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Maurice J.G. Bun

Tinbergen Institute

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Tinbergen Institute Amsterdam

Keizersgracht 482
1017 EG Amsterdam
The Netherlands
Tel.: +31.(0)20.5513500
Fax: +31.(0)20.5513555

Tinbergen Institute Rotterdam

Burg. Oudlaan 50
3062 PA Rotterdam
The Netherlands
Tel.: +31.(0)10.4088900
Fax: +31.(0)10.4089031

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Bias correction in the dynamic panel data model with a nonscalar disturbance covariance matrix

MAURICE J.G. BUN*
University of Amsterdam

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Abstract

Approximation formulae are developed for the bias of ordinary and generalized Least Squares Dummy Variable (LSDV) estimators in dynamic panel data models. Results from Kiviet (1995, 1999) are extended to higher-order dynamic panel data models with general covariance structure. The focus is on estimation of both short- and long-run coefficients. The results show that proper modelling of the disturbance covariance structure is indispensable. The bias approximations are used to construct bias corrected estimators which are then applied to quarterly data from 14 European Union countries. Money demand functions for $M1$, $M2$ and $M3$ are estimated for the EU area as a whole for the period 1991:I-1995:IV. Significant spillovers between countries are found reflecting the dependence of domestic money demand on foreign developments. The empirical results show that in general plausible long-run effects are obtained by the bias corrected estimators. Moreover, bias correction can be substantial underlining the importance of more refined estimation techniques. Also the efficiency gains by exploiting the heteroscedasticity and cross-correlation patterns between countries are sometimes considerable.

* Tinbergen Institute & Faculty of Economics and Econometrics, University of Amsterdam, Roetersstraat 11, 1018 WB Amsterdam, The Netherlands (MBUN@FEE.UvA.NL). I want to thank M.M.G. Fase and C.C.A. Winder for kindly making available the data used in this paper. I am indebted to Jan Kiviet for his constructive comments. This paper has been presented at the Ninth International Conference on Panel Data in Geneva (June 2000) and at the World Congress of the Econometric Society in Seattle (August 2000).

1. Introduction

In this study we analyse various least squares based estimation procedures for the dynamic panel data model with fixed individual effects and a nonscalar covariance matrix. Both the ordinary and generalized Least Squares Dummy Variables (LSDV) estimators are considered. The choice of the model and estimators is based mainly on the typical empirical study at hand, i.e. estimation of money demand functions in the area of the European Union (EU). The data are a cross-section of times series for 14 EU countries and the number of cross-section units N in the dataset is relatively small compared with the time dimension T. In an earlier simulation study (Bun and Kiviet, 1999), we found that in the first-order dynamic panel data model with a scalar covariance matrix the bias of least squares based techniques is relatively small compared to instrumental variables based methods when T is larger than N. Based on a mean squared error criterion, least squares methods are to be preferred in this case.

Notwithstanding the superior performance of least squares methods, they are still biased in dynamic models due to the inclusion of lagged dependent variable regressors and require T large for consistency. Earlier simulation studies (Bun and Kiviet, 1999; Judson and Owen, 1999) show that these biases can be substantial especially for persistent models, i.e. stable dynamic models where the coefficient of the lagged dependent variable is close to unity. As these models are commonly encountered in macro-economic applications like the empirical study on money demand, it is important to develop more accurate estimation procedures.

In this study, we will develop alternative coefficient estimators by applying bias corrections on original estimators. Using asymptotic expansion techniques, Kiviet (1995, 1999) derives an approximation formula for the bias of the ordinary LSDV estimator in the first-order stable dynamic panel data model with normal disturbances and a scalar covariance matrix. We use these and other results on bias approximation in higher-order dynamic regression models (Kiviet and Phillips, 1994) to develop bias expressions for the ordinary and generalized LSDV estimators in higher-order dynamic panel data models with general covariance structure. Both extensions are necessary to apply bias corrected estimators in empirical work. In the empirical study on EU wide money demand, for example, it turns out that the first-order dynamic model is not general enough to capture all the dynamic features in the data. Regarding the disturbance covariance structure, when analysing time series for a group of countries both cross-sectional heteroscedasticity and interdependencies between countries are likely to be present. Hence, disturbance vectors of different cross-section units have different variances and may be correlated. To the extent that the covariance matrix of the disturbances is nonscalar, one should explicitly take this into account in any inference procedure exploiting the panel nature of the data.

Apart from developing more accurate estimation procedures for the so-called short-run parameters, estimation of other model parameters is considered also. First, in the case of the money demand relationship the long-run effects are important for policymakers. Hence, a clear distinction is made between estimation of short- and long-run parameter vectors. The direct bias correction on long-run coefficients, proposed by Pesaran and Zhao (1999) in the context of the dynamic random coefficients model, is applicable here also. Second, in practice often particular linear restrictions are imposed on the parameters before estimation. For example, in empirical studies of money demand often long-run price or income homogeneity is imposed. Hence, we also develop bias approximation formulae for restricted estimators along the lines of Kiviet and Phillips (1994). Third, in the type of model analysed in this study, estimation of the variances of coefficient estimators by conventional asymptotic expressions can be dramatically inaccurate (Freedman and Peters, 1984; Beck and Katz, 1995). Hence, we make use of bootstrap procedures to estimate standard errors.

The bias expressions developed in this study will be used to construct bias corrected estimators and they will be applied in the empirical study on money demand. Various authors have estimated a money demand function based on aggregated time series for the whole EU area and tested the stability of this function through time (Kremers and Lane, 1990; Monticelli and Papi, 1996; Fase and Winder, 1998). All those studies use time series techniques, but considering the EU countries as a cross-section one can possibly use panel data techniques. We will examine to what extent panel data techniques are a valid alternative for estimating EU wide money demand functions. Using aggregated time series it has been found that EU wide money demand is more stable than money demand in the individual member countries. An explanation for this fact is that when using aggregated data spillover effects between countries are internalized. As money demand specifications for individual countries typically do not contain variables measuring foreign developments, specification bias may cause them to be less stable than aggregated money demand. Exploiting the panel nature of the data we try to identify spillovers between countries by specifying a general disturbance covariance structure and including foreign variables in the empirical specification.

Section 2 gives an outline of the model. In evaluating the bias terms of the estimators, a detailed knowledge of the stochastic structure of the model is needed and this is described in this section. In section 3 bias expressions for ordinary and generalized LSDV estimators will be developed. In section 4 we consider estimation under linear restrictions, while estimators of long-run parameters will be analysed in section 5. In section 6 estimation of asymptotic standard errors by using either analytical expressions or bootstrap procedures will be discussed. In section 7 the estimation techniques will be applied to estimate EU wide money demand functions for M1, M2 and M3. The emphasis is on the plausibility of coefficient estimates and effectiveness of bias approximations. Section 8 concludes.

2. Model

We consider the higher-order dynamic panel data model

$$y_{it} = \sum_{p=1}^{P} \gamma_p y_{i,t-p} + \beta' x_{it} + \eta_i + \varepsilon_{it}, \qquad i = 1, ..., N;\ t = 1, ..., T. \quad (2.1)$$

In this model the dependent variable y_{it} is regressed on a $K \times 1$ vector of explanatory variables x_{it} with parameter vector β, P lagged values of the dependent variable and an individual specific constant η_i. The explanatory variables in x_{it} are assumed to be strictly exogenous, i.e.

$$E\left[x_{it}\varepsilon_{js}\right] = 0, \qquad \forall i, j, t, s,$$

and the individual effects η_i are assumed fixed, but unknown. Note that both the univariate processes for y_{it} and the elements of x_{it} may contain unit roots. However, the relationship (2.1) between y_{it} and x_{it} is assumed to be stable. For $p = 1$ this implies $|\gamma_1| < 1$, but in higher-order models more complicated restrictions on the autoregressive coefficients are required for stability. Regarding the disturbances ε_{it} it will be assumed throughout that they are normally distributed. Moreover, they are uncorrelated through time, but we allow for heteroscedasticity across cross-section units and non-zero contemporaneous cross-correlations, i.e.

$$\begin{aligned} E\left[\varepsilon_{it}\right] &= 0, & \forall i, t, \\ E\left[\varepsilon_{it}\varepsilon_{jt}\right] &= \sigma_{ij}, & \forall i, j, t, \\ E\left[\varepsilon_{it}\varepsilon_{js}\right] &= 0, & \forall i, j, t \neq s. \end{aligned}$$

Stacking the observations over time we got

$$\begin{aligned} y_i &= \sum_{p=1}^{P} \gamma_p y_{i,-p} + X_i\beta + \eta_i \iota_T + \varepsilon_i \qquad (2.2) \\ &= W_i\delta + \eta_i \iota_T + \varepsilon_i, \end{aligned}$$

where $y_{i,-p} = (y_{i,1-p}, ..., y_{i,T-p})'$, $X_i = (x_{i1}, ..., x_{iT})'$, $\iota_T = (1, ..., 1)'$ a $T \times 1$ vector of ones, $\delta = (\gamma', \beta')'$, $\gamma = (\gamma_1, ..., \gamma_P)'$ and $W_i = [y_{i,-1} \vdots ... \vdots y_{i,-P} \vdots X_i]$.

Like Kiviet (1999) we decompose y into a relevant random component, denoted by a tilde, and irrelevant random plus deterministic components, denoted by a bar. The relevant random component is in some way related to the disturbance term ε_{it}, while the irrelevant component is not, i.e.

$$\begin{aligned}\tilde{y}_i &= \sum_{p=1}^{P} \gamma_p \tilde{y}_{i,-p} + \varepsilon_i, \qquad (2.3)\\ \bar{y}_i &= \sum_{p=1}^{P} \gamma_p \bar{y}_{i,-p} + X_i\beta + \eta_i \iota_T,\end{aligned}$$

where we use the assumption that we have fixed individual effects and only strict exogenous explanatory variables, i.e. $\tilde{X}_i = O$ and $\tilde{\eta}_i = 0$. For the initial values we assume

$$\begin{aligned}\tilde{y}_{i,1-p} &= 0, \qquad p = 1, ..., P, \qquad (2.4)\\ \bar{y}_{i,1-p} &= y_{i,1-p},\end{aligned}$$

so we condition on p fixed starting values. Introducing a $T \times T$ matrix L_T with ones on the first subdiagonal and zeros elsewhere and defining

$$\Gamma_T = \left(I_T - \sum_{p=1}^{P} \gamma_p L_T^p \right)^{-1}, \qquad (2.5)$$

we write for the relevant random components in (2.3)

$$\tilde{y}_i = \Gamma_T \varepsilon_i, \qquad i = 1, ..., N. \qquad (2.6)$$

To analyse the estimators in the next section we need a decomposition of the matrix $A_T W_i$ with $A_T = (I_T - \frac{1}{T}\iota_T \iota_T')$. We write

$$\begin{aligned}A_T \tilde{W}_i &= A_T[\tilde{y}_{i,-1} \vdots ... \vdots \tilde{y}_{i,-P} \vdots \tilde{X}_i]\\ &= \sum_{p=1}^{P} A_T L_T^p \Gamma_T \varepsilon_i e_p', \qquad (2.7)\end{aligned}$$

because $\tilde{X}_i = O$ and $A_T \tilde{y}_{i,-p} = A_T L_T^p \tilde{y}_i$ and where e_p is $(P + K) \times 1$ unit vector with its pth element equal to one.

