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Product Market Competition and Trade Union Structure

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Product market competition and trade union structure*

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Abstract

Trade unions tend to reduce the dispersion of wages among their members. Skilled workers may therefore have an incentive to separate from an encompassing union and organize into a separate craft union. This paper examines a theoretical model to gain insight into the structure of trade unions at a firm. We show that imperfect competition in the product market may drive skilled and unskilled workers together, even though unskilled workers use their political power in the trade union to extract rents from the skilled workers. Additionally, we examine the influence of several features of production technology on trade union structure.

Keywords: product market competition, trade union structure, wage dispersion.

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1 Introduction

Trade unions raise wages (see, e.g., Booth, 1995, for an overview of the empirical literature). The magnitude of their influence varies across firms and industries. Several empirical studies show that the characteristics of the product market that a firm operates in are crucially important for a union's ability to raise wages; see in particular Mishell (1986) and Stewart (1990). The degree of product market competition determines the firm's 'ability to pay'. A firm operating in a perfectly competitive product market can not afford to pay wages above the wages paid in competing firms, as it would be priced out of the market. Hence, an imperfectly competitive product market is a necessary condition for a union to exercise influence over the wage level in a firm.

A union must possess some bargaining power to capture a part of the profits a firm makes in an imperfectly competitive product market. The bargaining power of a union depends on how effectively a union can hamper production through a strike. A union which can paralyze production completely has a strong bargaining power. Hence, a trade union which encompasses all firm's workers may be more successful in raising wages than a small union surrounded by competing unions, as an encompassing union has a larger impact on production. In practice, however, we observe not only encompassing unions, but also small unions organized by craft, skill, or plant (see, e.g., Machin et al., 1993). An important reason may be that encompassing unions tend to decrease the dispersion of wages among their members.¹ Some types of workers may therefore be better off by organizing into a separate union than by joining an encompassing union.

This paper examines a theoretical model to gain insight into the structure of trade unions at a firm. In particular, we study how product market competition and several features of production technology affect the union formation decision. We develop a simple model in which the work force is divided into two groups of workers, skilled and unskilled. Workers can organize into two separate craft unions or they can join into one encompassing union. Workers can also decide not to form a union and to bargain individually with the employer over the wage. In an encompassing union, decisions are made by majority voting. Low-skilled workers

¹See, e.g., Freeman (1980, 1982), Gosling and Machin (1995), and Card (1996).

are assumed to outnumber the high-skilled workers. The unskilled use their political power in the trade union to extract rents from the skilled workers. This provides the skilled workers with an incentive to start their own craft union.

Our analysis builds on Horn and Wolinsky (1988). They examine under what conditions two groups of identical workers decide to form one encompassing union rather than two separate unions. They show that the union formation decision depends on the degree of substitutability between workers. The intuition is straightforward. An encompassing union covers the whole work force and therefore bargains with the firm over the division of the entire payoff from production. A separate union bargains with the firm over the division of the marginal contribution of its members. The marginal contribution of a separate union is higher, the stronger the complementarity between the groups of workers in production. Increasing the number of employees increases production more than proportionally if workers are sufficiently complementary. Therefore, it may be in the interest of workers to form two separate unions, each bargaining over its marginal contribution, rather than to form one encompassing union. Conversely, workers join forces into an encompassing union when they are close substitutes in production.

We examine how the conditions in the product market affect the union formation decision in a setting where workers differ in productivity. We show that the less competitive the product market, the more likely will workers unionize. More interestingly, we show that the conditions in the product market also affect the decision to organize into an encompassing union or into two craft unions. The reason is that imperfections in the product market give firms room to raise the price of their product in case of a strike by part of the work force. The price increase partially compensates the decrease in production when the members of a craft union strike. This does not hold for an encompassing union as a strike by all workers paralyzes production. Therefore, imperfect competition in the product market reduces the attractiveness of organizing into craft unions and strengthens the incentive to organize into an encompassing union. The productivity gap between skilled and unskilled gives skilled workers an incentive to separate from an encompassing union. The reason is that the unskilled workers use their political power in an encompassing union to raise their own wage at the expense of the skilled workers. Even though skilled workers are exploited in an encompassing union, a strongly imperfect prod-

uct market may drive the skilled and the unskilled workers together.² Lastly, a technology with decreasing returns to scale also encourages an encompassing union, as it reduces the marginal contribution of each group of workers relative to total production.

