TI 2006-010/1
Tinbergen Institute Discussion Paper

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A Simple Model of Self-Assessments*

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Abstract

We develop a simple model that describes individuals' self-assessments of their abilities. We assume that individuals learn about their abilities from appraisals of others and experience. Our model predicts that if communication is imperfect, then (i) appraisals of others tend to be too positive, and (ii) overconfidence leading to too much activism is more likely than underconfidence leading to too much passivity. The predictions of our model are consistent with findings in the social psychological literature.

Key words: self-assessments, learning about ability, coaching, overconfidence, underconfidence.

JEL Classification: D81, D82, D83.

*We are grateful to Josse Delfgaauw, Chaim Fershtman, Alexandra van Geen, Hans Hvide, Bauke Visser and seminar participants at Erasmus University Rotterdam for comments.

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1 Introduction

A person's perception of his abilities may have substantial consequences for his actions.[1] If Shakespeare had had a low perception of his writing ability, perhaps nobody would have known Hamlet. A young musician's impression of her talent for music may determine whether she decides to become a professional violinist. One possible reason why women disproportionately avoid career in science is that they underestimate their scientific reasoning ability (see Ehrlinger and Dunning, 2003). Given that people's assessments of their abilities are important for their choices, one would expect that economists have paid much attention to the relationship between people's perceptions of their own abilities (self concepts) and people's actual abilities (the self). After all, isn't economics pre-eminently the study of how people choose? However, until very recently, research on this relationship has been done predominantly by social psychologists. Their research has resulted in "a large, fascinating, yet sometimes confusing and contentious literature" (Sedikides and Strube, 1995, p. 1277). Baumeister (1998) gives a very interesting survey of this literature.

A high degree of consensus among social psychologists seems to exist on the following three findings. First, people often misjudge their own abilities. For example, College Board (1976-1977) reports small correlations between objective abilities and persons' perceptions of their own abilities for a wide range of domains (see also Kruger, 1999). Of course, this finding is the raison d'être of the existence of a voluminous literature on self-assessments. Second, although many individuals have distorted self-concepts, people are neither generally overconfident nor generally underconfident. It is well-known that for some dimensions a majority of people see themselves better as average (famous examples are intelligence, attractiveness and car driving). For other dimensions, a majority of people see themselves as worse than average (music, art, mechanics, chess playing).[2] Ackerman et al. (2002) report experimental results suggesting that for broad items, say intelligence, people have higher self-estimates of ability than for specific items, say being able to study long hours (see also Klar et al., 1996). The third important finding of social psychologists

[1] Phillips (1984) shows a strong correlation between children's subjective perceptions of their abilities and their achievement behavior.

[2] Of course, the observation that a majority of people see themselves better (worse) than the average does not imply the existence of overconfidence (underconfidence). It may also be the result of a skewed distribution of abilities.

is that accurate feedback on one's ability is rare (see Jones and Wortman, 1973). In particular, feedback tends to be too positive (Brown and Dutton, 1995; Felson, 1993). Own perceptions of abilities often do not resemble the way abilities are perceived by others. On the other hand, in their survey of the early literature Shrauger and Schoeneman (1979) conclude that own perceptions of abilities are closely related to how people believe they are perceived by others.

In this paper we develop a simple model that yields predictions that are broadly consistent with the three findings discussed above. Our model describes the interaction between a senior and her junior. The senior cares about her junior. She knows the junior's ability, but the junior does not know his own ability. In line with the self-assessment theory in social psychology (Trope, 1979; Dunning, 1995; Taylor et al., 1995), the junior wants to learn his ability to make better decisions in the future. Specifically, the junior has to make decisions on two successive tasks. For each task, the junior must choose between performing the task or not. Furthermore, if the junior chooses to perform the task, he has to determine how much effort to put in it. Effort and ability are complementary, in the sense that the higher is the junior's ability, the more effort he wants to put in a task. If the junior's ability is below a certain threshold, he should not perform the task at all. At the beginning of the game, the senior sends a noisy message to the junior. Noisy in the sense that with some probability the junior receives another message than the senior has sent. Apart from learning about his ability through his senior's message, the junior can learn his ability by doing. Performing the first task yields information about his ability which can be used when making a decision on the second task.

We derive the following results. First, the senior tends to deflate the ability of a junior who is just able to perform a task. For such a junior, the cost of overconfidence - too much effort - is higher than the cost of underconfidence - mistaken passiveness. Second, the senior is inclined to inflate the ability of a talented junior. For a very able junior, the cost of overconfidence is smaller than the cost of underconfidence. The senior wants to avoid a situation that a talented junior abstains from performing a task. She does so by exaggerating the junior's ability. Third, we show that on average the senior exaggerates a junior's ability. The reason for this result is that the cost of underconfidence (passiveness) is permanent, whereas the cost of overconfidence (too much effort) is temporary because of learning by doing. This last result is in line with Felson (1989) who observes that experience is a better predictor of self- appraisals than appraisals of others.

