.

If there is a high sector risk, the capacity of the insurance sector during crises can be of a concern. In this case the capacity of the insurance sector during crises needs to be augmented. There are two sources of insurance cover outside the commercial insurance sector: the capital market and public insurance pools.

The insurance against major catastrophe can e.g. be provided by insurance pools in which the governments and the private sector participate. This may be necessary if a full private solution is not achievable. The advantage of such a pool is that it gives certainty on the level of insurance to residents and firms. A government could also promise to help those hit by a disaster. The advantage of a fund, however, is that the government receives insurance fees and the costs are clear up-front. This way the impact on the government budget of disasters is limited.

Another way to limit the impact of disasters is to make an extensive use of the capital markets. An important element of insurance products is the provision of liquidity. Liquidity can also be provided by capital markets. Firms can create an internal insurance pool with the use of capital markets and can in this way reduce their dependence on external insurance companies. If there is a high degree of sector risk, i.e. the failure measure is larger than two, regulators may encourage firms and governments to exploit the latter options.

A low sector wide dependence corresponds to a high idiosyncratic risk of insurers. A high idiosyncratic risk for insurers can be a reason to study the prudential regulation at the firm level. Regulators may e.g. want to reduce the risk of individual insurers, by encouraging them to diversify their exposure, or by demanding a higher solvency at the firm level.

5 Measuring dependence

For our estimation approach we follow the approach of Hartmann et al. (2005). The empirical return series for each firm i are denoted with X_{it}, where the subscript t denotes the t^{th} elementh of the sample of returns. We take the negative of the empirical observations. Let $Q_{i(t)}$ represent the t-th ascending order statistic of X_{it}, with $t = 1, ..., T$, such that $X_{i(1)} > ... > X_{i(T)}$ and $p = t/T$, where p is the probability corresponding to the empirical loss quantile $Q_{i(t)}$. We are interested in the losses occuring with a small probability. This probability p corresponds to a high threshold Q_{ivar} (the Value at Risk or stress test level) for each firm i, above which losses occur with the probability p. We derive the extreme loss quantile Q_{ivar} corresponding to the probability p using the empirical distribution of the returns X_{it}. Since the empirical distribution is firm specific, the stress test level Q_{ivar} corresponding to the probability p differs for each firm

$$p = P\left\{X_{1t} > Q_{1var}\right\} = ... = P\left\{X_{it} > Q_{ivar}\right\} = ... = P\left\{X_{nt} > Q_{nvar}\right\}. \tag{15}$$

We are interested in the expected number of firms that crash, conditional on the crash of at least one firm,

$$E[\kappa|\kappa \geq 1] = \frac{np}{1 - P(X_1 \leq Q_{1var}, ..., X_n \leq Q_{nvar})}. \tag{16}$$

If $n = 2$ this measure reduces to the bivariate measure in (1). The measure gives us the expected number of institutions that crash (κ) given that at least one institution crashes, i.e., has a return exceeding Q_{ivar}, where Q_{ivar} is the quantile from the empirical distribution corresponding to the probability p in (15). In the denominator of (16) we therefore have different thresholds $\{Q_{1var}, ..., Q_{nvar}\}$. For estimation purposes it is convenient if we premultiply the empirical returns with Q_{1var}/Q_{ivar},

$$p = P\left\{X_i > Q_i\right\} = P\left\{\frac{Q_1}{Q_i}X_i > \frac{Q_1}{Q_i}Q_i\right\} = P\left\{Q_{1i}X_i > Q_1\right\}, \tag{17}$$

where $Q_{1i} = (Q_{1var}/Q_{ivar})$. This way we can rewrite the denominator of (16) as $1 - P(Q_{1i}X_1 \leq Q_{1var}, ..., Q_{1n}X_n \leq Q_1 var)$. This is equivalent to $P(max[Q_{1i}X_1, ..., Q_{1i}X_n] > Q_{1var})$. This probability corresponds to the quantile Q_{1var} of the empirical distribution function of the maxima, which is equivalent to $max[Q_{1i}X_1, ..., Q_{1n}X_n]$. Since we evaluate the limit behavior of (16), we take Q_{1var} close to the boundary of the sample and use the 10^{th} largest order statistic of the return series X_i. As a result, the probability in (15) corresponds to 10 divided by the sample size, N. In the Appendix we indicate the robustness of the procedure, using a higher or lower number of order statistics.

