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The notion that social networks in labor markets play a critical role is intuitively appealing and has attracted attention by the literature at least since Mark S. Granovetter (1973). The empirical literature has shown that reliance on informal methods varies across demographic groups, but has not yet fully clarified how the pattern of employment and earnings payoffs to networks varies across groups [Yannis M. Ioannides and Linda D. Loury (2004)]. Recent theoretical advances, including Antoni Calvó-Armengol (2004), whose antecedent is Scott A. Boorman (1975), and Calvó-Armengol and Matthew O. Jackson (2004) show that social networks may explain salient characteristics of the labor market such as positive correlation

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of employment across agents and time and duration dependence in the likelihood of obtaining a job.

Only about one-third of overall earnings inequality may be explained by individual characteristics as gender, education and age [Laurence F. Katz and David H. Autor (1999)]. Residual or within-group inequality, which has notably increased for the United States in the 1980s and 1990s, has been attributed to search frictions, *inter alia*. Such frictions may cause otherwise identical workers to earn different wages [Dale T. Mortensen (2003)].

This paper shows analytically that workers who are better connected socially experience lower unemployment rates and receive higher wage rates in the average. It represents social connections in the labor market by a random graph, with nodes as individuals and edges as connections. The number of others each individual is connected with, the "degree," obeys an arbitrary probability distribution. In this fashion this paper extends previous research that in some cases assumes *complete* networks (everyone connected to everyone else) and in other cases *balanced* networks (everyone connected to an equal number of others) [*c.f.* Calvó-Armengol and Yves Zenou (2005), C-A&Z below, for short].

I. Job Matching in a Random Social Network

We follow Christopher A. Pissarides (2000), as adapted by C-A&Z, and consider a large number of workers who are *ex ante* identical with respect to job performance. Similarly, firms are identical with respect to productivity. At the beginning of each discrete-time period, each worker may receive information about a job opening directly from an employer, with probability equal to the vacancy rate, v. Jobs break up at the end of each period with constant probability δ. If a worker is *employed* when the job opening information arrives, she passes it on to a randomly selected *unemployed* acquaintance. If none of her acquaintances are unemployed, the information is lost. Thus unemployed workers receive job information either directly or indirectly. Newly employed workers go through an one-period probation, during which their earnings are equal to y_0, which without loss of generality is set equal to 0. In the following period, workers' productivity becomes $y_1 > 0$ and stays at that level for the duration of employment. Thus newly employed workers have no incentive to use new job openings to increase their current wage. Employers have perfect information about workers' social connections when wage bargaining takes place.

Agents are matched with other workers by being embedded in an exogenous but random social network. Unlike C-A&Z, workers differ with respect to the number k of other workers each is in contact with: k, a worker's degree, has a frequency distribution function denoted by $\mathbf{p} = (p_0, p_1, \ldots, p_k, \ldots)$. Having more links is better for an individual but not necessarily socially efficient, as C-A&Z show, because vacancies may not be filled due to coordination failure. However, we show that this result is sensitive to the nature of the social network. E.g., in the case of a Poisson degree distribution the monotonicity of matching is effectively restored.

A property of our model is noteworthy. The probability that a particular worker, chosen randomly from among a given worker's contacts, is connected with k other workers is *not* equal to p_k [Mark E. J. Newman (2003)]. To see this consider that a worker with m other contacts is m times more likely to be reached than for a worker with one contact. So, the degree distribution of an worker thus selected is $f_k \equiv \frac{kp_k}{\sum_j jp_j}$. This *connection bias* is conceptually akin to length-biased sampling in unemployment statistics.

Connection bias allows us to derive that a randomly chosen social contact of a typical worker transmit job opening information to her with probability:

$$\tilde{q}(\bar{u}, \mathbf{p}, \mathbf{u}) = \frac{1}{\bar{u}E_k(k)} E_k \left\{ (1 - u^k) \left[1 - (1 - \bar{u})^k \right] \right\}, \tag{1}$$

where the probability for one's contacts to be unemployed may depend on the number of their own contacts j, u^j, and write $\mathbf{u} = (u^0, u^1, \ldots)$. The probability for a worker with λ contacts to receive job opening information through her social contacts is given by

$$P(\lambda, \bar{u}, v, \mathbf{p}, \mathbf{u}) = 1 - [1 - v\tilde{q}]^\lambda . \tag{2}$$

Note that this probability is decreasing with the covariance between a worker's employment rate and the employment rate of a worker's contacts. The larger this covariance, the higher the probability that an unemployed individual would hear from her social contacts about job openings. The higher the mean unemployment rate, the less important is the effect from the other contacts of one's own contacts, because they are themselves more likely to need the information.

