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Abstract

I present a model in which individuals compete for a prize by choosing to apply or not. Abilities are private information and in attempt to select the best candidate, the committee compares applicants with an imperfect technology. The choice of application cost, size of the prize and use of information technology are being characterized. In equilibrium, the number of applicants is stochastic and may overload the committee. I show that in spite of overload, the optimal cost (size of the prize) is decreasing (increasing) in market size. Furthermore I show when having a perfect information technology is not optimal.

JEL Classification: D82, D45

Keywords: asymmetric information, beauty contest design, award competition, information overload.

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1 Introduction

Consider a scientific committee that has the task to organize a contest for a research prize. Having no knowledge of the qualities of potential applicants, the committee faces the problem of advertising the prize and designing the application procedure in order to award the best candidate. With only a limited (time) capacity to read and extract information from the applications, the committee runs the potential risk of being overloaded with piles of applications, when too many individuals apply. At the same time, the committee wishes not to discourage applying too much, since in that case *nobody* will apply. The problem is thus to design the contest in such a way that attention will be devoted only to the better candidates in the population, while ensuring that some people do apply. The scholars know their own quality but are uncertain about the number of other applicants and their qualities. To decide whether to apply or not, individuals weigh their chances of winning and the size of the prize against the trouble of writing the application. To what extent should entry be troublesome?

It is this interaction that is the subject of this paper: the competition by a number of privately informed candidates for a single prize, awarded by a committee that has imperfect ability to rank applicants. This interaction characterizes a number of markets: examples include recruitment in the labour market, architectural competition, competition for licences, conference presentations and publications. I will answer the above question by relating the (optimal) cost of applying to the size of the market and the available selection technology.

Since the pioneering work of Spence (1973), asymmetric information in the above markets has been studied quite extensively, especially in the literature on signaling and mechanism design. There are two pillars that virtually all models in the traditional literature rest on: *(i)* the uninformed side of the market relies on incentives solely in order to mitigate informational asymmetry and *(ii)* these incentive mechanisms, like signaling, require the informed agents to be heterogeneous in some observable way, being for example their output or education level. Moreover, in the latter case the individual's cost of sending a certain signal must be correlated to the hidden information (*e.g.* ability), to enable separation.

In some markets, like prize competition, these two assumptions do not seem adequate. Turning to the first, it is widely observed that effort is spent on acquiring information about trading partners simply by investigating the candidates. For example, the assessment of job candidates is a widespread phenomenon. Firms try to discover ability of applicants by using interviews or assessment centers in their hiring decisions. In trade, buyers may audit and compare the features of different sellers. A program committee of a conference tries to select the best papers by *reading* the papers. [1]

Hence, it is plausible that for many types of information asymmetries, the uninformed uses some kind of information technology to obtain imperfect estimates of the other side of the market. Using these estimates, different candidates can then be compared. It is likely that the uninformed side of the market can hereby improve its decisions and this practice should be allowed for in a formal model of asymmetric information.

I now turn to the secondly mentioned pillar, taking education as an example of a signaling device, as in Spence (1973). I will argue that such a device may not always work and may leave the problem of asymmetric information unsolved. In the first place, when asserted that the cost of education decreases in ability, it is often overlooked that more able people incur higher opportunity costs of education, due to unfulfilled other profitable activities. When contented with this assertion, two additional objections can be made. Firstly, the contest may be specialized in such a way that the pool of potential applicants is *de facto* homogeneous. Think for example about a research grant for Ph.D. students: they all have a university degree and some working paper ready to send. Thus, sorting by education level may be too crude and not go far enough. Secondly, the timeframe of a contest does not always allow for applicants to adjust their (educational) choices to the menu of contracts offered by a firm. When the scientific committee announces the research prize, contestants will typically send existing papers, rather than write new ones. In conclusion, in real world markets costs may not always serve as a sorting device. [2]

I present a strategic model in which both aforementioned pillars are relaxed. In my framework, incentives and technology are both means of dealing with informational imperfections and I do not require applicants to have different costs of applying. The ability of each agent is modelled as an independent draw from a common pool of abilities. Each individual applicant knows his own ability, but does not know the abilities of the other applicants, nor the distribution of these rivals' abilities.

The committee strives for awarding the best candidate in the population, that is, the committee cares about the *rank* of the winner in the original population and is *ex ante* uninformed about individuals' abilities. It is thus implicit in my model that, although qualities are unknown in the selection stage, they matter later on. The assumption is natural in situations where the merit of the hired candidate is realized when he or she 'beats' the other candidates. The following example clarifies:

[1] In an overview article on signaling and screening, Riley (2001) asserts on this point that "*there is a strong incentive for the market to seek alternative means of information transmission* (for signaling or screening devices). *It is likely that in environments where this is the case, there will be evidence of direct testing, early monitoring, etc. -all provided to greatly reduce, if not eliminate, asymmetric information.*" (p. 474).

[2] Supporting this assertion, Riley (2001) emphasizes "*the need for further discussion of equilibrium in which screening/signaling costs are not perfectly negatively correlated with quality.*" (p. 475).

a university seeks to hire a PhD for a PostDoc position. Since publishing takes time, the quality of recent graduates is relatively hard to observe. After some years, however, it will become more clear which candidates have research potential, since time allowed them to publish. At that stage, quality is easier to observe and it is likely that the largest research funds and international standing can be gained only if the best candidate was hired.

In a sequential game, the committee announces a contest to a total of T agents. Participation in this contest is made troublesome by the committee and I denote the utility cost of applying by c. To control c, the committee can, for example, require all kinds of formalities to be fulfilled and numerous documents be sent. Then individuals decide whether to participate or not and finally, the committee processes N applications with a selection technology and allocates the award to one candidate.[3]

The endogenous choice of how many contestants to process must take into account the imperfect and costly nature of any information processing technology. I capture this fact by implicitly assuming a constraint on time, budget or attention available for processing information. More specifically, I model the allocation of scarce limited attention by letting the reliability of the ranking of candidates decrease in the quantity of applications processed. As a result, a trade-off emerges between including the best candidate in the investigation and identifying the best candidate. I will call the decreasing reliability of the ranking technology the *overload effect*.

My results about equilibrium behaviour in this game are the following. Firstly, to avoid overload, the contest organizer sometimes chooses to disregard some pieces of information and thus randomly chooses N applications out of the total number. The overload effect thus causes the contest organizer to leave unattended some potential trading alternatives.

Secondly, I find an equilibrium of 'favorable selection', in the sense that each candidate participates if and only if the ability is above some demarcation level. Separation emerges not due to some cost heterogeneity but due to the fact that $N > 1$: the winner is determined by comparison of informative estimates of ability.[4] Individuals with higher abilities have higher probabilities of winning and are therefore easier inclined to incur the cost c, while lower abilities will not find it worthwhile to apply. I will label this the *selection effect*. I thus introduce a new mechanism for a separating equilibrium to emerge.

