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Explaining Hedge Fund Investment Styles by Loss Aversion

Arjen Siegmann

André Lucas

Faculty of Economics and Business Administration, Vrije Universiteit Amsterdam

Tinbergen Institute

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Tinbergen Institute Amsterdam

Keizersgracht 482
1017 EG Amsterdam
The Netherlands
Tel.: +31.(0)20.5513500
Fax: +31.(0)20.5513555

Tinbergen Institute Rotterdam

Burg. Oudlaan 50
3062 PA Rotterdam
The Netherlands
Tel.: +31.(0)10.4088900
Fax: +31.(0)10.4089031

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Explaining hedge fund investment styles by loss aversion:

A rational alternative

Arjen Siegmann* • André Lucas*

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Abstract

Recent research reveals that hedge fund returns exhibit a range of different, possibly non-linear pay-off patterns. It is difficult to qualify all these patterns simultaneously as being rational in a traditional framework for optimal financial decision making. In this paper we present a simple model based on loss aversion that can accommodate for all of these pay-off structures in one unifying framework. We provide evidence that loss-aversion is a likely assumption for management as well as investor preferences. Following the current empirical literature, we solve a static asset allocation problem that includes a nonlinear instrument. We show analytically that four different pay-off functions may be rationally optimal. The key parameter in determining which of these four to choose in a specific setting, is the financial planner's surplus. The notion of surplus connects hedge fund manager's incentive schemes with the idea of mental accounting as proposed in recent behavioral finance research.

Keywords: hedge funds, performance measurement, loss aversion, behavioral finance.

JEL classification: G11,G23.

*Both authors are affiliated with the Vrije Universiteit Amsterdam and the Tinbergen Institute.

Correspondence to: A.H. Siegmann, Faculty of Economics and Business Administration, Finance Department, Vrije Universiteit, De Boelelaan 1105, 1081 HV Amsterdam, the Netherlands.

Phone: +31-(0)20-444 6034. Fax: +31-(0)20-444 6020, E-mail: asiegmann@feweb.vu.nl.

1 Introduction

There is a growing interest in hedge fund performance among investors, academics, and regulators alike. Investors and academics are intrigued by the unconventional performance characteristics of these funds. Regulators on the other hand are concerned with the market impact of hedge funds' reported speculative activities during major market events.

Most hedge funds not being formally regulated, are not limited in the type of assets they can hold. Moreover, they face less restrictions on short sales than standard mutual funds and can be highly leveraged and concentrated in specific sectors, countries and/or asset categories. The fund's management compensation is based on the fund's financial performance, something that is less common for conventional mutual funds. According to Edwards (1999), mutual fund managers are generally compensated a flat-fee structure of assets under management. A typical hedge fund, by contrast, charges a 1% fixed fee and 20% of profits.

Given the dramatic increase in the number of hedge funds over the past decade and their large degree of freedom in investment behavior, there has been much recent attention devoted to measuring the performance and return characteristics of hedge funds, see for example Amin and Kat (2001), Fung and Hsieh (1997), Fung and Hsieh (2001), Agarwal and Naik (2000b), and Mitchell and Pulvino (2001). Popular opinion has it that hedge funds, through their large freedom and degree of specialization, deliver exceptionally high returns and are market-neutral. That is, it is believed that hedge fund returns are not correlated with (stock)market returns. If this is true, this is an attractive feature for many institutional investors. The papers cited above concentrate on hedge funds' investment strategies, explaining hedge fund returns empirically. They focus on market efficiency and value-added created by hedge funds and their managers. They find that hedge funds do not outperform stock market returns in terms of average return and volatility. However, the standard method of measuring performance does not work for hedge funds.

A simple approach to performance measurement that works well for standard mutual funds is the asset class factor model of Sharpe (1992). Sharpe models the return R_i on a fund i as

$$R_i = w_{i1}F_1 + w_{i2}F_2 + \dots + w_{in}F_n + e_i, \quad (1)$$

where R_i is the vector of returns on fund i , F_j is the value of systematic factor j , and

e_i presents the non-systematic factors in the return for fund i . The w_{ij} s represent the sensitivities of the returns R_i to factor F_j . Sharpe selects 12 major asset classes for the factors F_1, \dots, F_{12} and finds a correlation coefficient of about 90% for a sample of US mutual funds. This implies that the return of mutual funds can be well approximated by a linear combination of returns on standard asset classes. This systematic part of the return is referred to as ‘style’ by Sharpe. The other part, the return variation that cannot be explained, is called ‘selection’. The excess return e_i should be attributed to the skills of management in selecting the individual securities, hence the name ‘selection’. The basic result of Sharpe that standard asset classes can explain most of the variation in mutual funds’ returns have been confirmed and extended by various other authors, for example Brown and Goetzmann (1997) and De Roon et al. (2000).

In contrast to the style regressions of type (1) for mutual funds, Fung and Hsieh (1997) find that for hedge funds the straightforward approach of Sharpe does not provide a good fit. These results are confirmed by Agarwal and Naik (2000a), who find low correlations with different indices. Also, Brealey and Kaplanis (2001) find evidence for changing factor loadings over time. Fung and Hsieh argue that the lack of fit of (1) for hedge funds is due to the extensive use of dynamic trading strategies. Mutual funds generally follow a relatively stable investment strategy, resulting in w_{ij} ’s between zero and one with modest time variation. By contrast, according to Fung and Hsieh hedge funds have weights w_{ij} between -10 and 10. In addition, the managers’ opportunism may cause the w_{ij} ’s to change quickly over time and across market conditions. This helps to explain why traditional Sharpe style regressions (1) fail dramatically for hedge funds.

The inappropriateness of Sharpe’s model for hedge funds, leads Fung and Hsieh (1997) to consider dynamic strategies. As there is an infinite number of possible dynamic trading strategies, factor analysis is used to determine the dominant styles in hedge funds. They find that adding the style factors to Sharpe’s model explains a significantly larger part of the return variation. Moreover, they identify distinctly nonlinear relations between their new factors and traditional asset class returns as used in a typical Sharpe style regression. Figure 1 presents four of the most prevalent examples of nonlinear relations as found by Fung and Hsieh.

