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Efficient Allocation of Land between Productive Use and Recreational Use

Eduardo L. Giménez[1,2]
Manuel González-Gómez[1]

[1] *Universidade de Vigo,* [2] *Tinbergen Institute*

Tinbergen Institute

The Tinbergen Institute is the institute for economic research of the Erasmus Universiteit Rotterdam, Universiteit van Amsterdam and Vrije Universiteit Amsterdam.

Tinbergen Institute Amsterdam

Keizersgracht 482
1017 EG Amsterdam
The Netherlands
Tel.: +31.(0)20.5513500
Fax: +31.(0)20.5513555

Tinbergen Institute Rotterdam

Burg. Oudlaan 50
3062 PA Rotterdam
The Netherlands
Tel.: +31.(0)10.4088900
Fax: +31.(0)10.4089031

Efficient Allocation of Land between Productive Use and Recreational Use.

An application to Galician case.*

Eduardo L. Giménez
and
Manuel González-Gómez

Universidade de Vigo

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Abstract

In this paper the efficient allocation of natural recreational areas is analysed. Natural recreational areas have the features of public goods. We present the efficient allocation of this non-excludable public good in a rational general equilibrium model with heterogeneous agents. This allows us to deal with the free-rider problem in the provision of the public good. This framework could be considered as a microfoundation of the López, Shah and Altobello (1994) model. In addition we study both the "existence" value and the "use" value of the recreational area in the same setting. A methodological critique is also made of previous empirical literature. It is suggested that our theoretical framework is a suitable starting point for further empirical research. Finally an empirical application for the Galician case is presented. Our results suggest that current allocations of land to natural recreational areas in Galiza are not efficient.

Keywords:

Land Allocation, Efficient Allocation, Natural Recreational Areas, Public Good, Social Planner Problem, Voluntary Contribution Competitive Equilibrium, Use Value, Existence Value. **JEL:** Q21, Q24, Q26, D51, D10.

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Address: Facultade de C.C.Económicas, 36200 Vigo (Galiza), Spain. Fax: 34-986.812401 e-mail: <egimenez@setei.uvigo.es>

1.- Introduction

People like spending time in natural recreational areas. Natural areas offer different valuable benefits (clean air, tranquillity) and facilities for recreation, observation of nature, etc. An interesting economic issue, both for citizens and for public authorities, is the socially optimal quantity of land given over to natural recreational areas. This socially optimal amount of land can vary from country to country and from region to region. It depends on total land available, alternative uses and individual preferences.

This paper is a study of the efficient allocation of natural areas to recreational uses. These have the feature of being a public good. We present the efficient allocation of this non-excludable public good in a static rational general equilibrium framework with heterogeneous agents. This set-up permits us to deal with the free-rider problem on the provision of the public good. To the best of our knowledge this is a novel approach in the literature that addresses this issue. Another innovative feature of our approach is that this setting considers both "existence" value (the valuation of the good *per se*) and "use" value (the valuation of the good in terms of direct consumption).

The model presents two types of agents. A finite number of households and a representative competitive firm. Households consume, supply inputs (labour and land) and enjoy their leisure time in natural recreational areas. There is no accumulation, therefore there are no savings. Households are endowed with land and time. Land has two alternative uses: recreational and productive (e.g. location for industry, agriculture activities, etc.). Time is allocated to labour activities or to leisure in natural areas. Households are heterogeneous both in respect of land endowment and preferences. A household derives welfare from the consumption of both the private good and the recreational good. The private good is produced by a competitive firm that utilises as inputs labour and "qualified" land (i.e., land "qualified" by the capital located there). The recreational good is the output of an individual subjective production function. Its inputs are the size of the natural areas and the leisure time spent on visits there. So the study of both "existence" value and "use" value can be integrated within the same approach.

The social planner problem and a voluntary contribution competitive equilibrium are examined in the same setting. First we analyse the social planner problem. The Pareto-efficient allocations are found. Then we present the voluntary contribution equilibrium, whereby households voluntarily contribute land to natural areas. Since the valuation of the recreational good differs among agents, some of them do not contribute at all. The *free-rider* problem arises at this point. It is well known that in the presence of this externality, competitive allocations are not Pareto-efficient (see Jacques Laffont, 1988).

Two strands of the literature have analysed the land allocation problem. On the one hand, a number of authors have studied the optimal depletion rate of natural resources and its determinants.[1] The second strand analyses the efficient allocation between land dedicated to productive activities and land dedicated to natural areas.[2] The present paper differs from previous literature in that it is based on a rational general equilibrium framework with heterogeneous agents.[3] This approach to modelling could be viewed in terms of a microfoundation of the López, Shah and Altobello (1994) model. We overcome the shortcomings of both the *ad-hoc* social planner function (with the implicit assumption of homogeneous agents) and the only "existence" value is considered for natural recreational areas.

The closest work to the one presented here is by Anas (1988). He presents a three-agent general equilibrium model to obtain socially optimal allocations, whereas our model is a two-agent economy with a voluntary contribution of the public good. Anas' third agent, the Public Lands Administration (PLA), is an agency authorised by the central government to supply and price the naturally preserved environment as a public good. This government agency has asymmetric information concerning households preferences in respect of the natural areas. Thus a revealed mechanism is proposed and a second-best solution is found. However households are assumed homogeneous, so the *free-rider* problem disappears. We differ on this point and also on the type of equilibrium found.

The contribution of the present paper is threefold. Firstly, both Pareto-efficient allocations and the optimal competitive allocations are studied in a unified framework. This was not possible in previous literature where an *ad hoc* social planner problem was assumed. Secondly, the public good problem can be addressed due to heterogeneity in households preferences concerning the recreational good. All previous literature considers the social planner function as a kind of *ad hoc* representative agent utility function, so the essence of the public good disappears. Thirdly, this individual rational decision-making set-up makes possible a study of the "use" value of natural recreational areas, as well as the "existence" value. Agents can choose to enjoy their time there deriving welfare.[4] Only "existence" value has been considered

[1]See, for example, Anthony C. Fisher, John V. Krutilla and Charles J. Cicchetti (1972); Simeon K. Ehui, Thomas W. Hertel and Paul V. Preckel (1990); Bruce A. Larson and Daniel W. Bromley (1990); Lars J. Olson (1992); Scott Barrett (1992); Edward B. Barbier (1994); Edward B. Barbier and Joanne C. Burgess (1997); or Bernardo Mueller (1997).

[2]See, for example, Alex Anas (1988); Kenneth E. McConnell (1989); A. Myrick Freeman III (1993, Ch.13); Rigoberto A. López, Farhed A. Shah and Marilyn A. Altobello (1994); or Robert T. Deacon (1995).

[3]Some authors such as Giancarlo Marini and Pasquale Scaramozzino (1995), Peter Burton (1996) and Lars J. Olson and Keith C.Knapp (1997) consider heterogeneity between two types of agents. Marini *et al* and Olson *et al* develop a dynamic overlapping generations model, whereas Burton utilises a static model with forestry industry and environmentalists. In both cases, nevertheless, homogeneity within each group exists.

[4]Our point is to focus on a deeper choice. Agents derive welfare from consumption and from leisure time spent in a recreational area (see Gary Becker, 1965, for a study of the complementarity of time spent on the consumption of goods). Thus land gives direct utility to agents through their choice of leisure ("use value", i.e., spending time in natural recreational areas) as opposed to their choice of no-leisure (i.e., work). This "use" value may affect the allocation of land to recreational

in the theoretical literature to date. However this approach could be considered as a weak theory of pricing since "only for a society with a high degree of ecological consciousness it is guaranteed that a positive amount of land is permanently devoted to recreational uses" (Santiago Rubio and Renan-U. Goetz, 1997, p.6).[5]

A consequence of this framework is to present a suitable starting point for empirical research. In this sense some methodological shortcomings of all previous empirical literature on this topic can be overcome. Although the theoretical background is the *ad-hoc* social planner function, the empirical literature takes data from real world competitive allocations.[6] This sheds doubts on the validity of these empirical analyses and any results found there.

A second part of our paper presents an empirical application of the theoretical framework. We analyse the efficiency of present allocations of land between productive use and natural recreational use in Galiza (a region in the northwest of Spain). Given that only aggregate data is available, it is assumed throughout our empirical research that individuals are modelled by a representative agent. Hence, in equilibrium, all individuals have the same strategy, i.e. to contribute to the public good with the same amount of land. Although the homogeneous agents assumption is a strong hypothesis (as the public good problem seems to disappear), it provides a first exploratory framework for an empirical study.

The empirical study is carried out in the following steps. Firstly, some functional forms are assumed. Then, from the social planner problem, efficient allocations are obtained, which depend on a set of functional form parameters. Secondly, the parameters for the Galician case are calibrated. Given that the social planner parameters are not known (if so we could parallel model outcome efficient allocations and current allocations), our strategy is to make use of data at the macroeconomic and the microeconomic levels. Macroeconomic data are taken to calibrate the parameters that would support the current allocations which are assumed as efficient. Parameters at the microeconomic level are estimated via the equivalent variation on the basis of willingness-to-pay data obtained in a survey carried out at Monte Aloia natural recreational area. Implicitly we are assuming that this survey is a representative sample of Galician citizens. Both sets of parameters are compared. Since these sets are far from being closed, it is suggested that current recreational land allocation in Galiza is not Pareto-efficient.

This work develops through the following sections. In Section 2 a survey of literature is made. In Section 3 we present a rational general equilibrium model with heterogeneous agents, where the natural recreational area is a non-excludable pub-

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use, in a way similar to other variables (e.g., productivity, interest rate, wages, etc.).

[5]However, it should be borne in mind that there are benefits other than recreational ones. Therefore a public decision to preserve recreational areas should take these other benefits into account.

[6]Few theoretical studies include empirical application. Exceptions include Fisher, Krutilla and Cicchetti (1972, Sec. III), López, Shah and Altobello (1994, Sec. III), or Barbier and Burgess (1997, Sec. IV) among others.

lic good. Social planner problem and competitive equilibrium problem are shown. Furthermore, a methodological critique of standard empirical research is made. In Section 4 an empirical application for Galician case is carried out. Finally, Section 5 summarises conclusions and indicates further research.

2.- A review of the literature on efficient land allocation.

Main bulk of literature on efficient land allocation considers that only "existence" of natural resources brings positive welfare. Below we briefly review both a static and a dynamic modelization.

A) Static models

We focus on McConnell (1989) and López *et al* (1994). Let L be the fixed area of land to be allocated to agricultural land L_a and urban land L_u: $L = L_a + L_u$. There are two kinds of benefits: agricultural benefits and urban benefits. Agricultural benefits to society come from production benefits PB (i.e., aggregate rents from land in agriculture) and amenity benefits AB, which represents the monetary value placed on the total aesthetic and other non-market benefits accruing to farmland net of any negative externalities resulting from the practice of agriculture (water pollution, generation of odours, ...). Thus the total social benefits from land are $SB = PB + AB$. Urban benefits UB, derived from urban activities, are measured by aggregate rents accruing from commercial and residential uses of land, net of negative externalities (pollution, noise or traffic congestion).