A few other papers examine the formation of trade unions. Westermarck (1999) extends the analysis of Horn and Wolinsky (1988) by introducing an arbitrary number of employees who differ in productivity. Westermarck's analysis builds on Jun (1989) who introduces bargaining between heterogenous workers in an encompassing union on the division of rents. In our paper, the division of rents between skilled and unskilled workers is determined by majority voting.³ Moreover, our paper pays explicit attention to the impact of product market competition on the union formation decision.⁴

The set up of the paper is as follows. The model is described in section 2. Next, in section 3 we derive the equilibrium wages when workers bargain individually with the firm, when workers organize into two separate unions, and when workers organize into an encompassing union. In section 4, we examine the union formation decision. Section 5 concludes.

2 The model

A firm produces output, denoted Q , using skilled and unskilled labor, denoted N_s and N_u , respectively. Production technology is described by a CES production function:

$$Q = [\delta N_s^{-\rho} + (1 - \delta) N_u^{-\rho}]^{-\frac{\gamma}{\rho}},$$

where $0 < \delta < 1$ is the distribution parameter, $-1 < \rho \neq 0$ is the substitution parameter, and $\gamma > 0$ accounts for returns to scale in production. We assume that

²Alternatively, Agell and Lommerud (1992) argue that insurance motives may give high-productivity workers an incentive to stay within an encompassing union which pursues an egalitarian wage policy. We abstract from uncertainty about worker's future skill.

³Trade unions can be viewed as political organizations which are democratically organized, see Freeman and Medoff (1984) and Booth (1995).

⁴Another paper worth mentioning is Davidson (1988). While we examine the incentives of heterogeneous workers of a particular firm to join in an encompassing union, he examines the incentives of homogeneous workers of different firms to join in an industrywide union.

$\delta > \frac{1}{2}$, skilled workers are more productive than unskilled workers. Production technology exhibits decreasing returns to scale if $\gamma < 1$, constant returns to scale if $\gamma = 1$, and increasing returns to scale if $\gamma > 1$. The parameter ρ determines the ease with which skilled labor can be substituted for unskilled labor.⁵ If $\rho \rightarrow -1$, the two inputs become perfect substitutes. If $\rho \rightarrow +\infty$, the production factors become perfect complements.

The firm operates in an imperfectly competitive product market. The demand for the firm's product is given by $Q^d = P^{-\eta}$, where P is the price of output and $\eta > 1$ is the absolute value of the price elasticity of demand. We use the price elasticity as a measure of product market competition. The larger is η , the more competitive is the product market. The firm is a price taker if $\eta \rightarrow +\infty$. The firm's profit V is equal to:

$$V = [\delta N_s^{-\rho} + (1 - \delta) N_u^{-\rho}]^{-\frac{\gamma}{\rho}(1 - \frac{1}{\eta})} - W_s N_s - W_u N_u,$$

where W_s denotes the wage for skilled workers and W_u the wage for unskilled workers. For simplicity, we abstract from fixed costs. The trade union (or trade unions) is a monopoly supplier of labor (of a particular type). Furthermore, we assume that the levels of employment N_s and N_u are set before the wage formation process.⁶

The firm's employees are organized either into one encompassing trade union or into two craft unions, each representing a skill type. Employees may also decide not to organize into a union. A worker prefers to bargain individually with the firm if the resulting wage is higher than or equal to the wage a worker could obtain by cooperating with other workers in a union. An encompassing union bargains with the firm over the wages of both the skilled and the unskilled workers. Craft unions bargain independently of each other with the firm over the level of the wage for their members. An encompassing union arises only if both the skilled and the unskilled workers are better off in an encompassing union than in a craft union.

⁵The elasticity of substitution $\sigma = \frac{1}{1+\rho}$.

⁶We have examined the case in which the firm can decide (after wages have been set) to employ only parts of the groups of skilled and unskilled workers. Unfortunately, however, we were not able to obtain analytical solutions without seriously compromising on the structure of the model. In a model of union formation with three homogenous workers, Westermarck (1999) studies the consequences of endogenous employment *before wages are set*. Credibility of the firm's commitment not to hire a part of the work force after wages are set may be problematic in practice, however.

Clearly, there is a conflict of interest among members in an encompassing union on the wage differential. This conflict can be resolved in several ways (e.g., through voting, bargaining). We assume that union members take decisions by majority voting and that the unskilled workers constitute a majority ($N_u > N_s$).⁷

For convenience, worker's utility is assumed to be linear in income.⁸ The worker's reservation utility is assumed to be zero.⁹

3 Wage formation

This section derives the equilibrium wages under three different bargaining structures: individual bargaining, two craft unions, and one encompassing union. Section 4 shows under what conditions a particular bargaining structure emerges.

