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Modelling Seasonalities in Nonlinear Inflation Rates Using SEASETARs*

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Abstract

In this paper, we present a new time series model, which describes self-exciting threshold autoregressive (SETAR) nonlinearity and seasonality simultaneously. The model is termed multiplicative seasonal SETAR (SEASETAR). It can be viewed as a special case of a general non-multiplicative SETAR model by imposing certain restrictions on the parameters of the latter model. Related to these restrictions, we introduce two $C(\alpha)$-type test statistics, one deals with gaps, and the other tests for multiplicative constraints in non-multiplicative SETAR models. These statistics form the basis of a new seasonality-test. We also present a model selection strategy. The usefulness of both non-multiplicative SETAR model and multiplicative SEASETAR models is examined by applying these models to five monthly series of inflation rates. It turns out that the test statistics mentioned above play an important role in finding the best model for the series. Also, the estimated models can be sensibly interpreted from an economic standpoint. Finally, to get a better understanding of the basic features underlying the fitted SEASETAR models a dynamic analysis is carried out. The results of this analysis can be used to generate more realistic future scenarios of outcomes in order to settle solvency margins in the insurance business.

Key Words: Gaps, inflation, multiplicative models, testing, seasonality, threshold autoregressions, Wilkie's model.

JEL: C22, C51, C52, C53, G14.

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1 Introduction

A complex problem for financial institutions is the management of their assets in such a way that their liabilities can be covered and their goals achieved. It is well-known that this problem can be solved by so-called asset liability management (ALM). An important aspect of ALM systems concerns the identification and modelling of certain key economic time series in order to generate scenarios that are logically consistent and based on sound economic principles; see, e.g., Ziemba and Mulvey (1998) for a survey on ALM. Scenario selection differs considerably from short-term forecasting. If the set of scenarios is rich enough one of these events will actually occur, but at the start of the investigation, there is uncertainty as which one it will be. Parameters of the scenario generator must fit past data and trends.

Broadly speaking, over the past decade, three techniques to generate long-term scenario simulations have become popular in financial practice. The first one is based on historical distributions and on bootstrapping. They both focus on modelling the historical behavior of a time series. The former technique assumes a known distribution function whereas the latter consists of extracting the distribution from the real data. The second technique of simulating long-term scenarios is by using vector autoregressive (VAR) models. One of the main advantage of these models is that they can be quickly adjusted to changing economic conditions. Also VAR models can be easily generalized to include some economic equilibrium conditions; see, e.g., Boender, van Aalst, and Heemskerk (1998), and Kim, Malz, and Mina (1999). Finally, the third technique is based on a cascade structure. Every level of the system includes a set of variables that affects lower ones. In this way there are some key variables which drive the fluctuations in other, less important, variables. This approach has been adopted by Wilkie (1995), Mulvey and Thorlacius (1998), Ranne (1998), and many others. As such it is commonly used in actuarial practice. It will also be the corner-stone for the analysis carried out in this paper.

Inflation is one of the most relevant variables in insurance and financial risk management. This is not only because inflation is considered theoretically and empirically as a driving-force of many other variables like interest rates and wages indices, but also because it is itself a risk factor for fixed returns assets and for inflation-linked liabilities. An extensive explanation of the importance of inflation in the insurance business can be found in Daykin, Pentikäinen, and Pesonen (1994).

One of the firsts attempts to model inflation rates is by Wilkie (1986). He fitted a linear autoregressive (AR) of order 1 to a series of yearly inflation rates for the UK. Wilkie (1995) extends his model by adding an autoregressive conditional heteroskedastic (ARCH) term to the error process in order to capture patterns of heteroskedasticity in the residuals. Both Wilkie (1995) and Whitten and Thomas (1999) comment that the AR-ARCH way of modelling inflation rates can generate unrealistic values in practice. As an alternative these latter authors advocate the use of the so-called self-exciting threshold autoregressive models (SETAR) models. Basically, a SETAR model is a piecewise linear AR model with non-smooth, possibly severe switches (or changes in levels) in linear relationships. This type of behaviour seems more realistic in practice. Indeed, Clarkson (1991) as well as Whitten and Thomas (1999) note the existence of different levels of inflation, one corresponding to a "normal level" or "quiescent phase" and the other to a "high level" or "excited phase" in the UK inflation rates. For risk management it is important to include these changes of levels in a time series model. If not, a high level of inflation might damage the solvency of a risk portfolio due to a dramatic increase of the liabilities of a company and, at the same time, a decrease of the premiums value and assets.

Although a model specified on the basis of yearly data is often sufficient for practical purposes there is also a need for a model with data measured at more frequent time intervals. One important motivation is to be closer to the real cash-flow generating process. Moreover, the modelling of inflation might be integrated in a cascade structure with other economic variables such as interest rates. This latter series has been modelled in the finance literature with data measured at more frequent time intervals than just one year; see, e.g., Chan, Karolyi, Longstaff, and Sanders (1992). Thus, in summary, there is a strong need for introducing a SETAR time series model which can adequately capture nonlinear features and seasonality in inflation rates simultaneously. This, indeed, is the main theme of the paper.

The rest of the paper is organized as follows. First, in Section 2, we introduce as a special case of a non-multiplicative SETAR model, the so-called multiplicative seasonal SETAR model. This will be done by imposing certain restrictions on the parameters of the non-multiplicative SETAR model. Related to these restrictions, we introduce two test statistics. The first tests for gaps in the non-multiplicative SETAR model (Subsection 2.2). The second statistic tests for multiplicative constraints in non-multiplicative SETAR models (Subsection 2.3). Both tests form the basis of a seasonality test presented in Subsection 2.4. Next, in Section 3, we discuss a selection procedure for fitting the "best" (SEA)SETAR model to an observed time series. In Section 4, we analyze five series of monthly inflation rates. Both non-multiplicative linear AR, multiplicative linear AR, non-multiplicative SETAR and multiplicative SEASETAR models will be fitted to the data and various restrictions on the model parameters will be tested. In Section 5 we investigate the dynamic characteristics underlying the fitted SEASETAR models through a simulation study. The final section contains some concluding remarks.

2 Detecting gaps and seasonality in SETAR models

2.1 Null hypotheses

For simplicity of presentation, but without loss of generality, the details of the test statistics are derived in this section for a two-regime SETAR model only. Specifically, a time series $\{Y_t\}$ is said to follow an unrestricted discrete-time stationary two-regime SETAR$(2; p_1 + sP_1, p_2 + sP_2)$ process if it satisfies the relation

$$Y_t = \begin{cases} \alpha_0^{(1)} + \sum_{i=1}^{p_1+sP_1} \alpha_i^{(1)} Y_{t-i} + \varepsilon_t^{(1)}, & \text{if} \quad Y_{t-d} \leqslant r, \\ \alpha_0^{(2)} + \sum_{i=1}^{p_2+sP_2} \alpha_i^{(2)} Y_{t-i} + \varepsilon_t^{(2)}, & \text{if} \quad Y_{t-d} > r. \end{cases} \tag{1}$$

Here $\{\varepsilon_t^{(j)}\}$ $(j = 1, 2)$ is a sequence of independent and identically distributed (*i.i.d.*) random variables with mean zero and variance σ_j^2 such that $\varepsilon_t^{(1)}$ and $\varepsilon_t^{(2)}$ are independent; d is a known positive integer called the delay parameter (or threshold lag); and r is the threshold. The non-negative integers p_j, P_j $(j = 1, 2)$, d, and s are assumed known and are such that $0 \leqslant p_2 \leqslant p_1$, $0 \leqslant P_2 \leqslant P_1$, $1 \leqslant d \leqslant \max(p_1 + sP_1, p_2 + sP_2)$, $s \geqslant \max(p_1, p_2)$. Note that in each regime (1) has a linear AR structure. For linear time series this latter process can be represented by the relation

$$Y_t = \alpha_0 + \sum_{i=1}^{p+sP} \alpha_i Y_{t-i} + \varepsilon_t \tag{2}$$

where ε_t are *i.i.d.* $N(0, 1)$ random variables.