Recently, several economic papers have appeared on the topic of judgement biases. Here we discuss some papers that focus on self-assessments of abilities. We thereby ignore the literature that is concerned with questions like when and why are people too certain about events or why are people overoptimistic about certain life-events (see Van den Steen, 2004). Why should economists be interested in self-assessments of abilities in the first place? At the beginning of this introduction, we have already mentioned that people's choices may depend on how they see themselves. Fang and Moscarini (2005) nicely illustrate this point in the context of a principal-agent problem. Assuming that effort and ability are complements, and that agents overestimate their abilities, they show that performance evaluations may reduce a firm's profit. The reason is that through performance evaluations agents may learn their actual abilities. Because agents on average overestimate their abilities, learning may reduce average effort.

In Fang and Moscarini, overconfidence is assumed, not explained. Let us now discuss economic studies that try to explain self-assessments. Two strands in this literature can be distinguished. In the first, people form beliefs about their abilities that are most useful to them. The benefit of a particular belief can be direct or indirect. A direct benefit exists when a positive view of your abilities makes you happier. In the social psychological literature, this is referred to as the self-enhancement approach to self-appraisals. Brunnermeier and Parker (2004) and Weinberg (2005) show that the optimal belief depends on the direct benefit of a positive self view and the cost of making incorrect decisions. In Hvide (2002), the benefit of a particular belief is indirect. He shows that overconfidence strengthens the agent's bargaining power versus firms. In Bénabou and Tirole (2002) time inconsistent preferences are the reason that people may want to forget information on their ability. Forgetting negative information on your ability makes that you feel better now. The cost of forgetting information is distorted future decision making. Time inconsistent preferences may imply that the present benefit outweighs the future cost. By relying on time-inconsistent preferences Bénabou and Tirole follow the self-enhancement approach. Concerning the supply of information on ability, Bénabou and Tirole focus on communication between two selves, your current self, who possesses information, and the future self who may receive information. Because of the time-inconsistent preferences, the two selves have conflicting preferences.

In the second strand of the literature, people learn their abilities. Zábojník (2004) presents a model in which an agent can choose for receiving a signal about his ability at the cost of foregone production. He shows that an agent keeps buying signals until his self-assessment is sufficiently favourable (see also Brocas and Carrillo, 2002). Compte and Postlewaite (2004) assume that an agent's confidence has a direct effect on his performance. The agent's confidence depends on his perception of the frequency of past successes. Thus agents learn by doing. Our model belongs to the second strand in the literature. The agent learns his ability from others and may learn by doing.

A drawback of most of the studies mentioned above is that they only explain overconfidence. It is true that many studies have reported a bias toward overconfidence. However, one cannot deny that people exist who are plagued by self-doubt, and hold unrealistically negative impressions of their abilities. This suggests that self–enhancement cannot be the only explanation of self-assessments. More generally, we need a theory that can explain the existence of both overconfidence and underconfidence.

The remainder of this paper is organized as follows. The next section presents the model. Section 3 discusses an equilibrium of the model. Section 4 discusses the consequences of relaxing two assumptions for our main results.

2 The Model

We consider a simple model of a senior (she) and a junior (he). The model consists of three stages. The last two stages represent the junior's future. In the first stage the senior coaches the junior. In each of the last two stages, the junior must independently make a binary decision, say, whether or not to perform some ambitious task.

At the beginning of the game, the junior's ability (the self), a, is drawn from a distribution, $f(a)$, on $[0, 1]$. The senior observes a, but the junior does not. The junior only knows $f(a)$. In stage 0, the senior tries to inform the junior about his ability by sending a message $m \in [0, 1]$. Communication is not perfect. We model this as follows. Let r denote the message the junior receives. We assume that r results from a continuous density function, $g_m(r)$ defined over $[0,1]$. Moreover, we assume that $g_m(r) > 0$ and that it has one maximum, defined by $g'_m(r = m) = 0$. This assumption implies that small communication errors are more likely to occur than large communication errors. Finally, we assume that $g_i(r = i) = g_j(r = j)$. This assumption ensures that in equilibrium $E(a \mid r)$ is an increasing function of m.

Our way of modeling the communication between the senior and the junior tries to capture the psychological model of the reflected appraisal process (see Kinch, 1963, and for a more recent discussion, Felson, 1993). This process consists of three elements. The first element is self-appraisal. Self-appraisal refers to the way a person views a certain feature of himself. Examples of features are academic ability, an ability to perform a task, physical attractiveness and popularity. In our model, self-appraisal is modeled as the junior's equilibrium belief about his ability. The second element is actual appraisals of others. In our model, this is denoted by m. The last element is reflected appraisals, meaning a person's perception of actual appraisals. In our model, this is denoted by r. Empirical research by social psychologists shows that there are only weak correlations between actual appraisals, m, and reflected appraisals, r (Felson, 1993). We capture this by $g_m(r)$. Furthermore, experimental research suggests that if reflected appraisals are taken into account, actual appraisals do not explain self-appraisals (Schrauger and Schoeneman, 1979). In our model, reflected appraisals lead to self-appraisals through Bayes' rule.

As mentioned above, in stage $t = 1$ and stage $t = 2$, the junior chooses whether or not to perform a task. In these stages, the junior works independently and cannot rely on the senior anymore. The payoff of performing a task depends on the junior's ability and his effort, $e_t > 0$:

$$U_t(X_t = 1) = ae_t - \frac{1}{2}e_t^2 \tag{1}$$

Not performing a task $X_t = 0$ yields,

$$U_t(X_t = 0) = z > 0 \tag{2}$$

The implication of z is that only if a exceeds a certain threshold, then the junior should perform the task. Throughout, we assume that $z < \frac{1}{2}$, implying that juniors exist who should perform the task.