The hedge fund industry already has its qualitative descriptors for certain types of hedge funds. These can be used to qualify the style factors. Comparable sets of qualifiers

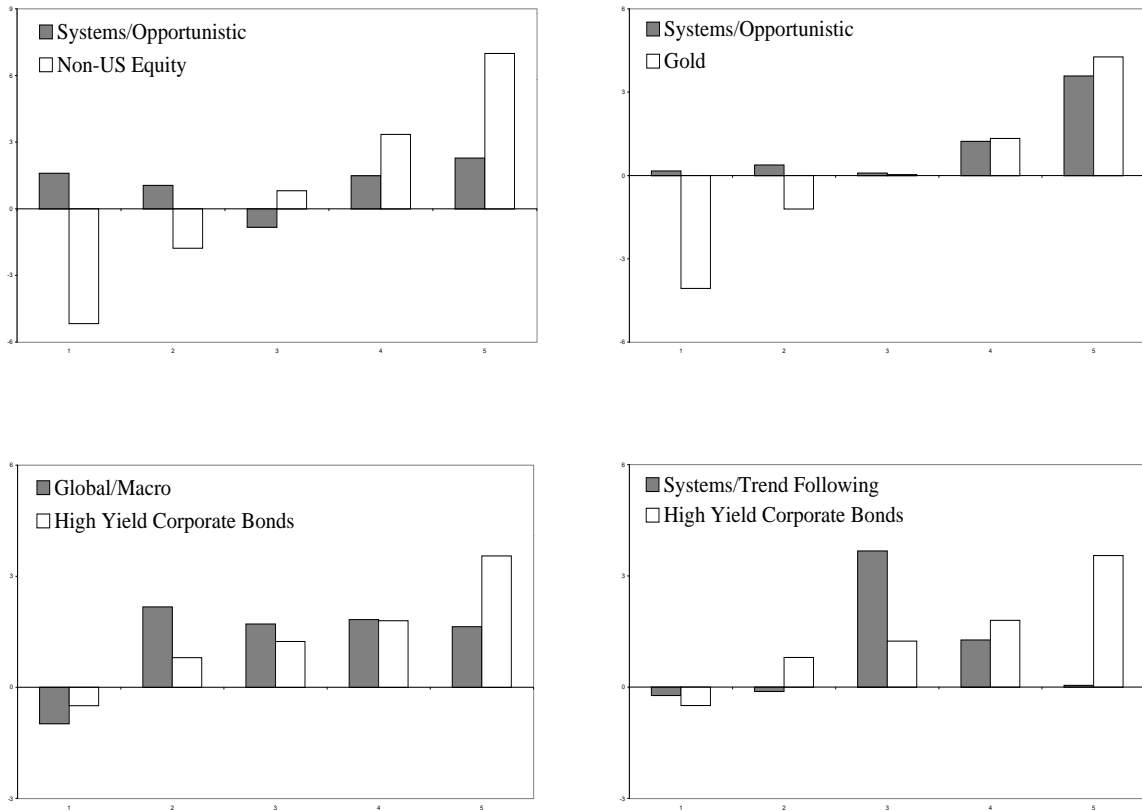


Figure 1: Empirical nonlinear relations between systematic hedge fund return factors and standard asset class returns.

The figures are based on data in Fung and Hsieh (1997), Table 1. We use their systematic risk factors for hedge fund returns corresponding to a specific qualitative descriptor. Given a factor return F_t and a traditional asset class return R_t , the pairs (F_t, R_t) are sorted on R_t and grouped into quintiles. For each quintile, the average return on F_t and R_t is plotted in the graphs.

are used throughout the hedge fund literature, see for example Osterberg and Thomson (1999), Edwards and Caglayan (2001) and Brown and Goetzmann (1997). In Figure 1 the qualifier for each style factor is from Fung and Hsieh and comes from the fund that has the highest correlation with that style factor. “Systems/Trend Following” refers to traders who use technical trading rules and are mostly trend followers. “Systems/Opportunistic” refers to technically driven traders who also take occasional bets on market events relying

on rule-based models. “Global/Macro” refers to managers who primarily trade in the most liquid markets in the world, such as currencies and government bonds, typically taking bets on macroeconomic events such as changes in interest rate policies and currency devaluations. These strategies rely mostly on their assessments of economic fundamentals.

Fung and Hsieh (1997) argue that Figure 1 shows option-like pay-off patterns in hedge fund returns. For example, the upper-left panel may be identified as a long straddle, whereas the upper-right and lower-left panels roughly correspond to long call and short put positions, respectively. Fung and Hsieh (2001) then proceed by including returns on buy and hold strategies of lookback-straddles. A similar approach is followed by Agarwal and Naik (2000b), who further show that the descriptive model also has significant out-of-sample forecasting power. Both these papers increase our understanding of the kind of strategies that hedge funds are likely to follow. There is an important question, however, that current research has not yet addressed. If hedge fund returns are so highly nonlinear and strategies are very different from standard buy-and-hold or constant-fraction portfolio strategies as derived in for example Merton (1990), what preferences drive the investment managers of these funds?

Some may argue that the observed patterns of hedge fund returns are merely a statistical artifact and that neither investors in hedge funds, nor hedge fund managers themselves, really know what type of pay-off pattern they generate. These arguments may be supported by the renowned secrecy surrounding hedge fund strategies and investment policies. In this paper we refrain from yielding directly to the argument of irrational investors. Instead, we offer a framework of rational, loss averse investors that optimally choose payoff patterns that are remarkably similar to those presented in Figure 1.

Loss aversion originates from the area of behavioral finance. As a perspective for explaining observed financial markets’ or institutions’ behavior it is rapidly finding its way into the finance literature. Originating with the work of Kahneman and Tversky (1979), we have seen its application to studying the equity premium puzzle, see Benartzi and Thaler (1995); rationalizing the momentum effect, see Shefrin and Statman (1985); explaining the behavior of asset prices in equilibrium, in particular boom and bust patterns, see Barberis et al. (2001) and Berkelaar and Kouwenberg (2000a); and explaining individual stock returns, see Barberis and Huang (2001). Building on the same framework and model specification for loss aversion as these earlier papers, we provide a rational

explanation for observed hedge fund payoff patterns.

Besides evidence from behavioral finance there is another *ex ante* reason to suspect that loss-aversion is an important phenomenon for hedge funds. This is the existence and structure of incentive fees. According to Edwards (1999), hedge funds pay managers large incentive fees as a fraction of the achieved return. Brown et al. (1999) mention common fees of 1% of funds under management and 20% of the profits, the same numbers as found by Liang (1999) in a large sample of hedge funds. Together with high incentive fees, however, investors usually require hedge fund managers to put a substantial amount of their own wealth in the fund. This requirement is obviously rooted in the preference of investors, who do not wish management to adopt a “recklessly risky” strategy. In conjunction with the use of high-water marks, managers can suffer a substantial personal loss if returns end up below a certain threshold level. As noted in Carpenter (2000), the most typical benchmark for hedge fund managers is a constant like the discounted value of current funds under management, or a benchmark representing a safe return, like a Treasury yield. We interpret the incentive scheme that is described above as (i) stimulating the manager to maximize expected fund wealth (or return) on the one hand, and (ii) making him loss-averse to avoid moral hazard problems, i.e. excessive risk taking. The contribution of this paper is that we combine these two effects into a model where the trade-off is between maximizing wealth and minimizing expected shortfall below a fixed target level.

