TI 2000-0042/1
Tinbergen Institute Discussion Paper

Consumer Search and the Size of Internet Markets

Maarten C.W. Janssen
José Luis Moraga

Tinbergen Institute

The Tinbergen Institute is the institute for economic research of the Erasmus Universiteit Rotterdam, Universiteit van Amsterdam and Vrije Universiteit Amsterdam.

Tinbergen Institute Amsterdam
Keizersgracht 482
1017 EG Amsterdam
The Netherlands
Tel.: +31.(0)20.5513500
Fax: +31.(0)20.5513555

Tinbergen Institute Rotterdam
Burg. Oudlaan 50
3062 PA Rotterdam
The Netherlands
Tel.: +31.(0)10.4088900
Fax: +31.(0)10.4089031

Pricing, Consumer Search and the Size of Internet Markets

Maarten Janssen* José Luis Moraga†
Erasmus University Rotterdam and Tinbergen Institute

May 2000

Abstract

Despite the mixed empirical evidence, many economists still hold to the view that Internet will promote competition between firms, thereby lowering prices and increasing economic welfare. This paper presents a search model that provides a different view. We analyze the market for a homogeneous good where some consumers are fully informed while others are not. Depending on the parameter values, there may be three types of equilibria and the comparative statics results are different for each of these equilibria. For example, a reduction in search cost may raise equilibrium prices when consumers' search intensity is low, but reduce prices when consumers search intensity is high. These different comparative statics results may explain the mixed empirical evidence found so far.

Keywords: Internet, price dispersion, search, search agents

JEL Classification: D40, D83, L13

*Department of Economics, H07-22, Postbox 1738, Erasmus University, 3000 DR Rotterdam, The Netherlands, Phone: 31-10-4082341, Fax: 31-10-4089149, E-mail: janssen@few.eur.nl

†Tinbergen Institute Rotterdam, Burg. Oudlaan 50, 3062 PA Rotterdam, Phone: 31 10 4088905, Fax: 31 10 4089031, E-mail: moraga@few.eur.nl. Moraga acknowledges financial support from the EU through the TMR post-doctoral grant No. ERB4001GT973264.

1 Introduction

The implications for economic performance of the rapid growth of Internet use to carry out economic transactions is a hotly debated issue in the media as well as in academic economics. The general consensus seems to be that the use of the Internet will promote economic efficiency and will reduce commodity prices. Bakos (1997), for example, argues that[1]

> "electronic marketplaces are likely to move commodity markets closer to the classical ideal of a Walrasian auctioneer where buyers are costlessly and fully informed about seller prices. ...we expect that electronic marketplaces typically will sway equilibria in commodity markets to favor the buyers, will promote price competition among sellers, and will reduce sellers' market power."

Vulkan (1999: F69-70) states a similar view:

> "search agents increase consumer's search power, and in general are thought of as increasing competitiveness in markets (at least markets for homogeneous goods)."

Two factors seem to be important in bringing markets closer to the competitive ideal. First, Internet diminishes consumers search cost, which reduces the market power of firms. The impact of a reduction in search cost can further be viewed from two perspectives. (i) Individual consumers spend less time searching for firms (and their prices) on the Internet than when they physically have to go to firms (or check newspapers and watch television). (ii) Moreover, the introduction of agent technology that allows electronic agents to search the web for firms and their prices implies that more and more consumers may have almost zero search cost. Second, the globalization of the world through Internet implies that markets become bigger and that more firms compete on the same market. This increased competition may also increase welfare and reduce commodity prices.[2]

Despite these historical predictions, recent empirical studies related to efficiency in Internet markets show a mixed evidence. Some studies (Lee, 1997, Bailey, 1998)

[1] See also Bailey and Bakos (1997).

[2] A reduction in search costs made possible through Internet may be compared to a reduction in transportation costs that historically has taken place through the use of faster ways of transportation (sailing ships, machine ships, trains, airplanes, etc.) and our model may be applied more broadly to study the impact of these changes as well.

find, for example, that prices in Internet are higher than corresponding prices in conventional markets. Other analyses (Brynjolfsson and Smith, 1999) find the reverse conclusion.[3] The effect of electronic marketplaces on price dispersion also seems to be ambiguous empirically. Bailey (1998) finds that price dispersion is not lower online than in traditional outlets. Other studies emphasize that on-line prices exhibit substantial price dispersion (Baye and Morgan, 2000; Clemons *et al.,* 1998).[4]

In this paper we scrutinize the theoretical arguments for the efficiency of electronic market places and show why the empirical findings are ambiguous. The main argument we put forward is that for different parameter values there exist qualitatively different type of equilibria and the comparative statics effects of lower search costs and an increase in the number of firms depends on the prevailing equilibrium (and thus on the parameter values). We investigate the equilibrium properties of a search model where N firms produce a homogeneous product and compete to sell their product to a number of consumers. There are two types of consumers. There are k informed consumers who have zero search cost and m less-informed consumers who have positive search costs, denoted by c. For simplicity, all consumers have identical willingness-to-pay, given by v. Firms simultaneously choose prices and put them on the web. Less-informed consumers decide how many searches to make before they know the prices firms set. These consumers can also decide to abstain from searching when they expect the price to be too high (or when they have high relative search cost). In equilibrium, consumer expectations are satisfied. Hence, the interaction of firms and consumers is modeled as a simultaneous move game, where (in equilibrium) the search behavior of consumers is influenced by the prices quoted by firms, and the price setting behavior of firms is influenced by the search behavior of consumers.

The arguments about the economic implications of the growth of Internet use can be studied in this model by investigating the comparative statics effects of changes in the parameters of the model. A reduction in search cost can be measured by a decrease in c and/or an increase in k relative to m. An increase in the size of the relevant market can be measured by an increase in the number of consumers, either in m and/or in k. Increased competition is captured by an increase in the number of firms N. The fact that consumers may decide not to search is also important

[3] Recent reports by consultant companies Ernst & Young, Forrester Research and Goldman Sachs have also reached opposite conclusions (OECD, 1999: 73).

[4] See Smith *et al.* (1999) for a recent overview of these empirical findings.

for evaluating the implications of Internet use as the "opening of markets" made possible through the Internet in fact means that consumers who previously did not search may now decide to search.[5]

The main results we obtain are as follows. Depending on the parameter values we obtain one (sometimes two) of three different types of equilibria. These equilibria all exhibit price dispersion. The main difference between these equilibria is given by the frequency with which the less-informed consumers search for prices. These consumers may search (i) with low intensity, i.e., randomize between one search and no search at all, (ii) with moderate intensity, i.e., making exactly one search, or (iii) with high intensity, i.e., randomize between one search and two searches. The equilibrium in which less-informed consumers search with low intensity has been disregarded by the search literature (see below). It exists when search costs are above a critical value, which approaches zero as N becomes large. The comparative statics results for the different parameters depend on which equilibrium prevails. We will summarize some of the more striking results here, leaving the rest of the results for the main body of the paper. A reduction in search cost increases expected prices when consumers' search intensity is low, but it decreases expected prices when consumers' search intensity is high. Firms' profits, in these cases, move in the same direction as expected prices. The reason for the first finding is that expected prices must be equal to $v - c$ in order to make less-informed consumers indifferent between searching for one price and not searching at all. If search costs decrease, more consumers will search (and make one link), which gives firms relatively more monopoly power, which in turn results in higher expected prices. The second result is more in accordance with conventional wisdom: if search cost fall consumers will search more intensively, i.e., make two searches in this case, which reduces the monopoly power of firms. Similar arguments hold true when the relative number of fully informed consumers increases, for example, through the advent and use of electronic agents. So our model helps explain the existing mixed evidence on the impact of search engines, shopbots, etc. on Internet markets. A number of interesting comparative statics results appear when we investigate the impact of an increase in the number of competitors N. A first thing to notice is that there exists an equilibrium where less-informed consumers randomize between making one search and not searching at

[5] Lohse *et al.* (1999), using the Wharton Virtual Test Market, an ongoing survey of Internet users, report a dramatic increase of online purchases from 1997 to 1998. Johnson *et al.* (2000), however, report that the buyers' actual search intensity over the Internet is rather low: 70% of CD shoppers, 70% of book purchasers and 36 % of travel shoppers were observed to visit just one site.

all for virtually any level of search costs, provided the number of competitors is large enough. This means that this type of equilibrium deserves important consideration in electronic markets. Second, when N becomes large, almost all less-informed consumers stop searching so that the number of transactions in the market diminishes and the surplus generated in the industry declines. Finally, equilibrium price dispersion increases with N so that our model provides an explanation for the notable price dispersion observed in Internet markets.

There is a vast literature on consumer search.[6] The papers that come closest to ours are a paper by Burdett and Judd (1983) and Stahl (1989).[7] Next, we describe these two papers in some detail and explain where our work differs from their analyses. Burdett and Judd show that equilibrium price dispersion may occur in *competitive* markets when consumers randomize between searching for two prices and searching for only one price, in a non-sequential fashion. For a range of parameter values two equilibria with price dispersion exist, one of which has lower expected prices and consumers searching more intensively than the other. As all their consumers are identical and have search cost bounded away from zero, their model has, in addition, one pure strategy equilibrium in which all firms charge the monopoly price (Diamond, 1971). Our paper also studies non-sequential search and the equilibria we obtain exhibit price dispersion in the vein of Burdett and Judd. Our model is, however, more suited to the study of the implications of the growth of Internet use for the following reasons. First, we present an *strategic* model where the implications of an increase in the number of firms in the relevant market may be studied by changing the parameter N. Second, we allow for the presence of fully informed consumers (without search cost). This implies that our model does not have a pure strategy equilibrium in which all firms charge the monopoly price, and also that all the equilibria of our model exhibit price dispersion. More importantly, the introduction of electronic search agents on the Internet makes it important to study the impact of a growing presence of fully informed consumers. Third, Burdett and Judd assume that each consumer makes at least one search. This implies that equilibrium price dispersion can only occur for sufficiently low search costs. As argued above, consumers are steadily entering electronic markets and thus we think that in the context of the Internet discussion, it is important to allow for the possibility

[6]See Stiglitz (1989) for a survey. More recent contributions include Burdett and Coles (1997), Fershtman and Fishman (1992) and McAfee (1995).

[7]An early paper with a model similar to ours is Varian (1980); however, he did not consider endogenous consumers search.

that consumers (previously) were searching, or searching with low intensity.