If the junior chooses $X_1 = 1$ in stage 1, then he observes his payoff and consequently infers a. If $X_1 = 0$, then the junior does not obtain new information about his ability. Our model thus allows for two ways of developing a self-concept: appraisal (the senior's message) and experience.

The senior cares about the junior. Her payoff is also given by (1) and (2). The problem of both the senior and the junior is that the latter should perform the task only if the task yields a payoff higher than z. This requires that the junior is sufficiently able. Another problem is that in case the junior performs the task, he must choose an effort level that accords with his ability.

Remark. In our model the senior provides information about the junior's ability by means of a simple message. For us, this simple message is a shortcut for something much broader. For example, a message may reflect the way the senior coaches a junior. A senior who gives the junior responsibilities may signal something else as a senior who always assists her junior.

3 Equilibrium

Our game is a dynamic cheap-talk game. To solve the game, we apply the standard Nash-Bayesian equilibrium concept, so that strategies are best responses to each other, given beliefs, and beliefs follow from the strategies according to Bayes' rule.[3]

3.1 Stage 2

Suppose that in stage 1 it is a best response of the junior to choose $X_1 = 1$ with $e_1 = a_r^e = E(a \mid r)$ if $r > r^*$, and to choose $X_1 = 0$ if $r \leq r^*$. When analyzing the second stage of the game, two cases have to be distinguished.

Case 1: $X_1 = 0$

If it were optimal for the junior to choose $X_1 = 0$ in stage 1, it is also optimal for him to choose $X_2 = 0$. To understand why, first recall that if $X_1 = 0$, the junior

[3] It is well-known that in cheap-talk games a pooling equilibrium always exists (see Crawford and Sobel, 1982). This is also true for our model. We ignore this pooling equilibrium.

does not learn his ability. Learning requires that $X_1 = 1$. As learning has not taken place, the junior's view on the project has not changed. Thus, there is an indirect benefit of performing the task in stage 1. By choosing $X_1 = 1$, the junior would have learned his ability. This knowledge could be used when making a decision on X_2. The total expected benefit of $X_1 = 1$ therefore exceeds the total expected benefit of $X_2 = 1$. Hence, if it were optimal for the junior to choose $X_1 = 0$, it is optimal for him to choose $X_2 = 0$ too.

Case 2: $X_1 = 1$

In this case, the junior has learned his ability. His decision on X_2 is then relatively easy. From (1) and (2) it is easy to see that if a junior opts for $X_2 = 1$, he chooses $e_2 = a$. Moreover, $X_2 = 1$ yields a higher payoff than $X_2 = 0$ if $\frac{1}{2}a^2 > z$ or $a > \sqrt{2z}$.

Notice that ideally the senior wants the junior to act similarly in stage 1. The senior knows the junior's ability from the beginning, and the senior's and junior's preferences are perfectly aligned. Consequently, if the senior's message were without any noise, $r = m$, then an equilibrium would exist in which the senior sends $m = a$. This strategy would induce the junior to act in his own interest. For future references, we would like to emphasize three features of the outcomes for the case that $r = m$. First, the senior's appraisal would not be biased. Second, underconfidence ($a > a_r^e$) or overconfidence ($a < a_r^e$) would not exist. Finally, learning by experience would not play a role. Performing the task in stage 1 would not deliver useful information for the junior's decision on the task in stage 2 in addition to the senior's message.

3.2 Stage 1

3.2.1 The Senior

For the moment we assume that in stage 1 it is a best response of the junior to choose $X_1 = 1$ with $e_1 = a_r^e$ if $r > r^*$, and to choose $X_1 = 0$ if $r \leq r^*$. Furthermore we assume that the junior's beliefs imply that a higher value of m increases a_r^e.

Let us begin by showing why truthfully revealing, that is *always* sending $m = a$, cannot be part of an equilibrium. First suppose that a is low. We have already established that if $a \leq \sqrt{2z}$, the junior should not perform the task. Therefore, for

$a \leq \sqrt{2z}$, the senior should minimize the probability that her junior performs the task. She does so by sending $m = 0$.

Now suppose that a is just above $\sqrt{2z}$. In that case, $e_1 = a$ would yield a payoff to the junior that is slightly higher than z in both periods. However, as a result of imperfect communication, it is unlikely that the junior actually chooses $e_1 = a$. In particular, overconfidence would result in $e_1 > a$ and consequently in $U_1(X = 1) < z$. The implication is that for a just above $\sqrt{2z}$ performing the task is likely to yield a payoff below $2z$. The more a deviates from $\sqrt{2z}$, the higher is the expected payoff of performing the task and the lower is the cost of overconfidence. More generally, a value of $a = a^* > \sqrt{2z}$ exists, for which the senior is indifferent between sending $m = 0$ and sending $m > 0$.