Necessary and sufficient conditions for the stationarity of SETAR models are available in the literature for only a few special cases. However, a practical and general way for checking stationarity follows from a direct analogue of a method for checking invertibility of nonlinear time series models proposed by De Gooijer and Brännäs (1995). Briefly stated, it consists of feeding *i.i.d.* innovations into the nonlinear model and then observing whether the model blows up or not. This approach will be applied in the empirical part of this paper.

Note that by imposing two different restrictions on the parameters of (1) the model becomes of special interest for analyzing seasonal time series with a period of seasonality s. Let B denote the lag operator such that $B^k Y_t = Y_{t-k}$. The first restriction is to set $P_j(s - p_j - 1)$ coefficients in the polynomial $\alpha^{(j)}_{p_1+sP_1}(B) = \sum_{i=1}^{p_j+sP_j} \alpha_i^{(j)} B^i$ $(j = 1, 2)$ equal to zero. These are the exclusion restrictions ("gaps") in regime j of the model (1), and a test for detecting such gaps may be regarded as test of the null hypothesis

$$H_{01}: \qquad \alpha^{(j)}_{is+\ell} = 0 \quad (0 \leqslant i \leqslant P_j - 1; p_j < \ell < s; j = 1, 2). \tag{3}$$

We shall refer to (1) together with the restriction (3) as the *non-multiplicative* SETAR model.

Not much research has been reported in the literature on testing hypothesis of the form (3) for linear time series models. Godolphin (1978) tested for gaps in purely linear moving average (MA) models. In fact, the test based on the null hypothesis H_0^*: $\rho_{is+\ell} = 0$ $(0 \leqslant i < Q; q < \ell < s)$, where ρ_i is the theoretical autocorrelation at lag i, and q and Q are the respectively the orders of the non-seasonal and seasonal MA polynomials. Harvey and Tomenson (1981) pointed out that a test for gaps places certain restrictions on the non-zero autocorrelations which are not reflected by the hypothesis H_0^*, resulting in a loss of information in Godolphin's test. These authors also compared the asymptotic relative efficiency (ARE) of a standard Lagrange multiplier (LM) test (see below) with the asymptotic ARE of Godolphin's test for a number of seasonal MA models. It appears that from their computed ARE values that the LM test is far more powerful than Godolphin's test. Unfortunately, Harvey en Tomenson's ARE results are wrong. Formula (3.1) in their paper is not correct because it should used second derivatives of the noncentrality parameter under H_0^*.

Thus for linear time series modelling, the problem of testing for gaps is still not solved in a satisfactory way. The problem is equally important when fitting SETAR models to time series. In fact, several empirical studies have appeared in the literature with non-seasonal SETAR models fitted to seasonal time series, having parameter values not statistically different from zero at intermediate, often non-seasonal, lags; see, e.g., Ray (1988, Table 8). Of course, related to the "gaps problem" is the important question whether a time series is seasonal or not. If seasonality is considered within the framework of SETAR modeling then it is reasonable to introduce a separate model specification with explicit seasonal components. Such a model follows directly as a generalization of the multiplicative seasonal AR model for linear time series processes. It will be termed *multiplicative* SEASETAR model and is given by

$$Y_t = \begin{cases} \alpha_0^{(1)} + \sum_{i=1}^{p_1} \sum_{j=1}^{P_1} \phi_i^{(1)} \Phi_j^{(1)} Y_{t-i-js} + \varepsilon_t^{(1)}, & \text{if} \quad Y_{t-d} \leqslant r, \\ \alpha_0^{(2)} + \sum_{i=1}^{p_2} \sum_{j=1}^{P_2} \phi_i^{(2)} \Phi_j^{(2)} Y_{t-i-js} + \varepsilon_t^{(2)}, & \text{if} \quad Y_{t-d} > r. \end{cases} \tag{4}$$

Note that the SEASETAR model (4) can be considered as a special case of the unrestricted model (1). In particular, (4) follows from (1) by imposing the $p_j P_j$ multiplicative constraints

$\alpha^{(j)}_{is+\ell} = \phi^{(j)}_\ell \Phi^{(j)}_i$ $(1 \leqslant i \leqslant P_j; 1 \leqslant \ell \leqslant p_j; j = 1, 2)$. A test for checking this second restriction on the parameters of (1) may regarded as a test of the null hypothesis

$$H_{02}: \qquad \alpha^{(j)}_{is+\ell} = \phi^{(j)}_\ell \Phi^{(j)}_i \quad (1 \leqslant i \leqslant P_j; 1 \leqslant \ell \leqslant p_j; j = 1, 2). \tag{5}$$

Now, given the hypotheses H_{01} and H_{02} above, the seasonality hypothesis is given by

$$H_0 = H_{01} \cap H_{02}.$$

Thus if one first tests for H_{01} and rejects it there is obviously no point in testing for H_{02}. Note that the above hypotheses can be straightforwardly modified to get a framework for testing gaps and seasonality in linear AR processes.

In the next three subsections we present three test statistics for each of the hypotheses H_{01}, H_{02}, and H_0, separately. Since, in all cases the tests are applied to two linear AR processes with the "switching" dynamics driven by the time series at time $t-d$, we drop for ease of notation the superscript (j) in the parameters $\alpha^{(j)}_i$, $\phi^{(j)}_i$, and $\Phi^{(j)}_i$. Also we drop the subscript j in the model orders p_j and P_j. For ease of deriving the tests it is further assumed that $\{\epsilon^{(1)}_t\} = \{\epsilon^{(2)}_t\} = \{\epsilon_t\}$ with $\{\varepsilon_t\}$ a sequence of *i.i.d.* $N(0, \sigma^2)$ random variables.

2.2 Testing for exclusion restrictions (gaps)

In this subsection we consider testing the hypothesis (3). To this end, we first define the parameter vector $\tau = (\tau_1', \tau_2')'$ where τ_1 contains the $p + P(p+1)$ free coefficients in the jth $(j = 1, 2)$ regime of the SETAR model (3) while τ_2 contains the coefficients restricted to zero by H_{01}. The dimension of τ_2 is $P(s-p-1)$. Let $k(\tau)$ and $I(\tau)$ stand for respectively the $p + sP$ score vector and the $(p+sP) \times (p+sP)$ information matrix of τ. Then, similar to the partition in the vector τ, we have

$$k(\tau) = \begin{pmatrix} k_1(\tau) \\ k_2(\tau) \end{pmatrix} \quad \text{and} \quad I(\tau) = \begin{pmatrix} I_{11}(\tau) & I_{12}(\tau) \\ I_{21}(\tau) & I_{22}(\tau) \end{pmatrix}.$$

Let $\tilde\tau = (\tilde\tau_1', \tilde\tau_2')'$ be a locally root n consistent estimator (lrnc) of τ. A computationally attractive special case of $\tilde\tau$ is obtained by choosing $\tilde\tau_2 = 0$. Using the method of scoring gives for τ the estimator

$$\begin{pmatrix} \hat\tau_1 \\ \hat\tau_2 \end{pmatrix} = \begin{pmatrix} \tilde\tau_1 \\ \tilde\tau_2 \end{pmatrix} - \begin{pmatrix} I_{11}(\tilde\tau) & I_{12}(\tilde\tau) \\ I_{21}(\tilde\tau) & I_{22}(\tilde\tau) \end{pmatrix}^{-1} \begin{pmatrix} k_1(\tilde\tau) \\ k_2(\tilde\tau) \end{pmatrix}. \tag{6}$$