The next section presents the model in its relation to other theoretical work in this area. From this model, we derive that the typical graphs of Figure 1 are exactly the four patterns that are optimal for a rational investor with mean-shortfall preferences. In Section 3 we highlight the model’s main implications. Section 4 concludes.

2 Model

In this section we introduce a simple model in which the hedge fund investment decision is represented through an investor with a loss-averse objective function. We consider an investor with initial wealth W_0 who optimizes expected utility defined over terminal wealth level W_1 . We choose wealth instead of return for ease of exposition, it is clear that there is an equivalent formulation in terms of possible return. A loss-averse ‘value

function' has been estimated by Kahneman and Tversky (1979), and has become a benchmark in itself for modelling behavioral preferences in a utility function framework, see for example Barberis et al. (2001) and Basak and Shapiro (2001). It is given by

$$V(W_0) = \mathbb{E} [((W_1 - W^B)^+)^{\gamma_1}] - \tilde{\lambda} \mathbb{E} [((W^B - W_1)^+)^{\gamma_2}], \quad (2)$$

where W^B is a benchmark level of wealth, $(x)^+$ denotes the maximum of 0 and x . Based on actual experiments with $W^B = W_0$, they obtain estimates $\tilde{\lambda} = 2.25$, $\gamma_1 = \gamma_2 = 0.88$. This objective is different from traditional utility functions as used in Merton (1990), in that utility is derived from deviations from the benchmark level W^B . Moreover, with γ_1 and γ_2 both smaller than 1, the value function implies risk-loving behavior in terms of losses, and risk-averse behavior in gains. In a finance context, this feature is confirmed in for example Shefrin and Statman (1985). Basak and Shapiro (2001) derive optimal investment policies in continuous-time under a constraint on expected shortfall (which can be compared to $\gamma_1 < 1$ and $\gamma_2 = 1$).

The objective function we propose to model investor preferences is a simplified version of the value function in (2). It is given by

$$\max \mathbb{E}[W_1] - \lambda \cdot \mathbb{E}[(W^B - W_1)^+], \quad (3)$$

where λ is the loss-aversion parameter. Our objective function (3) is obtained by setting $\gamma_1 = \gamma_2 = 1$ in (2). The investor thus faces a trade-off between expected wealth on the one hand, and expected shortfall below the benchmark wealth level W^B on the other hand. The objective function (3) thus contains an asymmetric or downside risk measure. If $W^B = W_0$, the investor weighs losses differently from gains. Instead of imposing a constraint on the optimization problem, we have incorporated the constraint in the objective function. This allows us to describe additional empirical features in hedge fund returns to the methodology followed by Basak and Shapiro. Barberis et al. (2001) use the expected shortfall measure in (3) as a risk measure to shed light on the behavior of firm-level stock returns in an asset-pricing framework. In their set up, W^B represents the *historical benchmark wealth level*, which may represent an average of recent portfolio wealth or the wealth at the end of a year. Berkelaar and Kouwenberg (2000b) use the more general specification (2) to solve a similar problem in continuous time.

Note that (3) is also relevant empirically. Sharpe (1998) explains how (3) is used by Morningstar to construct its 'risk-adjusted rating' for mutual funds. As these ratings

in turn profoundly influence the flow of money to a mutual fund, see Guercio and Tkac (2001), (3) de facto reflects actual preferences of at least part of the investment industry. Benartzi and Thaler (1995) also use (3) as an approximation to (2) to explain the equity premium puzzle.

To model the investment opportunities available to a hedge fund, we assume that the investor can select 3 assets, namely a risk-free asset, a linear risky asset, and an option on the risky asset. We label the risky asset as stock in the rest of this paper and normalize its initial price to 1. It should be kept in mind, however, that our results are not limited to stock investments. Alternative interpretations of the risky asset comprise stock indices, bonds or interest rates, and currencies. The stock has an uncertain pay-off u with distribution function $G(u)$. We assume that $G(\cdot)$ is defined on $(0, \infty)$, is twice continuously differentiable and satisfies $\mathbb{E}[u - r_f] > 0$, i.e., there is a positive equity premium. The option is modeled as a European call option on the stock with strike price x . Its current price is denoted by c . To avoid making a particular choice for the option's pricing model, we set the planning period equal to the option's time to maturity. The option's pay-off, R_c , is now completely determined by the stock return as $(u - x)^+$. We assume there is a positive risk premium for the option as well, i.e., $\mathbb{E}[(u - x)^+] > r_f$. Concentrating on hedge funds, we do not introduce any constraints on the positions the investor can take in any of these assets. We obtain

$$W_1 = W_0 r_f + X_0 \cdot (u - r_f) + X_1 \cdot (R_c - c \cdot r_f), \quad (4)$$

where r_f is the pay-off on the risk-free asset, X_0 is the number of shares, and X_1 the number of call options. The investor now maximizes (3) over $\{X_0, X_1\}$.

For ease of exposition we focus on the one-period model. The optimization problem introduced above is static. This may seem inappropriate for hedge funds, which are known to follow highly dynamic investment strategies. As argued in the introduction, however, there is ample empirical evidence that static models with non-linear instruments like options can explain a large part of the variation in hedge fund returns, both in-sample and out-of-sample, see Fung and Hsieh (1997) and Agarwal and Naik (2000b). Also, Siegmann and Lucas (2000) show that the solution to the multi-period version of (3) gives decision rules at each period that have the same shape as the solution to the one-period model. However, another objection to the present set-up might be that we only include a single option. The main advantage of focusing on one option only, is that we are

able to highlight the main features of the present model without introducing unnecessary complications. Even in this simple set-up, the model can describe several different pay-off patterns observed empirically. Moreover, Glosten and Jagannathan (1994) find that including more than one option in their contingent claims analysis does not give a better fit to their dataset of mutual fund returns. Similar results have been established by Agarwal and Naik (2000b), who show that for most hedge fund returns adding one option related factor in the Sharpe style regressions suffices to capture most of the non-linearity.

The following theorem gives our main result.

