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The Sustainability of the Pay-as-you-go System with Falling Birth Rates

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ABSTRACT

A model is presented that explains the mix between funded and unfunded pension systems. It turns out that total pension and the relative shares of the two systems may be explained and are determined by the population growth rate, technological growth, the time-preference discount rate, the relative risk aversion, the production function, and the degree of altruism. A fall in the population growth rate, even to negative values, will imply a reduction of the interest rate and an increase in the capital-output ratio, while the pension system will shift to more funding. A fall in the population growth rate will result in a reduction of average welfare and an increase in the income inequality between workers and retired people/individuals.

Keywords: Old-age pensions, pay-as-you-go, intergenerational transfers, retirement benefits, altruism

JEL classification: H55, D91, D64, J14, J26

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I. INTRODUCTION

Is the modern welfare state sustainable when the population growth rate tends to fall dramatically? In most European countries, until the nineties we saw a tendency towards an increasing role of the state. In recent years, however, the tide has turned and most countries with an ageing population try to reduce the role of the government. Some pessimists plead for a total dismantling of the welfare state. According to this paper, a fall in the population will lead to a trimming of the welfare state, but not to a total dismantling. We may expect and will have to accept a decrease in the average welfare level, as well as increasing inequality between the workers and the retired.

In this paper we shall consider the redistribution function of the government. Examples of redistribution are state pension schedules for the elderly and family allowances for families with children, social assistance, and provisions in kind (like health care, education, the provision of safety by the police and the army, etc.). These provisions are typically at non-market prices and frequently even at zero prices. The input for the system comes from tax revenues and social contributions. In this paper we shall restrict ourselves to the transfer between workers and retired, which we see as the prototypical characteristic of the modern welfare state.

In economics and political science there are elaborate theories explaining the existence and the tasks of a government and of a state. In this context we focus on one feature in particular: the state as the agent for *institutionalised altruism*. Most human beings are not perfect egoists. They are more or less altruistic as well. They tend to care for the weak and even for future generations. However, in a mass society it is very difficult for individuals to be active as altruists. As an individual, it is difficult to find the people who need (your) help, and if you have found any, to be sure that these are the most needy. Even if so, in our modern societies individuals are unable to give the necessary support as efficiently as specialised institutions may do. In short, the state is the individual's subcontractor for altruistic activities. Part of our tax is a payment for those activities. Due to that subcontract, we can think of ourselves as decent citizens. The existence of this arrangement frees us from the moral obligation to be altruistic as individuals as well.

This picture is incomplete, however. To be precise, we should distinguish between altruism towards the own family and well-known neighbours, and altruism for the unknown, anonymous people. We do not mean to say that individuals do not care for their kin or their friends anymore. We only try to make the point that altruism to the anonymous has been contracted out to the state. The legal force of the state can enforce non-market solutions and prohibit 'opting-out' by those citizens who happen to be the net-payers. It is the instrument by means of which the 'social compact', in the terms of Samuelson (1958), may be realized, which is favourable to all when compared to any market solution. Actually, most regulations might be replaced by private insurance, saving or private borrowing.

Apart from this role, the state also represents our own egoistic interests in those fields where the state can be a more efficient provider than the market. Given this specific role of the modern government, we assume that the government is an actor in the decision process, with its own utility function which we call the Social Welfare Function (SWF). The citizens define the SWF through the democratic process but the SWF is not necessarily identical with the utility functions of citizens. Citizens have their own *individual* welfare functions (IWF). Obviously, different groups will have different IWFs. We model the political decision process in the spirit of a Nash-Cournot equilibrium, where both parties optimize their own utility function with respect to the instrument variable(s) they can dispose of.

In this paper we look at a highly stylized overlapping generations model (Diamond, 1965), where we have only two population subgroups: the workers and the retired. Hence, the redistribution system is one of intergenerational transfers. The workers pay taxes at the rate θ out of their labour income w into the system, and the retired get a pension. This is a function of the current contributions, where we take into account that the retired generation is $1/(1+n)$ of the working generation, n being the population growth rate. We assume the system to be of the pay-as-you-go type (PAYG). Strictly speaking, old- age pensions are only one component of the PAYG-system. As there are flows between the two groups, and as part of these flows is in kind (e.g. old-age care, or education for the workers), it is fairly difficult to measure whether the net benefit goes to the workers or to the retired. That is, the sign of θ may be positive or negative. The value of θ is not an indication for the total volume of the flows between the two generations but it represents only the *difference* between the two flows.

The main question we look at is whether the model provides a unique θ, which defines a specific stationary redistribution among the generations. The second question is how this θ is affected by a change in the population growth rate n. The relevance of this question is evident, as in most Western countries the population growth rate is falling considerably (see Boeri, Bőrsch-Supan and Tabellini, 2001). As a consequence, the ratio between the numbers of retired and workers, the *dependency ratio,* is increasing dramatically, causing public unrest about the future sustainability of existing social security arrangements.

In our model we assume that individual workers behave non-altruistically. Hence, if it would not be compulsory, a PAYG-system, through which the net benefit goes to the retired, would be rather fragile. The workers determine their savings for their old age where they take θ as given. We assume a government with its own Social Welfare Function (SWF), which reflects the workers' interests *and* the interests of the retired. More generally speaking, the SWF embodies the egoistic and the altruistic motives. It seems probable that the relative weight of the egoistic motive may be much larger than that of the altruistic motive. In turn, the state maximizes its SWF with respect to θ where it takes individual savings S as given.