Stacking the observations also across individuals one gets

$$y = W\delta + S\eta + \varepsilon, \qquad (2.8)$$

where y and ε are $NT \times 1$ vectors, $\eta = (\eta_1, ..., \eta_N)'$ is a $N \times 1$ vector and $W = [W_1' \vdots ... \vdots W_N']'$ and $S = (I_N \otimes \iota_T)$ are $NT \times (K+P)$ and $NT \times N$ matrices respectively. The assumptions about ε can be written as

$$\varepsilon \sim \mathcal{N}(0, \Omega), \tag{2.9}$$

where $\Omega = \Sigma \times I_T$ with Σ a $N \times N$ matrix with typical element σ_{ij}. For the relevant stochastic components in AW we find from (2.7)

$$\begin{aligned} A\tilde{W} &= \sum_{p=1}^{P} AL^p \Gamma \varepsilon e_p' \\ &= \sum_{p=1}^{P} \Pi_p \varepsilon e_p', \end{aligned} \tag{2.10}$$

where $A = I_N \otimes A_T$, $L = I_N \otimes L_T$, $\Gamma = I_N \otimes \Gamma_T$ and $\Pi_p = AL^p \Gamma$.

Model (2.8) with (2.9) is a generalized normal regression model and in the next sections estimation of both short- and long-run coefficients will be considered. The elements of the parameter vector δ are called short-run coefficients and $\theta = \beta/(1 - \sum_{p=1}^{P} \gamma_p)$ is the vector of long-run effects.

3. Short-run coefficient estimators

3.1. LSDV estimator

The ordinary least squares estimator for δ in (2.8) is the familiar Least Squares Dummy Variables (LSDV) or fixed effect estimator. Using partitioned regression results it can be expressed as

$$\begin{aligned} \hat{\delta}_{LSDV} &= (W'M_S W)^{-1} W' M_S y \\ &= (W'AW)^{-1} W'Ay, \end{aligned} \tag{3.1}$$

where $M_S = I_{NT} - S(S'S)^{-1}S' = A$. Note that $A = I_N \otimes A_T$ is the well-known within transformation which wipes out the individual effects η.

The LSDV estimation error is

$$\hat{\delta}_{LSDV} - \delta = (W'AW)^{-1} W'A\varepsilon, \tag{3.2}$$

which depends in a non-linear way on the stochastic term ε. Defining $Q = E\left[W'AW\right]$ and using a similar approach as in Kiviet (1995, 1999), but now for fixed N, the first factor in (3.2) may be expanded as

$$(W'AW)^{-1} = Q^{-1} - Q^{-1}(W'AW - Q)Q^{-1} + O_p(T^{-2}), \tag{3.3}$$

Hence, for the bias of the LSDV estimator we find

$$E\left[\hat{\delta}_{LSDV}-\delta\right]=2Q^{-1}E\left[W'A\varepsilon\right]-Q^{-1}E\left[W'AWQ^{-1}W'A\varepsilon\right]+o(T^{-1}). \qquad (3.4)$$

In Appendix A it is shown that the approximation for the bias in the LSDV estimator equals

$$E\left[\hat{\delta}_{LSDV}-\delta\right]=B_{LSDV}(T^{-1})+o(T^{-1}), \qquad (3.5)$$

where

$$\begin{aligned}
B_{LSDV}(T^{-1}) \;=\; & \sum_p tr(\Pi_p\Omega)Q^{-1}e_p-\sum_p Q^{-1}\bar{W}'\Pi_p\Omega A\bar{W}Q^{-1}e_p \\
& -\sum_p tr\left[Q^{-1}\bar{W}'\Pi_p\Omega A\bar{W}\right]Q^{-1}e_p \\
& -2\sum_p\sum_r\sum_s q_{rs}tr(\Omega\Pi_p'\Pi_r\Pi_s\Omega)Q^{-1}e_p, \qquad (3.6)
\end{aligned}$$

with the indices p, r and s running from 1 to P and

$$\begin{aligned}
\bar{W} &= E(W), \\
Q &= \bar{W}'A\bar{W}+\sum_p\sum_r tr(\Pi_p'\Pi_r\Omega)e_pe_r', \\
q_{rs} &= e_r'Q^{-1}e_s.
\end{aligned}$$

Using this result an operational bias corrected estimator, denoted by LSDVc, can be constructed as

$$\hat{\delta}_{LSDVc}=\hat{\delta}_{LSDV}-\hat{B}_{LSDV}(T^{-1}), \qquad (3.7)$$

using any consistent preliminary estimators for δ and Ω in $\hat{B}(T^{-1})$, e.g based on LSDV residuals. The corrected LSDV estimator will be unbiased upto order $O(T^{-1})$, i.e.

$$E\left[\hat{\delta}_{LSDVc}-\delta\right]=o(T^{-1}). \qquad (3.8)$$

3.2. (Feasible) Generalized LSDV estimator

The ordinary LSDV estimator in (3.1) does not take the covariance structure of ε into account. Hence, we analyse also the generalized LSDV estimator of δ, denoted by $\hat{\delta}_{GLSDV}$. The GLSDV estimator of δ is

$$\hat{\delta}_{GLSDV}=(W'A\Omega^{-1}AW)^{-1}W'A\Omega^{-1}Ay, \qquad (3.9)$$

and its estimation error is

$$\hat{\delta}_{GLSDV} - \delta = (W'A\Omega^{-1}AW)^{-1}W'A\Omega^{-1}A\varepsilon. \tag{3.10}$$

Defining $A^* = A\Omega^{-1}A$ and $Q^* = E[W'A^*W]$ the following expansion is valid under the assumptions made in section 2, i.e.

$$(W'A^*W)^{-1} = Q^{*-1} - Q^{*-1}(W'A^*W - Q^*)Q^{*-1} + O_p(T^{-2}), \tag{3.11}$$

Hence, for the bias of the GLSDV estimator we find

$$E\left[\hat{\delta}_{GLSDV} - \delta\right] = 2Q^{*-1}E[W'A^*\varepsilon] - Q^{*-1}E\left[W'A^*WQ^{*-1}W'A^*\varepsilon\right] + o(T^{-1}). \tag{3.12}$$

In Appendix A the following approximation is derived for the bias in the GLSDV estimator, i.e.

$$E\left[\hat{\delta}_{GLSDV} - \delta\right] = B_{GLSDV}(T^{-1}) + o(T^{-1}), \tag{3.13}$$

where

$$\begin{aligned} B_{GLSDV}(T^{-1}) &= \sum_p tr(\Pi_p)Q^{*-1}e_p - \sum_p Q^{*-1}\bar{W}'\Omega^{-1}\Pi_p A\bar{W}Q^{*-1}e_p \\ &\quad - \sum_p tr\left(Q^{*-1}\bar{W}'\Omega^{-1}\Pi_p A\bar{W}\right)Q^{*-1}e_p \\ &\quad -2\sum_p\sum_r\sum_s q^*_{rs} tr(\Pi'_p\Pi_r\Pi_s)Q^{*-1}e_p, \end{aligned} \tag{3.14}$$

with

$$\begin{aligned} Q^* &= \bar{W}'A\Omega^{-1}A\bar{W} + \sum_p\sum_r tr(\Pi'_p\Pi_r)e_pe'_r, \\ q^*_{rs} &= e'_rQ^{*-1}e_s. \end{aligned}$$

In practice the GLSDV estimator cannot be calculated because Ω is unknown. We therefore analyse also the two-step feasible GLSDV estimator

$$\hat{\delta}_{FGLSDV} = (W'A\hat{\Omega}^{-1}AW)^{-1}W'A\hat{\Omega}^{-1}Ay, \tag{3.15}$$

where the covariance matrix Ω is consistently estimated using the LSDV residuals, i.e.

$$\begin{aligned} \hat{\Omega} &= \hat{\Sigma} \otimes I_T, \\ \hat{\sigma}_{ij} &= \frac{(y_i - W_i\hat{\delta}_{LSDV})'A_T(y_j - W_j\hat{\delta}_{LSDV})}{T}. \end{aligned} \tag{3.16}$$

The estimation error of the FGLSDV estimator is

$$\hat{\delta}_{FGLSDV} - \delta = (W'A\hat{\Omega}^{-1}AW)^{-1}W'A\hat{\Omega}^{-1}A\varepsilon, \tag{3.17}$$

In Appendix B it is shown that as long as a consistent estimator is used for Ω the bias in the FGLSDV estimator is

$$E\left[\hat{\delta}_{FGLSDV} - \delta\right] = B_{FGLSDV}(T^{-1}) + o(T^{-1}), \tag{3.18}$$

with

$$B_{FGLSDV}(T^{-1}) = B_{GLSDV}(T^{-1}) + o(T^{-1}). \tag{3.19}$$

In other words, the bias approximation to order $O(T^{-1})$ is equal for the FGLSDV and GLSDV estimators. Hence, we can construct an operational bias corrected estimator, denoted by FGLSDVc, as

$$\hat{\delta}_{FGLSDVc} = \hat{\delta}_{FGLSDV} - \hat{B}_{FGLSDV}(T^{-1}), \tag{3.20}$$

using any consistent preliminary estimators for δ and Ω in $\hat{B}_{FGLSDV}(T^{-1})$. The corrected FGLSDV estimator will be unbiased upto order $O(T^{-1})$, i.e.

$$E\left[\hat{\delta}_{FGLSDVc} - \delta\right] = o(T^{-1}). \tag{3.21}$$

4. Estimation under linear restrictions

In practice, model (2.8) is often estimated in a restricted version for several reasons. First, economic theory may a priori impose certain values for some of the model parameters. For example, in money demand studies often an income or price elasticity of unity is imposed. Second, econometric modelling may lead, for example, to the exclusion of variables from the most general model to arrive at a more parsimoneous specification. In this section we will develop bias expressions for the restricted LSDV, GLSDV and FGLSDV short-run coefficient estimators along the lines of Kiviet and Phillips (1994). Consider a set of J general linear restrictions on δ, i.e.

$$R\delta = r, \tag{4.1}$$

where R is $J \times (K + P)$ and $r = J \times 1$. Note that no restrictions regarding the individual constants will be considered as they are typically filtered out before estimation.

