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# Zipf's Law for Integrated Economies

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# THE DISTRIBUTION OF OUTPUT AND FACTORS IN INTEGRATED ECONOMIC AREAS: NEW PERSPECTIVES AND EVIDENCE

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**Abstract**

This paper derives and tests three theoretical predictions regarding the distribution of output and factors across members of an integrated economic area (IEA) in which goods and factors are mobile. First, we show that each member's shares of total IEA output and stocks of productive factors will be equal. If policies are largely harmonized across IEA members then this "equal-share" property implies that the growth in any member's output and factor shares can be considered a random outcome. If IEA member output and factor shares evolve as geometric Brownian motion with a lower bound then the limiting distribution of each share across IEA members will be a rank-share distribution that exhibits a property called Zipf's law. We then show this property implies that the limiting distribution of output shares depends only on the number of IEA members. We empirically examine for these theoretical predictions for two presumably integrated economic areas: U.S. states and E.U. countries. Our empirical findings strongly support Zipf's law for the distribution of output, physical capital and human capital across members of these two IEAs. Support is also obtained for the "equal-share" property and for the prediction that the distribution of output across IEA members depends only on the number of IEA members.

JEL Classification: E13, F15, F21, F22, O57

Keywords: growth, economic integration, factor price equalization, Zipf's law

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**The Distribution of Output and Factors in Integrated Economic Areas:**

**New Perspectives and Evidence**

The number of regional trade agreements has increased continuously since the early 1990s and many new initiatives for special association agreements are currently being negotiated (see WTO website). Institutional arrangements under which countries open their borders differ in reality. Most agreements are designed to increase international trade between markets but some, like the European Union, also allow greater mobility of productive factors within the integrated area and seek greater policy coordination across members. In the limit, such integration would be represented by a fully integrated economic area (IEA) in which there is free mobility of goods and factors among IEA members together with complete harmonization of economic and social policies. The economic union of U.S. states comes the closest to this ideal.

Prior work has demonstrated the potentially important role of trade[1] and factor mobility[2] as influences on economic growth, but less attention has been given to the question of how increased trade and factor mobility within integrated areas impacts the distribution of output across members of an IEA, and hence how these influences affect the relative economic position of members. Apart from being simply a question of distributional

[1] An extensive body of work has explored the role of international trade and of factor mobility as mechanisms generating endogenous economic growth. For example, Grossman and Helpman (1991) show that trade generally enhances growth, particularly when it facilitates the international transmission of knowledge. Similarly, Rivera-Batiz and Romer (1991) show that increased trade due to economic integration may have both level and growth effects depending upon the processes by which R&D and information flow across borders. Devereux and Lapham (1994) extend Rivera-Batiz and Romer's model to show that, even without knowledge flows, the balanced growth rate when there is free trade in goods alone exceeds that in autarky, provided that initial levels of national income differ across countries.

[2] For example, Baldwin and Martin (2004) examine the relationship between growth and the agglomeration of economic activity and find that it depends crucially on the extent of capital mobility between regions. Similarly, Viaene and Zilcha (2002) show that while complete capital market integration among countries has a positive effect on outputs, it does not raise long-run growth rates above autarky values. Instead, these growth rates are affected only by parameters that describe the accumulation of human capital.

consequences, analysis of this question has important implications for the nature of models that can be used to characterize the growth processes of IEA members.

Country shares of regional (or world) output, or shares of a region's total supplies of productive factors, have become increasingly important constructs in the international trade literature (e.g., Bowen et al. (1987), Helpman and Krugman (1985), Leamer (1984), Viaene and Zilcha (2002)). In this regard, below we derive and test three main theoretical predictions regarding the nature of the distribution of output and factors across IEA members. First, we demonstrate that within an IEA each member's share of total IEA output will equal its shares of total IEA stocks of each productive factor (i.e., physical capital and human capital). We label this result the "equal-share" relationship. This relationship also holds in the presence of technological differences or costs of factor mobility among members if outputs or inputs are properly measured to reflect such differences or costs. The second prediction regarding the distribution of economic activity is *Zipf's law*. Specifically, this law establishes a specific relationship among member shares, namely that the share of e.g. output of the largest member is twice that of the second largest member, thrice that of the third largest member, etc. Building on Gabaix (1999a), if output and factor shares are random and evolve as geometric Brownian motion with a lower bound, then the limiting distribution of these shares will exhibit Zipf's law. The last prediction is that, if Zipf's law holds, we show that the limiting values of each IEA member's output and factor shares are completely determined once the number of IEA members is specified.

Given the potential importance of these three properties, we empirically examine for their presence for two groupings of economic units: the 50 U.S. states and the District of Columbia (hereafter the 51 U.S. states) and 14 countries of the European Union (E.U.). The IEA consisting of the 51 U.S. states represents the highest form of integration and serves as benchmark of our empirical analysis. Our data on the output and factor stocks of each IEA member generally covers the period from 1965 to 2000. For both U.S. states and the E.U. countries, our empirical results convincingly support the theoretical predictions of Zipf's Law and the equal-share relationship as well as the prediction that the distribution of output (and of each factor) across IEA members depends only on the number of IEA members. While there may be several explanations for these surprising results, the evidence is consistent with a model that assumes that the growth process of the shares of IEA members is random and homogeneous across members.

The concept of equal-share is central to our analysis. One implication is that it sets a constraint on the long-run relative growth performance of IEA members. In particular, since the sum of output shares across all IEA members equals unity, the long-run expected growth rate of output shares must be zero. Therefore, it is not possible for every member of an IEA to sustain a positive rate of growth in its output share in the long run. A second implication is that, in any given time period, the relative growth performance of IEA members may simply be a random outcome that is contingent on alternative states of nature. The random behavior of IEA member's relative growth is more true the greater the extent of economic integration among members. Hence, it is more likely if members do not run independent monetary or exchange rate policies, when fiscal policies are constrained by institutions, when education systems are harmonized, and when successful local industrial policies are rapidly imitated. For example Ireland, although an E.U member, independently conducted in the 1980s and 1990s a number of policies (e.g., low corporate tax rate, education reforms, etc.) that differed significantly from those followed by other E.U. member states. These policies attracted multinationals in key sectors to Ireland, particularly from the U.S., and led some E.U. firms to relocate to Ireland. At the same time, Ireland's share of E.U.-15 GDP rose from 0.6% in 1980 to 1.2% in 2000. This increase in Ireland's share of E.U. GDP illustrates the differential impact of harmonized versus non-harmonized policies in an integrated economic area on a member country's share of output. Given its unusual success, E.U. (and OECD) members pressured Ireland to move its policies, particularly its tax regime, closer to the E.U. average.