We consider the dependence among pairs of firms and the dependence among 4 and 8 firms, i.e. $n = 2, 4$ and 8. In a bivariate setting we apply a different estimation approach. The probability that two firms realize a loss larger than 7.5% on a given day is estimated. The advantage of this approach is that the estimates are straightforward to interpret, since the estimates return the probability that the second firm of a pair realizes a loss larger than 7.5%, given that one firm realizes a loss of 7.5%. For the multivariate estimates we have to scale the variables as in (17) to be able to take the maximum in the denominator. See the Appendix for estimation details for the bivariate estimation.

6 Empirical results

In this section the dependence among insurance firms is measured. We estimate dependence between firms within the insurance sector and within the reinsurance sector. As a benchmark, we estimate dependence between large oil companies and the dependence between firms from entirely different sectors. The motivation for these benchmarks is given in the following. We present the estimated dependence among pairs of firms and the dependence among multiple firms.

6.1 Empirical benchmark

A benchmark is needed to interpret the estimated degree of dependence between insurers. We need a benchmark for dependence among firms in a sector with a high degree of dependence and a benchmark for the dependence among firms which are unrelated. The value of oil firms depends heavily on the price of oil. We therefore expect to find a high degree of dependence among oil firms. This sector is therefore the benchmark for a sector with high dependence. If dependence within the insurance sector is of the same magnitude, it is plausible that the returns of insurers are driven by a common factor, which is comparable to the price of oil in the oil sector.

We also estimate the dependence among firms from different sectors. This gives the value for the failure measure in case the returns of the firms are not driven by sector risks. This way we can compare the estimation results for the insurance sector with estimation results for a sector with high dependence and with results for firms from different sectors. The firms used for the estimation of the dependence across the different sectors are given in Table A.1 in the Appendix.

6.2 Independence

Suppose that the marginal returns of two insurers are independently distributed and rewrite the failure measure (1) under this assumption. Under the assumption of independently distributed returns for the firms, the denominator of the failure measure is equal to $1 - (P(F_1 \leq t) * P(F_2 \leq t))$. The failure measure for independently distributed returns then reads

$$E[\kappa|\kappa \geq 1] = \frac{P(F_1 > t) + P(F_2 > t)}{1 - (P(F_1 \leq t) * P(F_2 \leq t))}. \tag{18}$$

By estimating the univariate probabilities $P(F_i > t)$, and calculating (18) as if the returns are unrelated, we obtain yet another benchmark to judge the amount of dependency. Note that

$$\lim_{n \longrightarrow \infty} = \frac{P(F_1 > t) + P(F_2 > t)}{1 - (P(F_1 \leq t)P(F_2 \leq t))} = 1,$$

but at finite loss levels this measure is larger than 1 since $\frac{2(1-p)}{1-p^2} = \frac{2(1-p)}{(1-p)(1+p)} = \frac{2}{1+p} > 1$.

Since we know the univariate probability of an extreme loss in (15) is equal to the number of observation above the threshold Q_i, divided by the sample size N (10/2739), we can substitute for this in the failure measure

$$E[\kappa|\kappa \geq 1] = \frac{P(F_1 > t) + P(F_2 > t)}{1 - (P(F_1 \leq t) * P(F_2 \leq t))} = \frac{20/2739}{1 - ((2729/2739)^2)} = 1.0018.$$

If the marginal returns of two insurers are independently distributed, the conditional probability of a simultaneous crash is close to 0 $(1.0018 - 1)$. Hence the 0.0018 probability provides a lower bound benchmark.

6.3 Data

The dataset starts in January 1995 and ends in June 2005, because of data availability. The sample size N of daily data is equal to 2739. In the EU there are 4 reinsurers with stock price data available for the full sample. The selected 8 insurers, independent firms and oil firms are given in Table A.1. For the estimation of the dependence among 4 firms, only the first 4 firms in Table A.1 are used.

	Insurers	Reinsurers
Insurers	0.18	0.13
Reinsurers	0.13	0.12

Table 1: Average bivariate dependence (no scaling)

6.4 Dependence among pairs of firms

The failure measure in a bivariate setting provides the conditional probability of a crash of two firms. We estimate the bivariate conditional probabilities of all possible combinations of insurers, all combinations of reinsurers and combinations of insurers and reinsurers. In Table 1 we report the averages of the estimates, the pair wise estimates can be found in Table A.2 in the Appendix. From Table 1 we see that the average probability that two insurers realize an extreme loss, given the extreme loss of one of the two insurers is on average 18%. The probability that two reinsurers realize a large loss, given the large loss of one of the two reinsurers is equal to 12%. The dependence among insurers is thus higher than the dependence among reinsurers. The dependence among insurers and reinsurers is 13% and lower than the dependence among insurers. All values are much larger than the benchmark lower bound of 0.0018, indicating that there is considerable asymptotic dependence. Since the value for the insurance companies is much larger, we therefore conclude that the systemic risk in the insurance sector differs from the systemic risk in the reinsurance sector.