Since the null hypothesis H_{01} is equivalent to $\tau_2 = 0$, we are interested in finding an explicit expression for $\hat{\tau}_2$. Using the well-known formula for the inverse of a partitioned matrix, (6) yields

$$\hat{\tau}_2 = \tilde{\tau}_2 - I_{22\cdot 1}^{-1}(\tilde{\tau})\{I_{21}(\tilde{\tau})I_{11}^{-1}(\tilde{\tau})k_1(\tilde{\tau}) - k_2(\tilde{\tau})\}$$

where $I_{22\cdot 1}(\tilde{\tau}) = I_{22}(\tilde{\tau}) - I_{21}(\tilde{\tau})I_{11}^{-1}(\tilde{\tau})I_{12}(\tilde{\tau})$. Under H_{01} equation (6) defines an asymptotically efficient estimator of τ. Thus, if H_{01} is true then the statistic $n^{1/2}\hat{\tau}_2$ is asymptotically normal with mean vector zero and covariance matrix $I_{22\cdot 1}^{-1}(\tau)$. This yield the test statistic

$$C_1 = n\hat{\tau}_2' I_{22\cdot 1}(\tilde{\tau})\hat{\tau}_2 \tag{7}$$

which under H_{01} is asymptotically distributed as χ^2 with degrees of freedom equal to the dimension of τ_2, i.e. $P(s-p-1)$. Statistic (7) is locally asymptotically optimal in a class of so-called $C(\alpha)$ tests introduced by Neyman (1959). By optimal we mean that it has the same asymptotic distribution as the corresponding likelihood ratio, LM and Wald statistics both under the hypothesis H_{01}. From a sequential testing standpoint, the $C(\alpha)$ tests are particularly convenient since the computation of (7) does not require estimates which converge to a final value. The test statistic C_1 has good size and power properties as can be seen from the simulation results presented in the Appendix.

In the special case where $\tilde{\tau}_2 = 0$ and $\tilde{\tau}_1$ is the restricted maximum likelihood estimator, statistic C_1 becomes

$$C_1^* = nk_2'(\tilde{\tau})I_{22\cdot 1}^{-1}(\tilde{\tau})k_2(\tilde{\tau})$$

since $k_1(\tilde{\tau}) = 0$. This is the LM statistic for the hypothesis H_{01}. A statistic asymptotically equivalent to C_1 is obtained by replacing $I_{22.1}(\tilde{\tau})$ in (7) by $I_{22.1}(\hat{\tau})$ $(\hat{\tau} = (\tilde{\tau}_1', \hat{\tau}_2')')$. If the scheme (6) is iterated until convergence and if $\hat{\tau}$ denotes the final estimate then this form of the statistic C_1 is the Wald statistic for the hypothesis H_{01}.

Lrnc estimates of τ can be obtained in a number of different ways. Here we shall adopt the arranged autoregression approach proposed by Tsay (1989) which makes use of least squares estimates of the model parameters. The estimation of the parameters d and r will be discussed in more detail in Section 3.

2.3 Testing for multiplicative constraints

Now consider testing for H_{02} when H_{01} is assumed to be valid. We are thus concerned with the $p+P(p+1)$-dimensional parameter vector τ_1 given by

$$\tau_1 = (\alpha_1, \ldots, \alpha_p, \alpha_s, \alpha_{s+1}, \ldots, \alpha_{s+p}, \ldots, \alpha_{sP}, \alpha_{sP+1}, \ldots, \alpha_{sP+p})'.$$

Define the function

$$h_{i\ell}(\tau_1) = \alpha_{is+\ell} - \alpha_{is}\alpha_\ell, \quad (\ell = 1, \ldots, p; i = 1, \ldots, P).$$

Furthermore, set

$$h_{i\cdot}(\tau_1) = (h_{i1}(\tau_1), \ldots, h_{ip}(\tau_1))' \quad \text{and} \quad h(\tau_1) = (h_{1\cdot}(\tau_1'), \ldots, h_{P\cdot}(\tau_1'))'.$$

Now the null hypothesis (6) may be written as

$$H_{02} : h(\tau_1) = 0.$$

The function $h(\tau)$ has continuous partial derivatives of all order. From the definition of $h_{i\ell}(\tau_1)$ we obtain

$$\frac{\partial h_{i\ell}(\tau_1)}{\partial \alpha_r} = \begin{cases} -\alpha_{is}\delta_{ri} & r = 1, \ldots, p, \\ -\alpha_\ell \delta_{i1} & r = s, \\ \delta_{i1}\delta_{(r-s),\ell} & r = s+1, \ldots, s+p, \\ \vdots & \vdots \\ -\alpha_\ell \delta_{iP} & r = sP, \\ \delta_{iP}\delta_{(r-sP),\ell} & r = sP+1, \ldots, sP+p, \end{cases}$$

where $\delta_{r\ell}$ is Kronecker's delta. From this we can form the matrix $H_{i\cdot}(\tau_1) = \partial h_{i\cdot}(\tau_1)/\partial \tau_1'$ $(i = 1, \ldots, P)$ and

$$H'(\tau_1) = (H_{1\cdot}'(\tau_1), \ldots, H_{P\cdot}'(\tau_1)).$$

The elements of these matrices can be easily computed. Consider, for instance, the $(p+P(p+1)) \times p$ matrix $H_{1\cdot}'(\tau_1)$ given by

$$H_{1\cdot}'(\tau_1) = \begin{pmatrix} -\alpha_1 I_p \\ -\alpha_1 - \alpha_2 \cdots - \alpha_p \\ I_p \\ O_{(P-1)(p+1)\times p} \end{pmatrix}.$$

The matrix $H'_{i\cdot}(\tau_1)$ $(2 \leqslant i \leqslant P-1)$ can be obtained from $H'_{1\cdot}(\tau_1)$ by interchanging the positions of the rows $p+1, \ldots, 2p+1$ and $p+1+(i-1)(p+1), \ldots, p+i(p+1)$, respectively, and by replacing α_{1s} by α_{is}. Note that the matrix $H'(\tau_1)$ is of order $(p+P(p+1)) \times pP$. Since this matrix is of full rank, the general results about hypothesis testing and constrained estimation can be applied here.

As before, let $\tilde{\tau}_1$ be a lrnc consistent estimator of τ_1 and consider the equation

$$\begin{pmatrix} \hat{\tau}_1^* \\ \hat{\lambda} \end{pmatrix} = \begin{pmatrix} \tilde{\tau}_1 \\ 0 \end{pmatrix} - \begin{pmatrix} I_{11}(\tilde{\tau}_1) & H'(\tilde{\tau}_1) \\ H(\tilde{\tau}_1) & 0 \end{pmatrix}^{-1} \begin{pmatrix} k_1(\tilde{\tau}_1) \\ h(\tilde{\tau}_1) \end{pmatrix}, \tag{8}$$

where $\hat{\lambda}$ is the vector of Lagrange multipliers. As we have noticed earlier $\hat{\tau}_1^*$ is an asymptotically efficient estimator of τ_1 under the constraints $h(\tau_1) = 0$ whose validity may optimally be tested by testing the significance of the statistic $\hat{\lambda}$. To derive the test statistic denote

$$R(\tau_1) = -(H(\tau_1) I_{11}(\tau_1) H'(\tau)_1)^{-1} \quad \text{and} \quad Q(\tau_1) = -I_{11}(\tau_1) H'(\tau_1) R(\tau_1).$$

Then,

$$\hat{\lambda} = -Q'(\tilde{\tau}_1) k_1(\tilde{\tau}_1) - R(\tilde{\tau}_1) h(\tilde{\tau}_1)$$

and under H_{02} the statistic $n^{1/2}\hat{\lambda}$ is asymptotically normal with mean vector zero and covariance matrix $-R(\tau_1)$. This yields the test statistic

$$C_2 = -\hat{\lambda}' R(\tilde{\tau}_1)^{-1} \hat{\lambda}$$

which is asymptotically distributed as χ^2 with pP degrees of freedom under H_{02}; see, e.g., Saikkonen (1986). Since we know how to compute $I_{11}(\tilde{\tau}_1)$ and $H(\tilde{\tau}_1)$, all statistics involved in C_2 can be computed.