3.1 Individual bargaining

In the absence of unions, each worker bargains individually with the firm over his wage. We focus on an interior solution, that is, we assume that in equilibrium all of the workers reach an agreement with the firm. At any moment in time, both the firm and each of the workers can ask for a renegotiation. In equilibrium, wages are such that neither the firm nor any of the workers has an incentive to ask for a renegotiation. The firm's fall-back position in a bargain with a worker is the profit it would obtain if the worker quits the firm. The worker's fall-back position (his reservation utility) is assumed to be zero. The outcome of the bargain between the firm and a skilled worker is given by the generalized Nash bargaining solution:

$$\max_{\text{to } W_s} \phi = \left\{ \left[\delta N_s^{-\rho} + (1 - \delta) N_u^{-\rho} \right]^{-\frac{\gamma}{\rho} \left(1 - \frac{1}{\eta} \right)} - \left[\delta (N_s - 1)^{-\rho} + (1 - \delta) N_u^{-\rho} \right]^{-\frac{\gamma}{\rho} \left(1 - \frac{1}{\eta} \right)} - W_s \right\} W_s.$$

⁷Empirical studies of unions support the assumptions that unions take decisions through voting and that the unskilled constitute a majority, see Freeman (1980) and Freeman and Medoff (1984).

⁸Concavity of the utility function reduces the share of the rents that workers obtain, but does not affect the workers' incentives to form craft unions or an encompassing union.

⁹Non-zero reservation utility seriously complicates the analysis (cf. Horn and Wolinsky, 1988).

The first-order condition reads after some rewriting:

$$W_s = \frac{1}{2} \left\{ \left[\delta N_s^{-\rho} + (1 - \delta) N_u^{-\rho} \right]^{-\frac{\gamma}{\rho} \left(1 - \frac{1}{\eta}\right)} - \left[\delta (N_s - 1)^{-\rho} + (1 - \delta) N_u^{-\rho} \right]^{-\frac{\gamma}{\rho} \left(1 - \frac{1}{\eta}\right)} \right\}. \quad (1)$$

Equation (1) shows that the wage of a skilled worker is equal to a half of his marginal contribution to the firm's revenues. Similarly, we can derive:

$$W_u = \frac{1}{2} \left\{ \left[\delta N_s^{-\rho} + (1 - \delta) N_u^{-\rho} \right]^{-\frac{\gamma}{\rho} \left(1 - \frac{1}{\eta}\right)} - \left[\delta N_s^{-\rho} + (1 - \delta) (N_u - 1)^{-\rho} \right]^{-\frac{\gamma}{\rho} \left(1 - \frac{1}{\eta}\right)} \right\}. \quad (2)$$

Clearly, the wage of unskilled workers is always lower than the wage of skilled workers, since the skilled are more productive ($\delta > \frac{1}{2}$) and the unskilled outnumber the skilled ($N_u > N_s$).

3.2 Two craft unions

Suppose the unskilled workers are organized into union U and the skilled workers are organized into union S . Each craft union bargains separately with the firm over the wage of its own members. As in section 3.1, we assume that in equilibrium both craft unions reach an agreement with the firm. The firm's fall-back position in a bargain with a craft union is the profit it would obtain if the members of the craft union would be on strike. This follows the approach in Horn and Wolinsky (1988).

Consider the bargain between the firm and union S over the wage of the skilled workers. The firm's fall-back position is the profit it would obtain if only the unskilled workers are available for production. The fall-back position of the skilled workers is zero. Maximizing the Nash bargaining function towards W_s results in:

$$W_s = \frac{1}{2} \left(\frac{1}{N_s} \right) \left\{ \left[\delta N_s^{-\rho} + (1 - \delta) N_u^{-\rho} \right]^{-\frac{\gamma}{\rho} \left(1 - \frac{1}{\eta}\right)} - \left[(1 - \delta) N_u^{-\rho} \right]^{-\frac{\gamma}{\rho} \left(1 - \frac{1}{\eta}\right)} \right\}. \quad (3)$$

Union S captures half of the marginal contribution of the skilled workers and distributes the acquired rents equally over its members.