The remainder of the paper is as follows. Section 1 derives the theoretical result on the equality of output and factor shares for each member of a fully integrated economic area followed by first evidence in support of this result. Section 2 explains the emergence of a rank-share distribution that exhibits Zipf's law. Section 3 indicates the empirical specification used to test for Zipf's law, the data used, and the test results. Section 4 uses the evidence of Section 3 to further characterize integrated economic areas. This includes a formal test of the equal-share relationship together with the derivation and tests regarding the limiting distribution of shares. Section 5 summarizes and discusses the consequences of our findings.

## 1 Equality of Output and Factor Shares in an Integrated Economic Area

To demonstrate the equality of output and factor shares for each member of a fully integrated economic area (IEA) we consider an IEA that consists of $m = 1, \ldots, M$ members, each producing a single good by means of a constant return to scale production function of the form:

(1) $$Y_{mt} = F(K_{mt}, H_{mt}) \qquad m = 1, \ldots, M$$

where $Y_{mt}$ is the level of output, $K_{mt}$ the stock of physical capital, and $H_{mt}$ the stock of human capital, all for country $m$ at time $t$. The production function is assumed to satisfy all the neoclassical assumptions including diminishing marginal productivity with respect to each factor. For ease of exposition, the production function is assumed to take the Cobb Douglas form:[3]

(2) $$Y_{mt} = A_{mt} K_{mt}^{\alpha_m} H_{mt}^{1-\alpha_m} \qquad m = 1, \ldots, M,$$

[3] The Cobb-Douglas specification has wide empirical support (e.g., Mankiw et al. (1992)). The analysis using a constant elasticity of substitution (C.E.S.) form yields the same outcome.

Here $A_{mt}$ is a scale parameter and $\alpha_m$ is capital's share of total output. If physical capital and labor are perfectly mobile between the $M$ members then we would expect the marginal product of each factor to be equal. Barriers to capital mobility (e.g. corporate income tax differentials, capital controls) or labor mobility (e.g. language, different pension systems) would instead create persistent differences in factor rates of returns between members. Consider one reference member of this IEA that, without loss of generality, we take to be country $i$. Let $\lambda_{mt}^k$ and $\lambda_{mt}^h$ define the proportional difference in rates of return to physical capital and to human capital between any country $m$ and reference country $i$. The relation between rates of return to physical capital in the IEA can then be written as:

$$(3) \qquad v_1 \lambda_{1t}^k \frac{Y_{1t}}{K_{1t}} = \ldots = \frac{Y_{it}}{K_{it}} = \ldots = v_M \lambda_{Mt}^k \frac{Y_{Mt}}{K_{Mt}}$$

where $v_m = \alpha_m/\alpha_i$, implying $v_m = 1$ when $\alpha_m = \alpha_i$ ($m$ = 1, ..., $M$). Note that for $m = i$, $\lambda_{it}^k = 1$ and $v_i = 1$. Likewise, the relation between rates of return to human capital can be written:

$$(4) \qquad \omega_1 \lambda_{1t}^h \frac{Y_{1t}}{H_{1t}} = \ldots = \frac{Y_{it}}{H_{it}} = \ldots = \omega_M \lambda_{Mt}^h \frac{Y_{Mt}}{H_{Mt}}$$

where $\omega_m = (1-\alpha_m)/(1-\alpha_i)$, implying $\omega_m = 1$ when $\alpha_m = \alpha_i$ ($m$ = 1, ..., $M$). Note that for $m = i$, $\omega_i = 1$ and $\lambda_{it}^h = 1$. The ratio of (3) to (4) gives the following relationship between ratios of human to physical capital:

$$(5) \qquad \eta_1 \lambda_{1t} \frac{H_{1t}}{K_{1t}} = \ldots = \frac{H_{it}}{K_{it}} = \ldots = \eta_M \lambda_{Mt} \frac{H_{Mt}}{K_{Mt}} = \frac{\sum_{m=1}^{M} \eta_m \lambda_{mt} H_{mt}}{\sum_{m=1}^{M} K_{mt}}$$

where

$\eta_m = v_m / \omega_m = \alpha_m (1-\alpha_i) / \alpha_i \, (1-\alpha_m)$, implying $\eta_m = 1$ when $\alpha_m = \alpha_i$;

$\lambda_{mt} = \lambda_{mt}^k / \lambda_{mt}^h$, implying $\lambda_{mt} = 1$ when $\lambda_{mt}^k = \lambda_{mt}^h$.

Like in (5), we can rewrite (3) as:

(6) $$v_1 \lambda_{1t}^k \frac{Y_{1t}}{K_{1t}} = \ldots = \frac{Y_{it}}{K_{it}} = \ldots = v_M \lambda_{Mt}^k \frac{Y_{Mt}}{K_{Mt}} = \frac{\sum_{m=1}^{M} v_m \lambda_{mt}^k Y_{mt}}{\sum_{m=1}^{M} K_{mt}}$$

Combining (5) and (6) yields the following relationship between output and factor shares for reference member *i* of the IEA:

(7) $$\frac{Y_{it}}{\sum_{m=1}^{M} v_m \lambda_{mt}^k Y_{mt}} = \frac{K_{it}}{\sum_{m=1}^{M} K_{mt}} = \frac{H_{it}}{\sum_{m=1}^{M} \eta_m \lambda_{mt} H_{mt}} \qquad i = 1, \ldots, M$$

We term equation (7) the "equal-share relationship." This relationship determines the distribution of output and factors across the *M* members of an IEA. Expression (7) contains both observable variables ($Y_{mt}, K_{mt}, H_{mt}$) and unknown parameters $(\alpha_m, \lambda_m^k, \lambda_m^h)$. Differences in technology or factor market imperfections imply a multiplicative rescaling of the observable variables that is different for each ratio. For example, a difference in $\alpha$'s leaves the observed values (and shares) of physical capital unaffected but transforms the observed values of output and human capital in different ways (through $v_m$ and $\eta_m$ respectively). If we assume that the *M* members of the IEA share the same technology ($\eta_m = v_m = \omega_m = 1$), and that there is costless (perfect) mobility of factors ($\lambda_{mt}^k = \lambda_{mt}^h = 1$) between members, then we obtain the simplest expression of the equal-share relationship for any member *i*:

(8) $$\frac{Y_{it}}{\sum_{m=1}^{M} Y_{mt}} = \frac{K_{it}}{\sum_{m=1}^{M} K_{mt}} = \frac{H_{it}}{\sum_{m=1}^{M} H_{mt}} \qquad i = 1, \ldots, M$$

Hence, with perfect capital mobility and similar technology, each economy's share of total IEA output, and each economy's share of total IEA physical capital stock, at any date t equals its share of the total IEA stock of human capital.

