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The Shadow Price of Aircraft Noise Nuisance

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The shadow price of aircraft noise nuisance^{*}

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Abstract

This paper has a twofold objective. First, we develop a new method to assess the monetary value for individuals of external effects (viz., aircraft noise nuisance) which are not or only partly internalized in market prices. The method makes use of an ordinal index of life satisfaction as scored by individual respondents who are subjected in varying intensity to the external effect. Our second objective is to assess, with this method, to what extent noise nuisance effects around Amsterdam Airport Schiphol are internalized and what should be the monetary compensation for the nuisance. Such a compensation scheme depends on, among other things, the objective noise level, income and the presence of noise insulation. The results are both significant and plausible. The method is generally applicable.

1. Introduction

2. Short survey of the literature

3. The model

4. The empirical model and data

5. The resulting shadow prices

6. Conclusions

Bibliography

Appendix

^{*} This paper is partly based on the study "De schaduwprijs van geluidhinder door vliegtuigen rond Schiphol" (The shadow price of noise nuisance by airplanes around Schiphol) by Van Praag, Baarsma, Overtoom, Kok and Lambooy (1999, SEO), on the article "Vliegtuigen horen, geld zien" (Hearing airplanes in exchange for money) by Van Praag, Baarsma, Poot and Lambooy (1997, in: ESB), and on the dissertation by Baarsma (2000). We are grateful to Prof. J.G. de Wit and the staff of the Directorate General of Civil Aviation of the Dutch Ministry of Transport for giving us valuable comments. We thank Ingrid Overtoom and Marie-Louise Kok for their assiduous support in analyzing the data.

1. Introduction

Many city inhabitants are painfully aware that an airport is nearby. They do not only enjoy the advantages of an airport, but they also suffer from the noise made by the aircraft when the runway is near their place of living and is used for landing and taking-off.

Generally speaking, there is a negative external effect, which is caused by the airlines and/or by the airport. In the Amsterdam area the problem is heavily played up by environmentalist groups and inhabitants of the adjacent region. The solutions which are suggested vary from moving whole neighborhoods to other locations, leaving the original location as a non-housing area (as suggested by the Dutch Central Planning Bureau), to the more friendly idea of giving monetary compensation to inhabitants. A third solution is to reduce the problem by additional flight constraints, e.g., a ban on flights of heavy transport planes. A fourth solution is to ignore the problem. This solution, used for decades, does not work any longer.

Since Amsterdam Airport Schiphol is the only large-scale airport in the Netherlands and since it plays a major role as a hub airport in Western Europe, the second solution (of compensation to neighboring inhabitants) is now coming to the fore as a feasible solution (Van Praag et al., 1997). The compensations are to be paid either by the airlines, the airport, the state authorities, or a combination of these parties. The compensation should depend on the degree of noise pollution. In this paper we shall operationalize this idea of compensation on the basis of empirical estimates of the damage done to the inhabitants in the Schiphol area.

In Section 2 we give a short survey of the literature. In Section 3 we discuss some theoretical aspects of the model used. In Section 4 we consider the data and we formulate the empirical model, while Section 5 presents the empirical results and we consider the monetary compensation which follows from the model. Finally, section 6 concludes.

2. Short survey of the literature

Several other studies have been conducted on the subject of the valuation of aircraft noise nuisance. However, these studies either use revealed preference methods (like the hedonic price method), which value only part of the effects of noise

nuisance, or they use *direct* stated preference methods (like the contingent valuation method), which are unsuitable in the case of Schiphol, since noise nuisance is such a very sensitive subject, and because asking direct questions provokes strategic behavior. Below, some of these hedonic price studies and a contingent valuation study on aircraft noise nuisance are reviewed. Besides these methods, other methods could be used as well (e.g., Blomquist et al., 1988).

Hedonic price studies

Attempts to value people's preferences for peace and quiet have centered on the use of the hedonic price method. This method tries to impute a price for an environmental good by examining the effect that its presence has on a relevant market-priced good, like houses. In the case of aircraft noise nuisance, the method attempts to identify –with the use of certain statistical techniques– how much of a difference in housing prices is due to the level of noise nuisance, and to infer how much people are willing to pay for an improvement in that level.

Table 1 shows the results of various hedonic price surveys that have studied the effect of aircraft noise on residential property values. The price sensitivity with respect to aircraft noise is in most studies evaluated by the Noise Depreciation Index (NDI), which measures the change in property prices in terms of percentage for each unit of change in the noise level. The NDI is derived on the basis of a survey of the changes in property values over particular periods or geographical areas (Nelson, 1980, pp. 40-42). A hedonic price equation is specified with the property value (V) on the one hand, and a set of physical and locational housing characteristics (Z) and the level of noise nuisance (N) on the other hand: $V = V(Z, N)$. The measures of noise nuisance levels N differ between countries. For instance, the US noise descriptor is the Noise Exposure Forecast (NEF), the UK noise descriptor is the noise and number index (NNI), whereas the Dutch noise descriptor is the Kosten unit (Ku). The NDI is derived from $\partial V / \partial N$. The consensus view that seems to have emerged from the hedonic price studies is that aircraft noise has a negative and statistically significant effect on housing prices, i.e. NDI is around 0.6% on average (Collins and Evans, 1994, p. 175; Nelson, 1980, p. 46). This means that a house of, say, \$200,000 would sell for \$176,000 if located in a noisier zone with 20 units more noise nuisance.

Table 1: A summary of hedonic price studies and aircraft noise nuisance

Source: Nelson (1980, p.47-51); Pearce (1993, p. 72); Schipper (1997, p. 6).

Study location	NDI estimate	Study location	NDI estimate
<i>Australia</i>		<i>USA</i>	
Sydney	0.0-0.4*	Atlanta	0.64-0.67*
		Boston	0.8*
<i>Canada</i>		Dallas	0.6-2.3*
Edmonton	0.1-1.6*	Los Angeles	0.8-1.8*
Toronto	0.2-0.6**	New York	1.6-2.0*
Vancouver	0.65-0.9*	New Orleans	0.4*
		Minneapolis	0.6*
<i>UK</i>		Rochester	0.55-0.7*
Heathrow	0.2-0.3**	San Francisco	0.5-0.58*
Manchester	0.0-0.4**	Washington DC	1.06*

* noise nuisance is measured in NEF.

** noise nuisance is measured in NNI.

Contingent valuation studies

The contingent valuation method (CVM) to value noise nuisance has not been applied as often as hedonic pricing. The CVM uses surveys to find the willingness to pay (WTP) for a welfare gain or the willingness to accept (WTA) compensation for a welfare loss. Here only one study is reviewed that uses the CVM to value aircraft noise nuisance.

A CVM study was conducted in Israel by Feitelson et al. (1996). It estimated the effect of changes in aircraft noise exposure following an airport expansion on the WTP for residences. Home owners in three communities near a major airport where a significant expansion is planned, were asked to state their WTP for a four-bedroom single family residence located in an area with no aircraft noise at all. Next, they were asked to state their WTP for the same residence when it is located at sites subject to different levels of noise, expressed in yet another noise descriptor, viz. L_{dn} . A similar sequence of WTP questions was conducted for tenants in terms of monthly rent for a three-bedroom residence.