Theorem 2.1 *The optimal investment strategy for a finite solution to problem (3) is given by one of the following:*

$$I: \quad X_0^* = 0, \text{ and } X_1^* = S_0/c, \quad (5)$$

$$II: \quad X_0^* = -S_0/p, \text{ and } X_1^* = -X_0^*, \quad (6)$$

$$III: \quad \begin{pmatrix} X_0 + X_1 \\ -X_0 \end{pmatrix} = \frac{1}{A} \cdot \begin{pmatrix} x - \bar{u}_1 \\ \bar{u}_2 - x \end{pmatrix} \cdot S_0 \cdot r_f, \quad (7)$$

where $\bar{u}_1 < x < \bar{u}_2$, and $A > 0$. S_0 represents time 0 surplus, defined as $W_0 - W^B/r_f$. Through put-call parity, p is the price of a put option with strike price x as $p = x/r_f + c - 1$.

Proof: See appendix.

Theorem 2.1 states that if (3) has a finite solution, then the optimal investment strategy takes one out of three possible forms. A finite solution is ensured by a sufficiently high loss aversion parameter λ in (3). Unbounded solutions are less interesting in this setting, as they are not observed in practice.

Strategy I corresponds to a long position in the risk-free asset and the call option. The amount invested in the call option is exactly equal to the time 0 surplus. Strategy II corresponds to a short put position: an amount equal to the net shortfall is earned by selling puts. Strategy III is a condensed representation of either a long or short straddle¹ position. As A , $x - \bar{u}_1$, and $\bar{u}_2 - x$ are all assumed positive under III, the sign of $X_0 + X_1$ and $-X_0$ are completely determined by the sign of the surplus S_0 . If the surplus is positive, we obtain a long straddle position. There is a short position in stocks, $-X_0 > 0$,

¹We use the term straddle to denote a portfolio of long put and call positions, where the number of puts and calls are not necessarily equal.

which is offset by the long call position for sufficiently high stock prices, $X_0 + X_1 > 0$. Similarly, if the surplus is negative, we obtain a short straddle pay-off pattern. The appendix shows how \bar{u}_1 and \bar{u}_2 are derived from the model parameters and defines A as a function of \bar{u}_1 , \bar{u}_2 , and r_f only.

Theorem 2.1 shows that the optimal pay-offs from model (3) are exactly those found by Fung and Hsieh (1997) in Figure 1. To make this point even stronger, we conduct a simple numerical experiment. Using a lognormal $G(\cdot)$, we compute the optimal solution to (3) for different strike prices and surplus levels. Computing the optimum is straightforward. It can either be done by discretizing $G(\cdot)$, or by solving the first order conditions. The latter are derived in the appendix in order to prove Theorem 2.1 and are very easy to solve numerically for any $G(\cdot)$. Figure 2 presents the results. For a positive surplus, we obtain a long straddle or a long call strategy, depending on whether the strike price is low or high, respectively. For negative surplus levels, the short put and short straddle are optimal for low and high strike prices, respectively. The similarity between Figure 2 and Figure 1 is striking: the present simple set-up provides a unified framework that can explain a large portion of all the different pay-off patterns observed empirically. Strike prices and surplus levels determine which of the pay-offs is optimal in a particular setting.

It can be seen from Theorem 2.1 that higher absolute surplus levels $|S_0|$ lead to more ‘aggressive’ investment policies, i.e., larger investments in the risky asset. For example, for increasingly large and positive values of the surplus, the number of long straddles or long puts increases as well, resulting in a steeper pay-off over the non-flat segments of the pay-off pattern. A similar result holds if the surplus becomes increasingly negative.

3 Implications for hedge funds

In Section 2 we found the optimal investment policies for a loss averse investor in a static setting. As mentioned, these results can be linked to pay-off patterns generated by dynamic investment strategies following the ideas of Fung and Hsieh (1997). Their paper, however, is mainly empirical. In this section we provide additional arguments linking the results of Theorem 2.1 and Figure 2 to dynamic investment strategies that are actually used by hedge funds.

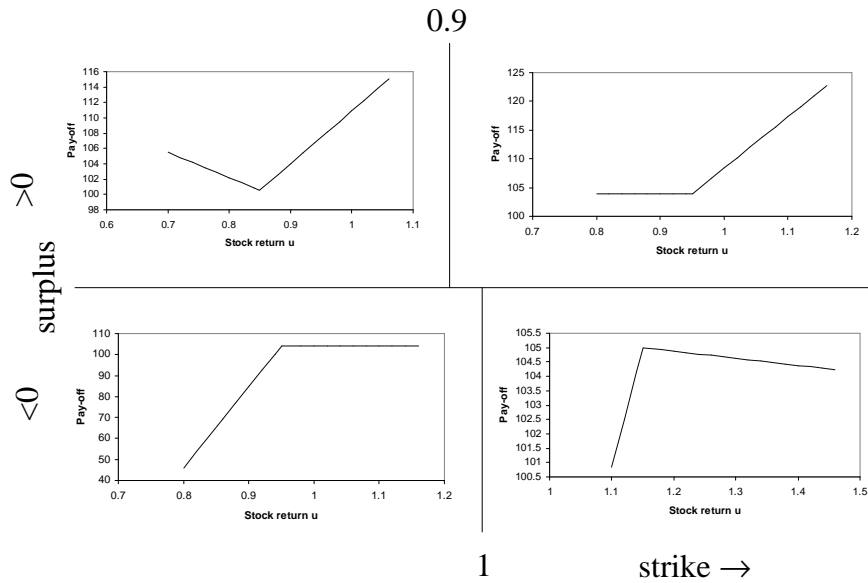


Figure 2: Characteristics of the optimal pay-offs as a function of the strike price x and the surplus $W_0 - W^B/r_f$.

The figure displays optimal pay-offs as a function of the risky return u for four different combinations of initial surplus S_0 and strike price of the option. Though the precise form and steepness of these four pay-offs may vary if other combinations of S_0 and strike are used, they are representative (in terms of positive/negative slope to the left/right of the strike) for the area in which they are plotted. These areas are bounded by the dashed lines in the figure. The horizontal line separates positive from negative surplus. The two vertical lines separate ‘high’ from ‘low’ strike prices (for positive and negative surplus, respectively).

The first dynamic strategy is that of a market timer as described by Merton (1981). Merton shows that the return pattern of such a strategy resembles a straddle on the traded asset. This idea is pursued further by Fung and Hsieh (2001), who find that using the return to a synthetic lookback straddle captures most of the variation in returns of Trend Following hedge funds. The upper-left panel in Figure 2 therefore clearly corresponds to at least one dynamic investment strategy.

Another popular dynamic investment strategy is portfolio insurance, see for example Leland (1980). The basic idea is to reduce to proportion of stock if prices fall, and to

increase it if prices rise. This mimics a delta hedge strategy of a call option, which is precisely the pay-off pattern given in the upper-right panel of Figure 2. Portfolio insurance strategies are especially attractive for financial institutions facing short-term restrictions on their asset value, like certain pension funds, as argued by Shefrin and Statman (1985), Benninga and Blume (1985), Brennan and Solanki (1981).