2.4 Testing for seasonality

In the previous two subsections we derived the statistic C_1 for the null hypothesis H_{01} and the statistic C_2 for the hypothesis H_{02} conditional that H_{01} is true. A test statistic for H_0 is given by

$$C = C_1 + C_2$$

which under H_0 is asymptotically distributed as χ^2 with degrees of freedom equal to the sum of the degrees of freedom of C_1 and C_2, i.e. $P(s-p-1) + pP$. This result may be obtained by noting that under H_0 the statistics C_1 and C_2 are asymptotically independent which follows from the general result of testing nested hypotheses; see, e.g., Graybill (1976, p. 308) where an analogue situation in the linear regression model is considered. Statistics C_1 and C_2 above are general $C(\alpha)$ type test statistics of Neyman (1959). As before LM and W type special cases may be obtained by choosing the initial estimator $\tilde{\tau}_1$ appropriately. In the Appendix the power properties of C_1 and C are studied in a small-scale simulation study.

3 Selection Procedure

A critical step in (SEA)SETAR modelling is choosing an appropriate model from a large set of candidate models in a systematic and reproducible way. Automatic model selection criteria, such as Akaike's information criterion (AIC) can then be used find a balance between model lack-of-fit and model complexity. For linear time series model selection, McQuarrie, Shumway, and Tsai (1997) obtained an "unbiased" version of AIC, AIC_u. For SETAR$(2; p_1, p_2)$ model selection this latter criterion can be defined as follows. Let n_j denote the number of observations belonging to the jth regime and let $\hat{\sigma}^2_{\varepsilon_j}$ be the corresponding residual variance ($j = 1, 2$). Then

$$AIC_u = AIC + \sum_{j=1}^{2} \Big\{ n_j \ln\{n_j/(n_j - p_j - 2)\} + 2(p_j + 2)(p_j + 3)/(n_j - p_j - 3) \Big\}, \qquad (9)$$

where

$$AIC = \sum_{j=1}^{2} \Big\{ n_j \ln \hat{\sigma}^2_{\varepsilon_j} + 2(p_j + 2) \Big\}.$$

Thus AIC_u penalizes models which are over-parametrized more strongly than AIC, especially as n increases, and so gives some value to model parsimony. This observation has been verified by De Gooijer (2001) in a simulation experiment with various SETAR models. Hence, we decided to adopt AIC_u as a model selection criterion.

Now, using (9), we propose the following procedure for selecting (SEA)SETAR models.

1. Fix the maximum number of regimes using prior information about the time series under study. As mentioned in Section 1, there is quite some empirical evidence that there are two levels of inflation, i.e. a normal inflation level as opposed to a high inflation level. As a consequence, we fixed the number of levels at two.

2. Fix the maximum delay, say d^*. Since the data under study are monthly observations, and our interest is in modelling seasonality in the series, it is reasonable to fix d^* at 12. Thus, a set $D = \{d : d = 1, \ldots, 12\}$ of possible delays will be entertained.

3. Fix the maximum AR seasonal orders, say P_j^*, $(j = 1, 2)$ in each regime. In many applications, linear seasonal AR models were fitted with either $P = 1$ or $P = 2$. To allows for some flexibility in the specification of the models we decided to fix P^* at two. The maximum order of the nonseasonal AR part, say p_j^* $(j = 1, 2)$, is fixed at $s - 1$. In this study $s = 12$.

4. Select an interval $[r_L, r_U]$ in which the threshold values are searched. In this study r_L and r_U are the 10th percentile and the 90th percentile of the empirical distribution of the series $\{Y_t\}$, respectively.

5. To guarantee that there are enough observations in each regime, search thresholds at the fixed interval width $(r_U - r_L)/[0.8n]$, with $[.]$ denoting the integer part, such that within each jth regime there are at least 20 observations. This approach results in a set of, say R_{j-1} candidate threshold values.

6. Select a set of candidate models, based on the $p_j^* + sP_j^*$ $(j = 1, 2)$ non-multiplicative seasonal SETAR models. Define a sequence in which parameters are included in the model from $(p, P) = (0, 0)$ up to (p_j^*, P_j^*). For example, for $P_j = 1$, the sequence begins with $p_j = 0$, and the model under study has the form

$$(1 - \alpha_{12} B^{12}) Y_t = \alpha_0 + \varepsilon_t.$$

   Next, attention will be focussed on a model with orders $(p_j, P_j) = (1, 1)$. In the third step, the model with order $(p_j, P_j) = (2, 1)$ will be fitted to the data, etc. This process continues till the maximum orders (p_j^*, P_j^*) are reached. Given two regimes, fixed delay parameter, $P_1^* = P_2^* = P$, and fixed threshold values there are $S = s^2 \times (P^* + 1)^2$ candidate models to represent the series at this step.

7. Calculate the minimum value of (9) over all $R \times S$ candidate models. At this point the best orders, say (p_0, P_0), and the best threshold value, say $\hat{r}$, are obtained given a fixed value of d.

8. Repeat steps 5–7 for each $d \in D$ up to d^*.

9. Finally, select that (SEA)SETAR model which has a minimum AIC_u value among the set of d^* best-fitted models.

Note that in the above procedure, with two regimes, the total number of models under investigation is equal to $d^* \times R \times S$, with $S = s^2 \times (P^* + 1)^2$. In the study reported below $S = 1296$.

4 Empirical Results

4.1 Data

The inflation rates analyzed in this article are four monthly OCDE data series covering the period January 1960 – July 1998 (475 observations) for the countries France, Spain, UK, and USA. Also included is the inflation rate for the Netherlands for the period April 1960 – July 1998 (472 observations). So in total a set of five series will be subject to investigation. We analyze the first differences, $(1 - B)$, of the natural logarithms of the original series. To check the series are indeed integrated of order 1, we computed two unit-root tests, i.e. the augmented Dickey-Fuller and the Phillips-Perron test. Both tests do not reject the null of no unit-root at the 5% level. A LR test was applied for discriminating SETAR models from linear AR models; see, e.g., Tong (1990). The test rejects linearity for all series, apart from the inflation rates of the UK, at the 5% significance level.

4.2 Linear non-seasonal and seasonal AR models

Table 1 displays the linear AR models fitted to each of the five inflation rate series. Parameter values significant at the 5% level are denoted by an asterisk. The last three lines contain respectively values of AIC_u, p-values of the gaps test statistic C_1, and p-values of the Lin-Mudholkar test statistic for normality against asymmetric alternatives; see Lin and Mudholkar (1980). The latter statistic is asymptotically Gaussian distributed with mean zero and unit variance, under the null hypothesis of Gaussian distributed residuals.

Note that the orders of the best-fitted linear models, using AIC_u as an order selection criterion, are quite different for the series. If we denote these orders by respectively $(\hat{p}, \hat{P})$ for the non-seasonal and seasonal AR part, they are respectively given by (1,6) for France, (7,2) for the Netherlands, (1,2) for Spain, (2,2) for the UK, and (9,1) for the USA. Furthermore, it is interesting to note that for the inflation rate series for the UK and Spain the gaps hypothesis is rejected at the 5% significance level. Clearly, the p-values of the test for detecting gaps for the remaining three series indicate strong evidence in favour of the hypothesis (3). Finally, the results of the Lin-Mudholkar test seem to suggest that all fitted linear AR models are far from optimal.

We have also fitted multiplicative seasonal AR models to the data. To save space, we summarize the main results. For all series the value of AIC_u is higher than the value of AIC_u is higher than the non-multiplicative one, however none of the p-values of the C statistics indicate the acceptannce of the seasonality hypothesis. Note that the linear fitted AR models given in Table 1 are not directly comparable with those obtained by Wilkie (1995) since the time period under study and the periodicity of the data are different. Nevertheless, it is interesting to see that for three countries, including the UK, the optimal seasonal orders are higher than one.