4.1. restricted LSDV estimator

The LSDV estimator of δ under the restrictions (4.1) is

$$\hat{\delta}_{LSDV,R} = \hat{\delta}_{LSDV} - (W'AW)^{-1}R'\left[R(W'AW)^{-1}R'\right]^{-1}(R\hat{\delta}_{LSDV} - r). \qquad (4.2)$$

Below we shall derive a bias expression upto order $O(T^{-1})$ for the estimator in (4.2). The relation between the unrestricted and restricted LSDV estimators can be written as

$$\hat{\delta}_{LSDV,R} - \delta = F_{(W)}(\hat{\delta}_{LSDV} - \delta), \qquad (4.3)$$

where $F_{(W)} = I - (W'AW)^{-1}R'\left[R(W'AW)^{-1}R'\right]^{-1}R$. Hence, the bias in the restricted LSDV estimator can be expressed as

$$E\left[\hat{\delta}_{LSDV,R} - \delta\right] = E\left[F_{(W)}(W'AW)^{-1}W'A\varepsilon\right]. \qquad (4.4)$$

The evaluation of the right hand side of (4.5) is not straightforward because the elements of $F_{(W)}$ are also stochastic. However, applying a similar reasoning as in the proof of Theorem 2 in Kiviet and Phillips (1994) it can be shown that

$$E\left[\hat{\delta}_{LSDV,R} - \delta\right] = -F_{(\bar{W})}Q^{-1}E\left[W'AWF_{(\bar{W})}Q^{-1}W'A\varepsilon\right] + o(T^{-1}), \qquad (4.5)$$

where $F_{(\bar{W})} = I - Q^{-1}R'\left[RQ^{-1}R'\right]^{-1}R$. The evaluation of the right hand side in (4.5) is now similar to earlier expectations and one gets

$$E\left[\hat{\delta}_{LSDV,R} - \delta\right] = B_{LSDV,R}(T^{-1}) + o(T^{-1}), \qquad (4.6)$$

where

$$\begin{aligned} B_{LSDV,R}(T^{-1}) &= \sum_p tr(\Pi_p\Omega)F_{(\bar{W})}Q^{-1}e_p \qquad (4.7)\\ &\quad -\sum_p F_{(\bar{W})}Q^{-1}\bar{W}'\Pi_p\Omega A\bar{W}F_{(\bar{W})}Q^{-1}e_p \\ &\quad -\sum_p tr\left[F_{(\bar{W})}Q^{-1}\bar{W}'\Pi_p\Omega A\bar{W}\right]F_{(\bar{W})}Q^{-1}e_p \\ &\quad -2\sum_p\sum_r\sum_s e_r'F_{(\bar{W})}Q^{-1}e_s tr(\Omega\Pi_p'\Pi_r\Pi_s\Omega)F_{(\bar{W})}Q^{-1}e_p. \end{aligned}$$

4.2. restricted (Feasible) Generalized LSDV estimator

The GLSDV estimator of δ under the restrictions (4.1) is

$$\hat{\delta}_{GLSDV,R} = \hat{\delta}_{GLSDV} - (W'A\Omega^{-1}W)^{-1}R'\left[R(W'A\Omega^{-1}W)^{-1}R'\right]^{-1}(R\hat{\delta}_{GLSDV} - r), \qquad (4.8)$$

which can be rearranged as

$$\hat{\delta}_{GLSDV,R} - \delta = F_{(W\Omega)}(\hat{\delta}_{GLSDV} - \delta), \tag{4.9}$$

where $F_{(W)} = I - (W'A\Omega^{-1}W)^{-1}R'\left[R(W'A\Omega^{-1}W)^{-1}R'\right]^{-1}R$. The bias of the restricted GLSDV estimator is

$$E\left[\hat{\delta}_{GLSDV,R} - \delta\right] = E\left[F_{(W\Omega)}(W'A\Omega^{-1}W)^{-1}W'A\Omega^{-1}\varepsilon\right], \tag{4.10}$$

which can be written as

$$E\left[\hat{\delta}_{GLSDV,R} - \delta\right] = -F_{(\bar{W}\Omega)}Q^{-1}E\left[W'A\Omega^{-1}WF_{(\bar{W}\Omega)}Q^{-1}W'A\Omega^{-1}\varepsilon\right] + o(T^{-1}), \tag{4.11}$$

where $F_{(\bar{W}\Omega)} = I - Q^{*-1}R'\left[RQ^{*-1}R'\right]^{-1}R$. The evaluation of the right hand side is now similar to earlier derivations and one gets

$$E\left[\hat{\delta}_{GLSDV,R} - \delta\right] = B_{GLSDV,R}(T^{-1}) + o(T^{-1}), \tag{4.12}$$

where

$$\begin{aligned} B_{GLSDV,R}(T^{-1}) \;=\; & \sum_p tr(\Pi_p)F_{(\bar{W}\Omega)}Q^{*-1}e_p \qquad (4.13)\\ & -\sum_p F_{(\bar{W}\Omega)}Q^{*-1}\bar{W}'\Omega^{-1}\Pi_p A\bar{W}F_{(\bar{W}\Omega)}Q^{*-1}e_p \\ & -\sum_p tr\left[F_{(\bar{W}\Omega)}Q^{*-1}\bar{W}'\Omega^{-1}\Pi_p A\bar{W}\right]F_{(\bar{W}\Omega)}Q^{*-1}e_p \\ & -2\sum_p\sum_r\sum_s e_r'F_{(\bar{W})}Q^{*-1}e_s tr(\Pi_p'\Pi_r\Pi_s)F_{(\bar{W}\Omega)}Q^{*-1}e_p. \end{aligned}$$

The restricted GLSDV estimator is not applicable because it depends on the unknown covariance matrix Ω. Hence, we consider the feasible GLSDV estimator under restrictions, i.e.

$$\hat{\delta}_{FGLSDV,R} = \hat{\delta}_{FGLSDV} - (W'A\hat{\Omega}^{-1}W)^{-1}R'\left[R(W'A\hat{\Omega}^{-1}W)^{-1}R'\right]^{-1}(R\hat{\delta}_{FGLSDV} - r). \tag{4.14}$$

In Appendix C it is shown is shown that

$$E\left[\hat{\delta}_{FGLSDV,R}\right] = B_{FGLSDV,R}(T^{-1}) + o(T^{-1}), \tag{4.15}$$

with

$$B_{FGLSDV,R}(T^{-1}) = B_{GLSDV,R}(T^{-1}) + o(T^{-1}), \tag{4.16}$$

i.e. the bias approximation to order $O(T^{-1})$ is the same for the restricted FGLSDV and GLSDV estimators. Hence, a bias corrected restricted FGLSDV estimator can be constructed with the expression in (4.13), which is unbiased upto order T^{-1}.

5. Long-run coefficients

The short-run estimators in the previous sections can be used to construct estimators for the long-run effects θ, which are defined by

$$\hat{\theta} = \hat{\beta}/(1 - \iota_P'\hat{\gamma}), \tag{5.1}$$

where $\hat{\beta}$ and $\hat{\gamma}$ are any of the estimators considered before and ι_P is a $P \times 1$ vector of ones. If bias corrected short-run estimators like (3.7) or (3.20) are used to correct for bias in (5.1) the resulting long-run estimator is called "naive" by Pesaran and Zhao (1999), which analyse several estimators of the long-run coefficients in the context of the dynamic random coefficient model. The "naive" or indirect way of bias correction in (5.1) does not lead to an estimator unbiased to order $O(T^{-1})$. Note that the estimation error $\hat{\delta} - \delta = O_p(T^{-\frac{1}{2}})$ irrespective of the estimator used. Hence, in general we can write

$$\begin{aligned}
\hat{\theta} &= \frac{\hat{\beta}}{(1 - \iota_P'\gamma) - \iota_P'(\hat{\gamma} - \gamma)} \qquad (5.2)\\
&= \frac{\hat{\beta}}{(1 - \iota_P'\gamma)}\left[1 - \frac{\iota_P'(\hat{\gamma} - \gamma)}{(1 - \iota_P'\gamma)}\right]^{-1}\\
&= \frac{\hat{\beta}}{(1 - \iota_P'\gamma)} + \frac{\hat{\beta}}{(1 - \iota_P'\gamma)}\frac{\iota_P'(\hat{\gamma} - \gamma)}{(1 - \iota_P'\gamma)} + \frac{\hat{\beta}}{(1 - \iota_P'\gamma)}\frac{(\iota_P'\hat{\gamma} - \iota_P'\gamma)^2}{(1 - \iota_P'\gamma)^2} + o_p(T^{-1})\\
&= \frac{\beta}{(1 - \iota_P'\gamma)} + \frac{1}{(1 - \iota_P'\gamma)}(\hat{\beta} - \beta) + \frac{\beta}{(1 - \iota_P'\gamma)^2}(\iota_P'\hat{\gamma} - \iota_P'\gamma)\\
&\quad + \frac{1}{(1 - \iota_P'\gamma)^2}(\hat{\beta} - \beta)(\iota_P'\hat{\gamma} - \iota_P'\gamma) + \frac{\beta}{(1 - \iota_P'\gamma)^3}(\iota_P'\hat{\gamma} - \iota_P'\gamma)^2 + o_p(T^{-1}).
\end{aligned}$$

Therefore, we find for the bias in estimating θ the following

$$\begin{aligned}
E\left[\hat{\theta} - \theta\right] &= \frac{1}{(1 - \iota_P'\gamma)}E(\hat{\beta} - \beta) + \frac{\beta}{(1 - \iota_P'\gamma)^2}E(\iota_P'\hat{\gamma} - \iota_P'\gamma) \qquad (5.3)\\
&\quad + \frac{1}{(1 - \iota_P'\gamma)^2}E(\hat{\beta} - \beta)(\iota_P'\hat{\gamma} - \iota_P'\gamma) + \frac{\beta}{(1 - \iota_P'\gamma)^3}E(\iota_P'\hat{\gamma} - \iota_P'\gamma)^2 + o(T^{-1}),
\end{aligned}$$

which is $O(T^{-1})$ because all explicit terms in (5.3) are in general non-zero and of order $O(T^{-1})$.

Pesaran and Zhao (1999) propose a direct way of bias correction. Using any original uncorrected estimator, i.e. (3.1), (3.15), (4.2) or (4.14) and rearranging (5.3) we find for the bias in the long-run coefficientvector θ

$$E\left[\hat{\theta} - \theta\right] = B_\theta(T^{-1}) + o(T^{-1}), \tag{5.4}$$

where

$$B_\theta(T^{-1}) = \frac{1}{(1-\iota_P'\gamma)^2}\left[(1-\iota_P'\gamma)\left(B_\beta + \theta\iota_P' B_\gamma\right) + Cov(\hat{\beta},\iota_P'\hat{\gamma}) + \theta Var(\iota_P'\hat{\gamma})\right], \tag{5.5}$$

with $B_\beta = E(\hat{\beta}-\beta)$ and $B_\gamma = E(\hat{\gamma}-\gamma)$. This can be used to construct corrected estimators of the long-run coefficients, which are unbiased upto order $O(T^{-1})$.