Convergence bets as a dynamic investment strategy typically generate a pay-off pattern resembling a short put position, see the lower-left panel in Figure 2. A good example of this type of strategy is merger arbitrage as documented by Mitchell and Pulvino (2001). By taking a long position in the stock of the target in a merger or takeover, and a corresponding short position in the stock of the acquirer in case payment is in stock rather than cash, Mitchell and Pulvino (2001) show that positive returns are possible in bull markets. These returns are largely uncorrelated with the market, i.e., they have a beta equal to zero. In bear markets, mergers are more likely to fail, such that the merger arbitrage strategy results in potentially large losses there. These losses usually correlate positively with the market. The correlation pattern documented by Mitchell and Pulvino is precisely that of a short put. It is not surprising, therefore, that including a put-option return in the Sharpe style regressions for merger arbitrage returns significantly increases the explanatory power. Mitchell and Pulvino also show that hedge funds specialized in merger arbitrage generate pay-offs very similar to short puts, albeit that they have a slightly positive beta in bull markets. Different interpretations of the short put pattern are also possible, for example, convergence bets in credit markets. In that case, the short put may be seen as a direct reflection of the credit spread, see Merton (1974). By taking off-setting positions in corporate and government bonds, or in bonds of governments with different credit ratings, one can lock in the spread in most cases. In case of default (of the long position), however, the strategy results in a large loss. Strategies of this type are known to have been implemented by, for example, LTCM.

The short straddle position in the lower-right panel is more difficult to link up with well-known dynamic investment strategies, and we will conjecture on some of the possible explanations for this further below. Thus far, we have established a link between the optimal pay-off patterns emerging from our model and the pay-offs on dynamic investment strategies that are documented, either empirically or theoretically. In the remainder of

this section, we further explore the validity and implications of our model for hedge funds.

It is clear from Figure 2 that the shape of the optimal pay-off crucially depends on two variables, namely the (sign of the) surplus and the location of the strike price. We start with discussing the former. Figure 2 clearly shows that there is a remarkable difference between strategies for a positive and negative surplus, respectively. The interpretation of the surplus in the context of hedge funds is not straightforward and crucially depends on the interpretation of the benchmark W_B . As discussed in the introduction and documented in Brown et al. (1999) and Brown et al. (1997), most hedge fund managers get their bonuses if the fund earns a return above high-water marks. The mark can be given by a treasury bill return, a fixed return, or a return based on average market performance. If, for example, the high-water mark is fixed or is linked to a treasury yield, a sufficiently high intermediate return over part of the measurement period brings the fund into a situation with a positive ‘surplus’. So with respect to the managers of the fund, we can interpret the surplus in terms of the return that is necessary to attain the high-water mark. Analogously, a negative intermediate return may jeopardize the manager’s future fee and bring him in a situation of a negative surplus. Alternatively, Barberis et al. (2001) and Barberis and Huang (2001) use the idea of a surplus together with mental accounting practices adopted by loss-averse investors. In their set-up, surplus represents the difference between the current price of a stock or fund and its historical benchmark. The historical benchmark may represent an average of recent stock prices, or some specific historical stock price, such as the price at the end of the year. The difference between the realized stock price and the benchmark, if positive, is the investor’s personal measure of how much ‘he is up’ on his investment and conversely, if negative, how much ‘he is down’. If this way of mental accounting is relevant for modeling investor preferences, then our results suggest that the behavior of hedge funds is in line with the preferences of those who invest in it. This fits in with the incentive schemes as discussed before, such that these schemes can be seen as the proper instrument for attaining alignment.

One of the results also mentioned in the previous section was that larger absolute values of the surplus $|S_0|$ result in steeper pay-off patterns. In particular, if the surplus becomes increasingly negative, our model predicts that a more aggressive investment strategy is adopted. Intuition for this can be found in Brown et al. (1997). They discuss the consequences of high-water mark thresholds used by hedge fund managers and note:

‘If a fund has a negative return, the manager is out of the money and presumably has an incentive to increase risk’. Theorem 2.1 formalizes this intuition.

Given the above interpretation of surplus, our model predicts that funds with high water marks are more likely to follow convergence strategies. Vice versa, if our proposed rational framework has empirical content, we would expect higher high-water marks for firms focusing on convergence bets. Some anecdotal empirical evidence supporting our claim is available for LTCM. Jorion (2000) explains that the core strategy of LTCM was a relative-value or convergence-arbitrage trade on credit spreads, while Edwards (1999) states that LTCM had one of the highest incentive fees in the industry.

The second important variable driving influencing the shape of the optimal pay-off pattern is the strike price x of the option. The strike price links to the dynamic strategy followed by the fund. In our framework, a fund focusing on a particular strategy is tantamount to the fund picking its strike price for the option. Again, our model gives rise to several implications that are corroborated by the empirical literature. For example, consider the lower-left panel in Figure 2. As mentioned earlier, the short put pay-off pattern corresponds to a trading strategy focusing on convergence bets. Following our model, such a strategy is only optimal if the strike is sufficiently low, i.e., if the put is sufficiently far out of the money. This is supported by the findings of Mitchell and Pulvino (2001). As mentioned earlier, they capture much of the variation in merger arbitrage returns by including a put option. It is remarkable to note that the optimal strike price of their option is at a return level of -4%. This resembles an out of the money put along the lines predicted by our present simple framework. Another example is given by the upper-left panel in Figure 2. Originally suggested by Merton (1981), Fung and Hsieh (2001) use the long straddle pay-off to describe the returns on market timers, i.e. those traders that buy when they believe the market goes up and sell short when they believe it going down. Our model implies that strike prices must not be too high in order for the long straddle to be optimal. Fung and Hsieh use at-the-money straddles in their empirical work. Our results suggest that out-of-the-money straddles would provide an even better fit in their regression model.

The above interpretation of the strike price as the chosen dynamic strategy of the fund raises the question what type of strategy will be chosen by investors if they can freely choose among different hedge fund styles. Extending the model (3) to optimize over

$\{X_0, X_1\}$ and x simultaneously leads to the optimality of the lower-left and upper-right panels in Figure 2, depending on whether the surplus is negative or positive, respectively. This has two implications. First, it may in part explain why the lower-right panel does not match any of the well-known dynamic investment styles used in the industry. It is simply not optimal to follow this strategy if the strategy choice is at the discretion of the hedge fund manager or the investor. The argument is strengthened by noting that the lower two panels describe a situation with a negative surplus. In such a setting, an efficient choice of the asset mix including the strike x is even more important than in a situation of a positive surplus. This may explain why we can observe the long straddle strategy (trend followers) empirically, while this is harder for the short straddle. Our second implication of the optimal choice of the strike x , however, is that the (long) straddle type pay-off patterns (upper-left panel in Figure 2) may not survive in the long run, either. As investors become more aware of the funds' properties and potential alternatives, our results suggests that they will shift out of trend following funds into long call strategies.