This Israeli study indicates that the difference in valuation for residences with no noise nuisance (50-55 L_{dn}) compared to residences with frequent and severe noise nuisance (70-75 L_{dn}) is 2.4-4.1% of the housing prices (for home owners) and 1.8-3.0% of the rents (for tenants). These noise depreciation indices (NDI) are higher than the values obtained in most hedonic price studies (around 0.6% on average). This is partly due to the fact that CVM estimates include the loss of non-use values, whereas the hedonic price estimates only identify market premiums. The authors also suggest another

explanation, viz. the fact that the WTP structures are kinked. This implies that, beyond a certain disturbance threshold, households are unwilling to pay anything for the residences. Hence, their valuation of (the reduction of) noise nuisance is so high that they are not willing to pay anything for a residence at a noisy location.

3. The model

The impact of external effects is best described by means of an indirect utility function (Deaton and Muellbauer, 1980) with two arguments, income y and noise z . In our case, z stands for the level of aircraft noise from which the individual suffers. The indirect utility function reads:

$$W = W(y, z) \quad (3.1)$$

If z causes a negative effect, we will have $\partial W / \partial z < 0$.

We assume $W(\dots)$ to be continuously differentiable in both variables. Let us consider the case where initially the income is y_0 with a noise level $z = 0$. If the level increases to z_1 , the income compensation or shadow price Δy for the noise is found from the equation:

$$W(y_0; 0) = W(y_0 + \Delta y, z_1) \quad (3.2)$$

The money amount Δy is the monetary compensation or shadow price we look for. We notice that there is no reason why Δy should be linear in z . We also note that Δy generally will depend on the income level. Finally, we notice that Δy may depend on the utility level as well.

If W depends on other variables, like the age of the individual (*age*) or the family size (*fs*), it follows that, generally speaking, the compensation scheme may depend on those other variables as well. Whether such variables are taken into account as a basis for compensation is a question of politics, administration costs, and negotiation power of the action group representing the interests of inhabitants and other parties, e.g., environmentalists.

In economic literature it is sometimes argued that external effects are ‘internalized’ by the market mechanism. For instance, let the rent of your house be $p(0)$ under ‘no noise’ conditions, and $p(z_1)$ under noise level z_1 . Using an indirect utility function $W(y, p; z)$ and assuming internalization by the market, there should hold:

$$W(y, p(0); 0) = W(y, p(z); z) \quad (3.3)$$

Where the rent difference $p(0) - p(z) > 0$ is actually the shadow price of the external negative effect z . Even more directly, if the rent difference would fully compensate the external effect, we would have:

$$W(y, 0) = W(y, z) \quad (3.4)$$

Where individuals with the same income would enjoy the same welfare level, irrespective of the external noise effect, as the rent difference would compensate the external effect. Hence, when the effect is fully internalized by the market mechanism, there will hold under *ceteris paribus* conditions:

$$W(y, 0) = W(y, z) = W(y) \quad (3.5)$$

Assuming that we have an empirically operational definition of well-being, by means of which we could observe well-being per individual household, this supplies us with a test instrument on the hypothesis that effects are fully internalized by the market. If $W(y; z) \neq W(y)$, it implies that the effect is not fully internalized. In reality we may expect that the external effect is *partly* internalized via prices. If we find a significant effect of z , this is always a *residual* external effect, as it is already partly internalized via prices.

Most economists are skeptical on the measurability and interpersonal comparability of well-being. In our sister disciplines of psychology and sociology, but also in health economics, this skepticism is not shared. Actually, the previous analysis in terms of a W -function does not lead to anything, if we do not define an empirical analogue. We suggest to use the so-called Cantril ladder question, originally devised by Cantril (1965), which runs as follows:

Figure 1: The Cantril ladder-of-life question

Here is a picture of a ladder, representing the ladder of life. Suppose we say that the top of the ladder (step 10) represents the best possible life for you, and the bottom (step 1) represents the worst possible life for you.
Where on the ladder do you feel you personally stand at the present time?
(Please cross one box only)

Best conceivable →	10	<input type="checkbox"/>
	9	<input type="checkbox"/>
	8	<input type="checkbox"/>
	7	<input type="checkbox"/>
	6	<input type="checkbox"/>
	5	<input type="checkbox"/>
	4	<input type="checkbox"/>
	3	<input type="checkbox"/>
	2	<input type="checkbox"/>
Worst conceivable →	1	<input type="checkbox"/>

This question module, or a modification of it as a horizontal scale, is since 1965 included as a matter of routine in many sociological and psychological surveys all over the world. The question is rather easy to answer and most respondents do answer the question. The answer may be explained to a satisfactory degree by utilizing standard econometric models.

This question asks respondents to evaluate their 'life as a whole' on a (0, 10)-scale, which is the usual scaling used in the Dutch schooling system and hence familiar to all respondents. Ten stands for 'excellent', and zero for the 'worst possible situation'. The obvious questions are now whether this measure leads to interpersonally comparable answers and whether the resulting function may be considered as a cardinal or ordinal measure of well-being.

Let us describe the respondent i 's objective life situation by (the vector) v_i , his personal characteristics like age and family size by (the vector) x_i , and his subjective evaluation of 'life as a whole' (well-being, for short) by W_i , then we might assume a relationship:

$$W_i = W(v_i, x_i) \tag{3.6}$$

Let us assume two individuals i and j , with $v_i = v_j$ and $x_i = x_j$. When there holds $W_i = W_j$, it follows that i and j , who are in identical circumstances v and x , evaluate their life identically on this scale. If this is true for all individuals who are in the same circumstances and if W varies with v and x , then it follows that (v,x) is a perfect predictor for the well-being of individuals. This does not imply that individual well-being is really well measured by the Cantril question. It only implies that (v,x) is a good predictor of the response to the Cantril question and that there is a functional relationship $W = W(v,x)$. However, in such a case we equate the levels of well-being with the response on the Cantril scale. In that case, there is perfect interpersonal comparability of the W -measure.

If we find that there is an imperfect fit but

$$W_i = W(v_i, x_i) + \varepsilon \quad (3.7)$$

where effects are significant, we again will assume interpersonal comparability. However, with respect to the imperfect fit we are not so sure whether this is caused by an imperfect specification and/or omission of variables, or whether it is caused by imperfect interpersonal comparability. We refer to Van Praag (1993, 1994) for empirical experiments on the translation of verbal labels like 'good', 'bad' into numerical ranking. It was shown there that most people tend to translate verbal labels into numbers in a similar way and hence that equal responses may be translated into equal verbal descriptions of feelings. This does not imply that feeling 'good' or 'bad' means the same to every respondent, but it is highly probable for individuals living and being brought up in the same language community.

Let us assume that well-being is interpersonally comparable by means of the Cantril question, then the question arises whether it is a cardinal measure, i.e. equal distances represent equal jumps in well-being. Although we believe this to be a reasonable assumption (cf. Van Praag, 1993, 1994), we do not have means to test that assumption. Hence it stays as an unproven assumption. However, does it matter in the perspective of this paper whether the measure is ordinal or cardinal?