PB, AB and UB are function of both land use and population P of the area. That is, $PB = PB(L_a, P)$; $AB = AB(L_a, P)$; $UB = UB(L_u, P)$. For the sake of modelling tractability, it is assumed that these functions are quasi-concave, and thus each of the partials PB_1, PB_2, AB_1, AB_2, UB_1 and UB_2 are positive[7] whereas PB_{11} and UB_{11} are negative (i.e., increasing at a diminishing rate). This follows from the neoclassical theory of production when land is viewed as a productive input. An increase in L_a should rose AB at diminishing rate (i.e., $AB_{11} < 0$) which follows from the theory of diminishing marginal utility as applied to amenity benefits from agricultural land.

If population is held constant the (socially) efficient problem is as follows:

$$\begin{aligned}\max_{L_a, L_u} \quad & SB(L_a, P) + UB(L_u, P) = [PB(L_a, P) + \theta AB(L_a, P)] + UB(L_u, P) \\ s.t. \quad & L = L_a + L_u \\ & \text{given } P \text{ constant}\end{aligned}$$

[7]Increased demand for housing and other services cause UB to increase, whilst the increase in AB occurs because amenity benefits have the features of a public good, so that the higher the population, the higher the total amenity benefits. The increase in PB is due to increase demand for agricultural products and productivity.

where θ is an indicator function such that when $\theta = 0$ then land allocation does not reflect any amenity benefits, and when $\theta = 1$ then amenity benefits are fully recognised.

The first order conditions for the optimum allocation are given by:

$$\begin{aligned} \frac{\partial PB(L_a, P)}{\partial L_a} + \theta \frac{\partial AB(L_a, P)}{\partial L_a} - r &\equiv 0 \\ \frac{\partial UB(L_u, P)}{\partial L_u} - r &\equiv 0 \\ r[L_a + L_u - L] &\equiv 0 \end{aligned}$$

where r is the Lagrangian multiplier. The López *et al* examines two cases:

1. *Amenity benefits do not affect land allocation* ($\theta = 0$). In this case the first order condition is changed to $\frac{\partial PB(L_a,P)}{\partial L_a} = r$. That is, the value of the marginal product of agricultural land equals the shadow price of land which, under competitive conditions, should equal the market rate of return for land. From this first identity a demand function for agricultural land is found and from the second, a supply function is obtained. Thus:

$$\begin{aligned} r &= D(L_a, P) \\ r &= S(L_a, P). \end{aligned}$$

   The intersection of demand and supply gives the solution L_a^M and r^M. (See Figure 1.)

2. *Amenity benefits are fully recognised* ($\theta = 1$). Taking into account the marginal amenity benefits at first-order condition, the downward-slopping function for L_a is given by $\frac{\partial SB(L_a,P)}{\partial L_a} = r$. As marginal amenity benefits are positive (i.e., $\frac{\partial AB(L_a,P)}{\partial L_a} > 0$), then the marginal social benefit curve lies at the right of the market demand curve for agricultural land. The demand and supply functions are now given by:

$$\begin{aligned} r &= MSB(L_a, P) \\ r &= S(L_a, P). \end{aligned}$$

   This solution gives L_a^*, the socially desirable values for agricultural land, and r^*, which can be interpreted as the social rate of return of land. (See Figure 1.)

At this point a critic arises. Since the efficient social planner problem (though not necessarily Pareto-efficient one) is an *ad-hoc* functional specification, it can not be established whether the market allocations are efficient or not. In other words, the implicit assumption is that competitive allocations are efficient as stated by the first theorem of welfare. This would not hold, however, in the presence of an externality –for example for a public good. In this case it is not clear that shadow price of land

r equals the marginal product of land in agriculture. Moreover doubts spring on any empirical research that makes use of real data (i.e., competitive market outcome) to estimate or calibrate under this framework, as in the case with e.g., López *et al* (1994, Section III).

B) Dynamic models

Usually these models try to calculate an optimal depletion rate of a natural resource (e.g., the Amazoon jungle) hence dynamics are necessary. Two papers by Barrett (1992) and Barbier (1994) are reviewed. These show an eclectic model from those presented by Fisher, Krutilla and Cicchetti (1972) and Krautkraemer (1985). The assumption underline Barrett's and Barbier's studies is that virgin rain forest are exploited both for their tropical hardwoods (a flow) and for agricultural land (a stock). Whilst incremental deforestation is not necessary for production, the consumption obtained by the incremental development is likely to be valued more highly when per capita consumption is very low –as it is in tropical rain forest countries– than when it is high –as it is in industrialised countries.

It is assumed a fixed total volume of a non-capable of regeneration natural resource $S > 0$ (stock of land, water stored in wetlands, etc. expressed as hectares of virgin rain forest). Let S_t be the stock of the resource at t. Let D_t be the amount of drained wetland resource (measured in hectares of managed forest or agricultural land). Finally, let $r_t \geq 0$ be the rate at which this stock is transformed or developed. Thus the dynamics of the stock are $\dot{S}_t = -r_t$

Societal consumption takes two forms –the depletion natural resource itself and the stock of the resource in its transformed or developed state: $D_t = D_0 + \int_0^t r_\tau d\tau$ and, as the total quantity of resource S is fixed $D_t - D_0 = S - S_t$. The transformed resource is given by a production function: $F(D_t) = F(S + D_0 - S_t) = f(S_t)$, where $F_D > 0$, $F_{DD} < 0$, $f_S < 0$ and $f_{SS} > 0$.

With c_t representing total consumption, and assuming that all output is consumed, then goods market equilibrium yields $c_t = \sigma e^{\gamma t} r_t + f(S_t)e^{\omega t}$, where γ is the rate of technical progress in the frontier sector (constant but not necessarily positive), ω is the rate of technical progress in the developed sector (also constant but not necessarily positive), and σ is a constant that converts the rate of depletion in the initial period into a consumption rate.[8] Population is supposed to be constant. Agent's welfare comes from direct consumption of goods and from the existence of the natural resource. Social Planner optimize agents' welfare through instantaneous social utility function $U(c_t, S_t)$. They assume $U_c, U_S > 0$; $U_{cc} \leq 0$; $U_{SS} < 0$; $U_{cS}, U_{Sc} = 0$; and $U_c(0, \cdot) = \infty$. The social planner's problem is

$$max_{\{r_t\}_{t=0}^{\infty}} \qquad \int_0^{\infty} U(c_t, S_t) e^{-\delta t} dt$$

[8]Fisher *et al* assume $\sigma = 0$, $f(S_t) > 0$ for $S_t < S + D_0$, whilst Krautkraemer assumes $\sigma = 1$ and $f(S_t) = 0$ for all S_t, $0 \leq S_t \leq S$ (i.e., frontier development is necessary for consumption to be positive).

$$
\text{s.t.} \quad \begin{aligned} \dot{S}_t &= -r_t \\ c_t &= \sigma e^{\gamma t} r_t + f(S_t) e^{\omega t} \\ &\text{given } \delta > 0,\ \gamma,\ \omega,\ \sigma \text{ and } S_0 = S. \end{aligned}
$$

It should be pointed out that here the existence of pristine land provides direct utility to agents, even if unutilised. Hence, in the case that any quantity of natural resource reports no welfare to agents (i.e., $U(c_t, S_t) = U(c_t, 0)$ for all $c_t \geq 0$), then all the land available would be dedicated to productive uses, i.e., $D_t = S$ for all t. Thus it can be concluded that results are very fragile, given that direct welfare derived from the "existence" use of natural resources is assumed completely *ad-hoc*.

3.- The model

A theoretical general equilibrium model with a public good and heterogeneous agents in both preferences and land endowments is presented.[9] This is done within a rational maximisation agent framework, where agents can choose to "enjoy" their free time in areas earmarked for recreational use, i.e., "use" value. This approach marks a departure from previous literature where only "existence" value is considered. For the sake of paralleling our results with previous literature, the closest microfoundation version of López, Shah and Altobello (1994) model is constructed.

3.1.- The agents

There are two types of agents in the economy: H households and a number of firms (whether timber, agricultural or industrial). Households are endowed with both land and time. They consume, supply inputs (labour and land) and enjoy leisure time in natural areas. There is no accumulation, so there are no savings. Households are the owners of the firms. In the interest of simplicity we assume that the competitive firms produce any output (whether agricultural or industrial) utilising as inputs labour and "qualified land" (i.e., land qualified by the capital located on it). Hence land is considered to be homogeneous, in the sense that any piece has the same competing uses: production uses and recreational uses.[10][11]

[9]This differs from a similar work by Anas (1988) where a homogeneous agent framework is considered.

[10]The realism of the model would be improved if it were possible to consider heterogeneous land. However a location problem is present. See Burton (1996) and Dean M. Hanink and Robert G. Cromley (1998) for two possible approaches.

[11]A further model could be constructed on the basis of four kind of agent exist, both rural and urban households and firms. Urban households work at urban firms and spend their leisure time in recreational areas. The urban firms' output could be simplified by assuming that labour is the only required input –or stock of capital could be assumed to be fixed. Rural households, on the other hand, owns the land. They hire both labour and land to rural firms, and also spent their leisure time in recreational areas. Agents, whether urban or rural can be assumed to have the same utility functions for both rural and urban outputs, but it could be assumed that their preferences

The household problem.

Households have endowments of time and land. Each household h has T units of time (i.e., a year, or any other delimited period) that is allocated between working time at firms n^h and leisure time l^h.[12] Households spend their leisure time travelling and in recreational areas. Thus l^h could be divided into λ_i^h, the leisure time dedicated by agent h to make the visit i to recreational area. Given that agent h makes V^h visits per year, then $l^h = \sum_{i=1}^{V^h} \lambda_i^h$. Finally $N = \sum_{h=1}^{H} n^h$ represents the total number of hours of working time in firms.

Household h is the owner of a ρ^h share of the total amount of land L (note that $\sum_{h=1}^{H} \rho^h = 1$). She supplies either qualified land L_u^h (i.e., productive land) or as an input L_a^h to produce the non-excludable recreational area. Hence the total amount of land L dedicated to private production use is $L_u = \sum_{h=1}^{H} L_u^h$, whereas that given over to produce the public good is $L_a = \sum_{h=1}^{H} L_a^h$.

Households also derive welfare from the consumption of the private good c produced on qualified land, and from the consumption of a recreational good g "produced" when time is spent at a recreational areas. Let us assume that household h's preferences can be represented by a utility function for both goods:[13][14] $U^h(c^h, g^h)$.