The properties of probability function $P(\lambda, \bar{u}, v, \mathbf{p}, \mathbf{u})$ are discussed in the remainder of this paper for a special case of (1), when each person assumes her contacts' unemployment rates are equal to the mean unemployment. This probability is, one, increasing and concave in the number of a worker's contacts; two, decreasing in the mean unemployment rate in the economy; and three, increasing in the vacancy rate. In contrast to C-A&Z, Ioannides and Adriaan R. Soetevent (2005), I&S for short, show that the function $P(\cdot)$'s being convex in $\bar{u}$ over some range depends on whether one's own number of contacts does not exceed a threshold value, which itself increases with the mean degree.

A. *The job matching function*

Defining the expected probability for a worker to hear of a vacancy either directly or indirectly as the *job matching function*, $m(\mathbf{u}, v, \mathbf{p}) = \sum_{\lambda=0}^{L-1} p_\lambda u_\lambda [v + (1-v)P(\lambda, \bar{u}, v, \mathbf{p})]$, we may write the rate at which vacancies are filled as:

$$\ell(\mathbf{u}, v, \mathbf{p}) = \frac{1}{v} \sum_{\lambda} p_\lambda u_\lambda h(\lambda, \bar{u}, v, \mathbf{p}), \tag{3}$$

where $h(\lambda, \bar{u}, v, \mathbf{p}) \equiv v + (1-v)P(\lambda, \bar{u}, v, \mathbf{p})$ stands for the probability that an unemployed worker with λ contacts hears of a vacancy. Proposition 2, I&S, *op. cit.*, proves that the probability that a worker hears of a vacancy is increasing in u_λ, and is increasing and strictly concave in the vacancy rate.

II. Labor Market Equilibrium

By adapting the Pissarides model as modified by C-A&Z, we work with the associated Bellman equations at the steady state for I_F^λ, the intertemporal profit for a job's being filled by a worker with λ contacts, and for I_V, the expected value of opening a vacancy at the beginning of a typical period. In our case the profit of a filled job depends, through the wage rate, on the number of contacts held by a worker who fills the job:

$$I_F^\lambda = y_1 - w_1^\lambda + \frac{1}{1+r}\left[(1-\delta)I_F^\lambda + \delta I_V\right],\ \forall\lambda; \tag{4}$$

$$I_V = -\gamma - \frac{1-\ell(\mathbf{u},v,\mathbf{p})}{1+r}I_V + \ell(\mathbf{u},v,\mathbf{p})\left[y_0 - w_0 + \frac{1}{1+r}\left((1-\delta)E_{g(\lambda)}\left[I_F^\lambda\right] + \delta I_V\right)\right], \tag{5}$$

where the expectation is taken with respect to $g(\lambda)$, a probability distribution function that is specified as follows. In order to account for connection bias — this time with regard to firms with vacancies being more likely to be filled by agents with more links — the expectation is taken with respect to the probability distribution function

$$g(\lambda;\mathbf{u},v,\mathbf{p}) = \frac{p_\lambda u_\lambda h(\lambda,\mathbf{u},v,\mathbf{p})}{m(\mathbf{u},v,\mathbf{p})}. \tag{6}$$

Equilibrium with free entry implies that the value to firms from opening a vacancy is driven down to 0, $I_V = 0$. Solving for I_F^λ yields: $I_F^\lambda = (y_1 - w_1^\lambda)\frac{1+r}{r+\delta}$. That is, the value of filling a vacancy is equal to the expected present value of a flow of net profit, adjusted for the probability of breakup. The labor demand equation is:

$$E_{g(\lambda)}\left[\left(y_1 - w_1^\lambda\right)\ell(\mathbf{u},v,\mathbf{p})\right] = \gamma\frac{r+\delta}{1-\delta}. \tag{7}$$

That is, the expected rate of profit per vacancy filled is equal to the amortized fixed costs of hiring, adjusted for the likelihood of jobs' breaking up.

A. *Wages*

Turning to the labor supply side, we index by λ the expected lifetime income of an employed and an unemployed worker at the beginning of a period and before vacancies are posted, I_E^λ and I_U^λ, respectively. The Bellman equations at the steady state imply that the gain for an unemployed worker with λ contacts from accepting employment is: $I_E^\lambda - I_U^\lambda = \frac{1+r}{r+\delta+(1-\delta)h(\lambda,\bar{u},v,\mathbf{p})}w_1^\lambda$.