Stahl (1989) studies a sequential model where strategically acting firms set prices before consumers search. Like in our model, there are fully informed consumers (with zero search cost) and less-informed consumers (with positive search cost). Less-informed consumers know the distribution of prices that are set and search sequentially, i.e., they first observe one price and then decide whether to observe more prices or not.[8] The first price quotation is observed for free, which implies that every consumer makes at least one search. The sequential nature of the model implies that there exists a unique equilibrium. In this equilibrium, less-informed consumers observe exactly one price quotation. The sequential nature of decisions and the fact that less-informed consumers make exactly one link to a firm gives the firms quite a bit of market power. There are two main differences between our model and Stahl's. First, in Stahl's model consumers passively observe the price distribution and decide on an optimal search strategy given this distribution. Firms take the reactions of consumers into account. In our model, in contrast, consumers are more active as search behavior and price distributions are simultaneously determined. Secondly, in our model consumers may "threaten" not to search, which gives them potential power vis-a-vis firms.

The remainder of the paper is organized as follows. Section 2 describes the model. We characterize the equilibria of our model in Section 3 and give the comparative statics analysis for the duopoly case in Section 4. The analysis of large markets is given in Section 5. We conclude in Section 6.

2 The Model

Consider a market for a homogeneous good. On the demand side of the market, there is a total of $m+k$ consumers who wish to purchase at most a single unit of the good. A number k of the consumers search for prices costlessly. We will refer to these consumers as *informed* consumers. The other m consumers must pay search cost $c > 0$ to observe a price quotation. These consumers, referred to as *less-informed* consumers, may decide to obtain several price quotations, say n, in which case they incur search cost equal to nc. For future reference, let $\lambda = k/(m+k)$ denote the proportion of informed consumers in the market, $0 < \lambda < 1$. All consumers are fully

[8] Morgan and Manning (1985) derive optimal search strategies which combine features of the fixed-sample-size search strategy and the sequential search strategy.

rational, i.e., informed consumers buy the good from the lowest priced store, while less-informed consumers acquire it from the store with the sampled lowest price. The maximum price any consumer is willing to pay for the good is $v > c$.

On the supply side of the market there are $N \geq 2$ firms. Firms produce the good at constant returns to scale and their identical unit production cost is normalized to zero, without loss of generality.

Firms and consumers play a simultaneous move game. An individual firm chooses its price taking price choices of the rivals as well as consumers' search behavior as given. Consumers form conjectures about the distribution of prices in the market and decide how many prices to observe before purchasing from the store with the lowest observed price. Let $F(p)$ denote the distribution of prices charged by a firm. Let μ_n denote the probability with which a less-informed consumer searches for n price quotations. We only consider symmetric equilibria. An equilibrium is a tuple $\{F(p), \overline{\pi}, \{\mu_n\}_{n=0}^{N}\}$ such that (a) $\pi(p) = \overline{\pi}$ for all p in the support of $F(p)$, (b) $\pi(p) \leq \overline{\pi}$ for all p, and (c) $\{\mu_n\}_{n=0}^{N}$ describes the optimal search behavior of less-informed consumers given that their conjectures about the price distribution are correct.

3 Equilibrium under duopoly

In this section we will concentrate on the case where $N = 2$ for the ease of exposition. In Section 5 we will present the N firm case and analyze the limit economy when N approaches infinity.

Taking as given the firms' choices of prices, a less-informed consumer must decide whether to visit no store, one store, or two stores. Then, a less-informed consumer's strategy is a probability distribution over these three events. Informed consumers observe all prices at no cost.

Our first observation is that there are no equilibria in which less-informed consumers decide to search for both stores' prices. Also, we note that there are no equilibria in the extreme opposite case, i.e., where less-informed consumers do not search at all.

Lemma 1 *If $c > 0$ and $k > 0$, (i) an equilibrium where $\mu_2 = 1$ does not exist. (ii) An equilibrium where $\mu_0 = 1$ does not exist either.*

Proof. (i) Suppose $\mu_2 = 1$. Then firms would charge Bertrand prices, i.e., $p_i = 0$, $i = 1, 2$. But if this is so, less-informed consumers would search only once, i.e., $\mu_2 = 0$. Thus, $\mu_2 = 1$ cannot be part of an equilibrium. (ii) Suppose $\mu_0 = 1$. Again, firms would charge Bertrand prices and therefore costly-search consumers would find beneficial to search at least once. Thus, $\mu_0 = 1$ cannot be part of an equilibrium either. ■

Lemma 1 shows that the following alternatives exhaust the equilibrium possibilities of consumers' search behavior: (a) $0 < \mu_1 \leq 1$, $\mu_0 + \mu_1 = 1$ (b) $0 < \mu_1 \leq 1$, $\mu_1 + \mu_2 = 1$.[9]

It is straightforward to see that there does not exist a symmetric equilibrium either where both stores charge a particular price with positive probability. If this were so, a small reduction in that price by one of the firms would be beneficial as it would attract all informed consumers. Thus, the only price that could be proposed as having positive probability is $p = 0$. However, since $\mu_0 < 1$ by Lemma 1, a single firm would make a positive (expected) profit by raising its price. The following Lemma summarizes.

Lemma 2 *Given the search behavior of the consumers, if $F(p)$ is an equilibrium price distribution, then it is atomless. Hence, there is no pure strategy equilibrium.*

Lemma 2 shows that equilibria must necessarily exhibit price dispersion. In what follows, we study equilibrium behavior of firms under each of the less-informed consumers' behavioral hypotheses.

Case a: $0 \leq \mu_0 < 1$, $\mu_0 + \mu_1 = 1$.

Consider first that consumers randomize between searching for one price quotation and not searching at all, i.e., $\mu_0 > 0$, $\mu_0 + \mu_1 = 1$. We only consider symmetric equilibria. Let $F(p)$ be the probability that firm i charges a price that is smaller than p. The expected payoff to firm i of charging price p when the rival chooses a random pricing strategy according to the cumulative distribution $F(\cdot)$ is

$$\pi_i(p, F(p)) = p\left[\frac{m\mu_1}{2} + k(1 - F(p))\right] \qquad (1)$$

[9] It may also happen that there exists an equilibrium for which $\mu_0 > 0$, $\mu_1 > 0$ and $\mu_0 + \mu_1 + \mu_2 = 1$. However, this equilibrium is non-generic in the sense that it imposes restrictions on the set of exogenous parameters that are only satisfied in a null set.

This profit expression is easily understood. A firm obtains a per consumer profit of p. The expected demand faced by a firm stems from the two different groups of consumers. A firm attracts the k fully informed consumers when it charges a lower price than the rival, which happens with probability $1 - F(p)$. A firm also serves the m less-informed consumers whenever they actively search for one price, with happens with probability μ_1, and, particularly, when they find its store, which occurs with probability one half.

In equilibrium, the firm must be indifferent between charging any price in the support of F. Hence, a price in the support of F must satisfy the first order condition

$$\frac{m\mu_1}{2} + k(1 - F(p)) - kpf(p) = 0, \tag{2}$$

where $f(p)$ denotes the density function associated with $F(p)$.

The maximum price a firm will ever charge is v since no buyer who observes a price above his/her reservation price will acquire the good. Also, the upper bound of the price distribution cannot be lower than v because a firm would gain by slightly raising its price. Thus, it must be the case that $F(v) = 1$. Solving the differential equation (2) with the boundary condition $F(v) = 1$ yields

$$F(p) = \frac{2k + m\mu_1}{2k} - \frac{m\mu_1}{2k}\frac{v}{p}. \tag{3}$$

Since F is a distribution function there must be some $\underline{p}$ for which $F(\underline{p}) = 0$. Solving for $\underline{p}$ one obtains the lower bound of the price distribution

$$\underline{p} = \frac{m\mu_1 v}{2k + m\mu_1}.$$

A mixed strategy over the support $\underline{p} \leq p \leq v$ according to the cumulative distribution function F specified above is an equilibrium if and only if consumers are indeed indifferent between searching for one price and none at all. Therefore, it must be the case that $v - E[p] - c = 0$, where E denotes the expectation operator.[10]

[10] It must further be checked that it is not profitable for consumers to search more than once, i.e., that $v - E[\min\{p_1, p_2\}] - 2c < 0$. We prove this in Fact 0 in the Appendix.

In other words, the following condition must be satisfied:[11]

$$1 - \frac{m\mu_1}{2k} \ln\left(\frac{2k + m\mu_1}{m\mu_1}\right) = \frac{c}{v} \tag{4}$$

Let us denote the left-hand-side of equation (4) as $\Phi(\mu_1; m, k)$. The following facts about the function Φ are proved in the Appendix:

Fact 1: $\frac{d\Phi}{d\mu_1} < 0$

Fact 2: $\frac{d^2\Phi}{d\mu_1^2} > 0$

Fact 3: $\lim_{\mu_1 \to 0} \Phi(\mu_1) = 1$

Fact 4: $\Phi(1) = \frac{2k - m\ln\left(\frac{m+2k}{m}\right)}{2k} > 0.$

Facts 1 to 4 allow us to represent condition (4) as shown in Figure 1. The decreasing and convex curve represents Φ as a function of μ_1. The flat line is just the right-hand side of (4). An equilibrium consumer's randomization probability is thus given by the intersection of curve Φ and c/v. Facts 1 to 4 also enable us to state that these two curves intersect once at most.

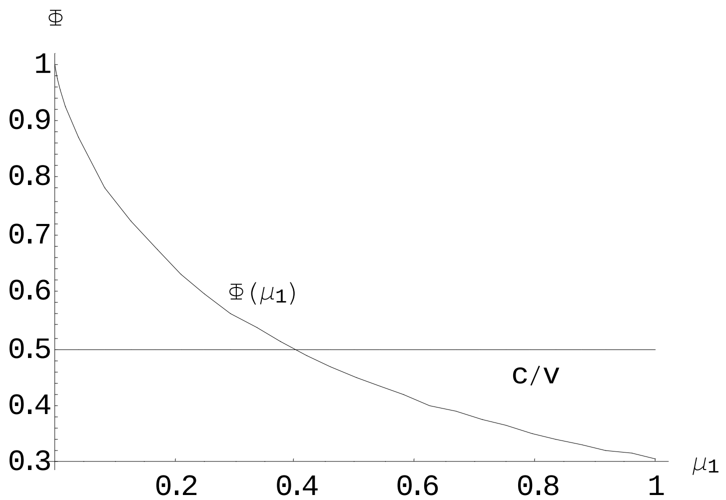

Figure 1: Buyers randomize between one-search and no-search (m=200, k=100)

The following proposition summarizes these findings:

[11] For current and future reference, let $H(p) = a - bv/p$ be a distribution function in the support $bv/a \leq p \leq v$, with $a - b = 1$. Then $E[p] = bv\ln[a/b]$ and $E[\min\{p_1, p_2\}] = 2bv(1 - b\ln[a/b])$.