Thirdly, the number of participants is stochastic in equilibrium and there is a *risk of delay* or breakdown in the contest. It might occur that in equilibrium, no candidate finds it worthwhile to participate as all candidates perceive their ability

[3] We abstract from the question whether ignoring applicants is possible, from an institutional perspective.

[4] The accuracy may be quite low.

level as too low, that is, below the demarcation level.

I finally study two design problems. The first insight is that the selection effect of c is used to balance the risk of delay and the overload effect. I then analyze how the demarcation level depends on the market size T and find that it increases in T, for fixed c. The latter finding implies that the larger the potential market, the smaller the expected share of the active market. In fact, for some parameters, the expected total number of applications may actually decrease in the number of potential applicants!

Thus the sheer number effect of market size T is not the only effect to consider when overload is feared: the competitive effect, due to strategic interaction, works in the opposite direction: it discourages applying. The above suggests that the risk of delay becomes an issue for larger markets. Indeed, even though the committee incurs overload, I find that the optimal application costs c decrease in the size of the market T.

Since in some contests not the trouble of applying (c) can be controlled easily, but committees rather invest in their selection technology, the second design problem I investigate is whether for fixed c and market size T, it is optimal to invest in a perfect selection technology. The surprising result I find is that when applying is troublesome enough, it is not optimal to have a perfect selection technology, even when this technology has zero costs. The reason is that the risk of delay becomes an issue when c is high and lowering the accuracy of the selection technology is then a way to dampen this risk.

The issues in this study relate to several strands in the literature. Firstly, it is related to the traditional signaling literature. I have already discussed the features that differentiate my analysis from this field. The paper that comes closest is Janssen (2002), who studies competition for a job and the effect of the number of competitors on signaling activity and the wage the firm sets.

Secondly, another vast area is that of contests and tournaments. A distinction should be made between mechanisms that have the purpose to select one candidate out of a group (*e.g.* whom to hire) and mechanisms that serve to induce a desired behaviour of all group members (*e.g.* work hard). The term 'contest' is often understood to include the latter purpose. I should therefore emphasize that I only study the selection role of contests. For the incentive role I refer to Lazear and Rosen (1981) and subsequent papers.[5]

Most papers that study contests as selection devices share the feature that contestants provide effort (or make expenditures) that increases their chances of winning the prize.[6] I have argued that a selection procedure does not always allow for con-

[5] A more recent contribution is Clark and Riis (1998) who study competition over more than one prize.

[6] Typically some contest success function is assumed that increases in effort or rent-seeking ex-

testants to adjust their efforts. In my setup, contestants merely decide whether to apply or not and it excludes effort considerations.

Thirdly, there is some literature that discusses imperfect information acquisition. A study that comes close to my model is Ficco (2004). His emphasis is on the conditions for information overload to occur in equilibrium, defined as a situation in which the number of candidates that applies is larger than the number that will be processed. An important feature that differentiates it from my work, is that the distribution of the rivals' abilities is known. Hence, there is no risk that nobody applies, and consequently the effect of costs to invoke a selection effect is unambiguous and is not an issue in his model. Malueg and Yongsheng Xu (1997) investigate the optimal acquisition of information in order to assign a worker to a job. Information acquisition also plays a role for prospective home buyers, for whom the beliefs of quality also depend on other consumers' decisions and the time the house has been on the market. This interaction is analyzed by Taylor (1999).

Finally, there are two ingredients of prize competition that dissociate it from a typical auction: *(i)* the (common) value is deterministic and *(ii)* the committee's main objective is not to raise revenue, but to allocate the prize (contract, licence etc.) to the best candidate.[7] It is also worth noting that auctions may have undesirable self-selection effects in procurement, as shown by Manelli and Vincent (1995).[8]

The paper is organized as follows. The next section discusses the ranking of candidates by using some technology. Section 3 analyses the game and is followed by Section 4 on contest design. Section 5 relates the market size to the competitiveness of the market and Section 6 concludes and discusses further work. The lengthier proofs are in the Appendix.

2 Selection Technology

In this section, I introduce a selection technology, the committee's corresponding preference relation and its use of the selection technology. Consider a population S with k members. Each member i of this population has a certain ability a_i, determined by an independent random draw from a common pool of abilities. Ability a_i is private information held by agent i and the realization of abilities is unknown.

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penditures. Recent contributions are Morgan (2003), who studies sequential expenditures by contestants, and Hvide and Kristiansen (2003), who study the degree of risk taking as a strategic variable.

[7] Moreover, auction mechanisms are not always available as prices are sometimes fixed due to regulation or other reasons, for example house rents in the Netherlands.

[8] In spite of these two important differences, a connection with the auction literature can be made, as there are some papers that study the role of participation cost and the potential number of bidders, see Levin and Smith (1994) and Menezes and Monteiro (2000). Both studies also find that participation should be costly.

In this section, I abstract from agents' incentives and I assume all k agents apply. The committee can make N random draws from this set of applying candidates, denote this sample by X_N, with $X_N \subseteq S$. The committee investigates the ability of all $i \in X_N$ and its decision is to choose a single element from X_N.[9]

One can think of various objectives for a committee to strive for. For example, to make sure that only candidates that exceed a treshold apply, or to maximize the expected ability of the winner. Instead, I assume that the objective of the committee is to maximize the probability that the best candidate will win the prize. As mentioned in the Introduction, good arguments can be given for this assumption. The committee strives for awarding the best candidate and obtains utility u_h in that event and u_l if another was awarded, where $u_h > u_l$, w.l.o.g. I normalize $u_l = 0$.[10]

As mentioned above, an important aspect I want to capture, is that the precision of the investigation depends on N. I introduce an assessment method that suggests one candidate from X_N as winner. With a certain probability, this candidate is indeed the best candidate and with the remaining probability this candidate is a random draw from X_N. The following definition is used throughout.

Definition *An imperfect selection technology* $\pi : \mathbb{N} \to [0,1]$ *suggests a winner. The probability that the winner is* $\arg\max_{i \in X_N} \{a_i\}$ *, that is, the best candidate, is* $\pi(N)$ *and the probability that the winner is a random draw from* X_N *is* $1 - \pi(N)$*.*

To incorporate the scarce nature of the capacity to process information we assume that technology π satisfies one of the two following conditions:

(A) $\pi(N+1) < \pi(N)$ *for all* N *and* $\lim_{N\to\infty} \pi(N) = 0$.

