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Abstract

Many economic organizations have some relational structure, meaning that economic agents do not only differ with respect to certain individual characteristics such as wealth and preferences, but also belong to some relational structure in which they usually take different positions. Two examples of such structures are communication networks and hierarchies. In the literature the distinction between these two types of relational structures is not always clear. In models of restricted cooperation this distinction should be defined by properties of the set of feasible coalitions. We characterize the feasible sets in communication networks and compare them with feasible sets arising from hierarchies.

Keywords: Communication, hierarchy, cooperative game, feasible set.
JEL Subject Classification: C71, D85

1 Introduction

Many economic organizations have some relational structure, meaning that economic agents do not only differ with respect to certain individual characteristics such as wealth and preferences, but also belong to some relational structure in which they usually take different positions. Two examples of such structures are communication networks and hierarchies. There is an extensive literature on communication networks, both from a cooperative and a non-cooperative point of view. However, studying hierarchical organizations is not yet well developed in economic theory, although some features of hierarchical organizations are studied, for example in principal-agent models and optimal control models. In cooperative game theory, attempts to study hierarchical organizations are made in the field of restricted cooperation. However, there is still a lot of confusion. For example, the question what is the difference between communication networks and hierarchies is not always clear. In the field of restricted cooperation the difference should be defined by properties of the sets of feasible coalitions. In this paper we characterize the feasible sets arising from communication networks and compare them with feasible sets arising from hierarchies.

Although there are different approaches to communication in the economic literature, there seems to be more consensus with respect to the definition and implications of communication than with respect to hierarchy. It seems that models of communication are based on the concept of 'connectedness'. Although hierarhical relations are usually between different types of agents (or agents having different roles), and thus are asymmetric relations, communication relations might be symmetric (and between similar type of agents) or asymmetric. An example of asymmetric communication relations is given by Dewatripont and Tirole (2005) who consider communication as a 'transfer of knowledge' between a sender and a receiver. They formulate a principal-agent model to communication as a moral hazard problem between this sender and receiver. Dessein (2002) extends the model of Crawford and Sobel (1982) and studies a principal-agent model of an organization where the principal can make a trade-off between delegation (implying a loss of control) and communication (implying a loss of information).

On the other hand, Bala and Goyal (2000) consider communication relations between similar agents and a communication link between two agents means that these two agents share their information with one another (the two-sided case) or the agent who builds the relation gets access to the information of the other (the one-sided case). In their model of network formation agents can unilateral decide to build or delete communication links. In the network formation model of Jackson and Wolinksy (1996) agents can unilateral decide to delete links, but for building links mutual agreement is needed. Their model applies the static model of restricted cooperation in cooperative games of Myerson (1977) where communication means that connectedness determines the possibilities of cooperation.

Although these are different models of communication, their seems to be some consensus in the sense that for each form of communication the underlying idea is connectedness: agents must be connected in order to share information or to be able to cooperate. There seems to be less consensus about the meaning of hierarchy in economic and political organizations. In the literature on hierarchies it is even not clear whether hierarchy implies authority or not, as expressed by Hart and Moore (2005). One of the first formal models of a hierarchical production organization is presented by Williamson (1967) where the depth of a hierarchical firm structure with constant span of control determines the total number of productive employees in a firm and thus the total profit that can be made. The hierarchical monitoring behind this model is specified in more detailed by, e.g. Calvo and Wellisz (1978, 1979). Other models use optimal control techniques to determine optimal hierarchies as information processing organizations, see e.g. Keren and Levhari (1979, 1983) and Radner (1992).

But even when agreeing on whether a hierarchy is about authority or not, the implications of authority differ across different models. Although there is a large literature on communication and hierarchies, few attempts are made to build a consistent theory on organizations comparing both types of relational structures. An attempt is made by Bolton and Dewatripont (1994) who describe a model where efficient (i.e. cost minimizing) information processing in a communication network implies some hierarchical structure in the sense that efficient networks take a pyramidal form. A similar result is obtained by Chwe (2000) who studies directed communication networks and shows that the minimal sufficient networks for coordination can be seen as hierarchies.

Another attempt is made in Demange (2004) in the field of restricted cooperation, but as we claim later this is not really about hierarchies but only about communication. In the underlying paper we present results in this field comparing communication with hierarchies. First we compare the model of restricted communication as developed by Myerson (1977) with the model of games with a hierarchical permission structure where hierarchical permission (or approval) relations determine cooperation restrictions. Then we show that the model of restricted communication has strong similarities with relational structures that can be represented by antimatroids (which generalize games with a hierarchical permission structure). We give full characterizations of communication feasible sets that differ from antimatroids only with respect to a union property and an accessibility property. To show that antimatroids are more general than games with a permission structure we also discuss ordered partition voting as an example of another hierarchical structure that can be represented as an antimatroid. Finally, we make some concluding remarks.