Even though both sectors deal with insurance, the interdependencies are higher in the insurance sector. This is somewhat surprising, given that the reinsurers take on risk from the different insurers. These risks are concentrated at reinsurance companies. Apparently, the connectedness stems from other sources of risk, such as investment risk, which may be more similar among insurers.

For individual firms, the conditional probability of a loss, given the loss of another firm, differs considerably from the sector averages. Since there are 8 insurers and 4 reinsurers in the dataset, we can estimate the bivariate conditional failure probability for each firm in combination with 11 other firms. This bivariate dependence is estimated and the average probability of a loss for a firm is reported in Table 2.

The interpretation of the bivariate probabilities is the probability that

Firm	mean prob.	Firm	mean prob.
ALLIANZ	0.17	MUNICH RE	0.16
ING	0.23	SWISS RE	0.13
GENERALI	0.08	HANNOVER RE	0.13
AXA	0.18	SCOR	0.10
AEGON	0.17		
AVIVA	0.14		
PRUDENTIAL	0.16		
ZFS	0.14		

Table 2: Bivariate dependence (no scaling)

Insurers	Insurance/Reinsurance	Firms
1.905	1.739	1.290
	$1 \leq E[\kappa \vert \kappa \geq 1] \leq 8$	
	Jan. 1995 - June 2005	

Table 3: Dependence among 8 companies

one of the two firms crash, given that the other crashes. If e.g. an insurer or reinsurer realizes a large loss, the probability that ING also realizes a loss is on average 23%, which is the highest probability in the table. If an insurer or reinsurer realizes a large loss, the probability that Generali also realizes a loss is low, on average 8%. A possible explanation for this result is that ING has more risk factors in common with other insurers than Generali. These common risk factors can be related to e.g. country risk and the riskiness of individual business lines.

6.5 Dependence among multiple firms

Our main research question concerns the sector wide dependence between insurance and reinsurance companies. We therefore do not only estimate the downside risk dependence among pairs of firms, but also among multiple firms. First, we estimate the dependence among eight firms. However, there are only four reinsurers and four oil firms in the dataset. Secondly, we therefore estimate the dependence among four firms to obtain the dependence among reinsurers and oil firms.

6.5.1 Dependence among eight firms

Recall that the failure measure returns the expected number of firms that crash, given the crash of one firm and is not smaller than 1. If there are eight firms, the failure measure can be at most equal to eight. The failure measure is estimated among 8 insurers and 8 firms from different sectors. Moreover, we are interested in the dependence between reinsurers. Since we have only 4 reinsurers, we estimate the dependence measure among a set of 4 reinsurers and 4 insurers, together a set of 8 firms. The results are presented in Table 3.

The dependence among the eight insurance firms is higher (1.905) than the dependence among the 8 firms from the different sectors (1.290). Dependence among the set of 4 insurers and 4 reinsurers (1.739) is thus lower than the dependence among insurers. This result supports the findings from the bivariate estimates. However, we can now interpret these results with the theoretical model which we developed in the previous section.

We use Proposition 2 to interpret the estimation results and argue that the impact of market shocks is of the same magnitude as the impact of idiosyncratic shocks. An estimate of 1.905 implies that the ratio of $\bar{\beta}^{\alpha}/\bar{i}$ is close to but smaller than 1. The estimate of 1.739 implies that the ratio of $\bar{\beta}^{\alpha}/\bar{i}$ is smaller than 1. This indicates that the impact of idiosyncratic risk in the reinsurance sector is larger than in the insurance sector.