4 Conclusions

In this article we discussed a very simple financial optimization model based on loss-aversion for which exactly four different pay-off patterns can be optimal. These patterns closely match the various patterns in hedge fund returns observed in empirical work. We provided several reasons to suspect that loss aversion is an important phenomenon in hedge funds. The most explicit piece of evidence given in the literature relates to incentive schemes for hedge fund managers: the limited liability for managers is reduced by requiring them to put a substantial amount of their own money at stake. In that sense, loss aversion is a result of the common 'put your money where your mouth is'-policy that hedge funds use to attract potential investors and signal their commitment.

Our model provides a unified, rational framework for explaining patterns in hedge fund returns. Using traditional utility functions that do not have the loss-aversion property, capturing the wide variety in hedge fund return patterns into one unifying framework is much more difficult. With respect to robustness, numerical computations reveal very similar patterns for alternative specifications for the loss-averse objective function. In particular, using the original specification of Kahneman and Tversky (1979) our findings

do not change significantly.

Interestingly, our model also gives rise to some new predictions pertaining to the relation between the incentive schemes and adopted dynamic strategies. Partial evidence supporting some of the predictions is available from empirical work in the literature. More detailed data sets, however, are needed to substantiate the empirical validity of some other implications of our theoretical model. Of course, in practice hedge fund managers may adopt a number of trading strategies and/or change strategies over time. It is clear that such complications fall outside the scope of our current simple modeling framework. We believe, however, that the model and its results provide a useful step in a further understanding of hedge funds.

Appendix

Proof of Theorem 2.1

We start by restating the optimization problem in (3) as

$$\max_{X_0, X_1} V(X_0, X_1), \tag{A1}$$

$$\text{with } V(X_0, X_1) = \mathbb{E}[W_1] - \lambda \cdot \mathbb{E}[(W^B - W_1)^+], \tag{A2}$$

subject to

$$W_1 = W_0 r_f + X_0 \cdot (u - r_f) + X_1 \cdot (R_c - c \cdot r_f). \tag{A3}$$

Define $p = x/r_f + c - 1$, the price of the put option corresponding to price of the call following from put-call parity, see e.g. Hull (1997). Define $R_{c,x,G(\cdot)}$ as the expected return on the call option with strike x on an asset with return $u \sim G(\cdot)$, given by $\mathbb{E}[(u - x)^+/c]$. For ease of notation, we drop the subscripts x and $G(\cdot)$. Likewise, we denote the expected return on the put option with R_p . To ensure a finite optimal solution we need the following assumptions.

A: $\lambda r_f G(x) > R_c - r_f$,

B: $\lambda \int_0^x (x - u)^+/p - r_f dG > -(R_p - r_f)$,

C: R_c is increasing in x .

The motivation for these two assumptions will follow from the proofs below. In short, assumptions A and B put a lower bound on the loss-aversion parameter λ to ensure that the trade-off between risk and return leads to a finite solution. Assumption C is tested empirically in Coval and Shumway (2001), who find that the expected return of S&P index option returns increases with the strike price.

There are four possible pay-off patterns resulting from the combination of a risk-free asset, a stock, and a call option on the stock, namely decreasing-decreasing(I), increasing-increasing(II), decreasing-increasing(III), increasing-decreasing(IV), where for example case (I) refers to a setting where the pay-off increases in u both before and after the strike price x .

pattern I (decreasing-decreasing)

Conditions for case I are $X_0 \leq 0$ and $X_0 + X_1 \leq 0$. The first order conditions in this case are given by

$$\frac{\partial V}{\partial X_0} = 0 \Rightarrow 0 = \mathbb{E}[u - r_f] + \lambda \int_{\bar{u}}^{\infty} u - r_f dG, \quad (\text{A4})$$

and

$$\frac{\partial V}{\partial X_1} = 0 \Rightarrow 0 = \mathbb{E}[(u - x)^+ - c \cdot r_f] + \lambda \int_{\bar{u}}^{\infty} (u - x)^+ - c \cdot r_f dG, \quad (\text{A5})$$

where \bar{u} is a constant depending on (X_0, X_1) . Using $\mathbb{E}[u] > r_f$ and $\mathbb{E}[(u - x)^+] > c \cdot r_f$, we obtain that the right-hand sides of both (A4) and (A5) are positive for any value of \bar{u} , such that there is no interior optimum. The solution in situation I is, therefore, to set $X_0^* = X_1^* = 0$.

pattern II (increasing, increasing)

Conditions for case II are $X_0 > 0$ and $X_0 + X_1 > 0$. First order conditions are given by the system

$$\begin{cases} \mathbb{E}[u - r_f] + \lambda \int_0^{\bar{u}} u - r_f dG = 0, \\ \mathbb{E}[(u - x)^+ - c \cdot r_f] + \lambda \int_0^{\bar{u}} (u - x)^+ - c \cdot r_f dG = 0, \end{cases} \quad (\text{A6})$$

where \bar{u} is again a constant depending on (X_0, X_1) . Each equation in (A6) has either zero or two solutions. The zero-solution case for the first order condition corresponds to unbounded solutions for the original optimization problem (A1), since the left-hand side in (A6) must then necessarily be positive. We have abstracted from unbounded solutions however. Since the

integrands in the two equations of (A6) are different, the two equations will not be satisfied for the same value of \bar{u} . Hence, the optimum is attained at the extremals. In this case, for II the extremals are defined by two sets of parameter values, given by

$$X_0 = 0, X_1 > 0, \quad (\text{A7})$$

or

$$X_0 > 0, X_0 + X_1 = 0. \quad (\text{A8})$$

Starting with the former, investing only in the call option implies an optimization problem with the following first order condition for an interior optimum:

$$\mathbb{E}[(u - x)^+ - c \cdot r_f] + \lambda \int_0^{\bar{u}} (u - x)^+ - c \cdot r_f dG = 0, \quad (\text{A9})$$

where $\bar{u} = (W^B - W_0 r_f)/X_1 + x + c \cdot r_f$. By definition, $\bar{u} \geq x$. Under assumption A, we find that the FOC is never fulfilled, i.e. the derivative with respect to X_1 is negative. Without an interior optimum, the optimal solution is given by

$$X_1^* = \begin{cases} (W_0 - W^B/r_f)/c & \text{if } W_0 > W^B/r_f, \\ 0 & \text{if } W_0 \leq W^B/r_f. \end{cases} \quad (\text{A10})$$

We call this the long call strategy.