Along the same line, we obtain:

$$W_u = \frac{1}{2} \left(\frac{1}{N_u} \right) \left\{ \left[\delta N_s^{-\rho} + (1 - \delta) N_u^{-\rho} \right]^{-\frac{\gamma}{\rho} \left(1 - \frac{1}{\eta}\right)} - \left[\delta N_s^{-\rho} \right]^{-\frac{\gamma}{\rho} \left(1 - \frac{1}{\eta}\right)} \right\}. \quad (4)$$

Union U captures half of the marginal contribution of the unskilled workers and distributes these rents over its members.

3.3 One encompassing union

Finally, suppose all employees are organized into one encompassing union. Wages W_s and W_u are determined in a bargain between the firm and a union representative. The union representative is a worker elected by majority rule. Since we assume that unskilled workers outnumber the skilled workers, the representative of an encompassing union maximizes the utility of the unskilled workers. We impose the condition that the wage of the skilled workers at least equals the wage of the unskilled workers. If $W_u > W_s$, then skilled workers would claim to be unskilled so as to earn a higher wage. As a consequence, forming an encompassing union implies full wage equality within the firm ($W_s = W_u$). Both the firm's fall-back position as well as the fall-back position of the encompassing union equals zero. Maximizing the Nash bargaining function results in:

$$W_s = W_u = \frac{1}{2} \left(\frac{1}{N_u + N_s} \right) [\delta N_s^{-\rho} + (1 - \delta) N_u^{-\rho}]^{-\frac{\gamma}{\rho} (1 - \frac{1}{\eta})}. \quad (5)$$

The encompassing union captures half of the total amount of firm's revenues. These rents are equally distributed among all employees.

4 Union formation decision

This section analyzes the union formation decision using the results of section 3. We first examine under what conditions it is profitable for employees to organize themselves.

Lemma 1 *Employees organize into a union if and only if the marginal revenue product declines with employment.*

Proof. See the appendix. ■

Lemma 1 states that employees organize into a union (encompassing or craft) only if the revenues of the firm show decreasing returns to the level of employment. The intuition is straightforward. Decreasing returns imply that at the margin a

single worker adds less to a firm's revenues than the average revenue per worker. The firm takes advantage of the low marginal contribution of a single worker if the firm bargains with each worker individually. Clearly, in case of decreasing returns, there are benefits from organizing into a union for the employees. A union bargains over the contribution to firm's revenues of all the union members and distributes the acquired rents equally among the members. From Lemma 1, it is clear that less intense competition in the product market fosters unionization. The reason is that inframarginal profits increase relative to marginal profits when the product market becomes less competitive. Employees prefer to bargain individually with the firm if an increase in employment leads to a more than proportional increase in the revenues of the firm. This is because the marginal contribution of a single worker then exceeds the average contribution of the members of a union.

Suppose the conditions in Lemma 1 are satisfied and, consequently, employees prefer unions. This raises the question whether employees organize into two separate craft unions or into one encompassing union. An encompassing union arises only if it is in the interest of both the skilled and the unskilled workers. Comparing equation (3) with equation (5), it follows that skilled workers prefer joining an encompassing union if:

$$[(1 - \delta)N_u^{-\rho}]^{-\frac{\gamma}{\rho}(1-\frac{1}{\eta})} - \left(\frac{N_u}{N_u + N_s}\right) [\delta N_s^{-\rho} + (1 - \delta)N_u^{-\rho}]^{-\frac{\gamma}{\rho}(1-\frac{1}{\eta})} > 0. \quad (6)$$

Comparing (4) to (5) shows that unskilled workers prefer joining an encompassing union if:

$$[\delta N_s^{-\rho}]^{-\frac{\gamma}{\rho}(1-\frac{1}{\eta})} - \left(\frac{N_s}{N_u + N_s}\right) [\delta N_s^{-\rho} + (1 - \delta)N_u^{-\rho}]^{-\frac{\gamma}{\rho}(1-\frac{1}{\eta})} > 0. \quad (7)$$

Since $\delta > \frac{1}{2}$ and $N_u > N_s$, condition (6) is always more restrictive than condition (7). Hence, unskilled workers are willing to form an encompassing union for a broader range of parameter values than skilled workers. The reason is that the unskilled workers use their political power to extract rents from the skilled workers so that an encompassing union reduces wage dispersion. Proposition 1 describes under what conditions an encompassing union is more likely to arise.