2 Communication and hierarchies in cooperative games

A situation in which a finite set of players can obtain certain payoffs by cooperation can be described by a *cooperative game with transferable utility*, or simply a TU-game, being a pair (N, v), where $N \subseteq \mathbb{N}$ is a finite set of players and $v\colon 2^N \to \mathbb{R}$ is a *characteristic function* on N satisfying $v(\emptyset) = 0$. For any *coalition* $S \subseteq N$, $v(S)$ is the *worth* of coalition S, meaning that the members of coalition S can obtain a total payoff of $v(S)$ by agreeing to cooperate.

Main question in cooperative games is to determine the distribution of payoffs over individual players. A *payoff vector* $x \in \mathbb{R}^n$ of an n-player TU-game (N, v) is an n-dimensional vector giving a payoff $x_i \in \mathbb{R}$ to any player $i \in N$. A *solution* for

TU-games is a mapping f that assigns to every game (N, v) a set of payoff vectors $f(N, v) \subseteq \mathbb{R}^n$. A famous and widely applied solution is the *Core* which assigns to every game the set of efficient and coalitionally stable payoff vectors, i.e. $Core(N, v) = \{x \in \mathbb{R}^n \mid \sum_{i \in N} x_i = v(N) \text{ and } \sum_{i \in S} x_i \geq v(S) \text{ for all } S \subset N\}$.

2.1 Communication

In a TU-game any subset $S \subseteq N$ is assumed to be able to form a coalition and earn the worth $v(S)$. However, models have been developed in which there are restictions on the feasibility of coalitions. One of the most well-known restrictions on coalition formation are communication restrictions meaning that a coalition S is feasible if and only if the players in S are connected within a given *communication network* on the set of players. In Myerson (1977) this communication network is represented as an undirected graph on the set of players.

An *undirected graph* is a pair (N, G) where N is the set of nodes and $G \subseteq \{\{i, j\} | i, j \in N, \ i \neq j\}$ is a collection of subsets of N such that each element of G contains precisely two elements of N. The elements of G represent bilateral communication links and are refered to as *edges* or *links*. Since the nodes in a graph represent the positions of players in a communication network we refer to the nodes as players. A sequence of k different players $(i_1, \ldots, i_k)$ is a *path* in (N, G) if $\{i_h, i_{h+1}\} \in G$ for $h = 1, \ldots, k-1$. Two distinct players i and j, $i \neq j$, are *connected* in graph (N, G) if and only if there is a path $(i_1, \ldots, i_k)$ with $i_1 = i$ and $i_k = j$. A coalition $S \subseteq N$ is *connected* in graph (N, G) if every pair of players in S is connected by a path that only contains players from S, i.e. for every $i, j \in S, \ i \neq j$, there is a path $(i_1, \ldots, i_k)$ such that $i_1 = i, \ i_k = j$ and $\{i_1, \ldots, i_k\} \subseteq S$. A maximally connected subset of coalition S in (N, G) is called a *component* of S in that graph, i.e. $T \subseteq S$ is a component of S in (N, G) if and only if T is connected in $(N, G(S))$ and for every $h \in S \setminus T$ the coalition $T \cup \{h\}$ is not connected in $(N, G(S))$, where $G(S) = \{\{i, j\} \in G | \{i, j\} \subseteq S\}$ is the set of links between players in S. A sequence of players $(i_1, \ldots, i_k, i_1)$ is a *cycle* in (N, G) if $(i_1, \ldots, i_k)$ is a path in (N, G), and $\{i_k, i_1\} \in G$. A graph (N, G) is *cycle-free* when it does not contain any cycle. A player $i \in N$ is called a *pending player* if it is connected to exactly one other player, i.e. if $|\{g \in G \mid i \in g\}| = 1$. Note that a cycle-free communication graph has at least two pending players. A graph that is connected and cycle-free is called a *tree*.