The wage rate w_1^λ is determined from a Nash bargain, in which workers' power is denoted by $\beta \in [0,1]$. The wage rate maximizes $(I_E^\lambda - I_U^\lambda)^\beta(I_F^\lambda - I_V)^{1-\beta}$. This yields a wage rate schedule, conditional on the number of contacts by a worker:

$$w_1^\lambda = \frac{\beta(r+\delta) + \beta(1-\delta)h(\lambda,\bar{u},v,\mathbf{p})}{r+\delta+\beta(1-\delta)h(\lambda,\bar{u},v,\mathbf{p})}y_1. \tag{8}$$

It follows that a worker may extract a better bargain the more options she has in the form of a greater number of contacts. This is reflected in the wage bargain via the probability that an unemployed worker hears of a vacancy through the social structure. Also, the greater is β, a worker's bargaining power, the higher the wage rate.

B. *Steady-state labor market equilibrium*

Job creation and job destruction are assumed to take place as follows. At the beginning of each period, some of the unemployed find jobs. At the end of each period, employed workers, which includes newly hired and incumbent employees, lose their jobs because jobs break up randomly, with probability δ. At the steady state for each type of worker, the flow into unemployment from the breakup of jobs, $\delta p_\lambda(1-u_\lambda)$, equals that of out of unemployment because of job taking, $(1-\delta)p_\lambda u_\lambda h(\lambda,\bar{u},v,\mathbf{p})$. We thus arrive at individual Beveridge curves, one for each worker type:

$$u_\lambda = \frac{\delta}{\delta+(1-\delta)h(\lambda,\bar{u},v,\mathbf{p})},\ \forall\lambda. \tag{9}$$

This is an inverse relationship between unemployment and vacancy rates. Workers with more social contacts incur lower unemployment rates.

A steady state equilibrium of this economy must satisfy the labor demand equations, the individual Beveridge equations, and the wage functions, (7–9). However, it suffices to find equilibrium values of the unemployment rates for all worker types and the vacancy rate: $(\mathbf{u}^*, v^*)$. Using (8) in (7) and recalling the definition of the function $h(\cdot)$, yields a condition for the equilibrium vacancy rate:

$$E_{g(\lambda)}\left[\frac{\ell(\mathbf{u},v,\mathbf{p})}{r+\delta+\beta(1-\delta)[v+(1-v)P(\lambda,\bar{u},v,\mathbf{p})]}\right]y_1 = \frac{\gamma}{(1-\beta)(1-\delta)}. \tag{10}$$

I&S give sufficient conditions, in terms of an upper bound for the job breakup probability as a function of all parameters of the model and of a lower bound that involves endogenous variables, for existence and uniqueness of the labor market equilibrium given any degree distribution $\mathbf{p}$.

C. *Numerical Results for Poisson Degree Distributions*

We calibrate the model for Poisson degree distributions and with other parameters chosen, following Francois Fontaine (2005), so as to match US data. We find that as the network becomes denser (greater mean degree), mean unemployment falls and mean wage rate increases. Unemployment and wage rates of the least connected workers are adversely affected by increases in overall network density. See Figures 1 and 2. The average unemployment rate among workers without connections is three to four times as large as the unemployment rate of the most well connected. The wage rates of the latter are 15% to 25% higher. The equilibrium vacancy rate falls with network density, because of the higher wage rates firms have to pay due to workers' increased bargaining power. Interestingly, the matching function monotonically increases with network density. This is in contrast to C-A&Z, who identify a critical network density above which matching declines. So, at least with random graphs with Poisson distributions, the monotonicity of the Pissarides matching function is restored. See I&S for full details.

III. Conclusions

Wage dispersion in our model is generated by differences in the number of contacts workers have with other workers. We contrast by briefly considering closely related previous literature. Mortensen and Tara Vishwanath (1994) obtain wage dispersion because job offers obtained through referrals from employed social contacts are higher than those received directly from employers. Unlike our model, theirs does not incorporate competition for job-related information among an informed worker's social contacts, nor does it deal with workers' social networks as such. Variation in the number of links between *firms* and workers is emphasized by Kenneth J. Arrow and Ron Borzekowski (2004). They use simulations to show that 15% of the unexplained variation in wages may be explained by the number of ties between firms and workers. Fontaine, *op. cit.,* models the evolution of employment and wages in a complete social network. He shows by simulation that stochastic matching leads to a stationary distribution that is associated with significant wage differentials among otherwise identical workers. Samuel Bentolila, Claudio Michelacci and Javier Suarez (2004) test a Pissaridesstyle matching model that trades off higher productivity in the "formal" economy against an effect of personal contacts in shortening unemployment spells at the cost of a lower wage rate. Using United States and European Union data, they find a wage discount of 3% to 5% for jobs found through personal contacts. Their regressions control for industries and occupations, and for measures of cognitive ability and own demographic characteristics. They attribute the wage discount to occupational mismatch. Research that allows for individual background characteristics to influence connectedness, and combines with referrals, workers' links to firms and assortative matching deserves attention in the future.