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in terms of the public good are different. Given that agents are heterogeneous and that a market for the public good does not exist, allocations will not be Pareto-efficient. This is explained by the fact that whilst urban households derive welfare from the existence of recreational areas they have no choice on terms of its allocation –since it belongs to Rural households. A version of the Grooves mechanism could be applied in this case to achieve a socially optimal allocation of land uses. (See Burton, 1996.)

[12] Here we also differ from Anas (1988) where supply of labour is inelastic, even though leisure time is needed to enjoy recreational areas. Perhaps it was assumed because leisure time is a small proportion of total available time, T.

[13] This utility function is close to that of Anas (1988) in a homogeneous agents set-up: $\mathcal{U}(c, L_H, VL_p, L_p)$. The utility function arguments are the following: the quantity of a composite commodity is c; L_H^h is the (aggregate) quantity of the consumer's private land for housing; V is the total number of recreational visits to the natural environment; and total recreational enjoyment is VL_p, where L_p is the level of environmental quality in the preserved lands. In this specification even if the consumer does not recreate (i.e., $V^h = 0$), the natural environment still yields utility in the form of both option and existence value. Here we assume heterogeneity in agents and do not consider private land for housing (i.e., $L_H^h = 0$). Hence $U^h(c^h, g^h) \equiv \mathcal{U}(c^h, 0, V^h L_p, L_p)$, where g^h depends on the number of visits to recreational areas by agent h, given by leisure time spent at them, l^h, and the quality of the natural recreational area, L_p.

[14] In fact a (somewhat strong) initial assumption could be made concerning individuals. They could be assumed to allocate their wealth to goods produced by natural areas (agricultural goods and recreational activities) and by non-natural areas (e.g., industrial goods). Likewise, they could allocate time to the production of these two kinds of good, and the remaining time to leisure (i.e., to visit the recreational areas).

A quasilinear utility function for goods not produced in natural areas c_{NN} could also be assumed, given that agricultural goods c_N and the recreational good g represent a small part of the total expenditure: $\mathcal{U}(c_{NN}, c_N, g) = U(c_N, g) + c_{NN}$. This is similar to the López *et al* (1994) model. They present the following objective function $(PB + \theta AB) + UB$, where PB is agricultural goods production and AB are the amenity benefits at the recreational areas. So $PB + \theta AB = U(c_N, g)$. Finally, UB could be considered as all the remaining goods not produced in natural areas, c_{NN}.

In the interest of modelling simplicity we will assume that this is an increasing and strictly concave function (i.e., U_c^h, U_g^h are positive, and U_{cc}^h, U_{gg}^h are negative).

The amount of recreational good "consumed" g^h is a production function that depends on the proportion of land dedicated to recreational use L_p (i.e., natural recreational areas) and the time spent there, $l^h - d^h$. Since in this paper transportation costs are ignored, then $l^h - d^h = l^h$. Individuals have the same utility from visiting large recreational areas fewer times, as from visiting smaller areas on more occasions. Thus there is a degree of substitution between large spaces and leisure time spent at recreational areas. Hence $g_{L_p}^h$, g_l^h are both positive. The natural recreational area L_p has the features of a public good. It is produced using non-productive land according to a non-increasing returns technology characterised by $L_p = G(L_a)$, with $G' > 0$ and $G'' \leq 0$.[15] Here the input land L_a is taken to be positive.[16]

Household wealth comes from the payments for labour (real wages $\frac{w}{P}$) and for qualified land hired to firms (at a real interest rate of $\frac{r}{P}$). Agent h's restrictions are the following:

$$
\begin{aligned}
c^h &= \frac{w}{P} n^h + \frac{r}{P} k L_u^h && (1)\\
n^h &= T - l^h && (2)\\
g^h &= g^h(l^h, L_p) && (3)\\
L_a^h + L_u^h &= \rho^h L && (4)\\
L_p &= G\left(L_a^h + \sum_{j \neq h} L_a^j\right) && (5)\\
L_a^h &\geq 0 && (6)
\end{aligned}
$$

where k represents the capital on qualified productive land L_u (that is to say, the aggregate stock of capital is $\mathcal{K} = kL_u$). We differ slightly from public goods literature since agents use their wealth only to consume the private good.[17]

The interpretation in our paper is that the production of market goods is global. Then $PB+UB$ can be considered equivalent to the utility derived from consumption of any produced good c and AB as equivalent to the utility from the public good g, thus $(PB + UB) + \theta AB \equiv U(c, g)$.

[15] In Anas (1988) the environmental quality L_p is measured by a homogeneous function of the first degree $\mathcal{G}(V, L_a)$, where $V = \sum_{h=1}^{H} V^h$ is the total number of recreational visits per year. Some assumptions are made as follows: $\mathcal{G}(V, 0) = 0$, $\mathcal{G}_V < 0$, $\mathcal{G}_{L_a} > 0$, $\mathcal{G}_{VV}, \mathcal{G}_{L_a L_a} < 0$. This modelling allows for the possibility of congestion of the public good. Here we consider an uncongested environmental quality, i.e., $G(L_p) \equiv \mathcal{G}(0, L_p)$.

[16] In order to produce a public good in the form of a "recreational area", probably some input from private good whould be required (e.g., forest clearing, fire system protection, etc.) and possibly labour.

[17] Household budget restrictions could be viewed in another way as follows. There are two goods, the consumption good and the recreational good. Thus there exists an opportunity cost in

The firms problem.

There exist a number of perfectly competitive firms. Given that constant returns of scale for technology are assumed, a single aggregate firm could be considered.[18] This representative firm maximises its (nominal) returns by demanding labour N and qualified land to produce both agricultural (also forest goods) and industrial goods:

$$\max_{L_u,N} \Pi\left(kL_u, N\right) \quad = \quad PF\left(kL_u, N\right) - wN - rkL_u \tag{8}$$

Since perfectly competitive conditions and constant returns of scale are assumed, then the firm's returns will be zero: $\Pi\left(kL_u, N\right) = 0$.

3.2.- Social planner Pareto-efficient problem and competitive equilibrium.

3.2.1.- The social planner Pareto-efficient problem.

The social planner maximises the agents' weighted welfare function subject to feasible clearing market conditions. That is to say, consumption of goods equals production, and the land dedicated to recreational use L_a plus that dedicated to productive activities L_u are equal to L, the total amount of land available.

$$\begin{array}{rrcl} \max_{\left\{c^h,l^h\right\}_{h=1}^{H},L_a,L_u,L_p,Y} & \multicolumn{3}{l}{\sum_{h=1}^{H} \alpha_h U^h\left(c^h, g^h(l^h, L_p)\right)} \\ \text{s.t.} & \sum_{h=1}^{H} c^h & = & Y \\ & n^h + l^h & = & T \quad \text{for } h = 1, ...H \\ & L_a + L_u & = & L \\ & Y & = & F(L_u, \sum_{h=1}^{H} n^h) \\ & L_p & = & G(L_a) \end{array}$$

where α_h is the weighting assigned to household h by the planner. Given our desire to parallel results with those of López *et al*, the social planner problem is modelled

monetary terms. So (2) and (4) could be substituted back into budget restriction (1)

$$c^h = \frac{w}{P}(T - l^h) + \frac{r}{P}k\rho^h(L - L_a^h).$$

This would imply that:

$$c^h + \left[\frac{w}{P}l^h + \frac{r}{P}k\rho^h L_a^h\right] = \frac{w}{P}T + \frac{r}{P}k\rho^h L$$

Given that the "production" of the recreational good requires leisure and recreational areas, both factors are remunerated on their own opportunity cost basis. If perfect competition and constant returns of scale are also assumed, then $g^h = \left[\frac{w}{P}l^h + \frac{r}{P}k\rho^h L_a^h\right]$. Hence

$$c^h + g^h \quad = \quad \frac{w}{P}T + \frac{r}{P}k\rho^h L \tag{7}$$

[18] Price-acceptant and without any capacity to affect prices.

as closely as possible on their model. Hence the social planner maximises each agents' welfare in consumption c^h, leisure time l^h, productive land L_u and natural recreational land L_a. Following several substitutions the social problem can be represented as:[19]

$$\begin{aligned}\max_{\left\{c^h,l^h\right\}_{h=1}^{H},L_a,L_u} \quad & \sum_{h=1}^{H}\alpha_h U^h\left(c^h, g^h(G(L_a),l^h)\right)\\ s.t. \quad & L_a+L_u=L\\ & \sum_{h=1}^{H}c^h = F\left(L_u,\sum_{h=1}^{H}(T-l^h)\right)\end{aligned}$$

The first order conditions are:

$$\sum_{h=1}^{H}\alpha_h\frac{\partial U^h(c^h,g^h)}{\partial g}\frac{\partial g^h(l^h,L_p)}{\partial L_p}G'(L_a)-r \;=\; 0 \tag{9}$$

$$\alpha_h\frac{\partial U^h(c^h,g^h)}{\partial c}\frac{\partial F(L_u,N)}{\partial L_u}-r \;=\; 0 \quad \text{for } h=1,...H \tag{10}$$

$$-\frac{\partial U^h(c^h,g^h)}{\partial c}\frac{\partial F(L_u,N)}{\partial N}+\frac{\partial U^h(c^h,g^h)}{\partial g}\frac{\partial g^h(l^h,L_p)}{\partial l} \;=\; 0 \quad \text{for } h=1,...H \tag{11}$$

$$r[L_a+L_u-L] \;=\; 0 \tag{12}$$

where r is the Lagrangian multiplier, which is positive as long as the restriction (12) binds. Equations (9) and (10) are a version of the Bowen-Lindahl-Samuelson condition:

$$\sum_{h=1}^{H}\frac{\frac{\partial U^h(c^h,g^h)}{\partial g}\frac{\partial g^h(l^h,L_p)}{\partial L_p}}{\frac{\partial U^h(c^h,g^h)}{\partial c}} = \frac{\frac{\partial F(L_u,N)}{\partial L_u}}{G'(L_a)} \tag{13}$$

That is to say, the sum over all consumers of marginal rates of substitution between the public good and the private good must be equal to the marginal rate of transformation in production between these two goods. The other equilibrium condition is equation (11), where the marginal productivity of labour equals the marginal productivity of leisure for any agent h.

[19] Observe here that if we set the egalitarian (social) problem, i.e. $\alpha_h = 1$ for all h, and if the utility function is assumed to be quasilinear $U^h(c^h,g^h) = c^h + \theta u^h(g^h)$, then we can obtain a microfoundated version of López et al (1994) social planner problem:

$$\begin{aligned}\max_{\{l^h\}_{h=1}^{H},L_a,L_u} \quad & F\left(L_u,\sum_{h=1}^{H}(T-l^h)\right)+\theta\sum_{h=1}^{H}u^h\left(g^h(G(L_a),l^h)\right)\\ s.t. \quad & L_a+L_u=L\end{aligned}$$

where the first term represents production benefits and urban benefits $PB+UB$, and the second part is amenity benefits AB, where θ is the indicator function that represents the social recognition of this benefit.