A triple (N, v, G) with (N, v) a TU-game and (N, G) an undirected graph on N is called a *communication situation*. In the communication situation (N, v, G) players can cooperate if and only if they are able to communicate with each other, i.e. a coalition S is feasible if and only if it is connected in (N, G). In other words, the set of feasible coalitions in a communication situation (N, v, G) is the set of coalitions $\mathcal{F}_G \subseteq 2^N$ given by

$$\mathcal{F}_G = \{S \subseteq N \mid S \text{ is connected in } (N, G)\}.$$

We refer to this set as the *communication feasible set* of communication graph (N, G). Myerson (1977) introduces the *restricted game* of a communication situation (N, v, G) as the TU-game (N, v_G) in which every feasible coaliton S can earn its worth $v(S)$. Whenever S is not feasible it can earn the sum of the worths of its components in (N, G). Denoting the components of $S \subseteq N$ in (N, G) by $C_G(S)$, the restricted game (N, v_G) corresponding to communication situation (N, v, G) thus is given by $v_G(S) = \sum_{T \in C_G(S)} v(T)$ for all $S \subseteq N$. As a solution Myerson (1977) proposes to take for every communication situation the *Shapley value* (Shapley (1953)) of the corresponding restricted game, a solution that is later named the *Myerson value* for communication situations. Alternatively, Le Breton, Owen and Weber (1992) and Demange (1994, 2004) consider the Core of the restricted game for the special class of communication situations where the game is superadditive and the communication graph is cycle-free, respectively, a tree.

2.2 Hierarchies

The concept of restricted communication as reviewed in the previous subsection is widely accepted in the literature. So, the ability to communicate is considered to be fully determined by the connectedness of the players in the communication graph. But what do we mean when we speak about a hierarchy? Although, as mentioned in the introduction, in the literature various attempts to capture the idea of a hierarchy are made, the concept of hierarchy does not seem to be so clearly understood as communication. In the field of restricted cooperation a model that tries to answer this question is that of a *(cooperative) game with a permission structure*. In a game with a permission structure it is assumed that players who participate in a cooperative TU-game are part of a hierarchical organization in which there are players that need permission from certain other players before they are allowed to cooperate. For a finite set of players N such a hierarchical organization is represented by a directed graph (N, D) with $D \subseteq N \times N$, referred to as a *permission structure* on N. The directed links $(i, j) \in D$ are called *arcs*. The players in $F_D(i) := \{j \in N \mid (i, j) \in D\}$ are called the *followers* of player i, while the players in $P_D(i) := \{j \in N \mid (j, i) \in D\}$ are called the *predecessors* of i. (Note that $j \in F_D(i)$ if and only if $i \in P_D(j)$.) A sequence of different players $(i_1, \ldots, i_k)$ is a *directed path* between players i and j, $i \neq j$, in a permission structure (N, D) if $i_1 = i$, $i_k = j$ and $(i_h, i_{h+1}) \in D$ for all $1 \leq h \leq k - 1$. Here we only consider *acyclic* permission structures, i.e. we assume that there exists no directed path $(i_1, \ldots, i_k)$ with $(i_k, i_1) \in D$. Note that in an acyclic permission structure D there always exists at least one player with no predecessors, i.e. $TOP(D) := \{i \in N \mid P_D(i) = \emptyset\} \neq \emptyset$. We refer to these players as the *top-players* in the permission structure.

Two approaches to games with a permission structure are considered. In the conjunctive approach as developed in Gilles, Owen and van den Brink (1992) and van den Brink and Gilles (1996), it is assumed that each player needs permission from *all* its predecessors before it is allowed to cooperate. This implies that a coalition $S \subseteq N$ is feasible if and only if for every player in the coalition it holds that all its predecessors belong to the coalition. The set of feasible coalitions in this approach thus is given by

$$\Phi_D^c := \{S \subseteq N \mid P_D(i) \subset S \text{ for all } i \in S\},$$

which we refer to as the *conjunctive feasible set* of D.

Alternatively, in the disjunctive approach as developed in Gilles and Owen (1994) and van den Brink (1997), it is assumed that each player (except the top-players) needs permission from *at least one* of its predecessors before it is allowed to cooperate with other players. Consequently, a coalition is feasible if and only if every player in the coalition (except the top-players) has at least one predecessor who also belongs to the coalition. Thus, the feasible coalitions are the ones in the set

$$\Phi_D^d := \{S \subseteq N \mid P_D(i) \cap S \neq \emptyset \text{ for all } i \in S \setminus TOP(D)\},$$

which we refer to as the *disjunctive feasible set* of D.

An approach using restricted games similar to the approach described in the previous subsection for communication situations assigns to every coalition in a game with a permission structure the worth of its largest feasible subset[2].

[2] These largest feasible subsets are well defined by the sets Φ_D^c and Φ_D^d being closed under union.