Proposition 3 *Let* $1 \geq \frac{c}{v} \geq \frac{2k+m\ln\left(\frac{m+2k}{m}\right)}{2k}$. *Then an equilibrium of the game described above exists where the less-informed consumers randomize between searching for one price quotation with probability* μ_1^*, *and not searching at all with probability* $1-\mu_1^*$, *where* $\mu_1^* \in (0,1]$ *solves*

$$1 - \frac{m\mu_1}{2k} \ln\left(\frac{2k+m\mu_1}{m\mu_1}\right) = \frac{c}{v},$$

and firms randomly select prices from the set $p \in \left[\frac{m\mu_1^* v}{2k+m\mu_1^*}, v\right]$ *according to the cumulative distribution function*

$$F(p) = \frac{2k+m\mu_1^*}{2k} - \frac{m\mu_1^*}{2k}\frac{v}{p}.$$

There is at most one such equilibrium.

Case b: $0 \leq \mu_2 < 1$, $\mu_1 + \mu_2 = 1$.

We now turn to the case where less-informed consumers randomize between searching for one price quotation with probability μ_1, and searching for two price quotations, with the remaining probability $1-\mu_1$.[12] The expected payoff to firm i of charging price p when the rival chooses a random pricing strategy according to the cumulative distribution function $G(p)$ and less-informed consumers search as specified above is

$$\pi_i(p, G(p)) = p\left[\frac{m\mu_1}{2} + (k + m(1-\mu_1))(1-G(p))\right] \tag{5}$$

This profit function can be easily interpreted. A firm makes a per consumer profit of p. The firm's expected number of consumers is $m\mu_1/2 + (k+m(1-\mu_1))(1-G(p))$. The first summand of (5) stems from the less-informed consumers when they search for only one price, which happens with probability μ_1. A firm attracts these $m\mu_1$ consumers with probability one half. The second summand of (5) comes from the fully informed consumers as well as the less-informed consumers when they search for two prices, which happens with probability $1-\mu_1$. A firm attracts these consumers, a total of $k + m(1-\mu_1)$, when it charges a lower price than the rival, which occurs with probability $1-G(p)$.

[12] For ease of exposition, we maintain the notation used so far in the sense that μ_1 denotes the probability with which buyers search for one price. However, unlike in case a above, $1-\mu_1$ denotes now the probability with which consumers search for two prices.

In equilibrium, the firm must be indifferent between charging any price in the support of G. Hence, a price in the support of G must satisfy the first order condition

$$\frac{m\mu_1}{2} + (1 - G(p) - pg(p))(m(1-\mu_1) + k) = 0, \tag{6}$$

where $g(p)$ denotes the density function associated to $G(p)$.

As mentioned above, the upper bound of an equilibrium price distribution must be v. Solving the differential equation (6) with the boundary condition $G(v) = 1$ yields

$$G(p) = \frac{m + 2k + m(1-\mu_1)}{2(k + m(1-\mu_1))} - \frac{m\mu_1}{2(k + m(1-\mu_1))}\frac{v}{p}.$$

Since G is a distribution function, there must be some $\underline{\underline{p}}$ for which $G(\underline{\underline{p}}) = 0$. Solving for $\underline{\underline{p}}$ one obtains

$$\underline{\underline{p}} = \frac{m\mu_1}{m + 2k + m(1-\mu_1)}.$$

A mixed strategy over the support $\underline{\underline{p}} \leq p \leq v$ according to the cumulative distribution function G specified above is an equilibrium if and only if consumers are indeed indifferent between searching for only one price and searching for two prices.[13] Therefore it must be the case that

$$v - E[p] - c = v - E[\min\{p_1, p_2\}] - 2c$$

In other words, the following must be satisfied (see footnote 11):

$$\frac{m\mu_1}{2(k + m(1-\mu_1))}\left[\frac{m+k}{m(1-\mu_1)+k}\ln\left(\frac{m + 2k + m(1-\mu_1)}{m\mu_1}\right) - 2\right] = \frac{c}{v} \tag{7}$$

To analyze the consumers' stability condition (7), let us denote the left-hand-side of this equation as $\Gamma(\mu_1; m, k)$. The following facts, proved in the Appendix, are useful in what follows:

Fact 5: $\Gamma(1) = \frac{m\left((m+k)\ln\left[\frac{m+2k}{m}\right] - 2k\right)}{2k^2} > 0.$

Fact 6: $\lim_{\mu_1 \to 0} \Gamma(\mu_1) = 0$

Fact 7: $\left.\frac{d\Gamma}{d\mu_1}\right|_{\mu_1 = 1} > 0$ iff $\frac{k}{m+k} > \overline{\lambda}$,

[13] In addition, we must be sure that no consumer gains by making no search, i.e., it must be that case that $v - E[p] - c > 0$. This is trivially satisfied.

where $\overline{\lambda}$ is defined as the solution to equation[14]

$$\ln\left[\frac{1+\lambda}{1-\lambda}\right]-\frac{2\lambda(2+\lambda)}{2+\lambda-\lambda^2}=0.$$

Fact 8: $\frac{d^2\Gamma}{d\mu_1^2}<0.$

Facts 5 to 8 illustrate that the shape of function $\Gamma(\cdot)$ depends on parameter constellation. On one hand, when the percentage of informed consumers is large enough, Fact 7 together with Fact 8 indicate that Γ is an increasing and concave function of μ_1, as represented in Figure 2. In such a case, an equilibrium is given by the intersection of curve $\Gamma(\mu_1)$ with the line c/v. It is easily seen that there is at most one such equilibrium.

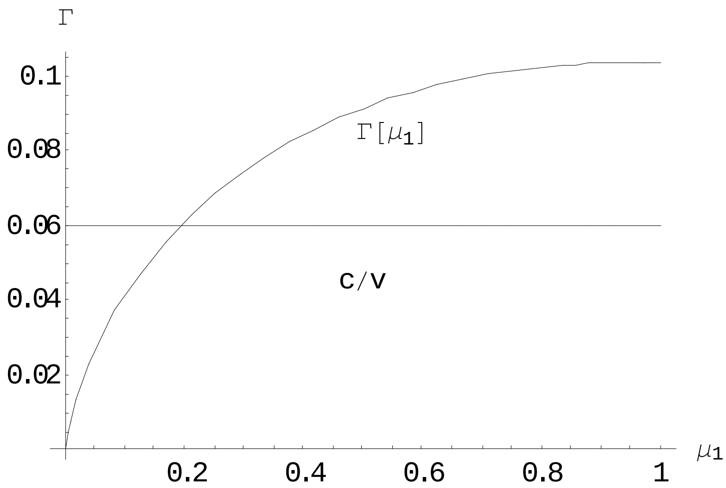

Figure 2: Buyers randomize between one-search an two-searches (m=50, k=250).

On the other hand, when the percentage of informed consumers is small, Facts 5 to 8 imply that the curve $\Gamma(\mu_1)$ is first increasing and afterwards decreasing. The strict concavity of $\Gamma(\cdot)$ ensures that there is some unique μ_1 for which $\Gamma(\cdot)$ reaches a maximum. The shape of $\Gamma(\cdot)$ when k is small relative to m is illustrated in Figure 3. This graph shows that for small enough c/v there may be either one equilibrium or two equilibria, depending on the relative size of the search cost of less-informed consumers.

[14] It can be shown numerically that there is a unique solution to this equation, which is approximately equal to $\overline{\lambda}=0.634816$.

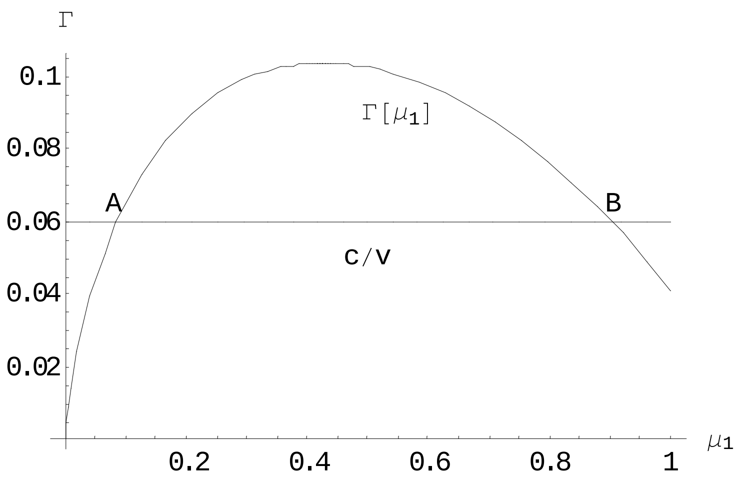

Figure 3: Buyers randomize between one-search and two-searches (m=250, k=50).

Let $\overline{\Gamma}$ denote the maximum value of $\Gamma(\cdot)$, i.e., $\overline{\Gamma} = \max_{\mu_1 \in (0,1]} \Gamma(\mu_1)$. Upon observing Figures 2 and 3, we can conclude that:

Proposition 4 *Let $\overline{\Gamma} \geq \frac{c}{v} \geq 0$. Then either one equilibrium or two equilibria of the game described above exist where the less-informed consumers randomize between searching for one price with probability μ_1^* and searching for two prices with probability $1 - \mu_1^*$, where $\mu_1^* \in (0, 1]$ is the solution to*

$$\frac{m\mu_1^*}{2(k + m(1 - \mu_1^*))} \left[\frac{m + k}{m(1 - \mu_1^*) + k} \ln \left(\frac{m + 2k + m(1 - \mu_1^*)}{m\mu_1^*} \right) - 2 \right] = \frac{c}{v},$$

and firms randomly select prices from the set $p \in \left[\frac{m\mu_1^*}{m+2k+m(1-\mu_1^*)}, v \right]$ according to the cumulative distribution function[15]

$$G(p) = \frac{m + 2k + m(1 - \mu_1^*)}{2(k + m(1 - \mu_1^*))} - \frac{m\mu_1^*}{2(k + m(1 - \mu_1^*))} \frac{v}{p}.$$

There is at most one such equilibrium that is stable.