In this paper we are basically interested in the trade-off between Δz and Δy in $W = W(y,z)$, where income y is a dimension of the personal characteristic x , and z is a dimension of the objective situation v . Now, if we look at another ordinal specification of W , say, $W = \varphi(W(z,y))$, where φ is a monotonically increasing function, it is obvious that

if $W(z + \Delta z, y + \Delta y) = W(z, y)$ (3.8)

then $\varphi [W(z + \Delta z, y + \Delta y)] = \varphi [W(z, y)]$ (3.9)

Hence, our conclusion is that we do not need to assume cardinality to derive trade-off ratios and hence, that we do not have to make a statement on the cardinal nature of the Cantril well-being measure either. It is only interpersonal comparability that matters.

4. The empirical model and data

The geographical region of interest is an area of 50 square kilometres around Amsterdam Schiphol Airport. The area is closely monitored on aircraft noise. The average noise burden for each zip code is known and monitored in terms of Kosten units, named after the late Professor Kosten, who devised the measure in the sixties. The measure is a composite formula, built up from the maximum noise in Db, the frequency of that noise level and weighted by the period of the day and night. For values of Ku under 20, the noise measurement becomes less reliable, according to some experts, as aircraft noise will be confused with the measurement of the blowing wind, street noise, et cetera.

The number of observations on which this analysis is based is 1,400 individuals dispersed over the Schiphol area. Those individuals sent in an anonymous mail questionnaire. As the zip code (on the average covering about 10 to 12 households) was asked in the questionnaire, it was possible to relate the Ku-information per zip code with the individual reactions.

As a considerable part in the area is noise-conscious and/or is heavily opposed to expansion of the airport and air traffic, it is to be expected that explicit questions on noise pollution will yield strategic responses, i.e., individuals will exaggerate the problems. Hence, the questionnaire was cast in the form of a questionnaire about the living satisfaction, dealing with a variety of aspects of life.

Consider now the Cantril question. The answers to the Cantril question are a discrete and ordinal variable. In order to be able to run an OLS regression, the Cantril variable W is monotonically transformed to a $[-\infty, +\infty]$ scale, following the

procedure described by Plug and Van Praag (1995). This monotonic transformation method replaces the values of W from 1 to 10 by numbers W^* , defined as:

$$W^* = N^{-1} \left(\sum_{j=1}^{W-1} p_j + \frac{1}{2} p_W; 0, 1 \right) \quad (4.1)$$

where N stands for the standard normal distribution, and p_W is the sample fraction of individuals who responded level W ($= 1, \dots, 10$). It is easily seen that this is a monotonically increasing transformation of W . We call this transformation the empirical normal transformation.¹

We selected the following explanatory variables:

- net monthly household income ($\ln y$)
- family size ($\ln fs$ and $(\ln fs)^2$)
- interaction term of income and family size ($\ln y * \ln fs$)
- age of the respondent ($\ln age$ and $(\ln age)^2$)
- noise² in terms of Kosten units ($\ln Ku$)
- interaction term of a dummy for noise insulation (Ins) and noise in terms of Ku ($Ins * \ln Ku$)

The Cantril question has already been estimated by Plug (1997), Plug and Van Praag (1995), Van Praag and Plug (1998). In these publications, a noise effect was not included as the data sets used did not contain such variables. Using the variables listed above, the Cantril measure of well-being W^* is explained by:

$$W^* = \beta_0 + \beta_1 \ln y + \beta_2 \ln fs + \beta_3 (\ln fs)^2 + \beta_4 \ln y * \ln fs + \beta_5 \ln age + \beta_6 (\ln age)^2 + \beta_7 \ln Ku + \beta_8 Ins * \ln Ku \quad (4.2)$$

¹ If W would be observed on a continuous scale, the empirical distribution function would be denoted by $F(W)$. Then W^* would be defined as $W^* = N^{-1}(F(W))$. This monotonical transformation would imply that W^* is normally distributed on the $(-\infty, +\infty)$ -axis.

² The Kosten unit is based on log(decibels), the flight frequency, while a penalty weight is assigned to evening and night flights.

The effect of income is of course expected to be positive. The family-size effect is ambiguous. For all parents there is a finite optimum, and if the number of children rises above that optimum, children become more or less ‘undesired’. Using the log-parabolic specification, such an optimum is found as the solution of the equation

$$\beta_2 + 2\beta_3 \ln fs + \beta_4 \ln y = 0 \quad (4.3)$$

with the explicit solution:

$$\ln fs = \frac{-\beta_4 \ln y - \beta_2}{2\beta_3} \quad (4.4)$$

The solution may be smaller than two, in which case we assume that the optimum number of children is zero.³ From the equation that specifies ‘lnfs’ it is obvious that the interaction term of ‘lny’ with ‘lnfs’ is quite important. It presumes that the optimum number of children depends on the financial situation of the household.

Furthermore, it seems safe to assume that well-being is age-dependent. As we do not know the relationship, we choose for a flexible form by adding a log-quadratic form. We choose for the logarithm of age instead of age, although age is used in much of the literature (Mincer, 1963). In our view, ln(age) is more reasonable, as years are perceived as running quicker as one grows older.

Next, two variables describing the respondents’ living situations are included in the model, viz. the level of aircraft noise nuisance and the presence of noise insulation. Obviously, the effect of aircraft noise nuisance on well-being is expected to be negative. The interaction term *Ins*lnKu* is included in the model since we assume that the size of the negative noise effect will do less harm if the house has noise insulation and, hence, that well-being is positively affected by the presence of noise insulation. The resulting estimates for this equation are presented in table 2.

³ Family size is defined as follows: one + partner + number of children living at home.

Table 2: Estimation of the well-being equation with the variable Ku

Variable	Parameter estimate	Standard deviation	t-value
intercept	2.6708	1.7360	1.5385#
lny	0.4035	0.0688	5.8619
lnfs	- 1.8240	0.6997	- 2.6070
(lnfs) ²	- 0.1352	0.1055	- 1.2818#
lny*lnfs	0.2517	0.0880	2.8601
lnage	- 3.2883	0.9520	- 3.4543
(lnage) ²	0.4412	0.1298	3.3989
lnKu	- 0.0308	0.0233	- 1.3225#
lns*lnKu	0.0500	0.0209	2.4002
N= 1,075		R ² = 0.1590	

Not significantly different from 0 at a 5% level.

Looking at these results (on which we shall not comment at this point but later on, when discussing the results in the tables 4 and 5 below), we see that the (external) effect of noise nuisance is not significant. It follows that our first attempt to identify the external effect has not been rewarded.

The reason is that it is not the *objectively* measurable aircraft noise nuisance that matters but the *subjectively* perceived nuisance, which partly depends on non-acoustic factors. For instance, if an individual is at home during the daytime, it stands to reason that aircraft noise will have a larger impact than when he or she is not. The concept that will co-determine well-being is the subjective variable *perceived noise*, which we call *noise* for short.

This variable *noise* is based on the answers to question 25 in the survey. This question asks respondents to indicate the extent to which several sound sources (enumerated in the question) cause noise nuisance at their place of living. These noise sources relate to, among other things, trains, neighbors, industry and airplanes. The answer to this question is given on a discrete 5-value scale, indicating that the respondent “never” experiences noise nuisance up to a situation in which the respondent “always” experiences noise nuisance. In the area around Schiphol, which we considered, we found the following distribution of noise:

Table 3: Aircraft noise nuisance (To what extent do airplanes cause noise nuisance at your place of living?)