Proposition 1 *If the condition in Lemma 1 is satisfied, an encompassing union is*

more likely to arise if:

- i) the product market is less competitive ($\eta \rightarrow 1$),*
- ii) the workers are more easily substitutable ($\rho \rightarrow -1$),*
- iii) the technology exhibits stronger decreasing returns to scale ($\gamma \rightarrow 0$), and if*
- iv) the productivity differences are smaller ($\delta \rightarrow \frac{1}{2}$).*

Proof. See the appendix. ■

Proposition 1 describes four factors which affect union formation. We explain the role of these factors in detail. The fall-back position of the firm when bargaining with an encompassing union is zero. The fall-back position of the firm when bargaining with a craft union is the revenues if the members of the craft union would leave the firm. The firm then continues production at a lower level since only the members of the other craft union work. In a perfectly competitive product market, this decrease in production does not affect the price. In an imperfectly competitive product market, the firm partially compensates for the loss in production by increasing the price. This improves the fall-back position of the firm in the bargain with a craft union. Craft unions therefore obtain a smaller part of the rents. Product market competition does not affect the fall-back position of the firm when workers are organized into an encompassing union. The reason is that an encompassing union can stop all production. Hence, the incentive to form an encompassing union is stronger when the competition in the product market is less intense. A strong complementarity in production between the two groups of workers encourages the forming of craft unions. The firm's revenues in case only one group of workers is available for production are low when the two groups are complementary in production. Consequently, the fall-back position of the firm bargaining with a craft union is weak and thus provides the workers with an incentive to form craft unions.¹⁰ Stronger decreasing returns to scale give workers an incentive to organize into an encompassing union as it reduces the marginal contribution of each group relative to the total value of production.

¹⁰Note that for any $\rho > 0$, it holds that if either N_s or N_u approaches zero, the production Q approaches zero. Hence, the fall-back position of the firm bargaining with a craft union is zero, as in the bargain with an encompassing union. Then, two craft unions arise which capture half of total firm's revenues each. For any $\rho \in (-1, 0)$ either one union or two unions may arise.

Lastly, the difference in productivity between skilled and unskilled workers affect the decision to unite. Skilled workers have an incentive to start their own craft union because unskilled workers reduce wage dispersion in an encompassing union. This incentive is stronger the larger the productivity gap between skilled and unskilled workers. Although skilled workers are exploited in an encompassing union, they may have an incentive to cooperate with the unskilled. The decision whether to join an encompassing union or to form two craft unions ultimately depends on the combination of the factors mentioned in Proposition 1.

5 Concluding remarks

Trade unions compress the distribution of wages among their members. High skilled workers may therefore have an incentive to separate from an encompassing union and organize into a craft union. This paper has examined conditions under which an encompassing union arises rather than two separate craft unions. We have shown that the bargaining position of a craft union depends on the degree of competition in the product market and on several features of the production technology. Imperfect competition in the product market and a production technology characterized by decreasing returns to scale give incentives to join forces, as they weaken the consequences for the firm of a strike by craft union members. In contrast, a strike by a part of the work force greatly reduces a firm's revenues if workers are complementary in production, thereby improving the bargaining position of a craft union. The incentive to organize into a craft union is also stronger, the larger is the productivity differential between skilled and unskilled workers.

Our model certainly overstates the union's ability to reduce wage dispersion among their members. First, different ways of deciding upon the wage policy of the union, through for example a bargain or requiring unanimity, may result in larger wage inequality in an encompassing union than under majority voting. Second, the incentive of an encompassing union to compress wages may be lower when the firm can adjust employment after wages are set. The reason is that an increase in the wage of the unskilled, at the expense of the skilled, induces the firm to employ less unskilled workers. Including these features certainly complicates the analysis but need not affect our main results qualitatively. We conjecture that skilled workers

will have a stronger incentive to join in an encompassing union as unskilled workers are less able or less willing to extract rents. The comparative static results regarding product market competition and production technology may, nevertheless, still hold.

Our analysis has clear testable implications. It would be particularly interesting to test whether the intensity of competition in the product market has an effect on the number of trade unions which bargain with the firm or industry. Our theoretical analysis suggests that imperfect competition in the product market not only affects the decision to form a union but also affects the incentive for workers with different skills to join forces.

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Appendix

Proof of Lemma 1

Proof. Define firm's revenues $[\delta N_s^{-\rho} + (1 - \delta)N_u^{-\rho}]^{-\frac{\gamma}{\rho}(1 - \frac{1}{\eta})}$ as a function of the levels of employment $f(N_s, N_u)$.