4.3 Non-multiplicative SETAR models

Table 2 provides information on the best-fitted non-multiplicative SETAR models. We see that, in terms of AIC_u values, there is a considerable improvement in model fit for all series over their corresponding linear AR models given in Table 1. Denoting the optimal model orders by $(\hat{p}_1, \hat{P}_1, \hat{p}_2, \hat{P}_2)$, we obtain the following results for the five series: (0,2,6,1) for France, (0,2,1,2) for the Netherlands, (0,2,1,0) for Spain, (1,2,1,1) for the UK, and (3,1,3,0) for the USA. The corresponding optimal values for the delay d and the threshold r, denoted by $(\hat{d}, \hat{r})$, are respectively given by: (11, 0.00264) for France, (6, 0.00002) for the Netherlands, (8, 0.01018) for Spain, (6, 0.00751) for the UK, and (6, 0.00194) for the USA. Further note that in the first regime the gaps hypothesis (3) is not rejected at the 5% level for the UK, and the USA. In contrast, the models fitted to the series of France, the Netherlands, and the UK in the second regime do not seem to provide sufficient evidence against H_{01}. Thus, in all these cases, the series are likely to follow a seasonal multiplicative model and testing for gaps and multiplicative constraints jointly $(H_{01} \cap H_{02})$ using the test statistic C may well reveal this behaviour. Apart from the residuals of the estimated model for the USA, the p-values of the Lin-Mudholkar still indicate some model inadequacies.

Unfortunately, when thresholds models are used in practice there is often not much guidance for the choice of the delay d. For this reason, the estimation of the "best" value of d is a part of the selection procedure discussed in Section 3. It is well-known that seasonality induces large sample autocorrelations not only at the seasonal lags, but also at the half-seasonal and at the quarter-seasonal lags. It is interesting to note that, apart from Spain, the values for d are in two cases equal to six and in one case close to 12 (France).

On examining the threshold values $\hat{r}$, there appears to be a difference between the five fitted models. On the one hand, the values of $\hat{r}$ obtained for France and the Netherlands represent a lower-regime with inflation rates close to zero or negative. The fitted models only have parameters at the seasonal lags 12 and 24. This regime applies to about one-third of the available data. In the upper-regime, the fitted models have significant parameters at both seasonal and non-seasonal lags for about two-third of the observations. Thus, this latter regime seems to reflect the normal phase for both series. On the other hand, the models fitted to the series of Spain, the UK, and the USA have about one-fourth of the observations in the upper-regime. Clearly, this regime seems to correspond with the excited phase discussed above. This result is further supported by the estimated standard deviations $\hat{\sigma}_j$ $(j = 1, 2)$. As is known SETAR models can also capture conditional variances when fitted models have different standard deviations. In all cases we found significant differences between the $\hat{\sigma}_j$ associated with regime j. For France, and the Netherlands, the lower-regime has the highest standard deviation. Thus there is an inverse relationship between the level of inflation and its volatility. For the other countries, this is just the reverse.

Finally, it is interesting to compare the threshold value for the UK with the one obtained by Whitten and Thomas (1999), using yearly data for a different time period. These authors suggest a threshold value of 10% to partition the annual rates which implies an upper-regime with only 8 observations. If we recompute our result on an annual basis, the optimal threshold value corresponds to an annual rate slightly higher than 9% and the number of yearly observations is almost equal to ten. Thus, both approaches give almost similar results for the threshold value.

4.4 Multiplicative SEASETAR models

Table 3 provides summary information on the best-fitted multiplicative SEASETAR estimated for four inflation rates. The series for Spain is not included here, since the results of the test statistic C_2 indicated that this series cannot be represented by a multiplicative SEASETAR model. We used the same model orders $(\hat{p}_1, \hat{P}_1, \hat{p}_2, \hat{P}_2)$ as for the non-multiplicative SETAR models. Also we adopted the values for $(\hat{d}, \hat{r})$ given in the previous subsection. Note that, in terms of AIC_u, there is a considerable improvement in model fit for all series as opposed to the corresponding non-multiplicative SETAR models given in Table 2. Again, apart for the residuals of the estimated model for the USA, the residuals show some problems although there has been some improvements.

In scenario simulation it is often useful to have mean values of the series in each regime. For economic interpretation we report annualized values and between brackets the fitted ones: 2.69%, 5.37%, $(0.22\%, 0.45\%)$ for France, 0.94%, 5.41%, $(0.08\%, 0.45\%)$ for the Netherlands, 6.21%, 13.36%, $(0.52\%, 1.11\%)$ for Spain, 4.29%, 12.02%, $(0.36\%, 1.00\%)$ for the UK, 1.02%, 1.83% and $(0.08\%, 0.15\%)$

Table 4 shows the dominant roots for both the non-multiplicative SETAR and the multiplicative SEASETAR models presented in respectively Tables 2 and 3. In all cases the roots of the characteristic polynomials indicate that the local dynamics in the regimes is stationary. For France the upper-regime of the multiplicative SEASETAR model has a root with modulus 101.89, indicating a cycle of about 8 years. Note, that for the models fitted to the series of France, the Netherlands, the UK, and the USA the dominant roots for the non-multiplicative and the corresponding multiplicative models are very close to each other. Thus the basic dynamics of these models will approximately be the same. For the Netherlands and the UK we see roots corresponding to a period length of three, six, and twelve in the upper-regime of the fitted multiplicative models. Interestingly, this observation is supported by the p values of the seasonality test C given in Table 3. There we see that for these two countries the seasonality hypothesis cannot be rejected in the second regime.

5 Dynamic Analysis

To understand the basic dynamics of the best-fitted multiplicative SEASETAR models, we simulated each model 10 times. Figure 1 shows plots of the series. In each case 120 observations are generated. This set of data corresponds closely to long-term goals one often has in mind in financial risk management, assuming that there is a monthly periodicity in the data. The series generated for Spain is based on the non-multiplicative SETAR model given in Table 2. Clearly, the simulated series exhibit the seasonal periodicity in the models. Note, that the range of the fluctuations in the USA-series (Fig. 1.a) is about four to ten times smaller than the range of fluctuations in the other four series. Traditionally European countries have not succeeded in inflation control as this has been the case in the USA.

Figure 2 shows plots of the systematic part (skeleton) part of the best-fitted multiplicative SEASETAR model. The seasonal periodicity is clearly visible for the Netherlands and the UK. This may also be noted from Figure 3, which shows scatter plots of (Y_t, Y_{t-1}) for these two series.

6 Concluding Remarks

In this paper we introduced a new SETAR-type nonlinear model which can be used to characterize seasonalities in time series. We also proposed three tests: one for detecting gaps, one for detecting multiplicative constraints, and one for testing seasonality in (non)linear (SET)AR models. All tests have good power and size properties. Further, a selection procedure for (SEA)SETAR modelling has been presented. When we applied these tools to five series of monthly inflation rates, we noted that (SEA)SETAR models have a good in-sample fit as compared to linear seasonal and non-seasonal AR models.

The fitted (SEA)SETAR models have been used to generate more realistic future scenarios (out-of-sample) for the monthly inflation rates. These scenarios can be used for projection and dynamic solvency-testing purposes. They represent the real-world distribution for this concrete risk factor. Thus the scenarios are not "arbitrage-free" probabilities by construction and cannot be used for valuation of asset and liability cash-flows. However, they can be made "arbitrage-free" by adjusting them. This requires estimating a market price of risk for the inflation rate. Indeed, inflation rate can be regarded as the driving-force of many other variables. Hence, we aim to extend the methodology presented here to a multivariate setting.