Proposition 4 indicate that when the number of informed consumers is not very large, in particular when $k/(m + k) < \overline{\lambda}$ (see Fact 7), our economy may have two equilibria where the less-informed consumers randomize between searching for one price and searching for two prices. However, only one equilibrium is stable. Points

[15] *Note that when we substitute $k = 0$, the price distribution is exactly equal to the price distribution found by Burdett and Judd (1983) for a competitive market. Hence, this type of equilibrium is quite robust to the number of firms present in the market.*

A and B in Figure 3 depict the two different equilibria. It is easily seen that the equilibrium denoted by the point B in the graph is not stable. The argument is as follows. To the left of point B, the expected value consumers derive from searching for two prices instead of searching for one price is larger than its associated cost. Therefore, consumers would increase the probability with which they search for two prices, down to the point where such expected benefits and costs are equal, i.e. down to point A in Figure 3. Note that the opposite holds to the right of B, which should lead consumers to search less intensively. These observations question the stability of the equilibrium represented by the point B in Figure 3.[16] A similar argument shows that the equilibrium depicted by point A in Figure 3 is a stable equilibrium. In what follows, for our comparative statics results, we will concentrate on this stable equilibrium.

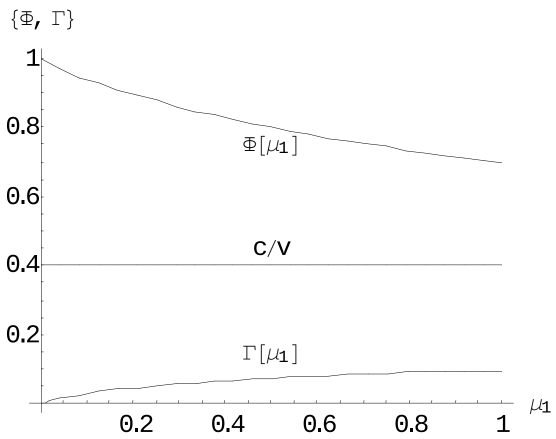

Figure 4a

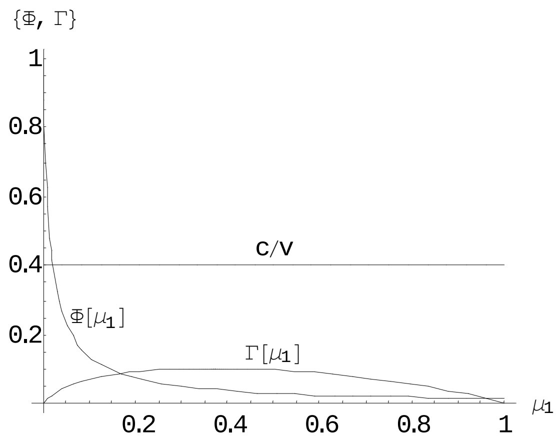

Figure 4b

We are now ready to provide the complete characterization of stable equilibria in our model. The next result states that, depending on parameter constellations, for every possible search cost level, there may be either a single equilibrium, or two stable equilibria. The two possibilities are illustrated in Figures 4a and 4b. On the left-hand side, Figure 4a shows the case where the relative number of informed consumers is sufficiently large ($m = 50$, $k = 250$). In this case, for large search cost parameters there is a unique equilibrium where less-informed consumers randomize between searching for one price and not searching at all. As the search cost falls, these consumers find it beneficial to search more intensively. Indeed, for intermediate search cost levels, the only equilibrium is such that the less-informed consumers

[16] See Fershtman and Fishman (1992).

search for one price with probability one. Finally, when the search cost is sufficiently low, consumers randomize between searching for one price and searching for two prices in the only stable equilibrium.

Figure 4b illustrates the case where the relative number of informed consumers is small ($m = 295$, $k = 5$). In this case, again, for high search costs, there is a single equilibrium where consumers randomize between searching for one price and none at all. However, for low search costs there may be two equilibria: the equilibrium previously mentioned and one in which consumers randomize between one and two searches. Finally, for very low search costs, there may be two equilibria too: one where buyers search for one price with probability one, and another in which consumers randomize between searching for one price and searching for two prices. With the help of Fact 9, proved in the appendix, the following result summarizes:

Fact 9: $\Phi(1) - \Gamma(1) > 0$.

Theorem 5 *(A) Suppose* $\Phi(1) > \overline{\Gamma}$. *Then:*
(A.1) For $1 > \frac{c}{v} \geq \Phi(1)$, *there is a single equilibrium in which less-informed buyers randomize between searching for one price and not searching at all. This equilibrium is characterized in Proposition 3.*
(A.2) For $\Phi(1) \geq \frac{c}{v} \geq \overline{\Gamma}$, *there is only one equilibrium in which less-informed consumers search for one price with probability one. This equilibrium is characterized by substituting* $\mu_1 = 1$ *either in Proposition 3, or in Proposition 4.*
(A.3) For $\overline{\Gamma} \geq \frac{c}{v} \geq \Gamma(1)$, *there is (i) an equilibrium in which consumers search for one price with probability one (characterized as in A.2), and (ii) a stable equilibrium in which consumers randomize between searching for one price and searching for two prices (Proposition 4).*
(A.4) Finally, for $\Gamma(1) \geq \frac{c}{v} > 0$, *there is only one stable equilibrium where consumers randomize between searching for one price and for two prices (Proposition 4).*

[B] Suppose $\Phi(1) < \overline{\Gamma}$. *Then:*
(B.1) For $1 > \frac{c}{v} \geq \overline{\Gamma}$, *there is a single equilibrium in which less-informed buyers randomize between searching for one price and not searching at all (Proposition 3).*
(B.2) For $\overline{\Gamma} \geq \frac{c}{v} \geq \Phi(1)$, *there is (i) an equilibrium in which consumers randomize between searching for one price and none at all (Proposition 3), and (ii) a stable equilibrium where buyers randomize between searching for one price and for two prices (Proposition 4).*
(B.3) For $\Phi(1) \geq \frac{c}{v} \geq \Gamma(1)$, *there is (i) an equilibrium where buyers search for one price with probability one (characterized as in A.2), and (ii) a stable equilibrium*

where less-informed buyers randomize between one search and two searches (Proposition 4).
(B.4) Finally for $\Gamma(1) \geq \frac{c}{v} > 0$, *there is a single stable equilibrium where buyers randomize between searching for one price and for two prices (Proposition 4).*

4 Comparative statics

In this section, we study the impact of changes in the parameters of the model. As argued above, a reduction in search cost is captured in our model either by a reduction in c or by an increase in the number of informed consumers k, keeping the total number of consumers $m + k$ constant. An increase in the size of the market is captured by an increase in the total number of consumers $m + k$. This can be further decomposed in an increase in the number of less-informed consumers, and in an increase in the number of full informed consumers. The impact of changes in the number of competitors N will be studied in the next section.

Interestingly, the impact of parameter changes depends on which equilibrium prevails in our economy. Next, we study the comparative statics effects of changes in the parameters of the model on the different equilibria described above. In doing so, the following facts prove useful: Let p be distributed according to $H(p) = a - bv/p$ in the support $bv/a \leq p \leq v$, with $a - b = 1$. Then:

Fact 10: $\frac{dE[p]}{db} > 0$

Fact 11: $\frac{dE[\min\{p_1,p_2\}]}{db} > 0$.

The effects of a reduction in search cost c:
We start by considering a reduction in search costs. Consider first the equilibrium where consumers randomize between searching for one price and not searching at all (see Proposition 3). From Figure 1, it is clear that a reduction in search costs results in an increase of μ_1, i.e., less-informed consumers search more intensively. Since in equilibrium it must be the case that $v - E[p] - c = 0$, it is clear that as result of a fall in c, the price that the less-informed consumers expect increases! The intuition behind this surprising result is easily understood: when less-informed consumers search more intensively, sellers have monopoly power over more buyers, as these consumers search for one price at most. In the equilibrium under consideration, less-informed consumers do not exercise "price comparisons", and thus are prepared to accept relatively high prices.

Our next observation is that a decrease in the less-informed consumers' search cost also increases the price expected by the fully informed consumers. Interestingly, the fact that less-informed consumers participate in the market with higher probability exerts a negative externality on the informed consumers by increasing firms' incentives to raise prices. This is easily seen by noting that $b = m\mu_1/2k$ in this equilibrium, and employing Fact 11.

Given that expected prices raise, one should expect the impact of a reduction in search cost on firms' expected benefits to be positive. Indeed, in equilibrium firms obtain profits $\pi = vm\mu_1/2$ (see (1)), which increase with μ_1 as a result of a fall in c.

We finally pay attention to the effects of a reduction of search cost on social welfare. Since less-informed consumers randomize between searching for one price and none at all, expected social welfare is equal to $W = kv + (v-c)m\mu_1$. A decrease in c is welfare improving *per se* because the inefficiency present in our market mitigates. However, here a fall in c implies, in addition, that more poorly informed consumers become active in the market. Therefore, from a social point of view, a reduction of search costs is clearly beneficial. An important observation in line is that all the additional surplus generated by a search cost reduction is captured by the firms!

We now turn to study the effects of a reduction in search cost in the equilibrium where less-informed consumers search for one price with probability one (see Figure 4). Trivially, a small change in c leaves less-informed consumers' behavior unchanged. This implies that expected prices and firms' profits remain the same. Social welfare $W = kv + m(v - c)$ however raises, but this is just due to the fact that less-informed consumers incur lower costs to discover prices.

Finally, we analyze the effects of a reduction in search cost on the stable equilibrium characterized by consumers' randomization between searching for one price and searching for two prices (see Proposition 4 and Figures 2 and 3). In this case, less-informed consumers search more intensively as a response to a reduction in c. This increases the extent to which "price comparisons" occur in the market, which unavoidably increases price competition between the firms. In this case $b = m\mu_1/(2(k+m(1-\mu_1))$ and it is easily seen that $db/d\mu_1 = m(m+k)/(2(k+m(1-\mu_1)^2) > 0$. This together with Facts 10 and 11 prove that expected prices for both types of consumers decrease.

From (5), one can see that equilibrium profits $\pi = vm\mu_1/2$ decrease as μ_1 decreases. Finally the total surplus in the market is $W = kv + m(v - 2c + \mu_1 c)$. The effects of a reduction in c on social welfare are given by $dW/dc = m(-2+d\mu_1/dc+\mu_1)$, whose sign we have been unable to characterize.

These findings are summarized below in Table 1.

An increase in the relative number of informed consumers λ:
To capture the effects of the introduction of search engines, shopbots, gatekeepers, etc., i.e., electronic search agents which automatically search the web for prices in Internet markets, we study next the impact of an increase in the relative number of completely informed consumers. To do so, we recall that λ denotes the ratio of informed consumers in the economy, i.e., $\lambda = k/(m+k)$. Hence, we next study how our economy is affected by an increase in λ which leaves the total number of buyers constant.