6. Estimation of asymptotic standard errors

Standard errors of coefficient estimators can be estimated by either using asymptotic variance expressions following from limiting distributions or applying bootstrap procedures. To save space we will discuss both approaches for the LSDV and FGLSDV estimators only, but similar results can be derived for bias corrected, restricted or long-run estimators. Let us define

$$\begin{aligned} \Upsilon_{WAW} &= \operatorname*{plim}_{T\to\infty} \frac{1}{T} W'AW \\ \Upsilon_{WA\Omega W} &= \operatorname*{plim}_{T\to\infty} \frac{1}{T} W'A\Omega W \\ \Upsilon^*_{WAW} &= \operatorname*{plim}_{T\to\infty} \frac{1}{T} W'A\Omega^{-1} W. \end{aligned} \tag{6.1}$$

>From the usual asymptotic reasoning it follows that

$$\sqrt{T}\left(\hat{\delta}_{LSDV} - \delta\right) \underset{T\to\infty}{\overset{d}{\longrightarrow}} \mathcal{N}\left[0,\ \Upsilon^{-1}_{WAW}\Upsilon_{WA\Omega W}\Upsilon^{-1}_{WAW}\right], \tag{6.2}$$

and

$$\sqrt{T}\left(\hat{\delta}_{GLSDV} - \delta\right) \underset{T\to\infty}{\overset{d}{\longrightarrow}} \mathcal{N}\left[0,\ \Upsilon^{*-1}_{WAW}\right]. \tag{6.3}$$

Furthermore, assuming

$$\begin{aligned} \operatorname*{plim}_{T\to\infty} \frac{1}{T} W'A(\hat{\Omega} - \Omega^{-1})W &= 0 \\ \operatorname*{plim}_{T\to\infty} \frac{1}{T} W'A(\hat{\Omega}^{-1} - \Omega^{-1})\varepsilon &= 0, \end{aligned}$$

one gets

$$\sqrt{T}\left(\hat{\delta}_{FGLSDV} - \delta\right) \underset{T\to\infty}{\overset{d}{\longrightarrow}} \mathcal{N}\left[0,\ R^{*-1}_{WAW}\right]. \tag{6.4}$$

Regarding restricted estimators or long-run estimators analogous limiting results can be derived. Note that bias corrected estimators have the same limiting behavior as their uncorrected counterparts.

The limiting distributions above can be used to approximate the asymptotic distributions of the LSDV and FGLSDV estimators, i.e.

$$\begin{aligned} \hat{\delta}_{LSDV} &\underset{T\to\infty}{\overset{a}{\sim}} \mathcal{N}\left[\delta,\ V_{LSDV}\right], \\ \hat{\delta}_{FGLSDV} &\underset{T\to\infty}{\overset{a}{\sim}} \mathcal{N}\left[\delta,\ V_{FGLSDV}\right], \end{aligned} \tag{6.5}$$

with

$$\begin{aligned} V_{LSDV} &= (W'AW)^{-1}W'A\Omega W(W'AW)^{-1}, \\ V_{FGLSDV} &= (W'A\Omega^{-1}W)^{-1}. \end{aligned} \tag{6.6}$$

Hence, using a consistent estimator for Ω the asymptotic variance matrix of the LSDV or FGLSDV estimator can be estimated consistently with the expressions in (6.6).

Various authors (Freedman and Peters, 1984; Beck and Katz, 1995) note that for the FGLSDV estimator the expression in (6.6) is very inaccurate in finite samples, i.e. true standard deviations are underestimated dramatically using conventional first-order asymptotic approximations. An alternative approach is using bootstrap procedures to estimate standard errors. We propose the following parametric resampling scheme, i.e.

- Obtain the estimators $\hat{\delta}$ and $\hat{\Sigma}$ (LSDV, FGLSDV)
- Take a random sample $\varepsilon^{(b)} \sim \mathcal{N}\left[0, \hat{\Sigma} \otimes I_T\right]$
- Calculate $Ay^{(b)} = AW^{(b)}\hat{\delta} + A\varepsilon^{(b)}$
- Estimate the model with the resampled data $(Ay^{(b)}, AW^{(b)})$ giving the bootstrap estimator $\hat{\delta}^{(b)}$

Remark that due to the presence of lagged values of y in the regressor matrix W a recursive sampling scheme is used. Because the normality assumption is used in the derivation of the bias expressions we employ this assumption here and use a parametric bootstrap procedure contrary to Freedman and Peters (1984).

Considering the bootstrap LSDV and FGLSDV estimators, it can be shown that the following limiting results hold, i.e.

$$\sqrt{T}\left(\hat{\delta}^{(b)}_{LSDV} - \delta\right) \underset{T\to\infty}{\overset{d}{\longrightarrow}} \mathcal{N}\left[0,\ \Upsilon^{-1}_{WAW}\Upsilon_{WA\Omega W}\Upsilon^{-1}_{WAW}\right], \tag{6.7}$$

$$\sqrt{T}\left(\hat{\delta}^{(b)}_{FGLSDV} - \delta\right) \underset{T\to\infty}{\overset{d}{\longrightarrow}} \mathcal{N}\left[0,\ R^{*-1}_{WAW}\right]. \tag{6.8}$$

Hence, we can use the bootstrap estimator of the asymptotic variance matrix instead of the expressions in (6.6). Repeating the steps above B times, B realisations of $\hat{\delta}^{(b)}$ are created and

$$\hat{V}_{LSDV} = \frac{1}{B-1}\sum_{b=1}^{B}\left[\hat{\delta}_{LSDV}^{(b)} - \frac{1}{B}\sum_{b=1}^{B}\hat{\delta}_{LSDV}^{(b)}\right]\left[\hat{\delta}_{LSDV}^{(b)} - \frac{1}{B}\sum_{b=1}^{B}\hat{\delta}_{LSDV}^{(b)}\right]', \tag{6.9}$$

$$\hat{V}_{FGLSDV} = \frac{1}{B-1}\sum_{b=1}^{B}\left[\hat{\delta}_{FGLSDV}^{(b)} - \frac{1}{B}\sum_{b=1}^{B}\hat{\delta}_{FGLSDV}^{(b)}\right]\left[\hat{\delta}_{FGLSDV}^{(b)} - \frac{1}{B}\sum_{b=1}^{B}\hat{\delta}_{FGLSDV}^{(b)}\right]',$$

are the bootstrap estimators of the asymptotic variance matrices of $\hat{\delta}_{LSDV}$ and $\hat{\delta}_{FGLSDV}$. The results in Freedman and Peters (1984) show that for the FGLSDV estimator the bootstrap variance estimator in (6.9) underestimates the true covariance matrix much less than the conventional expression in (6.6). Regarding bias corrected, restricted or long-run estimators similar resampling schemes, but now conditional on those estimators, can be exploited to calculate bootstrap standard errors.

7. The demand for money in the European Union

In this section the performance of the various estimators will be illustrated with an empirical application. Money demand in the European Union is analysed by panel data techniques. Earlier empirical studies (Kremers and Lane, 1990; Monticelli and Papi, 1996; Fase and Winder, 1998) use aggregated time series to estimate EU wide money demand functions, but considering the EU countries as a cross-section one can possibly use panel data techniques.

As compared with the aggregate time series approach the use of panel data techniques is different at least in three respects. First of all, it is not necessary to convert money stock and income measures for the different countries into one common currency as is the case for the aggregated time series approach. As long as a suitable conversion measure and functional form are chosen, the individual constants in the panel data model will absorb the effects of this conversion.

Second, as in dynamic panel data models the individual effects are typically filtered out before estimation, the cross-section dimension in the panel implies extra data to estimate the same number of unknown parameters. Hence, it seems possible to use fewer time observations as in the aggregate time series approach. To the extent that one is primarily interested in a description of the very near past this is convenient, because especially the short-run parameters of the money demand relationship may not have been constant over the last few decades.

Third, an important specification issue in modeling money demand for a group of countries is interdependence between individual countries money demand. Spillover effects between countries arise as a result of international integration. Also financial integration increases the elasticity of national money demand with respect to the return on foreign assets resulting in increased currency substitution. Hence, apart from domestic variables national money demand is likely to depend on foreign variables. The aggregated time series approach may overcome these specification problems because it internalizes interdependencies between individual countries. However, it is clearly not capable of adressing the presence and importance of any spillover effects. In contrast, panel data models allowing for interdependencies between country specific disturbance terms are better suited for this purpose. Also, foreign variables can be explicitly incorporated as explanatory variables.

We shall examine the possibility to estimate standard money demand functions for the European Union using panel data techniques. More in particular, the effectiveness of bias corrected estimators will be examined. Next, we will focus on spillovers between countries.

7.1. standard money demand functions

The dataset used is from Fase and Winder (1998) and contains time series on several variables for Belgium (BE), Denmark (DK), Germany (GE), United Kingdom (UK), Finland (FIN), France (FR), Greece (GR), Ireland (IE), Italy (IT), The Netherlands (NL), Austria (AT), Portugal (PT), Spain (SP) and Sweden (SWE). Together with Luxembourg, which is not included in the dataset, these countries currently form the European Union. The time series of the variables have quarterly frequency, are not seasonally adjusted and are collected over the period 1970-1995. The variables in the dataset are M1, M2, M3, real GNP, GNP deflator, short- and long-term interest rates.

For each of the definitions of money stock specification (2.1) is estimated using

$$x_{it} = (\ln gnp_{it}, \ln gnp_{i,t-1}, rs_{it}, rs_{i,t-1}, rl_{it}, rl_{i,t-1}, ir_{it}, ir_{i,t-1}, s_{1,t}, s_{2,t}, s_{3,t})' \quad (7.1)$$

and where the dependent variable y_{it} is the logarithm of real money stock, i.e. $\ln(M1/P)_{it}$, $\ln(M2/P)_{it}$ or $\ln(M3/P)_{it}$. The explanatory variables are contemporaneous and one-period lagged values of real income (gnp), short- (rs) and long-term (rl) interest rates and the inflation rate (ir). To account for seasonal patterns a set of seasonal dummy variables (s_1, s_2 and s_3) is included. Furthermore, lagged values of the dependent variable are incorporated to model autoregressive dynamic adjustments. Separate regressions for the individual countries, which are not reported here, suggest to include one lagged value for the $M1$ specification and to use two lagged values for the $M2$ and $M3$ specifications. Hence, the dimension of the parameter vector δ is $K+1$ for the $M1$ specification and $K+2$ for $M2$ and $M3$ with $K = 11$.