Finally, substituting of L_u for equation (12) in the remaining equations, and similarly leisure for (11) in equations (9) and (10), we have what López *et al* call a "demand function for land in agricultural sector", equation (9), and a "supply function of land in agriculture", equation (10). That is,

$$\begin{aligned} r &= D(L_a) \\ r &= S(L_a) \end{aligned}$$

The Pareto-efficient allocation of recreational land $\hat{L}_a$ and its shadow price $\hat{r}$ is obtained from the intersection of these functions.

3.2.2.- A voluntary-contribution equilibrium.

The household h's problem.

Household h maximises the utility $U^h(c^h, g^h)$ subject to her restrictions (1)-(6).

$$\begin{array}{lrcl} \max_{c^h, l^h, L_a^h, L_u^h, L_p} & U^h\left(c^h, g^h(l^h, L_p)\right) & & \\ \text{s.t.} & c^h & = & \frac{w}{P} n^h + \frac{r}{P} k L_u^h \\ & n^h + l^h & = & T \\ & L_a^h + L_u^h & = & \rho^h L \\ & L_a^h & \geq & 0 \\ & L_p & = & G\left(L_a^h + \sum_{j \neq h} L_a^j\right) \\ \text{given} & L_a^j & & \text{for } j \neq h \end{array}$$

Again, with a view to paralleling the López *et al* results, the competitive problem is constructed as closely as possible to the competitive version of their model. Hence, agent h maximises leisure time l^h and land dedicated both to productive use L_u^h and recreational use L_a^h. Following several substitutions, the problem can be presented as:

$$\begin{array}{ll} \max_{l^h, L_a^h, L_u^h} & U\left(\frac{w}{P}(T - l^h) + \frac{r}{P} k L_u^h, g^h\left(l^h, G(L_a^h + \sum_{j \neq h} L_a^j)\right)\right) \\ \text{s.t.} & \begin{array}{rcl} L_a^h + L_u^h & = & \rho^h L \\ L_a^h & \geq & 0 \\ L_a^j & & \text{for } j \neq h \end{array} \\ \text{given} & \end{array}$$

The first order conditions are as follows:

$$L_a^h \left[\frac{\partial U^h(c^h, g^h)}{\partial g} \frac{\partial g^h(l^h, L_p)}{\partial L_p} G'(L_a) - \lambda^h\right] = 0 \tag{14}$$

$$\frac{\partial U^h(c^h, g^h)}{\partial c} \frac{r}{P} k - \lambda^h = 0 \tag{15}$$

$$-\frac{\partial U^h(c^h, g^h)}{\partial c} \frac{w}{P} + \frac{\partial U^h(c^h, g^h)}{\partial g} \frac{\partial g(l^h, L_p)}{\partial l} = 0 \tag{16}$$

$$\lambda^h [L_a^h + L_u^h - \rho^h L] = 0 \tag{17}$$

where $\lambda^h \geq 0$ is the multiplier. We thus obtain the supply functions for labour, productive land and natural resource land:

$$\begin{aligned} L_a^s &= S_a\left(\frac{r}{P}, \frac{w}{P}\right) \\ L_u^s &= S_u\left(\frac{r}{P}, \frac{w}{P}\right) \\ (T - l^s) &= S_n\left(\frac{r}{P}, \frac{w}{P}\right) \end{aligned}$$

The representative firm's problem.
The representative firm maximisation of profits are given by (8),

$$\max_{\{L_u^h, n^h\}_{i+1}^h, Y} \quad Y \quad -\frac{w}{P}\sum_{h=1}^{H} n^h - \frac{r}{P}k\sum_{h=1}^{H} L_u^h \qquad (18)$$
$$Y = F\left(kL_u, N\right)$$

The demand functions for labour and productive land are obtained as follows:

$$\begin{aligned} \frac{\partial F(L_u, N)}{\partial N} &= \frac{w}{P} \\ \frac{\partial F(L_u, N)}{\partial L_u} &= \frac{r}{P}k \end{aligned}$$

Voluntary-contribution equilibrium

A voluntary-contribution equilibrium E is a set of goods, time and land allocations and factor prices $\left\{\left\{c^{*h}, n^{*h}, l^{*h}, L_a^{*h}, L_u^{*h}\right\}_{h=1}^{H}, L_p^*, \{\left(\frac{w}{P}\right)^*, \left(\frac{r}{P}\right)^*\}\right\}$ such that: [1] for each agent, $\left\{\left\{c^{*h}, n^{*h}, l^{*h}, L_a^{*h}, L_u^{*h}\right\} L_p^*\right\}$ is a solution to agent h's maximisation problem, given both other agents' contributions of natural recreational land L_a^j with $j \neq h$, and the equilibrium prices $\{\frac{w}{P}, \frac{r}{P}\}$; and [2] markets clear: $\sum_{h=1}^{H} c^h = F(kL_u, N)$, $n^h + l^h = T$ for all h, and $\sum_{h=1}^{H}\left(L_a^h + L_u^h\right) = \sum_{h=1}^{H}\rho^h L = L$.

Thus, for each agent $h = 1, ..., H$, (16) must hold in equilibrium

$$\begin{aligned} \frac{\partial U^h(c^h, g^h)}{\partial g}\frac{\partial g^h(l^h, L_p)}{\partial l} &= \frac{\partial U^h(c^h, g^h)}{\partial c}\frac{\partial F(L_u, N)}{\partial N} \\ \sum_{h=1}^{H}\left(L_a^h + L_u^h\right) &= \sum_{h=1}^{H}\rho^h L = L \end{aligned}$$

We also find that at least a number of agents J, with $1 \leq J \leq I$ contribute to the production of the public good, i.e. $L_a^h > 0$ in equation (14). This contribution verifies that the marginal cost of the public good measured in terms of the private good (and taking the other agents' voluntary contributions as given), i.e. $\frac{\partial F(L_u,N)}{\partial L_u}/G'(L_a)$, is equal to her marginal rate of substitution

$$\frac{\frac{\partial U^j(c^j,g^j)}{\partial g}\frac{\partial g^j(l^j,L_p)}{\partial L_p}}{\frac{\partial U^j(c^j,g^j)}{\partial c}} = \frac{\frac{\partial F(L_u,N)}{\partial L_u}}{G'(L_a)}$$

for all $j = 1, ...J$ such that $L_a^j > 0$. The remainder of the $k = J+1, ...I$ *free-rider* agents do not contribute any land to public good production, thus $L_a^k = 0$ in equation (14). For them, this condition is verified with inequality, given that the marginal cost of the public good measured in terms of the private good (and taking the other agents' voluntary contributions as given) is greater than his marginal rate of substitution.

It can be observed that these conditions are clearly different from the Bowen-Lindahl-Samuelson condition represented by (13). No agent considers the benefits to other agents of the output obtained by its own contribution. Given that this is true for each consumer, then consumers as a group contribute less than the amount desirable for Pareto optimality (i.e., $L_a^* < \hat{L}_a$).

Some remarks on other equilibrium

Anas (1988) presents a three-agent general equilibrium model to obtain socially optimal allocations, instead of our two-agent economy with a voluntary contribution of the public good. His third agent, the Public Lands Administration (PLA), is an agency authorised by the central government to supply and price the naturally preserved environment as a public good.

The decentralised problem is close to the problem described here. The firm's problem is the same as in our study. A representative household h maximises consumption c^h, the number of visits V^h, the labour supplied n^h and the environmental quality L_p, subject to the budget constraint

$$tV^h + \sigma L_p + c^h = \frac{w}{P}n^h + \frac{r}{P}L$$

where t is the real fee consumer h pays for a ticket to make one of her V^h visits to the natural environment, and σ is a real tax price charged to each consumer per unit of natural environmental quality. The PLA decides the quantity of land L_a that is to be preserved and the maximum number of visits $V = \sum_{h=1}^{H} V^h$ that can be permitted to this land:

$$\max_{V,L_a} \; tV + \sigma H G(L_a) - \frac{r}{P}L_a$$

An equilibrium is found.[20] The government agency has asymmetric information concerning households preferences in respect of natural areas. Since households do

[20] It should be pointed out here that this equilibrium could not be Pareto comparable with the one found in the present paper.

not purchase natural environment quality in the market, the level of demand L_p^d cannot be observed, nor it is possible for the government to verify that demand is the same as the supply L_p^s, given σ. So there is a difficulty in implementing a demand revelation mechanism that led the author to consider a second-best solution.

3.2.3.- Paralleling first-order conditions for both the social planner and competitive general equilibrium problems.

López *et al* state that from the social planner first-order conditions a demand and supply function can be obtained. The key to how this is possible is if the social planner does not take prices into account. They claim that the multiplier r is "the rate of the marginal product of land in agriculture equals the shadow price of land which, under competitive conditions, should equal the market rate of return on land." (p.55) Consequently r is the price of land. Nevertheless, since their framework cannot present a competitive equilibrium version, it is not possible for them to substantiate their statement. The general equilibrium framework studied in this paper permits us to gauge to what extent this multiplier is in fact a price. Recall that (10) is a Pareto-efficient condition (Pareto allocations are denoted by caps), and (15) is a decentralised equilibrium condition for each h (competitive allocation are denoted by stars):

$$\begin{aligned} \alpha^h \frac{\partial U^h(\hat{c}^h, \hat{g}^h)}{\partial c} \frac{\partial F(\hat{L}_u, \hat{N})}{\partial L_u} - \hat{r} &= 0 \\ \frac{\partial U^h(c^{*h}, g^{*h})}{\partial c} \left(\frac{r}{P}\right)^* k - \lambda^{*h} &= 0 \end{aligned}$$

From the firm's problem we obtain $\frac{\partial F(L_u^*, N^*)}{\partial L_u} = \left(\frac{r}{P}\right)^* k$. In the presence of the public good we have shown that $L_u^* > \hat{L}_u$. So even with an egalitarian social function (i.e., $\alpha^h = 1$ for all h) and a quasilinear utility function (i.e., $U^h(c^h, g^h) = c^h + \theta u^h(g^h)$) the social planner multiplier is not the equivalent of rents from land in competitive equilibrium:[21]

$$\hat{r} = \frac{\partial F(\hat{L}_u, \hat{N})}{\partial L_u} \neq \frac{\partial F(L_u^*, N^*)}{\partial L_u} = \left(\frac{r}{P}\right)^* k$$

Furthermore, the inequality is undefined because it is not clear, for example, that $\hat{N} > N^*$. Consequently, the indications are that the statement by López *et al* (1994) and the remaining empirical work using data from real world (i.e., competitive equilibrium allocations) could be void of meaning.

[21] Even if $\partial^2 F / \partial L_u^2 < 0$ nothing can be said about inequalities, since N^* can be higher than $\hat{N}$.

4.- On the efficient allocation of land between productive uses and recreational uses. An application to Galiza.