Example 2.1 For the permission structure (N, D) given by $N = \{1, 2, 3, 4\}$ and $D = \{(1, 2), (1, 3), (2, 4), (3, 4)\}$ we have $\Phi_D^c = \{\{1\}, \{1, 2\}, \{1, 3\}, \{1, 2, 3\}, \{1, 2, 3, 4\}\}$ and $\Phi_D^d = \Phi_D^c \cup \{\{1, 2, 4\}, \{1, 3, 4\}\}$. □

Why do these models capture the idea of a hierarchy? Since we are in the field of restricted cooperation, the answer must be found in the properties of the feasible set. Algaba, Bilbao, van den Brink and Jiménez-Losada (2004) show that the conjunctive and disjunctive feasible sets are *antimatroids* being sets of feasible coalitions satisfying the following properties (see, Dilworth (1940) and Edelman and Jamison (1985)). A set of feasible coalitions $\mathcal{F} \subseteq 2^N$ satisfies *accessibility* if every nonempty feasible coalition has at least one player that can leave the coalition leaving behind a feasible subcoalition, i.e. $S \in \mathcal{F}$, $S \neq \emptyset$, implies that there exists an $i \in S$ such that $S \setminus \{i\} \in \mathcal{F}$. A set of feasible coalitions $\mathcal{F} \subseteq 2^N$ satisfies *closedness under union* if the union of any two feasible coalitions is also feasible, i.e. $S, T \in \mathcal{F}$ implies that $S \cup T \in \mathcal{F}$. Together with the empty set being feasible these two properties define an antimatroid. Additionally we require the feasible set $\mathcal{F} \subseteq 2^N$ to be *normal* meaning that every player belongs to at least one feasible coalition, i.e. for every $i \in N$ there exists an $S \in \mathcal{F}$ such that $i \in S$.

Definition 2.2 *A set of feasible coalitions $\mathcal{F} \subseteq 2^N$ is a* **normal antimatroid** *if it contains the empty set and satisfies normality, accessibility and closedness under union.*

A player $i \in S$ is called an *endpoint* of $S \in \mathcal{F}$ if $S \setminus \{i\} \in \mathcal{F}$. Note that by accessibility every feasible coalition in an antimatroid has at least one endpoint. Clearly, antimatroids have some hierarchical flavour. Besides the example of permission structures mentioned above, another example is given by ordered partition voting introduced in Section 4. But do they fully capture the idea of a hierarchy? This question is still unanswered. But let us for the moment compare antimatroids with communication feasible sets.

3 A comparison between hierarchies and communication

Looking at the properties of normal antimatroids, it can easily be verified that communication feasible sets contain the empty set and satisfy normality and accessibility. In fact, they satisfy the stronger *2-accessibility* meaning that every feasible coalition with two or more players has at least two players that can leave the coalition leaving behind a feasible coalition, i.e. $S \in \mathcal{F}$ with $|S| \geq 2$, implies that there exist $i, j \in S$, $i \neq j$, such that $S \setminus \{i\}, S \setminus \{j\} \in \mathcal{F}$. Communication feasible sets are not closed under union. However, Algaba, Bilbao, Borm and López (2001) show that they satisfy the weaker property of *union stability* meaning that the union of two feasible coalitions having a nonempty intersection is also feasible, i.e. $S, T \in \mathcal{F}$ with $S \cap T \neq \emptyset$ implies that $S \cup T \in \mathcal{F}$. It turns out that weakening closedness under union to union stability, and strengthening accessibility to 2-accessibility characterizes the communication feasible sets.

Theorem 3.1 *Let $\mathcal{F} \subseteq 2^N$ be a set of feasible coalitions. Then $\mathcal{F}$ is the communication feasible set of some communication graph if and only if $\mathcal{F}$ contains the empty set and satisfies normality, 2-accessibility and union stability.*

Proof. A communication feasible set containing the empty set and satisfying normality and union stability follows from Algaba, Bilbao, Borm and López (2001). To show that it satisfies 2-accessibility, let $S \subseteq N$ be connected in communication graph (N, G) with $|S| \geq 2$. If $(N, G(S))$ contains a cycle, say $(i_1, \ldots, i_k)$, $k \geq 3$, then for each pair of consecutive players (i_h, i_{h+1}), $h \leq k-1$, in this cycle it holds that $S \setminus \{i_h, i_{h+1}\}$ is connected in (N, G), and thus i_h and i_{h+1} are both endpoints of S. Otherwise, if $(N, G(S))$ is cycle-free then there are at least two pending players in $(N, G(S))$, say i_1 and i_2. But then $S \setminus \{i_1, i_2\}$ is connected in (N, G) (or the empty set if $|S| = 2$), and thus i_1 and i_2 are both endpoints of S.

To show that every feasible set satisfying the properties mentioned in the theorem implies that it must be the communication feasible set of some communication graph, suppose that $\mathcal{F} \subseteq 2^N$ satisfies these properties. We must prove that there is a communication graph (N, G) such that $\mathcal{F}$ is the set of connected coalitions in (N, G), i.e. $\mathcal{F} = \mathcal{F}_G$. Take $G^{\mathcal{F}} = \{S \in \mathcal{F} \mid |S| = 2\}$. By $\mathcal{F}$ containing the empty set and satisfying normality it is sufficient to show that for every $S \subseteq N$ with $|S| \geq 2$, it holds that $S \in \mathcal{F}$ if and only if S is connected in $(N, G^{\mathcal{F}})$.