(B) For some capacity constraint $\widehat{N} \in \mathbb{N}$ *and constant* $\overline{\pi} \in [0,1]$,

$$\pi(N) = \begin{cases} \overline{\pi} & if\ N \leq \widehat{N} \\ 0 & if\ N > \widehat{N} \end{cases}.$$

Technology (A) exhibits the trade-off in quantity and quality of information processing most straightforwardly, whereas technology (B) is a step-wise approach and can be the result of outsourcing the test to an assessment center, with whom a contract $\left\{\overline{\pi}, \widehat{N}\right\}$ is agreed that specifies a reliability and a capacity of the test.

The above definition implies that, when k candidates apply and N will be processed, the probability of awarding the best in S equals:

$$\phi(N; k, \pi) \equiv \pi(N) \frac{N}{k} + [1 - \pi(N)] \frac{1}{k}. \qquad (1)$$

[9] We assume it is not possible to award a candidate $i \notin X_N$ that was not processed.

[10] In the model, both the preference relation and the technology treat all candidates but the best in the same way. It can be seen that in a setting where all ranks are selected and evaluated differently, all the insights of the paper would obtain. Such a setting would only complicate matters needlessly.

This can be understood as follows. With probability $\pi(N)$, the best in X_N will be chosen and since the best in S is included in X_N with probability $\frac{N}{k}$ the first term follows. With probability $1 - \pi(N)$, the winner was a random draw from X_N and in this case the probability that the best is drawn is $\frac{1}{k}$.

The trade-off in information processing can now be seen in the first term: (a) as more candidates are being assessed the more likely it is that a really good candidate is amongst them (the fraction $\frac{N}{k}$) but as (b) the limited attention is divided over more applications, the more superficial the assessment will be, obscuring the informativeness of every single candidate's application ($\pi(N)$ decreases). Hence, this trade-off is one of processing many alternatives and the informativeness of the investigation. The problem of choosing how many alternatives to process can then be written as follows:

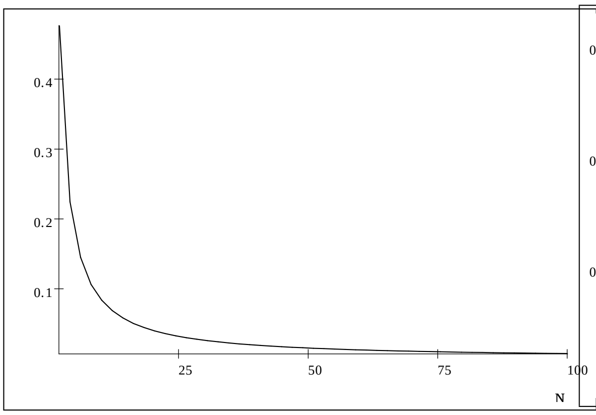
$$\max_{1 \leq N \leq k} \quad u_h \phi(N; k, \pi) \tag{2}$$

A few observations about the solution can be made. Firstly, since ϕ is a probability, it exists. Secondly, the solution to the unconstrained problem, say N^*, is independent of k and fully determined by π. Therefore I can write the solution to (2) as $N = \min\{k, N^*\}$: N^* tells the committee how many applications to process *at most*. Thirdly, the variable u_h has no impact of interest and will therefore be normalized to $u_h = 1$. Finally, since $\phi(2; k, \pi) > \phi(1; k, \pi)$, I have that $N^* > 1$. These results hold for any specification of π. In summary:

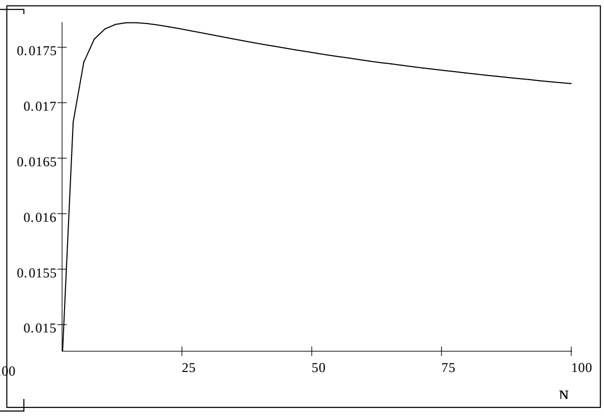
Lemma 2.1 *The committee processes at most N^* applications, ignoring applications randomly if necessary, where N^* is the solution to (2). N^* is larger than 1, independent of the number of applications actually received and fully determined by the selection technology π.*

The following example illustrates:

Example *Suppose $\pi(N) = N^g$ for some $g < 0$. Then the following are graphs of $\pi(N)$ and $\phi(N; k, \pi)$, respectively.*



$\pi(N) = N^{-1.07}$.



$\phi(N; 100, \pi(N))$ for $\pi(N) = N^{-1.07}$.

In the above example, $N^* = 15$. I thus conclude that imperfect decision making may require to process a finite amount of information, implying that some sources are left unattended.

3 The Game

Above we studied the problem of the committee in isolation from applicants' incentives. In this section I introduce the entire game and its equilibrium. There is a population of T agents. The committee announces to all T agents a contest $C \left\langle \frac{c}{V}, \pi; T \right\rangle$ for prize V that costs c to participate in. The timing of events is as follows:

0. The committee determines and announces the contest C.
   Nature draws abilities $\{a_1, a_2, ..., a_T\}$ from a uniform distribution with support $[0, 1]$.

1. Each agent i privately learns a_i and decides whether to participate (*i.e.* to apply) or not. By participating in the contest he incurs cost c.

2. The committee observes the number of applications and decides how many to investigate, N.

3. The investigation of the committee yields a winner and the prize is awarded.

We will postpone the determination of c/V and π in stage 0 to the next section. Before analyzing stages 1 and 2 in detail, I introduce a first result:

Lemma 3.1 *Let $\pi(N) > 0$. If in an equilibrium of the game a candidate with ability a' applies then all candidates with ability $a_i > a'$ apply as well.*

Proof. First note that all candidates have the same costs and benefits of applying and that they only differ in their chances to win. Hence, I prove the claim by arguing that the probability to be chosen by the committee increases in true ability a_i on $[a', 1]$. The probability to be chosen is equal to the probability to be suggested as winner by π. The winner, in turn, is either determined correctly or by chance. Thus, the higher the ability, the higher the probability to win. Furthermore, note that $\pi(N)$ is independent of a_i and then the result follows. ■

This result implies that there exists a demarcation level α such that a candidate i applies if and only if $a_i \geq \alpha$. I will now analyze Stages 2 and 1 subsequently. Let us denote by $S \subseteq T$ the subset of candidates that send an application to the contest organizer in Stage 1, with $\mid S \mid = k$. The contest organizer has to decide in Stage 2 on N, the number of applications to consider. S consists of the k best candidates in T and Lemma 2.1 applies.