Now consider high values of a. Let us start with $a = 1$. Clearly, in that case, the senior has no incentive to send $m < 1$. She wants the junior to perform the task with $e_1 = 1$. The best the senior can do is sending $m = 1$. Now suppose that a is slightly below 1. Through the posterior beliefs, sending $m < 1$, in expectation, impels the junior to choose a lower level of effort than sending $m = 1$. We now argue that also in this case it is optimal for the senior to send $m = 1$. The reason is twofold. First, by sending $m = 1$, the senior maximizes the probability that the junior performs the task ($\Pr(r > r^* \mid m)$ increases in m). Staying passive rather than performing the task does not only affect the payoff in period 1. As learning by doing does not take place when $X_1 = 0$, period 2 payoff falls from $\frac{1}{2}a^2$ to z. The benefit of sending $m = 1$ instead of $m < 1$ is thus an increase in the probability that the junior learns. The cost of sending $m = 1$ is a higher probability that the junior exerts too much effort because of overconfidence ($E(a \mid m = 1) > a$). However, as a is close to 1, both the probability of overconfidence and the cost of overconfidence are low. The second reason why the senior may want to send $m = 1$ is that communication errors are to some extent systematic. When a is close to 1, noise of communication is likely to imply that $a_r^e < a$. To compensate, the senior sends $m = 1$.

From the above discussion it follows that for high values of a, there are two benefits of sending $m = 1$ rather than sending $m < 1$: first, it increases the probability of learning, and second, it may correct a systematic error. The magnitude of these benefits increases in a. The cost of sending $m = 1$ is a higher probability of overconfidence. This cost diminishes as a increases. More generally, there exists a value of $a = a^{**}$ for which the senior is indifferent between sending $m = 1$ and sending $m < 1$.

So far, we have established three ranges of a: $a \leq a^*$, $a \geq a^{**}$ and $a^* < a < a^{**}$. For each of the first two ranges, the senior does not discriminate among types. For $a \leq a^*$, she sends $m = 0$, and for $a \geq a^{**}$, she sends $m = 1$. For $a^* < a < a^{**}$, the senior neither wants to protect the junior fully against overconfidence nor wants to protect the junior fully against passiveness. That is, the senior trades off the costs of overconfidence and the costs of passiveness. Clearly, the higher is a, the higher are the costs of passiveness, and the lower are the costs of overconfidence. For $a^* < a < a^{**}$, the senior's strategy can now be characterized by $m(a)$ with $m \in (0,1)$ and $m'(a) > 0$. For a's just above a^*, m is smaller than a, whereas for a's just below a^{**}, m is larger than a.

3.2.2 The Junior

After the junior has received r, he forms a belief about a. Let the density function $h_r(a)$ [with cumulative distribution $H_r(a)$] denote this belief. As $m'(a) \geq 0$ and the probability of large errors exceeds the probability of small errors, the expected value of a, $a_r^e = \int_0^1 a h_r(a)\, da$, is an increasing function of r. It is now easy to characterize the junior's best response. First, suppose that $X_1 = 1$. Then, (1) implies that the junior chooses $e_1 = a_r^e$. Second, the junior chooses $X_1 = 1$ if and only if $r > r^*$, with r^* solving

$$\frac{1}{2}\left[\int_0^1 a h_{r^*}(a)\, da\right]^2 + H_{r^*}\left(\sqrt{2z}\right) z + \left[1 - H_{r^*}\left(\sqrt{2z}\right)\right] \frac{\int_{\sqrt{2z}}^1 \frac{1}{2} a^2 h_{r^*}(a)\, da}{\int_{\sqrt{2z}}^1 h_{r^*}(a)\, da} = 2z \quad (3)$$

The first term of (3) simply denotes the expected period 1 payoff when $e_1 = a_r^e$. The second and third term of (3) denote the expected period 2 payoff for a junior who has exerted effort in period 1. Because the junior only exerts effort in period 2 if $a > \sqrt{2z}$, together the second and third term are larger than z. This reflects learning by doing. The implication is that the junior chooses $X_1 = 1$ even if the expected payoff for period 1 is lower than z. The difference between the first term of (3) and z can be interpreted as the price the junior is willing to pay for learning his ability.

Finally, we have to ensure that $0 \leq r^* \leq 1$. Two conditions must hold. First, for $r = 1$, the left-hand side of (3) must be larger than $2z$. Second, for $r = 0$, the left-hand side of (3) must be smaller than $2z$. These conditions require that communication is not too noisy. In case communication is very noisy, it is optimal for the junior not to rely on his senior when deciding on whether or not to perform the task. For instance, if for $r = 1$, the left-hand side of (3) is lower than $2z$, the junior always abstains from performing the task. In that case, only a pooling equilibrium exists. If for $r = 0$ the left-hand side of (3) is larger than $2z$, then the junior always performs the task. The senior's message may affect the junior's effort. If an interior solution of r^* exists, then the equilibrium is best characterized as a semi-separating equilibrium. Some types of seniors choose the same action, while other types of seniors choose different actions.

The discussion above can be summarized by the following proposition.