(i) Take $S \in \mathcal{F}$. We prove that S is connected in $(N, G^{\mathcal{F}})$ by induction on $|S|$. If $|S| = 2$ then S is connected in $(N, G^{\mathcal{F}})$ by definition of $G^{\mathcal{F}}$. Proceeding by induction, suppose that $S' \in \mathcal{F}$ is connected in $(N, G^{\mathcal{F}})$ whenever $|S'| < |S|$. By 2-accessibility

there exist $i, j \in S$ such that $S \setminus \{i\}, S \setminus \{j\} \in \mathcal{F}$. The induction hypothesis implies that $S \setminus \{i\}$ and $S \setminus \{j\}$ are both connected in $(N, G^{\mathcal{F}})$. But then there exists an $h \in S \setminus \{i, j\}$ such that there is a path from i to h and from j to h using only players from S. This implies that there is a path from i to j in S, and thus S is connected in $(N, G^{\mathcal{F}})$.
(ii) Take $S \subseteq N$ connected in $(N, G^{\mathcal{F}})$. We must prove that $S \in \mathcal{F}$. If $|S| = 2$ then $S \in \mathcal{F}$ by definition of $G^{\mathcal{F}}$. If $|S| > 2$ then S is the union of all links in S, i.e. $S = \bigcup\{\{i, j\} \in G^{\mathcal{F}} \mid \{i, j\} \subset S\}$. Since all these links $\{i, j\}$ belong to $\mathcal{F}$ by definition of $G^{\mathcal{F}}$, and S is connected, union stability implies that $S \in \mathcal{F}$. □

Obviously, closedness under union implies union stability, and 2-accessibility implies accessibility. Thus, comparing Theorem 3.1 with Definition 2.2, the difference between antimatroids and communication feasible sets is that antimatroids satisfy the stronger union property, while communication feasible sets satisfy the stronger accessibility property. Note that, given accessibility and closedness under union, normality implies that $N \in \mathcal{F}$ as is the case for antimatroids (and thus for conjunctive and disjunctive feasible sets). Given 2-accessibility and union stability, normality implies that $\{i\} \in \mathcal{F}$ for all $i \in N$ as is the case for communication feasible sets.

As mentioned in the introduction, Demange (2004) claims that hierarchies can lead to group stability in case the game is superadditive[3]. To verify if that paper is really about hierarchies we now can verify the properties of the feasible set. In Demange (2004) the players belong to a hierarchy which can be represented by a directed graph with a tree structure. In terms of acyclic permission structures this means that there is a unique top-player $i_0 \in N$ and all other players have exactly one predecessor, i.e. $|TOP(D)| = 1$ and $|P_D(i)| = 1$ for all $i \in N \setminus TOP(D)$. Consequently, for every $i \in N \setminus \{i_0\}$ there is a directed path from i_0 to i. (Note that in this case the conjunctive and disjunctive feasible sets are the same.) In Demange (2004) the feasible sets (or *teams*) are those coalitions S such that for every pair of players $i, j \in S$ either there is a directed path from i to j, or there is a directed path from j to i, or there is another player $h \in S \setminus \{i, j\}$ such that there is a directed path from h to i and from h to j. In fact, as also mentioned by Demange (2004), the feasible sets are exactly the communication feasible sets in the underlying undirected communication graph (N, G) with $G = \{\{i, j\} \subseteq N \mid i \neq j \text{ and } \{(i, j), (j, i)\} \cap D \neq \emptyset\}$. Note that this

[3]A TU-game (N, v) is superadditive if $v(S \cup T) \geq v(S) + v(T)$ for all $S, T \subseteq N$ with $S \cap T = \emptyset$.

communication graph is a tree.

We end this section by characterizing the feasible sets that can be communication feasible sets as appearing in Demange (2004), i.e. that can arise from communication graphs with a tree structure. In order to do that we generalize paths in communication structures. Let $\mathcal{F} \subseteq 2^N$. A coalition $S \in \mathcal{F}$ is a *2-path* in $\mathcal{F}$ if it has exactly two endpoints. The 2-path $S \in \mathcal{F}$ is called a *$\{i,j\}$-path* in $\mathcal{F}$ if it has i and j as endpoints. Now, a set of feasible coalitions $\mathcal{F} \subseteq 2^N$ satisfies the *2-path property* if for every $i,j \in N$, $i \neq j$, there is at most one $\{i,j\}$-path. Finally, we say that a set of feasible coalitions $\mathcal{F} \subseteq 2^N$ is *connected* if for every $i,j \in N$ there is an $S \in \mathcal{F}$ with $\{i,j\} \subseteq S$.