I now turn to stage 1, the incentive to apply. Due to Lemma 3.1 we can restrict the quest for equilibrium strategies to those of the form "apply if and only if $a_i \geq \alpha$" and I will now investigate how the demarcation level α is determined.[11] We have that k denotes the unknown total number of applications. In determining the expected payoff of applying, the expectation about the number of rivals applying $(k-1)$ will turn out to play a major role. A potential contestant i with ability $a_i \in [0, \alpha]$ has to conjecture two uncertain events to decide whether or not to apply: $i \in X_N$ (being processed) and the event of being selected (being labeled as winner). I now proceed by investigating how the probability to win for such a type is determined.

First, given that $i \in S$, $\Pr(i \in X_N \mid i \in S) = \frac{N}{k}$. Second, given that i applies $(i \in S)$ and that i will be investigated $(i \in X_N)$, i wins if the test with technology π suggests him as best candidate.

Consider first the case $1 < k \leq N^*$. All k candidates will be processed and thus agent i will be considered surely. If the winner is determined correctly, he looses for sure since at least one other candidate applied and this other candidate has a higher ability. If the winner is determined by chance, he wins with probability $\frac{1}{k}$. Thus, the probability to win for given $k-1$ in case all will be processed equals

$$[1 - \pi(k)] \frac{1}{k} \text{ for } 1 < k \leq N^*,$$

and when $k > N^*$, then i will be considered with probability $\frac{N^*}{k}$ and only N^* candidates will be processed, hence the probability to win is then:

$$\frac{N^*}{k} [1 - \pi(N^*)] \frac{1}{N^*} \text{ for } k > N^*.$$

Now since, if $k = 1$, agent i is the only one and wins for sure, we can write the probability to win for all types $a_i \in [0, \alpha]$ as:

$$\Gamma(\alpha, \pi, N^*, T) \equiv \sum_{k=1}^{T} P(k-1) F(k-1), \text{ where } F(k-1) = \begin{cases} 1 & \text{if } k = 1 \\ \frac{1-\pi(N)}{k} & \text{if } k > 1 \end{cases}, \quad (3)$$

$N = \min\{k, N^*\}$ and $P(k-1) \equiv \binom{T-1}{k-1} \alpha^{T-k} (1-\alpha)^{k-1}$ denotes the density function of $k-1$.

To relate the demarcation α that arises in equilibrium to the other variables of the model, it is important how $\Gamma(\alpha, \cdot)$ depends on α. When α increases, the probability that a given rival candidate applies decreases and thus, probability mass shifts to lower k. How this effects $\Gamma(\alpha, \cdot)$ depends on the shape of $F(k)$, as can been seen in expression (3). For $k > 1$ and $N^* > T$, we have that $F(k-1) = (1 - \pi(k))/k$

[11] Note that we have in fact two kinds of contestant-strategy that are part of an equilibrium: "apply iff $a_i > \underline{a}$" and "apply iff $a_i \geq \underline{a}$". The distinction is unimportant.

and two counterforces are at work: when k decreases, the easier it is to be selected when the selection is random, but the lower the probability that the selection will be random since the technology is more accurate for lower k. Thus, both numerator and denominator of F increase in k. To characterize equilibrium I want Γ to be monotone, however. The following assumption resolves this:

Assumption 1 Let $\hat{k} = (T-1)(1-\alpha)+1$. Technology π is such that

$$F(k-1) \geq F\left(\hat{k}-1\right) \quad \forall\, k \leq \hat{k},$$
$$F(k-1) < F\left(\hat{k}-1\right) \quad \forall\, k > \hat{k}.$$

It is immediate that a constant technology (type B) always satisfies this assumption (since π does not vary). For type (A), we have that $\widehat{k}-1$ is the expected number of competitors so this condition requires that, for a non-applying agent, the chance to win if $k-1$ is below (above) average is higher (lower) than the chance to win if $k-1$ is average.[12] The assumption allows us to show:

Lemma 3.2 *The probability* $\Gamma(\alpha, \pi, N^*, T)$ *increases in* α.

The following Theorem characterizes the equilibrium for the game.

Theorem 3.3 *(i) Suppose* $(1-\pi(N^*))/T < c/V < 1$, *then in every subgame perfect equilibrium of the game, only agents with ability* $a_i \geq \alpha$ *apply, where* $\alpha > 0$ *is the unique solution to*

$$\Gamma(\alpha, \pi, N^*, T) = c/V. \tag{4}$$

(ii) Suppose that $c/V \leq (1-\pi(N^*))/T$. *Then an equilibrium in which all agents apply exists.*
In both cases the committee processes at most a finite number of applications, which is fully determined by π.

Proof. In Lemma 3.1 I have demonstrated the existence of a demarcation level α as stated. Furthermore, the uniqueness of the solution $\Gamma(\alpha, \pi, N^*, T) = c/V$ follows from Lemma 3.2.
(i) I proceed by showing that in every equilibrium: (a) $\alpha > 0$ and (b) the determination of demarcation level α as in (4).
(a) When every candidate applies ($\alpha = 0$), the probability distribution $P(m)$ collapses such that all probability mass shifts to $m = T-1$, hence $\Gamma(0, \pi, N^*, T) = (1-\pi(N^*))/T < c/V$, from which it follows that a sender with $a_i = \varepsilon$, for small

[12] Technologies characterized by a very steep decrease of π (the ability to select the best deteriorates very fast in the number processed) might not satisfy this assumption. What $\Gamma(\alpha, \cdot)$ might look like in that case is left as an exercise.

$\varepsilon > 0$, is strictly better off by not applying, yielding a contradiction.
(b) Denote the demarcation level by $\widetilde{a}$. We will rule out the situations $\Gamma(\widetilde{a}, \pi, N^*, T) < c/V$ and $\Gamma(\widetilde{a}, \pi, N^*, T) > c/V$. The first implies that $\widetilde{a}$ is strictly better off by not applying and the second point implies that all in $[0, \widetilde{a}]$ apply. Both points contradict the fact that $\widetilde{a}$ is a demarcation level.
(ii) That an equilibrium with all T candidates applying exists easily follows. ■

I note that non-applying agents have the same incentive to apply as the candidate with ability level α. The reason for this is the following. Incentives to apply are constant in a_i for $a_i < \alpha$ as low-ability people can only be chosen if the winner is determined by chance or if $k = 1$, and hence, their winning is independent of their true ability. I therefore assign a strategy to low-ability people not to apply, although they do not have a strict incentive to do so, namely they are indifferent, as agent α would be. I have shown that this profile of strategies is the only subgame perfect equilibrium profile.

The equilibrium proposed above shows how information asymmetry can be overcome, when the market does not allow for signaling. Because the uninformed compares 2 or more candidates (with a precision that may be quite low), the higher types have a higher probability of winning. As a result, only the higher types are willing to incur the participation costs and separation is induced. I will label this the *selection effect* of participation costs c.