Proposition 1 *Suppose a value of r^*, $0 < r^* < 1$, for which (3) holds. An equilibrium exists in which (i) the junior chooses $X_2 = 0$ if $X_1 = 0 \vee a < \sqrt{2z}$, and chooses $X_2 = 1$ with $e_2 = a$ if $X_1 = 1 \wedge a \geq \sqrt{2z}$; (ii) the junior chooses $X_1 = 1$ with $e_1 = \int_0^1 a h_r(a)\, da$ if $r > r^*$, and $X_1 = 0$ if $r \leq r^*$; (iii) the senior sends $m = 0$ if $a \leq a^*$, $m = 1$ if $a \geq a^{**}$, and $0 < m(a) < 1$ with $m'(a) > 0$ if $a^* < a < a^{**}$, and $0 < a^* \leq a^{**} < 1$; and (iv) posterior beliefs, $h_r(a)$, result from the senior's strategy according to Bayes' rule.*

How does Proposition 1 relate to the psychological literature discussed in the introduction? One finding by social psychologists was that people often misjudge their own abilities. In our model, juniors may initially misjudge their abilities, but in the end active juniors learn their abilities. The reason is that by performing a task the junior becomes fully informed about his ability.[4] Just after the juniors have received the senior's messages, they misjudge their abilities. In line with the

[4]If we relax this assumption (active juniors receive only a noisy signal), then juniors misjudge their abilities also at the end of the game.

second finding of the social psychological literature, then some juniors overestimate their abilities, whereas others underestimate their abilities. Finally, our results are consistent with the finding that appraisals by others are not accurate. To avoid overconfidence, appraisals are sometimes too negative, but to avoid passiveness appraisals can also be too positive. To produce more comparative-static results, we consider an example in the next section.

4 An Example

Assume that a is drawn from a uniform distribution on $[0,1]$, that is $f(a) = 1$. Moreover, assume that the senior can send three messages, $m \in \{l, n, h\}$, $m = l$, meaning "low ability", $m = n$, meaning "normal ability", and $m = h$, meaning "high ability". The junior receives $r \in \{l, n, h\}$. Because of noise of communication r may deviate from m. As to this noise, we assume:

Assumption 1 $\Pr(r = l \mid m = l) = \Pr(r = n \mid m = n) = \Pr(r = h \mid m = h) = \alpha$

Assumption 2 $\Pr(r = n \mid m = l) = \Pr(r = n \mid m = h) = 1 - \alpha$

Assumption 3 $\Pr(r = l \mid m = n) = \Pr(r = h \mid m = n) = \frac{1}{2}(1 - \alpha)$

Assumption 4 $\alpha > \frac{1}{2}$.

Assumption 1 states that the probability that the junior receives the correct message equals α, and that this probability is independent of m. Assumption 2 and 3 imply that small communication errors are more likely than large ones. Notice that by sending $m = l$ ($m = h$), the senior can avoid that the junior receives $r = h$ ($r = l$).[5] The assumption that $\alpha > \frac{1}{2}$ implies that the probability that $r = m$, is higher than the probability that $r \neq m$. This assumption ensures that if the senior wants the junior to receive message $r = i$ the best she can do is sending message $m = i$.

Finally, we simplify the model of the previous section by restricting the choice of effort to three alternatives, $e_t \in \left\{0, \frac{1}{2}, 1\right\}$. As before, $e_t = 0$ amounts to maintaining

[5] We have set $\Pr(r = h \mid m = l) = \Pr(r = l \mid m = h)$ at zero rather than at small positive values to reduce notation. What matters for the results is that by choosing $m = h$ ($m = l$), the senior minimizes the probability that the junior receives $r = l$ ($r = h$).

status quo, and learning requires $e_t > 0$. As in the present model the junior cannot learn his ability by exerting an infinitesimal level of effort, we can assume that $z = 0$. Let us now discuss the equilibrium for this example.

4.1 Stage 2

Suppose that in stage 1 it is a best response of the junior to act in line with the message he has received: choose $X_1 = 0$ if $r = l$, $X_1 = 1$ with $e_1 = \frac{1}{2}$ if $r = n$, and $X_1 = 1$ with $e_1 = 1$ if $r = h$. Suppose $X_1 = 0$. Then, as in the general model, it is also optimal for the junior to choose $X_2 = 0$. Now suppose $X_1 = 1$, implying that the junior learns his ability. His decision then depends on the answer to the question: for which values of a should the junior choose $X_2 = 0$, $X_2 = 1$ with $e_2 = \frac{1}{2}$, or $X_2 = 1$ with $e_2 = 1$? Let a_L denote the value of a for which the junior, knowing a, is indifferent between $X_2 = 0$ and $X_2 = 1$ with $e_2 = \frac{1}{2}$. a_L follows from [see (1) and (2) with $z = 0$]

$$\frac{1}{2}a - \frac{1}{8} = 0 \Rightarrow a = a_L = \frac{1}{4} \tag{4}$$

Furthermore, let a_H denote the value of a for which the junior, knowing a, is indifferent between $X_2 = 1$ with $e_2 = \frac{1}{2}$ and $X_2 = 1$ with $e_2 = 1$. a_H follows from

$$\frac{1}{2}a - \frac{1}{8} = a - \frac{1}{2} \Rightarrow a = a_H = \frac{3}{4} \tag{5}$$

Equations (4-5) imply that in stage 2 all three options may be optimal for the junior. For $a \in [0, \frac{1}{4}]$, the junior chooses $X_2 = 0$; for $a \in (\frac{1}{4}, \frac{3}{4})$, the junior chooses $X_2 = 1$ with $e_2 = \frac{1}{2}$; and for $a \in [\frac{3}{4}, 1]$, the junior chooses $X_2 = 1$ with $e_2 = 1$.