Theorem 3.2 *Let $\mathcal{F} \subseteq 2^N$ be a set of feasible coalitions. Then*
(i) $\mathcal{F}$ is the communication feasible set of some cycle-free communication graph if and only if $\mathcal{F}$ contains the empty set and satisfies normality, 2-accessibility, union stability and the 2-path property.
(ii) $\mathcal{F}$ is the communication feasible set of some communication tree if and only if $\mathcal{F}$ contains the empty set and satisfies normality, 2-accessibility, union stability, the 2-path property and is connected.

Proof. (i) Let (N,G) be a cycle-free communication graph. $\mathcal{F}_G$ containing the empty set and satisfying normality, 2-accessibility and union stability follows from Theorem 3.1. $\mathcal{F}_G$ satisfying the 2-path property follows since the unique $\{i,j\}$-path in $\mathcal{F}_G$, $i,j \in N$, is the path $(i_1,\dots,i_k)$ in (N,G) with $i_1 = i$ and $i_k = j$.
To prove the 'if' part, suppose that $\mathcal{F} \subseteq 2^N$ satisfies the properties mentioned in statement (i) of the theorem. We already showed in Theorem 3.1 that $\mathcal{F}$ is the set of communication feasible coalitions corresponding to a communication graph (N,G). Suppose that (N,G) has a cycle. Then there exists a sequence of different players $(i_1,i_2,...,i_k) \in N$ such that $\{i_k,i_1\} \in G$. Take any $l \in \{2,...,k\}$. Then $\{i_1,...,i_l\}$ and $\{i_l,...,i_k,i_1\}$ both are $\{i_1,i_l\}$-paths in $\mathcal{F}$, yielding a contradiction with the 2-path property.

(ii) Let (N,G) be a communication tree. $\mathcal{F}_G$ containing the empty set and satisfying normality, 2-accessibility, union stability and the 2-path property follows from statement (i) of the theorem. $\mathcal{F}_G$ satisfying connectedness follows since $N \in \mathcal{F}_G$ if G is a tree.

To prove the 'if' part, suppose that $\mathcal{F} \subseteq 2^N$ satisfies the properties mentioned in statement (ii) of the theorem. Above we already showed that $\mathcal{F}$ is the set of communication feasible coalitions corresponding to a cycle-free communication graph (N, G). Suppose that (N, G) is not connected. Then there are at least two components T^1, T^2 in (N, G). Take $i \in T^1$ and $j \in T^2$. Since every $S \subseteq N$ with $\{i, j\} \subseteq S$ is not feasible, the communication feasible set $\mathcal{F}_G$ is not connected, yielding a contradiction. □

It can be verified that the feasible sets in Demange (2004) satisfy the properties of Theorem 3.2.(ii), and thus we conclude that paper is about communication and not about hierarchies.

4 Another hierarchical structure: ordered partition voting

We claimed that antimatroids express hierarchical structures and we discussed permission structures as examples. Question is if antimatroids are really more general than permission structures. Put differently, are there antimatroids that cannot be the conjunctive or disjunctive feasible set of some acyclic permission structure. The answer is yes. An example is *ordered partition voting* which describes a situation in which there is an ordered partition of the player set N, such that different levels of approval are distinguished. To activate players in a particular level, a qualified majority approval in every higher level is necessary. Formally, an ordered partition voting situation is a triple (N, P, q) where $P = (P_1, ..., P_m)$ is an ordered partition of the player set N (i.e. $P_k \cap P_l = \emptyset$ for all $k, l \in \{1, \ldots, m\}$, $k \neq l$, and $\bigcup_{k=1}^m P_k = N$), and for each 'level' P_k, $k \in \{1, ..., m-1\}$, in the partition there is a quota $q_k \in \mathbb{N}$. Now, a coalition S is feasible if and only if for all elements of the partition P, except the lowest level that is represented in S, at least the quota is present. For $S \subseteq 2^N$ let $l(S) = \max\{l \in \{1, ..., m\} | S \cap P_l \neq \emptyset\}$ be the lowest level present in S. Given ordered partition voting situation (N, P, q) with $P = (P_1, ..., P_m)$ and $q = (q_1, \ldots, q_{m-1})$, the set of feasible coalitions $\mathcal{F}_{(P,q)}$ is defined as

$$\mathcal{F}_{(P,q)} = \{S \subseteq N | \text{ for all } k \in \{0, 1, ..., l(S) - 1\} \text{ it holds that } |S \cap P_k| \geq q_k\},$$

where $P_0 = \emptyset$ and $q_0 = 0$.