In what follows I characterize the contest design. Since variations in c and π will have no effect on the incentive to apply as long as $c/V \leq (1 - \pi(N^*))/T$, I will focus on the properties of the equilibrium described under (i) of the Theorem, rather than (ii).

4 Contest Design

In this section I consider two design problems: (i) for given market size and selection technology, how troublesome should applying be made and (ii) for given market size and cost of applying, is it optimal to have a perfect selection technology ?

Since what matters in equilibrium condition (4) is the ratio c/V, all my results on c can be applied to V as well.[13] For simplicity, we normalize $V = 1$. The committee's objective function is given by:

$$U(c, \pi; T) \equiv \sum_{k=1}^{N^*} P(k; \alpha)\,\phi(k; k, \pi) + \sum_{k=N^*+1}^{T} P(k; \alpha)\,\phi(N^*; k, \pi),$$

where, with a slight change in notation from (3), $P(k; \cdot)$ denotes the probability that

[13]This is the case to the extent that V is not a transfer.

k out-of T apply. There is a probability that *no one* applies ($k = 0$) and when that happens, the contest is delayed. I will label this the *risk of delay* and the payoff in that event is normalized to 0.[14]

For $k \geq 1$ the above expression can be understood by recalling that when k agents apply, the probability to award the winner is $\phi(\cdot)$, as was elucidated in (1). Note that when $k > N^*$, the test capacity is overloaded and then the best candidate might not be considered, and I will label this the *overload effect*.

In the remainder of the paper I will study the interaction between the selection effect, the overload effect and the risk of delay to characterize the optimal design of the contest. First observe that Lemma 3.2 and equilibrium equation (4) imply that $\frac{\partial \alpha}{\partial c} > 0$. This leaves us with investigating one other variable, market size T.

For an individual candidate, the number of potential competitors affects the chance of winning. First, recall that a type $a_i \in [0, \alpha]$ can only win if the test picks up a candidate randomly. Then, as T increases, for given demarcation α more candidates apply, and the discussion that proceeds assumption 1 is again relevant: the increase in k makes it more difficult to be selected if selection is random, but increases the probability that the selection is in fact random. The latter effect stimulates low-ability types to apply. By assumption 1 the first effect dominates and consequently I can show:

Lemma 4.1 *The demarcation level α increases in T.*

We now use this result to characterize the two design problems.

Designing c

For a given technology π, I relate the optimal cost c to the market size T. If the demarcation level α would be independent of T, one might conclude that, since for given α, k increases in T, optimal cost increase in T. The intuition would be that the larger the market, the more troublesome applying should be, in order to avoid overload. However, since Lemma 4.1 shows that α does depend on T, this reasoning lacks the strategic effect of the market size. To accurately incorporate the fact that $U(c, \pi; T)$ depends on T via α as well as directly via $P(k; \cdot)$, I resort to a type (B) technology.[15] Doing so, I find that:

Proposition 4.2 *Suppose that the selection technology is of type (B). There exist $\widetilde{\pi} \leq 1$ such that if $\overline{\pi} < \widetilde{\pi}$, then $c(T+1) < c(T)$ for all T.*

The above Proposition demonstrates that the strategic effect of T inverts the logic that in larger markets it should be more troublesome to apply: even though overload constitutes a problem, application cost should decrease in market size!

[14] The dynamic process of delay and the resulting updating of information is left out of the model.

[15] The difficulty with a type (A) technology is that Γ contains a sum.

Designing π

It can easily be verified that when $N^* > T$ and $\overline{\pi} = 1$ (perfect information acquisition), $c(T) = 0$ for all T. In many prize competitions however, the utility cost of applying cannot be controlled to that extent. Therefore, I now look at the design of π : this variable can be controlled by investing time and budget in the selection technology. When c is given, I investigate whether it is optimal to have perfect information acquisition:

Proposition 4.3 *There exists $\widetilde{c} < 1$, such that if $c > \widetilde{c}$, the committee prefers an imperfect selection technology over a perfect one with $\overline{\pi} = 1$ and $N^* > T$.*

To give an idea about what the level of $\widetilde{c}$ might be, I computed it for a market with $T = 50$ potential applicants: when the utility cost of applying for the prize amounts to more than 20% of the value of winning it, the committee prefers having an imperfect selection technology to a perfect one! The reason is that a very accurate selection technology works discouraging and by lowering it, the risk of delay is dampened. Note that the result does not require the technology to be costly.

5 Market Size

The analysis above shows how a committee best balances the risk of receiving too many applications with the risk of receiving no applications. Surprisingly, when the market size increases and the information technology is poor (low π), the selection effect of c should not be used to discourage applications, but to encourage them. Indeed, when information acquisition is perfect, participation cost c should be zero. Conversely, when c cannot be that low, information acquisition should not be perfect. This points out that the risk of market break down might increase with market size and invalidates the commonly held belief that the more players there are, the more competitive the outcome will be.

Since additionally, perfect information acquisition is believed to enable lush competition, I will in this section use the model to investigate the hypothesis that having perfect information and a large number of competitors will lead to a competitive outcome. We denote by $E(k \mid c, T)$ the expected number of applications:

Proposition 5.1 *Let $\pi(\cdot)$ be of type (B) with $\overline{\pi} = 1$.*
(i) The risk of break down $P(0; \cdot)$ increases in T;
(ii) For all finite T, there exist $\widehat{c}(T) < 1$ such that $E(k \mid c, T) > \lim_{T\to\infty} E(k \mid c, T)$ for $c \in (\widehat{c}(T), 1]$.

The proposition shows that having a larger market does not imply having a more competitive outcome (understood as the expected number of participants), even when processing capacity is unbounded. This is a relevant insight for those designing markets. Applied to the labour market, the above implies that in times of high unemployment, people could actually apply less in the aggregate, due to the low expectations they have about their chances of being accepted!

My result corresponds to findings in the consumer search literature in which the expected price is related to the number of firms in the market. Janssen and Moraga-Gonzalez (2004) show for example that an increase in the number of firms may lead to an increase in the expected price. In their study, some consumers engage in costly search for prices, others are fully-informed and firms randomize their price. Now when the number of rival firms increases, it becomes less likely to be the lowest-priced firm (*i.e.* to win the competition for selling to the informed consumers) and in that case a firm can be better off by targeting the consumers that search for one price only, instead of targeting the fully-informed consumers. Hence, the expected price increases.

6 Concluding Remarks

A model of prize competition was presented. My model intends to describe markets where the contest is specialized in such a way that the applicants cannot be sorted on the basis of heterogenous variables. I show that in this case, the uninformed market side finds an alternative means of information acquisition: directly comparing candidates by some technology. As a result, better candidates are more likely to win and separation is induced. A novel feature for the traditional asymmetric information literature is that both technology and incentives reduce informational asymmetry. My model thus fills the gap in markets where the traditionally studied mechanisms are not applicable.