Appendix: Two Simulation Experiments

Two small-scale simulation experiments will be conducted in this Appendix. First, we examine the finite-sample performance of the gaps test C_1 and compare it with results obtained for the LM test C_1^*. Second, we are concerned with the performance of the seasonality test C. Recall that this latter test is composed of the gaps test C_1 and the test for multiplicative constraints C_2. In both experiments, the results presented here will be for linear AR models. They are fairly representative for other models, including SETAR.

For the first simulation experiment we considered the following model

$$(1 - 0.5B - \alpha_2 B^2 - \alpha_3 B^3 - 0.5B^4 + 0.25B^5)Y_t = \varepsilon_t \tag{10}$$

with $\varepsilon_t \sim i.i.d.N(0,1)$. Four sets of parameter values (α_1, α_2) are defined: i) (0.15, -0.25); ii) (0.25, -0.25); iii) (-0.25, 0); and iv) (0, -0.25). The corresponding AR models will be denoted by respectively M1, M2, M3, and M4. Under the null hypothesis $\alpha_2 = \alpha_3 = 0$, (10) may be written as

$$(1 - 0.5B)(1 - 0.5B^4)Y_t = \varepsilon_t. \tag{11}$$

We shall refer to this model as M0. The first simulation experiment is performed with 10,000 replications.

Figure 4 shows size-power trade-off curves of the test statistics C_1 and C_1^* for a sample size $n = 100$. The curves are generated by varying the critical value for the tests. The upper right-hand corner of the graph corresponds to a critical value of zero. Both size and power are 1 at this point. The lower left-hand corner corresponds to a very large critical value, so large that the test statistics will never exceed it. Both size and power are 0 at this point. It is clear from Figure 4 that, for any given model, the power of the test C_1 exceeds that of C_1^*. Table 5 contains results on the size of the tests for sample sizes 100, 200, and 300. Note, that the empirical size of the test C_1 is close to the nominal ones for sample size 300, whereas this is not the case for C_1^*.

Using 1,000 replications, Table 6 contains results of the second simulation experiment for four different multiplicative seasonal AR models and sample sizes 100 and 200. Note that Model I) coincides with (11). Also, note that similar to the experiment above, each multiplicative seasonal AR model has a corresponding non-multiplicative AR model. For these latter models column 3, denoted by CO, gives the number of times the correct order was identified by AIC_u. Column 4, denoted by NRG, gives the number of times the gaps hypothesis is not rejected by C_1 for those models identified as having the correct order. Hence, this parts concerns testing for H_{01}. As such the simulation results are completely in line with those presented in Table 5.

Now, in the second step, consider the seasonality hypothesis H_0. To this end, the specification strategy continues with only those series not rejecting the gaps hypothesis (column 4). Of these series we summarized in column 5 (NRM) the number of times the multiplicative constraints are not rejected by C_2. Finally, column 6, contains the number of times the seasonality hypothesis is not rejected. Obviously, the test statistic C_2 for multiplicative constraints seems to be too strong in rejecting the null hypothesis H_2 as compared to the statistic C for testing seasonality.

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Table 1: Estimation results for linear AR models; * denotes significant at the 5% level.

	i	France	Netherlands	Spain	UK	USA
		AR parameter α_i				
	0	0.0001	0.0007*	0.0007	0.0006	0.0001*
	1	0.2658*	0.0638	0.2222*	0.3686*	0.2379*
	2	0.0639	-0.0561		0.1348*	0.2218*
	3	0.1720*	0.1020*			-0.0115
	4	-0.0206	0.0581			0.0416
	5	0.0422	-0.0402			0.0569
	6	0.1109*	0.0571			0.0388
	7		0.0661			0.1005*
	8					0.0431
	9					0.1513*
	12	0.2252*	0.3313*	0.1571*	0.4234*	0.1945*
	13	-0.0544	-0.0346	-0.0062	-0.1391*	-0.0430
	14	-0.0299	0.0340		-0.0905	-0.1270*
	15	-0.0027	-0.1181*			0.0783
	16	-0.0016	-0.0115			0.0344
	17	-0.0370	0.0630			-0.0806
	18	0.1173*	0.0995*			-0.0566
	19		0.1663*			-0.0747
	20					0.0982*
	21					-0.0582
	24		0.2595*	0.1869*	0.2481*	
	25		0.0509	-0.0089	-0.1284*	
	26		-0.0372		-0.0438	
	27		-0.0506			
	28		-0.0245			
	29		-0.0088			
	30		-0.1220*			
	31		-0.0955*			
AIC_u		-5389	-4769	-4590	-4774	-6565
p-value of C_1		0.1348	0.5250	0.0000	0.0064	0.4718
p-value of Lin-Mudholkar test		0.0089	0.0270	0.0003	0.0000	0.0009

Table 2: Estimation results for non-multiplicative SETAR models; * denotes significant at the 5% level.

Regime		SETAR parameter $\phi_i^{(j)}$				
(j)	i	France	Netherlands	Spain	UK	USA
Lower (1)	0	-0.0002	0.0011*	0.0009*	0.0004	0.0000
	1				0.3603*	0.3043*
	2					0.1277*
	3					-0.1136*
	12	0.2216*	0.0074	0.2582*	0.4499*	0.2344*
	13				-0.1294*	-0.0796
	14					-0.0856*
	15					0.1710*
	24	0.2623*	0.3719*	0.2211*	0.1945*	
	25				-0.1079*	
p-value of C_1		0.0000	0.0000	0.0000	0.1424	0.0817
Upper (2)	0	0.0003	0.0005*	0.0067*	0.0022*	0.0004
	1	0.4340*	0.0252	0.4016*	0.3510*	0.0789
	2	0.0681				0.4134*
	3	0.1624*				0.2400*
	4	-0.1108				
	5	0.0260				
	6	0.1920*				
	12	0.2761*	0.5090*		0.5006*	
	13	-0.1551*	-0.0450		-0.2770*	
	14	-0.0147				
	15	0.0120				
	16	-0.0049				
	17	-0.0348				
	18	0.1096*				
	24		0.1405*			
	25		0.0637*			
p-value of C_1		0.2930	0.9601		0.0578	
AIC_u		-5384	-4832	-4601	-4849	-6682
p-value of Lin-Mudholkar test		0.0199	0.0246	0.0004	0.0000	0.2765

Table 3: Estimation results for multiplicative SEASETAR models; seasonal AR parameters are denoted by S1 and S2; * denotes significant at the 5% level.

Regime		AR and seasonal AR parameters				
(j)	i	France	Netherlands	Spain	UK	USA
Lower (1)	0	0.0011*	0.0003	0.0019*	0.0007*	0.0002*
	1				0.3987*	0.3309*
	2					0.2320*
	3					0.0678
	12	0.2047*	0.1943*	0.3281*		
	24	0.3180*	0.4342*	0.3010*		
	S1				0.4987*	0.3932*
	S2				0.1945*	
p-value of C_2					0.1204	0.0000
p-value of C					0.0964	0.0000
Upper (2)	0	0.0002	0.0012*	0.0067*	0.0033*	0.0004
	1	0.4728*	0.1741*	0.4016*	0.3813*	0.0846
	2	0.0434				0.4057*
	3	0.2418*				0.2467*
	4	-0.1751*				
	5	-0.0324				
	6	0.2990*				
	S1	0.4013*	0.5035*		0.4674*	
	S2		0.1767*			
p-value of C_2		0.0000	0.0010		0.1855	
p-value of C		0.0000	0.3400		0.0513	
AIC_u		-5452	-4903	-4622	-4928	-6751
p-value of Lin-Mudholkar test		0.0002	0.0364		0.0062	0.1668

Table 4: Dominant roots and periods for the best-fitted (non-)multiplicative (SEA)SETAR models given in Tables 2 and 3.