Consider first the equilibrium in which the less-informed consumers randomize between searching for one price and none at all (see Proposition 3 and Figure 1). One can apply the implicit function theorem to equation (4) to obtain

$$\frac{d\mu_1}{d\lambda} = \frac{\mu_1}{\lambda(1-\lambda)} > 0. \tag{8}$$

This means that an increase in the relative number of fully informed consumers results in an increase in the search intensity of the less-informed consumers μ_1. The intuition is that a larger proportion of well informed consumers in the market makes searching more attractive for the less-informed consumers, as the former buyers put pressure on firms to cut prices. This in turn implies that there are more consumers over whom the firms can exert monopoly power, which gives firms an incentive to raise prices. Interestingly, these two opposite forces cancel away so that expected prices remain constant! To see this, note that since $v - E[p] - c = 0$ in equilibrium, and since neither v nor c varies, an increase in λ must be accompanied by an increase in μ_1 in a manner such that $E[p]$ does not change. It is also easily seen that the price that informed consumers expect, i.e., $E[\min\{p_1, p_2\}]$, does not change either. To see this, note that in this case $b = (1-\lambda)\mu_1/2\lambda$. Using (8), just a little algebra shows that $db/d\lambda = 0$, which implies that the expected minimum price remains constant too!

Profits of the firms are $\pi = v(m+k)(1-\lambda)\mu_1/2$. Since μ_1 changes as a response to an increase in λ, the comparative statics impact of an increase in the relative number of informed buyers is given by

$$\frac{d\pi}{d\lambda} = \frac{v(m+k)}{2}\left[-\mu_1 + (1-\lambda)\frac{d\mu_1}{d\lambda}\right]. \tag{9}$$

Using (8) we obtain $d\pi/d\lambda = v(m+k)\mu_1(1-\lambda)/2\lambda > 0$, i.e., an increase in the relative number of fully informed consumers increases firms' profits! This surprising result is interpreted as follows. From above we know that expected price and expected minimum price remain constant. However, there are more less-informed consumers who are buying, which implies that firms better off!

It is also remarkable that social welfare increases as a result of an increase in the relative number of well informed consumers. To see this, note that $W = (m + k)\left[\lambda v + (v - c)(1 - \lambda)\mu_1\right]$. Taking into account (8) we can compute $dW/d\lambda = (m + k)[v + (v - c)\mu_1(1 - \lambda)/\lambda] > 0$. Interestingly, social welfare increases because the less-informed consumers participate more actively in the market (it is easily seen that $m\mu_1$ increases). Observe, however, that a great deal of the increase in social welfare is captured by the firms.

Consider next the equilibrium where consumers search for one price with probability one (Figure 4). An increase in the ratio of informed consumers does not affect the search behavior of less-informed consumers. However, the market becomes more competitive because relatively more consumers exercise price comparisons. Indeed, equilibrium profits are $\pi = v(m + k)(1 - \lambda)/2$, which decline with λ. The price expected by the less-informed consumers falls. To see this, note that here $b = (1 - \lambda)/2\lambda$, $db/d\lambda = -1/2\lambda^2 < 0$ and $dE[p]/db > 0$ (see Fact 10). Analogously, using Fact 11, one sees that the price expected by the informed consumers also decreases. Social welfare is now equal to $W = (m + k)[v - c(1 - \lambda)]$, which increases with λ because relatively fewer consumers incur the search cost. In summary, the increase in the relative number of informed consumers increases welfare and benefits the demand side of the market in this case.

We finally consider the equilibrium where the less-informed consumers randomize between searching for one price and searching for two prices (Proposition 4 and Figures 2 and 3). Applying the implicit function theorem to equation (7), one obtains

$$\frac{d\mu_1}{d\lambda} = \frac{\mu_1}{1 - \lambda} > 0. \tag{10}$$

This implies that an increase in the proportion of well informed consumers makes the less-informed consumers searching less intense, i.e., they will exercise price comparisons more rarely. Surprisingly, it turns out that the behavior of less-informed consumers compensates away the pressure that the presence of relatively more informed consumers puts on the firms to cut prices. To see this, note that $b = (1 - \lambda)\mu_1/(2(\lambda + (1 - \lambda)(1 - \mu_1))$ in this case. Using (10), it is easily seen that

$db/d\lambda = 0$. This implies that neither the price expected by less-informed consumers who search only once, nor the price expected by buyers who search twice changes!

Firms profits equal $\pi = (m+k)v(1-\lambda)\mu_1/2$, and it is easily checked that $d\pi/d\lambda = 0$. Surprisingly, firms' profits remain the same when the relative number of informed consumers increases!

Finally, we pay attention to social welfare $W = (m+k)[\lambda v + (1-\lambda)(v - 2c + \mu_1 c)]$. A little algebra shows that $dW/d\lambda = 2(m+k)c > 0$, which implies that social welfare increases when relatively more consumers become well informed in the market. Two sources are behind this increase in surplus. On the one hand, a direct effect, i.e., fewer consumers pay search costs. On the other hand, an indirect effect, i.e. less-informed consumers search less intensively and the economy further saves in search costs. Remarkably, this surplus is captured by the consumers.

All these comparative statics results are summarized below in Table 1.

The comparative statics of a change in λ assumes that the size of the market does not vary. Internet markets, however, are characterized for the elimination of geographical boundaries, time restrictions, etc. Next, we briefly study the impact of an increase in the number of consumers. Of course, results are very different if new cohorts of well informed consumers enter the market, as compared to the case where the new consumers are poorly informed and must incur search costs to discover prices. The impact of an increase in the number of well informed consumers k is easily retrieved by taking into account that $d\lambda/dk = m/(m+k)^2 > 0$, and then using the comparative statics effects of a change in λ. Analogously, the impact of an increase in the number of less-informed consumers is easily obtained by noting that $d\lambda/dm = -k/(m+k)^2 < 0$, and using the results above. To economize on space, we summarize the results below in Table 1 and skip the details.

The following Table summarizes the comparative statics results. The equilibrium where consumers randomize between no search and one search is represented by $\mu_0 > 0$; the equilibrium where buyers search for one price with probability one is represented by $\mu_1 = 1$; finally, the equilibrium where consumers randomize between one search and two searches is represented by $\mu_2 > 0$. Expected prices, profits and welfare are represented by p, π, and W respectively. An upwards (downwards) arrow means that the variable increases (falls); the symbol "$-$" means that the variable remains constant; finally, a question mark "?" stands when the direction of change of a variable remains ambiguous to us.

	$\mu_0 > 0$			$\mu_1 = 1$			$\mu_2 > 0$		
	p	π	W	p	π	W	p	π	W
$\downarrow c$	$\uparrow$	$\uparrow$	$\uparrow$	–	–	$\uparrow$	$\downarrow$	$\downarrow$	?
$\uparrow \lambda$	–	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	–	–	$\uparrow$
$\uparrow k$	–	$\uparrow$	$\uparrow$	$\downarrow$	–	$\uparrow$	–	$\uparrow$	$\uparrow$
$\uparrow m$	–	–	–	$\uparrow$	$\uparrow$	$\uparrow$	–	$\uparrow$	?

Table 1: Summary of Comparative Statics Results

5 Large Markets

So far, we have characterized the different types of possible equilibrium configurations and studied their comparative statics properties in the case of duopoly. We will now analyze the properties of equilibria when there is a larger number of firms, denoted by N, in the market. As above, we will first characterize the type of equilibria that may result, and then investigate the comparative statics. As Lemmas 1 and 2 also hold true for general values of N, we restrict attention to the two cases analyzed in Section 3.

Case a: $0 < \mu_1 \leq 1$, $\mu_0 + \mu_1 = 1$.

Consider first that consumers randomize between searching for one price quotation and not searching at all. The expected payoff to firm i of charging price p when competitors choose a random pricing strategy according to the cumulative distribution function $F(p)$ is

$$\pi_i(p, F(p)) = p\left[\frac{m\mu_1}{N} + k(1 - F(p))^{N-1}\right]. \tag{11}$$

Equation (11) is interpreted in a similar way as expression (2). When there are N firms in the market, since informed consumers only buy from the lowest priced store, a firm has a chance $(1 - F(p))^{N-1}$ of getting the informed consumers at its store. The less-informed consumers show up at a firm with probability μ_1/N.

In equilibrium, a firm must be indifferent between charging any price in the support of F and charging a price equal to v. Hence, we must have that

$$p\left[\frac{m\mu_1}{N} + k(1 - F(p))^{N-1}\right] = \frac{\mu_1 m v}{N},$$

which can be solved for $F(p)$ to obtain

$$F(p) = 1 - \sqrt[N-1]{\frac{\mu_1 m(v-p)}{Nkp}}.$$

The lower bound of the price support is easily found by solving $F(\underline{p}) = 0$. It obtains $\underline{p} = m\mu_1 v/(kN + m\mu_1)$.

In equilibrium, consumers must indeed be indifferent between searching for one price and not searching at all, i.e. $v - E[p] - c = 0$. We would first like to establish how Proposition 3 and the corresponding Figure 1 generalize to the case where $N > 2$. The following three observations are important for this end. First, note that if $\mu_1 = 0$, then $F(p) = 1$ for all $p > 0$ and $E(p) = 0$. In other words, if less-informed consumers do not search at all, then the only relevant buyers are the fully informed consumers, which implies that all firms choose prices equal to zero. Second, $F(p)$ is decreasing in μ_1, and $\underline{p}$ is increasing in μ_1. These two facts imply that $E(p)$ is increasing in μ_1. That is, the larger the number of consumers who make one search, the higher the prices that firms charge, as the firms have monopoly power over more buyers. Third, for any given $\mu_1 > 0$, $F(p)$ goes to zero as N becomes larger and larger. In other words, if more and more firms become active in the market, the prices that firms will charge become larger and larger. This effect is also found by Rosenthal (1980), Stiglitz (1987) and Stahl (1989), among others: when there are more competitors in the market and when the search behavior of consumers would remain constant, the chance of being the lowest priced firm becomes exponentially smaller and firms concentrate more and more on the less-informed customers, over whom they have some monopoly power. This in turn implies that for a given μ_1 expected prices converge to v as N goes to infinity. These findings allow us to state the following proposition.