Relationship (8) has an important implication. It contrasts the policies pursued in isolation by any given IEA member with those that are instead pursued jointly (harmonized) across members. For example (8) does not change when a coordinated education policy by all IEA members increases their human capital by the same proportion. In contrast, the same policy implemented by only one member would increase that member's share of total IEA human capital (as long as this policy is not imitated by other members). Hence, if IEA members have harmonized economic and social policies (e.g., fiscal, education, industrial policies) then the equal-share property implies that the relative performance of each member remains unaffected by these policies. In this sense, member shares can be considered a random variable whose outcome is dependent on the particular state of nature at time *t*. Such randomness can easily be understood from the fact that various kinds of random shocks, like discoveries, weather, or natural disasters, including some that are specific to a particular member, would give rise to new and different sets of shares for all members.

In Section 3 we report tests of the null hypothesis given by (8) against the alternative hypothesis given by (7). As a prelude to that analysis, we provide here a first indication of the potential validity of the equal-share relationship by examining a "weak" form of this relationship, namely, that there will be conformity between (pair-wise) rankings of the output and factor shares across members of a given integrated area. Table 1 provides evidence of this weaker proposition by reporting Spearman rank correlation coefficients for pair-wise rankings of the shares of output, physical capital and human capital across members of the 51 U.S. states and the 14 E.U. countries in 1990, 1995 and 2000, years for which overlapping data on output, physical capital and human capital are available.[4] All rank correlations are positive and significant for both U.S. states and E.U. countries, supporting a "weak" form of

[4] For U.S. states our data consists of annual cross-sections covering 1990 to 2000. For E.U. countries the cross-sections are instead equally spaced at 5-year intervals and generally cover the period from 1965 to 2000. Section 3 gives a complete description of these data.

the equal-share relationship: that there will be conformity between (pair-wise) rankings of the output and factor shares across members of a given integrated area.

[Insert Table 1 here]

## 2 Rank-Share Distributions and Zipf's Law

A rank-share distribution describes a particular relationship between the share and rank of a variable across a set of observational units. It is related to the concept of a rank-size distribution. For instance, a rank-size distribution for city size exists if the relationship between the natural logarithm of city size and of rank based on size is linear and exhibits a negative slope; Zipf's law arises when this slope value equals -1.

The existence of Zipf's law for city sizes is a widely documented empirical regularity (e.g., see Brakman et al. (2001), Fujita et al. (1999), Gabaix (1999b), Gabaix and Ioannides (2004), Eeckhout (2004) and Rose (2005)). Several explanations have been advanced for the observed regularity of Zipf's law with respect to the distribution of city sizes. Some argue it constitutes an optimal spatial pattern that arises when congestion and urbanization externalities interact as part of the process of development and growth of cities. Such forces are usually found in core models of urban and regional growth (e.g., see Eaton and Eckstein (1997), Black and Henderson (1999), Brakman et al. (1999)). Others have stressed more mechanical forces that often involve a random growth process for city size. A recent example is Gabaix (1999a), who draws on Gibrat's law[5] to assume that cities follow a random but common growth process. Normalizing city population by a country's total population, Gabaix shows (his Proposition 1) that if population shares evolve as geometric Brownian motion with an infinitesimal lower bound then the steady state distribution of population shares will be a rank-size distribution that exhibits Zipf's law.

[5] Gibrat's law (Gibrat, 1931) states that firm growth is independent of firm size.

As previously noted, the equal-share property for members of an IEA together with an assumed harmonization of IEA members' economic policies implies that the relative performance of any one IEA member can be considered a random variable. Given this, we can assume like in Gabaix (1999a) that the share of variable *j* (e.g., *j* = output) evolves as geometric Brownian motion with a lower bound,[6] and moreover, that the distribution of growth rates of these shares is common to all IEA members (i.e., Gibrat's law).[7] These assumptions imply that the limiting distribution of the shares of variable *j* across IEA members will be a rank-share distribution that exhibits Zipf's law.

To understand the implications of Zipf's law, consider again an IEA consisting of *M* members. Let $S_{mj}$ denote member *m*'s share of the total IEA amount of variable *j* (*j* = output (*y*), physical capital (*k*) or human capital (*h*)) and let $R_{mj}$ denote the rank of member *m* in the ranking of shares of variable *j* across all members ($m = 1,\ldots, M$). Assume $R_{mj} = 1$ for the member with the largest share of variable *j* and $R_{mj} = M$ for the member with the lowest share of variable *j*. If variable *j* has a rank-share distribution then we can write:

(9) $$S_{mj} = \gamma_j \left(R_{mj}\right)^{\beta_j},$$

where $\beta_j < 0$ is the "power-law exponent" and $0 < \gamma_j < 1$ is the share of variable *j* for the IEA member with the highest rank (i.e., when $R_{mj} = 1$). A power law implies a specific relationship among shares: $S_{1j} / S_{2j} = 2^{-\beta_j}, S_{1j} / S_{3j} = 3^{-\beta_j}, \ldots, S_{1j} / S_{Mj} = \mathrm{M}^{-\beta_j}$. Zipf's law corresponds to $\beta_j = -1$, which simplifies the relationship among member shares, namely: $S_{1j} = 2S_{2j} = 3S_{3j} = \ldots = \mathrm{M}S_{Mj}$. This states that the share value of the highest ranked country is twice the share value of the second ranked country, etc.

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[6] One needs to prevent output and factors from falling below some lower bound in order to obtain a power law. Otherwise the distribution would be lognormal. A lower bound makes sense in integrated areas as important income transfers are institutionalized to prevent states/regions/countries to vanish. For example, the E.U. maintains a social fund and a regional fund.

[7] The equal-share relationship implies that the common expected rate of growth is zero since the sum over *i* of the output and factor shares in (8) must be one.

To gain insight into how a rank-share distribution that exhibits Zipf's law emerges we simulate the evolution of the distribution of output shares across U.S. states, allowing the number of years simulated to be 20, 50, 75, 100, 150, 200, 250 and 300. For each simulation, each U.S. state is initially assumed to have same level of output and hence the same share $S_{my}$ (i.e. $S_{my}$ = 0.0196 = 1/51). Starting from these common share values, the output shares are then specified to evolve randomly over time as geometric Brownian motion with a lower bound.[8] At annual intervals ($t$) during a simulation period, the output shares are used as data to estimate rank-share equation (9) and to test if the estimated $\beta_y$ is statistically different from -1.[9] This allows us to determine the point in time at which the distribution of output shares conforms to a rank-share distribution exhibiting Zipf's law.

Our simulations indicate that Zipf's law emerges after 75 to 150 years depending on the assumed volatility of the growth rate of the shares: the higher the volatility, the faster is convergence to Zipf's law.[10] Figure 1 summarizes the results of the 200 year simulation by showing the evolution of the maximum, median and minimum output share, and the point in time after which the distribution of the output shares exhibits, statistically, Zipf's law.