Country	Regime	Dominant Root	Modulus	Period
		Non-multiplicative		
France	Lower	$-0.48142 - 0.83384i$	0.96284	3.0
		-0.96284	0.96284	
	Upper	0.99114	0.99114	
Netherlands	Lower	$-0.48005 - 0.83148i$	0.96011	3.0
		-0.96011	0.96011	
	Upper	0.97684	0.97684	
Spain	Lower	$-0.48026 - 0.83184i$	0.96052	3.0
		-0.96052	0.96052	
	Upper	0.4016	0.40160	
UK	Lower	-0.97417	0.97417	
	Upper	-0.95536	0.95536	
USA	Lower	0.92585	0.92585	
	Upper	-0.80500	0.80500	
		Multiplicative		
France	Lower	$-0.48392 - 0.83817i$	0.96784	3.0
	Upper	$0.96276 - 0.05944$	0.96459	101.89
Netherlands	Lower	$-0.48887 - 0.84674i$	0.97773	3.0
		-0.97773	0.97773	
	Upper	$-0.84473 - 0.4877i$	0.97541	2.4
		$-0.4877 - 0.84473i$	0.97541	3.0
		$8.3236 \times 10^{-7} - 0.97541i$	0.97541	4.0
		$0.48771 - 0.84473i$	0.97541	6.0
		$0.84473 - 0.4877i$	0.97541	12.0
		0.97541	0.97541	
		-0.97541	0.97541	
UK	Lower	$-0.84607 - 0.48848i$	0.97696	2.4
		$-0.48848 - 0.84607i$	0.97696	3.0
		$-2.6943 \times 10^{-6} - 0.97696i$	0.97696	4.0
		$0.48848 - 0.84607i$	0.97696	6.0
		$-0.84607 - 0.48848i$	0.97696	12.0
		-0.97696	0.97696	
	Upper	$-0.81284 - 0.46929i$	0.93859	2.4
		$-0.46929 - 0.81284i$	0.93859	3.0
		$0.4693 - 0.81284i$	0.93859	6.0
		$0.81284 - 0.46929i$	0.93859	12.0
		0.93859	0.93859	
USA	Lower	0.92524	0.92524	
	Upper	-		

Table 5: Size of C_1 and C_1^* for model M0.

	0.10%		2.50%		5.00%		95.00%		97.50%		99.00%	
n	C_1	C_1^*	C_1	C_1^*	C_1	C_1^*	C_1	C_1^*	C_1	C_1^*	C_1	C_1^*
100	2.36	5.14	4.16	8.94	6.69	14.33	94.25	99.39	97.22	99.87	99.02	99.99
200	1.78	4.40	3.34	7.67	5.58	12.70	94.70	99.01	97.41	99.80	99.02	99.97
300	1.55	4.11	3.00	7.77	5.02	12.46	94.21	98.78	97.17	99.58	98.86	99.91

Table 6: Number of times the test statistics C_1, C_2, and C do not reject their corresponding null hypotheses.

Model	n	CO	NRG	NRM	NRS
I) $(1-0.5B)(1-0.5B^4)Y_t = \varepsilon_t$	100	926	888	613	783
	200	992	939	694	850
II) $(1-0.6B)(1-0.4B^{12})Y_t = \varepsilon_t$	100	841	551	355	467
	200	942	607	409	535
III) $(1-0.6B-0.3B^2)(1-0.5B^6)Y_t = \varepsilon_t$	100	406	376	132	193
	200	759	680	392	399
IV) $(1-0.8B)(1-0.5B^7)Y_t = \varepsilon_t$	100	941	684	354	505
	200	988	693	385	538

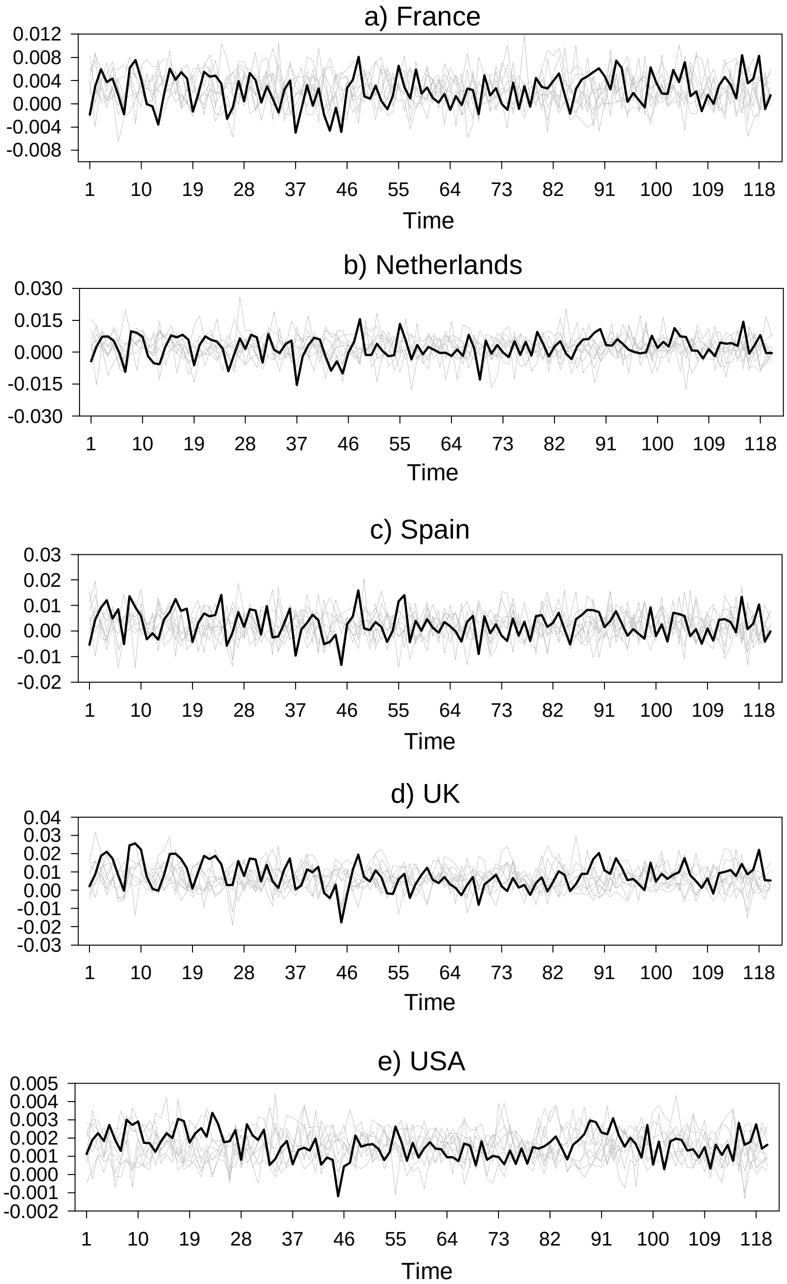

Figure 1: *Ten realisations of the fitted multiplicative SEASETAR models given in Table 3. For Spain the results are based on the non-multiplicative SETAR model in Table 2; solid black lines show one realisation of the fitted models.*