The shaping of the SWF depends on the way in which the government is formed. In a Western-style democracy, the SWF will be the result of parliamentary consensus or at least will reflect the opinions of a majority. Under a dictatorship, the SWF is likely to reflect the opinions of the dictator and his clique only. Moreover, the government may (and in most democracies will) assign a weight to the interests of future generations. Hence, in this arrangement the government acts as the defender of the weak (the retired and the unborn). The relative weight of the retired is δ, which we call the *altruism* coefficient. The period-utility functions of the workers and those of the retired, denoted by U_1 and U_2, may differ. We specify the period-utility function as one exhibiting Constant Relative Risk Aversion (CRRA) with the parameter γ representing the Arrow-Pratt measure of constant relative risk aversion (Pratt, 1964; Arrow, 1971). If $\gamma = 1$, this function is the traditional logarithmic utility function.

If both parties, the individual workers and the government, maximize their objective functions, we get a stationary equilibrium with a non-zero θ. If $\delta = 0$, we find the traditional golden rule path, characterized by $r = n + g$ (Samuelson, 1958), where g stands for autonomous labour productivity growth. In all other cases we find $r \geqq n + g$. The solution of the system, especially for θ, appears to depend on n, g as usual, and on the value of α, the capital elasticity in the Cobb-Douglas production function, the risk aversion γ, the individual time discount ratio ρ, and the weighting system δ in the government's SWF.

In contrast to most of the literature, where $n > 0$ is assumed as a matter of routine, present reality demonstrates that negative growth is a real possibility. Our model admits for (moderate) negative growth as well.

In section 3 we develop our model. In section 4 we solve that model. In section 5 we consider the consequences as to income inequality between the workers and the retired. In Section 6 we consider the transition problem. In section 7 we discuss some numerical results. We end with a discussion and some conclusions.

II. SETTING IN THE LITERATURE

Before embarking on our model and findings, let us have a look at the position of this paper in the literature, without claiming completeness. Although, strictly speaking, this paper only deals with intergenerational transfers in general and not with old-age pensions only, in the abstract traditional setting which we also employ in this paper, there is no way to make a distinction between the wider and the narrower concept. The only difference between social subgroups that we distinguish in the framework of this paper, is with respect to the status of 'being retired' or 'working'.

The seminal paper in the overlapping generations literature is that by Samuelson (1958)[2]. He finds that, in a more-generation model, the equilibrium interest rate would be the ('biological') population growth rate; it would equal the social optimum interest rate which Samuelson sees as an astonishing result. However, in that world there are no durable goods which can be handed over to the next generation, which implies that no trade takes place between individuals of different ages. Samuelson demonstrated that the introduction of a 'social compact' could make the social optimum a stable and achievable equilibrium. Looking at a similar problem, Aaron (1966) showed that pay-as-you-go social security improves the utility of each individual, if the growth rate of the economy is larger than the (exogenously given) interest rate.

The basic paper in the OLG-literature is Diamond (1965). It introduces production, and the general equilibrium interest rate is not necessarily efficient anymore. There is no collective social security system; instead there is a government issuing debt and collecting taxes. Through debt management, the government is able to eliminate dynamic inefficiency in an OLG-economy.

[2] Allais (1947) predates some of Samuelson's results.

Samuelson (1975) introduces a compulsory social security system, which is a mix of funding and pay-as-you-go. He finds that such a system may, under certain assumptions, perfectly replace private voluntary savings.

Feldstein (1985) doubts the realism of Samuelson. We quote Feldstein (p.304):

> "More generally, as Samuelson has noted, a social security trust fund could acquire enough capital to bring the economy to golden-rule efficiency. In general, this would require that the social security obligations are more than fully funded and may require the trust fund to own the nation's capital stock. As a practical matter, however, the social security program in the United States and in many other countries operates on a pay-as-you-go basis without a capital fund".

Feldstein also considers the possibility of a mix between a funded and a PAYG-system. He looks for an optimal security system. That is, he looks for an optimum θ, where he admits for the possibility of myopia (that is, that individuals "underestimate their future social security benefits"). However, Feldstein makes the restrictive assumption that "to avoid the problem of an endogenous and varying rate of return, I assume that the marginal product of capital remains constant". This is tantamount to assuming an exogenous wage and interest rate and a capital/labor ratio. However, one of the most interesting and relevant effects of a PAYG-system is its influence on those variables. A similar restrictive assumption is basic for Homburg's (1990) analysis, where he shows "that a PAYG-system may be converted into a capital reserve system without inflicting damage upon anyone" (see also Brunner, 1996).

Another, more modern strand in the literature tries, since Browning (1975), to describe and explain the existence of intergenerational flows, and unfunded old-age pensions in particular, by taking recourse to political decision theory. They assume a one-dimensional political spectre, where the median voter is the crucial opinion maker. Most of the times the focus is on different incomes, due to different skills or abilities (see e.g. Tabellini, 2000, or Conesa and Krueger, 1999). The result is that, where a coalition of old people and the poorest part of the young generation are voting for a social security scheme running on a pay-as-you-go (PAYG) basis, the median voter is decisive. Casamatta, Cremer and Pestieau (1999) considered the case where voters differ with respect to productivity and with respect to age. They found that in such a case the winning coalition will include the medium-wage workers rather than the low-wage workers. For our paper the Median Voter (MV) theory is irrelevant, as the population is split up into two brackets only: the workers and the retired. Normally, the workers will have the majority and there is no median voter on a continuous spectrum. Workers would always have it their way. This does not mean that the MV-theory is irrelevant in more complex situations, but in the basic situation that we consider here, that of a two-generations world, we do not see a role for it.