[Insert Figure 1 about here]

[8] Following Grabaix (1999a), each share evolves as $dS_{myt}/S_{myt} = \mu dt + \sigma dB_t$ if $S_{myt} > \min(S_{myt})$ where $\min(S_{myt})$ is the lower bound. Alternatively, the increment in each share is $dS_{myt}/S_{myt} = \max[\mu dt + \sigma dB_t, 0]$ for $S_{myt} \leq \min(S_{myt})$. Here $\mu < 0$ is a negative drift, $\sigma$ is the standard deviation (volatility), $B_t$ is a Wiener process. The term $dB_t$ is then the increment of the process, defined in continuous time as $dB_t = \varepsilon_t(dt)^{1/2}$. Since $\varepsilon_t$ has zero mean and unit standard deviation, $E[dB_t] = 0$ and $Var(dB_t) = dt$. The increment $dB_t$ is approximated by a running sum of 730 discrete increments ("shocks") since our simulations assume one calendar year is 365 days ($dt$ = 1/365) and we arbitrarily assume two random shocks (two draws of $\varepsilon_t$) on each day. We set $\mu$ = -0.01, $\min(S_{myt})$ = 0.001, and variously, $\sigma$ = 0.04, 0.05 and 0.07.
[9] The estimation procedures used are those detailed in Section 3 below.
[10] On the other hand, with lower volatilities (i.e., 0.01, 0.02 and 0.03) convergence is not obtained even after 300 years.

## 3 Empirical Analysis

To formally assess the hypothesis that output and factor shares conform to a rank-share distribution that exhibits Zipf's law we can take the natural logarithm of each side of (9) to obtain:

$$\text{(10)} \qquad \log\left(S_{mj}\right) = \theta_j + \beta_j \log\left(R_{mj}\right) + u_{mj} \qquad m = 1,...,M; j = y,k,h$$

where $\theta_j = \log(\gamma_j) < 0$ and $u_{mj}$ is the error term. Estimates of the intercept and slope parameters in (10) are crucial to our analysis and are obtained by regressing the share of variable *j* on variable *j*'s rank value across a given set of IEA members.

We estimate (10) separately for the output share, physical capital share and human capital share with respect to two presumably integrated economic areas: the 51 U.S. states and 14 E.U. countries.[11] Given estimates of (10), evidence against Zipf's law is assessed by testing if the estimated slope coefficient is significantly different from minus one. However, as Gabaix and Ioannides (2004) and Nishiyama and Osada (2004) recently demonstrate, both the OLS estimate of $\beta_j$ in (10) and its associated standard error will be biased downward, with these biases diminishing as the number of observational units ($M$) increases. Hence, without some correction for these inherent biases one is likely to more often reject Zipf's law when it is in fact true.

To correct for these biases, we follow Gabaix and Ioannides (2004, p. 10) and conduct, for the cases $M$ = 14 (E.U. countries) and $M$ = 51 (U.S. States), a Monte Carlo analysis of the OLS slope estimates derived from (10) under the assumption that Zipf's law holds.[12] The difference between the Zipf's law true slope value (-1) and the average of the

[11] The countries are Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal, Spain, Sweden and United Kingdom. Luxembourg is excluded for lack of data on human capital. Given the small scale of Luxembourg's economy relative to other E.U. countries this omission is unlikely to affect the results.

[12] Briefly, for a given sample size $M$ (either $M$ = 14 or $M$ = 51), 100,000 Monte Carlo simulations are performed drawing from an exact power law with coefficient –1 (Zipf's Law). This involved drawing $M$ i.i.d. variables $v_{m,}$

Zipf's law OLS slope estimates (-1.172 for $M$ = 14 and -1.081 for $M$ = 51) gives an estimate of the downward bias, which is 0.172 for $M$ = 14 and 0.081 for $M$ = 51. Given these estimates of the bias for each $M$, an estimate of the true slope coefficient is obtained by adding the estimated bias to the OLS estimate derived from (10).

To obtain a bias adjusted estimate of the standard error we follow Nishiyama and Osada (2004) and use the asymptotic approximation to the true standard error of the OLS slope estimate given as $-\hat{\beta}_j \sqrt{2/M}$, where $\hat{\beta}_j$ is the OLS estimate of the slope in (10).[13] The test statistic formed using these bias corrected values has asymptotically a normal distribution (Nishiyama and Osada, 2004).

**Data**

Our data set consists of data in a given year on the output and stocks of human and physical capital for the 51 U.S. states and for 14 E.U. countries. Due to limitations on sourcing data for human capital, the data on U.S. states are restricted to annual observations from 1990 to 2002. The data for E.U. countries are restricted to consist of observations equally spaced at 5 year intervals over the period 1960 to 2000. The following provides more details regarding data methods and sources.

For U.S. states, output for the period 1990 to 2000 is measured by real gross state product as reported annually by the U.S. Bureau of Economic Analysis.[14] For E.U. countries, output is measured by real gross domestic product (GDP), derived from data on real GDP per

uniformly distributed in the interval [0, 1], and then constructing sizes $L_m = 1/v_m$. The sizes $L_m$ are then normalized into shares $S_m$ that were then ordered and assigned a rank value $R_m$. We then perform 100,000 OLS regressions using the specification $\log(S_m) = \theta + \beta \log(R_m) + u_i$. The complete results are available from the authors upon request.

[13] Another method for estimating the parameters of a power law distribution is the maximum likelihood Hill estimator (Hill, 1975). However, as Gabaix and Ioannides (2004) remark, the properties of the Hill estimator in finite samples can be "very worrisome," and in particular their theoretical results predict a large bias in parameter estimates and associated standard errors in small samples. We computed the Hill estimators (results not shown) and indeed found very high downward biases in both parameter estimates and standard errors.

[14] Data on gross state product available at http://www.bea.doc.gov/bea/regional/gsp

capita and population given in the Penn World Tables 6.1 (Heston, Summers and Aten, 2002).

For both U.S. states and E.U. countries, the human capital stock is measured by the number of persons with at least secondary level of education. For U.S. states, data on educational attainment by state are taken from the U.S. Bureau of the Census.[15] These data are available only every 10 years, which limit the data on human capital for U.S. states to two years: 1990, and 2000.

For the E.U. countries, human capital stock is measured by multiplying the percentage of the population having at least a secondary level of education times a country's total population. Data on rates of educational attainment are taken from Barro and Lee (1993, 1996, and 2000); country population data are from Heston, Summers and Aten (2002).[16] The educational attainment data are only available every 5 years, which limit the E.U. data on human capital to five-year intervals from 1960 to 2000.