In order to make valid inference with panel data techniques both parameter constancy through time and over countries must hold to some extent. To avoid parameter variability through time, we have chosen to analyse a relatively short time span, i.e. only the years after the German reunification in 1990 are considered and the sample period is 1991:I-1995:IV. As far as parameter constancy over countries is concerned, it is reasonable to assume that by taking a recent period the problem of parameter heterogeneity across countries is mitigated. We are therefore confident to impose common slope vectors, but allow for individual specific effects.

The number of countries analysed is $N = 14$. For $M1$ one period is lost in constructing the lagged value of the dependent variable, so for this specification the first estimation period is 1991:II and $T = 19$. The estimation period for $M2$ and $M3$ is 1991:III-1995:IV because one extra period is lost in constructing the two-period lagged value of the dependent variable and $T = 18$. In all tables only the bootstrap standard errors are given, because, as argued before, standard analytical variance expressions may be inaccurate here. The number of bootstrap replications used is 100.

For $M1$ the estimation results of the short-run coefficients are in Table 1, while Table 2 gives the estimates for the long-run coefficients. Regarding the short-run coefficients the bias corrected LSDV and FGLSDV estimators produce in general a higher autoregressive coefficient than the original estimators, while the bias correction in the other coefficients seems to be small. Considering the variance estimators the decrease in variance is apparant when using the FGLSDV estimator compared with the LSDV estimator. The table with long-run coefficients reasserts these efficiency gains.

The results for $M2$ are in Tables 3 and 4. As noted before, a two-period lagged value of the dependent variable is included also, so the bias corrections according to (3.6) and (3.14) have been applied for $P = 2$. The sum of the autoregressive coefficients is considerably higher than the same parameter for $M1$ implying more persistence in the demand for $M2$ as compared with $M1$. Again the difference in accuracy of ordinary and generalized LSDV is apparent, i.e. estimated standard deviations are lower for the FGLSDV estimator. The long-run coefficients in Table 4 are again plausible.

Tables 5 and 6 contain the estimation results for $M3$. The same remarks on the specification of $M2$ can be made for $M3$ too. For $M3$ the long-run effect of inflation has the a priori expected negative sign although it is still poorly determined. Also the coefficient of the two-period lagged dependent variable is small and not significant despite its significance in some of the individual country regressions.

We compare the general pattern of the long-term estimates with the (semi) elasticities found in earlier research based on the aggregated time series approach.

Overviews of these results can be found in Fase and Winder (1993) and Monticelli and Papi (1996). If not restricted to one, the income elasticity is often found close to unity for $M1$ and larger than one for both $M2$ and $M3$. In this study, the long-run estimates of the various corrected estimators reflect this pattern in general. As far as the interest rate semi-elasticities are concerned, in general they are close to the estimates found in earlier studies. The only exception is the long-term interest rate effect for $M2$, which is found to be particularly strong as compared with other studies. Considering the inflation rate no significant long-run effects have been found contrary to earlier studies.

To illustrate the empirical performance of the restricted bias corrected estimators we impose the restrictions that the long-run income elasticity is unity. The estimation results for the long-run parameters in Table 2, 4 and 6 show that this restriction is not rejected for all three monetary aggregates. In Table 7 the estimation results for the long-run parameters are shown for $M3$. The estimation results show hardly any differences with the pattern of the unrestricted estimators. Similar results are found for short-run estimators and alternative monetary aggregates.

7.2. spillover effects

We will analyse possible spillover effects between countries in two ways. First, following Lane and Poloz (1992) and Angeloni et al. (1994) the covariance matrix of the disturbances is analysed more thoroughly to detect any interdependencies between countries. Second, we will focus on the importance of currency substitution and include foreign interest rates into the national money demand equations (Lane and Poloz, 1992).

The likelihood ratio (LR) test for the null hypothesis of zero cross-correlations, i.e. Σ is a diagonal matrix Σ_d, is

$$LR = T\left(\ln|\tilde{\Sigma}_d| - \ln|\hat{\Sigma}|\right), \tag{7.2}$$

where Σ_d is estimated under the restrictions. Under the null hypothesis the statistic LR is χ^2 distributed with $\frac{1}{2}N(N-1)$ degrees of freedom. The values of LR for the money demand equations estimated in the previous subsection are 327.35, 277.47 and 274.26 for $M1$, $M2$ and $M3$ respectively, clearly rejecting the null hypothesis. Hence, we conclude that interdependencies between countries are important.

Next, we include foreign interest rates in the specifications to quantify the effects of portfolio diversification on domestic money demand. We include a weighted average of foreign short-term interest rates of other EU countries, i.e.

$$\begin{aligned} rs_{it}^{f} &= \sum_{j=1}^{N} w_{jt} rs_{jt} - w_{it} rs_{it}, \\ rl_{it}^{f} &= \sum_{j=1}^{N} w_{jt} rl_{jt} - w_{it} rl_{it}, \\ w_{it} &= \frac{c_i gnp_{it}}{\sum_{j=1}^{N} c_j gnp_{jt}} \end{aligned} \tag{7.3}$$

where the constant c_i is the nominal exchange rate of country i against the DMark in 1985. The weights are depending on national incomes of the individual countries denoted in a single currency. For each of the definitions of money stock similar specifications as before are estimated using

$$\begin{aligned} M1 &: \ln gnp, rs, (rs^f - rs), s_1, s_2, s_3, \\ &: \ln gnp, rl, (rl^f - rl), s_1, s_2, s_3, \\ M2, M3 &: \ln gnp, rs, rl, (rs^f - rs), (rl^f - rs), s_1, s_2, s_3, \end{aligned} \tag{7.4}$$

as explanatory variables.

The estimation results are in Table 8. To save space only the implied long-run effects of the corrected $FGLSDV$ estimator are given, the pattern of the short-run estimates and the differences between estimators are similar as the results in the previous subsection. The estimation results show that especially foreign long-term interest rates do influence domestic money demand. If foreign long-term interest rates increase relative to local rates domestic money demand drops. These results are robust to the monetary aggregate used. Regarding short-term interest rates no significant long-run effects are found.

8. Concluding Remarks

In this study we analyse various least squares based estimation procedures for the dynamic panel data model with fixed individual effects and a nonscalar covariance matrix. Because of the typical dimensions of the panel at hand, which is dominated by its time dimension, least squared based methods are used instead of instrumental variables techniques. The latter are commonly used in the typical small T, large N panel.

Despite its superior performance in this type of panel, least squares estimators are biased in dynamic models and the bias may be substantial in finite samples. Hence, approximation formulae for the bias of the various estimators are developed upto order $O(T^{-1})$ using results of Kiviet (1995, 1999) and related work on bias approximation. The resulting bias expression are then used to construct bias corrected estimators. From the bias approximations it is seen that falsely assuming a scalar covariance matrix will lead to corrected estimators, which still contain a bias term of order $O(T^{-1})$. This result underlines the importance of taking into account the true covariance structure of the disturbances.

With panel data techniques money demand functions for $M1$, $M2$ and $M3$ are estimated for the EU area as a whole. As far as we know, until now only aggregate time series studies have been undertaken in this area. As is shown by the estimation results the bias terms can be substantial in this type of data reasserting the importance of more refined estimation techniques. The efficiency gains of exploiting the heteroscedasticity and cross-correlation patterns between countries are sometimes considerable.

The empirical results show that panel data estimators produce plausible long-run effects commonly found in other empirical studies on money demand. As such, the panel data approach is a valuable alternative to the aggregate time series approach. Moreover, exploiting the cross-sectional variation in the panel we are able to identify and estimate any interdependencies between countries. Significant spillover effects between EU countries are found as the cross-correlations in the disturbance covariance matrix are significantly different from zero. Also foreign long-term interest rates turn out to have explanatory power for domestic money demand.

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A. Bias in the ordinary and generalized LSDV estimators

In this appendix the expressions (3.6) and (3.14) are derived. Using the decomposition of W into a irrelevant and relevant stochastic part, i.e. $W = \bar{W} + \tilde{W}$, and exploiting the normality of ε we have

$$\begin{aligned} E\left[W'A\varepsilon\right] &= E\left[\bar{W}'A\varepsilon\right] + E\left[\tilde{W}'A\varepsilon\right] \\ &= \sum_{p=1}^{P} tr(\Pi_p\Omega)e_p, \end{aligned} \tag{A.1}$$

using $A\tilde{W} = \sum_{p=1}^{P} \Pi_p \varepsilon e_p'$. For Q we write

$$\begin{aligned} Q &= E\left[W'AW\right] \\ &= \bar{W}'A\bar{W} + E\left[\tilde{W}'A\tilde{W}\right] \\ &= \bar{W}'A\bar{W} + \sum_p \sum_r tr(\Pi_p'\Pi_r\Omega)e_p e_r'. \end{aligned} \tag{A.2}$$

Omitting terms with zero moments we have also

$$\begin{aligned} E\left[W'AWQ^{-1}W'A\varepsilon\right] &= \bar{W}'A\bar{W}Q^{-1}E\left[\tilde{W}'A\varepsilon\right] + \bar{W}'E\left[A\tilde{W}Q^{-1}\tilde{W}'A\varepsilon\right] \\ &\quad + E\left[\tilde{W}'A\bar{W}Q^{-1}\bar{W}'A\varepsilon\right] + E\left[\tilde{W}'A\tilde{W}Q^{-1}\tilde{W}'A\varepsilon\right] \\ &= \bar{W}'A\bar{W}Q^{-1}\sum_p tr(\Pi_p\Omega)e_p \\ &\quad + \sum_p \bar{W}'\Pi_p\Omega A\bar{W}Q^{-1}e_p + \sum_p tr\left[Q^{-1}\bar{W}'\Pi_p\Omega A\bar{W}\right]e_p \\ &\quad + \sum_p \sum_r \sum_s q_{rs}\left[tr(\Pi_p'\Pi_r\Omega)tr(\Pi_s\Omega) + 2tr(\Omega\Pi_p'\Pi_r\Pi_s\Omega)\right]e_p \\ &= \sum_p tr(\Pi_p\Omega)e_p + \sum_p \bar{W}'\Pi_p\Omega A\bar{W}Q^{-1}e_p \\ &\quad + \sum_p tr\left[Q^{-1}\bar{W}'\Pi_p\Omega A\bar{W}\right]e_p \\ &\quad + 2\sum_p \sum_r \sum_s q_{rs}tr(\Omega\Pi_p'\Pi_r\Pi_s\Omega)e_p, \end{aligned} \tag{A.3}$$

where $q_{rs} = e_r'Q^{-1}e_s$ and we have used $\bar{W}'A\bar{W} = Q - \sum_s \sum_r tr(\Pi_s'\Pi_r\Omega)e_s e_r'$. Using (A.1) and (A.3) in (3.4) the result in (3.6) readily follows.