In this paper we consider a closed economy where each citizen tries to maximize his utility. The retired have no power, except via their influence on the government. The interests of the presently retired are a component of the government's SWF. We find that, in such a state, a mix of the two systems (viz. individual savings *and* a PAYG-system of intergenerational transfers) is the likely outcome, although the precise outcome strongly depends on the nature of the citizens' utility functions, the weight distribution in the SWF, the demography, and the nature of technology. It is difficult to evaluate the result as an *optimal* mix, because it is the result of a compromise between the citizens and the government. Hence, we would characterize this paper as a positive rather than as a normative paper.

III. THE MODEL

We assume that the life of individuals is split up into two periods. During the first period they are working and during the second period they are retired. Let us assume a natural population growth rate of n per period.

The population L_t grows according to the equation:

$$L_t = (1 + n) * L_{t-1}$$

Each individual embodies a number of A_t labour efficiency units, which number grows per period by a factor $(1 + g)$ due to labour-augmenting technological growth. Hence, the number of efficiency units per worker develops according to

$$A_{t+1} \;=\; (1+g) * A_t$$

The sum of the efficiency units over the population as a whole gives the labour force in terms of efficiency units. The total growth rate of the labour force in terms of efficiency units is denoted by ν

$$(1+\nu) = (1+g) * (1+n)$$

We assume that the wage rate and the interest rate are w and r per efficiency unit. Capital per efficiency unit is k. We assume that there is only one good, which can be used either as capital or as a consumption good. A constant returns to scale production function of the Cobb-Douglas type, with capital elasticity α, is postulated.

III.1. The Citizen

A worker's consumption equals

$$C_{t,1} = \{(1-\theta)w_t - S_t)\}A_t \tag{1}$$

where the transfer ratio θ is that fraction of his/her wage rate, which is transferred to or received on balance from the state and where S_t stands for savings. As said before, this fraction can be positive or negative. In our notation, the subscript t stands for the date, and i $(i = 1, 2)$ stands for the status of 'working' or 'retired'. Notice that S is an amount per labour efficiency unit. Similarly, the consumption of the currently retired is

$$C_{t,2} = \{\theta(1+\nu)w_t + (1+r)S_{t-1}\}A_{t-1} \tag{2}$$

We assume for the (working) individual a separable utility function, where both period-utility functions are concave but not necessarily identical, reflecting possibly different need structures, depending on the age of the individual. More precisely, we have a two-period utility function

$$U_1(C_{t,1}(S_t)) + \rho * U_2(C_{t+1,2}(S_t)) \tag{3}$$

where ρ is the *individual* time preference discount factor.

Let us now become more specific by adopting a Constant Relative Risk Aversion (CRRA) utility specification for both period-utility functions of the type:

$$U(C) = \frac{C^{1-\gamma}}{1-\gamma} \tag{4}$$

where $\gamma > 0$ is the coefficient of relative risk aversion. In the limiting case of $\gamma = 1$, this function tends to be the logarithmic utility function. This CRRA-function is well-known from the literature (Arrow, 1971 and Pratt, 1964).

We assume for the worker that he has only one variable which he can influence, viz. his savings S. We also assume that the individual worker takes his wage rate, the interest rate, the growth rate and the transfer ratio as given. Hence, the savings decision is optimized by

$$\max U_1(C_{t,1}(S_t)) + \rho * U_2(C_{t+1,2}(S_t)) \tag{5}$$

We notice that, due to the concavity, there is only one optimal S for a given θ. The retired have no instrument variable to be optimized and, hence, there utility is just $U_2(C_{t,2}(S_{t-1}))$.

In this simple model there is no independent role for the retired. Their interests have to be represented by the government.

III.2. The Production Side

We assume that workers save S_t per efficiency unit over their working life. Summing the investments of all workers, we get the aggregate capital stock

$$K_t = A_{t-1}L_{t-1}S_{t-1} \tag{6}$$

If S is constant over time, the capital per worker grows by $(1+g)$, and total capital grows by $(1+\nu)$.

We assume a constant returns to scale technology. For convenience sake, we choose a Cobb-Douglas production function with capital elasticity α. When assuming efficient production, the interest and wage rate will be

$$\begin{aligned} r_t &= \alpha k_t^{\alpha-1} \\ w_t &= (1-\alpha)k_t^{\alpha} \end{aligned}$$

The capital per labour efficiency unit, denoted by k_t in period t, equals

$$k_t = S_{t-1}/(1+\nu) \tag{7}$$

III.3. The Government

The second player in this game is the government. There is a government sector, which taxes the workers $\theta^{(w)}w$ and the retired -$\theta^{(r)}w$ per efficiency unit. The tax revenue is spent on government production, like education and infrastructure, and on an income redistribution between the workers and the retired. As we assume that the government does not save, we have the balance equation

$$\theta_t^{(w)} L_t A_t - \theta_t^{(r)} L_{t-1} A_{t-1} = 0 \tag{8}$$

It follows that, if the retired profit on balance, the workers will pay, and *vice versa.* In most developed economies, the main redistribution systems will include old-age retirement pensions and children benefits. One might think that the redistribution will always be in favour of the retired, but in this wider framework this is by no means obvious. For each cohort we get the identity

$$\theta_t^{(w)} = \theta_t^{(r)}(1+\nu) \tag{9}$$

Although we are just looking at *net* transfers, expressed as a share of gross wages per labour efficiency unit, from now on we will call θ the PAYG tax rate.