Annual estimates of U.S. state physical capital stocks from 1990 to 2000 are derived from BEA (2002) estimates of the total U.S. physical capital stock in each of nine one-digit industrial sectors that together comprise all economic activity.[17] The national industry physical capital stocks are allocated to each state by multiplying an industry's total capital stock[18] by the industry's contribution to a state's total income.[19] For each state, these industry capital stock estimates are then summed to obtain an estimate of a state's total stock of physical capital.[20] The calculation performed for each state *m* at time *t* can be expressed as

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[15] Decennial census dataset are available at http://factfinder.census.gov

[16] Other studies using the Barro-Lee data include Rajan and Zingales (1998), Ramey and Ramey (1995), Barro (1999), Easterly and Levine (1998), Hall and Jones (1999) and Sachs and Warner (1995).

[17] The sectors (BEA code) are Farming (81), Agricultural services, forestry, fishing & other (100); Mining (200); Construction (300); Manufacturing (400); Transportation(500); Wholesale and retail trade (610); Finance, insurance and real estate (700); and Services (800).

[18] Data on state physical capital stocks by industry were taken from U.S. Fixed Assets Tables, available at http://www.bea.doc.gov/bea/dn/faweb

[19] Annual data on state value added available at http://www.bea.doc.gov/bea/regional/spi

[20] This procedure follows that used by Munnel (1990) and Garofalo and Yamarik (2002).

$$k_{mt} = \sum_{j=1}^{9}\left[K_{jt}\left(\frac{y_{mjt}}{Y_{mt}}\right)\right]$$

In this equation, $k_{mt}$ is the stock of physical capital in state *m*, $y_{mjt}$ is industry *j's* value added in state *m* (*m* = 1, …, 51), $Y_{mt}$ is total value added in state *m*, and $K_{jt}$ is the national physical capital stock in industry *j* (*j* = 1,…, 9). This procedure assumes the capital-to-output ratio within industry *j* (i.e., $k_{mjt}/y_{mjt}$) is the same across U.S. states, that is, $k_{mjt}/y_{mjt} = K_{mt}/Y_{mt}$.

Estimates of E.U. country physical capital stocks for the period 1965 to 1990 are constructed by multiplying the Penn World Tables 5.6 (Heston and Summers, 1991a and 1991b) data on population, physical capital stock per worker and real GDP per capita and then dividing the result by real GDP per worker. Timmer et al. (2003) provides data on E.U. country physical capital stocks for 1980-2000.[21] These data for 1995 and 2000 are combined with the computed estimates for 1965-1990 to yield data on physical capital stocks at five year intervals between 1965 to 2000, inclusive.[22]

## Results

Table 2 reports the OLS and bias corrected estimates of (10) for the share of output, physical capital and human capital for the sample of U.S. states; Table 3 reports OLS and bias corrected estimates for the sample of E.U. countries.[23] Over all results, the adjusted $R$-squares range from 0.791 to 0.945 indicating a strong relationship between the share and rank of each variable.

For U.S. States, the column labeled "Bias Corrected Slope" in Table 2 reports the estimated slope value corrected for bias expected when OLS is used to estimate (10). Based on this bias corrected slope value, the column labeled "Z-statistic Testing Slope = -1"

[21] Physical capital database available at http://www.ggdc.net/dseries/growth-accounting.shtml
[22] Estimation was conducted using both sets of data for E.U. countries. No qualitative difference in results was found for the years in which data were available from both sources (i.e., 1980, 1985 and 1990). For these three years we therefore report only the results using the capital stock data from Timmer et al. (2003).
[23] The standard errors associated with the OLS estimates are "robust" in the sense of White (1980).

indicates strong support for the hypotheses that the output and factor shares for U.S. states conform to a rank-share distribution that exhibits Zipf's law; in no instance can we reject (at the 5% level) the hypothesis that the slope coefficient is significantly different from –1. This is strong evidence that, for U.S. States, each of the three share distributions exhibit Zipf's law.

For E.U countries, the column labeled "Bias Corrected Slope" in Table 3 reports the estimated slope value corrected for the bias expected when OLS is used to estimate (10). Based on this bias corrected slope value, the column labeled "Z-statistic Testing Slope = -1" indicates strong support for the hypotheses that the output and factor shares for E.U. countries conform to a rank-share distribution that exhibits Zipf's law: in no instance can we reject (at 5% level) the hypothesis that the slope coefficient is significantly different from -1.

[Insert Tables 2 and 3 about here]

These findings for U.S. states and for E.U. countries are striking empirical results. For comparison, we preformed the same analysis and tests (results not shown) with respect to a grouping of 30 developing countries as well as a "world" of 55 countries and found no evidence to support Zipf's law at the usual levels of significance.

## 4 Further Characterization of Integrated Economic Areas

The empirical findings of the preceding section have further implications regarding the characterization of integrated economic areas. One implication is the potential empirical validity of the equal-share relationship as derived in (8) since, if a power law holds (not necessarily Zipf), the output shares, or the shares for any given factor, are proportional across IEA members. Hence, if the equal-share relationship holds for one member then it must also hold for all other IEA members.[24] A second implication is that if Zipf's law holds then the limiting share values across IEA members are completely determined once the number of IEA members is specified. Each of these implications is investigated below.

### The Equal-Share Relationship

A strong test for the equal-share relationship involves the null hypothesis given by equation (8) against the alternative hypothesis given by (7). Evidence in favor of the equal-share relationship is obtained in two steps. First one tests for homogeneity of the OLS slope estimates (i.e., whether $\beta_y = \beta_k = \beta_h$) to verify that the distributions of shares come from a common power-law distribution. Second, one tests for intercept homogeneity across the three share equations (i.e., whether $\theta_y = \theta_k = \theta_h$) to examine if the equal-share relationship holds with respect to the highest ranked member of each IEA (i.e., California for U.S. states and Germany for E.U. countries). Failure to reject the null hypothesis would imply that technological differences and factor market imperfections are not strong enough to prevent the equal-share relationship from holding in a statistical sense.

Table 4 reports $p$-values for testing the hypotheses of slope and intercept homogeneity across the three share distributions in each sample year.[25] For U.S. states we cannot reject the hypotheses of intercept equality and slope equality in either of the two years for which data were available on all three shares (1990 and 2000). This result supports the equal-share relationship for U.S. states. The results for E.U. countries also indicate support for the equal-share relationship. We remark that the slope homogeneity tests use the OLS slope estimates uncorrected for bias. However, correcting for the expected downward bias would only strengthen support for the equal-share relationship found here.

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[24] Equally, it can be demonstrated that the equal-share property obtains if one assumes that output shares alone exhibit Zipf's law and that IEA members have identical, homogenous of degree one, production functions.

[25] These tests were performed by establishing, in each year, a system comprising the three share equations but without initially imposing any cross-equation parameter restrictions.