Now for the second case of extremals in situation I:

Define $X_2 = X_0 + X_1$, and $p = (x + c \cdot r_f - r_f)/r_f$. Condition for an interior optimum is

$$\mathbb{E}[(x - u)^+ - p \cdot r_f] + \lambda \int_0^{\bar{u}} (x - u)^+ - p \cdot r_f dG = 0, \quad (\text{A11})$$

where $\bar{u} = (W_0 r_f - W^B)/X_2 + x - p \cdot r_f$. By definition, $\bar{u} \leq x$. Under assumption B, the FOC has no solution, i.e. the derivative with respect to X_2 is positive.

Without an interior optimum, the optimal solution is given by

$$X_2^* = \begin{cases} (W^B/r_f - W_0)/p & \text{if } W_0 < W^B/r_f, \\ 0 & \text{if } W_0 \geq W^B/r_f. \end{cases} \quad (\text{A12})$$

We call this the short put strategy.

For $W_0 \geq W^B/r_f$ the long call strategy has a higher objective value than the short put. This is seen from the objective values, which are $W_0 r_f + (W_0 - W^B/r_f)(R_c - r_f)$ for the long call versus the short put value of $W_0 r_f$.

For $W_0 < W^B/r_f$ the short put strategy has higher objective value than the long call. This is seen from the objective values, which are $W_0 r_f + \lambda \cdot (W_0 r_f - W^B)$ for the long call versus the short put value that is larger than $W_0 r_f + \lambda \cdot (W_0 r_f - W^B) \cdot G(x)$. The last inequality follows from Assumption B.

pattern III (decreasing-increasing: straddle)

Situation III is characterized by $X_0 < 0$, $X_0 + X_1 > 0$.

The two values for which $W_1 = W^B$ are given by $\bar{u}_1 < x$ and $\bar{u}_2 > x$. They are defined as

$$\bar{u}_1 = \frac{W^B - W_0 \cdot r_f - X_0 \cdot r_f - X_1 \cdot c \cdot r_f}{X_0} \quad (\text{A13})$$

$$\bar{u}_2 = \frac{W^B - W_0 \cdot r_f - X_0 \cdot r_f - X_1 \cdot x - X_1 \cdot c \cdot r_f}{X_0 + X_1} \quad (\text{A14})$$

The first order conditions are given by

$$\frac{\partial V}{\partial X_0} = \mathbb{E}[u - r_f] + \lambda \cdot \int_{\bar{u}_1}^{\bar{u}_2} (u - r_f) dG = 0, \quad (\text{A15})$$

$$\frac{\partial V}{\partial X_1} = \mathbb{E}[(u - x)^+ - c \cdot r_f] + \lambda \cdot \int_{\bar{u}_1}^{\bar{u}_2} ((u - x)^+ - c \cdot r_f) dG = 0. \quad (\text{A16})$$

$$\text{s.t.} \quad \bar{u}_1 < x < \bar{u}_2. \quad (\text{A17})$$

It can be checked that under the current assumptions the Hessian is negative definite. If the FOC is fulfilled for a feasible (X_0, X_1) , it constitutes a local optimum. Note that if the FOC is satisfied, the value of the objective function can be written as

$$W_0 r_f + \lambda \cdot r_f \cdot (W_0 - W^B/r_f) \cdot (G(\bar{u}_2^*) - G(\bar{u}_1^*)) \leq W_0 r_f + \lambda \cdot S_0. \quad (\text{A18})$$

The function value of the optimum in situation II for positive surplus is given by the value of the long call strategy as

$$W_0 r_f + (W_0 - W^B/r_f) \cdot \mathbb{E}[(u - x)^+ / c - r_f] = W_0 r_f + S_0 \cdot [R_c - r_f]. \quad (\text{A19})$$

Using assumption C, which says that $R_c - r_f$ is increasing in x , we find that for $x \rightarrow 0$ and $S_0 > 0$ the straddle pay-off is better than the long call pay-off of case II.

pattern IV (increasing, decreasing: short straddle)

Situation IV is similar to case III and characterized by $X_0 > 0$, $X_0 + X_1 < 0$. \bar{u}_1 and \bar{u}_2 are the same as in situation III.

The first order conditions are given by

$$\frac{\partial V}{\partial X_0} = \mathbb{E}[u - r_f] + \lambda \cdot \int_0^{\bar{u}_1} (u - r_f) dG + \lambda \cdot \int_{\bar{u}_2}^{\infty} (u - r_f) dG, \quad (\text{A20})$$

$$\begin{aligned} \frac{\partial V}{\partial X_1} &= \mathbb{E}[(u - x)^+ - c \cdot r_f] \\ &+ \lambda \cdot \int_0^{\bar{u}_1} ((u - x)^+ - c \cdot r_f) dG + \lambda \cdot \int_{\bar{u}_2}^{\infty} ((u - x)^+ - c \cdot r_f) dG. \end{aligned} \quad (\text{A21})$$

For the moment, it is left as an exercise to the reader to verify that the Hessian is negative definite. If the FOC is fulfilled for a feasible (X_0, X_1) , it constitutes a local optimum. Note that if the FOC is satisfied, the value of the objective function can be written as

$$W_0 r_f + \lambda \cdot (W_0 r_f - W^B) \cdot (1 - (G(\bar{u}_2^*) - G(\bar{u}_1^*))). \quad (\text{A22})$$

The function value of the optimum in situation II for negative surplus is given by the short put strategy as

$$\begin{aligned} W_0 r_f + X_2 \cdot p \cdot \left(\mathbb{E}[R_p - r_f] + \lambda \cdot \int_0^x (x - u)^+ - p \cdot r_f dG \right) \\ + \lambda \cdot (W_0 r_f - W^B) \cdot G(x), \end{aligned} \quad (\text{A23})$$

which is, according to Assumption B, larger than

$$W_0 r_f + \lambda \cdot (W_0 r_f - W^B) \cdot G(x). \quad (\text{A24})$$

This implies that there is a strike y such that for strikes $x < y$, the short put strategy of pattern II has a higher objective value than the current short-straddle pattern IV.