Agricultural and cattle activities were both traditionally important to the Galician rural economy. The utilization of natural areas was an integral part of economic activity: this involved using the land, pasture for cattle, vegetation for manure and wood. This "factory" yielded an output in the form of intermediate goods for the rural community. The scenario in terms of competing uses for land has changed considerably in recent decades. As has happened in many other developed countries, the importance of agriculture as a productive sector has gradually fallen (39% in 1957 to 8% in 1995). Migrations to cities and changes in productive processes for agriculture and cattle made land less necessary. Then forest administration and wood pulp companies began to demand these areas to forest them with non-autochthonous fast-growing productive species: formerly maritime pine and more recently eucalyptus and insignis pine. By 1995 the eucalyptus and pine (both insignis and maritime) had come to represent 29% and 41% of forested land, respectively. The areas with forest plantations are also those with the greatest population density and of most importance in terms of tourism. These factors generate demand for land for recreational uses, specially on autochthonous forestlands. However the percentage of land dedicated in Galiza to natural spaces (0.7%) is less than Spain's and other European countries' mean.[22] Thus, given the current allocation of land a prime objective is to know whether the quantity of land dedicated to natural recreational use is efficient.

A second objective of this work is to present an empirical application of the theoretical framework decribed in the previous section. We study the efficiency of the present allocation of land in Galiza between productive uses and natural recreational uses. This depends on individual preferences and on the productivity of land of these competing uses. Although several authors and politicians claim for an increase in natural recreational areas in Galiza, this is the first empirical study made concerning social efficiency allocations.

We focus on Monte Aloia recreational area (746 hectares), a fairly typical woodland in Galiza. Unlike productive plantation areas, this woodland, like other natural recreational areas in this region, has longer rotation periods, lower density of trees as well as having infrastructures for the development of leisure activities.

4.1.- The functional forms

Firstly some functional forms for the production and utility functions are chosen in order to obtain a parametrized solution.

[22] Spain: 5.7%; Portugal: 6.8%; France: 8.2%; United Kingdom: 10.6%; Germany: 11.7 %; Norway: 15.5%. See Meixide Vecino and Hernández Pousa (1997) p.163.

The production functions

Private good.- It is assumed that technology can be represented by a Cobb-Douglas constant scale returns production function. The required inputs are qualified land kL_u (i.e., the amount of land dedicated to production), and labour N:

$$F(kL_u, N) = A(kL_u)^{\alpha} N^{1-\alpha} \tag{19}$$

Public good.- The public good is produced directly from non-productive land[23] $L_p = G(L_a) \equiv L_a$.

The utility function

The objective is to calibrate the model for the Galician case. Thus our choice has to be consistent with the data available. At microlevel data from the González-Gómez (1998) survey at Monte Aloia recreational area are utilised, in which individual preferences are revealed. The question put to interviewees was: "How much are you willing to pay to continue visiting the natural recreational area the *same* number of times?" This kind of survey is common in public good valuation (Arrow *et al*, 1993). Welfare Theory on revealed preferences indicates that this question assumes that agents' preferences are represented by a quasilinear utility function. It could be the case for goods where expenditure is small in comparison with total expenditure on private good. *Compensating variation* and *equivalent variation* are the same (see Figure 2). So the following quasilinear utility function will be assumed as a representation of preferences[24]

$$U^h(c^h, g^h) = c^h + \theta^h Lng^h$$

The valuation of recreational areas depends on the portion of land dedicated to these areas L_a and on the leisure time to enjoy them: $l - d$, with d representing the time given over to travelling. Since there is a degree of substitution between large natural recreational areas and leisure time spent in them, the "valuation" function of the natural resource is assumed to be represented by a Cobb-Douglas constant scale returns function[25][26]

$$g(L_p, l^h) = BL_a^{\beta}(l^h - d^h)^{1-\beta}$$

In this case there is no time given over to travelling, thus $l^h - d^h = l^h$. Observe that for the case of $\beta \in (0, 1)$ this functional form permits us only to study the "use" value of recreational areas in isolation, or else the "use" value and the "existence" value jointly. We could set $\beta = 1$ in order to study "existence" value only. To be consistent with data where only "use" value is available, we will consider the former.

[23] Remember that we are considering an uncongested non-excludable public good. Making use of Anas (1988)'s notation, $G(L_a) = \mathcal{G}(0, L_a) \equiv CL_a$, where C is a constant.

[24] Here we endeavour to keep as close as possible to the López *et al* framework. See footnote 14.

[25] All the production functions are of the same type in keeping with modelling practices.

[26] This kind of utility function assumes that these "intermediate" goods are neutral (i.e. the demand for each good is not affected by the price of the other goods).

Finally, it must be pointed out that heterogeneity between agents comes from the individual subjective weighting of the public good θ^h. And so the parameters β and B, at the "valuation" function of the natural resource, are assumed to be the same for all agents.

4.2.- The social planner Pareto-efficient problem and competitive equilibrium.

In order to be consistent with the data available, the social planner and the competitive problem for the representative agent are also studied.

4.2.1.- The social planner Pareto-efficient problem

Social planner first order conditions (9)-(12) for the assumed functional forms are as follows

$$\begin{aligned} \alpha_h \theta^h \beta \frac{1}{L_a} - r &= 0 \\ \alpha_h A \frac{(kL_u)^\alpha (T-l^h)^{1-\alpha}}{kL_u} \alpha k - r &= 0 \\ -\frac{A(kL_u)^\alpha (T-l^h)^{1-\alpha}(1-\alpha)}{T-l^h} + \theta^h(1-\beta)\frac{1}{l^h} &= 0 \\ r[L_a + L_u - L] &= 0 \end{aligned}$$

Versions of (13) and (11) can be obtained by substituting L_u in the last condition and by deleting r in the first and the second conditions:

$$\begin{aligned} \theta^h \beta \frac{1}{L_a} &= A\alpha k^\alpha (L-L_a)^{-(1-\alpha)}(T-l^h)^{1-\alpha} \\ \theta^h (1-\beta)\frac{1}{l^h} &= A(1-\alpha)[k(L-L_a)]^\alpha (T-l^h)^{-\alpha} \end{aligned}$$

Dividing both we obtain:

$$L_a = L \frac{1}{\frac{T-l^h}{l^h}\frac{\alpha}{1-\alpha}\frac{\beta}{1-\beta}}$$

So making substitutions in any of them, we have:

$$\left(\frac{(1-\alpha)A}{\theta^h(1-\beta)}\right)^{\frac{1}{\alpha}} kL\alpha(1-\beta){l^h}^{\frac{1}{\alpha}} = T\alpha(1-\beta) + l^h(\beta-\alpha)$$

See Figure 3. And so, there exists an efficient allocation of leisure $\hat{l}^h$ if:

$$\left(\frac{(1-\alpha)A}{\theta^h(1-\beta)}\right)^{\frac{1}{\alpha}} kL\alpha(1-\beta)T^{\frac{1}{\alpha}} > T\beta(1-\alpha)$$

which is to say, if $\left(\frac{A}{\theta^h}\right)^{\frac{1}{\alpha}} \left(\frac{(1-\alpha)T}{1-\beta}\right)^{\frac{1-\alpha}{\alpha}} \frac{kL}{\beta} > 1$ then the efficient allocations of land are $\hat{L}_a$ and $\hat{L}_u$.

4.2.2.- Voluntary-contribution competitive equilibrium

Given the functions assumed we can obtain household supply functions for labour, recreational land and consumption from the representative agent's first order conditions:

$$
\begin{aligned}
L_a^s &= \frac{\theta^h \beta}{\frac{r}{P}k} && (20)\\
l^s &= \frac{\theta^h(1-\beta)}{\frac{w}{P}} && (21)\\
c &= \frac{w}{P}T + \frac{r}{P}kL - \theta^h && (22)\\
L &= L_a + L_u
\end{aligned}
$$

Meanwhile, the firm's demand function for labour and for productive land are given by:

$$
\begin{aligned}
\frac{w}{P} &= \frac{A(kL_u^d)^\alpha (T-l^d)^{1-\alpha}}{T-l^d}(1-\alpha) && (23)\\
\frac{r}{P}k &= \frac{A(kL_u^d)^\alpha (T-l^d)^{1-\alpha}}{L_u^d}\alpha && (24)\\
A(kL_u^d)^\alpha (T-l^d)^{1-\alpha} &= \frac{w}{P}(T-l^d) + \frac{r}{P}kL_u^d && (25)
\end{aligned}
$$

Voluntary-Contribution Equilibrium

A voluntary-contribution equilibrium E is a set of goods, time and land allocations and factor prices $\{c^*, g^*, n^*, l^*, L_a^*, L_u^*, \{\left(\frac{w}{P}\right)^*, \left(\frac{r}{P}\right)^*\}\}$ such that: [1] $\{c^*, g^*, n^*, l^*, L_a^*, L_u^*\}$ is a solution to the agent's maximisation problem, given the equilibrium price $\{\frac{w}{P}, \frac{r}{P}\}$; and [2] goods and input markets clear $c = F(kL_u, T-l)$, $l^s = l^d$, $L_u^s = L_u^d$ and $L_a^s = L_a^d$.

Thus we have, from the supply and demand functions for labour given by (21), (23) and (25):

$$\frac{w}{P}T\alpha = \alpha\theta^h(1-\beta) + \frac{r}{P}k(L-L_a)(1-\alpha) \qquad (26)$$

From the supply and demand for recreational land given by (20), (24) and (25):

$$\frac{r}{P}kL(1-\alpha) = (1-\alpha)\theta^h\beta + \frac{w}{P}(T-l)\alpha \qquad (27)$$

And finally, from the demand and supply for the private good (22) and (25), we have:

$$\frac{w}{P}T + \frac{r}{P}kL - \theta^h = A(k(L-L_a))^\alpha (T-l)^{1-\alpha} \qquad (28)$$

There are thus four unknowns, $\{\frac{w}{P}, \frac{r}{P}, L_a, l\}$ and four equations (25) to (28). From (26) we obtain the amount of land dedicated to recreational use L_a as a function of prices:

$$L_a = L - \frac{\frac{w}{P}T\alpha - \alpha\theta^h(1-\beta)}{\frac{r}{P}k(1-\alpha)}$$

and likewise, from (27) we obtain leisure l as function of prices:

$$l = T - \frac{\frac{r}{P}kL(1-\alpha) - (1-\alpha)\theta^h\beta}{\frac{w}{P}\alpha}$$

Substituting both into (25) and (28) we obtain:

$$\begin{aligned} \frac{w}{P}T + \frac{r}{P}kL - \theta^h &= A\left(\frac{r}{P}kL - \theta^h\beta\right)\left(\frac{r}{P}\right)^{-\alpha}\left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha}\left(\frac{w}{P}\right)^{-(1-\alpha)} \\ \alpha\frac{w}{P}T &= (1-\alpha)\frac{r}{P}kL + \theta^h(\alpha-\beta) \end{aligned}$$