[Insert Table 4 about here]

**Limiting Distribution of Shares**

Let $V_{mj}$ denote the level of variable $j$ for member $m$. Assume, without loss of generality, that member 1 has the highest value of variable $j$ and let $\delta_{mj}$ be member $m$'s value of variable $j$ relative to that of member 1 (i.e., $\delta_{mj} = V_{mj} / V_{1j}$), so that $\delta_{1j} = 1$. Now order the values of variable $j$ in descending order. This ordering of the values of variable $j$ across the $m = 1, \ldots, M$ members can be written:

$$(11) \quad V_{1j} > \delta_{2j} V_{1j} > \delta_{3j} V_{1j} > \ldots > \delta_{Mj} V_{1j}.$$

Since the total IEA amount of variable $j$ is then $(1 + \delta_{2j} + \delta_{3j} + \ldots + \delta_{Mj})V_{1j}$, (11) implies the following relations between member ranks and shares:

$$(12) \quad \begin{aligned} &Rank\ 1: \quad S_{1j} = \frac{1}{1+\delta_{2j}+\delta_{3j}+\ldots+\delta_{Mj}}; \\ &Rank\ 2: \quad S_{2j} = \frac{\delta_{2j}}{1+\delta_{2j}+\delta_{3j}+\ldots+\delta_{Mj}}; \\ &Rank\ 3: \quad S_{3j} = \frac{\delta_{3j}}{1+\delta_{2j}+\delta_{3j}+\ldots+\delta_{Mj}}; \\ &\quad \vdots \qquad\qquad\qquad \vdots \\ &Rank\ \mathrm{M}: \quad S_{Mj} = \frac{\delta_{Mj}}{1+\delta_{2j}+\delta_{3j}+\ldots+\delta_{Mj}}. \end{aligned}$$

Expressions (12) indicate that the sequence of shares $S_{mj}$ is a Harmonic series, where each share value $S_{mj}$ depends on the values of the $\delta$'s and the number of members $M$. Accepting our preceding empirical evidence that the distribution of shares exhibits Zipf's law then $\delta_{2j} = 1/2$, $\delta_{3j} = 1/3$, $\delta_{4j} = 1/4$, etc., then the theoretical shares in (12) depend only on the number of IEA members and therefore can be computed once the number of members ($M$) is specified. For example, the theoretical share values for the $M = 51$ U.S. states are: 0.2213, 0.1106, 0.0738, 0.0553, …, 0.0043. For the $M = 14$ E.U. countries the theoretical share values are: 0.3075, 0.1538, 0.1025, 0.0769, …, 0.0220.

To test whether observed shares conform to those theoretically expected using (12), Table 5 reports simple correlations between the natural logarithms of the actual and expected shares for U.S. states and E.U countries in 1990 and 2000. The correlations range from 0.9176 to 0.9619 and all are highly significant, indicating a strong positive relationship between actual and theoretical shares.

[Insert Table 5 about here]

These simple correlations indicate a significant association among shares, but they do not indicate overall conformity of the actual and theoretical share distributions, that is, whether the actual and expected shares come from the same distribution. To test this, we use the non-parametric two-sample Kolmogorov-Smirnov test. In this test, the null hypothesis is that both sets of shares come from a common distribution against the alternative hypothesis that they do not. The results, shown in Table 6, convincingly fail to reject the null hypothesis, suggesting that the actual and theoretical shares arise from the same distribution.

[Insert Table 6 about here]

## 5 Summary and Concluding Remarks

This paper derived three theoretical predictions regarding the distribution of output and factors across members of an integrated economics area and then empirically examined for these predictions with respect to two integrated economics areas: the 51 U.S. states and 14 E.U. countries. Our data generally covered the period from 1965 to 2000. In all cases our empirical results supported the theoretical predictions. Specifically, our empirical results strongly supported the prediction that the distribution of output and factor shares would exhibit Zipf's law. While there may be several explanations for this empirical finding, the evidence on the empirical significance of Zipf's law is consistent with a model that assumes that the growth process of the shares of integrated economic area (IEA) members is random and homogeneous across members.

Our empirical results also supported the prediction of an equal-share relationship: each IEA member's shares of IEA output and IEA factor stocks will be equal. Evidence of the equal-share relationship has several implications regarding the characterization of an integrated economic area. First, the empirical significance of the equal-share relationship is consistent with the proposition that the relative growth performance of IEA members is largely random and hence strongly dependent on particular states of nature. Such randomness will be more true the greater the extent of economic integration among members, perhaps most exemplified by the IEA comprising U.S. states. Hence, it is more likely to be true the more harmonized are education systems and fiscal codes, when members do not run independent monetary policies, and when industrial policies are quickly imitated across members. Second the equal-share relationship addresses Lucas' (1990) question as to why capital does not flow from rich to poor countries. Namely, an economy with a low level (and hence low share) of human capital will also have a low share of physical capital, and also a low share of output. Finally, if the equal-share relationship holds, then all members of an integrated economic area will have the same output per efficiency unit of labor (i.e., human capital). This implication is the essence of the absolute convergence hypothesis (Barro and Sala-i-Martin (2004)), here interpreted in terms of efficiency units of labor, not in per capita terms.

Finally, we derived the result that when Zipf's law holds the values of the output and factor shares of IEA members are completely determined once the number of members is specified. These shares are limiting values that derive from the relative position (rank) of each member, and they would be expected to emerge as an IEA approaches full integration. Nonetheless, our empirical results indicated statistically significant agreement between the actual share values and the theoretically expected share values.

In providing evidence of Zipf’s law and the equal-share relationship with respect to members of an IEA, this paper indicates that these empirical characterizations should be kept in mind when studying the implications of alternative policies on the relative growth of members of an integrated economic area.