4.2 The Senior

Generally, the senior wants the junior to choose $X_1 = 0$ for low values of a, to choose $X_1 = 1$ with $e_1 = \frac{1}{2}$ for intermediate values of a, and to choose $X_1 = 1$ with $e_1 = 1$ for high values of a. Against this background, it is natural to assume that the senior's strategy can be represented by:

$$m(a) = \begin{cases} m = l & \text{if } a \leq a^* \\ m = n & \text{if } a^* \leq a < a^{**} \\ m = h & \text{if } a \geq a^{**} \end{cases} \tag{6}$$

Let us first establish that $\alpha < 1$ has consequences for the senior's strategy.

Lemma 1 *Suppose that the junior chooses $X_1 = 0$ if $r = l$, $X_1 = 1$ with $e_1 = \frac{1}{2}$ if $r = n$, and $X_1 = 1$ with $e_1 = 1$ if $r = h$. Then, $a_L < a^*$ and $a^{**} < a_H$.*

Proof. Suppose that $a = a_L$. Then, for $\alpha = 1$, both players are indifferent between $X_1 = 0$ and $X_1 = 1$ with $e_1 = \frac{1}{2}$. Clearly, $X_1 = 1$ with $e_1 = 1$ yields a lower payoff. The inequality $\alpha < 1$ implies that $m = n$ may impel the junior to choose $X_1 = 1$ with $e_1 = 1$. Sending $m = l$, by contrast, never leads the junior to choose $X_1 = 1$ with $e_1 = 1$. Hence, the senior strictly prefers sending $m = l$ to sending $m = n$. Now suppose that $a = a_H$. Then, both players are indifferent between $X_1 = 1$ with $e_1 = \frac{1}{2}$ and $X_1 = 1$ with $e_1 = 1$, while $X_1 = 0$ yields a lower payoff. By sending $m = h$, the senior can avoid that the junior chooses the inferior option $X_1 = 0$. Hence, the senior strictly prefers sending $m = h$ to sending $m = n$. Q.E.D.

To understand the intuition behind Lemma 1, first consider a junior whose ability is just above a_L. In this situation, the senior ideally wants the junior to perform the task with moderate effort. However, the senior really wants to prevent the junior to exert too much effort. The costs of overconfidence are much higher than the costs of underconfidence. To avoid overconfidence, and in turn $e_1 = 1$, the senior sends $r = l$. Now consider a junior whose ability is just below a_H. In that situation, the worst case is that the junior chooses not to perform the task. Then, the benefits of the project in period 1 are foregone, and the junior will not learn his ability. Clearly, the cost of underconfidence are now higher than the costs of overconfidence. By sending $m = h$ the senior is sure to avoid a situation in which the junior chooses not to perform the task.

Let us now determine the equilibrium values of a^* and a^{**}. For $a = a^*$, the senior is indifferent between sending $m = l$ and sending $m = n$. Suppose that the senior sends $m = l$. Then, with probability α, the junior receives $r = l$ and does not perform the task. With probability $1-\alpha$, the junior receives $r=n$ and chooses $X_1=1$ with $e_1=\frac{1}{2}$. In that case, the junior learns that $a=a^*>a_L$, implying that he will also choose $X_2=1$ with $e_2=\frac{1}{2}$ in stage 2. Thus, $m=l$ yields an expected payoff to the junior equal to

$$2(1-\alpha)\left(\frac{1}{2}a^*-\frac{1}{8}\right) \tag{7}$$

Now suppose that the senior sends $m=n$. Then, with probability $\frac{1}{2}(1-\alpha)$ the junior receives $r=l$ and does not perform the task. With probability α the junior receives $r=n$, and performs the task with $e_1=\frac{1}{2}$. Finally, with probability $\frac{1}{2}(1-\alpha)$, the junior performs the task with effort $e_1=1$. In the latter two cases, the junior learns that $a=a_L$, leading him to choose $X_2=1$ with $e_2=\frac{1}{2}$ in stage 2. Sending $m=n$ yields an expected payoff to the junior equal to

$$\frac{3}{4}a^*+\frac{1}{4}\alpha a^*-\frac{5}{16}+\frac{1}{16}\alpha \tag{8}$$

It is easy to verify that (7) equals (8) for

$$a^*=\frac{1+3\alpha}{20\alpha-4} \tag{9}$$

Equation (9) illustrates the tradeoff the senior faces when she must choose between sending $m=l$ and sending $m=n$. On the one hand, she knows that the junior is (just) sufficiently able to perform the task when $a=a^*$. Therefore, the more confident the senior is that $m=n$ induces the junior to choose $X_1=1$ with $e=\frac{1}{2}$ (that is the higher is α), the more she tends to send $m=n$. On the other hand, the senior fears that by sending $m=n$ the junior will put too much effort on the task.