For the bias in the GLSDV estimator the expectations in (3.12) have to be evaluated. Now

$$E\left[W'A^*\varepsilon\right] = E\left[\bar{W}'A\Omega^{-1}A\varepsilon\right] + E\left[\tilde{W}'A\Omega^{-1}A\varepsilon\right]$$

$$= \sum_{p=1}^{P} E\left[\varepsilon'\Omega^{-1}\Pi_p\varepsilon\right] e_p$$
$$= \sum_{p=1}^{P} tr(\Pi_p)e_p, \tag{A.4}$$

and for Q^* we write

$$\begin{aligned} Q^* &= \bar{W}'A\Omega^{-1}A\bar{W} + E\left[\tilde{W}'A\Omega^{-1}A\tilde{W}\right] \\ &= \bar{W}'A\Omega^{-1}A\bar{W} + \sum_p\sum_r tr(\Pi_p'\Pi_r)e_pe_r'. \end{aligned} \tag{A.5}$$

Also we write omitting terms with zero moments

$$\begin{aligned} E\left[W'A^*WQ^{*-1}W'A^*\varepsilon\right] &= \bar{W}'A^*\bar{W}Q^{*-1}E\left[\tilde{W}'A^*\varepsilon\right] + \bar{W}'E\left[A^*\tilde{W}Q^{*-1}\bar{W}'A^*\varepsilon\right] \\ &\quad + E\left[\tilde{W}'A^*\bar{W}Q^{*-1}\bar{W}'A^*\varepsilon\right] + E\left[\tilde{W}'A^*\tilde{W}Q^{*-1}\tilde{W}'A^*\varepsilon\right] \\ &= \bar{W}'A^*\bar{W}Q^{*-1}\sum_p tr(\Pi_p)e_p \\ &\quad + \sum_p \bar{W}'\Omega^{-1}\Pi_pA\bar{W}Q^{*-1}e_p + \sum_p tr\left(Q^{*-1}\bar{W}'\Omega^{-1}\Pi_pA\bar{W}\right)e_p \\ &\quad + \sum_p\sum_r\sum_s q^*_{rs}\left[tr(\Pi_p'\Pi_r)tr(\Pi_s) + 2tr(\Pi_p'\Pi_r\Pi_s)\right]e_p \\ &= \sum_p tr(\Pi_p)e_p + \sum_p \bar{W}'\Omega^{-1}\Pi_pA\bar{W}Q^{*-1}e_p \\ &\quad + \sum_p tr\left(Q^{*-1}\bar{W}'\Omega^{-1}\Pi_pA\bar{W}\right)e_p \\ &\quad + 2\sum_p\sum_r\sum_s q^*_{rs}tr(\Pi_p'\Pi_r\Pi_s)e_p, \end{aligned} \tag{A.6}$$

where $q^*_{rs} = e_r'Q^{*-1}e_s$ and we have used $\bar{W}'A^*\bar{W} = Q^* - \sum_s\sum_r tr(\Pi_s'\Pi_r)e_se_r'$. Hence, inserting (A.4) and (A.6) in (3.12) the bias expression in (3.14) follows.

B. Bias in the feasible generalized LSDV estimator

We give in this appendix a proof of (3.19), i.e. the bias approximations of the GLSDV and FGLSDV estimators are the same upto order $O(T^{-1})$. The estimation error of the FGLSDV estimator (3.17) consists of two factors. The first factor in (3.17) can be expressed as

$$\begin{aligned} W'A(\hat{\Sigma}^{-1}\otimes I_T)AW &= W'A(\Sigma^{-1}\otimes I_T)AW \\ &\quad + W'A((\hat{\Sigma}^{-1} - \Sigma^{-1})\otimes I_T)AW, \end{aligned} \tag{B.1}$$

with

$$\begin{aligned} W'A(\Sigma^{-1}\otimes I_T)AW &= Q^* + \bar{W}'A(\Sigma^{-1}\otimes I_T)A\tilde{W} + \tilde{W}'A(\Sigma^{-1}\otimes I_T)A\bar{W} \\ &\quad + \left(\tilde{W}'A(\Sigma^{-1}\otimes I_T)A\tilde{W} - E\left[\tilde{W}'A(\Sigma^{-1}\otimes I_T)A\tilde{W}\right]\right) \\ &= Q^* + A_1 + A_2 + A_3, \qquad \text{(B.2)} \end{aligned}$$

where $Q^* = O(T)$ and A_1, A_2 and A_3 are $O_p(T^{\frac{1}{2}})$. For the second term in (B.1) we will need the following

$$\begin{aligned} \hat{\Sigma} &= E'A_TE/T + O_p(T^{-1}) \qquad \text{(B.3)} \\ E &= (\varepsilon_1, ..., \varepsilon_N), \end{aligned}$$

where ε_i is the $T \times 1$ disturbance vector belonging to individual i. A proof of a similar result is given in Kiviet et al. (1995). Now $\hat{\Sigma} - \Sigma$ can be replaced by $E'A_TE/T - \Sigma$ without changing the order of the approximation, i.e

$$E'A_TE/T = \Sigma + O_p(T^{-\frac{1}{2}}), \qquad \text{(B.4)}$$

and we may write

$$\hat{\Sigma}^{-1} = \Sigma^{-1} - \Sigma^{-1}\left[\frac{E'A_TE}{T} - \Sigma\right]\Sigma^{-1} + O_p(T^{-1}). \qquad \text{(B.5)}$$

Exploiting (B.5) the second term in (B.1) becomes

$$\begin{aligned} W'A((\hat{\Sigma}^{-1} - \Sigma^{-1})\otimes I_T)AW &= -W'A\left(\Sigma^{-1}\left[\frac{E'A_TE}{T} - \Sigma\right]\Sigma^{-1}\otimes I_T\right)AW + O_p(1) \\ &= -\bar{W}'A\left(\Sigma^{-1}\left[\frac{E'A_TE}{T} - \Sigma\right]\Sigma^{-1}\otimes I_T\right)A\bar{W} \\ &\quad -\tilde{W}'A\left(\Sigma^{-1}\left[\frac{E'A_TE}{T} - \Sigma\right]\Sigma^{-1}\otimes I_T\right)A\tilde{W} + O_p(1) \\ &= A_4 + A_5 + O_p(1), \qquad \text{(B.6)} \end{aligned}$$

where A_4 and A_5 are $O_p(T^{\frac{1}{2}})$. Hence, from (B.1), (B.2) and (B.6) we find

$$\begin{aligned} W'A(\hat{\Sigma}^{-1}\otimes I_T)AW &= Q^* + \sum_{i=1}^{5} A_i + O_p(1) \\ &= \left(I + \left(\sum_{i=1}^{5} A_i + O_p(1)\right)Q^{*-1}\right)Q^*, \qquad \text{(B.7)} \end{aligned}$$

and

$$\left(W'A(\hat{\Sigma}^{-1}\otimes I_T)AW\right)^{-1} = Q^{*-1}\left(I+\left(\sum_{i=1}^{5}A_i+O_p(1)\right)Q^{*-1}\right)^{-1}$$
$$= Q^{*-1}-Q^{*-1}\left(\sum_{i=1}^{5}A_i+O_p(1)\right)Q^{*-1}+O_p(T^{-2})$$
$$= Q^{*-1}-Q^{*-1}\left(\sum_{i=1}^{5}A_i\right)Q^{*-1}+O_p(T^{-2}). \qquad \text{(B.8)}$$

The second factor in (3.17) can be written as

$$W'A(\hat{\Sigma}^{-1}\otimes I_T)A\varepsilon = W'A(\Sigma^{-1}\otimes I_T)A\varepsilon+W'A((\hat{\Sigma}^{-1}-\Sigma^{-1})\otimes I_T)A\varepsilon$$
$$= \bar{W}'A(\Sigma^{-1}\otimes I_T)A\varepsilon+\tilde{W}'A(\Sigma^{-1}\otimes I_T)A\varepsilon$$
$$-\bar{W}'A\left(\Sigma^{-1}\left[\frac{E'A_TE}{T}-\Sigma\right]\Sigma^{-1}\otimes I_T\right)A\varepsilon$$
$$-\tilde{W}'A\left(\Sigma^{-1}\left[\frac{E'A_TE}{T}-\Sigma\right]\Sigma^{-1}\otimes I_T\right)A\varepsilon+O_p(T^{-\frac{1}{2}})$$
$$= A_6+A_7+A_8+A_9+O_p(T^{-\frac{1}{2}}), \qquad \text{(B.9)}$$

where A_6, A_7 are $O_p(T^{\frac{1}{2}})$ and A_8 and A_9 are $O_p(1)$. From (B.8) and (B.9) the estimation error of the FGLSDV estimator is

$$\hat{\delta}_{FGLSDV}-\delta = Q^{*-1}\left(\sum_{i=6}^{9}A_i\right)$$
$$-Q^{*-1}\left(\sum_{i=1}^{5}A_i\right)Q^{*-1}\left(\sum_{i=6}^{7}A_i\right)+O_p(T^{-\frac{3}{2}}), \qquad \text{(B.10)}$$

using the fact that A_8 and A_9 are $O_p(1)$ and Q^{*-1} is $O(T^{-1})$. Evaluating the expectation of the estimation error in (B.10) we got many terms. Noting that

$$E\left[\hat{\delta}_{GLSDV}-\delta\right]=Q^{*-1}E\left[A_6+A_7\right]-Q^{*-1}E\left[(A_1+A_2+A_3)Q^{*-1}\left(A_6+A_7\right)\right]+o(T^{-1}), \qquad \text{(B.11)}$$

we have the following

$$E\left[\hat{\delta}_{FGLSDV}-\hat{\delta}_{GLSDV}\right]=Q^{*-1}E\left[A_8+A_9\right]-Q^{*-1}E\left[(A_4+A_5)Q^{*-1}\left(A_6+A_7\right)\right]+o(T^{-1}). \qquad \text{(B.12)}$$

We have to evaluate the expectations of the six remaining terms on the right hand side in (B.12). It is easily seen that $E\left[A_8\right]=0$ and $E\left[A_4Q^{*-1}A_6\right]=0$. We will sketch the proof that the other four expectations are all of order $O(T^{-1})$, so premultiplied by Q^{*-1} their contribution is $o(T^{-1})$. In the following all summations run from 1 to N except the index p, which runs from 1 to P. Consider first

$$\begin{aligned} E\left[A_9\right] &= -E\left[\tilde{W}'A\left(\Sigma^{-1}\left[\frac{E'A_TE}{T}-\Sigma\right]\Sigma^{-1}\otimes I_T\right)A\varepsilon\right] \\ &= -E\left[\tilde{W}'A\left(\Sigma^{-1}\frac{E'A_TE}{T}\Sigma^{-1}\otimes I_T\right)A\varepsilon\right] \\ &\quad +E\left[\tilde{W}'A(\Sigma^{-1}\otimes I_T)A\varepsilon\right]. \end{aligned} \tag{B.13}$$