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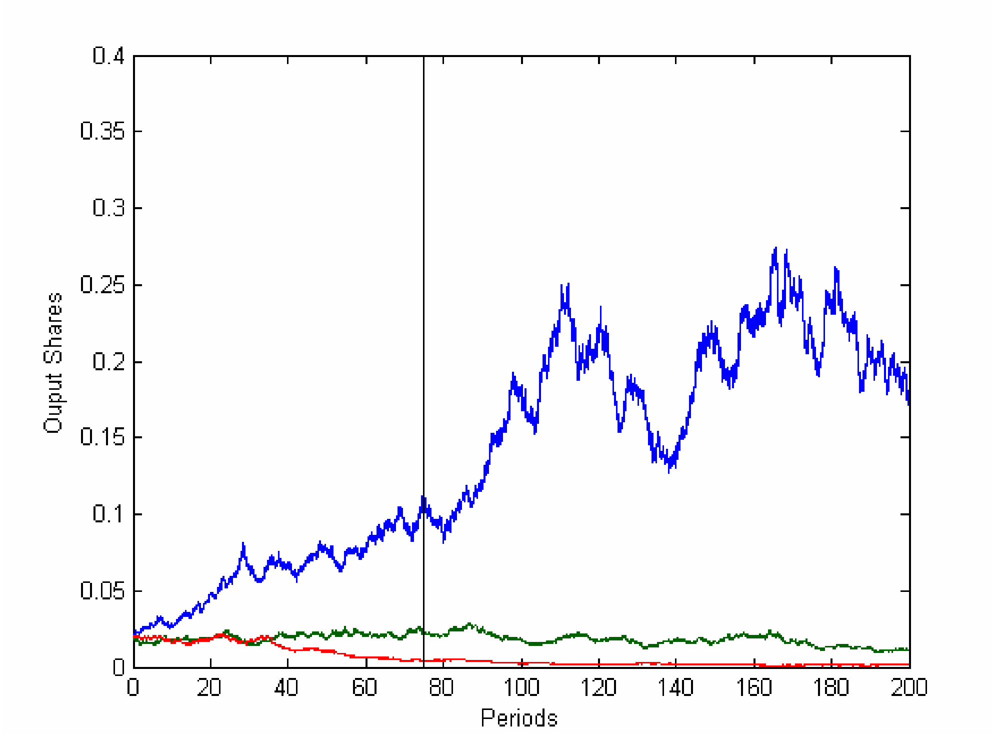
**Table 1. Spearman Rank Correlations for Output, Physical Capital and Human Capital Shares across U.S. States and E.U. Countries**

| Integrated Economic Area | Year | Spearman Rank Correlation between Shares of | | |
|---|---|---|---|---|
| | | Output and Physical Capital | Output and Human Capital | Physical And Human Capital |
| U.S. States [a] | 1990 | 0.987 | 0.977 | 0.980 |
| | 1995 | 0.991 | n.a. | n.a. |
| | 2000 | 0.992 | 0.981 | 0.978 |
| E.U. Countries [b] | 1990 | 0.956 | 0.776 | 0.829 |
| | 1995 | 0.960 | 0.851 | 0.837 |
| | 2000 | 0.956 | 0.820 | 0.881 |

[a] N = 51 in each year; coefficients whose absolute value exceeds 0.326 are significantly different from zero at the 1% level; critical values of the spearman rank correlation tests are obtained from Zar (1972).

[b] N = 14 in each year; coefficients whose absolute value exceeds 0.626 are significantly different from zero at the 1% level; critical values of the spearman rank correlation tests are obtained from Zar (1972).

**Figure 1. Simulated Path of the Maximum, Median and Minimum Output Share across 51 U.S. States** [a]




[a] Simulation over 200 years assuming that each state begins with the same output share and that the evolution of the shares then follows geometric Brownian motion with a lower bound. For this simulation, drift $\mu = -0.01$, lower bound $\min(S_{myt}) = 0.001$, volatility $\sigma = 0.07$. The vertical line indicates the starting period at which the power law exponent ($\beta_{yt}$) is no longer significantly different from –1 (Zipf's law holds).

**Table 2. OLS and Bias Corrected Estimates of Rank-Share Relationships for U.S. States**

| Variable | Year | OLS Intercept [a] | OLS Slope [b] | Bias-corrected Slope [c] | Z-statistic Testing Slope = -1 [d] | OLS Adj. $R^2$ |
|---|---|---|---|---|---|---|
| **Output Share (M=51)** | 1990 | -1.179 (0.248) | -1.101 (0.081) | -1.020 | -0.092 | 0.887 |
| | 1991 | -1.194 (0.248) | -1.093 (0.081) | -1.012 | -0.055 | 0.884 |
| | 1992 | -1.199 (0.252) | -1.090 (0.082) | -1.009 | -0.042 | 0.883 |
| | 1993 | -1.207 (0.258) | -1.085 (0.084) | -1.004 | -0.019 | 0.881 |
| | 1994 | -1.208 (0.265) | -1.084 (0.086) | -1.003 | -0.014 | 0.876 |
| | 1995 | -1.209 (0.265) | -1.083 (0.086) | -1.002 | -0.009 | 0.874 |
| | 1996 | -1.205 (0.267) | -1.085 (0.087) | -1.004 | -0.019 | 0.872 |
| | 1997 | -1.192 (0.271) | -1.091 (0.088) | -1.010 | -0.046 | 0.868 |
| | 1998 | -1.173 (0.272) | -1.100 (0.088) | -1.019 | -0.087 | 0.868 |
| | 1999 | -1.168 (0.271) | -1.103 (0.088) | -1.022 | -0.101 | 0.866 |
| | 2000 | -1.164 (0.266) | -1.106 (0.087) | -1.025 | -0.114 | 0.868 |
| **Physical Capital Share (M=51)** | 1990 | -1.199 (0.246) | -1.092 (0.080) | -1.011 | -0.051 | 0.892 |
| | 1991 | -1.207 (0.247) | -1.089 (0.080) | -1.008 | -0.037 | 0.891 |
| | 1992 | -1.200 (0.251) | -1.092 (0.081) | -1.011 | -0.051 | 0.892 |
| | 1993 | -1.197 (0.257) | -1.093 (0.083) | -1.012 | -0.055 | 0.890 |
| | 1994 | -1.196 (0.266) | -1.092 (0.086) | -1.011 | -0.051 | 0.884 |
| | 1995 | -1.173 (0.275) | -1.102 (0.089) | -1.021 | -0.096 | 0.879 |
| | 1996 | -1.168 (0.276) | -1.105 (0.089) | -1.024 | -0.110 | 0.878 |
| | 1997 | -1.126 (0.286) | -1.125 (0.093) | -1.044 | -0.198 | 0.870 |
| | 1998 | -1.126 (0.283) | -1.126 (0.091) | -1.045 | -0.202 | 0.876 |
| | 1999 | -1.108 (0.283) | -1.135 (0.092) | -1.054 | -0.240 | 0.875 |
| | 2000 | -1.093 (0.282) | -1.143 (0.091) | -1.062 | -0.274 | 0.880 |
| **Human Capital Share (M=51)** | 1990 | -1.244 (0.280) | -1.064 (0.091) | -0.983 | 0.081 | 0.854 |
| | 2000 | -1.264 (0.293) | -1.054 (0.096) | -0.973 | 0.129 | 0.839 |

[a] OLS standard errors in parentheses. All intercept coefficients significantly different from zero at 1% level.
[b] OLS standard errors in parentheses. All slope coefficients significantly different from zero at 1% level.
[c] Computed as the OLS slope estimate plus 0.081 (the bias).
[d] Computed as the OLS slope estimate plus 0.081 (the bias) minus –1 divided by the asymptotic approximation of the true standard error (given as minus the OLS slope estimate times 0.198 = $(2/51)^{0.5}$). All slope coefficients are not significantly different from –1 at the 5% level.