For $a=a^{**}$, the senior is indifferent between sending $m=h$ and sending $m=n$. Lemma 1 states that for $a=a^{**}$, a junior, knowing his ability, chooses to perform the task with moderate effort. Thus, if the junior chooses $X_1=1$ in stage 1, he will choose $X_2=1$ with $e_2=\frac{1}{2}$ in stage 2. Consequently, sending $m=n$ yields a payoff equal to (8) with a^{**} instead of a^*. Moreover, straightforward algebra shows that sending $m=h$ yields a payoff to both the senior and the junior equal to

$$a^{**}+\frac{1}{2}\alpha a^{**}-\frac{1}{4}-\frac{3}{8}\alpha \tag{10}$$

Equation (8), with a^* replaced by a^{**}, equals (10) for

$$a^{**} = \frac{7\alpha - 1}{4(1+\alpha)} \tag{11}$$

Equation (11) shows that the lower is α, the more the senior is inclined to send $m = h$. In deciding to send $m = h$ or $m = n$, the senior compares two costs. First, the costs of underconfidence. By sending $m = n$, the senior runs the risk that the junior receives $r = l$ and consequently does not perform the task in stage 1. In that case, the junior does not learn her ability and will not perform the task in stage 2 either. Second, the costs of overconfidence. By sending $m = h$ the junior will expend too much effort (recall $a^{**} < a_H = \frac{3}{4}$). Notice that this cost is limited to stage 1. As learning by doing takes place, the junior will choose $X_2 = 1$ with $e_2 = \frac{1}{2}$ in stage 2.

The following proposition summarizes the above discussion.

Proposition 2 *Suppose that the junior chooses $X_1 = 0$ if $r = l$, $X_1 = 1$ with $e_1 = \frac{1}{2}$ if $r = n$, and $X_1 = 1$ with $e_1 = 1$ if $r = h$. Then, the senior sends $m = l$ if $a \leq \frac{1+3\alpha}{20\alpha-4}$, $m = h$ if $a \geq \frac{7\alpha-1}{4(1+\alpha)}$ and $m = n$ otherwise.*

A direct implication of the above proposition is that if $\alpha = \frac{1}{2}$, then the senior never sends $m = n$.

4.3 Evaluation of the Example

Let us now go back to the social psychologists' findings on self-assessments discussed in the introduction. Data on self-assessments are usually based on experiments. In these experiments, persons - often undergraduates - are asked to rate a certain skill on some scale. Researchers use different scales. For example, Kruger (1999) uses a scale from 1 to 10, while Ehrlinger and Dunning (2003) use a 3-point scale in one of their experiments. How would juniors from our model rate themselves?

In our example 25 percent of the juniors should not perform the task; 50 percent should perform the task with moderate effort; and 25 percent should perform the task with high effort. Suppose that the juniors are asked to rate their abilities in stage 1, that is, after they have received message r. In line with the senior's message space we assume a 3-point scale: low ability, normal ability, and high ability. It seems natural to assume that when asked to their ability, the juniors from our model report the message they have received from their senior. This means that if $\alpha = 1$, abilities and self-assessments do not differ (see Figure 1)

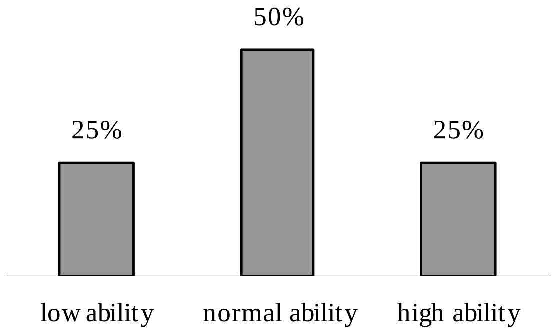

Figure 1

Now suppose that $\alpha = 0.7$. With the help of Proposition 2, it is easy to calculate the frequencies with which the senior sends the three possible messages. Figure 2 gives the distribution of m.

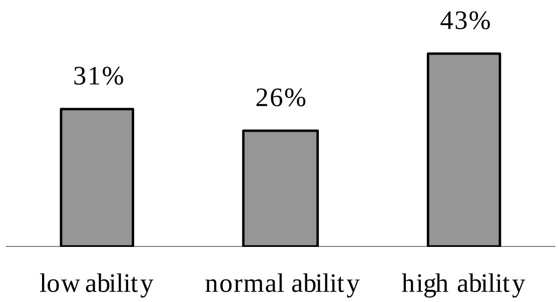

Figure 2

Using Assumption (1-3) and the percentages given in Figure 2, we can calculate the messages the juniors receive and thus their self-assessments. Figure 3 gives the results.

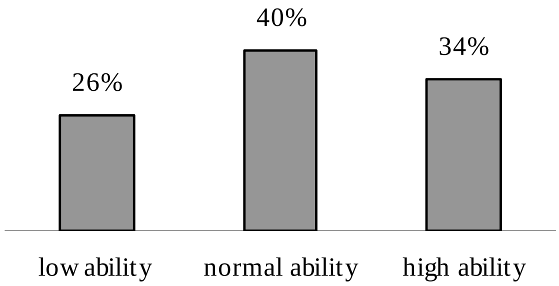

Figure 3

A comparison between Figure 1 and Figure 3 shows that the tails in Figure 3 are much thicker than in Figure 1. Almost 35 percent of the juniors report that they have a high ability, while only 25 percent of the juniors actually have a high ability. Another feature of Figure 3 is that the distribution of self-assessments is skewed to the right. It is tempting to conclude from this skewness that our model predicts inflated self-assessments. As the distribution of actual abilities is symmetric, there is an "above average effect". However, as long as the juniors make correct statistical inferences, overconfidence (or underconfidence) does not exist on average. To understand why, consider a junior who has received $r = h$. If rational, this junior takes into account that (1) the senior may have sent $m = n$ or $m = h$; and (2) the senior has sent a message in line with the values of a^* and a^{**} reported in Proposition 1. In that case, the junior does not make a systematic error when assessing his ability.