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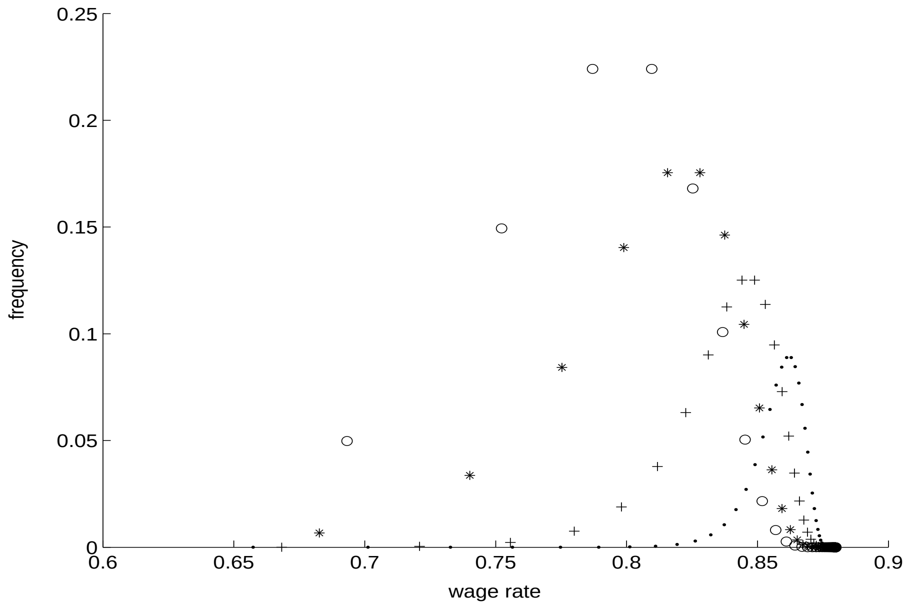

Figure 1: Wage dispersion when the degree distribution is Poisson, $p_k = e^{-\theta}\theta^k/k!; \theta = 3$ (o); $\theta = 5$ $(*)$; $\theta = 10$ $(+)$ and $\theta = 20$ $(\cdot)$.

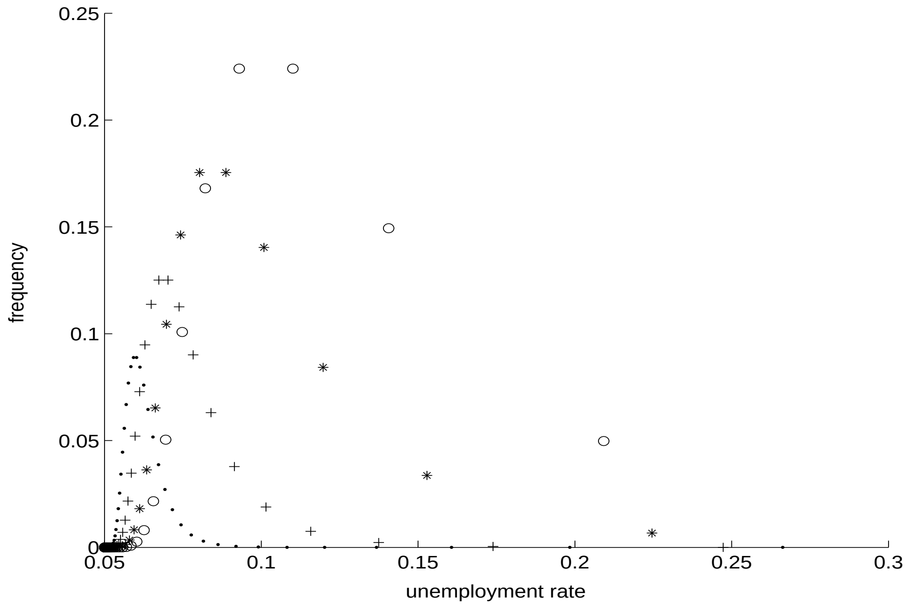

Figure 2: Unemployment distribution when the degree distribution is Poisson, $p_k = e^{-\theta}\theta^k/k!; \theta = 3$ (o); $\theta = 5$ $(*)$; $\theta = 10$ $(+)$ and $\theta = 20$ $(\cdot)$.