Defining ξ_j as the jth column of Σ^{-1} and σ^{ij} as its ij^{th} element we write for the first term in (B.13)

$$E\left[\tilde{W}'A\left(\Sigma^{-1}\frac{E'A_TE}{T}\Sigma^{-1}\otimes I_T\right)A\varepsilon\right]=\sum_i\sum_j E\left[\xi_i'\frac{E'A_TE}{T}\xi_j\tilde{W}_i'A_T\varepsilon_j\right]. \tag{B.14}$$

Evaluating a particular term in (B.14) we got

$$\begin{aligned} E\left[\xi_i'\frac{E'A_TE}{T}\xi_j\tilde{W}_i'A_T\varepsilon_j\right] &= \sum_r\sum_s E\left[\sigma^{ri}\sigma^{sj}\frac{\varepsilon_r'A_T\varepsilon_s}{T}\tilde{W}_i'A_T\varepsilon_j\right] \\ &= \sum_r\sum_s E\left[\sigma^{ri}\sigma^{sj}\frac{\varepsilon_r'A_T\varepsilon_s}{T}\sum_p\varepsilon_i'\Pi_{pT}'\varepsilon_je_p\right] \\ &= \frac{1}{T}\sum_r\sum_s\sum_p\sigma^{ri}\sigma^{sj}E\left[\varepsilon_r'A_T\varepsilon_s\varepsilon_i'\Pi_{pT}'\varepsilon_j\right]e_p \\ &= \frac{1}{T}\sum_r\sum_s\sum_p\sigma^{ri}\sigma^{sj}\sigma_{rs}\sigma_{ij}\left(tr(A_T)tr(\Pi_{pT})+2tr(\Pi_{pT})\right)e_p \\ &= \sum_r\sum_s\sum_p\sigma^{ri}\sigma^{sj}\sigma_{rs}\sigma_{ij}tr(\Pi_{pT})e_p+O(T^{-1}) \\ &= E\left[\xi_i'\Sigma\xi_j\tilde{W}_i'A_T\varepsilon_j\right]+O(T^{-1}). \end{aligned} \tag{B.15}$$

where $\Pi_{pT}=A_TL_T^p\Gamma_T=O(1)$. Hence, substituting (B.15) into (B.14)

$$E\left[\tilde{W}'A\left(\Sigma^{-1}\frac{E'A_TE}{T}\Sigma^{-1}\otimes I_T\right)A\varepsilon\right]=E\left[\tilde{W}'A\left(\Sigma^{-1}\otimes I_T\right)A\varepsilon\right]+O(T^{-1}), \tag{B.16}$$

and now it is easily seen from (B.13) and (B.16) that $E\left[A_9\right]$ is of order $O(T^{-1})$.

Next consider

$$E\left[A_4Q^{*-1}A_7\right] = -E\left[\bar{W}'A\left(\Sigma^{-1}\left[\frac{E'A_TE}{T}-\Sigma\right]\Sigma^{-1}\otimes I_T\right)A\bar{W}Q^{*-1}\tilde{W}'A(\Sigma^{-1}\otimes I_T)A\varepsilon\right]$$

$$= -E\left[\bar{W}'A\left(\Sigma^{-1}\left[\frac{E'A_TE}{T}\right]\Sigma^{-1}\otimes I_T\right)A\bar{W}Q^{*-1}\tilde{W}'A(\Sigma^{-1}\otimes I_T)A\varepsilon\right]$$
$$+\bar{W}'A\left(\Sigma^{-1}\otimes I_T\right)A\bar{W}Q^{*-1}E\left[\tilde{W}'A(\Sigma^{-1}\otimes I_T)A\varepsilon\right]. \quad (B.17)$$

For the first term in (B.17) we write

$$\bar{W}'A\left(\Sigma^{-1}\left[\frac{E'A_TE}{T}\right]\Sigma^{-1}\otimes I_T\right)A\bar{W}Q^{*-1}\tilde{W}'A(\Sigma^{-1}\otimes I_T)A\varepsilon$$
$$= \sum_i\sum_j\sum_k\sum_l \xi_i'\frac{E'A_TE}{T}\xi_j\bar{W}_i'A_T\bar{W}_jQ^{*-1}\sigma^{kl}\tilde{W}_k'A_T\varepsilon_l, \quad (B.18)$$

with

$$\xi_i'\frac{E'A_TE}{T}\xi_j\bar{W}_i'A_T\bar{W}_jQ^{*-1}\sigma^{kl}\tilde{W}_k'A_T\varepsilon_l$$
$$= \sum_p\sum_r\sum_s \sigma^{ri}\sigma^{sj}\frac{\varepsilon_r'A_T\varepsilon_s}{T}\bar{W}_i'A_T\bar{W}_jQ^{*-1}\sigma^{kl}\varepsilon_k'\Pi_{pT}\varepsilon_le_p. \quad (B.19)$$

The expectation of a particular term in (B.19) is

$$E\left[\sigma^{ri}\sigma^{sj}\frac{\varepsilon_r'A_T\varepsilon_s}{T}\bar{W}_i'A_T\bar{W}_jQ^{*-1}\sigma^{kl}\varepsilon_k'\Pi_{pT}\varepsilon_le_p\right]$$
$$= \frac{1}{T}\sigma^{ri}\sigma^{sj}\sigma^{kl}\sigma_{rs}\sigma_{kl}\left(tr(A_T)tr(\Pi_{pT})+2tr(\Pi_{pT})\right)\bar{W}_i'A_T\bar{W}_jQ^{*-1}e_p$$
$$= \sigma^{ri}\sigma^{sj}\sigma^{kl}\sigma_{rs}\sigma_{kl}tr(\Pi_{pT})\bar{W}_i'A_T\bar{W}_jQ^{*-1}e_p+O(T^{-1}). \quad (B.20)$$

Using this result and follow the same steps back it follows that $E\left[A_4Q^{*-1}A_7\right]$ is of order $O(T^{-1})$. In the same fashion the expectations of the remaining two terms $A_5Q^{*-1}A_6$ and $A_5Q^{*-1}A_7$ can be shown to be of order $O(T^{-1})$ too.

Having derived the order of magnitude of the expectations of the several terms on the right hand side of (B.12) and noting that Q^{*-1} is of order $O(T^{-1})$, it is straightforward to see that

$$E\left[\hat{\delta}_{FGLSDV}-\delta\right]-E\left[\hat{\delta}_{GLSDV}-\delta\right]=o(T^{-1}), \quad (B.21)$$

Hence, the result in (3.19) readily follows or, in other words, the magnitude of the bias upto order $O(T^{-1})$ is the same for the GLSDV and FGLSDV estimators.

C. Bias in the restricted feasible generalized LSDV estimator

In this appendix a proof of (4.16) is given, i.e. the bias approximations of the restricted GLSDV and FGLSDV estimators are the same upto order $O(T^{-1})$. We can write for the restricted FGLSDV estimator in (4.14) the following

$$\hat{\delta}_{FGLSDV,R} - \delta = F_{(W\hat{\Omega})}(\hat{\delta}_{FGLSDV} - \delta), \tag{C.1}$$

where $F_{(W\hat{\Omega})} = I - (W'A\hat{\Omega}^{-1}W)^{-1}R'\left[R(W'A\hat{\Omega}^{-1}W)^{-1}R'\right]^{-1}R$. Defining $C = (W'A\Omega^{-1}W)^{-1}$ we have

$$\begin{aligned} W'A\hat{\Omega}^{-1}W &= W'A\Omega^{-1}W + W'A(\hat{\Omega}^{-1} - \Omega^{-1})W \\ & \left[I + W'A(\hat{\Omega}^{-1} - \Omega^{-1})WC\right]C^{-1}, \end{aligned} \tag{C.2}$$

where C and $W'A(\hat{\Omega}^{-1} - \Omega^{-1})W$ are of order $O_p(T^{-1})$ and $O_p(T^{1/2})$ respectively. Hence, we may expand

$$\begin{aligned} \left(W'A\hat{\Omega}^{-1}W\right)^{-1} &= C\left[I - W'A(\hat{\Omega}^{-1} - \Omega^{-1})WC\right] \\ & +O_p(T^{-2}). \end{aligned} \tag{C.3}$$

As a result of this, we have

$$\begin{aligned} R\left(W'A\hat{\Omega}^{-1}W\right)^{-1}R' &= RCR' - RCW'A(\hat{\Omega}^{-1} - \Omega^{-1})WCR' \\ & +O_p(T^{-2}), \end{aligned} \tag{C.4}$$

and

$$\begin{aligned} \left[R\left(W'A\hat{\Omega}^{-1}W\right)^{-1}R'\right]^{-1} &= [RCR']^{-1} \\ & + [RCR']^{-1} RCW'A(\hat{\Omega}^{-1} - \Omega^{-1})WCR' [RCR']^{-1} \\ & +O_p(1). \end{aligned} \tag{C.5}$$

Using the expansions in (C.3) and (C.5) we can write for $F_{W\tilde{\Omega}}$ the following

$$\begin{aligned} F_{(W\hat{\Omega})} &= I - \left[C - CW'A(\hat{\Omega}^{-1} - \Omega^{-1})WC + O_p(T^{-2})\right] \\ & R'\left[(RCR')^{-1} + (RCR')^{-1} RCW'A(\hat{\Omega}^{-1} - \Omega^{-1})WCR' (RCR')^{-1} + O_p(1)\right]R \\ &= F_{W\Omega} + F_{W\Omega}CW'A(\hat{\Omega}^{-1} - \Omega^{-1})WCR' (RCR')^{-1} R + O_p(T^{-1}). \end{aligned} \tag{C.6}$$

For the second factor in the estimation error (C.1) we can write

$$\hat{\delta}_{FGLSDV} - \delta = CW'A\Omega^{-1}\varepsilon + O_p(T^{-1}) \tag{C.7}$$

so we have for the estimation error in (C.1) the following

$$\begin{aligned} F_{(W\hat{\Omega})}(\hat{\delta}_{FGLSDV} - \delta) &= F_{W\Omega}(\hat{\delta}_{GLSDV} - \delta) \\ & +F_{W\Omega}CW'A(\hat{\Omega}^{-1} - \Omega^{-1})WCR' (RCR')^{-1} R \\ & CW'A\Omega^{-1}\varepsilon + o_p(T^{-1}) \end{aligned} \tag{C.8}$$

so we can write for the bias in the restricted FGLSDV estimator

$$\begin{aligned} E\left[\hat{\delta}_{FGLSDV,R} - \delta\right] &= E\left[\hat{\delta}_{GLSDV,R} - \delta\right] \\ &\quad + E[F_{W\Omega} C W' A (\hat{\Omega}^{-1} - \Omega^{-1}) W C R' \left(R C R'\right)^{-1} R \\ &\quad C W' A \Omega^{-1} \varepsilon] + o(T^{-1}). \end{aligned} \tag{C.9}$$

We will now show that the second term in (C.9) is actually $o(T^{-1})$, which establishes the result of (4.16).