Proposition 6 *For any v, c such that $v > c > 0$, there exist an N^* such that for all $N > N^*$, an equilibrium of the game described above exists where firms randomly select prices from the set $p \in \left[\frac{\mu_1 mv}{Nk+\mu_1 m}, v\right]$ according to the cumulative distribution function*

$$F(p) = 1 - \sqrt[N-1]{\frac{\mu_1 m(v-p)}{Nkp}},$$

and the less-informed consumers randomize between searching for one price quotation with probability μ_1, and not searching at all with probability $1 - \mu_1$, where

$\mu_1 \in (0,1)$ *solves* $v - E[p] - c = 0$.[17] *There is at most one such equilibrium.*

Case b: $0 < \mu_1 \leq 1$, $\mu_1 + \mu_2 = 1$

Consider now the case where costly-search consumers randomize between searching for two price quotations with some probability and searching for only one price quotation, with the remaining probability. The expected payoff to firm i of charging price p when rivals choose a random pricing strategy according to the cumulative distribution function $G(p)$ and costly-search consumers search as specified above is

$$\pi_i(p, G(p)) = p\left[k(1-G(p))^{N-1} + \frac{2m}{N}(1-\mu_1)(1-G(p)) + \frac{m\mu_1}{N}\right].$$

As before, in equilibrium, a firm must be indifferent between charging any price in the support of F and charging a price equal to v. Hence, we must have that

$$p\left[\frac{m\mu_1}{N} + \frac{2m}{N}(1-\mu_1)(1-G(p)) + k(1-G(p))^{N-1}\right] = \frac{\mu_1 m v}{N}. \tag{12}$$

An equilibrium in this case is given by a pair $(\mu_1, G(p))$ which simultaneously solves (12) and equation $v - E[p] - c = v - E[\min\{p_i, p_j\}] - 2c$.

Unfortunately, an explicit solution of equation (12) does not exist for general values of N. What we can do is give an approximate solution for $G(p)$ for large N. For any given value of the other parameters and for large N, the term $k(1-G(p))^{N-1}$ approaches zero, and the price distribution solution to (12) is approximately equal to

$$G(p) = 1 - \frac{\mu_1(v-p)}{2(1-\mu_1)p} = \frac{2-\mu_1}{2(1-\mu_1)} - \frac{\mu_1}{2(1-\mu_1)}\frac{v}{p},$$

with support $(\underline{p}, v)$ where $\underline{p} = \mu_1 v/(2-\mu_1)$. Note that this solution is similar to the one obtained by Burdett and Judd (1983) for competitive markets without fully informed consumers. When there are many firms, the influence of the fully informed consumers becomes negligible.

[17] *We also have to guarantee that consumers do not want to make two searches. This follows from the following considerations. First, if there is more price dispersion (fact which will become more evident later), consumers have more incentives to search. Second, Figure 5 shows that there is more price dispersion when N increases and, in the limit, firms randomize with probabilities α and $(1-\alpha)$ over prices 0 and v such that $E[p] = (1-\alpha)v = v - c$. If consumers had an incentive to deviate and search for two prices, they would certainly do so in the limit economy. However, the equilibrium condition for less-informed consumers in the limit economy, $(1-\alpha)v = v - c$, together with $E[p] - E[\min(p_1, p_2)] = \alpha(1-\alpha)v$, implies that consumers strictly prefer not to make two searches.*

Using the same techniques as in Section 3, we have the following condition characterizing the equilibrium behavior of less-informed consumers:

$$\frac{\mu_1}{2(1-\mu_1)}\left[\frac{1}{(1-\mu_1)}\ln\frac{2-\mu_1}{\mu_1}-2\right]=\frac{c}{v}.$$

Employing the same notation as above we can summarize in the following proposition.

Proposition 7 *Let $N \to \infty$. Let $\overline{\Gamma} \geq \frac{c}{v} \geq 0$. Then an equilibrium of the game described above exist where the less-informed consumers randomize between searching for two prices with probability $1-\mu_1$ and searching for only one price with probability μ_1, where $\mu_1 \in (0,1)$ is the solution to*

$$\frac{\mu_1}{2(1-\mu_1)}\left[\frac{1}{(1-\mu_1)}\ln\frac{2-\mu_1}{\mu_1}-2\right]=\frac{c}{v},$$

and firms randomly select prices from the set $p \in \left[\frac{\mu_1 v}{2-\mu_1}, v\right]$ according to the cumulative distribution function

$$G(p)=\frac{2-\mu_1}{2(1-\mu_1)}-\frac{\mu_1}{2(1-\mu_1)}\frac{v}{p}.$$

There is only one stable equilibrium.

Note that for large N, generic values of the parameters such that less-informed consumers search for one price for sure do not exist. That is, for all values of the c/v, less-informed consumers either randomize between one search and no search (Proposition 6), or randomize between one and two searches (Proposition 7).

5.1 Comparative statics

The comparative statics of the equilibria when changes in c, k and m are considered coincide with those above presented for the case of $N = 2$. Here, we only consider the impact of an increase in N. We will concentrate on the equilibrium in which less-informed consumers randomize between one search and no search, as we only have a limit distribution for the other equilibrium configuration. The first thing to note is that the expected price does not change with N, as consumers must be indifferent between making no search and one search. To keep expected price constant when N increases, the probability μ_1 with which less-informed consumers search for one price has to become smaller and smaller. This means that less-informed buyers search with lower intensity as N increases. This is easy to see upon observing Figure 5a below. This graph shows that the function $(v - E(p))/v$ shifts downwards as the number of competitors N raises. As a result, it follows that fewer consumers search and buy, and thus total surplus declines. Indeed total surplus equals $W = kv + m\mu_1(v - c)$ and only μ_1 changes (decreases) as N increases. It is interesting to disentangle what happens to the surplus of the informed consumers and to profits of firms. Unfortunately, it is difficult to get an analytic expression for $E[\min\{p_1, p_2, ..., p_N\}]$. However, observe that the lowest price of the support of the price distribution $\underline{p} = m\mu_1 v/(Nk + m\mu_1)$ is decreasing in N. Further, as N increases the chance that the lowest price in the distribution is chosen by at least one firm also increases, which already suggests that the informed consumers can buy at lower prices. This suggestion is strengthened when considering Figure 5b that shows the equilibrium price distributions for different numbers of competitors N. The graph shows that as N increases, firms put more weight on low prices (close to marginal cost) and high prices (monopoly prices), thereby increasing the variance of price distribution.[18] This means that price dispersion increases and that informed consumers buy at lower prices indeed, and that their surplus increases. Accordingly, firms profits decrease for two reasons. First, the informed consumers buy at lower prices and, second, there are fewer less-informed consumers active in the market.

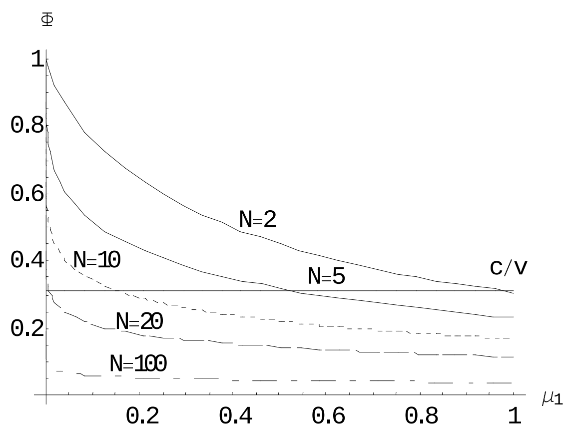

Figure 5a

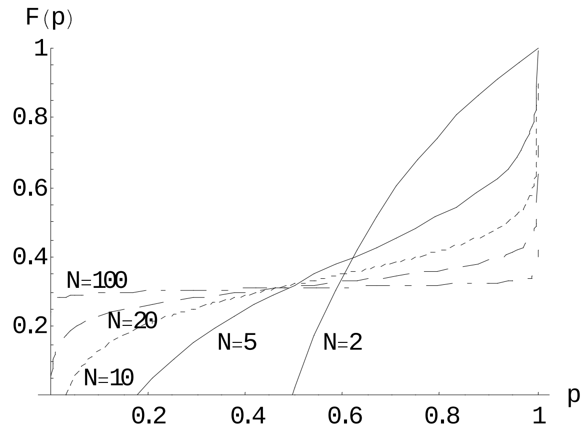

Figure 5b

In summary, what happens when N increases is three related things. First, there exists an equilibrium where consumers randomize between making one search

[18] Hence, as compared to other search models, this equilibrium does not exhibit a Diamond (1971) type of result as N becomes very large (in e.g. Stiglitz, 1987 and Stahl, 1989 prices increase to their monopoly levels when the number of firms goes to infinity).

and making no search even for very small values of the search cost. This means that it is an important equilibrium to consider in electronic markets. Second, when N increases the probability that less-informed consumers search goes down and eventually approaches zero. This means that eventually almost all consumers left in the market are the fully informed ones, which implies that the number of transactions in the market shrinks and the surplus generated in the industry declines. Third, the model gives an explanation for the substantial price dispersion observed in Internet markets. As the number of competitors raises, firms increasingly choose prices either close to their marginal cost or close to the monopoly level. This turns out to be an outcome of the equilibrium balance between the trade-off of giving less-informed consumers an incentive to search, and the tendency of firms to increase their prices, as the probability of being the lowest priced firm gets smaller and smaller.

5.2 Extension

In this section, we discuss a possible extension related to differences in consumer characteristics. So far, we have assumed that all consumers have the same willingness to pay for the good, and that all less-informed buyers have identical search costs. In what follows, we allow for consumer heterogeneity and show to what extent the "shrinking of market" outcome previously discussed is robust. To this end, assume first that apart from the k informed consumers there are I different groups of less-informed consumers, and that each group has a different valuation for the good. In particular, suppose there are m_i consumers with valuation v_i, where $v_1 < v_2 < ... < v_I$.

We shall first establish the conditions under which an equilibrium exists where the maximum price a firm sets equals v_1 and all consumers in the other groups $i \geq 2$ search at least once for sure. We shall then argue that as N gets larger the conditions will be violated and firms will have an incentive to raise their prices such that lowest valuation consumers will drop out of the market. The same process will continue as N gets even larger until only the highest valuation consumers will actively be searching in the market. We shall then conclude that the "shrinking of market" effect found in the previous section holds true in a more general setting.

Let us then first construct an equilibrium in which the maximum price a firm sets equals v_1, the lowest valuation consumers randomize between searching for one price and not searching at all, and the rest of consumers in groups $i = 2, ..., I$ make one search for sure. The expected payoff to firm i of charging price p_i when competitors choose a random pricing strategy according to $F(p)$ is:

$$\pi_i(p_i, F(p)) = p\left[\frac{\mu_1^1 m_1 + \sum_{i\neq 1} m_i}{N} + k(1-F(p))^{N-1}\right], \tag{13}$$

where μ_1^1 denotes the probability with which a v_1 consumer chooses to search for one price. Equation (13) is similar to equation (11). It only differs from (11) in that the less-informed consumers in groups $2, ..., I$ whose valuations lie above v_1 make one search. Proceeding as above, a solution for $F(p)$ obtains:

$$F(p) = 1 - \sqrt[N-1]{\frac{(\mu_1^1 m_1 + \sum_{i\neq 1} m_i)(v_1 - p)}{Nkp}}.$$

The proposed behavior of less-informed consumers is optimal if there exists a μ_1^1, with $0 < \mu_1^1 < 1$, such that $v_1 - E[p] - c = 0$. It is easy to see that this implies that $v_i - E[p] - c > 0$ for $i = 2, ..., I$. Further, note that the condition that none of the less-informed groups of consumers prefer not to make two links,

$$v_i - E[p] - c > v_i - E[\min\{p_1, p_2\}] - 2c,$$

is independent of v_i. So, if less-informed consumers group 1 does not want to form two links, then none of the other less-informed buyers would do it. It is however difficult to provide an exact expression for $E[\min\{p_i, p_j\}]$ but similar arguments as in the previous subsection show that no less-informed consumer would search twice.