From this last equation we obtain wages as a function of the interest rate, and so substitution gives:

$$\left(\frac{r}{P}kL - \theta^h\beta\right) = \frac{A\alpha}{\left(\frac{r}{P}\right)^{\alpha}\left(\frac{\theta^h(\alpha-\beta)+(1-\alpha)\frac{r}{P}kL}{(1-\alpha)T}\right)^{1-\alpha}}\left(\frac{r}{P}kL - \theta^h\beta\right)$$

Two solutions are possible as follows:
1) $\frac{r}{P}kL - \theta^h\beta = 0$. Hence $\left(\frac{r}{P}\right)^* = \frac{\theta^h\beta}{kL}$. But this yields a corner solution where all available land is given over to recreational use, i.e. $L_u^* = 0$, $L_a^* = L$, $l^* = T$, $n^* = 0$, $c^* = 0$. Being inconsistent with the assumption that expenditure on the public good is lower than expenditure on the private good, this possibility is removed.
2) $A\alpha = \left(\frac{r}{P}\right)^{\alpha}\left(\frac{\theta^h(\alpha-\beta)+(1-\alpha)\frac{r}{P}kL}{(1-\alpha)T}\right)^{1-\alpha}$. Which is to say,

$$\frac{\theta^h(\alpha-\beta)}{\alpha} + \frac{r}{P}kL = T\,(A\alpha)^{\frac{1}{1-\alpha}}\left(\frac{r}{P}\right)^{-\frac{\alpha}{1-\alpha}} \tag{29}$$

It can be observed that the right-hand side is an increasing function in $\frac{r}{P}$ and that the left-hand side is a decreasing function. Thus a solution exists $\left(\frac{r}{P}\right)^{**} > \left(\frac{r}{P}\right)^*$. (See Figure 4.) This result give us optimal allocation and prices l^{**}, L_a^{**} and L_u^{**}.

4.3.- The questionnaire

Personal interviews were carried out at Monte Aloia natural recreational area over 1994 and 1995. In order to obtain a conservative design (Arrow *et al*, 1993), the recommendations of the influential report on contingent valuation by the U.S. National Oceanic and Atmospheric Administration (NOAA) were followed. A pretest of 25 questionnaires and 402 final interviews were conducted. Both the days for interviewing and the interviewees were selected on a random basis.

Visitors were invited to assess a change from the current situation of free access to the recreational area to a hypothetical scenario whereby visitors would have to pay. The interviewees were asked: "How much are you willing to pay to continue visiting the natural recreational area the same number of times?" The questionnaire also asked for socio-economic information on users (e.g., income, household size, etc.), details of use of the recreational area (e.g., frequency of use, opportunity cost) and attributes of the area they valued (landscape, tranquillity, etc.).

4.4.- Empirical results

Utilising the theoretical framework from the previous section and applying, the functional forms assumed, demand and supply functions and equilibrium allocations are obtained. All these depend on model parameters, thus the following had to be calibrated: the individual subjective weighting of the public good, θ^h; the individual productivity of the natural recreational area when it is utilised, β; individual productivity of leisure time spent at the recreational area, $1-\beta$; the residual of this productivity, B; the productivity of capital when consumption goods are produced, α; the productivity of labour when consumption goods are produced, $1-\alpha$; and the residual of production of the consumption good, A.

The only source of information at microeconomic level is the data obtained from the interviewees in the above mentioned Monte Aloia survey. Three hypothesis in this empirical study are stated. Firstly, there is an implicit assumption concerning visitors' behaviour. Given the data available, it is assumed that the people interviewed at Monte Aloia are a representative sample. On this basis it is also assumed that average length of a visit to Monte Aloia and the willingness to pay per hour of visit are approximately the same for all the Galician natural recreational areas. Secondly, although "existence" value may also be important in valuing the public good, only "use" value is considered. This is so as to fit data into the theoretical model. Thirdly, given that there is no information about "existence" value both from users and non-users, non-users are excluded from this study. The model assigns for these agents a zero equivalent valuation (EV), i.e. $EV = 0$. However, the proportion of the non-users in the total population is unknown.

Calibration at macroeconomic level

Data for the Galician economy in 1993 can be found in Table 1 of Appendix 1. The total number of leisure hours spent at recreational areas per person at Monte Aloia, as well as at other natural recreational areas in this region, is taken from the survey at Monte Aloia. (assuming that all recreational areas were visited for the same number of hours).[27] Applying (24) we obtain α as follows:

$$\alpha = 1 - \frac{\text{Compensation of Employees}}{\text{NVA}_{fc}}$$

The Solow residual A is obtained from (19) as the ratio:

$$A = \frac{\text{NVA}_{fc}}{\mathcal{K}^{\alpha} n^{1-\alpha}}.$$

Thus, from the goods market equilibrium, following (19) and (22), we obtain: $\theta_{MAC} = \frac{w}{P}T + \frac{r}{P}\mathcal{K} - C$. Finally from (29) we have:

$$\beta_{MAC} = \frac{\alpha}{\theta_{MAC}} \left[\theta_{MAC} + \frac{r}{P}\mathcal{K} - T\left(A\alpha\right)^{\frac{1}{1-\alpha}} \left(\frac{r}{P}\right)^{-\frac{\alpha}{1-\alpha}} \right].$$

As a residual we have B. From (7) and (22) we calculate that $g = \theta_{MAC}$, thus $B = \frac{\theta_{MAC}}{L_a^{\beta_{MAC}} l^{1-\beta_{MAC}}}$. The results can be summarised as follows:

α	A	θ_{MAC}	β_{MAC}	B
0.4107	0.02097	41649.94	0.23406	0.01217

Calibration at microeconomic level

The relevant data are shown in Table 2 of Appendix 2. The median statistic is chosen because of its robustness. From the equivalent variation of assumed quasi-linear preferences, a combination (β, θ^h) yielding efficient allocations is obtained. Since β is assumed to be identical for all agents, this parameter is taken from the macroeconomic data (i.e, $\beta = \beta_{MAC}$). Thus, an individual subjective valuation for the recreational good θ^h can be obtained. This parameter corresponds with an efficient allocation for each individual from the following equivalent variation:[28]

$$\theta^h(1-\beta) = \Omega^h$$

where for every agent h in the survey,[29]

$$\Omega^h = \frac{EV^h}{\frac{\left(\frac{w}{P}\right)^h}{\left(\frac{w}{P}\right)^h + \tau^h} - Ln\frac{\left(\frac{w}{P}\right)^h}{\left(\frac{w}{P}\right)^h + \tau^h} - 1}$$

[27] 24 hours were obtained as the median number of hours spent in recreational areas per person in a year, where the mean was 34,69 (41,54).

[28] See Appendix 4 for the theoretical analysis.

[29] The total number of valid interviews was reduced to 313 interviewees with positive equivalent variation. Visitors with zero willingness-to-pay were excluded from the sample for technical reasons (the individual parameter θ^h cannot be computed).

and

$$\left(\frac{w}{P}\right)^h = \frac{\text{Household } h \text{ Monthly Income}}{\text{Number of hours worked per month} \times \text{Household size}}$$
$$\tau^h = \text{Household } h \text{ WTP per hour of visit}$$
$$EV^h = \text{Household } h \text{ WTP} \times \text{Number of visits per household } h$$

Discussion

Both sets of parameters are juxtaposed. Figure 5 illustrates a histogram comparing the distribution of individual valuation of the recreational good at the microeconomic level θ^h, with the aggregate valuation of the recreational good at macroeconomic level θ_{MAC}. Recall that a common land contribution to the production of recreational good, $\beta_{MAC} = 0.23406$ is taken. Our conclusion, based on the result obtained, is that approximately 91% of individuals value the recreational good more than what would be expected from the present allocation of the land (see Appendix 3.- Table 3). This would suggest that the present allocation of land to recreational uses in Galiza –1.09% of total available land– is below the social optimum.[30] Nevertheless this result must be interpreted with care. Non-users are not taken into account in the present empirical study. If they were, the θ-distribution would present many zeros, as $EV = 0$ for non-users. This would reduce considerably the percentage of individuals that value the recreational good at the aggregate level. Whether "existence" value were also to be considered the θ-distribution would increase, and the net effect is unknown.

5.- Conclusions and extensions.

The objective of this research is to analyse the efficient allocation to land to recreational use. As an application of this theoretical framework, land allocation in Galiza was selected for study, using an approach that departs from standard literature. The theoretical contribution of this paper is threefold. Firstly, Pareto-efficient allocations for the social planner problem and optimal competitive allocations are studied in a unified framework. Secondly, heterogeneity among agents is introduced in a rational equilibrium framework. This is crucial for the study of externalities (e.g., a public good), so that the *free-rider* problem can be addressed. Thirdly, the individual rational decision-making set-up makes possible the study both the "use"

[30] Two alternative proposals were also analysed. The first is a proposal from the Spanish government to Nature Network 2000 and the second, a recommendation to Spain by OECD (1997). The first proposal assigns 61977 ha to "recreational areas" in Galiza. This represents 2,10 % of total available land. Our results show that at this level of allocation 88.18% of individuals would place a higher value on the recreational good than what would be expected from aggregate data. The OECD proposal is to dedicate 14% of total available land (i.e., 412673.38 ha for Galician case). Our experiment shows that this approaches the socially optimal allocation, since at this level only 57% of individuals place a higher value on the recreational good than would be expected from the current allocation. If only "use" value is taking into account, this recommendation seems to be closer to the socially optimal allocation.

value and the "existence" value of natural recreational areas.

Consequently this framework overcomes some methodological shortcomings of all previous empirical literature. Although the theoretical background is the *ad-hoc* social planner function the empirical literature takes data from real world competitive allocations. This sheds doubts on the validity of these empirical analysis and any results found there.

A second part of this work involved an empirical study of land allocation in Galiza within a restricted representative agent framework. The conclusion arrived at is that the current amount of land dedicated to natural recreational areas is not efficient, given that it is lower than what is considered to be Pareto-efficient. As this is a standard public good problem, competitive equilibrium is usually non-efficient, given that in the real world agents behave strategically.[31] Thus competitive equilibrium allocation of the public good is lower than the social planner's, as observed in the real world. Along these lines several mechanisms could be recommended in order to obtain efficient allocations. For example, Lindahl personalized prices, Grooves mechanism (see Burton, 1996) or government provision of the public good financed by a tax revenue system (see Anas, 1988). Since the public good market fails, intervention in the market by the regulator (Xunta de Galicia, the regional authority) is necessary in order to obtain the efficient allocation of natural areas.