The explanation for the larger impact of idiosyncratic risk in the reinsurance sector is that the risk exposure of reinsurers is more heterogenous than the risk exposure of insurers. Dependence can arise out of the same investments or out of the same insurance risks. Possibly the insurance risks of reinsurers differ to a larger extent than the insurance risks in the insurance sector. Since we do not have information on the insurance portfolio of the companies, it is difficult to validate this explanation. Another possible explanation is that the investment risks in the insurance sector are more alike than in the reinsurance sector. This may originate from the interest rate risk of life insurance policies, which is a relatively large risk for direct insurance companies. Another possibility is that the equity investments made by (life) insurance companies may be larger than the equity investments made by reinsurers. Losses on the stock market may therefore have a larger impact

Insurers	Reinsurers	Oil	Firms
1.481	1.333	1.538	1.053
$1 \leq E[\kappa \mid \kappa \geq 1] \leq 4$			
Jan. 1995 - June 2005			

Table 4: Dependence among 4 companies

on insurers. If regulators deem the sector risk as high, these possible explanations offer a starting point for regulators to reduce the dependence among firms, by reducing the common risk exposures.

Dependence among firms from different sectors is lower than the dependence in the insurance sector, since the estimated dependence among eight firms is only 1.290. The idiosyncratic risk is the most important risk factor for the firms from different sectors and is a factor 3.45 times bigger ($\bar{\beta}^{\alpha} = 3.45\bar{i}$, since $\bar{\beta}^{\alpha}/\bar{i} = 0.29$). The common factors $\bar{\beta}^{\alpha}$ are clearly of importace for the risks in the insurance sector.

6.5.2 Dependence among four firms

We also estimate the dependence among four firms. The estimates are given in Table 4. The failure measure estimate for four insurers is 1.481. The estimate of the dependence among four firms from different sectors is 1.053. It is clear that dependence in the tails is much smaller for firms from different sectors than among insurance companies. An extreme negative return of one of the firms from unrelated sectors is almost unrelated to the losses of the other firms. The downside risk dependence between four reinsurance companies is 1.333. This is lower than the dependence among insurers. Dependence among the tail risk of large oil companies (1.538) is of the same order as dependence in the insurance sector, and much higher than among the independent firms. Sector risk in the insurance sector is therefore of the same magnitude as sector risk in the oil sector.

The result of 1.481 implies that idiosyncratic risk is very relevant. For large n and t, the failure measure is equal to 1 plus the ratio of the average market risk and the average idiosyncratic risk for the insurers, i.e. $E[\kappa|\kappa \geq t] = 1 + \bar{\beta}^{\alpha}/\bar{i} = 1.481$. Since the ratio is smaller than 2, idiosyncratic risk appears more important than market risk. Since the expected number of firms from different sectors realizing a loss is equal to 1.053, which is close to 1, it is evident that there are hardly any common risk factors causing joint losses.

The tail dependence among firms from different sectors is very low. Even though the firms are from different sectors, they can be exposed to similar risks. The stock market crash of September 11, 2001 had an impact on all stock prices. Such broad macro shocks could have resulted in a higher dependence in stock prices among firms from the different sector. The failure estimator could have returned a higher value than the estimated 1.053.

In a way it is remarkable that the dependence among reinsurers is lower than among insurance companies. Since reinsurers provide insurance against major catastrophes, they can receive claims arising out of the same (natural) catastrophe, such as hurricanes. However, it seems that these simultaneous losses are smaller than we expected.

One can argue that the dependence within the oil sector should be relatively high, since the results of oil companies are driven by changes in oil price. The oil price has a major impact on profits and losses for companies in the oil sector. Even though it is difficult to point at a single factor causing the dependence among insurers, such as the oil price for oil companies, there should be a similar explanation for the dependence among insurers. The estimates are larger than the estimates in the four firm setting. The expected number of insurers that crash, given the crash of one insurers increased from 1.481 to 1.905. However, the increase is limited, if we consider that the maximum possible value for the failure measure doubled from 4 to 8.

7 Conclusion

There is an increasing interest in the impact of extreme losses of insurers for the stability of the financial system. To this end the downside risk dependence between the losses of insurance companies is investigated. We provide an explanation for a similar exposure to losses, based on the idea that multiple insurers carry similar risks.

For the design of optimal regulation, it matters if regulators have to deal with sector wide risk or firm specific risk. When firms are exposed to similar risks, all insurers realize losses on either their assets or liabilities, during a crisis. We model and estimate the effect of risk diversification on downside risk for the insurance sector.

The probability that multiple insurers realize a loss is relevant for our understanding of insurance sector risk. We specify a two factor model, where the downside risk of a firm is determined by common risk factors and idiosyncratic risk. The impact of market risk and idiosyncratic risk on the expected number of firms that crash is investigated. We define a conditional failure measure based on the expected number of firms that crash, conditional on a large loss of one firm. We proof that the measure converges to the ratio of idiosyncratic and common risk factors. This ratio is therefore an indicator for the importance of sector risks.