Finally, the pricing behavior of firms is optimal if it is not beneficial for them to switch to a price distribution with a higher upper bound, i.e., v_2, v_3, etc. Deviating to such a price distribution would yield a pay-off of $(v_2 \sum_{i\neq 1} m_i)/N$, or $(v_3 \sum_{i\neq 1,2} m_i)/N$, and so on. Deviating to, for example v_2, is not profitable if

$$v_1\left[\frac{\mu_1^1 m_1 + \sum_{i\neq 1} m_i}{N}\right] \geq v_2\left[\frac{\sum_{i\neq 1} m_i}{N}\right], \tag{14}$$

which reduces to $(v_2 - v_1)\sum_{i\neq 1} m_i \leq \mu_1^1 m_1 v_1$. This condition seems independent of N, but however, we know from the previous section that μ_1^1 decreases in N and, further, that it approaches zero for very large N (whenever the above configuration is indeed an equilibrium). This means that there exists a large enough N for which a necessary condition for the above proposal to be an equilibrium is violated. The v_1 consumers are not served in equilibrium.

The same argument holds true for any equilibrium construction where firms choose prices smaller than or equal to v_i, $i = 2, ..., I$. To keep consumers with valuations v_i, μ_i^1 becomes smaller when N increases. Eventually, a necessary condition analogous to (14) for such a proposal to be equilibrium is violated. Hence, for sufficiently large N the only equilibrium with consumers randomizing between making one search and not searching at all is to have all consumers in groups $i = 1, ..., I-1$ not searching at all and buyers in group I randomizing between making one search and making no search. Moreover, from the previous section we know that for large enough N such an equilibrium exists for any strictly positive value of the search cost parameter c. Hence, we can conclude that the "shrinking of the market" argument made in the previous section generalizes to the case with consumer heterogeneity. Moreover, as in the previous section, price dispersion increases with N.

Finally, it is worth noting that if we allow consumers to have different search costs, the argument also holds true and that when N gets larger and larger the consumer with the highest valuation and the lowest search cost will be the only type of consumer who searches for one price with positive probability.

6 Conclusion

In this paper we have investigated the common sense arguments that with a decrease in search costs and with more intense competition commodity prices decrease and total welfare generated in the market increases. We have found that this argument misses two important points. First, when search costs decrease, the number of consumers that are willing to search for commodity prices may increase thereby increasing the size of the relevant market. If firms have some monopoly power over these "new" consumers, then prices may actually go up instead of going down. A similar argument can be made if the proportion of consumers with zero search cost increases, for instance as a result of the availability of search engines in Internet markets. Second, when the number of competitors increases the probability that a firm charges the lowest price decreases exponentially thereby providing incentives to raise prices (in an attempt to extract surplus from the consumers who do not search intensively and thus do not exercise price comparisons). This may give consumers incentives to drop out of the market and firms consequently respond by increasing price dispersion. The former effect reduces total welfare.

It is an empirical question whether these points are important in actual markets where search costs go down and more competitors enter the relevant market. The empirical evidence on the effects of lower search costs because of the increased use of Internet on commodity prices seems to be quite mixed (see the Introduction). Also, substantial price dispersion seems to continue to prevail in Internet markets. The arguments provided in this paper may give a rationale for this mixed evidence.

7 Appendix

Next, we provide the proof of Facts stated without demonstration above.

Proof of Fact 0 in Footnote 10: Consider the equilibrium where consumers randomize between searching for one price and none at all (see Proposition 3). Let us prove that consumers do not find profitable to search for two prices. To prove this, we must check that in equilibrium $v - E[\min\{p_1, p_2\}] - 2c < 0$. We can use Footnote 11 above to write this inequality as

$$v - 2bv(1 - b\ln[a/b]) - 2c < 0. \tag{15}$$

In equilibrium, $v - E[p] - c = 0$, i.e., $v - c = bv\ln[a/b]$. We can substitute this relationship into (15) to obtain $v - 2c(1+b) < 0$, or

$$\frac{c}{v} > \frac{1}{2(1+b)}.$$

Since $v - E[p] - c = 0$ in equilibrium, $c/v = 1 - b\ln[a/b] = 1 - b\ln[(1+b)/b]$. From (3), notice that $b = m\mu_1/2k > 0$. So, we need to check that

$$\frac{1+2b}{2b(1+b)} > \ln\left[\frac{1+b}{b}\right] \tag{16}$$

for all $b > 0$. Three observations prove that (16) is indeed satisfied:
(i) $\lim_{b\to 0} \frac{\ln\left[\frac{1+b}{b}\right]}{\frac{1+2b}{2b(1+b)}} > 0$,
(ii) $\lim_{b\to\infty} \frac{\ln\left[\frac{1+b}{b}\right]}{\frac{1+2b}{2b(1+b)}} = 1$
(iii) the left-hand side of (16) $\frac{1+2b}{2b(1+b)}$ decreases at the rate $\frac{2b^2+2b+1}{2b^2(1+b)^2}$, which is larger than the rate at which the right-hand side $\ln\left[\frac{1+b}{b}\right]$ diminishes, namely, $1/b(1+b)$. The proof is now complete.

Proof of Fact 1: $\frac{d\Phi}{d\mu_1} < 0$.
Recall that

$$\Phi(\mu_1) = 1 - \frac{m\mu_1}{2k}\ln\left(\frac{2k + m\mu_1}{m\mu_1}\right)$$

Note that

$$\frac{d\Phi}{d\mu_1} = \frac{m\left[2k - (2k+m\mu_1)\ln\left(\frac{2k+m\mu_1}{m\mu_1}\right)\right]}{2k(2k+m\mu_1)}.$$

The sign of $\frac{d\Phi}{d\mu_1}$ is negative if and only if

$$\frac{2k}{2k+m\mu_1} < \ln\left(\frac{2k+m\mu_1}{m\mu_1}\right) \tag{17}$$

Denote the left-hand side of (17) as $h_1(\mu_1)$ and its right hand side as $h_2(\mu_1)$. Note that $h_1'(\mu_1) = \frac{-2km}{(2k+m\mu_1)^2}$ and that $h_2'(\mu_1) = \frac{-2km}{m\mu_1(2k+m\mu_1)}$. It is evident that $h_1'(\mu_1) > h_2'(\mu_1)$, which implies that the left-hand side of (17) decreases at a lower speed than its right-hand side. Therefore, if (17) holds for $\mu_1 = 1$, it will also be satisfied for any $\mu_1 \in (0,1)$. We now employ the definition of $\lambda = k/(m+k)$, $0 < \lambda < 1$. Dividing both sides of equation (17) by $m+k$ and substituting $\mu_1 = 1$, equation (17) can be rewritten as

$$\ln\frac{1+\lambda}{1-\lambda} > \frac{2\lambda}{1+\lambda}. \tag{18}$$

To see that (18) holds, notice that:
(i) $\lim_{\lambda\to 0}\left(\ln\frac{1+\lambda}{1-\lambda}\right) = \lim_{\lambda\to 0}\left(\frac{2\lambda}{1+\lambda}\right) = 0$
(ii) the left-hand side of (18) $\ln\frac{1+\lambda}{1-\lambda}$ increases at the rate $\frac{2}{(1-\lambda)^2}$, which is larger than the rate $\frac{2}{(1+\lambda)^2}$ at which its right-hand side $\frac{2\lambda}{1+\lambda}$ increases. This completes the proof of Fact 1.

Proof of Fact 2: $\frac{d^2\Phi}{d\mu_1^2} > 0$.
To prove Fact 2, it is enough to compute

$$\frac{d^2\Phi}{d\mu_1^2} = \frac{2km}{\mu_1\left(2k+m\mu_1\right)^2} > 0$$

Proof of Fact 3: $\lim_{\mu_1\to 0}\Phi(\mu_1) = 1$

Note that $\lim_{\mu_1\to 0}\Phi(\mu_1) = 1 - \lim_{\mu_1\to 0}\frac{\ln\left(\frac{2k+m\mu_1}{m\mu_1}\right)}{\frac{2k}{m\mu_1}}$. Applying L'Hopital rule we have $\lim_{\mu_1\to 0}\Phi(\mu_1) = 1 - \lim_{\mu_1\to 0}\frac{m\mu_1}{2k+m\mu_1} = 1$.

Proof of Fact 4: $\Phi(1) = \frac{2k - m\ln\left(\frac{m+2k}{m}\right)}{2k} > 0$.

Using the definition of λ given above, it is enough to prove that

$$\frac{2\lambda}{1-\lambda} > \ln\left[\frac{1+\lambda}{1-\lambda}\right] \tag{19}$$

for all $\lambda \in (0,1)$. The following two observations show that (19) holds:
(i) the left-hand side $\frac{2\lambda}{1-\lambda}$ increases at the rate $\frac{2}{(1-\lambda)^2}$, which is higher than the rate at which the right-hand side raises, namely $\frac{2}{1-\lambda^2}$.
(ii) Then, it is enough to note that $\lim_{\lambda\to 0}\frac{2\lambda}{1-\lambda} = \lim_{\lambda\to 0}\ln\left[\frac{1+\lambda}{1-\lambda}\right] = 0$.

Proof of Fact 5: $\Gamma(1) = \frac{m\left((m+k)\ln\left[\frac{m+2k}{m}\right]-2k\right)}{2k^2} > 0$.

To see that $\Gamma(1) > 0$, we need to prove that $\ln\left[\frac{m+2k}{m}\right] > 2k/(m+k)$. Using λ, this inequality writes $\ln\left[\frac{1+\lambda}{1-\lambda}\right] > 2\lambda$. A little more simple algebra shows that this inequality holds for all $\lambda \in (0,1)$.

Proof of Fact 6: $\lim_{\mu_1\to 0}\Gamma(\mu_1) = 0$.