Government represents the interests of both the currently living and the future cohorts. How this works out is a question of politics, for instance, depending on the vote distribution. The government also assumes all variables as given, except for the transfer ratio θ, which is subject to legislation. Hence, it maximizes a government objective function of the following type:

$$W = \{U_1(C_{t,1}(\theta)) + \rho U_2(C_{t+1,2}(\theta))\} + \delta U_2(C_{t,2}(\theta)) \tag{10}$$

The first two terms represent the utility function of the present workers. As the present workers are interested in their *lifetime welfare,* their relevant utility function contains a part which refers to their future life in retirement. The third term stands for the utility of the currently retired. The weight δ reflects the relative weight that the government, as the representative of the retired and of the altruist workers, assigns to the retired. The weight distribution will depend on the relative population share of the retired, which is $1/(2+n)$, and on the political weight of the retired in the political system.

Although governments are, as a rule, most interested in their own election period, they are not so short-sighted that they would not take the interests of future generations into no account at all. We suggest the following extended SWF for the less short-sighted government:

$$[U_1(C_{t,1}(\theta)) + \rho U_2(C_{t+1,2}(\theta))] + \varphi_1[U_1(C_{t+1,1}(\theta)) + \rho U_2(C_{t+2,2}(\theta))] + \delta U_2(C_{t,2}(\theta)) \tag{11}$$

where we assume in this example that the government is sensitive to the interest of the first-next unborn generation. For the traditional logarithmic utility function and its generalization, the CRRA function, the generalization is rather easy.

For a CRRA function we have

$$U(C_{t+1}) = U(C_t).(1+g)^{1-\gamma} \tag{12}$$

where we assume that, in the steady state, individual consumption grows at the rate $(1+g)$ per period.

The consumption growth is only caused by the fact that, during each period, the individual embodies more labour efficiency units. A similar observation can be made for the 'retirement part'. Hence, it follows that we may rewrite the above extended social welfare function as

$$(1+\varphi_1(1+g)^{1-\gamma})[U_1(C_{t,1}(\theta))+\rho U_2(C_{t+1,2}(\theta))]+\delta U_2(C_{t,2}(\theta)) \tag{13}$$

Similarly, if we would have a social welfare function of the type

$$\sum_i \varphi_i[U_1(C_{t+i,1}(\theta))+\rho U_2(C_{t+1+i,2}(\theta))]+\delta U_2(C_{t,2}(\theta)) \tag{14}$$

where more than one unborn generation is included, we may rewrite it as

$$\sum_i (\varphi_i(1+g)^{(1-\gamma)i})[U_1(C_{t,1}(\theta))+\rho U_2(C_{t+1,2}(\theta))]+\delta U_2(C_{t,2}(\theta)) \tag{15}$$

We notice that the weights may follow the usual exponential pattern, but that this is not necessary. If it does, we have $\varphi_i = \varphi^i$. Then we call φ the *political* time preference discount rate. Notice that there is no reason why it should be equal to the individual time preference discount rate ρ. The only thing which is obviously needed, is that the sum of the weights is finite. We shall assume that

$$\sum (\varphi_i(1+g)^{(1-\gamma)i}) < \infty$$

This is the analogue of the transversality condition in the variational calculus approach.

It is obvious that the SWF may include an infinity of future generations.

We may normalize this welfare function by setting the first coefficient equal to one and we write the government's objective function or SWF as

$$[U_1(C_{t,1}(\theta))+\rho U_2(C_{t+1,2}(\theta))]+\tilde{\delta}.U_2(C_{t,2}(\theta)) \tag{16}$$

where $\tilde{\delta} = 1/\sum(\varphi_i(1+g)^{(1-\gamma)i})$.

The first term represents the lifetime utility of the present (and future) working generations, while the second term stands for the utility of the currently retired. The weight $\tilde{\delta}$ reflects the weight which the government assigns to the retired when compared to the present (and future) working generations, whose weight equals one. This weight depends on the relative population share of the currently retired, which is $1/(2+n)$, and on the political weight of the currently retired in the political system.

IV. THE EQUILIBRIUM

We are now looking for the existence of a stationary equilibrium. If we assume that both the individual citizens and the government attempt to optimize their objective functions, we get two first-order conditions.