	Total	No noise insulation	Noise insulation is present
Never	159 (11.4) *	}	}
Sometimes	460 (32.9)	} ⇒ 647 (73.5)	} ⇒ 233 (26.5)
Regularly	261 (18.6)	}	}
Often	263 (18.8)	170 (64.6)	93 (35.4)
Always	185 (13.2)	96 (51.9)	89 (48.1)

* Percentages taken from the sample size (N = 1,400). Due to missings, the percentages do not add up to 100%.

The table indicates that in an area of 50 square kilometres around the airport, many sites exist where airplanes do not come over at all or at least do not cause any noise nuisance.⁴ It also shows that 26.5% of the Schiphol respondents who answer “never”, “sometimes” or “regularly” to question 25d have noise insulation in their homes. This figure rises to 35.4% for the Schiphol respondents who marked the option “often” in question 25d, and to 48.1% for the respondents who marked the option “always”. We note that these percentages cannot be interpreted in a univocal way, since in some of these cases the noise insulation may have been installed compulsorily, as part of the insulation program of Schiphol Airport.

The variable *noise* depends on objective circumstances, especially the objectively measurable noise level in Ku *and* individual variables *x*, like family size, housing expenses et cetera (see table 5 below). If we include the intermediate variable *noise*, the specification of well-being reads as follows:

$$W^* = \beta_0 + \beta_1 \ln y + \beta_2 \ln fs + \beta_3 (\ln fs)^2 + \beta_4 \ln y * \ln fs + \beta_5 \ln age + \beta_6 (\ln age)^2 + \beta_7 noise + \beta_8 Ins * noise \quad (4.5)$$

$noise = f(Ku, x)$

In this specification we suppose that well-being is indirectly, and not directly, influenced by changes in the level of Ku, viz. via the intermediate variable *noise*. We replace the objective variable by a subjective variable *noise*. The noise nuisance depends on Ku *and* on individual characteristics.

⁴ The distance of 50 kilometres is dictated by the fact that Ku-measurement is only performed for that region.

Noise is an ordinal variable as well: if we replace *noise* by $\psi(\text{noise}) = \text{noise}^*$, where $\psi(\cdot)$ is a monotonic transformation, it is obvious that the terms in W^* , involving *noise*, may be rewritten as:

$$\beta_7 \text{noise} + \beta_8 \text{Ins} * \text{noise} = \beta_7 \psi^{-1}(\text{noise}^*) + \beta_8 \text{Ins} * \psi^{-1}(\text{noise}^*) \quad (4.6)$$

It follows that the transformation of *noise* into *noise** implies only a change in the functional specification of W^* as a function of *noise*. We apply a similar 'empirical-normal' transformation on *noise*.

Evidently, we have to find the effect of the objectively measured *Ku* level on well-being. Hence, we have to assume that perceived noise is a function of *Ku* and other variables *x*, that is, $\text{noise} = \text{noise}(Ku, x)$ in equation (4.5). The effect of changes in *Ku* on well-being is now assessed through a two-stage model. If the noise level changes from Ku^{old} to Ku^{new} , the equivalent income change ($y^{new} - y^{old}$) may be calculated from the equation:

$$W^*(\text{noise}(Ku^{old}), y^{old}) = W^*(\text{noise}(Ku^{new}), y^{new}) \quad (4.7)$$

The next step is to substitute the objective variable *Ku* in equation (4.2) by the subjective variable *noise* (4.5) in the equation explaining well-being. The resulting estimates for this equation are presented in table 4.

Table 4: Estimation of the well-being equation with the intermediate variable noise

Variable	Parameter estimate	Standard deviation	t-value
intercept	3.3052	1.7465	1.8924#
lny	0.3963	0.0691	5.7352
lnfs	- 1.9125	0.7031	- 2.7199
(lnfs) ²	- 0.1215	0.1054	- 1.1530#
lny*lnfs	0.2641	0.0882	2.9952
lnage	- 3.6482	0.9604	- 3.7985
(lnage) ²	0.4954	0.1308	3.7864
noise	- 0.1395	0.0416	- 3.3551
lns*noise	0.1178	0.0575	2.0489
N= 1,039		R ² = 0.1645	

Not significantly different from 0 at a 5% level.

Let us start to notice that the coefficients in tables 4 and 2 hardly differ, except for the noise coefficient. The variable *noise* has now a significant and negative influence on well-being. The positive and significant interaction term of noise insulation with noise nuisance (*Ins*noise*) indicates that, if the house does *not* have noise insulation, the effect of noise nuisance on well-being is -0.1395, whereas this effect decreases to -0.0217 (0.1175 - 0.1395), if the house does have noise insulation. Apparently, insulation does not fully mitigate the effects of aircraft noise on well-being.

Net monthly income has a positive and significant impact on well-being. The family-size effects *Infs* and (*Infs*)² are negative, but the latter is not significant. The coefficient of the interaction term with income (*Infs*Iny*) is positive and significant. The combined effect of all three variables describing the family size indicates that an optimal family size exists and that this size increases with income. The impact of age on well-being is negative in the relevant age range.

This result, that insulation does not fully mitigate the effect of noise, was also found in the contingent valuation study conducted by Feitelson et al. (1996, p. 11) discussed in section 2. Moreover, in a Regioplan study where the nature and the extent of the complaints about aircraft noise nuisance in the Schiphol region are studied, it was found that noise insulation does decrease the number of complaints about aircraft noise nuisance, but that it does not eliminate all complaints (Hulshof and Noyon, 1997, p. 73).

Now we estimate the relation between (the empirically transformed) noise and *Ku*. Using the specification discussed above, the intermediate variable *noise* is explained with the following variables:

- family size (*Infs*)
- monthly housing expenses (*InHe*)
- dummy for presence at home during the day (*home*)
- noise in terms of Kosten units (*InKu*)
- dummy for presence of balcony (*Bal*)
- dummy for presence of garden (*Gar*)

The resulting estimates for this equation are shown in table 5. Since the *Ku* values lower than 20 are –according to experts– not wholly reliable, we used a correction method in order to correct for the measurement error related to these lower *Ku* values (cf. appendix).

Table 5: Estimation of the intermediate variable 'noise'

Variable	Parameter estimate	Standard deviation	t-value
intercept	- 1.4008	0.5451	- 2.5699
<i>Infs</i>	0.0483	0.0817	0.5910#
<i>InHe</i>	0.1082	0.0839	1.2901#
<i>Home</i>	0.1399	0.0733	1.9087#
<i>InKu</i>	0.3696	0.0147	25.1200
<i>Bal</i>	0.1098	0.0615	1.7840#
<i>Gar</i>	0.2513	0.0987	2.5451
N= 1,132		R ² = 0.1772	

Not significantly different from 0 at a 5% level.

The influence of family size on *noise* is positive (the larger the household, the more annoyance by the aircraft noise) but not very significant. Such a positive effect of family size on annoyance by aircraft noise is also found in the Regioplan publication mentioned earlier (Hulshof and Noyon, 1997). A special complaints bureau exists that residents can call to lodge a complaint about aircraft noise nuisance. The researchers found that in the group that has actually lodged a complaint, 14% comes from individuals living alone (versus 29% in the group of non-complainers), 35% comes from people who are living together, as partners, without having any children (versus 30% in the group of non-complainers), and 46% comes from couples with children (versus 36% of the non-complainers) (ibidem, p. 100 and p. 152). However, both our and Regioplan's results are contradicted by another Dutch survey in the Schiphol region, which has been carried out by TNO-PG and RIVM (1998). They found a negative influence of the family size on the annoyance by aircraft noise, i.e. individuals living alone are more often annoyed by aircraft noise than people in households with more than one member.