While juniors do not necessarily make systematic errors on self-assessments, they do make systematic errors on the choice of task. In our model overconfidence induces juniors to put too much effort in the task. Underconfidence induces juniors to remain passive, while they would have benefited from performing the task. Misperceptions of abilities are partly caused by noise in the communication ($\alpha < 1$). For example, a moderately talented junior may abstain from performing a task because he mistakenly infers from his senior's message that he is untalented. More interesting are the cases in which the junior's misperception is "intended by the senior". We have seen that when $a \in [a^{**}, a_H)$, the senior sends a message which is likely to induce a junior to expend too much effort on the task. Such a junior is overconfident. Notice that overconfidence is temporary. Because the junior performs the task, he will learn his ability. This feature of the model is consistent with the observation that "performance is a better predictor of self-appraisals than the appraisals of others" (Felson, 1989, p. 965). When $a \in (a_L, a^*]$ the senior is likely to induce a junior who would benefit from performing the task to abstain from performing the task. In that case, the junior is underconfident. As for $a \in (a_L, a^*]$ underconfidence leads to passiveness, underconfidence is permanent. Also notice that because underconfidence leads to passiveness and overconfidence leads to activism, underconfidence is relatively hidden. It is easier to observe that somebody has overestimated his ability than that somebody has underestimated his ability.

In Figure 2, the distribution of the senior's messages is skewed to the right. To show that this result does not depend on the specific value of α chosen, we compare the length of the interval $(a_L, a^*]$ with the length of the interval $[a^{**}, a_H)$. Straightforward algebra shows that $a_H - a^{**} = \frac{(1-\alpha)}{1+\alpha} > a^* - a_L = \frac{1-\alpha}{10\alpha-2}$ for $\frac{1}{2} \leq \alpha < 1$. Hence, consistent with the empirical findings, our model predicts that on average appraisals are too positive. The reason for this result is that the costs of causing overconfidence (too much effort) are lower than the costs of underconfidence (passiveness). As discussed above, overconfidence disappears in stage 2, but underconfidence does not disappear in stage 2. Indeed, one can verify that if we eliminate stage 2 from our model, then we obtain $a_H - a^{**} = \frac{1}{4\alpha} - \frac{1}{4} = a^* - a_L$.

5 Concluding Remarks

We have developed a simple model of self-assessments in order to explain some observations made by social psychologists. In this model, a junior has limited information about his ability. He can learn about his ability by information provided by a senior and by experience. We have shown that when communication between the senior and the junior is noisy, the senior may have an incentive to give too positive appraisals to more talented juniors and too negative appraisals to less talented juniors. On average, the senior's appraisals are too positive. Concerning self-assessments, our model predicts the well-known "above average effect": the distribution of self-assessments is skewed to the right, while the distribution of actual abilities is symmetric. Nevertheless, in our model some juniors believe they are less able than they actually are. In this respect, our model deviates from most other economic models of self-assessments that predict that all agents are overconfident (references are in the introduction). Finally, we have argued that when overconfidence or underconfidence matters for behaviour, there is an important difference between the two. Underconfidence leads to passiveness, which obstructs learning by experience. In contrast, overconfidence leads to activism, which enhances learning by experience. The implication is that underconfidence is more permanent, while overconfidence is more temporary.

In order to highlight the role of imperfect communication and the possibility of learning by doing in the self-appraisal process, we have made several restrictive assumptions. We have already discussed some of them. Let us elaborate on two other ones.

First, we have assumed that by performing the task, the junior fully learns his ability. In many situations, this assumption is not realistic. However, assuming that by performing the task the junior receives a noisy signal about his ability rather than a fully informative signal does not affect our results qualitatively. Relative to underconfidence, overconfidence remains a temporary phenomenon. Things become more complicated when the degree of learning depends on effort. One can imagine situations in which the degree of learning is positively related to effort. In that case, the junior will be more biased towards performing the task with higher effort.

A second important assumption is that the junior's and senior's preferences are perfectly aligned. A natural extension of our model is to allow for conflicting preferences. Suppose, for example, that the senior's preferences are identical to those of the junior, except that the senior attaches less cost to the junior's effort: $U_t^m (X_t = 1) = ae_t - \lambda_m e_t^2$, with $\lambda_m < \frac{1}{2}$. The junior's preferences are still represented by (1). It is easy to verify that in the resulting model, three types of equilibria exist. A separating equilibrium exists if λ_m is close to $\frac{1}{2}$. The outcomes are similar to those discussed in the previous section, save that the senior has a stronger incentive to exaggerate her junior's ability. Thus, $\lambda_m < \frac{1}{2}$ strengthens the senior's tendency towards too positive feedback. If λ_m is smaller than a certain threshold, it is not a best reply for the junior anymore to act in accordance with his senior's message. The senior inflates the junior's ability too much. For moderate values of λ_m a partially separating equilibrium exists, in which the senior only sends two messages, say $m = n$ and $m = h$. Finally, for very small values of λ_m, it is a best response for the junior to ignore the senior's message completely, and to base his decision on the task on his prior information. That is, only a pooling equilibrium exists. The upshot of this discussion is that $\lambda_m < \frac{1}{2}$ increases the senior's incentive to give too positive feedback. This stronger incentive may partially or fully distort communication between the senior and the junior.

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