The first-order condition for the worker is

$$\frac{\partial U_t}{\partial S} = U'_{t,1}(-A_t) + \rho U'_{t+1,2}(1+r)A_t = 0 \tag{17}$$

where the prime term denotes the first derivative. Division by A_t gives

$$\rho U'_{t+1,2}(1+r) = U'_{t,1} \tag{18}$$

The first-order condition for the government reads

$$\frac{\partial W_t}{\partial \theta} = U'_{t,1}(-wA_t) + \rho U'_{t+1,2}(1+\nu)wA_t + \delta.U'_{t,2}((1+\nu)wA_{t-1}) = 0 \tag{19}$$

After simplification we get

$$-U'_{t,1} + \rho U'_{t+1,2}(1+\nu) + \delta.U'_{t,2}(1+n) = 0 \tag{20}$$

Now we may consider the two first-order conditions as two equations in the unknowns S and θ. Combining the two conditions (18) and (20), we find

$$U'_{t+1,2}(\rho(1+r) - \rho(1+\nu)) = \delta U'_{t,2}(1+n) \tag{21}$$

Using the CRRA utility specification, we find

$$(C_{t+1,2}/C_{t,2})^{-\gamma} = \frac{\delta(1+n)}{\rho(r-\nu))} \tag{22}$$

Taking into account that per capita consumption grows by $(1+g)$, we get

$$(1+g)^{-\gamma} = \frac{\delta(1+n)}{\rho(r-\nu)} \tag{23}$$

which yields an explicit solution

$$r = v + \frac{\delta(1+n)(1+g)^{\gamma}}{\rho} \tag{24}$$

It follows that the interest rate r is defined by the effective growth rate g, the population growth rate n, the individual time discount rate ρ, and the political weight δ given to the currently retired cohort. If we replace δ by its generalization $\tilde{\delta}$, it is obvious that the interest of future generations will count as well. Most surprisingly, we find that for this specification there is no direct relation between the interest rate and the transfer ratio θ (cf. Diamond, 1965). It is also surprising that the equilibrium interest rate value does not depend on the technology level α. The classical golden rule path, where $r = \nu$, is only realized when $\delta = 0$ and $\rho = 1$. As $(1+n) > 0$ by definition, in all other cases we find $r > \nu$.

Knowing r, we find the associated k, w, S (all per efficiency unit) by using the CD-technology. We have $r = \alpha * k^{\alpha-1}$ and $k = \sqrt[\alpha-1]{\frac{r}{\alpha}}$ when firms maximize their profits. It follows that

$$k = \sqrt[(\alpha-1)]{\frac{v + \frac{\delta(1+n)(1+g)^{\gamma}}{\rho}}{\alpha}} \tag{25}$$

and by the other first-order condition. for profit maximization:

$$w = (1-\alpha).(\frac{v + \frac{\delta(1+n)(1+g)^{\gamma}}{\rho}}{\alpha})^{\frac{\alpha}{\alpha-1}} \tag{26}$$

It is rather evident that these computations would change if we were to assume another constant returns to scale production function like Constant Elasticity of Substitution. Finally we have

$$\frac{S_{t-1}}{1+v} = k_t \tag{27}$$

The resulting gross savings ratio is

$$\frac{S}{w} = \frac{(1+v)k}{(1-\alpha)k^{\alpha}} = \frac{(1+v)}{1-\alpha}k^{1-\alpha} \tag{28}$$

Substitution for k yields

$$\frac{S}{w} = \frac{\alpha}{1-\alpha} * \frac{1+v}{v + \frac{\delta(1+n)(1+g)^{\gamma}}{\rho}} \tag{29}$$

The transfer ratio θ is now derived from the first-order condition of the individual maximization problem. We get

$$\frac{[((1-\theta)w - S)A]^{-\gamma}}{[((1+v)\theta w + (1+r)S)A]^{-\gamma}} = \rho(1+r) \tag{30}$$

After some simplifications we find

$$\theta = \frac{1}{1 + (1+v)\rho^{-1/\gamma}(1+r)^{-\frac{1}{\gamma}}}[1 - \frac{S}{w} * (1 + \rho^{-1/\gamma}(1+r)^{\frac{\gamma-1}{\gamma}})] \tag{31}$$

This apparently yields a linear relation between θ and S (cf. Feldstein, 1996). It is only apparent as r and w are endogenous and vary with S as well. In the steady state, w, r, k, θ, S are constant per efficiency unit.

General SWF

The solution may be generalized for the general SWF as

$$r = v + \frac{\tilde{\delta}(1+n)(1+g)^{\gamma}}{\rho} \tag{32}$$

We see that, if the number of future generations included rises, the relative weight of the currently retired will dwindle to zero. We already saw that, in the case of $\delta = 0$, we get the golden rule path $r = \nu$. Hence, the more generations we include, the more we converge to the golden rule path. So, as we expand the planning horizon, we approach a 'dynastic' model, in which dynamic inefficiency vanishes. This result is intuitively plausible and conforms to the literature of dynastic models following the Ramsey-Cass-Koopmans model. We notice that the sequence $\{\varphi_i\}$ will usually, but not always, follow a traditional geometric pattern. The analysis holds for a general pattern.

Let us reconsider $\widetilde{\delta} = 1/\sum(\varphi_i(1+g)^{(1-\gamma)i})$. We shall assume that $\varphi_i = \varphi^i(1+n)^i$, which implies that future generations are weighted according to their size. In that case, the denominator is a geometric series with ratio $\varphi(1+n)(1+g)^{1-\gamma}$. In order to have the sum bounded, this ratio has to be smaller than one.

The infinite series adds up to the reciprocal of the ratio, hence

$$\widetilde{\delta} = \varphi(1+n)(1+g)^{1-\gamma} < 1$$

As we assign a weight of one (by normalisation) to the present working generation, it follows that the problem can only be solved if the relative total weight, assigned to all future generations, is smaller than the weight assigned to the currently working generation.

We see that in this case, where the weight is made dependent on n, the equilibrium interest rate will become quadratic in n.

V. THE RESULTING INCOME DISTRIBUTION.

In this economy we distinguish just two groups, the workers and the retired. Their net incomes are the consumption levels $C_{t,1}$ and $C_{t,2}$, respectively. Now we are interested in the mean income of the population and in how it is divided between the two groups. We are especially interested in the inequality of the income distribution according to the definition by Atkinson (1970).

The production per (working) labour efficiency unit is $y = k^\alpha$. Given the fact that for each worker there are $(1+n)^{-1}$ retired, the average income in this society will be

$$\overline{y} = \frac{k^{\alpha}}{1 + (1+n)^{-1}}$$

There is no equal distribution in this society. According to the first order condition for individuals, there holds

$$\rho U'_{t+1,2}(1+r) = U'_{t,1} \tag{33}$$

and accordingly

$$\rho U'_{t,2}(1+r)(1+g)^{-\gamma} = U'_{t,1} \tag{34}$$

This gives for the *benefit ratio* b between the retired and the workers

$$b = \frac{C_{t,2}}{C_{t,1}} = \{\rho(1+r)\}^{-1/\gamma}(1+g) \tag{35}$$

It follows that the average net income is

$$\frac{C_{t,1} + (1+n)^{-1} b * C_{t,1}}{1 + (1+n)^{-1}} = \overline{y} \tag{36}$$

The average utility $\overline{W}$ is equal to

$$\begin{aligned} \overline{W} &= \frac{U(C_{t,1}) + (1+n)^{-1} b^{(1-\gamma)} U(C_{t,1})}{(1 + (1+n)^{-1})} \\ &= U(C_{t,1}) \frac{\{1 + (1+n)^{-1} b^{(1-\gamma)}\}}{(1 + (1+n)^{-1})} \\ &= U(C_{t,1}[\frac{\{1 + (1+n)^{-1} b^{(1-\gamma)}\}}{(1 + (1+n)^{-1})}]^{1/(1-\gamma)}) \end{aligned} \tag{37}$$

The Equal Distribution Equivalent y_{EDE} according to Atkinson is the income which, when given to everybody, would yield the same average welfare as does the present unequal distribution . Hence, we have

$$y_{EDE} = C_{t,1}[\frac{\{1 + (1+n)^{-1}b^{(1-\gamma)}\}}{(1 + (1+n)^{-1})}]^{1/(1-\gamma)} \tag{38}$$

The Atkinson measure of inequality is

$$\begin{aligned} A &= 1 - \frac{y_{EDE}}{\overline{y}} \\ &= 1 - [\frac{\{1 + (1+n)^{-1}b^{(1-\gamma)}\}}{(1 + (1+n)^{-1})}]^{1/(1-\gamma)} / \frac{1 + (1+n)^{-1}b}{1 + (1+n)^{-1}} \end{aligned}$$

where

$$b = \{\rho(1+r)\}^{-1/\gamma}(1+g)$$

The result of this section is that average welfare and the inequality between the two age brackets appear to be determined (among other things) by the population growth rate .

VI. DYNAMICS AND TRANSITION

In this section we will look at the transition dynamics when the growth rate falls from $n^{(0)}$to $n^{(1)}$. In that case we have a new equilibrium, where all endogenous variables have changed from $r^{(0)}$to $r^{(1)}$, etc. The issue now is how the transition will go and, more in particular, whether the transition process will converge to the new equilibrium. We shall assume throughout that the individual citizens will not anticipate on the change but will only react to it. The active role is with the government. The government faces the problem that the old balance in the social security fund no longer holds, and thus the government has to adapt the tax rate. The citizens reacting on that adapt their savings.

Let us assume an initial equilibrium with a capital per efficiency unit of

$$k^{(0)} = S^{(0)}/(1+\nu^{(0)})$$

This equation states that the savings in the previous period are used as the capital of the following period. As the number of efficiency units increases per period at a rate of $\nu^{(0)}$, the savings per efficiency unit have to be spread as capital over more efficiency units in the next period. Let us now assume that in period 1 the growth rate falls to $\nu^{(1)}$. The immediate effect will be that

$$k_1 = S^{(0)}/(1+\nu^{(1)})$$

As $\nu^{(1)} < \nu^{(0)}$, it follows that the workers in period 1 get an unexpected boon. They have more capital to work with than the preceding generations. Consequently, their wages w_1 will be higher than the equilibrium wage $w^{(0)}$, and the interest rate will be lower $r_1 < r^{(0)}$. It follows that the first generation after the fall of the birth rate will get a feeling of euphoria, and that was precisely the feeling of the nineties, although there were also other causes like technological innovations in the upswing of the Kondratiev cycle.