Consider a skilled worker. A skilled worker prefers joining a craft union to individual bargaining if:

$$\frac{1}{N_s} [f(N_s, N_u) - f(0, N_u)] > f(N_s, N_u) - f(N_s - 1, N_u),$$

which is satisfied if and only if $f(\cdot)$ is a concave function in the level of N_s , that is, if the marginal revenue product declines in employment. A skilled worker prefers joining an encompassing union to individual bargaining if:

$$\frac{1}{N_s + N_u} f(N_s, N_u) > f(N_s, N_u) - f(N_s - 1, N_u).$$

This condition only holds if $f(\cdot)$ is a concave function in the level of N_s . Concluding, concavity of the revenue function of the firm is a necessary and sufficient condition for skilled workers to organize themselves. Skilled workers may organize themselves into a craft union or they may decide to join the unskilled into an encompassing union.

Along the same line of reasoning, we can show that unskilled workers prefer organizing themselves if and only if $f(\cdot)$ is a concave function in the level of N_u . ■

Proof of Proposition 1

Proof. First, note that if $\rho > 0$, then firm's revenues approach zero if either N_s or N_u approaches zero. As a consequence, both condition (6) and (7) are violated and two craft unions arise.

Now, consider the case where $-1 < \rho < 0$. We can restrict attention to condition (6) as condition (7) is always less restrictive than (6). Rewriting condition (6) gives:

$$\left(\frac{N_u + N_s}{N_u} \right) - \left[\frac{\delta N_s^{-\rho} + (1 - \delta)N_u^{-\rho}}{(1 - \delta)N_u^{-\rho}} \right]^{-\frac{\gamma}{\rho}(1 - \frac{1}{\eta})} > 0.$$

Let us first examine the effect of γ and η . For convenience, define $\alpha = -\frac{\gamma}{\rho}(1 - \frac{1}{\eta})$, where $\alpha > 0$ since $\gamma > 0$, $-1 < \rho < 0$ and $\eta > 1$. Both γ and η affect condition (6)

only through α . Taking the derivative with respect to α gives:

$$\frac{d}{d\alpha} = - \left[\frac{\delta N_s^{-\rho} + (1 - \delta) N_u^{-\rho}}{(1 - \delta) N_u^{-\rho}} \right]^\alpha \ln \left[\frac{\delta N_s^{-\rho} + (1 - \delta) N_u^{-\rho}}{(1 - \delta) N_u^{-\rho}} \right] < 0.$$

An increase in α makes condition (6) more restrictive. Hence, an increase in γ or an increase in η makes condition (6) more restrictive and, consequently, an encompassing union less likely to arise.

Next, consider the impact of ρ . Taking the derivative with respect to ρ gives:

$$\begin{aligned} \frac{d}{d\rho} &= \frac{\gamma}{\rho} \left(1 - \frac{1}{\eta} \right) \frac{\delta}{1 - \delta} \left(\frac{N_u}{N_s} \right)^\rho \left[\frac{\delta N_s^{-\rho} + (1 - \delta) N_u^{-\rho}}{(1 - \delta) N_u^{-\rho}} \right]^{-\frac{\gamma}{\rho} (1 - \frac{1}{\eta}) - 1} \ln \left(\frac{N_u}{N_s} \right) + \\ &\quad - \frac{\gamma}{\rho^2} \left(1 - \frac{1}{\eta} \right) \left[\frac{\delta N_s^{-\rho} + (1 - \delta) N_u^{-\rho}}{(1 - \delta) N_u^{-\rho}} \right]^{-\frac{\gamma}{\rho} (1 - \frac{1}{\eta})} \ln \left[\frac{\delta N_s^{-\rho} + (1 - \delta) N_u^{-\rho}}{(1 - \delta) N_u^{-\rho}} \right] < 0, \end{aligned}$$

where we use $N_u > N_s$ and hence $\ln \left(\frac{N_u}{N_s} \right) > 0$. An increase in ρ makes it less likely that workers join in an encompassing union.

Finally, consider the impact of δ on condition (6). Rewrite condition (6) gives:

$$\left(\frac{N_u + N_s}{N_u} \right)^{\frac{1}{\alpha}} - \left[\frac{\delta N_s^{-\rho} + (1 - \delta) N_u^{-\rho}}{(1 - \delta) N_u^{-\rho}} \right] > 0.$$

Taking the derivative with respect to δ gives:

$$\frac{d}{d\delta} = - \left(\frac{N_s}{N_u} \right)^{-\rho} \frac{1}{(\delta - 1)^2} < 0.$$

Clearly, the skilled have a stronger incentive to separate from an encompassing union if their productivity level increases. ■