The technology is imperfect and as a result, resources will not be spend on *every* application received. This resembles the practice in selection procedures in which numerous applications are first ranked on rather trivial, uninformative attributes and consequently, costly interviews are given to a select few.

The nature of information asymmetry I use deserves some discussion. In my setup, besides not knowing which candidate has which ability (as in most models), it is also unknown whether there is an applicant out there with ability above some given level.[16] For an applicant, knowing her own ability, this implies that she does not know whether a *better* candidate considers applying. Especially in those situations where the population of potential applicants is not too small (in that case people may know each other too well) one cannot be sure that one of the applicants possesses any pre-specified ability.

[16] Again, as exception, I have this feature in common with Janssen (2002).

The finding that there is a positive probability that *nobody* applies, is driven by the fact that realizations of ability are unknown. As argued above, for many markets, not requiring players to know the distribution is appropriate, and hence, the contingency that no one applies is an important one. A contest designer must simply take into account that it might happen that nobody finds it worthwhile to apply: all agents perceive their chance of winning too low compared to the prize. If no one applies, information about abilities in the population is revealed and lower quality candidates are encouraged to apply. This dynamic process, which is left out of the theoretical model, delays the contest.

The variables c and V of the model are not treated as transfers but instead incurred, resp. enjoyed, only by the agents. They lend themselves therefore for various, non-monetary interpretations. The cost parameter c for example, can be seen as the trouble of collecting the application documents and the value of the prize V can be seen as prestige. In case of journal submissions, the cost c can be seen as the waiting time for an editorial decision.

The above mentioned selection and overload effects and risk of delay were analyzed and showed that the actual level of competition may vary with market size in an unanticipated way. Insights about optimal contest design that we obtained are firstly, that the larger the market, the easier applying should be, in order to mitigate the discouraging effect of competition. This effect of market size dominates the potential overload of applications. Secondly, when the utility cost of applying is fixed and high enough, an accurate selection technology also discourages applicants. In that case, it is in the interest of the committee to have a less than perfect technology.

Besides the typical contests for prizes and contracts, these insights can be applied to the allocation of public assets, such as frequencies and licences for example, when they are procured via a beauty contest rather than via an auction. In this light, I should emphasize that the committee's sole objective in this model is to allocate the prize to the best possible candidate, rather than revenue maximization. A comparison of the two mechanisms constitutes an avenue for future research.

Appendix

We investigate the impact on $\Gamma(\cdot)$ of marginal changes in α and T :

Lemma 3.2 *The probability* $\Gamma(\alpha, \pi, N^{*}, T)$ *increases in* α.

Proof. For convenience, we denote $m \equiv k-1$. First note that:

$$\frac{\partial P(m,\alpha)}{\partial \alpha} \begin{cases} >0 \text{ for } m<\widehat{m} \\ =0 \text{ for } m=\widehat{m} \\ <0 \text{ for } m>\widehat{m}, \quad \text{where } \widehat{m}=(T-1)(1-\alpha). \end{cases} \tag{5}$$

This easily follows from the fact that:

$$\frac{\partial P(m,\alpha)}{\partial\alpha}=\binom{T-1}{m}\alpha^{T-m-1}(1-\alpha)^m\left(\frac{(T-1)(1-\alpha)-m}{\alpha(1-\alpha)}\right).$$

Let $P'=\frac{\partial P(m,\alpha)}{\partial\alpha}$, the claim can then be stated as

$$\frac{\partial\Gamma(\cdot)}{\partial\alpha}=\sum_{m=1}^{\widehat{m}}P'F(m)+\sum_{m=\widehat{m}+1}^{T-1}P'F(m)+\frac{\partial P(0,\alpha)}{\partial\alpha}>0.$$

By Assumption 1 and (5) this is surely the case if $F(\widehat{m})\sum_{m=1}^{T-1}P'+\frac{\partial P(0,\alpha)}{\partial\alpha}>0$ which implies

$$F(\widehat{m})\sum_{m=0}^{T-1}P'+(1-F(\widehat{m}))\frac{\partial P(0,\alpha)}{\partial\alpha}=(1-F(\widehat{m}))\frac{\partial P(0,\alpha)}{\partial\alpha}>0,$$

now since $F(\widehat{m})<1$ this inequality holds. This completes the proof. ■

Lemma 4.1 *The demarcation level α increases in T.*

Proof. A change in T changes only the probability distribution and the possible values m can take. Therefore, let us denote the distribution of m when there are $T+1$ senders in total by $\widetilde{P}(m)$, (omitting the argument α for convenience):

$$\begin{aligned}P(m)&=\binom{T-1}{m}\alpha^{T-1-m}(1-\alpha)^m,\\ \widetilde{P}(m)&=\binom{T}{m}\alpha^{T-m}(1-\alpha)^m=\frac{\alpha T}{T-m}P(m),\end{aligned}$$

hence

$$\begin{aligned}\Gamma(\alpha,\pi,N^*,T+1)&=\sum_{m=1}^{N^*-1}\widetilde{P}(m)\left\{\frac{(1-\pi(m+1))}{m+1}\right\}+\\&+\sum_{m=N^*}^{T-1}\widetilde{P}(m)\left\{\frac{(1-\pi(N^*))}{m+1}\right\}+\widetilde{P}(T)\frac{1-\pi(N^*)}{T+1}+\alpha P(0),\end{aligned}$$

and thus we need to show that

$$\Gamma(\alpha,\pi,N^*,T+1)-\Gamma(\alpha,\pi,N^*,T)=\sum_{m=1}^{N^*-1}\left[\widetilde{P}(m)-P(m)\right]\left\{\frac{(1-\pi(m+1))}{m+1}\right\}+$$
$$\sum_{m=N^*}^{T-1}\left[\widetilde{P}(m)-P(m)\right]\left\{\frac{(1-\pi(N^*))}{m+1}\right\}+\widetilde{P}(T)\frac{1-\pi(N^*)}{T+1}-(1-\alpha)P(0)<0$$

Now note that $\widetilde{P}(m)-P(m)=\frac{m-(1-\alpha)T}{T-m}P(m)$ such that the coefficients on $F(m)$

are negative for small m and positive for large m. Indeed, we can use the same method as in Lemma 3.2, employing again (5) and Assumption 1.