Furthermore, the results indicate that the higher the housing expenses, the more someone is annoyed by aircraft noise nuisance. Also, individuals who are at home during the day on weekdays experience more aircraft noise nuisance than people who are not at home during that period. However, both effects are not significant at a 5% significance level, while the effect of the variable *Home* is significant at a 10% significance level.

Next, three variables describing the respondents' living situations are included in the model, viz. the level of aircraft noise nuisance, the presence of a balcony and the presence of a garden. Of course, the effect of aircraft noise nuisance on *noise* is positive, but what is more important: it is a significant effect. The dummy variable *balcony* is 1 if a balcony is present and 0 otherwise. The same applies to the dummy variable *garden*. It appears that the presence of a garden significantly increases the extent to which individuals are annoyed by aircraft noise. The effect of the presence of a balcony is also positive but not significant at a 5% level (at a 10% level of significance the effect is significant).

Since *noise* is positively related to the noise level in *Ku*, well-being is negatively related to the noise level in *Ku*. Using this specification of well-being, it is now possible to compute monetary compensations for changes in the noise level in *Ku*.

5 The resulting shadow prices

On the basis of tables 4 and 5, we are now able to derive shadow prices for changes in the noise level measured in *Ku*. We may write schematically $W^* = W^*(y, noise(Ku, x), z)$, where y stands for income, *noise* is the perceived noise nuisance, which is a function of the noise level in *Ku* and of other variables x , and finally a variable z , including family size (*fs*) and age. The shadow price Δy , needed to compensate a noise increase of ΔKu , is now calculated from the equation

$$W^*(y + \Delta y, noise(Ku + \Delta Ku, x), z) = W^*(y, noise(Ku, x), z) \quad (5.1)$$

Dropping all non-relevant terms in (4.5), this boils down to the equation

$$\begin{aligned} & (\beta_1 + \beta_4 \ln fs)^* (\ln y + \Delta \ln y) + (\beta_7 + \beta_8 Ins)^* (noise(Ku + \Delta Ku)) = \\ & (\beta_1 + \beta_4 \ln fs)^* \ln y + (\beta_7 + \beta_8 Ins)^* noise(Ku) \\ & (\beta_1 + \beta_4 \ln fs)^* \Delta \ln y = -(\beta_7 + \beta_8 Ins)^* 0.3696 (\Delta \ln Ku) \end{aligned} \quad (5.2)$$

or

where $\beta_1, \beta_4, \beta_7, \beta_8$ are given in table 4 and the coefficient 0.3696 is taken from table 5.

Equation 5.2 may be rewritten as

$$\frac{\partial \ln y}{\partial \ln Ku} = -\frac{(\beta_7 + \beta_8 Ins)}{(\beta_1 + \beta_4 \ln fs)} \quad (5.3)$$

The first point that follows from equation (5.3) is that the price is not a constant, i.e., the compensation is not linear in Ku . The change from 20 Ku to 30 Ku is equivalent to the change from 30 Ku to 45 Ku . So, it is the relative changes that count. This is not surprising as nearly every psycho-physical stimulus is translated on a logarithmic scale. This is the celebrated Weber-Fechner law.

Similarly, the compensation in money depends on the initial income level. Here, it is also found that the relative changes count. The expression $\partial \ln y / \partial \ln Ku$ is an elasticity. Politically, this implies that the compensation for noise nuisance depends on income, where richer people are entitled to a higher compensation in money terms. Politically, this is hard to defend but not impossible. It is actually the same mechanism which makes a progressive income tax acceptable. The pain of an income loss of \$100 is smaller if someone has an income of \$2000, than if one earns an income of \$1000. Similarly, a compensation of \$100 means less to somebody with \$2000 than for an individual earning \$1000.

From equation (5.3) it is obvious that the compensation (elasticity) depends on the fact whether or not the house is insulated against noise. The compensation needed is much smaller when the house is insulated ($Ins = 1$). Finally, the compensation depends on the family size. As this is not a politically relevant parameter, we fix the value of fs at the sample average of 2.2585.

Two values result for the elasticity ($\partial \ln y / \partial \ln Ku$), viz., a noise elasticity without noise insulation

$$-\frac{(\beta_7)}{(\beta_1 + \beta_4 \ln fs)} = -\frac{(-0.1395)}{(0.3963 + 0.2641 \cdot 2.2585)} = 0.1405 \quad \text{standard deviation} = 0.0484$$

and a noise elasticity with noise insulation

$$-\frac{(\beta_7 + \beta_8)}{(\beta_1 + \beta_4 \ln fs)} = -\frac{(-0.1395 + 0.1178)}{(0.3963 + 0.2641 \cdot 2.2585)} = 0.0218 \quad \text{standard deviation} = 0.0550$$

We see that the first elasticity is significantly positive, but that the second elasticity does not differ significantly from zero.

5.1 Compensation scheme differentiated for income positions

We may now tabulate the money amounts. The first columns of the tables below show the net monthly income positions of a household. In the next four columns the compensation amounts for particular changes in noise levels are presented.

Table 6 gives the results for non-insulated houses. We see that at a monthly income of f1,500 a household would have to be compensated with f64.57 per month for a noise increase from 20 to 30 Ku. A change from 20 to 40 Ku would require f64.57 + f39.54 = f104.11 per month.

Table 6: Monetary compensation if noise insulation is not present (differentiated for income positions)

	20 → 30 Ku	30 → 40 Ku	40 → 45 Ku	40 → 50 Ku
f 1,500	f 64.57	f 39.54	f 14.92	f 27.89
f 2,000	f 86.10	f 52.72	f 19.89	f 37.19
f 3,000	f 129.15	f 79.08	f 29.84	f 55.78
f 4,000	f 172.20	f 105.43	f 39.79	f 74.38
f 5,000	f 215.25	f 131.79	f 49.74	f 92.97
f 6,000	f 258.30	f 158.15	f 59.68	f 111.57
f 7,500	f 322.88	f 197.69	f 74.61	f 139.46
f10,000	f 430.50	f 263.59	f 99.47	f 185.95
f12,500	f 538.12	f 329.48	f 124.34	f 232.43
f15,000	f 645.75	f 395.38	f 149.21	f 278.92

Table 7 shows the amounts for houses *with* insulation. These amounts are much smaller. For instance, at the same income level of f1,500 the compensation would be only f9.86. This implies also that the value of insulation at this level would be f64.57 – f9.86 = f54.72. Under pressure of inhabitants, the airport authorities are obliged to insulate dwellings which are in high Ku areas (>45 Ku).