Insurers limit and diversify their risk exposure by reinsurance contracts. We investigate the dependence among reinsurers, to understand if risk in the reinsurance sector is similar to insurance risks. When the dependence between pairs of firms is investigated, we conclude that reinsurance sector risk differs from insurance sector risk.

The conditional failure measure is also estimated to understand dependence in the insurance sector. It is found that common risk factors are an important source of risk in the insurance sector and to a smaller extent in the reinsurance sector. Idiosyncratic risk is relatively important for the reinsurance sector.

Dependence in the insurance sector is of the same order as in the oil sector. This implies there is a similar factor driving the returns in the insurance sector as in the oil sector. Tail dependence is relatively high in the insurance sector, when compared to a portfolio of stocks from different sectors.

References

[1] Buchanan, B., T. Arnold, Lance Nail, Beware of the ides of March: The demise of HIH Insurance, Research in International Business and Finance, 17, 2003

[2] Feller, W., An Introduction to Probability Theory and its Applications, Vol.II 2nd ed. Wiley, New York, 1971

[3] Forbes, K., and R. Rigobon, No contagion, only interdependence: measuring stock market comovements, Journal of Finance, 57, 2223-2262, 2002

[4] FSF, Press release, ref.no. 17/2002E, 4 September 2002

[5] Geluk, J.L. and C.G. de Vries, Weighted sums of subexponential random variables and asymptotic dependence between returns on reinsurance equities, Insurance: Mathematics and Economics, 38, 39-56,2006

[6] IAIS, Global reinsurance market report 2003, December 2004

[7] Joint Forum, Credit risk transfer, Bank for International Settlements, Basel, March 2005

[8] OECD, Insurance and Financial Market Issues Related to the Management of Large-Scale Disasters , Financial Market Trends 86, Paris, March 2004

[9] Ross, S.A., The arbitrage theory of capital asset pricing, Journal of Economic Theory, 13, 314-360, 1976

[10] Sharpe, W.F., Capital asset prices: a theory of market equilibrium under conditions of risk, Journal of Finance, 19, 425-442, 1964

[11] Swiss Re, Reinsurance-a systemic risk?, Sigma No. 5/2003, 2003

[12] Swiss Re, World insurance in 2004: growing premiums and stronger balance sheets, Sigma No 2/2005, 2005

[13] Xin, H., Statistics of Bivariate Extreme Values, Ph. D thesis, Tinbergen Institute Research Series, Erasmus University Rotterdam, 1992

[14] Vaughan, T.M., Financial stability and insurance supervision: the future of prudential supervision, The Geneva Papers on Risk and Insurance, 29, 258-272, April 2004

INSURERS	REINSURERS	OIL FIRMS	INDEPENDENT FIRMS
ALLIANZ	MUNICH RE	BP	RD SHELL
ING	SWISS RE	TOTAL	SAP
GENERALI	HANNOVER RE	RD SHELL	L'OREAL
AXA	SCOR	REPSOL	TELEFONICA
AEGON			BMW
AVIVA			NOKIA
PRUDENTIAL			BASF
ZFS			PHILIPS ELECTRONICS

Table A.1: Selected firms

A Data

The companies used for the estimation of dependence within the different sectors and between pairs of insurers and reinsurers are given in Table A.1.

B Bivariate estimation

In this section we elaborate on the bivariate estimation technique employed in the paper. We first rewrite the failure measure and turn it into an estimator.

From elementary probability theory we know that $P(X_1 \leq t, X_2 \leq t) = 1 - P(\max[X_1, X_2] > t)$ and $P(X_1 > t) + P(X_2 > t) = P(\max[X_1, X_2] > t) + P(\min[X_1, X_2] > t)$. One can therefore rewrite the conditional expectation as follows

$$E[\kappa|\kappa \geq 1] = \frac{P(X_1 > t) + P(X_2 > t)}{1 - P(X_1 \leq t, X_2 \leq t)} = 1 + \frac{P(\min[X_1, X_2] > t)}{P(\max[X_1, X_2] > t)}. \tag{19}$$