**Table 3. OLS and Bias Corrected Estimates of Rank-Share Relationships for E.U. Countries**

| Variable | Year | OLS Intercept [a] | OLS Slope [b] | Bias-corrected Slope [c] | Z-statistic Testing Slope = -1 [d] | OLS Adj. $R^2$ |
|---|---|---|---|---|---|---|
| **Output Share (M=14)** | 1960 | -0.645 (0.397) | -1.461 (0.192) | -1.289 | -0.523 | 0.908 |
| | 1965 | -0.665 (0.416) | -1.435 (0.204) | -1.263 | -0.485 | 0.889 |
| | 1970 | -0.699 (0.433) | -1.406 (0.212) | -1.234 | -0.440 | 0.867 |
| | 1975 | -0.742 (0.435) | -1.366 (0.211) | -1.194 | -0.376 | 0.859 |
| | 1980 | -0.755 (0.419) | -1.357 (0.202) | -1.185 | -0.361 | 0.870 |
| | 1985 | -0.763 (0.417) | -1.354 (0.199) | -1.182 | -0.356 | 0.872 |
| | 1990 | -0.772 (0.420) | -1.346 (0.198) | -1.174 | -0.342 | 0.872 |
| | 1995 | -0.777 (0.405) | -1.343 (0.187) | -1.171 | -0.337 | 0.878 |
| | 2000 | -0.857 (0.376)* | -1.272 (0.170) | -1.100 | -0.208 | 0.885 |
| **Physical Capital Share (M=14)** | 1965 | -0.816 (0.417) | -1.293 (0.217) | -1.121 | -0.248 | 0.851 |
| | 1970 | -0.825 (0.396) | -1.275 (0.208) | -1.103 | -0.214 | 0.858 |
| | 1975 | -0.836 (0.388)* | -1.262 (0.203) | -1.090 | -0.189 | 0.858 |
| | 1980 | -0.760 (0.484) | -1.332 (0.245) | -1.160 | -0.318 | 0.828 |
| | 1985 | -0.732 (0.404)* | -1.358 (0.205) | -1.186 | -0.362 | 0.870 |
| | 1990 | -0.670 (0.398) | -1.418 (0.206) | -1.246 | -0.459 | 0.873 |
| | 1995 | -0.632 (0.330) | -1.457 (0.174) | -1.285 | -0.518 | 0.908 |
| | 2000 | -0.658 (0.382) | -1.431 (0.186) | -1.259 | -0.479 | 0.904 |
| **Human Capital Share (M=14)** | 1960 | -0.147 (0.448) | -2.103 (0.287) | -1.931 | -1.171 | 0.791 |
| | 1965 | -0.343 (0.341) | -1.890 (0.184) | -1.718 | -1.005 | 0.880 |
| | 1970 | -0.529 (0.280)* | -1.639 (0.176) | -1.467 | -0.754 | 0.865 |
| | 1975 | -0.642 (0.236)** | -1.518 (0.126) | -1.346 | -0.603 | 0.928 |
| | 1980 | -0.683 (0.239)** | -1.433 (0.122) | -1.261 | -0.482 | 0.933 |
| | 1985 | -0.747 (0.185)** | -1.409 (0.092) | -1.237 | -0.445 | 0.945 |
| | 1990 | -0.895 (0.191)** | -1.241 (0.112) | -1.069 | -0.147 | 0.912 |
| | 1995 | -0.897 (0.201)** | -1.225 (0.115) | -1.053 | -0.114 | 0.912 |
| | 2000 | -0.905 (0.196)** | -1.215 (0.110) | -1.043 | -0.094 | 0.919 |

[a] OLS standard errors in parentheses. Significantly different from zero at ** = $p < 0.05$ or * = $p < 0.10$.
[b] OLS standard errors in parentheses. All slope coefficients significantly different from zero at the 1% level.
[c] Computed as the OLS slope estimate plus 0.172 (the bias).
[d] Computed as the OLS slope estimate plus 0.172 (the bias) minus –1 divided by the asymptotic approximation of the true standard error (given as minus the OLS slope estimate times $0.3779 = (2/14)^{0.5}$). All slope coefficients are not significantly different from –1 at the 5% level.

**Table 4. Results Testing the Equal-Share Relationship**

| Integrated Economic Area | Year | *p*-values Testing Across-equation Homogeneity | |
|---|---|---|---|
| | | Intercepts | Slopes |
| U.S. States | 1990 | 0.9680 | 0.9014 |
| | 2000 | 0.8241 | 0.5964 |
| E.U. Countries | 1965 | 0.6063 | 0.0445 [a] |
| | 1970 | 0.8011 | 0.2797 |
| | 1975 | 0.8619 | 0.3655 |
| | 1980 | 0.9689 | 0.8461 |
| | 1985 | 0.9969 | 0.9305 |
| | 1990 | 0.8111 | 0.6034 |
| | 1995 | 0.7124 | 0.3697 |
| | 2000 | 0.7291 | 0.4072 |

[a] Cross-equation homogeneity rejected at 5% level.

**Table 5. Correlation between Logarithm of Actual and Theoretical Output and Factor Shares for U.S. States and E.U. Countries, 1990 and 2000**

| **Integrated Economic Area** | **Year** | **Correlation between Logarithms of Actual and Theoretical Shares** | | |
|---|---|---|---|---|
| | | Output | Physical Capital | Human Capital |
| **U.S. States** | 1990 | 0.9429 | 0.9456 | 0.9258 |
| | 2000 | 0.9332 | 0.9393 | 0.9176 |
| **E.U. Countries** | 1990 | 0.9392 | 0.9397 | 0.9397 |
| | 2000 | 0.9453 | 0.9548 | 0.9619 |

**Table 6. Two-Sample Kolmogorov-Smirnov Tests between Actual and Theoretical Output and Factor Shares**

| **Integrated Economic Area** | **Year** | **Kolmogorov-Smirnov *D*-Statistic between Actual Shares and Theoretical Shares** [a] | | |
|---|---|---|---|---|
| | | Output | Physical Capital | Human Capital |
| **U.S. States** | 1990 | 0.2157** | 0.2157** | 0.2353** |
| | 2000 | 0.2353** | 0.2745* | 0.2157** |
| **E.U. Countries** | 1990 | 0.3571** | 0.3571** | 0.3571** |
| | 2000 | 0.3571** | 0.3571** | 0.2143** |

[a] Unable to reject that both the actual and theoretical shares come from a common distribution if the *D*-statistic is lower than a critical value; for U.S. states this critical value is 0.3228 at 1% level and 0.2693 at 5% level; for E.U. countries, this critical value is 0.6161 at 1% level and 0.5140 at 5% level.
** Cannot reject that shares come from a common distribution at 5% level
* Cannot reject that shares come from a common distribution at 1% level