**Table 7: Monetary compensation if noise insulation is present
(differentiated for income positions)**

	20 → 30 Ku	30 → 40 Ku	40 → 45 Ku	40 → 50 Ku
f 1,500	f 9.86	f 6.08	f 2.31	f 4.30
f 2,000	f 13.14	f 8.10	f 3.08	f 5.73
f 3,000	f 19.71	f 12.15	f 4.61	f 8.60
f 4,000	f 26.28	f 16.20	f 6.16	f 11.47
f 5,000	f 32.85	f 20.25	f 7.70	f 14.33
f 6,000	f 39.42	f 24.30	f 9.24	f 17.20
f 7,500	f 49.28	f 30.38	f 11.54	f 21.50
f10,000	f 65.71	f 40.51	f 15.39	f 28.67
f12,500	f 82.13	f 50.64	f 19.24	f 35.84
f15,000	f 98.56	f 60.76	f 23.09	f 43.00

Now the question arises whether it would be cheaper to pay the compensation or to insulate the house. By subtracting table 7 from table 6, we find the value of insulation on a monthly basis.

Clearly, noise insulation is a capital investment. Using an interest-rate of 5%, a monthly amount of f54.72 is equivalent to a capital expenditure of $20 \cdot 12 \cdot f54.72 = f13,132.80$. It follows that authorities should insulate the dwellings of households earning f1,500 per month experiencing a noise increase from 20 to 30 Ku, if the costs of insulation are below this amount of f13,132.80.

Table 8: The value of noise insulation (differentiated for income positions)

	20 → 30 Ku	30 → 40 Ku	40 → 45 Ku	40 → 50 Ku
f 1,500	f 54.72	f 33.46	f 12.61	f 23.59
f 2,000	f 72.96	f 44.62	f 16.82	f 31.46
f 3,000	f 109.44	f 66.92	f 25.22	f 47.18
f 4,000	f 145.92	f 89.23	f 33.63	f 62.91
f 5,000	f 182.39	f 111.54	f 42.04	f 78.64
f 6,000	f 218.87	f 133.85	f 50.45	f 94.37
f 7,500	f 273.59	f 167.31	f 63.06	f 117.96
f10,000	f 364.79	f 223.08	f 84.08	f 157.28
f12,500	f 455.99	f 278.85	f 105.10	f 196.60
f15,000	f 547.18	f 334.62	f 126.12	f 235.92

5.2 Compensation scheme differentiated to housing expenses

It was already hinted at that a compensation scheme that depends on the income level might not be politically acceptable. An alternative way to set such a scheme uses the housing expenses (*housing exp*) as a key determinant. Here we have to distinguish between home owners and tenants. For home owners in the Schiphol region, we found the following relationship:

$$\ln(\textit{housing exp}) = 0.34 + 0.39 \ln y - 0.27 \ln(\textit{Tor}) + 0.30 \ln(\textit{asking price}) \quad (5.4)$$

(8.42) (-13.71) (6.12)

$R^2 = 0.393$; N = 615

The current market value of the house is denoted by *asking price*. The variable *Tor* stands for the time of residence. For home owners, the negative effect of *Tor* on *He* may be explained as follows. In the Netherlands, houses are in very short supply and prices rose by 5 to 20% per year during the last 15 years. There is also an annual general inflation fluctuating between 2 and 10% over that period. The housing expenses are for a good deal based on historical costs, while loans are paid off over the years. It follows that nominal housing expenses of home owners tend to fall with the time of residence.

We may now predict income (y_{pred}) if we know the housing expenses, the time of residence and the current market value of the dwelling. We find

$$\ln y_{pred} = (0.39)^{-1} [\ln(\textit{housing exp}) - 0.34 + 0.27 \ln(\textit{Tor}) - 0.30 \ln(\textit{asking price})] \quad (5.5)$$

Departing from tables 6 and 7 and replacing the income by predicted income y_{pred} we find tables 9 (insulation not present) and 10 (insulation present). These tables are calculated on the assumption that the household has lived in the house for five years ($Tor = 5$).

**Table 9: Compensation for home owners if noise insulation is not present
(differentiated for net monthly housing expenses and asking price)**

	20 → 30 Ku	30 → 40 Ku	40 → 45 Ku	40 → 50 Ku
Asking price f150,000				
Housing expenses f 500	f 37.59	f 23.01	f 8.69	f 16.24
f 750	f 117.19	f 71.75	f 27.08	f 50.62
f1,000	f 262.59	f 160.78	f 60.68	f 113.42
Asking price f400,000				
Housing expenses f1,000	f 114.70	f 70.23	f 26.50	f 49.54
f1,500	f 357.62	f 218.97	f 82.64	f 154.47
f2,000	f 801.34	f 490.64	f 185.16	f 346.12

**Table 10: Compensation for home owners if noise insulation is present
(differentiated for net monthly housing expenses and asking price)**

	20 → 30 Ku	30 → 40 Ku	40 → 45 Ku	40 → 50 Ku
Asking price f150,000				
Housing expenses f 500	f 5.74	f 3.54	f 1.34	f 2.50
f 750	f 17.89	f 11.03	f 4.19	f 7.80
f1,000	f 40.08	f 24.71	f 9.39	f 17.49
Asking price f400,000				
Housing expenses f1,000	f 17.51	f 10.79	f 4.10	f 7.64
f1,500	f 54.58	f 33.65	f 12.79	f 23.82
f2,000	f 122.31	f 75.40	f 28.65	f 53.36

Similarly, we may 'predict' household incomes from monthly rents. For tenants we find

$$\ln(\text{housing exp}) = 4.05 + 0.31 \ln y - 0.01 \ln(\text{Tor}) \quad (5.6)$$

(10.60) (-0.43)

$$R^2 = 0.185; N = 496$$

$$\ln y_{pred} = (0.31)^{-1} [\ln(\text{housing exp}) - 4.05 + 0.01 \ln(\text{Tor})] \quad (5.7)$$

The effect of *Tor* on housing expenses is not significant. The main reason for this non-significance is that all Dutch rents are fixed per housing unit. The initial rent level of the housing unit is increased each year by a nationally by law fixed percentage α , which approximately equals the nominal growth rate of income.

Hence, staying in one apartment for a long time does not automatically reduce the relative rent/income ratio, apart from incidental cases where the income grows considerably.

On the basis of these results, we can now compute the amounts of compensation differentiated for rent (tenants), again assuming that the household has lived in the house for five years. Table 11 gives the compensations if noise insulation is not present, and table 12 if noise insulation is present.

Table 11: Compensation for tenants if noise insulation is not present (differentiated for net monthly rent)

	20 → 30 Ku	30 → 40 Ku	40 → 45 Ku	40 → 50 Ku
Rent:				
f300	f 11.87	f 7.27	f 2.74	f 5.13
f500	f 53.61	f 32.82	f 12.39	f 23.16
f750	f 177.37	f 108.60	f 40.98	f 76.61

Table 12: Compensation for tenants if noise insulation is present (differentiated for net monthly rent)

	20 → 30 Ku	30 → 40 Ku	40 → 45 Ku	40 → 50 Ku
Rent:				
f300	f 1.81	f 1.12	f 0.42	f 0.79
f500	f 8.18	f 5.04	f 1.92	f 3.57
f750	f 27.07	f 16.69	f 6.34	f 11.81

5.3 The costs of compensation to society

An important policy question now is what the total amount of compensation would be for compensating the population living around Schiphol for the noise nuisance they suffer. This means that we have to compute the amount per household in the area involved, taking into account that different households have different incomes and experience different levels of Ku. Subsequently, the amounts for all households concerned have to be added together.