Finally, some extensions might be made to the paper. Firstly where the data availability makes it possible, the consideration of heterogeneity in the empirical applications would improve the results. Secondly, we are dealing with a quantity of homogeneous areas. However, from a competitive use point of view, land is heterogeneous. Thus, an interesting applied economic question would be to divide the region into individual parcels (pixels) of land. Then, via a geographical information system (GIS), to assign the land to competing uses using a system of suitability scores (see Burton, 1996, Peter J. Parks, Edward B. Barbier and Joanne C. Burgess, 1998, and specially Hanink and Cromley, 1998). Finally, the effect on equilibrium allocation when transportation costs and entry fees are taken into account could be studied. Likewise, this methodology permits the establishment of a microeconomic foundation for demand both for natural recreational areas and the number of visits, of the kind that is often used *ad-hoc* in the literature.

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[31] As indicated above 91% of interviewees in the Monte Aloia survey require a greater amount of recreational area than at present. This suggests that a number of free-riders are hidden behind them.

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Appendices.

Appendix 1.- Table 1.- Macroeconomic Data for the Galician Economy, 1993.

$\mathcal{K} = kL_u$	9366394.79	Net Capital Stock Private and Public (in a wide sense) (Mass *et al*, 1997)
$NVA_{fc} = Y$	3107703	Net Value Added t factor cost (BBV, 1997)
$\frac{n}{N}$	1710	Number of working hours per worker (IGE, 1997)
N	593796	Number of workers (BBV, 1997)
$n = N\frac{n}{N}$	1015391160	Total number of hours worked in 1993
$\frac{w}{P}n$	1831373	Compensation of Employees (BBV, 1997)
$\frac{w}{P}$	0.0018036	Hourly wage per worker
$\frac{l}{N}$	24	Number of leisure hours spent on recreational areas per person (see Table 2, [3*b*])
$l = N\frac{l}{N}$	14251104	Total number of hours spent at recreational areas
$T = l + n$	1029642264	Total number of working and leisure hours per year
$OS = \frac{r}{P}\mathcal{K}$	1276330	Net Operating Surplus (BBV, 1997)
L_u	2573324.9	Land in productive (industrial and agriculture) use, hectares (MAPA, 1995)
L_a	32151.1	Land in recreational land use, hectares (CAGM, 1991)
$L = L_a + L_u$	2605476	Total land available, hectares
$k = \frac{\mathcal{K}}{L_u}$	3.63980	Productivity of productive land

Comments.-
Except where indicated, all data are from 1993 at current millions pesetas.
Consellería de Agricultura, Gandeiria e Montes (1991) is denoted by (CAGM, 1991)

Appendix 2.- Table 2.- González-Gómez Survey (1998). Selected data.

	Mean	Median	Minumum	Maximum
[1] Number of visits per year	5.83 (7.7196)	3	1	55
[2] Average length per visit (hours)	4.21 (2.707)	4	0.25	8
[3] Total hours of visits per year = [1] × [2]	33.152 (43.876)	10	0.25	312
[4] Willingness-to-pay per visit (pesetas)	445.75 (455.222)	300	25	4000
[5] Willingness-to-pay per hour of visit (pesetas) τ^h	68.41 (84.630)	40	1.54	500
[6] Household h monthly income (pesetas)	217252 (111.003)	200000	100000	400000
[7] Household size	2.06 (1.103)	2	1	9
[8] EV^h = [1] × [5] (pesetas)	216.39 (381.289)	100	8.33	3000
[9] Wage per hour per member of household h (pesetas)	886.39 (0.579)	701.7	77.97	3508.77
[10] Ω^h	780446.618 (2412086.11)	215972.8	5680.63	24716583
[1*b*] Number of visits to all recreational areas per year	8.82 (8.8938)	5	1	61
[3*b*] Total hours of visits to all recreational areas per year= [1] × [2]	34.59 (41.507)	24	0.25	328

Comments.-
The data [1] − [10] refer to visitors to Monte Aloia. The number of valid cases was

313 (53 cases were excluded for zero willingness-to-pay). The figures in parenthesis in the Mean column represent the standard deviations.

[9] is expressed as $\frac{w}{P} = \frac{[6]}{\frac{\frac{n}{N}}{1710} \times [7]}$, where $\frac{n}{N} = 1710$ is taken from Table 1.

For [3b] it is assumed that the average length per visit to all natural recreational areas is the same.

Appendix 3.- Table 3.- Distribution of individual θ^h for visitors to the Monte Aloia recreational area.

	Frequency	% accumulated
8103.08	2	0.64%
22026.46	13	4.79%
41649.94	13	8.95%
162754.79	74	32.59%
442413.39	94	62.62%
1202604.28	66	83.71%
3269017.37	34	94.57%
8886110.52	12	98.40%
24154952.75	2	99.04%
65659969.14	3	100.00%

The study was done given a $\beta = 0.23406$.

Appendix 4.- Measuring individuals welfare: Equivalent Variation.

The purpose of this appendix is to furnish a conceptual basis for the empirical analysis of the welfare corresponding to interviewees at natural recreational areas. We mainly follow the analysis by Varian (1984, Chapter 7). This can be done making use equivalent variation and compensating variation. Which measure is the most appropriate depends on the circumstances involved and what question one is trying to answer. If one is considering trying to arrange for some compensation scheme at the new prices, then the compensating variation seems reasonable. However, if one is simply trying to get a reasonable measure of "willingness to pay," the equivalent variation is probably better. This is so for two reasons. First, the equivalent variation measures the income change at *current* prices, and it is much easier for decision makers to judge the value of an euro at current prices than at some hypothetical prices. Second, if we are comparing more than one proposed policy change, the compensating variation keeps changing the base prices while the equivalent variation keeps the base prices fixed at the status quo. Thus, the equivalent variation is more suitable for comparisons among a variety of projects.

In our application the functional form for the utility function is quasilinear, so both are equal. This study tries to obtain the possible values for the parameters of the particular utility function assumed for agents. The empirical study is made on the basis of the survey data obtained at Monte Aloia. The data provides information concerning individual wealth and willingness-to-pay per visit. It has to be borne in mind that the agents take as given the amount of land at Monte Aloia (denoted by $\bar{L}_a$). Thus agents only maximise their utility with respect to consumption and leisure.

The concepts applied in the study are defined as follows:

The Indirect Utility Function $v(p,y)$. "It is the maximum utility achievable at given prices p and income y"

$$v(p,y) \equiv \begin{cases} \max_x & u(x) \\ \text{s.t.} & px = y \end{cases}$$

The Expenditure Function $e(p,U)$. "It is the minimal amount of income necessary to achieve utility U at prices p"

$$e(p,U) \equiv \begin{cases} \min_z & pz \\ \text{s.t.} & u(z) \geq U \end{cases}$$

The Direct Compensated Function $m(p,x)$. "It is the money needed at prices p to be as well off as to be by consuming the bundle of goods x"

$$m(p,x) \equiv e(p,u(x)) = \begin{cases} \min_z & pz \\ \text{s.t.} & u(z) \geq u(x) \end{cases}$$

The Indirect Compensated Function $\mu(p;q,y)$. "Measure how much money one would need at prices p to be as well off as one would be facing prices q and having income y"

$$\mu(p;q,y) \equiv e(p,v(q,y)) = \begin{cases} \min_z & pz \\ \text{s.t.} & u(z) \geq v(q,y) = \begin{cases} \max_x & u(x) \\ \text{s.t.} & qx = y \end{cases} \end{cases}$$

We start out from the status-quo situation where any agent h does not pay for any of her V^h visits to the natural recreational area. As there are two goods -consumption and leisure- the status-quo prices are $p^0 = \left(1, \frac{w}{p}\right)$. For the functional forms presented in section 4, let it be assumed that each visit lasts the same length of time, i.e. $\lambda_i^h = \lambda$ for $i = 1, 2, \ldots V^h$ and for all agent h. If an entry fee t is established for each visit then, given that leisure is dearer, prices will change to $p^1 = \left(1, \frac{w}{p} + \tau\right)$ where $\tau = \frac{t}{\lambda}$.

Given the data by the survey, our goal is to obtain the Equivalent Variation for agent h:

$$EV^h = M^h - \mu(p^0; p^1, M^h)$$

This uses the status-quo prices as the base and poses what income change at current prices would be *equivalent* to the proposed change. In order to answer this question the indirect compensated function is calculated $\mu(p^0;p^1,M^h) = e(p^0, v(p^1,M^h))$, but the indirect utility function $v(p^1,M^h)$ must first be obtained.

For the functional forms described in section 4, the indirect utility function for prices p^1 and wealth $M = \frac{w}{P}T + \frac{r}{P}\bar{L}_u$ are calculated

$$\begin{aligned} v(p^1,M) &\equiv \begin{cases} \max_{c,l} & c + \theta Ln\left[BL_a^{\beta} l^{1-\beta}\right] \\ \text{s.t.} & c + \left(\frac{w}{P} + \tau\right) l = \frac{w}{P}T + \frac{r}{P}k\bar{L}_u \equiv M \end{cases} \\ &\equiv \begin{cases} \max_{c,l} & c + \theta Lnl + \bar{\pi} \\ \text{s.t.} & c + \left(\frac{w}{P} + \tau\right) l = M \end{cases} \end{aligned}$$

where $\bar{\pi} = \theta Ln\left(B\bar{L}_a^{\beta}\right)$. From first order conditions we obtain $l^{*1} = \frac{\theta(1-\beta)}{\frac{w}{P}+\tau}$ and $c^{*1} = M - \theta(1-\beta)\frac{\frac{w}{P}}{\frac{w}{P}+\tau}$.

Thus

$$v(p^1, M) = c^{*1} + \theta Lnl^{*1} + \bar{\pi} = M - \theta(1-\beta)\left[Ln\frac{\theta(1-\beta)}{\frac{w}{P}+\tau} - \frac{\frac{w}{P}}{\frac{w}{P}+\tau}\right] + \bar{\pi}$$

Substitution of this at the expenditure function $\mu(p^0; p^1, M) \equiv e(p^0, v(p^0, M))$

$$e(p^0, v(p^0, M)) = \begin{cases} \min_{c,l} & c + \left(\frac{w}{P}+\tau\right) l \\ \text{s.t.} & c + \theta Lnl + \bar{\pi} \geq M - \theta(1-\beta)\left[Ln\frac{\theta(1-\beta)}{\frac{w}{P}+\tau} - \frac{\frac{w}{P}}{\frac{w}{P}+\tau}\right] + \bar{\pi} \end{cases}$$

From the first order conditions we obtain:

$$\begin{aligned} \hat{l}^* &= \frac{\theta(1-\beta)}{\frac{w}{P}} \\ \hat{c}^* &= M - \theta(1-\beta)\left[Ln\frac{\theta(1-\beta)}{\frac{w}{P}+\tau} - \frac{\frac{w}{P}}{\frac{w}{P}+\tau}\right] - \theta(1-\beta)Ln\frac{\theta(1-\beta)}{\frac{w}{P}} \end{aligned}$$

Then the Equivalent Variation is

$$EV = M - \mu(p^0; p^1, M) = \theta(1-\beta)\left[\frac{\frac{w}{P}}{\frac{w}{P}+\tau} - Ln\frac{\frac{w}{P}}{\frac{w}{P}+\tau} - 1\right]$$

There will be an individual h relationship between parameters θ and β, since agents are heterogeneous.

$$EV^h = \hat{\theta}^h(1-\hat{\beta})\left[\frac{\left(\frac{w}{P}\right)^h}{\left(\frac{w}{P}\right)^h + \tau^h} - Ln\frac{\left(\frac{w}{P}\right)^h}{\left(\frac{w}{P}\right)^h + \tau^h} - 1\right]$$

From the survey, individuals revealed the following: Entry fee for a single visit (in pesetas); number of visits per year; length spent at Monte Aloia recreational area (in hours); monthly average income per family (in pesetas); and the household size.