Hence, the workers' consumption in period 1 will be

$$C_{1,1} = \{(1-\theta_1)w_1 - S_1\}A_1 \tag{39}$$

The consumption of the retired will be

$$C_{1,2} = \{\theta_1(1+\nu_1)w_1 + (1+r_1)S_0\}A_{1-1} \tag{40}$$

More generally we get the dynamic equations

$$C_{t,1} = \{(1-\theta_t)w_t(S_{t-1}) - S_t\}A_t \tag{41}$$

and

$$C_{t,2} = \{\theta_t(1+\nu_1)w_t(S_{t-1}) + (1+r_t(S_{t-1}))S_{t-1}\}A_{t-1} \tag{42}$$

Now, assuming the same mechanisms as described in section 3, the workers optimize their savings S_t by solving the first-order condition

$$\rho U'_{t+1,2}(1+r_t) = U'_{t,1} \tag{43}$$

while the government looks for the optimal θ_t by solving

$$-U'_{t,1} + \rho U'_{t+1,2}(1+\nu^{(1)}) + \delta . U'_{t,2}(1+n^{(1)}) = 0 \tag{44}$$

As before, after some substitutions we find

$$(C_{t+1,2}/C_{t,2})^{-\gamma} = \frac{\delta(1+n^{(1)})}{\rho(r_t - \nu^{(1)})} \tag{45}$$

However, on the transition path consumption will not grow at the rate g. Hence, we do not find the elegant equilibrium solution for the interest rate of equation (24). Instead, we find

$$C_{t+1,2} = \{\frac{\delta(1+n^{(1)})}{\rho(r_t - \nu^{(1)})}\}^{-1/\gamma} . C_{t,2} \tag{46}$$

which yields after substitution

$$((1-\theta_t)w_t(S_{t-1}) - S_t)(1+g) - \{\frac{\delta(1+n^{(1)})}{\rho(r_t(S_{t-1}) - \nu^{(1)})}\}^{-1/\gamma}((1-\theta_{t-1})w_t(S_{t-1}) - S_{t-1}) = 0 \tag{47}$$

which is a linear equation in S_t and θ_t.

Similarly, after some substitution the first-order condition yields

$$(1-\theta_t)w_t - S_t - \rho^{-1/\gamma}(\theta_t(1+\nu^{(1)})w_t + (1+r_t)S_t) = 0 \tag{48}$$

which is linear in S_t and θ_t.

Note that the 'coefficients' in these equations depend on S_{t-1}. This gives the dynamics of the system, which is non-linear.

We have simulated the system for various values of the parameters and we have found that it converges rapidly to the new equilibrium.

VII. EVALUATION AND DISCUSSION

It is now time to look at the outcomes of this model when filled in with specific parameter values. There are six basic parameters in the model. The first one, evidently, is population growth n. The time dimension of this model differs from the usual one. An annual population growth rate of 1% is equivalent to a growth rate of $1.01^{35} - 1 = 0.4166$, if we assume that a period (or generation) stands for 35 years. Similarly, a negative growth rate of 1% is equivalent to a negative rate of 1-$(1/1.01^{35}) = -30\%$. This implies that we have to choose our values carefully and that period values have to be translated in annual equivalents. The same considerations hold for the technological growth rate g, the time preference discount rate ρ, and the altruism coefficient δ, if it refers to future generations. The two coefficients which have no time dimension are the capital elasticity α and the relative risk aversion coefficient γ. We take as our basic parameter configuration

$$\begin{aligned} n &= -0.3, 0.0, 0.3 \\ \delta &= 0.4 \\ g &= 0.35 \\ \gamma &= 2. \\ \rho &= 0.35 \\ \alpha &= 0.2 \end{aligned}$$

where we allow n to vary. If we assume a period length of 35 years, the average length until retirement is 17.5 years. Hence, a value of $\rho = 0.35$ corresponds with an annual time discount of slightly less than 6 %. We take the altruism parameter somewhat higher as it refers to the present generation of retired. We explicitly point out that reliable estimates of ρ and δ are not known to us. The same holds for γ, although it is conventional wisdom to set it at about 2. However, Blake (1996) estimates values of between 4 and 8 (see also Drèze, 2000).

The outcomes for this parameter configuration, where we apply three population growth rates, are given in table 1. It is obvious that a number of outcomes are interesting. Here we present only the most interesting ones. First, we are interested in the equilibrium interest rate, which we annualise by taking the 35-root. The second outcome is the capital-output ratio (k/y), where we take into account that capital has no time dimension while output has. For instance, the capital-output ratio calculated for a period of six months is twice the usual k/y ratio calculated on a full year's production. Similarly, in our case we have to multiply the k/y ratio by 35 to make the period k/y ratio comparable to the annual k/y ratio. The savings ratio S/w and the tax rate θ do not pose any dimension problems. The same holds for the funding ratio, which we define as the fraction of the retirement income which stems from own savings (funded income), that is

$$fr = \frac{(1+r)S}{((1+v)\theta w + (1+r)S)} \tag{49}$$

Then we look at the benefit ratio, which we define as

$$b = C_{t,2}/C_{t,1}$$

The last two outcomes deal with average consumption and with the income inequality as defined by Atkinson. Note that this inequality is in fact the 'between-group' inequality in the case where there are only two groups.

For this configuration we find the following outcomes (see table 1).

TABLE 1.
SOME OUTCOMES OF THE MODEL FOR VARYING GROWTH RATES

n	-1.1%	0%	0.75%
interest r	2.6 %	3.6 %	4.3%
Capital/Output ratio k/y	4.64	2.88	2.08
Savings ratio S/w	17%	14 %	12%
tax ratio θ	17.0%	19.1%	19.0 %
funding ratio fr	71%	64%	63%
C_1	0.317	0.287	0.272
C_2	0.220	0.233	0.248
benefit ratio	69.4%	81.2%	91.5%
average welfare $\overline{W}$	-3.975	-3.888	-3.831
income inequality	.032	.011	.002

It is obvious that, with other parameter choices, we would generate different outcomes. Therefore, it does not make sense to do many simulations, the more so as the reader himself can easily compute values for other configurations. Nevertheless, we have found a considerable variation in the outcomes as a reaction to different input values.