Suppose we set a critical Ku limit of x Ku, for example. What is the percentage of households having a noise nuisance level worse than x Ku, and how high would be the amount to compensate for the exceeding nuisance? In table 13 below we have done this for a number of critical levels.

Table 13: Total yearly amount of compensation

x Ku	Number of households concerned ¹	Average monthly compensation per household concerned	Total yearly amount of compensation
> 20 Ku	134,705 (16.3%)	f 114.78	f 185.54 million
> 25 Ku	49,052 (5.9%)	f 76.93	f 45.28 million
> 30 Ku	10,041 (1.2%)	f 67.49	f 8.13 million
> 35 Ku	5,086 (0.6%)	f 61.49	f 3.75 million
> 40 Ku	3,511 (0.4%)	f 46.28	f 1.95 million

¹ Of the total population in the Schiphol region.

To be precise, we have computed the total yearly compensation necessary to bring down the nuisance level for all people suffering from a damage level of over x Ku, to the chosen level of x Ku. If we choose x to be 20, the resulting amount is f114.78.

The table shows that the average monthly amount of compensation for a bottom level of 20 Ku is higher than the average amounts for higher critical levels. That is logical, because the higher the critical level, the smaller the number of Ku that are compensated. This is shown even more clearly in column 4 of the table, where the total amount of annual compensation is mentioned. This is because the number of households exposed to over 20 Ku is much higher than the number of households exposed to higher critical levels, because it encompasses all the higher critical levels.

6. Conclusions

In this paper we estimated the shadow price of an external effect, viz., of aircraft noise nuisance. An external effect is always a residual effect which is left after taking into account that prices, in this case housing prices and rents, partially reflect the impact of the external effect. Our first finding is that housing prices and rents do not appear to internalize the external effect completely.

Methodologically, the paper is innovating, since it uses the Cantril ladder-of-life question as information on well-being, while recognizing the ordinal character of the Cantril index. It also estimates a two-equation model, where both variables to be explained are ordinal and one of the two variables figures as explanatory variable for the other. In the appendix we show how to deal with an heteroscedastic measurement error in the objective noise levels in Kosten units (below 20 Ku).

The monetary compensations found, say shadow prices, are derived from that model and they differ according to whether or not the house is noise insulated. The compensations depend either on household income, reflecting the falling marginal utility of income (Gossen's first law), or on the housing expenses.

We do believe that this is the first time that external effects have been monetarily measured by means of using the Cantril question. It is obvious that this external effect could only be measured by the circumstance that noise nuisance varied a lot over the region and that the nuisance was pretty well registered according to zip codes, making it possible to connect objective noise nuisance with the subjective feelings of the individuals living there.

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Appendix

Specification of the Unreliability of the Ku Values lower than 20

Technical experts suspect that the objective measurement of noise in Kosten units (Ku) is not correct, if the noise level is below 20 Ku. More specifically, the non-reliability of this measurement procedure increases, the lower the true Ku value is. This appendix deals with the problem how to detect the (heteroscedastic) measurement error and how to correct for it.

We assume that noise nuisance (y) depends on a number of variables $x = (x_1, \dots, x_k)$ and the objective noise level K measured in Ku, or, more precisely, the logarithm of the value in Ku.⁵ We denote the variable K by:

$$K = \ln(\text{noise in Ku})$$

We model

$$y = \beta' x + \beta_K K + \varepsilon \quad (\text{A.1})$$

where β stands for a k -vector of regression coefficients and β_K is the specific coefficient corresponding to the variable K . The variable y to be explained attains only five verbal values, viz. “always”, “often”, “regularly”, “sometimes”, or “never”, which have to be translated to some numerical scale by a monotonically increasing transformation. Due to the ordinal character, the choice of that transformation is in a certain sense irrelevant, as we explained in the main text (cf. section 6.6). The OLS estimator, arrived at by minimizing the sum of squared residuals,

$$\sum_{n=1}^N \varepsilon_n^2$$

where N stands for the number of observations, is:

$$b = (X'X)^{-1} X'y \quad (\text{A.2})$$

where Σ_{xx} stands for the sample covariance matrix of the vector (x, K) , then we have, using the traditional textbook notation $(X'X) = N*\Sigma_{xx}$. Similarly, we have $(X'y) = N*\Sigma_{xy}$,

⁵ The variable noise nuisance ' y ' is called '*noise*' in the main text.

and Σ_{xy} stands for the covariance vector y with (x, K)

Hence we may rewrite

$$b = \Sigma_{XX}^{-1} \Sigma_{Xy} \quad (\text{A.3})$$

Let us now assume that noise cannot be exactly measured if the noise level is below 20 Ku. In that case we meet a serious problem. Let the true value be denoted by ξ , but we observe

$$K = \xi + \eta \quad (\text{A.4})$$

where η stands for the measurement error or unreliability (in terms of percentages, since K is measured in logarithms). We assume, as usual, that $E(\eta) = 0$ and $\text{var}(\eta) = \sigma_\eta^2 > 0$. Moreover, η is not correlated with ε and ξ . It follows that our matrix Σ_{XX} differs from the matrix $\Sigma_{\xi\xi}$. More specifically, consider the last diagonal element of Σ_{XX} , say σ_{KK} . We have

$$\sigma_{KK} = \sigma_{\xi\xi} + \sigma_\eta^2 \quad (\text{A.5})$$

and consequently $\Sigma_{XX} \neq \Sigma_{\xi\xi}$. Our quest should be for

$$b_{true} = \Sigma_{\xi\xi}^{-1} \Sigma_{\xi y} \quad (\text{A.6})$$

But as $\Sigma_{\xi\xi}$ is replaced by Σ_{XX} , it follows that the OLS estimate, say b_{OLS} , differs from the true b , since

$$b_{true} = \Sigma_{\xi\xi}^{-1} \Sigma_{\xi y} \neq \Sigma_{XX}^{-1} \Sigma_{Xy} = b_{OLS}$$

We say that b_{OLS} is biased due to the measurement error. Note that not only β_K is incorrectly estimated; this holds for *all* components of the vector β . We assume here that the other variables have zero measurement error, and, hence

$$\Sigma_{XX} = \Sigma_{\xi\xi} + I_K \sigma_\eta^2 \quad (\text{A.7})$$

where I_k is a zero matrix except for the diagonal element corresponding to the variable K , which is equal to one. We also assume that η is not correlated with the error term ε . Hence, we have $\Sigma_{xy} = \Sigma_{\xi y}$. It follows that, if we would know σ_η^2 , we may correct Σ_{xx} for the σ_η^2 effect and we then have

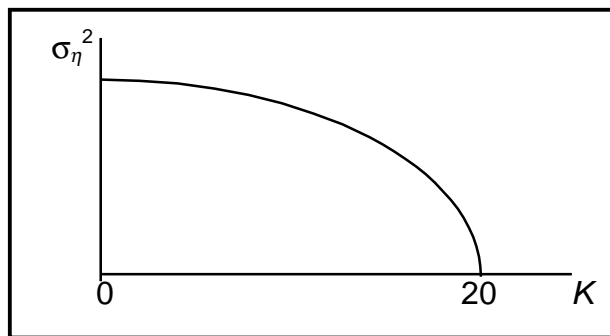
$$b_{true} = \left(\Sigma_{xx} - I_K \sigma_\eta^2 \right)^{-1} \Sigma_{xy} \quad (\text{A.8})$$

However, we do not know a priori what σ_η^2 is, and therefore this correction is generally impossible. Nonetheless, in the present case, we may identify σ_η^2 . We assume, with the technical experts, that $\sigma_\eta^2 = 0$ if $Ku \geq 20$, and $\sigma_\eta^2 > 0$ if $Ku < 20$. It may be inferred that σ_η^2 depends on the true noise level ξ , which is consistently estimated by its observed counterpart K . We then have $E(K) = \xi$. Hence, we write σ_η^2 as a decreasing function of K , or, more precisely, we assume:

$$\sigma_\eta^2(K) = 0 \text{ if } Ku \geq 20 \quad \text{and} \quad \sigma_\eta^2(K) = \sigma_\eta^2 \ln(21Ku) \text{ if } Ku < 20 \quad (\text{A.9})$$

Here σ_η^2 is a constant. We assume that, if K falls, $\sigma_\eta^2(K)$ increases, not linearly but logarithmically. We sketch this behavior in figure A.1.