A sufficient condition for the inequality above is then:

$$\sum_{m=1}^{\widehat{m}}\left[\widetilde{P}(m)-P(m)\right]F(\widehat{m})+\sum_{m=\widehat{m}+1}^{T-1}\left[\widetilde{P}(m)-P(m)\right]F(\widehat{m})+$$
$$+\widetilde{P}(T)\frac{1-\pi(N^{*})}{T+1}-(1-\alpha)P(0)<0$$

Repeatedly rewriting of LHS we get:

$$\begin{aligned}
& F(\widehat{m})\sum_{m=1}^{T-1}\left[\widetilde{P}(m)-P(m)\right]+\widetilde{P}(T)\frac{1-\pi(N^{*})}{T+1}-(1-\alpha)P(0)\\
= & F(\widehat{m})\left\{\sum_{m=1}^{T-1}\widetilde{P}(m)-\sum_{m=0}^{T-1}P(m)\right\}+\widetilde{P}(T)\frac{1-\pi(N^{*})}{T+1}-(1-\alpha-F(\widehat{m}))P(0)\\
= & \widetilde{P}(T)\left\{\frac{1-\pi(N^{*})}{T+1}-F(\widehat{m})\right\}-(1-\alpha)(1-F(\widehat{m}))P(0)<0,
\end{aligned}$$

and since $\frac{1-\pi(N^{*})}{T+1}=F(T)<F(\widehat{m})$ this inequality is satisfied. ■

The next two statements are on contest design. For easy reference, we state the objective function for a type (B) technology:

$$\begin{aligned}
U(c,\pi;T) = & \pi\left[1-\alpha^{T}\right]+(1-\pi)\left\{\sum_{k=1}^{T}\binom{T}{k}\frac{1}{k}(1-\alpha)^{k}\alpha^{T-k}\right\}+\\
& +\pi\left\{\sum_{N^{*}+1}^{T}\binom{T}{k}(1-\alpha)^{k}\alpha^{T-k}\left[\frac{N^{*}}{k}-1\right]\right\}.
\end{aligned}$$

Proposition 4.2 *Suppose that the selection technology is of type (B). There exists* $\widetilde{\pi}\leq 1$ *such that if* $\overline{\pi}<\widetilde{\pi}$, *then* $c(T+1)<c(T)$ *for all* T.

Proof. Take the first-order condition w.r.t. α for a maximum of U :

$$\begin{aligned}
& -T\pi\alpha^{T-1}+(1-\pi)\frac{T}{\alpha}\left\{\sum_{k=1}^{T}\binom{T}{k}\frac{1}{k}(1-\alpha)^{k}\alpha^{T-k}-\frac{1-\alpha^{T}}{T(1-\alpha)}\right\}+\\
& +\pi\sum_{N+1}^{T}\binom{T}{k}\alpha^{T-k}(1-\alpha)^{k}\frac{T(1-\alpha)-k}{\alpha(1-\alpha)}\left[\frac{N}{k}-1\right]=0, \qquad (6)
\end{aligned}$$

The proof consists of three steps.

Step (1).

Denote by $G(T)=\sum_{k=1}^{T}\binom{T}{k}\frac{1}{k}(1-\alpha)^{k}\alpha^{T-k}$ and by $H(T)=\sum_{N^{*}+1}^{T}\binom{T}{k}(1-\alpha)^{k}\alpha^{T-k}\left[\frac{N^{*}}{k}-1\right]$.

We then obtain that (6) implies:

$$c(T) = (1-\pi) G(T) + \pi \left(\alpha^{T-1} - \alpha^{T}\right) + \pi \frac{\alpha}{T} H(T).$$

Step (2).

Rewrite $G(T)$:

$\sum_{k=1}^{T} \binom{T}{k} \frac{1}{k} (1-\alpha)^k \alpha^{T-k} = \alpha^T \sum_{k=1}^{T} \binom{T}{k} \frac{1}{k} \left(\frac{1-\alpha}{\alpha}\right)^k$. Now let $z = \frac{1-\alpha}{\alpha}$, then we get: $G(T) = \alpha^T \sum \binom{T}{k} \frac{1}{k} z^k$. Now since $\frac{1}{k} z^k = \int_0^z x^{k-1} dx$, we get $\alpha^T \sum \binom{T}{k} \int_0^z x^{k-1} dx =$
$\alpha^T \int_0^z \left(\sum_0^T \binom{T}{k} x^{k-1} - \frac{1}{x}\right) dx =$
$\alpha^T \int_0^z \left(\frac{1}{x} \sum_0^T \binom{T}{k} x^k - 1\right) dx =$

$$\alpha^T \int_0^z \frac{1}{x} \left((1+x)^T - 1\right) dx = \alpha^T \int_0^{\frac{1-\alpha}{\alpha}} \frac{(1+x)^T - 1}{x} dx.$$

It is convenient to eliminate α from the bounds of integration. Let $y = \frac{\alpha}{1-\alpha} x$ and $dx = \frac{1-\alpha}{\alpha} dy$, then the above can be rewritten as $\alpha^T \int_0^1 \frac{\left(1+\frac{1-\alpha}{\alpha} y\right)^T - 1}{\frac{1-\alpha}{\alpha} y} \frac{1-\alpha}{\alpha} dy$ or:

$$G(T) = \int_0^1 \left[(\alpha + (1-\alpha) y)^T - \alpha^T\right] \frac{dy}{y}.$$

Using this expression, take the first-order condition w.r.t. α :

$$\frac{\partial U}{\partial \alpha} = -T\pi\alpha^{T-1} + (1-\pi) \frac{dG}{d\alpha} + \pi H(T) = 0 \text{ or}$$

$$-T\pi\alpha^{T-1} + (1-\pi) \left\{\int_0^1 \frac{dy}{y} \left(T[\alpha + (1-\alpha) y]^{T-1} (1-y) - T\alpha^{T-1}\right)\right\} + \pi H(T) = 0 \Rightarrow$$

$$-\pi\alpha^{T-1} + \frac{\pi}{T} H(T) + (1-\pi) \int_0^1 \frac{dy}{y} [\alpha + (1-\alpha) y]^{T-1} - \alpha^{T-1} =$$
$$(1-\pi) \int_0^1 dy \, [\alpha + (1-\alpha) y]^{T-1} \Longrightarrow$$

$$(1-\pi) G(T-1) - \pi\alpha^{T-1} + \frac{\pi}{T} H(T) = (1-\pi) \frac{1-\alpha^T}{T(1-\alpha)}$$

Hence, f.o.c. (6) when the market size equals T implies that:

$$c(T) = (1-\pi) G(T-1) + \frac{\pi}{T} H(T).$$

Step (3).