Most variables are monotonous functions of most input variables, but this is not always true. Obviously, as we give the explicit formulae for most of the outcomes, we may analyze the derivatives and investigate their signs. We did not do that, but, after a number of numerical simulations, we come to the following qualitative table, where the upward arrow means an increasing relation, whereas a downward arrow indicates a decreasing relationship, and a question mark stands for ambiguity.

TABLE 2.
QUALITATIVE RELATIONSHIPS

	r	k/y	S/w	θ	fr	b.r.	ineq.
n	$\nearrow$	$\searrow$	$\searrow$	$\nearrow$	$\searrow$	$\nearrow$	$\searrow$
δ	$\nearrow$	$\searrow$	$\searrow$	$\nearrow$	$\searrow$	$\nearrow$	?
g	$\nearrow$	$\searrow$	$\searrow$	$\nearrow$	$\nearrow$	$\searrow$	$\nearrow$
γ	$\nearrow$	$\searrow$	$\searrow$	$\nearrow$	$\searrow$	$\nearrow$	$\nearrow$
ρ	$\searrow$	$\nearrow$	$\nearrow$	$\searrow$	$\nearrow$	$\nearrow$	$\nearrow$
α	constant.	$\nearrow$	$\nearrow$	$\searrow$	$\nearrow$	constant.	constant.

The model we described in this paper is obviously still far removed from everyday reality. First, the restriction to a two-period instead of an annual model is unrealistic. In this way we have no room for the possibility that different birth cohorts, e.g., the thirty-years old and the sixty-years old, may have different interests, reflected by different utility functions. The second major shortcoming is the assumption that all people are homogenous. Other points of critique are easily conceived.

Nevertheless, we see this model as a good starting point to get more insight into the pension problem and the problem of intergenerational transfers. The model is not aimed to be normative. It aims at a *description* of reality. It is intended to be a positive theory. As such, even in this very abstract form it is not unpromising. Moreover, unlike most models in this field, it includes the case of (moderately) negative population growth rates, except if δ=0. Negative population growth is or will become the relevant situation in most developed economies for decades to come.

The most interesting point is that the model leads to a unique equilibrium. There appears to be a strict relationship between the values of the input parameters and the resulting output. This is most important when we try to get insight into the effects of changes in the population growth n.

As the values of n, g and α are directly observable, this also implies a strict relationship between δ, γ and ρ on the one hand, and the output parameters on the other hand. By comparing some of the outcomes with comparable observable variables in real life, like the interest rate, the savings rate, etc., we find an indirect way to 'estimate' the values of the underlying unobservable parameters δ, γ and ρ. Obviously, this 'estimation' is crude and lacks the 'finesse' of what econometricians call estimation techniques. However, looking at a rather complicated non-linear model, where we try to generate six or more output parameters by choosing values for three unknown input parameters, it is rather remarkable that we find an intuitively plausible input parameter configuration which succeeds in doing just that. Moreover, if we were to repeat this technique on various real economies, for instance with different demographies, etc., thus generating real 'observations', it would be possible to adapt basic but unknown input parameters of the model to reality, in such a way that the adaptation (in terms of calculated residuals) becomes optimal. However, this is still a long way ahead and beyond the scope of this paper.

It seems premature to draw political consequences from this model, as the level of abstraction is too high. But nevertheless, it raises some questions that are worthwhile thinking about. The first point is that the room for political choices with respect to intergenerational transfers in general, and the pension system in particular, is extremely limited. This holds especially for the benefit ratio and the funding ratio, which cannot be set from outside. Under *ceteris paribus* conditions, this implies that an autonomous fall in the population growth rate, even to the point of becoming firmly negative, must have far-reaching consequences. As we see from tables 1 and 2, a decrease in the growth rate will induce a fall in the interest rate and, consequently, a more capital-intensive way of production. This is of course in line with the fact that the labour supply is reduced. It also follows that the intergenerational transfer rate θ decreases. The funded pension system (or relying on one's own savings) gets relatively more weight. Not unexpectedly, the relative net income position of the working generation increases. What is somewhat unexpected, is that average welfare decreases and income inequality increases if the growth rate falls.

In the light of this model, the severe problem of the ageing societies in Western and Eastern Europe, but also in Japan and China, has to be re-evaluated. The structures where old-age pensions are fixed by law on a pay-as-you-go system are untenable in the long run. In real life those structures are endogenous and depend notably on the growth rate n. If n changes, those structures have to be changed as well, except if the change of the growth rate would be coupled with a change of political preferences as reflected by δ. This is rather unlikely. In the case where n falls, they have to change in the direction of more funding, although, as our model also shows, in all realistic situations a mix between the two systems is the final solution. It follows that it is dangerous to fix social security in the form of intergenerational transfers by law, without realizing that the structure will change, and has to do so, if n or one of the other basic parameters changes, at the cost of reaching a Pareto-non-optimal situation which is not sustained by the behaviour of the parties involved. Ideally, such a law should include a *variable* tariff $\theta(n)$. The same holds for the sensitive benefit ratio. It cannot be fixed by law at, say, 70 %, unless that figure happens to be the result of the system.

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