Figure A.1: Behavior of $\sigma_\eta^2(K)$



By making this assumption we are actually introducing an additional unknown σ_η^2 . In the literature and practical experience, there is no reliable estimate of σ_η^2 . It is only suggested that the relative measurement error below 20 may become quite considerable, especially for low values of Ku . The reason is that aircraft noise cannot be sharply distinguished on the ground from wind noise and traffic noise, if

the specific aircraft noise is moderate.

We may simultaneously estimate b_{true} and σ_η^2 as follows. Consider the model

$$y = \beta_{K,\text{true}}\xi + \varepsilon \quad (\text{A.10})$$

where, without loss of generality, we drop the other variables x for ease of exposition. We may rewrite

$$\begin{aligned} y &= \beta_K(\xi + \eta) + \varepsilon - \beta_K\eta \\ &= \beta_K K + (\varepsilon - \beta_K\eta) \\ y - \beta_{K,\text{true}}K &= \varepsilon - \beta_K\eta \end{aligned} \quad (\text{A.11})$$

We note that

And, hence, that the residual depends on ε and η . It follows that

$$E(y - \beta X) = 0$$

and

$$\text{var}(y - \beta X) = \text{var}(\varepsilon) + \beta_K^2 \sigma_\eta^2(K) \quad (\text{A.12})$$

If $\sigma_\eta^2(K)$ varies with K we have heteroscedastic unreliability. The previous equation implies that

$$(y - \beta X)^2 \approx \sigma_\varepsilon^2 + \beta_K^2 \sigma_\eta^2 \ln(21 - Ku)D(20 - Ku) + v \quad (\text{A.13})$$

where $D(20 - Ku) = 1$ for $Ku \leq 20$ and $D(20 - Ku) = 0$ for $Ku > 20$, and v is a disturbance error. If we assume β to be known, then we may estimate σ_ε^2 and σ_η^2 by OLS using the squared residuals from the equation (A.1) to estimate

$$(y - \beta X)^2 = \sigma_\varepsilon^2 + B \ln(21 - Ku)D(20 - Ku) + v \quad (\text{A.14})$$

then $\sigma_\eta^2 = B / \beta_K^2$. Hence we may identify the (k+1) unknowns (β and σ_η^2) from the equation system

$$\begin{aligned} (\sum_{XX} - I_K \overline{\sigma_\eta^2}) b_{true} &= \sum_{XY} \\ \beta_K^2 \sigma_\eta^2 &= B \end{aligned} \tag{A.15}$$

where we denote

$$\overline{\sigma_\eta^2} = \frac{1}{N} \sigma_\eta^2 \sum_{n=1}^N \ln(21 - Ku) D(20 - Ku)$$

and β_K stands for the coefficient corresponding to K .

This system is solved taking into account the heteroscedasticity of the measurement error. The results are presented in table A.1. Note that equation A.14 is re-estimated in each round with the previous β -values.

Equation A.14 is estimated in the first round as:

$$\begin{aligned} (y - \beta K)^2 &= 0.4322 + 0.0133 [\ln(21 - Ku) D(20 - Ku)] \\ &\quad (16.51) \quad (1.543) \\ N &= 1183; R^2 = 0.0020 \end{aligned}$$

The final estimate is:

$$\begin{aligned} (y - \beta K)^2 &= 0.4218 + 0.0577 [\ln(21 - Ku) D(20 - Ku)] \\ &\quad (14.01) \quad (3.76) \\ N &= 1183; R^2 = 0.0118 \end{aligned}$$

It is evident that the error is heteroscedastic, witness the strongly significant coefficient 0.0577.

In a similar way we compare in table A.1 the true and naive regression results for equation A.14.

In the first column of table A.1 we present the 'true' regression coefficients β_{true} . The

standard deviations are assessed by the usual delta-method. The second column presents the naive OLS regression results, viz., if we do not correct for the measurement error in K . The naive estimators are the starting-point for the iterative estimation method.

We see that the naive and true coefficient estimates do not differ considerably except for β_K , which increases from 0.1998 to 0.3696.

Table A.1: True and naive regression results

True regression results			Naive regression results		
Variables	Parameter estimate	Standard deviation	Variables	Parameter estimate	Standard deviation
Intercept	- 1.4008	0.5451	Intercept	- 1.0105	0.2496
ln[family size]	0.0483	0.0817	ln[family size]	0.0593	0.0461
ln[housing expenses]	0.1082	0.0839	ln[housing expenses]	0.1037	0.0370
presence at home on weekdays	0.1399	0.0733	presence at home on weekdays	0.1545	0.0545
ln[noise in Ku]	0.3696	0.0147	ln[noise in Ku]	0.1998	0.0172
presence of balcony	0.1098	0.0615	presence of balcony	0.1033	0.0465
presence of garden	0.2513	0.0987	presence of garden	0.2816	0.0536
N = 1,132 $R_{\text{true}}^2 = 0.1772^*$			N = 1,132 $R_{\text{naive}}^2 = 0.1786$		

* If no measurement error exists R^2 is computed as follows: $1 - ((\sigma_\varepsilon^2) / \sigma_y^2)$ which would equal 0.2411. The true explained variance is given by

$$R_{\text{true}}^2 = 1 - \frac{\sigma_\varepsilon^2 + B\sigma_\eta^2}{\sigma_y^2}$$

It is also interesting to see what the measurement error implies. For a given Ku value we may calculate a $1^*\sigma^2$ -confidence interval for this true value. Table A.2 gives these minimum and maximum values for our study. Note that the deviation in terms of percentages is largest for 1 Ku and lowest for 19 Ku.

Table A.2: Minimum and maximum Ku values

Ku	Minimum Ku	Maximum Ku
19	11.0618	32.6370
18	9.1091	35.5690
17	7.9099	36.5365
16	7.0161	36.4874
15	6.2854	35.7972
14	5.6553	34.6578
13	5.0932	33.1817
12	4.5799	31.4415
11	4.1035	29.4868
10	3.6559	27.3532
9	3.2313	25.0673
8	2.8256	22.6498
7	2.4358	20.1166
6	2.0594	17.4808
5	1.6946	14.7531
4	1.3398	11.9421
3	0.9939	9.0555
2	0.6558	6.0994
1	0.3247	3.0793