Theorem 4.1 *Let (N, P, q) be an ordered partition voting situation. Then $\mathcal{F}_{(P,q)}$ is a normal antimatroid on N.*

Proof. Clearly, the empty set belongs to $\mathcal{F}_{(P,q)}$. Normality of $\mathcal{F}_{(P,q)}$ follows since N is feasible. Accessibility of $\mathcal{F}_{(P,q)}$ follows since $S \in \mathcal{F}_{(P,q)}$ and $i \in S \cap P_{l(S)}$ implies that $S \setminus \{i\} \in \mathcal{F}_{(P,q)}$. To show that $\mathcal{F}_{(P,q)}$ is closed under union take $S, T \in \mathcal{F}_{(P,q)}$. Then $l(S \cup T) = \max\{l(S), l(T)\}$, and $|(S \cup T) \cap P_{l(S \cup T)}| = |(S \cap P_{l(S\cup T)}) \cup (T \cap P_{l(S\cup T)})| \geq q_{l(S\cup T)}$ since $\max\{|(S \cap P_{l(S\cup T)})|, |(T \cap P_{l(S\cup T)})|\} \geq q_{l(S\cup T)}$. □

Besides showing that for every permission structure (N, D) it holds that Φ^c_D and Φ^d_D are normal antimatroids, Algaba, Bilbao, van den Brink and Jiménez Losada (2004) also characterize those antimatroids that can be the conjunctive or disjunctive feasible set of some acyclic permission structure. They use the following notions for a feasible set $\mathcal{F} \subseteq 2^N$ which generalize the concept of a directed path in a permission structure. A coalition $S \in \mathcal{F}$ is a *path* in $\mathcal{F}$ if it has a unique endpoint. (Recall from Section 2 that a feasible coalition with exactly two endpoints was called a 2-path.) The path $S \in \mathcal{F}$ is called a *i-path* in $\mathcal{F}$ if it has $i \in S$ as its unique endpoint. The conjunctive feasible set of any acyclic permission structure is a normal antimatroid such that every player $i \in N$ has a unique i-path in $\mathcal{F}$.[4] Clearly, this property is not satisfied by all disjunctive feasible sets as can be seen from Example 2.1 where $\{1, 2, 4\}$ and $\{1, 3, 4\}$ are both 4-paths in Φ^d_D. Further Algaba, Bilbao, van den Brink and Jiménez Losada (2004) show that the disjunctive feasible set of any acyclic permission structure is a normal antimatroid such that deleting the unique endpoint of any path leaves behind a feasible coalition that is again a path. This property is not satisfied by all conjunctive feasible sets as can be seen from Example 2.1 where $\{1, 2, 3, 4\}$ is the unique 4-path in Φ^c_D, but $\{1, 2, 3\}$ is not a path. The antimatroids $\mathcal{F}_{(P,q)}$ that are obtained from ordered partition voting situations need not satisfy these additional path properties, and thus cannot be the conjunctive or disjunctive feasible set of some acyclic permission structure.

Example 4.2 Consider $N = \{1, 2, 3, 4, 5\}$, $P = (P_1, P_2)$ with $P_1 = \{1, 2, 3\}$ and $P_2 = \{4, 5\}$, and $q_1 = 2$. The set of feasible coalitions $\mathcal{F}_{(P,q)}$ consists of all subsets of $\{1, 2, 3\}$,

[4]In fact, this additional property characterizes those normal antimatroids that can be the conjunctive feasible set of some acyclic permission structure. Such antimatroids are also known as *poset antimatroids*. Alternatively they are characterized as those normal antimatroids that are *closed under intersection*.

the coalitions in the set $\{\{1,2,4\},\{1,2,5\},\{1,3,4\},\{1,3,5\},\{2,3,4\},\{2,3,5\}\}$ and all coalitions with at least four players. Then $\{1,2,4\},\{1,3,4\},\{2,3,4\} \in \mathcal{F}_{(P,q)}$ are 4-paths, so $\mathcal{F}_{(P,q)}$ cannot be the conjunctive feasible set of some acyclic permission structure. Consider the 4-path $S = \{1,2,4\}$. Since $S \setminus \{4\} = \{1,2\}$ is not a path, $\mathcal{F}_{(P,q)}$ cannot be the disjunctive feasible set of some acyclic permission structure. □

5 Concluding remarks

In this paper we characterized the communication feasible sets that can be the set of connected coalitions in a communication network and compared these properties to those of an antimatroid which expresses a hierarchical structure. We showed that these sets differ with respect to the accessibility and union property they satisfiy, where communication feasible sets satisfy the stronger accessibility property, while hierarchies (i.e. antimatroids) satisfy the stronger union property.