>From Appendix B we have

$$\begin{aligned} C &= Q^{*-1} + O_p(T^{-3/2}), \\ W' A (\hat{\Omega}^{-1} - \Omega^{-1}) W &= A_4 + A_5 + O_p(1), \\ W' A \Omega^{-1} \varepsilon &= A_6 + A_7 + O_p(1), \end{aligned}$$

so

$$[RCR']^{-1} = \left[R Q^{*-1} R'\right]^{-1} + O_p(T^{1/2}), \tag{C.10}$$

and

$$F_{W\Omega} = I - Q^{*-1} R' \left[R Q^{*-1} R'\right]^{-1} R + O_p(T^{-1/2}). \tag{C.11}$$

Hence, we can write

$$\begin{aligned} & F_{W\Omega} C W' A (\hat{\Omega}^{-1} - \Omega^{-1}) W C R' \left(R C R'\right)^{-1} R C W' A \Omega^{-1} \varepsilon \\ = & \left(I - Q^{*-1} R' \left[R Q^{*-1} R'\right]^{-1} R\right) Q^{*-1} (A_4 + A_5) Q^{*-1} \\ & R' \left(R Q^{*-1} R'\right)^{-1} R Q^{*-1} (A_6 + A_7) + o_p(T^{-1}) \\ = & F_{Q^*} Q^{*-1} (A_4 + A_5)(I + F_{Q^*}) Q^{*-1} (A_6 + A_7) + o_p(T^{-1}), \end{aligned} \tag{C.12}$$

with $F_{Q^*} = I - Q^{*-1} R' \left[R Q^{*-1} R'\right]^{-1} R$. Using similar derivations as for $E\left[A_4 Q^{*-1} A_6\right]$ in Appendix B it can be derived that

$$\begin{aligned} E\left[A_4 (I + F_{Q^*}) Q^{*-1} A_6\right] &= O(T^{-1}) \\ E\left[A_4 (I + F_{Q^*}) Q^{*-1} A_7\right] &= O(T^{-1}) \\ E\left[A_5 (I + F_{Q^*}) Q^{*-1} A_6\right] &= O(T^{-1}) \\ E\left[A_5 (I + F_{Q^*}) Q^{*-1} A_7\right] &= O(T^{-1}), \end{aligned}$$

Noting that $F_{Q^*} Q^{*-1}$ is $O(T^{-1})$ the second term in (C.9) is $o(T^{-1})$ and we have

$$E\left[\hat{\delta}_{FGLSDV,R} - \delta\right] = E\left[\hat{\delta}_{GLSDV,R} - \delta\right] + o(T^{-1}), \tag{C.13}$$

which establishes the result in (4.16).

Table 1: Estimation results of the short-run coefficients for $M1$*

	$LSDV$	$LSDVc$	$FGLSDV$	$FGLSDVc$
$\ln(M1/P)_{i,t-1}$	0.63	0.73	0.64	0.68
	(0.08)	(0.08)	(0.06)	(0.06)
$\ln gnp_{it}$	0.28	0.25	0.28	0.27
	(0.04)	(0.04)	(0.03)	(0.03)
$\ln gnp_{i,t-1}$	0.02	-0.00	0.02	0.01
	(0.05)	(0.04)	(0.03)	(0.03)
rs_{it}	-0.64	-0.56	-0.47	-0.43
	(0.23)	(0.21)	(0.19)	(0.18)
$rs_{i,t-1}$	0.21	0.25	0.07	0.07
	(0.23)	(0.21)	(0.18)	(0.17)
rl_{it}	-0.55	-0.49	-0.40	-0.38
	(0.32)	(0.28)	(0.26)	(0.25)
$rl_{i,t-1}$	0.35	0.32	0.13	0.14
	(0.36)	(0.32)	(0.30)	(0.29)
ir_{it}	-0.30	-0.28	-0.25	-0.24
	(0.24)	(0.22)	(0.18)	(0.18)
$ir_{i,t-1}$	0.30	0.28	0.26	0.25
	(0.21)	(0.19)	(0.15)	(0.15)

* N=14, T=19, P+K=12

* Figures in parentheses are bootstrap standard errors

Table 2: Long-run estimates for $M1$*

	$LSDV$	$LSDVc$	$FGLSDV$	$FGLSDVc$
gnp	0.81	0.87	0.83	0.87
	(0.14)	(0.16)	(0.12)	(0.12)
rs	-1.16	-1.14	-1.12	-1.14
	(0.43)	(0.45)	(0.35)	(0.35)
rl	-0.54	-0.61	-0.75	-0.75
	(0.55)	(0.59)	(0.46)	(0.46)
ir	-0.02	-0.03	0.03	0.01
	(0.48)	(0.51)	(0.39)	(0.39)

* Figures in parentheses are bootstrap standard errors

Table 3: Estimation results of the short-run coefficients for $M2$*

	$LSDV$	$LSDVc$	$FGLSDV$	$FGLSDVc$
$\ln(M2/P)_{i,t-1}$	0.91	0.99	0.91	0.95
	(0.11)	(0.11)	(0.09)	(0.09)
$\ln(M2/P)_{i,t-2}$	-0.09	-0.08	-0.11	-0.11
	(0.10)	(0.10)	(0.08)	(0.08)
$\ln gnp_{it}$	0.21	0.18	0.20	0.19
	(0.04)	(0.04)	(0.03)	(0.03)
$\ln gnp_{i,t-1}$	-0.04	-0.06	-0.04	-0.05
	(0.04)	(0.04)	(0.04)	(0.04)
rs_{it}	-0.05	-0.03	-0.03	-0.02
	(0.16)	(0.15)	(0.13)	(0.13)
$rs_{i,t-1}$	0.36	0.32	0.35	0.33
	(0.16)	(0.15)	(0.13)	(0.13)
rl_{it}	-0.54	-0.51	-0.44	-0.43
	(0.19)	(0.17)	(0.16)	(0.16)
$rl_{i,t-1}$	-0.03	0.05	-0.14	-0.10
	(0.24)	(0.22)	(0.19)	(0.19)
ir_{it}	0.28	0.29	0.28	0.28
	(0.20)	(0.18)	(0.17)	(0.17)
$ir_{i,t-1}$	-0.16	-0.17	-0.26	-0.25
	(0.16)	(0.15)	(0.13)	(0.13)

* N=14, T=18, P+K=13

* Figures in parentheses are bootstrap standard errors

Table 4: Long-run estimates for $M2$*

	$LSDV$	$LSDVc$	$FGLSDV$	$FGLSDVc$
gnp	0.95	1.11	0.78	0.86
	(0.20)	(0.24)	(0.18)	(0.18)
rs	1.73	2.34	1.63	1.84
	(0.51)	(0.59)	(0.43)	(0.43)
rl	-3.21	-4.00	-2.96	-3.17
	(0.71)	(0.85)	(0.60)	(0.62)
ir	0.66	0.89	0.10	0.17
	(0.59)	(0.66)	(0.49)	(0.51)

* Figures in parentheses are bootstrap standard errors

Table 5: Estimation results of the short-run coefficients for $M3$*

	$LSDV$	$LSDVc$	$FGLSDV$	$FGLSDVc$
$\ln(M3/P)_{i,t-1}$	0.84	0.93	0.82	0.83
	(0.12)	(0.12)	(0.10)	(0.10)
$\ln(M3/P)_{i,t-2}$	0.02	0.02	0.03	0.03
	(0.11)	(0.11)	(0.09)	(0.09)
$\ln gnp_{it}$	0.20	0.17	0.20	0.20
	(0.04)	(0.04)	(0.03)	(0.03)
$\ln gnp_{i,t-1}$	-0.07	-0.09	-0.06	-0.06
	(0.04)	(0.04)	(0.03)	(0.03)
rs_{it}	-0.05	-0.02	-0.08	-0.07
	(0.15)	(0.14)	(0.13)	(0.13)
$rs_{i,t-1}$	0.21	0.20	0.29	0.29
	(0.14)	(0.13)	(0.12)	(0.11)
rl_{it}	-0.44	-0.40	-0.31	-0.31
	(0.19)	(0.17)	(0.16)	(0.16)
$rl_{i,t-1}$	0.18	0.20	0.03	0.03
	(0.24)	(0.22)	(0.20)	(0.20)
ir_{it}	-0.18	-0.14	-0.18	-0.18
	(0.16)	(0.15)	(0.14)	(0.13)
$ir_{i,t-1}$	0.16	0.14	0.14	0.13
	(0.14)	(0.13)	(0.11)	(0.11)

* N=14, T=18, P+K=13

* Figures in parentheses are bootstrap standard errors

Table 6: Long-run estimates for $M3$*

	$LSDV$	$LSDVc$	$FGLSDV$	$FGLSDVc$
gnp	0.93	1.14	0.97	1.02
	(0.26)	(0.33)	(0.23)	(0.23)
rs	1.15	1.84	1.43	1.52
	(0.67)	(0.83)	(0.57)	(0.57)
rl	-1.83	-2.45	-1.91	-1.98
	(0.92)	(1.15)	(0.78)	(0.80)
ir	-0.12	-0.10	-0.30	-0.32
	(0.64)	(0.77)	(0.54)	(0.55)

* Figures in parentheses are bootstrap standard errors

Table 7: Restricted long-run estimates for $M3$*

	$LSDV$	$LSDVc$	$FGLSDV$	$FGLSDVc$
gnp	1.00	1.00	1.00	1.00
	(-)	(-)	(-)	(-)
rs	1.26	1.63	1.48	1.49
	(0.53)	(0.60)	(0.54)	(0.55)
rl	-1.94	-2.26	-1.93	-1.96
	(0.87)	(1.02)	(0.88)	(0.90)
ir	-0.11	-0.11	-0.31	-0.31
	(0.68)	(0.77)	(0.58)	(0.59)

* Figures in parentheses are bootstrap standard errors

* Unit long-run income elasticity is imposed

Table 8: Long-run $FGLSDVc$ estimates for specifications in (7.4)

	gnp	rs	rl	$rs^f - rs$	$rl^f - rl$
$M1$	0.87	-1.13		0.53	
	(0.12)	(0.20)		(0.56)	
$M1$	1.00		-2.92		-1.42
	(0.11)		(0.36)		(0.85)
$M2$	0.96	2.25	-4.59	1.01	-3.96
	(0.19)	(0.46)	(0.57)	(0.95)	(1.07)
$M3$	0.93	1.30	-3.87	0.27	-3.79
	(0.19)	(0.48)	(0.68)	(0.90)	(1.19)

* Figures in parentheses are bootstrap standard errors