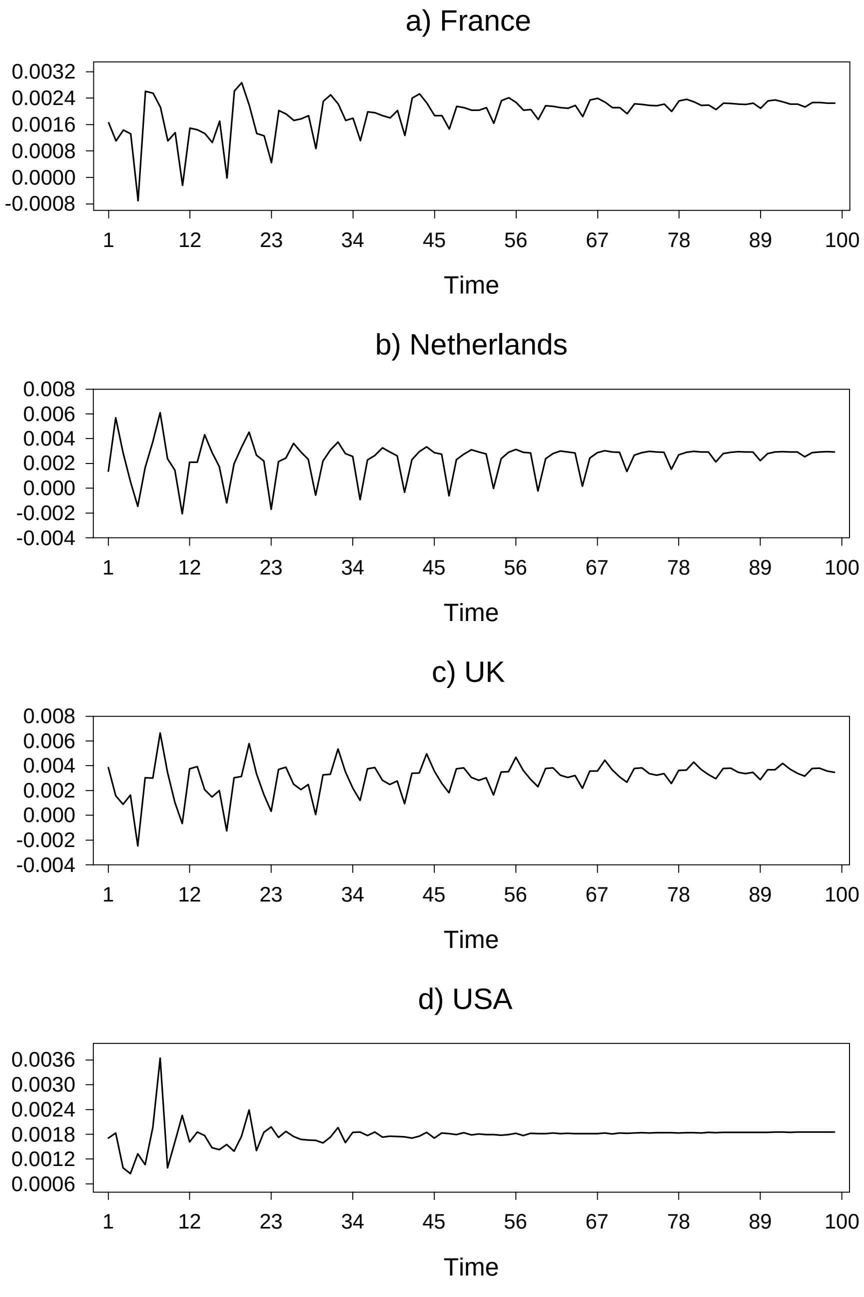

Figure 2: *Plots of the systematic parts of the fitted multiplicative SEASETAR models given in Table 3.*

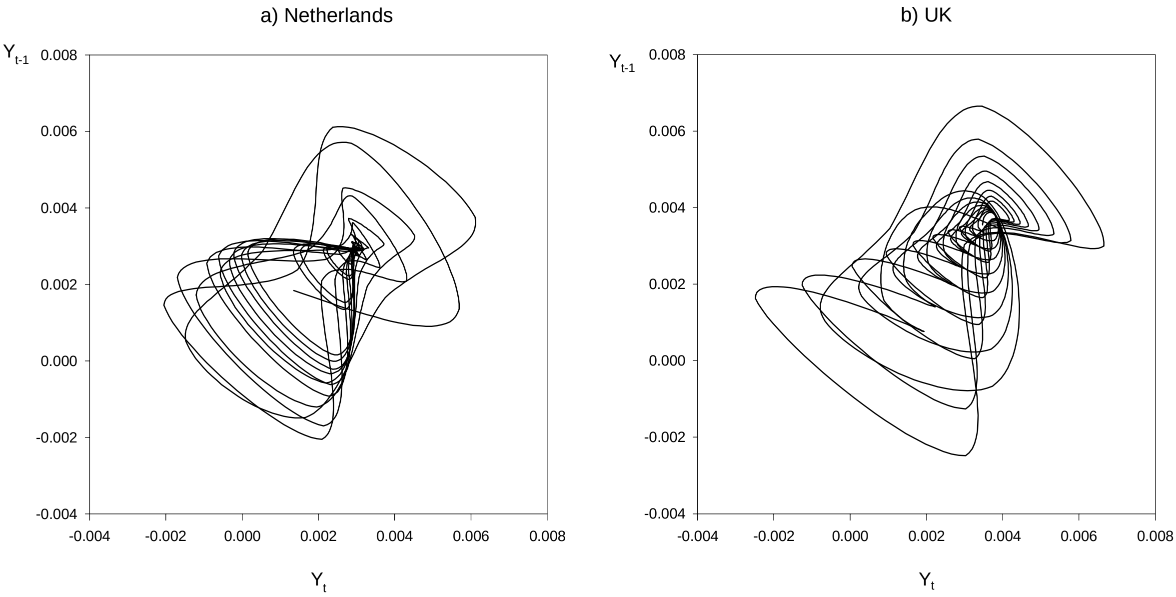

Figure 3: *Scatter plots for the systematic parts of the multiplicative SEASETAR models fitted to the series of the Netherlands and the UK.*

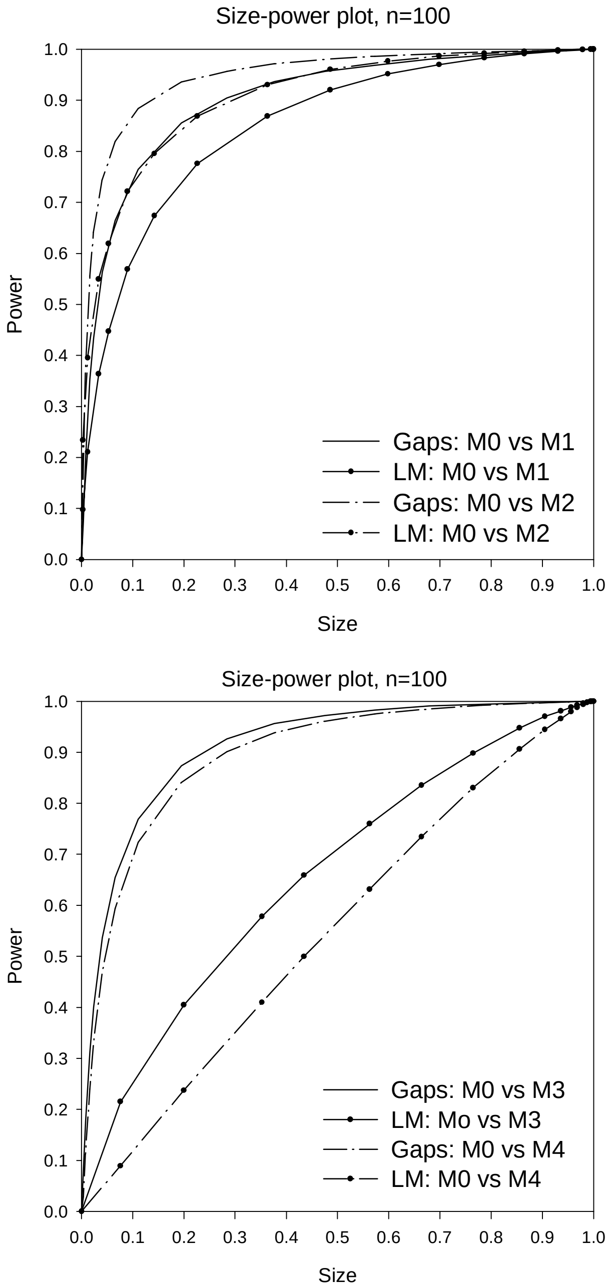

Figure 4: *Size-power trade-off curves for test statistics* C_1 *(gaps) and* C_1^* *(LM) for models M0 vs M1, M2, M3, and M4.*