The estimation of the probability of multiple crashes can thus be reduced to the estimation of two univariate probabilities. This greatly facilitates the empirical analysis, since one can proceed on basis of the previously described univariate estimation methods for the minimum and maximum return series. We use the notation P_{min} for $P(\min[X_1, X_2] > t)$ and the corresponding notation for the maximum. If the tail index α is identical for the minimum (α_i) and maximum (α_a) series, we obtain the following non-parametric estimator

$$E[\kappa|\kappa \geq 1] = 1 + \frac{\widehat{P}_{\min}}{\widehat{P}_{\max}}. \tag{20}$$

	ALLIANZ	ING	GENERALI	AXA	AEGON	AVIVA	PRUDENTIAL	ZFS	MUNICHRE	SWISSRE	HANNOVERRE	SCOR	Mean
ALLIANZ	1.00	0.29	0.11	0.24	0.17	0.19	0.22	0.15	0.22	0.12	0.07	0.08	0.17
ING	0.29	1.00	0.06	0.26	0.38	0.26	0.29	0.16	0.32	0.11	0.19	0.16	0.23
GENERALI	0.11	0.06	1.00	0.14	0.03	0.05	0.05	0.06	0.05	0.13	0.12	0.05	0.08
AXA	0.24	0.26	0.14	1.00	0.31	0.09	0.19	0.21	0.16	0.14	0.14	0.10	0.18
AEGON	0.17	0.38	0.03	0.31	1.00	0.11	0.23	0.19	0.13	0.09	0.14	0.11	0.17
AVIVA	0.19	0.26	0.05	0.09	0.11	1.00	0.19	0.15	0.19	0.13	0.07	0.09	0.14
PRUDENTIAL	0.22	0.29	0.05	0.19	0.23	0.19	1.00	0.12	0.17	0.12	0.14	0.08	0.16
ZFS	0.15	0.16	0.06	0.21	0.19	0.15	0.12	1.00	0.07	0.26	0.07	0.10	0.14
MUNICHRE	0.22	0.32	0.05	0.16	0.13	0.19	0.17	0.07	1.00	0.12	0.19	0.15	0.16
SWISSRE	0.12	0.11	0.13	0.14	0.09	0.13	0.12	0.26	0.12	1.00	0.13	0.04	0.13
HANNOVERRE	0.07	0.19	0.12	0.14	0.14	0.07	0.14	0.07	0.19	0.13	1.00	0.12	0.13
SCOR	0.08	0.16	0.05	0.10	0.11	0.09	0.08	0.10	0.15	0.04	0.12	1.00	0.10

Table A.2: Bivariate conditional expectation t=0.075

In Slijkeman et al. (2005) we show that (20) can be calculated using a simple counting procedure for the minima and maxima. We must take t large, since we are interested in the limit behavior of (19). We take t equal to 7.5%, since this corresponds to a value at the bound of the sample. The estimated dependence among all possible combinations are given in Table A.2.

C Multivariate results for different quantiles

For the estimation of the dependence we have to determine a threshold Q_1. For the estimation of the dependence among 4 and 8 firms, the threshold corresponding to the $10th$ largest order statistic of the univariate return series X_i was taken. In this section we present the estimation results for a higher and lower threshold, Q_1. The returns corresponding to the 5^{th}, 20^{th} and 30^{th} order statistic are taken as a threshold. This threshold is subsequently taken for the estimation of (16). The results for the dependence among 4 firms are given in Table A.3, the results for the dependence among 8 firms are given in Table A.4. When the dependence is estimated for a higher threshold (i.e. larger losses), this dependence is a bit lower, but remains of the same order. When the dependence for a lower threshold is evaluated, this no longer corresponds to the dependence among extreme returns. The 30^{th} largest return in 10 years can hardly be considered as an extreme return. Even for this lower threshold however, the conclusions on the relative importance of sector risks for the different sectors do no change considerably. Thus our procedure appears robust against the threshold selection.

Order statistic of Q_1	Insurers	Reinsurers	Oil	Firms
5	1.538	1.333	1.333	1.053
10	1.481	1.333	1.538	1.053
20	1.600	1.379	1.600	1.159
30	1.690	1.412	1.500	1.212
$1 \leq E[\kappa \vert \kappa \geq 1] \leq 4$				
Jan. 1995 - June 2005				

Table A.3: Dependence among 4 companies

Order statistic of Q_1	Insurers	Insurance/Reinsurance	Firms
5	1.739	1.600	1.250
10	1.905	1.739	1.290
20	2.133	1.928	1.404
30	2.222	2.069	1.538
$1 \leq E[\kappa \vert \kappa \geq 1] \leq 8$			
Jan. 1995 - June 2005			

Table A.4: Dependence among 8 companies