These characterizations imply that Demange (2004) makes a conceptual mistake by claiming that hierarchies can lead to group stability, whereas it is shown that restricted communication yields stability in case the game is superadditive and the communication graph has a tree structure. As also follows from Section 4 of Demange (2004), this result should be restated in the sense that the *hierarchical outcomes*[5] that are defined in that paper are extreme points of the Core of the restricted game (N, v_G). As a consequence we conclude that the results in Demange (2004) are about communication networks and not about hierarchies. Nonemptyness of the core under these circumstances has already been shown in Le Breton, Owen and Weber (1992) and Demange (1994). Demange (2004) already mentions that the Core of the restricted game can have extreme points that are not hierarchical outcomes. This leads to other interesting questions such as what properties has the set valued solution that assigns to every superadditive game with a communication tree (or more general cycle-free graph) the convex hull of the hierarchical outcomes. Clearly, by Demange (2004)'s results this is a nonempty subset of the core of the restricted game (N, v_G).

Under weaker conditions on the communication graph but stronger conditions on the game, van den Nouweland and Borm (1991) also show nonemptyness of the Core of the restricted game. In particular, they show that the restricted game is

[5]We refer to Demange (2004) for the definition of hierarchical outcome.

convex[6] whenever the original game is convex and the communication graph is cycle-complete[7]. Since every convex game has a nonempty Core, this implies that the Core of the restricted game (N, v_G) is nonempty under these conditions.

Results on games with a permission structure and solutions can be found in the above mentioned literature. In particular, van den Brink and Gilles (1996) and van den Brink (1997) give axiomatic characterizations of *permission values* being solutions of games with a permission structure that are obtained by applying the Shapley value to certain restricted games, in a similar way as done in Myerson (1977) for communication situations. The distinction between conjunctive and disjunctive permission structures is closely related to the concepts of hierarchies and polyarchies in the sense of Sah and Stiglitz (1986) who consider a collective decision maker that has to choose whether to accept or reject a project proposal. In a *hierarchy* a project proposal is accepted if and only if all individuals accept. As an alternative to a hierarchy, they also consider a *polyarchy* in which a project proposal is accepted if and only if at least one individual accepts. In this sense a polyarchy as a collective decision making organization is closely related to the disjunctive approach, while a hierarchy is closely related to the conjunctive approach.

In the particular case of firm hierarchies games with a permission structure have the same basic assumption as formulated by Rajan and Zingales (1998, 2001) who put the *control of access to a productive asset* as a central issue in firm hierarchies. This in cotrast to models of *incomplete contracts* which tries to explain the distribution of residual rights concerning the control over non-contractable assets taking the *ownership of assets* as a central feature (see, e.g., Grossman and Hart (1986), Hart and Moore (1990, 1999), and Maskin and Tirole (1999)).

In Hart and Moore (2005) a hierarchy determines decision-making authority with respect to different assets in the sense that the higher an agent is in the chain of command of a certain asset, the more easy it can decide on the use of this asset. Their assumption that 'Access to assets is determined by a hierarchical structure. That is, each asset a_k has a chain of command, a list L_k, that ranks agents by seniority over that asset' differs from 'permission' in the sense that in our framework a senior coordinator exercises authority when a junior coordinator must ask its approval. Junior

[6] A TU-game (N, v) is convex if $v(S \cup T) + v(S \cap T) \geq v(S) + v(T)$ for all $S, T \subseteq N$.

[7] A communication graph is cycle-complete if, whenever there is a cycle, the subgraph restricted to the players in that cycle is complete, i.e. if there is a cycle $(i_1, \ldots, i_k, i_1)$ then $\{\{i,j\} \mid \{i,j\} \subseteq \{i_1, \ldots, i_k\}\} \subseteq G$.

coordinators obtain access to the asset when they have approval from their senior superiors (in the conjunctive or disjunctive sense). In Hart and Moore (2005) a junior coordinator gets access only if its more senior coordinators do not exercise authority over the asset.

We do not claim that we developed a full theory of restricted cooperation, including communication and hierarchy, in this paper. However, we started with comparing these two organizational structures which belong to the most encountered in economic and political organizations. We agree with Chwe (2000) that although 'collective action depends on both social structure and individual incentives, these integral aspects have been formalized separately, in the fields of social network theory and game theory'. Whereas Chwe (2000) considers these integral aspects together in a noncooperative model, in this paper we took a step to consider communication and hierarchy together in a cooperative approach. This is necessary to put various models on relational structures in economic and political organizations into perspective and to build a consistent organization theory. Game theory provides useful tools for this purpose.

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