Having found the optimal pay-offs for each pair (x, S_0) , we end with defining A in Theorem 2.1. From the definition of \bar{u}_1 and \bar{u}_2 in (A13) and (A14), we can write X_0 and X_1 as a function of \bar{u}_1 and \bar{u}_2 in the following way:

$$\begin{pmatrix} X_0 + X_1 \\ -X_0 \end{pmatrix} = \frac{1}{A} \cdot \begin{pmatrix} x - \bar{u}_1 \\ \bar{u}_2 - x \end{pmatrix} \cdot S \cdot r_f, \quad (\text{A25})$$

where $A = -(\bar{u}_2 - x)(x - \bar{u}_1) + cr_f(x - \bar{u}_1) + (\bar{u}_2 - x)pr_f$, and $pr_f = x + cr_f - r_f$. As the long straddle pay-off is only optimal for positive surplus, we find $A > 0$. This concludes the proof. ■

References

- Agarwal, V. and N. Y. Naik (2000a, Spring). On taking the "alternative" route: The risks, rewards, and performance persistence of hedge funds. *Journal of Alternative Investments* 2(4), 6–23.
- Agarwal, V. and N. Y. Naik (2000b, July). Performance evaluation of hedge funds with option-based and buy-and-hold strategies. London Business School, Working paper No. 300, July 2000.
- Amin, G. S. and H. M. Kat (2001). Hedge fund performance 1990-2000. do the 'money machines' really add value? Working paper, ISMA Centre, University of Reading.
- Barberis, N. and M. Huang (2001). Mental accounting, loss aversion, and individual stock returns. *The Journal of Finance* 56(4), 1247–1292.
- Barberis, N., M. Huang, and T. Santos (2001). Prospect theory and asset prices. *Quarterly Journal of Economics* 66(1), 1 – 53.
- Basak, S. and A. Shapiro (2001). Value-at-risk based risk management: Optimal policies and asset prices. *Review of Financial Studies* 14(2), 371–405.
- Benartzi, S. and R. H. Thaler (1995). Myopic loss aversion and the equity premium puzzle. *Quarterly Journal of Economics* 110, 73–92.
- Benninga, S. and M. Blume (1985). On the optimality of portfolio insurance. *Journal of Finance* 40(5), 1341–1352.
- Berkelaar, A. and R. Kouwenberg (2000a). From boom til bust: How loss aversion affects asset prices. Econometric Institute Report EI-2000-21/A, Erasmus University Rotterdam.
- Berkelaar, A. and R. Kouwenberg (2000b). Optimal portfolio choice under loss aversion. Econometric Institute Report EI 2000-08/A.
- Brealey, R. A. and E. Kaplanis (2001). Changes in the factor exposures of hedge funds. London Business School, IFA Working Paper 320, January 2001.
- Brennan, M. and R. Solanki (1981). Optimal portfolio insurance. *Journal of Financial and Quantitative Analysis* 16, 279–3–00.

- Brown, S. J. and W. N. Goetzmann (1997). Mutual fund styles. *Journal of Financial Economics* 43, 373–399.
- Brown, S. J., W. N. Goetzmann, and R. G. Ibbotson (1999). Offshore hedge funds: Survival and performance, 1989-95. *Journal of Business* 72(1), 91–117.
- Brown, S. J., W. N. Goetzmann, and J. Park (1997). Conditions for survival: Changing risk and the performance of hedge fund managers and CTAs. Yale University, unpublished.
- Carpenter, J. N. (2000, October). Does option compensation increase managerial risk appetite? *The Journal of Finance* 55(5), 2311–2331.
- Coval, J. D. and T. Shumway (2001). Expected option returns. *The Journal of Finance* 56(3), 983–1009.
- De Roon, F. A., T. E. Nijman, and J. R. Ter Horst (2000, March). Evaluating style analysis. Erasmus University Rotterdam, ERIM Report Series ERS-2000-11-F&A.
- Edwards, F. R. (1999, Spring). Hedge funds and the collapse of long-term capital management. *Journal of Economic Perspectives* 13(2), 189–210.
- Edwards, F. R. and M. O. Caglayan (2001). Hedge fund performance and manager skill. *The Journal of Futures Markets* 21(11), 1003–1028.
- Fung, W. and D. A. Hsieh (1997). Empirical characteristics of dynamic trading strategies: The case of hedge funds. *The Review of Financial Studies* 10(2), 275–302.
- Fung, W. and D. A. Hsieh (2001, Summer). The risk in hedge fund strategies: Theory and evidence from trend followers. *The Review of Financial Studies* 14(2), 313–341.
- Glosten, L. and R. Jagannathan (1994). A contingent claim approach to performance evaluation. *Journal of Empirical Finance* 1, 133–160.
- Guercio, D. D. and P. A. Tkac (2001). Star power: The effect of morningstar ratings on mutual fund flows. Federal Reserve Bank of Atlanta, Working Paper 2001-15, August 2001.
- Hull, J. C. (1997). *Options, Futures, and Other Derivatives*. Prentice-Hall Inc.
- Jorion, P. (2000, September). Risk management lessons from long-term capital management. *European Financial Management* 6, 277–300.

- Kahneman, D. and A. Tversky (1979, March). Prospect theory: An analysis of decision under risk. *Econometrica* 47, 263–291.
- Leland (1980). Who should buy portfolio insurance? *The Journal of Finance* 35(2), 581–594.
- Liang, B. (1999). On the performance of hedge funds. Weatherhead school of management, Case Western Reserve university, working paper March 1999.
- Merton, R. (1990). *Continuous-Time Finance*. Padstow, UK: Basil Blackwell Inc.
- Merton, R. C. (1974). On the pricing of corporate debt: The risk structure of interest rates. *The Journal of Finance* 29, 449–470.
- Merton, R. C. (1981). On market timing and investment performance. i. an equilibrium theory of value for market forecasts. *Journal of Business* 54(3), 363–406.
- Mitchell, M. and T. Pulvino (2001, December). Characteristics of risk and return in risk arbitrage. *Journal of Finance* 56(6), 2135 – 2175.
- Osterberg, W. P. and J. B. Thomson (1999, May 1). The truth about hedge funds. Federal Reserve Bank of Cleveland, Economic Commentary Series.
- Sharpe, W. F. (1992). Asset allocation: Management style and performance measurement. *Journal of Portfolio Management* 18, 7–19.
- Sharpe, W. F. (1998, July/August). Morningstar’s risk-adjusted ratings. *Financial Analysts Journal*, 21–33.
- Shefrin, H. and M. Statman (1985). The disposition to sell winners too early and ride losers too long: Theory and evidence. *Journal of Finance* 40(3), 777–790.
- Siegmann, A. H. and A. Lucas (2000). Analytic decision rules for financial stochastic programs. Tinbergen Institute Discussion Paper no. 00-041/2.