Using λ we have $\lim_{\mu_1\to 0}\Gamma(\mu_1) = \lim_{\mu_1\to 0}\frac{-2+2(1-\lambda)\mu_1+\ln\left[\frac{2}{(1-\lambda)\mu_1}-1\right]}{2\left(\frac{1}{(1-\lambda)\mu_1}-1\right)}$. Applying the L'Hopital rule we obtain $\lim_{\mu_1\to 0}\Gamma(\mu_1) = \lim_{\mu_1\to 0}\frac{-(1-\lambda)^2\mu_1^2+\frac{(1-\lambda)\mu_1}{2-(1-\lambda)\mu_1}}{1-(1-\lambda)^2\mu_1^2} = 0$.

Proof of Fact 7: It can be seen that

$$\frac{d\Gamma}{d\mu_1} = \frac{\mu_1}{2(1-(1-\lambda)\mu_1)^3\,(2-(1-\lambda)\mu_1)} \left(2(3-4(1-\lambda)\mu_1+(1-\lambda)^2\mu_1^2) - (2+(1-\lambda)\mu_1-(1-\lambda)^2\mu_1^2)\ln\left[\frac{2-(1-\lambda)\mu_1}{(1-\lambda)\mu_1}\right]\right)$$

At the point $\mu_1 = 1$, we obtain

$$\left.\frac{d\Gamma}{d\mu_1}\right|_{\mu_1=0} = \frac{2\lambda(2+\lambda)+(\lambda^2-\lambda-2)\ln[\frac{1+\lambda}{1-\lambda}]}{2\lambda^3(1+\lambda)}.$$

$d\Gamma/d\mu_1|_{\mu_1=0} > 0$ if and only if $2\lambda(2+\lambda)/(2+\lambda-\lambda^2) > \ln[\frac{1+\lambda}{1-\lambda}]$. Using the software Mathematica 3.0, one can solve this inequality numerically. It obtains that $\overline{\lambda} \approx 0.634816$.

Proof of Fact 8: $\frac{d^2\Gamma}{d\mu_2^2} < 0$.

Using λ the second derivative of $\Gamma(\mu_1)$ with respect to μ_1 can be written as

$$\frac{d^2\Gamma}{d\mu_1^2} = \frac{\mu_1[2A+(1-\lambda)\mu_1(2+(1-\lambda)\mu_1)(2-(1-\lambda)\mu_1)^2B]}{(1-\lambda)(1-(1-\lambda)\mu_1)^4(2-(1-\lambda)\mu_1)^2},$$

where $A = -1 - 6(1-\lambda)\mu_1 + 13(1-\lambda)^2\mu_1^2 - 7(1-\lambda)^3\mu_1^3 + (1-\lambda)^4\mu_1^4$ and $B = \ln\left[\frac{2-(1-\lambda)\mu_1}{(1-\lambda)\mu_1}\right]$. Figure 6 shows this second derivative in the space $(\mu_1, \lambda) \in (0,1] \times (0,1)$. Upon observation of this graph, it is clear that $d^2\Gamma/d\mu_1^2 < 0$.

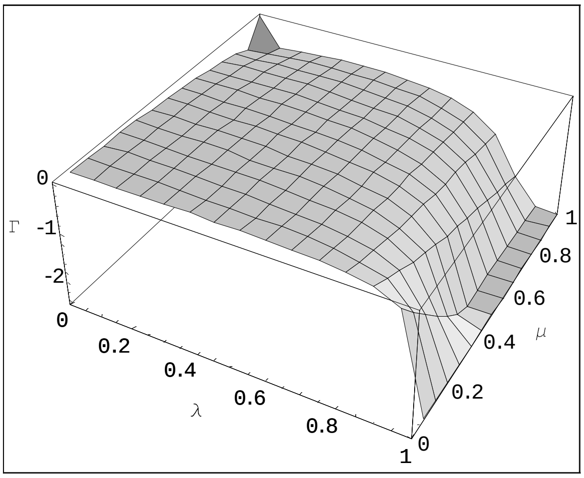

Figure 6

Proof of Fact 9: $\Phi(1) - \Gamma(1) > 0$.

$$\begin{aligned}\Phi(1) - \Gamma(1) &= \frac{2k - m\ln\left[\frac{m+2k}{m}\right]}{2k} - \frac{m((m+k)\ln\left[\frac{m+2k}{m}\right] - 2k)}{2k^2} \\ &= \frac{2k(m+k) - m(m+2k)\ln\left[\frac{m+2k}{m}\right]}{2k^2}.\end{aligned}$$

Using the fact that $\lambda = k/(m+k)$, we can write

$$\Phi(1) - \Gamma(1) = \frac{2\lambda - \ln\left[\frac{1+\lambda}{1-\lambda}\right](1-\lambda^2)}{2\lambda^2},$$

expression which is positive for all $\lambda \in (0,1)$.

Proof of Fact 10: $\frac{dE[p]}{db} > 0$

Using Footnote 10, we can compute

$$\frac{dE[p]}{db} = v\left(\ln\left[\frac{1+b}{b}\right] - \frac{1}{1+b}\right). \tag{20}$$

Note that $\lim_{b\to 0}\ln\left[\frac{1+b}{b}\right] > \lim_{b\to 0}\frac{1}{1+b}$. Note also that the first summand of (20) decreases at the rate $1/(b(1+b))$ while the second summand does so at the lower rate $1/(1+b)^2$. Then, the fact that

$$\lim_{b\to\infty} \frac{\ln\left[\frac{1+b}{b}\right]}{\frac{1}{1+b}} = \lim_{b\to\infty} \frac{1+b}{b} = 1$$

proves the result.

Proof of Fact 11: $\frac{dE[\min\{p_1,p_2\}]}{db} > 0$.

To see this note that $E[\min\{p_1, p_2\}] = 2b(v - E[p])$ (see footnote 11). Using (20), it is easily seen that

$$\frac{dE[\min\{p_1,p_2\}]}{db} = 2v\left(\frac{1+2b}{b(1+b)} - 2\ln\left[\frac{1+b}{b}\right]\right). \tag{21}$$

Notice that the first summand of (21) decreases at the rate $(2b^2+2b+1)/(b^2(1+b)^2)$ while the second summand does so at the lower rate $2/(b(1+b))$. This implies that $\lim_{b\to 0} \frac{1+2b}{b(1+b)} > \lim_{b\to 0} 2\ln\left[\frac{1+b}{b}\right]$. Since μ_1 increases with c and b increases with μ_1, to prove that $dE[\min\{p_1,p_2\}]/db > 0$ is then enough to observe that

$$\lim_{b\to\infty} \frac{\frac{1+2b}{b(1+b)}}{2\ln\left[\frac{1+b}{b}\right]} = \lim_{b\to\infty} \frac{2b^2+2b+1}{2b(1+b)} = 1.$$

References

[1] Michael R. Baye and John Morgan: "Information Gatekeepers on the Internet and the Competitiveness of Homogeneous Product Markets," forthcoming *American Economic Review.*

[2] Joseph Bailey and Yannis Bakos: "An Exploratory Study of the Emerging Role of Electronic Intermediaries," *International Journal of Electronic Commerce* 1-3, 7-20, 1997.

[3] Yannis Bakos: "Reducing Buyer Search Costs: Implications for Electronic Marketplaces," *Management Science* 43-12, 1997.

[4] Eric Brynjolfsson and Michael Smith: "Frictionless Commerce? A comparison of Internet and Conventional Retailers," forthcoming *Management Science.*

[5] Kenneth Burdett and Kenneth L. Judd: "Equilibrium Price Dispersion," *Econometrica* 51-4, 955-969, 1983.

[6] Kenneth Burdett and Melvyn G. Coles: "Steady State Price Distributions in a Noisy Search Equilibrium," *Journal of Economic Theory* 72, 1-32, 1997

[7] Eric Clemons, Il-Horn Hann and Lorin Hitt: "The Nature of Competition in Electronic Markets: An Empirical Investigation of On-line Travel Agent Offerings," mimeo, The Wharton School, University of Pennsylvania, 1999.

[8] Peter A. Diamond: "A Model of Price Adjustment," *Journal of Economic Theory* 3, 156-168, 1971.

[9] Chaim Fershtman and Arthur Fishman: "Price Cycles and Booms: Dynamic Search Equilibrium," *American Economic Review* 82-5, 1221-1233, 1992.

[10] Eric J. Johnson, Wendy Moe, Peter Fader, Steven Bellman and Jerry Lohse: "On the Depth and Dynamics of World Wide Web Shopping Behavior," mimeo prepared for the USC Conference Understanding Consumer Behavior on the Internet, Columbia Business School, Columbia University, 2000.

[11] Ho Geun Lee: "Do Electronic Marketplaces Lower the Price of the Goods?," *Communications of the ACM* 41-12, 1997.

[12] Gerald L. Lohse, Steven Bellman, Eric Johnson: "Consumer Buying Behavior on the Internet: Findings from Panel Data," Wharton Forum of Electronic Commerce, The Wharton School, University of Pennsylvania, 1999.

[13] R. Preston MacAfee: "Multiproduct Equilibrium Price Dispersion," *Journal of Economic Theory* 67-1, 83-105, 1995.

[14] Peter Morgan and Richard Manning: "Optimal Search," *Econometrica* 53-4, 923-944, 1985.

[15] OECD Report: *The Economic and Social Impacts of Electronic Commerce: Preliminary Findings and Research Agenda*, OECD Publications, France, 1999.

[16] Robert, W. Rosenthal: "A Model in Which an Increase in the Number of Sellers Leads to a Higher Price," *Econometrica* 48, 1575-1579, 1980.

[17] Michael Smith, Joseph Bailey and Eric Brynjolfsson: "Understanding Digital Markets: Review and Assessment," forthcoming in Eric Brynjolfsson and Brian Kahin, eds., *Understanding the Digital Economy*, The MIT Press, 1999.

[18] Dale O. Stahl II: "Oligopolistic Pricing with Sequential Consumer Search," *American Economic Review 79-4*, 700-712, 1989.

[19] Joseph E. Stiglitz: "Competition and the Number of Firms in a Market: Are Duopolies More Competitive than Atomistic Markets?," *Journal of Political Economy 95*, 1041-1061, 1987.

[20] Joseph E. Stiglitz: "Imperfect Information in the Product Market," in R. Schmalensee and R. D. Willig, eds., *Handbook of Industrial Organization*, North-Holland, New-York, 769-847, 1989.

[21] Hal R. Varian: "A Model of Sales," *American Economic Review 70-4*, 651-659, 1980.

[22] Nir Vulkan: "Economics Implications of Agent Technology and E-Commerce," *Economic Journal*, 109, F67-F90, 1999.