Now, when convenient, denote by $\alpha(T)$ the level of α that satisfies the f.o.c. when the market volume is T. From Steps (1) and (2) we know:

$$\begin{aligned} c(T) &= (1-\pi) G(T, \alpha(T)) + \pi \left(\alpha^{T-1} - \alpha^T\right) + \pi \frac{\alpha}{T} H(T) \\ c(T+1) &= (1-\pi) G(T, \alpha(T+1)) + \frac{\pi}{T+1} H(T+1). \end{aligned}$$

Thus, to prove the result we will show that:

$$(1-\pi) G(T, \alpha(T)) + \pi \left(\alpha^{T-1} - \alpha^T\right) + \pi \frac{\alpha}{T} H(T) > \\ (1-\pi) G(T, \alpha(T+1)) + \frac{\pi}{T+1} H(T+1) \text{ or}$$

$$G(T, \alpha(T)) - G(T, \alpha(T+1)) > \frac{\pi}{1-\pi} \left(\frac{H(T+1)}{T+1} - \left(\alpha^{T-1} - \alpha^T\right) - \frac{\alpha}{T} H(T) \right).$$

Note that, for $\alpha(T)$ we have $\frac{\partial G}{\partial \alpha} = \frac{\pi}{1-\pi}\left(T\alpha^{T-1} - H\right)$ due to (6). Since $\frac{\pi}{1-\pi}$ increases, we have that $\frac{\partial G}{\partial \alpha}$ goes to zero when π goes to zero. This, in turn, implies that when π goes to zero, $G(T, \alpha(T))$ goes to the maximum of G w.r.t. α. Now since $\alpha(T+1)$ is different from $\alpha(T)$ in a discrete way, we can conclude that there exists a $\hat{\pi}$ such that LHS of the latter inequality is positive for all $0 \leq \pi \leq \hat{\pi}$. Finally, since the RHS equals zero for $\pi = 0$, we can conclude that there exists $\widetilde{\pi} > 0$ such that the inequality holds for all $\pi \in [0, \widetilde{\pi}]$. This completes the proof. ■

Proposition 4.3 *There exists $\widetilde{c} < 1$, such that if $c > \widetilde{c}$, the committee prefers an imperfect selection technology over a perfect one with $\overline{\pi} = 1$ and $N^* > T$.*

Proof. We will investigate $\partial U / \partial \pi$ in the point $\overline{\pi} = 1$ and $N^* > T$, and use the same notation as in the previous proof. We need to show that:

$$\frac{\partial U}{\partial \pi} = 1 - \alpha(\pi)^T - T\alpha(\pi)^{T-1} \frac{\partial \alpha}{\partial \pi}\pi - G + \frac{\partial G}{\partial \pi}(1-\pi) + H + \frac{\partial H}{\partial \pi}\pi < 0$$

Since $N^* > T$, the last two terms vanish and then substituting $\pi = 1$ we get

$$1 - \alpha(\pi)^T - T\alpha(\pi)^{T-1} \frac{\partial \alpha}{\partial \pi} - G < 0. \tag{7}$$

First, we investigate $\frac{\partial \alpha}{\partial \pi}$. The demarcation α depends on π via, (4) and we thus use the Implicit Function Theorem to obtain that, in $\pi = 1$:

$$\frac{\partial \alpha}{\partial \pi} = -\frac{\alpha^{T-1} - \frac{1-\alpha^T}{T(1-\alpha)}}{(T-1)\alpha^{T-2}},$$

thus $T\alpha^{T-1}\frac{\partial\alpha}{\partial\pi} = \frac{1}{T-1}\left(\frac{\alpha\left(1-\alpha^T\right)}{1-\alpha} - T\alpha^T\right)$. Inequality (7) thus becomes

$$\begin{aligned} 1-\alpha^T - G &< \frac{1}{T-1}\left(\frac{\alpha\left(1-\alpha^T\right)}{1-\alpha} - T\alpha^T\right) \text{ or} \\ 1-G &< \frac{1}{T-1}\frac{\alpha-\alpha^T}{1-\alpha}. \end{aligned} \tag{8}$$

Now from the proof of Proposition 4.2 we know that
$G = \int_0^1 \left[(\alpha + (1-\alpha)y)^T - \alpha^T\right]\frac{dy}{y}$ and $\frac{dG}{d\alpha} = \int_0^1 \frac{dy}{y}\left(T\left[\alpha+(1-\alpha)y\right]^{T-1}(1-y) - T\alpha^{T-1}\right)$.
Label LHS and RHS of (8) by $LHS(\alpha)$ and $RHS(\alpha)$, respectively. We will now show that (i) $LHS(1) = RHS(1)$ and (ii) $\frac{dLHS}{d\alpha}(1) > \frac{dRHS}{d\alpha}(1)$.
(i) It is easily shown that $G(\alpha = 1) = 0$ and RHS can be written as $\frac{1}{T-1}\alpha\frac{1-\alpha^{T-1}}{1-\alpha}$. By applying l'Hopital's rule once we get that $\lim_{\alpha\to 1}\frac{1-\alpha^{T-1}}{1-\alpha} = T-1$, showing that $RHS(1) = 1$.
(ii) We have $\frac{dG}{d\alpha}(1) = \int_0^1 \frac{dy}{y}(T(1-y) - T) = -T$ and thus $dLHS/d\alpha = T$. Now, $\frac{dRHS}{d\alpha}(1) = \frac{1}{T-1}\lim_{\alpha\to 1}\frac{1-T\alpha^{T-1}}{1-\alpha}$ and by applying l'Hopital's rule twice we obtain that the limit equals $\frac{dRHS}{d\alpha}(1) = T/2$.

Due to points (i) and (ii) there exists $\alpha' < 1$ such that (8) holds for all $\alpha > \alpha'$ and the fact that α monotonically increases in c implies the existence of $\widetilde{c}$ as stated.
■

Proposition 5.1 *Let $\pi(\cdot)$ be of type (B) with $\overline{\pi} = 1$.*
(i) The risk of break down $P(0;\cdot)$ increases in T;
(ii) For all finite T, there exist $\widehat{c}(T) < 1$ such that $E(k \mid c, T) > \lim_{T\to\infty} E(k \mid c, T)$ for $c \in (\widehat{c}(T), 1]$.

Proof. (i) In equilibrium, $\alpha(T)^{T-1} = c$ and hence $P(0;\cdot) = \alpha(T)^T = \alpha(T)c$. Now since RHS increases in T, LHS must increase in T as well.
(ii) We first show that $E(k \mid c,T)$ converges to $-\ln(c)$. We have $E(k \mid c, T) = T\left[1-(c)^{1/(T-1)}\right]$. Write this as $\frac{T}{1/\left[1-(c)^{1/(T-1)}\right]}$ and note that both numerator and denumerator converge to infinity. The limit can then be obtained by repeatedly applying l'Hôpital's rule.

Next, we show

$$T\left(1-c^{1/(T-1)}\right) > -\ln(c).$$

First observe that both sides are decreasing in c. Then we will prove by showing that $LHS > RHS$ for c high enough. We have that $\frac{\partial LHS}{\partial c} = \frac{-T}{T-1}c^{-T/(T-1)}$ and $\frac{dRHS}{dc} = -\frac{1}{c}$ and $LHS(1) = RHS(1)$ and thus the inequality is satisfied in a left neighbourhood of $c = 1$. ■

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