This allow to obtain (in pesetas-per-hour): τ^h the ratio between the willingness-to-pay fee and the length of time per visit; $\overline{VE}^h$ the entry fee by the number of visits to the natural recreational area; and $\left(\frac{w}{P}\right)^h$ the ratio between total monthly income divided by 1710/12 (number of working hours per month) by the household size.

With this values we obtain a combination of possible values for parameters for each individual $\hat{\theta}^h$ and $\hat{\beta}$: $\hat{\theta}^h(1-\hat{\beta}) = \Omega^h$, with $\Omega^h = \frac{EV^h}{\frac{\left(\frac{w}{P}\right)^h}{\left(\frac{w}{P}\right)^h+\tau^h} - Ln\frac{\left(\frac{w}{P}\right)^h}{\left(\frac{w}{P}\right)^h+\tau^h} - 1}$.

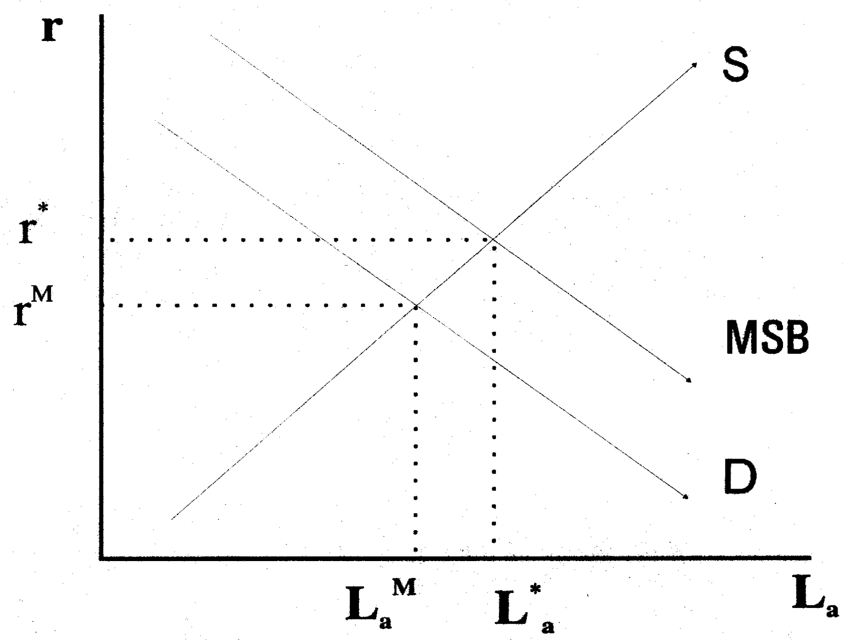

Figure 1:

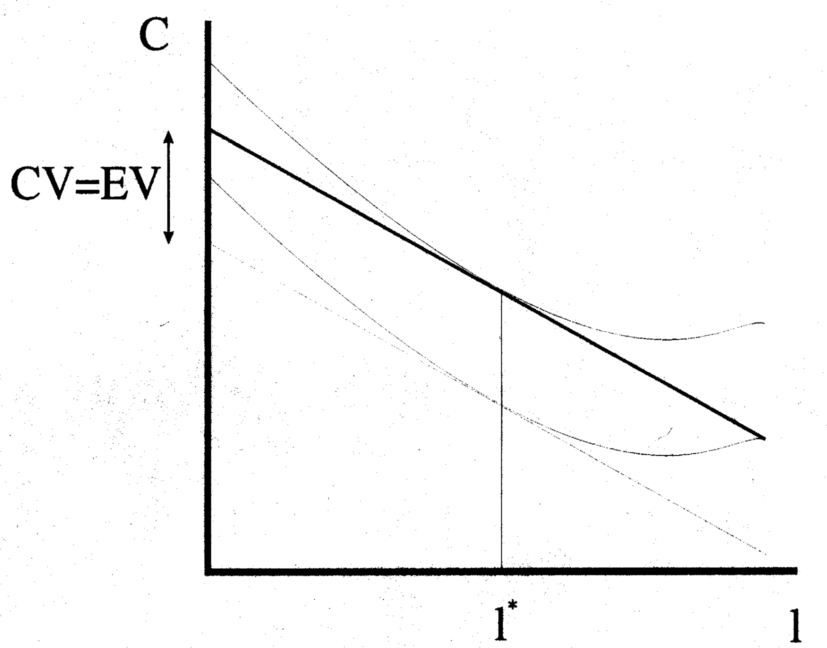

Figure 2:

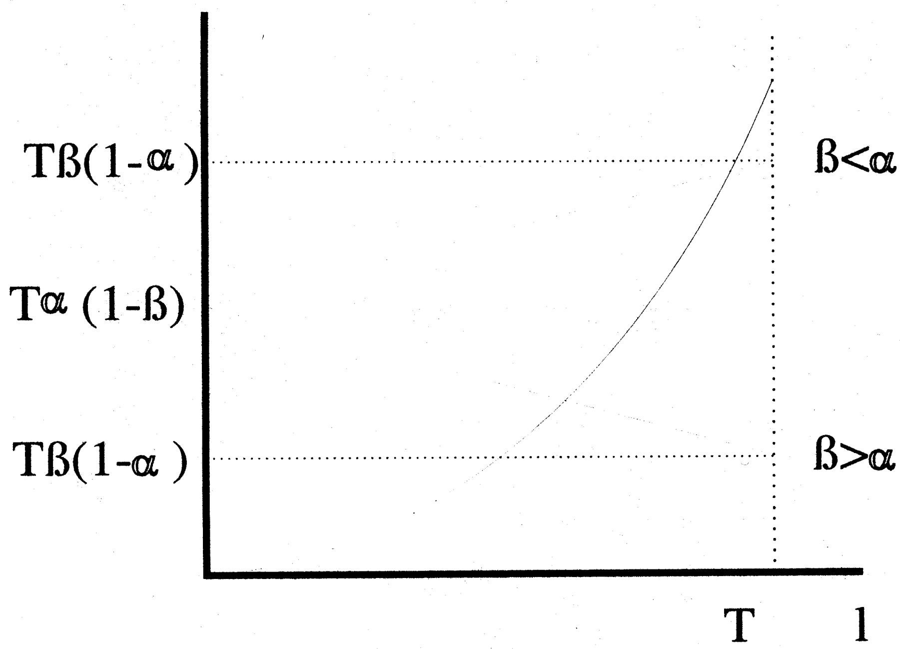

Figure 3:

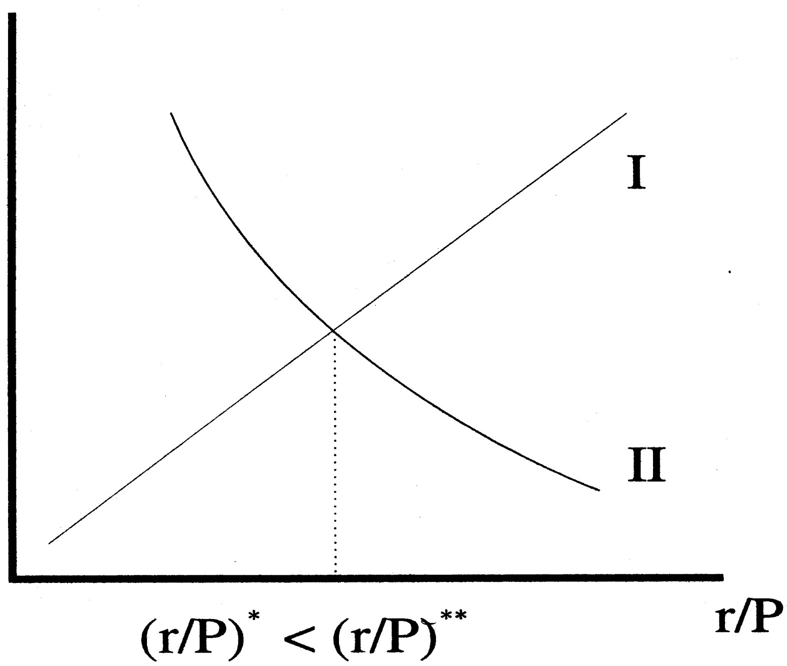

Figure 4:

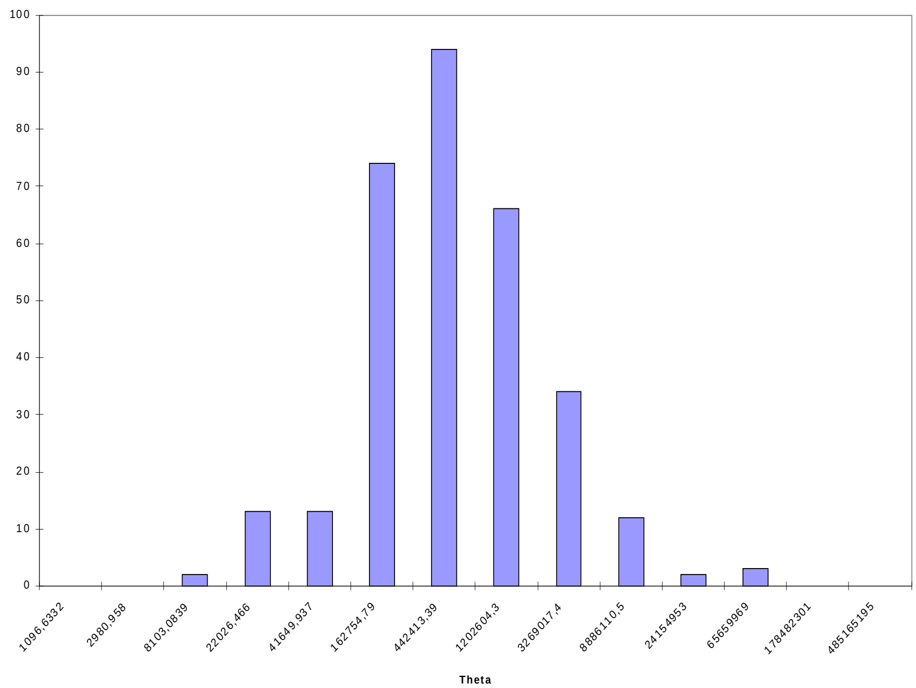
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Figure 5: